Convert from rnc to generic coordinates

The following code is based on the `gen2rnc.tex` code.

It is common to do some computations in a local RNC. Doing so makes various parts of the computations much easier to manage than in the original non-RNC coordinates. One simple example is the proof of the second Bianchi identities.

This code develops the inverse transformation, that is from the local RNC coordinates back to generic coordinates. The key equation (drawn form `gen2rnc.tex`) is

$$x^a_j = x^a_i + y^a - \sum_{k=2}^{\infty} \frac{1}{k!}\, \Gamma^a_{\underline{b}_k} y^{\cdot \underline{b}_k} \tag{1}$$

In `gen2rnc.tex` this equation was solved for the RNC coordinates y given the generic coordinates x_j and x_i. Here we will instead take x_i and y as given and use this equation to compute x_j. The first change we will make is to replace x_j with x (as the subscript j serves no useful purpose).

Thus our job will be to compute

$$x^a = x^a_i + y^a - \sum_{k=2}^{\infty} \frac{1}{k!}\, \Gamma^a_{\underline{b}_k} y^{\cdot \underline{b}_k} \tag{2}$$

given x_i and y. The generalised connections will be computed recursively by

$$\Gamma^a{}_{b\underline{c}d} = \Gamma^a{}_{(b\underline{c},d)} - (n+1)\Gamma^a{}_{p(\underline{c}}\Gamma^p{}_{bd)} \tag{3}$$

As noted in `gen2rnc.tex`, the generalised connections will scale with the expensions parameter ϵ according to

$$\Gamma^a{}_{bc} = \mathcal{O}\left(\epsilon\right)\ , \qquad \Gamma^a{}_{bcd} = \mathcal{O}\left(\epsilon^2\right)\ , \qquad \Gamma^a{}_{bcde} = \mathcal{O}\left(\epsilon^3\right)\ , \qquad \text{etc.}$$

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).

D{#}::Derivative.
\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

g_{a b}::Metric.
g^{a b}::InverseMetric.
g_{a}^{b}::KroneckerDelta.
g^{a}_{b}::KroneckerDelta.
\delta^{a}_{b}::KroneckerDelta.
\delta_{a}^{b}::KroneckerDelta.

R_{a b c d}::RiemannTensor.
R^{a}_{b c d}::RiemannTensor.
R_{a b c}^{d}::RiemannTensor.

A^{a}::Depends(\partial{#}).

g_{a b}::Depends(\partial{#}).
R_{a b c d}::Depends(\partial{#}).
R^{a}_{b c d}::Depends(\partial{#}).

Q^{a}_{b c}::Depends(\partial{#}).

Q^{a}_{b c}::TableauSymmetry(shape={2}, indices={1,2}).
Q^{a}_{b c d}::TableauSymmetry(shape={3}, indices={1,2,3}).
Q^{a}_{b c d e}::TableauSymmetry(shape={4}, indices={1,2,3,4}).
Q^{a}_{b c d e f}::TableauSymmetry(shape={5}, indices={1,2,3,4,5}).
Q^{a}_{b c d e f g}::TableauSymmetry(shape={6}, indices={1,2,3,4,5,6}).

Q^{p}_{a b}::Weight(label=numQ,value=1).
Q^{p}_{a b c}::Weight(label=numQ,value=2).
Q^{p}_{a b c d}::Weight(label=numQ,value=3).
Q^{p}_{a b c d e}::Weight(label=numQ,value=4).
Q^{p}_{a b c d e f}::Weight(label=numQ,value=5).

def product_sort (obj):
```

```
    substitute (obj,$ A^{a}                   -> A001^{a}             $)
    substitute (obj,$ x^{a}                   -> A002^{a}             $)
    substitute (obj,$ g^{a b}                 -> A003^{a b}           $)
    substitute (obj,$ Q^{p}_{a b}             -> A004^{p}_{a b}       $)
    substitute (obj,$ Q^{p}_{a b c}           -> A005^{p}_{a b c}     $)
    substitute (obj,$ Q^{p}_{a b c d}         -> A006^{p}_{a b c d}   $)
    substitute (obj,$ Q^{p}_{a b c d e}       -> A007^{p}_{a b c d e} $)
    substitute (obj,$ Q^{p}_{a b c d e f}     -> A008^{p}_{a b c d e f} $)
    sort_product   (obj)
    rename_dummies (obj)
    substitute (obj,$ A001^{a}                -> A^{a}                $)
    substitute (obj,$ A002^{a}                -> x^{a}                $)
    substitute (obj,$ A003^{a b}              -> g^{a b}              $)
    substitute (obj,$ A004^{p}_{a b}          -> Q^{p}_{a b}          $)
    substitute (obj,$ A005^{p}_{a b c}        -> Q^{p}_{a b c}        $)
    substitute (obj,$ A006^{p}_{a b c d}      -> Q^{p}_{a b c d}      $)
    substitute (obj,$ A007^{p}_{a b c d e}    -> Q^{p}_{a b c d e}    $)
    substitute (obj,$ A008^{p}_{a b c d e f}  -> Q^{p}_{a b c d e f}  $)

    return obj

def truncateQ (obj,n):

    ans = Ex(0)

    for i in range (0,n+1):
       foo := @(obj).
       bah  = Ex("numQ = " + str(i))
       keep_weight (foo, bah)
       ans = ans + foo

    return ans

# A^{a} = dx^a/ds

Gamma := Q^{d}_{a b} A^{a} A^{b}.

dAds  := A^{c} \partial_{c}{A^{d}}-> - @(Gamma).
```

```
# ==============================================================
eq0 := @(Gamma).                   # cdb (eq0.000,eq0)

# ==============================================================
eq1 := A^{c} \partial_{c}{@(eq0)}.      # cdb (eq1.000,eq1)

distribute      (eq1)                   # cdb (eq1.001,eq1)
unwrap          (eq1)                   # cdb (eq1.002,eq1)
product_rule    (eq1)                   # cdb (eq1.003,eq1)
distribute      (eq1)                   # cdb (eq1.004,eq1)
substitute      (eq1,dAds)              # cdb (eq1.005,eq1)
distribute      (eq1)                   # cdb (eq1.006,eq1)
eq1 = truncateQ (eq1,5)                 # cdb (eq1.007,eq1)
sort_product    (eq1)                   # cdb (eq1.008,eq1)
rename_dummies  (eq1)                   # cdb (eq1.009,eq1)
canonicalise    (eq1)                   # cdb (eq1.010,eq1)

# ==============================================================
eq2 := A^{c} \partial_{c}{@(eq1)}.      # cdb (eq2.000,eq2)

distribute      (eq2)                   # cdb (eq2.001,eq2)
unwrap          (eq2)                   # cdb (eq2.002,eq2)
product_rule    (eq2)                   # cdb (eq2.003,eq2)
distribute      (eq2)                   # cdb (eq2.004,eq2)
substitute      (eq2,dAds)              # cdb (eq2.005,eq2)
distribute      (eq2)                   # cdb (eq2.006,eq2)
eq2 = truncateQ (eq2,5)                 # cdb (eq2.007,eq2)
sort_product    (eq2)                   # cdb (eq2.008,eq2)
rename_dummies  (eq2)                   # cdb (eq2.009,eq2)
canonicalise    (eq2)                   # cdb (eq2.010,eq2)

# ==============================================================
eq3 := A^{c} \partial_{c}{@(eq2)}.      # cdb (eq3.000,eq3)

distribute      (eq3)                   # cdb (eq3.001,eq3)
unwrap          (eq3)                   # cdb (eq3.002,eq3)
product_rule    (eq3)                   # cdb (eq3.003,eq3)
```

```
distribute      (eq3)                   # cdb (eq3.004,eq3)
substitute      (eq3,dAds)              # cdb (eq3.005,eq3)
distribute      (eq3)                   # cdb (eq3.006,eq3)
eq3 = truncateQ (eq3,5)                 # cdb (eq3.007,eq3)
sort_product    (eq3)                   # cdb (eq3.008,eq3)
rename_dummies  (eq3)                   # cdb (eq3.009,eq3)
canonicalise    (eq3)                   # cdb (eq3.010,eq3)

# ============================================================
eq4 := A^{c} \partial_{c}{@(eq3)}.      # cdb (eq4.000,eq4)

distribute      (eq4)                   # cdb (eq4.001,eq4)
unwrap          (eq4)                   # cdb (eq4.002,eq4)
product_rule    (eq4)                   # cdb (eq4.003,eq4)
distribute      (eq4)                   # cdb (eq4.004,eq4)
substitute      (eq4,dAds)              # cdb (eq4.005,eq4)
distribute      (eq4)                   # cdb (eq4.006,eq4)
eq4 = truncateQ (eq4,5)                 # cdb (eq4.007,eq4)
sort_product    (eq4)                   # cdb (eq4.008,eq4)
rename_dummies  (eq4)                   # cdb (eq4.009,eq4)
canonicalise    (eq4)                   # cdb (eq4.010,eq4)
```

$$\texttt{eq0.000} := Q^{d}{}_{ab} A^{a} A^{b}$$

$$\texttt{eq1.000} := A^c\partial_c\big(Q^d{}_{ab}A^aA^b\big)$$

$$\texttt{eq1.001} := A^c\partial_c\big(Q^d{}_{ab}A^aA^b\big)$$

$$\texttt{eq1.002} := A^c\partial_c\big(Q^d{}_{ab}A^aA^b\big)$$

$$\texttt{eq1.003} := A^c\left(\partial_c Q^d{}_{ab}A^aA^b + Q^d{}_{ab}\partial_c A^aA^b + Q^d{}_{ab}A^a\partial_c A^b\right)$$

$$\texttt{eq1.004} := A^c\partial_c Q^d{}_{ab}A^aA^b + A^cQ^d{}_{ab}\partial_c A^aA^b + A^cQ^d{}_{ab}A^a\partial_c A^b$$

$$\texttt{eq1.005} := A^c\partial_c Q^d{}_{ab}A^aA^b - Q^a{}_{ce}A^cA^eQ^d{}_{ab}A^b - Q^b{}_{ec}A^eA^cQ^d{}_{ab}A^a$$

$$\texttt{eq1.006} := A^c\partial_c Q^d{}_{ab}A^aA^b - Q^a{}_{ce}A^cA^eQ^d{}_{ab}A^b - Q^b{}_{ec}A^eA^cQ^d{}_{ab}A^a$$

$$\texttt{eq1.007} := A^c\partial_c Q^d{}_{ab}A^aA^b - Q^a{}_{ce}A^cA^eQ^d{}_{ab}A^b - Q^b{}_{ec}A^eA^cQ^d{}_{ab}A^a$$

$$\texttt{eq1.008} := A^aA^bA^c\partial_c Q^d{}_{ab} - A^bA^cA^eQ^a{}_{ce}Q^d{}_{ab} - A^aA^cA^eQ^b{}_{ec}Q^d{}_{ab}$$

$$\texttt{eq1.009} := A^aA^bA^c\partial_c Q^d{}_{ab} - A^aA^bA^cQ^e{}_{bc}Q^d{}_{ea} - A^aA^bA^cQ^e{}_{cb}Q^d{}_{ae}$$

$$\texttt{eq1.010} := A^aA^bA^c\partial_a Q^d{}_{bc} - 2\,A^aA^bA^cQ^d{}_{ae}Q^e{}_{bc}$$

$$\texttt{eq2.000} := A^{c}\partial_{c}\big(A^{a}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} - 2\,A^{a}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf}\big)$$

$$\texttt{eq2.001} := A^{c}\partial_{c}\big(A^{a}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf}\big) - 2\,A^{c}\partial_{c}\big(A^{a}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf}\big)$$

$$\texttt{eq2.002} := A^{c}\partial_{c}\big(A^{a}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf}\big) - 2\,A^{c}\partial_{c}\big(A^{a}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf}\big)$$

$$\begin{aligned}\texttt{eq2.003} := {} & A^{c}\left(\partial_{c}A^{a}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} + A^{a}\partial_{c}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} + A^{a}A^{b}\partial_{c}A^{f}\partial_{d}Q^{d}{}_{bf} + A^{a}A^{b}A^{f}\partial_{cd}Q^{d}{}_{bf}\right)\\ & - 2\,A^{c}\left(\partial_{c}A^{a}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} + A^{a}\partial_{c}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} + A^{a}A^{b}\partial_{c}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} + A^{a}A^{b}A^{f}\partial_{c}Q^{d}{}_{ae}Q^{e}{}_{bf} + A^{a}A^{b}A^{f}Q^{d}{}_{ae}\partial_{c}Q^{e}{}_{bf}\right)\end{aligned}$$

$$\begin{aligned}\texttt{eq2.004} := {} & A^{c}\partial_{c}A^{a}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} + A^{c}A^{a}\partial_{c}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} + A^{c}A^{a}A^{b}\partial_{c}A^{f}\partial_{d}Q^{d}{}_{bf} + A^{c}A^{a}A^{b}A^{f}\partial_{cd}Q^{d}{}_{bf} - 2\,A^{c}\partial_{c}A^{a}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf}\\ & - 2\,A^{c}A^{a}\partial_{c}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} - 2\,A^{c}A^{a}A^{b}\partial_{c}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} - 2\,A^{c}A^{a}A^{b}A^{f}\partial_{c}Q^{d}{}_{ae}Q^{e}{}_{bf} - 2\,A^{c}A^{a}A^{b}A^{f}Q^{d}{}_{ae}\partial_{c}Q^{e}{}_{bf}\end{aligned}$$

$$\begin{aligned}\texttt{eq2.005} := {} & -Q^{a}{}_{ce}A^{c}A^{e}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} - Q^{b}{}_{ec}A^{e}A^{c}A^{a}A^{f}\partial_{d}Q^{d}{}_{bf} - Q^{f}{}_{ce}A^{c}A^{e}A^{a}A^{b}\partial_{d}Q^{d}{}_{bf} + A^{c}A^{a}A^{b}A^{f}\partial_{cd}Q^{d}{}_{bf} + 2\,Q^{a}{}_{cg}A^{c}A^{g}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf}\\ & + 2\,Q^{b}{}_{gc}A^{g}A^{c}A^{a}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} + 2\,Q^{f}{}_{cg}A^{c}A^{g}A^{a}A^{b}Q^{d}{}_{ae}Q^{e}{}_{bf} - 2\,A^{c}A^{a}A^{b}A^{f}\partial_{c}Q^{d}{}_{ae}Q^{e}{}_{bf} - 2\,A^{c}A^{a}A^{b}A^{f}Q^{d}{}_{ae}\partial_{c}Q^{e}{}_{bf}\end{aligned}$$

$$\begin{aligned}\texttt{eq2.006} := {} & -Q^{a}{}_{ce}A^{c}A^{e}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} - Q^{b}{}_{ec}A^{e}A^{c}A^{a}A^{f}\partial_{d}Q^{d}{}_{bf} - Q^{f}{}_{ce}A^{c}A^{e}A^{a}A^{b}\partial_{d}Q^{d}{}_{bf} + A^{c}A^{a}A^{b}A^{f}\partial_{cd}Q^{d}{}_{bf} + 2\,Q^{a}{}_{cg}A^{c}A^{g}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf}\\ & + 2\,Q^{b}{}_{gc}A^{g}A^{c}A^{a}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} + 2\,Q^{f}{}_{cg}A^{c}A^{g}A^{a}A^{b}Q^{d}{}_{ae}Q^{e}{}_{bf} - 2\,A^{c}A^{a}A^{b}A^{f}\partial_{c}Q^{d}{}_{ae}Q^{e}{}_{bf} - 2\,A^{c}A^{a}A^{b}A^{f}Q^{d}{}_{ae}\partial_{c}Q^{e}{}_{bf}\end{aligned}$$

$$\begin{aligned}\texttt{eq2.007} := {} & A^{c}A^{a}A^{b}A^{f}\partial_{cd}Q^{d}{}_{bf} - Q^{a}{}_{ce}A^{c}A^{e}A^{b}A^{f}\partial_{d}Q^{d}{}_{bf} - Q^{b}{}_{ec}A^{e}A^{c}A^{a}A^{f}\partial_{d}Q^{d}{}_{bf} - Q^{f}{}_{ce}A^{c}A^{e}A^{a}A^{b}\partial_{d}Q^{d}{}_{bf} - 2\,A^{c}A^{a}A^{b}A^{f}\partial_{c}Q^{d}{}_{ae}Q^{e}{}_{bf}\\ & - 2\,A^{c}A^{a}A^{b}A^{f}Q^{d}{}_{ae}\partial_{c}Q^{e}{}_{bf} + 2\,Q^{a}{}_{cg}A^{c}A^{g}A^{b}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} + 2\,Q^{b}{}_{gc}A^{g}A^{c}A^{a}A^{f}Q^{d}{}_{ae}Q^{e}{}_{bf} + 2\,Q^{f}{}_{cg}A^{c}A^{g}A^{a}A^{b}Q^{d}{}_{ae}Q^{e}{}_{bf}\end{aligned}$$

$$\begin{aligned}\texttt{eq2.008} := {} & A^{a}A^{b}A^{c}A^{f}\partial_{cd}Q^{d}{}_{bf} - A^{b}A^{c}A^{e}A^{f}Q^{a}{}_{ce}\partial_{d}Q^{d}{}_{bf} - A^{a}A^{c}A^{e}A^{f}Q^{b}{}_{ec}\partial_{d}Q^{d}{}_{bf} - A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{ce}\partial_{d}Q^{d}{}_{bf} - 2\,A^{a}A^{b}A^{c}A^{f}Q^{e}{}_{bf}\partial_{c}Q^{d}{}_{ae}\\ & - 2\,A^{a}A^{b}A^{c}A^{f}Q^{d}{}_{ae}\partial_{c}Q^{e}{}_{bf} + 2\,A^{b}A^{c}A^{f}A^{g}Q^{a}{}_{cg}Q^{d}{}_{ae}Q^{e}{}_{bf} + 2\,A^{a}A^{c}A^{f}A^{g}Q^{b}{}_{gc}Q^{d}{}_{ae}Q^{e}{}_{bf} + 2\,A^{a}A^{b}A^{c}A^{g}Q^{d}{}_{ae}Q^{e}{}_{bf}Q^{f}{}_{cg}\end{aligned}$$

$$\begin{aligned}\texttt{eq2.009} := {} & A^{a}A^{b}A^{c}A^{e}\partial_{cd}Q^{d}{}_{be} - A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{bc}\partial_{f}Q^{d}{}_{ae} - A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{cb}\partial_{d}Q^{d}{}_{fe} - A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{ce}\partial_{d}Q^{d}{}_{bf} - 2\,A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{be}\partial_{c}Q^{d}{}_{af}\\ & - 2\,A^{a}A^{b}A^{c}A^{e}Q^{d}{}_{af}\partial_{c}Q^{f}{}_{be} + 2\,A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{be}Q^{d}{}_{fg}Q^{g}{}_{ac} + 2\,A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{eb}Q^{d}{}_{ag}Q^{g}{}_{fc} + 2\,A^{a}A^{b}A^{c}A^{e}Q^{d}{}_{af}Q^{f}{}_{bg}Q^{g}{}_{ce}\end{aligned}$$

$$\begin{aligned}\texttt{eq2.010} := {} & A^{a}A^{b}A^{c}A^{e}\partial_{ab}Q^{d}{}_{ce} - A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{ab}\partial_{f}Q^{d}{}_{ce} - 4\,A^{a}A^{b}A^{c}A^{e}Q^{f}{}_{ab}\partial_{c}Q^{d}{}_{ef}\\ & - 2\,A^{a}A^{b}A^{c}A^{e}Q^{d}{}_{af}\partial_{b}Q^{f}{}_{ce} + 2\,A^{a}A^{b}A^{c}A^{e}Q^{d}{}_{fg}Q^{f}{}_{ab}Q^{g}{}_{ce} + 4\,A^{a}A^{b}A^{c}A^{e}Q^{d}{}_{af}Q^{f}{}_{bg}Q^{g}{}_{ce}\end{aligned}$$

$$
\begin{aligned}
\texttt{eq3.010} := {} & A^a A^b A^c A^e A^f \partial_{abc} Q^d{}_{ef} - A^a A^b A^c A^e A^f \partial_g Q^d{}_{ab} \partial_c Q^g{}_{ef} - 6\, A^a A^b A^c A^e A^f \partial_d Q^d{}_{bg} \partial_c Q^g{}_{ef} - 3\, A^a A^b A^c A^e A^f Q^g{}_{ab} \partial_{cg} Q^d{}_{ef} \\
& - 6\, A^a A^b A^c A^e A^f Q^g{}_{ab} \partial_{ce} Q^d{}_{fg} - 2\, A^a A^b A^c A^e A^f Q^d{}_{ag} \partial_{bc} Q^g{}_{ef} + 2\, A^a A^b A^c A^e A^f Q^g{}_{ab} Q^h{}_{cg} \partial_h Q^d{}_{ef} + 6\, A^a A^b A^c A^e A^f Q^g{}_{ab} Q^h{}_{ce} \partial_g Q^d{}_{fh} \\
& + 12\, A^a A^b A^c A^e A^f Q^g{}_{ab} Q^h{}_{cg} \partial_e Q^d{}_{fh} + 6\, A^a A^b A^c A^e A^f Q^g{}_{ab} Q^h{}_{ce} \partial_f Q^d{}_{gh} + 6\, A^a A^b A^c A^e A^f Q^d{}_{gh} Q^g{}_{ab} \partial_c Q^h{}_{ef} + 2\, A^a A^b A^c A^e A^f Q^d{}_{ag} Q^h{}_{bc} \partial_h Q^g{}_{ef} \\
& + 8\, A^a A^b A^c A^e A^f Q^d{}_{ag} Q^h{}_{bc} \partial_e Q^g{}_{fh} + 4\, A^a A^b A^c A^e A^f Q^d{}_{ag} Q^g{}_{bh} \partial_c Q^h{}_{ef} - 12\, A^a A^b A^c A^e A^f Q^d{}_{gh} Q^g{}_{ab} Q^h{}_{ci} Q^i{}_{ef} - 4\, A^a A^b A^c A^e A^f Q^d{}_{ag} Q^g{}_{hi} Q^h{}_{bc} Q^i{}_{ef} \\
& - 8\, A^a A^b A^c A^e A^f Q^d{}_{ag} Q^g{}_{bh} Q^h{}_{ci} Q^i{}_{ef}
\end{aligned}
$$

$$
\begin{aligned}
\texttt{eq4.010} := {} & A^a A^b A^c A^e A^f A^g \partial_{abce} Q^d{}_{fg} - 4\, A^a A^b A^c A^e A^f A^g \partial_d Q^h{}_{bc} \partial_{eh} Q^d{}_{fg} - A^a A^b A^c A^e A^f A^g \partial_h Q^d{}_{ab} \partial_{ce} Q^h{}_{fg} - 12\, A^a A^b A^c A^e A^f A^g \partial_d Q^h{}_{bc} \partial_{ef} Q^d{}_{gh} \\
& - 8\, A^a A^b A^c A^e A^f A^g \partial_d Q^d{}_{bh} \partial_{ce} Q^h{}_{fg} - 6\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_{ceh} Q^d{}_{fg} - 8\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_{cef} Q^d{}_{gh} \\
& + 8\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_i Q^d{}_{ch} \partial_e Q^i{}_{fg} + A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_i Q^d{}_{ce} \partial_h Q^i{}_{fg} + 4\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_i Q^d{}_{ce} \partial_f Q^i{}_{gh} \\
& + 12\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_h Q^d{}_{ci} \partial_e Q^i{}_{fg} + 24\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_c Q^d{}_{hi} \partial_e Q^i{}_{fg} + 8\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_c Q^d{}_{ei} \partial_h Q^i{}_{fg} \\
& + 32\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} \partial_c Q^d{}_{ei} \partial_f Q^i{}_{gh} - 2\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} \partial_{bce} Q^h{}_{fg} + 2\, A^a A^b A^c A^e A^f A^g Q^h{}_{ai} \partial_h Q^d{}_{bc} \partial_e Q^i{}_{fg} \\
& + 16\, A^a A^b A^c A^e A^f A^g Q^h{}_{ai} \partial_b Q^d{}_{ch} \partial_e Q^i{}_{fg} + 6\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} \partial_a Q^h{}_{bc} \partial_e Q^i{}_{fg} + 2\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} \partial_b Q^i{}_{ce} \partial_i Q^h{}_{fg} \\
& + 12\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} \partial_b Q^h{}_{ci} \partial_e Q^i{}_{fg} + 8\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ch} \partial_{ei} Q^d{}_{fg} + 3\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} \partial_{hi} Q^d{}_{fg} \\
& + 24\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} \partial_{fh} Q^d{}_{gi} + 24\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ch} \partial_{ef} Q^d{}_{gi} + 12\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} \partial_{fg} Q^d{}_{hi} \\
& + 8\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{ab} \partial_{ce} Q^i{}_{fg} + 6\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^i{}_{bc} \partial_{ei} Q^h{}_{fg} + 12\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^i{}_{bc} \partial_{ef} Q^h{}_{gi} \\
& + 4\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{bi} \partial_{ce} Q^i{}_{fg} - 4\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ch} Q^j{}_{ei} \partial_j Q^d{}_{fg} - 2\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} Q^j{}_{hi} \partial_j Q^d{}_{fg} \\
& - 16\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} Q^j{}_{fh} \partial_j Q^d{}_{gi} - 24\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} Q^j{}_{fh} \partial_i Q^d{}_{gj} - 12\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} Q^j{}_{fg} \partial_h Q^d{}_{ij} \\
& - 32\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ch} Q^j{}_{ei} \partial_f Q^d{}_{gj} - 16\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} Q^j{}_{hi} \partial_f Q^d{}_{gj} - 48\, A^a A^b A^c A^e A^f A^g Q^h{}_{ab} Q^i{}_{ce} Q^j{}_{fh} \partial_g Q^d{}_{ij} \\
& - 24\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{aj} Q^j{}_{bc} \partial_e Q^i{}_{fg} - 8\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{ab} Q^j{}_{ce} \partial_j Q^i{}_{fg} - 32\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{ab} Q^j{}_{ce} \partial_f Q^i{}_{gj} \\
& - 4\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^i{}_{bc} Q^j{}_{ei} \partial_j Q^h{}_{fg} - 12\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^i{}_{bc} Q^j{}_{ef} \partial_i Q^h{}_{gj} - 24\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^i{}_{bc} Q^j{}_{ei} \partial_f Q^h{}_{gj} \\
& - 12\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^i{}_{bc} Q^j{}_{ef} \partial_g Q^h{}_{ij} - 16\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{ab} Q^i{}_{cj} \partial_e Q^j{}_{fg} - 12\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{ij} Q^i{}_{bc} \partial_e Q^j{}_{fg} \\
& - 4\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{bi} Q^j{}_{ce} \partial_j Q^i{}_{fg} - 16\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{bi} Q^j{}_{ce} \partial_f Q^i{}_{gj} - 8\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{bi} Q^i{}_{cj} \partial_e Q^j{}_{fg} \\
& + 24\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{aj} Q^i{}_{bk} Q^j{}_{ce} Q^k{}_{fg} + 16\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{ab} Q^i{}_{jk} Q^j{}_{ce} Q^k{}_{fg} + 32\, A^a A^b A^c A^e A^f A^g Q^d{}_{hi} Q^h{}_{ab} Q^i{}_{cj} Q^j{}_{ek} Q^k{}_{fg} \\
& + 24\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{ij} Q^i{}_{bc} Q^j{}_{ek} Q^k{}_{fg} + 8\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{bi} Q^i{}_{jk} Q^j{}_{ce} Q^k{}_{fg} + 16\, A^a A^b A^c A^e A^f A^g Q^d{}_{ah} Q^h{}_{bi} Q^i{}_{cj} Q^j{}_{ek} Q^k{}_{fg}
\end{aligned}
$$

```
def reformat (obj):
   bah := @(obj).
   distribute     (bah)
   bah = product_sort (bah)
   rename_dummies (bah)
   canonicalise   (bah)
   factor_out     (bah,$A^{a?}$)
   substitute     (bah,$A^{a}->y^{a}$)
   substitute     (bah,$Q^{a}_{b c}->\Gamma^{a}_{b c}$)
   ans := @(bah).
   return ans

eq0 = reformat(eq0)  # cdb (eq0.100,eq0)
eq1 = reformat(eq1)  # cdb (eq1.100,eq1)
eq2 = reformat(eq2)  # cdb (eq2.100,eq2)
eq3 = reformat(eq3)  # cdb (eq3.100,eq3)
eq4 = reformat(eq4)  # cdb (eq4.100,eq4)

checkpoint.append (eq0)
checkpoint.append (eq1)
checkpoint.append (eq2)
checkpoint.append (eq3)
checkpoint.append (eq4)
```

Convert from local RNC coords (y) to generic (x)

$$x^a = x^a_i + \overset{0}{x}{}^a - \overset{1}{x}{}^a - \overset{2}{x}{}^a - \overset{3}{x}{}^a - \overset{4}{x}{}^a - \overset{5}{x}{}^a$$

$$\overset{0}{x}{}^a = y^a$$

$$2!\,\overset{1}{x}{}^a = y^a y^b \Gamma^d{}_{ab}$$

$$3!\,\overset{2}{x}{}^a = y^a y^b y^c \left(\partial_a \Gamma^d{}_{bc} - 2\,\Gamma^d{}_{ae}\Gamma^e{}_{bc}\right)$$

$$4!\,\overset{3}{x}{}^a = y^a y^b y^c y^e \left(\partial_{ab}\Gamma^d{}_{ce} - \Gamma^f{}_{ab}\partial_f\Gamma^d{}_{ce} - 4\,\Gamma^f{}_{ab}\partial_c\Gamma^d{}_{ef} - 2\,\Gamma^d{}_{af}\partial_b\Gamma^f{}_{ce} + 2\,\Gamma^d{}_{fg}\Gamma^f{}_{ab}\Gamma^g{}_{ce} + 4\,\Gamma^d{}_{af}\Gamma^f{}_{bg}\Gamma^g{}_{ce}\right)$$

$$\begin{aligned}
5!\,\overset{4}{x}{}^a = y^a y^b y^c y^e y^f \big(&\partial_{abc}\Gamma^d{}_{ef} - \partial_g\Gamma^d{}_{ab}\partial_c\Gamma^g{}_{ef} - 6\,\partial_a\Gamma^d{}_{bg}\partial_c\Gamma^g{}_{ef} - 3\,\Gamma^g{}_{ab}\partial_{cg}\Gamma^d{}_{ef} - 6\,\Gamma^g{}_{ab}\partial_{ce}\Gamma^d{}_{fg} - 2\,\Gamma^d{}_{ag}\partial_{bc}\Gamma^g{}_{ef} + 2\,\Gamma^g{}_{ab}\Gamma^h{}_{cg}\partial_h\Gamma^d{}_{ef} + 6\,\Gamma^g{}_{ab}\Gamma^h{}_{ce}\partial_g\Gamma^d{}_{fh} \\
&+ 12\,\Gamma^g{}_{ab}\Gamma^h{}_{cg}\partial_e\Gamma^d{}_{fh} + 6\,\Gamma^g{}_{ab}\Gamma^h{}_{ce}\partial_f\Gamma^d{}_{gh} + 6\,\Gamma^d{}_{gh}\Gamma^g{}_{ab}\partial_c\Gamma^h{}_{ef} + 2\,\Gamma^d{}_{ag}\Gamma^h{}_{bc}\partial_h\Gamma^g{}_{ef} + 8\,\Gamma^d{}_{ag}\Gamma^h{}_{bc}\partial_e\Gamma^g{}_{fh} + 4\,\Gamma^d{}_{ag}\Gamma^g{}_{bh}\partial_c\Gamma^h{}_{ef} - 12\,\Gamma^d{}_{gh}\Gamma^g{}_{ab}\Gamma^h{}_{ci}\Gamma^i{}_{ef} \\
&- 4\,\Gamma^d{}_{ag}\Gamma^g{}_{hi}\Gamma^h{}_{bc}\Gamma^i{}_{ef} - 8\,\Gamma^d{}_{ag}\Gamma^g{}_{bh}\Gamma^h{}_{ci}\Gamma^i{}_{ef}\big)
\end{aligned}$$

$$\begin{aligned}
6!\,\overset{5}{x}{}^a = y^a y^b y^c y^e y^f y^g \big(&\partial_{abce}\Gamma^d{}_{fg} - 4\,\partial_a\Gamma^h{}_{bc}\partial_{eh}\Gamma^d{}_{fg} - \partial_h\Gamma^d{}_{ab}\partial_{ce}\Gamma^h{}_{fg} - 12\,\partial_a\Gamma^h{}_{bc}\partial_{ef}\Gamma^d{}_{gh} - 8\,\partial_a\Gamma^d{}_{bh}\partial_{ce}\Gamma^h{}_{fg} - 6\,\Gamma^h{}_{ab}\partial_{ceh}\Gamma^d{}_{fg} - 8\,\Gamma^h{}_{ab}\partial_{cef}\Gamma^d{}_{gh} \\
&+ 8\,\Gamma^h{}_{ab}\partial_i\Gamma^d{}_{ch}\partial_e\Gamma^i{}_{fg} + \Gamma^h{}_{ab}\partial_i\Gamma^d{}_{ce}\partial_h\Gamma^i{}_{fg} + 4\,\Gamma^h{}_{ab}\partial_i\Gamma^d{}_{ce}\partial_f\Gamma^i{}_{gh} + 12\,\Gamma^h{}_{ab}\partial_h\Gamma^d{}_{ci}\partial_e\Gamma^i{}_{fg} + 24\,\Gamma^h{}_{ab}\partial_c\Gamma^d{}_{hi}\partial_e\Gamma^i{}_{fg} + 8\,\Gamma^h{}_{ab}\partial_c\Gamma^d{}_{ei}\partial_h\Gamma^i{}_{fg} + 32\,\Gamma^h{}_{ab}\partial_c\Gamma^d{}_{ei}\partial_f\Gamma^i{}_{gh} \\
&- 2\,\Gamma^d{}_{ah}\partial_{bce}\Gamma^h{}_{fg} + 2\,\Gamma^h{}_{ai}\partial_h\Gamma^d{}_{bc}\partial_e\Gamma^i{}_{fg} + 16\,\Gamma^h{}_{ai}\partial_b\Gamma^d{}_{ch}\partial_e\Gamma^i{}_{fg} + 6\,\Gamma^d{}_{hi}\partial_a\Gamma^h{}_{bc}\partial_e\Gamma^i{}_{fg} + 2\,\Gamma^d{}_{ah}\partial_b\Gamma^i{}_{ce}\partial_i\Gamma^h{}_{fg} + 12\,\Gamma^d{}_{ah}\partial_b\Gamma^h{}_{ci}\partial_e\Gamma^i{}_{fg} + 8\,\Gamma^h{}_{ab}\Gamma^i{}_{ch}\partial_{ei}\Gamma^d{}_{fg} \\
&+ 3\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\partial_{hi}\Gamma^d{}_{fg} + 24\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\partial_{fh}\Gamma^d{}_{gi} + 24\,\Gamma^h{}_{ab}\Gamma^i{}_{ch}\partial_{ef}\Gamma^d{}_{gi} + 12\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\partial_{fg}\Gamma^d{}_{hi} + 8\,\Gamma^d{}_{hi}\Gamma^h{}_{ab}\partial_{ce}\Gamma^i{}_{fg} + 6\,\Gamma^d{}_{ah}\Gamma^i{}_{bc}\partial_{ei}\Gamma^h{}_{fg} + 12\,\Gamma^d{}_{ah}\Gamma^i{}_{bc}\partial_{ef}\Gamma^h{}_{gi} \\
&+ 4\,\Gamma^d{}_{ah}\Gamma^h{}_{bi}\partial_{ce}\Gamma^i{}_{fg} - 4\,\Gamma^h{}_{ab}\Gamma^i{}_{ch}\Gamma^j{}_{ei}\partial_j\Gamma^d{}_{fg} - 2\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\Gamma^j{}_{hi}\partial_j\Gamma^d{}_{fg} - 16\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\Gamma^j{}_{fh}\partial_j\Gamma^d{}_{gi} - 24\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\Gamma^j{}_{fh}\partial_i\Gamma^d{}_{gj} - 12\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\Gamma^j{}_{fg}\partial_h\Gamma^d{}_{ij} \\
&- 32\,\Gamma^h{}_{ab}\Gamma^i{}_{ch}\Gamma^j{}_{ei}\partial_f\Gamma^d{}_{gj} - 16\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\Gamma^j{}_{hi}\partial_f\Gamma^d{}_{gj} - 48\,\Gamma^h{}_{ab}\Gamma^i{}_{ce}\Gamma^j{}_{fh}\partial_g\Gamma^d{}_{ij} - 24\,\Gamma^d{}_{hi}\Gamma^h{}_{aj}\Gamma^j{}_{bc}\partial_e\Gamma^i{}_{fg} - 8\,\Gamma^d{}_{hi}\Gamma^h{}_{ab}\Gamma^j{}_{ce}\partial_j\Gamma^i{}_{fg} - 32\,\Gamma^d{}_{hi}\Gamma^h{}_{ab}\Gamma^j{}_{ce}\partial_f\Gamma^i{}_{gj} \\
&- 4\,\Gamma^d{}_{ah}\Gamma^i{}_{bc}\Gamma^j{}_{ei}\partial_j\Gamma^h{}_{fg} - 12\,\Gamma^d{}_{ah}\Gamma^i{}_{bc}\Gamma^j{}_{ef}\partial_i\Gamma^h{}_{gj} - 24\,\Gamma^d{}_{ah}\Gamma^i{}_{bc}\Gamma^j{}_{ei}\partial_f\Gamma^h{}_{gj} - 12\,\Gamma^d{}_{ah}\Gamma^i{}_{bc}\Gamma^j{}_{ef}\partial_g\Gamma^h{}_{ij} - 16\,\Gamma^d{}_{hi}\Gamma^h{}_{ab}\Gamma^i{}_{cj}\partial_e\Gamma^j{}_{fg} - 12\,\Gamma^d{}_{ah}\Gamma^h{}_{ij}\Gamma^i{}_{bc}\partial_e\Gamma^j{}_{fg} \\
&- 4\,\Gamma^d{}_{ah}\Gamma^h{}_{bi}\Gamma^j{}_{ce}\partial_j\Gamma^i{}_{fg} - 16\,\Gamma^d{}_{ah}\Gamma^h{}_{bi}\Gamma^j{}_{ce}\partial_f\Gamma^i{}_{gj} - 8\,\Gamma^d{}_{ah}\Gamma^h{}_{bi}\Gamma^i{}_{cj}\partial_e\Gamma^j{}_{fg} + 24\,\Gamma^d{}_{hi}\Gamma^h{}_{aj}\Gamma^i{}_{bk}\Gamma^j{}_{ce}\Gamma^k{}_{fg} + 16\,\Gamma^d{}_{hi}\Gamma^h{}_{ab}\Gamma^i{}_{jk}\Gamma^j{}_{ce}\Gamma^k{}_{fg} + 32\,\Gamma^d{}_{hi}\Gamma^h{}_{ab}\Gamma^i{}_{cj}\Gamma^j{}_{ek}\Gamma^k{}_{fg} \\
&+ 24\,\Gamma^d{}_{ah}\Gamma^h{}_{ij}\Gamma^i{}_{bc}\Gamma^j{}_{ek}\Gamma^k{}_{fg} + 8\,\Gamma^d{}_{ah}\Gamma^h{}_{bi}\Gamma^i{}_{jk}\Gamma^j{}_{ce}\Gamma^k{}_{fg} + 16\,\Gamma^d{}_{ah}\Gamma^h{}_{bi}\Gamma^i{}_{cj}\Gamma^j{}_{ek}\Gamma^k{}_{fg}\big)
\end{aligned}$$