Side calculations for detg2.tex

This is a short computation for the expansions of $\sqrt{-\det g(x)}$ and $\log\left(-\det g(x)\right)$ in powers of R and its derivatives. The results wil be used in `detg2.tex`.

The starting point is to write $\det g(x)$ in the form

$$-g(x) = 1 + a\epsilon^2 + b\epsilon^3 + c\epsilon^4 + d\epsilon^5 + \mathcal{O}\left(\epsilon^6\right)$$

where ϵ is introduced as an expansion parameter and where a, b, c and d are simple expressions in R and its derivatives. These can be read off directly from the expansion for $\det g(x)$ given in `detg2.tex`.

```python
from sympy import *

eps, a, b, c, d = symbols('\epsilon a b c d')


ans = sqrt(1+a*eps**2+b*eps**3+c*eps**4+d*eps**5)   # py (ans.001,ans)
ans = ans.series(eps, 0, 6)                          # py (ans.002,ans)
ans = simplify(ans)                                  # py (ans.003,ans)

det = 1 + eps**2*a + eps**3*b + eps**4*c + eps**5*d   # py (det.001,det)
# this foo will be used in detg2.tex
foo = Rational(1,2) + det/2 - (eps**4*a**2)/8 - (eps**5*a*b)/4   # py (foo.001,foo)
err = simplify (ans-foo)   # py (err.001,err)
```

$$\texttt{ans.001} := \sqrt{\epsilon^5 d + \epsilon^4 c + \epsilon^3 b + \epsilon^2 a + 1}$$

$$\texttt{ans.002} := 1 + \frac{a}{2}\epsilon^2 + \frac{b}{2}\epsilon^3 + \epsilon^4\left(-\frac{a^2}{8} + \frac{c}{2}\right) + \epsilon^5\left(-\frac{a}{4}b + \frac{d}{2}\right) + O\left(\epsilon^6\right)$$

$$\texttt{ans.003} := 1 + \frac{a}{2}\epsilon^2 + \frac{b}{2}\epsilon^3 + \frac{\epsilon^4}{8}\left(-a^2 + 4c\right) + \frac{\epsilon^5}{4}\left(-ab + 2d\right) + O\left(\epsilon^6\right)$$

$$\texttt{det.001} := \epsilon^5 d + \epsilon^4 c + \epsilon^3 b + \epsilon^2 a + 1$$

$$\texttt{foo.001} := -\frac{a}{4}\epsilon^5 b + \frac{d}{2}\epsilon^5 - \frac{\epsilon^4}{8}a^2 + \frac{c}{2}\epsilon^4 + \frac{b}{2}\epsilon^3 + \frac{a}{2}\epsilon^2 + 1$$

$$\texttt{err.001} := O\left(\epsilon^6\right)$$

And while we're here, let's also expand $\log(g)$ in powers of $R, \nabla R$ etc.

```
from sympy import *

eps, a, b, c, d = symbols('\epsilon a b c d')

ans = log(1+a*eps**2+b*eps**3+c*eps**4+d*eps**5)    # py (ans.101,ans)
ans = ans.series(eps, 0, 6)                         # py (ans.102,ans)
ans = simplify(ans)                                 # py (ans.103,ans)

det = 1 + eps**2*a + eps**3*b + eps**4*c + eps**5*d   # py (det.001,det)
# this foo will be used in detg2.tex
foo = -1 + det - (eps**4*a**2)/2 - (eps**5*a*b)       # py (foo.002,foo)
err = simplify (ans-foo)   # py (err.002,err)
```

$$\texttt{ans.101} := \log\left(\epsilon^5 d + \epsilon^4 c + \epsilon^3 b + \epsilon^2 a + 1\right)$$

$$\texttt{ans.102} := \epsilon^2 a + \epsilon^3 b + \epsilon^4\left(-\frac{a^2}{2} + c\right) + \epsilon^5\left(-ab + d\right) + O\left(\epsilon^6\right)$$

$$\texttt{ans.103} := \epsilon^2 a + \epsilon^3 b + \epsilon^4\left(-\frac{a^2}{2} + c\right) + \epsilon^5\left(-ab + d\right) + O\left(\epsilon^6\right)$$

$$\texttt{det.001} := \epsilon^5 d + \epsilon^4 c + \epsilon^3 b + \epsilon^2 a + 1$$

$$\texttt{foo.002} := -\epsilon^5 ab + \epsilon^5 d - \frac{\epsilon^4}{2}a^2 + \epsilon^4 c + \epsilon^3 b + \epsilon^2 a$$

$$\texttt{err.002} := O\left(\epsilon^6\right)$$