

# Convert from rnc to generic coordinates

The following code is based on the `gen2rnc.tex` code.

It is common to do some computations in a local RNC. Doing so makes various parts of the computations much easier to manage than in the original non-RNC coordinates. One simple example is the proof of the second Bianchi identities.

This code develops the inverse transformation, that is from the local RNC coordinates back to generic coordinates. The key equation (drawn from `gen2rnc.tex`) is

$$x_j^a = x_i^a + y^a - \sum_{k=2}^{\infty} \frac{1}{k!} \Gamma_{b_k}^a y^{b_k} \quad (1)$$

In `gen2rnc.tex` this equation was solved for the RNC coordinates  $y$  given the generic coordinates  $x_j$  and  $x_i$ . Here we will instead take  $x_i$  and  $y$  as given and use this equation to compute  $x_j$ . The first change we will make is to replace  $x_j$  with  $x$  (as the subscript  $j$  serves no useful purpose).

Thus our job will be to compute

$$x^a = x_i^a + y^a - \sum_{k=2}^{\infty} \frac{1}{k!} \Gamma_{b_k}^a y^{b_k} \quad (2)$$

given  $x_i$  and  $y$ . The generalised connections will be computed recursively by

$$\Gamma_{bcd}^a = \Gamma_{(b\bar{c},d)}^a - (n+1)\Gamma_{p(\bar{c}}^a \Gamma_{bd)}^p \quad (3)$$

As noted in `gen2rnc.tex`, the generalised connections will scale with the expansions parameter  $\epsilon$  according to

$$\Gamma_{bc}^a = \mathcal{O}(\epsilon) \ , \quad \Gamma_{bcd}^a = \mathcal{O}(\epsilon^2) \ , \quad \Gamma_{bcde}^a = \mathcal{O}(\epsilon^3) \ , \quad \text{etc.}$$

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{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).

D{#}::Derivative.
\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

g_{a b}::Metric.
g^{a b}::InverseMetric.
g_{a}^{b}::KroneckerDelta.
g^{a}_{b}::KroneckerDelta.
\delta^{a}_{b}::KroneckerDelta.
\delta_{a}^{b}::KroneckerDelta.

R_{a b c d}::RiemannTensor.
R^{a}_{b c d}::RiemannTensor.
R_{a b c}^{d}::RiemannTensor.

A^{a}::Depends(\partial{#}).

g_{a b}::Depends(\partial{#}).
R_{a b c d}::Depends(\partial{#}).
R^{a}_{b c d}::Depends(\partial{#}).

Q^{a}_{b c}::Depends(\partial{#}).

Q^{a}_{b c}::TableauSymmetry(shape={2}, indices={1,2}).
Q^{a}_{b c d}::TableauSymmetry(shape={3}, indices={1,2,3}).
Q^{a}_{b c d e}::TableauSymmetry(shape={4}, indices={1,2,3,4}).
Q^{a}_{b c d e f}::TableauSymmetry(shape={5}, indices={1,2,3,4,5}).
Q^{a}_{b c d e f g}::TableauSymmetry(shape={6}, indices={1,2,3,4,5,6}).

Q^{p}_{a b}::Weight(label=numQ,value=1).
Q^{p}_{a b c}::Weight(label=numQ,value=2).
Q^{p}_{a b c d}::Weight(label=numQ,value=3).
Q^{p}_{a b c d e}::Weight(label=numQ,value=4).
Q^{p}_{a b c d e f}::Weight(label=numQ,value=5).

def product_sort (obj):

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substitute (obj,$ A^{a}          -> A001^{a}          $)
substitute (obj,$ x^{a}          -> A002^{a}          $)
substitute (obj,$ g^{a b}        -> A003^{a b}        $)
substitute (obj,$ Q^{p}_{a b}    -> A004^{p}_{a b}    $)
substitute (obj,$ Q^{p}_{a b c}  -> A005^{p}_{a b c}  $)
substitute (obj,$ Q^{p}_{a b c d}-> A006^{p}_{a b c d}  $)
substitute (obj,$ Q^{p}_{a b c d e}-> A007^{p}_{a b c d e} $)
substitute (obj,$ Q^{p}_{a b c d e f}-> A008^{p}_{a b c d e f} $)
sort_product (obj)
rename_dummies (obj)
substitute (obj,$ A001^{a}      -> A^{a}          $)
substitute (obj,$ A002^{a}      -> x^{a}          $)
substitute (obj,$ A003^{a b}    -> g^{a b}        $)
substitute (obj,$ A004^{p}_{a b}-> Q^{p}_{a b}    $)
substitute (obj,$ A005^{p}_{a b c}-> Q^{p}_{a b c}  $)
substitute (obj,$ A006^{p}_{a b c d}-> Q^{p}_{a b c d} $)
substitute (obj,$ A007^{p}_{a b c d e}-> Q^{p}_{a b c d e} $)
substitute (obj,$ A008^{p}_{a b c d e f}-> Q^{p}_{a b c d e f} $)

return obj

def truncateQ (obj,n):

    ans = Ex(0)

    for i in range (0,n+1):
        foo := @(obj).
        bah  = Ex("numQ = " + str(i))
        keep_weight (foo, bah)
        ans = ans + foo

    return ans

# A^{a} = dx^a/ds

Gamma := Q^{d}_{a b} A^{a} A^{b}.

dAds := A^{c} \partial_{c}{A^{d}}-> - @(Gamma).

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# =====
eq0 := @(Gamma).                # cdb (eq0.000,eq0)

# =====
eq1 := A^{c} \partial_{c}{@(eq0)}.    # cdb (eq1.000,eq1)

distribute      (eq1)                # cdb (eq1.001,eq1)
unwrap          (eq1)                # cdb (eq1.002,eq1)
product_rule    (eq1)                # cdb (eq1.003,eq1)
distribute      (eq1)                # cdb (eq1.004,eq1)
substitute      (eq1,dAds)           # cdb (eq1.005,eq1)
distribute      (eq1)                # cdb (eq1.006,eq1)
eq1 = truncateQ (eq1,5)              # cdb (eq1.007,eq1)
sort_product    (eq1)                # cdb (eq1.008,eq1)
rename_dummies  (eq1)                # cdb (eq1.009,eq1)
canonicalise    (eq1)                # cdb (eq1.010,eq1)

# =====
eq2 := A^{c} \partial_{c}{@(eq1)}.    # cdb (eq2.000,eq2)

distribute      (eq2)                # cdb (eq2.001,eq2)
unwrap          (eq2)                # cdb (eq2.002,eq2)
product_rule    (eq2)                # cdb (eq2.003,eq2)
distribute      (eq2)                # cdb (eq2.004,eq2)
substitute      (eq2,dAds)           # cdb (eq2.005,eq2)
distribute      (eq2)                # cdb (eq2.006,eq2)
eq2 = truncateQ (eq2,5)              # cdb (eq2.007,eq2)
sort_product    (eq2)                # cdb (eq2.008,eq2)
rename_dummies  (eq2)                # cdb (eq2.009,eq2)
canonicalise    (eq2)                # cdb (eq2.010,eq2)

# =====
eq3 := A^{c} \partial_{c}{@(eq2)}.    # cdb (eq3.000,eq3)

distribute      (eq3)                # cdb (eq3.001,eq3)
unwrap          (eq3)                # cdb (eq3.002,eq3)
product_rule    (eq3)                # cdb (eq3.003,eq3)

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distribute      (eq3)                # cdb (eq3.004,eq3)
substitute      (eq3,dAds)            # cdb (eq3.005,eq3)
distribute      (eq3)                # cdb (eq3.006,eq3)
eq3 = truncateQ (eq3,5)               # cdb (eq3.007,eq3)
sort_product    (eq3)                # cdb (eq3.008,eq3)
rename_dummies  (eq3)                # cdb (eq3.009,eq3)
canonicalise    (eq3)                # cdb (eq3.010,eq3)

# =====
eq4 := A^{c} \partial_{c}{@(eq3)}.      # cdb (eq4.000,eq4)

distribute      (eq4)                # cdb (eq4.001,eq4)
unwrap          (eq4)                # cdb (eq4.002,eq4)
product_rule    (eq4)                # cdb (eq4.003,eq4)
distribute      (eq4)                # cdb (eq4.004,eq4)
substitute      (eq4,dAds)            # cdb (eq4.005,eq4)
distribute      (eq4)                # cdb (eq4.006,eq4)
eq4 = truncateQ (eq4,5)               # cdb (eq4.007,eq4)
sort_product    (eq4)                # cdb (eq4.008,eq4)
rename_dummies  (eq4)                # cdb (eq4.009,eq4)
canonicalise    (eq4)                # cdb (eq4.010,eq4)

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$$\text{eq0.000} := Q^d_{ab} A^a A^b$$

$$\text{eq1.000} := A^c \partial_c (Q^d_{ab} A^a A^b)$$

$$\text{eq1.001} := A^c \partial_c (Q^d_{ab} A^a A^b)$$

$$\text{eq1.002} := A^c \partial_c (Q^d_{ab} A^a A^b)$$

$$\text{eq1.003} := A^c (\partial_c Q^d_{ab} A^a A^b + Q^d_{ab} \partial_c A^a A^b + Q^d_{ab} A^a \partial_c A^b)$$

$$\text{eq1.004} := A^c \partial_c Q^d_{ab} A^a A^b + A^c Q^d_{ab} \partial_c A^a A^b + A^c Q^d_{ab} A^a \partial_c A^b$$

$$\text{eq1.005} := A^c \partial_c Q^d_{ab} A^a A^b - Q^a_{ce} A^c A^e Q^d_{ab} A^b - Q^b_{ec} A^e A^c Q^d_{ab} A^a$$

$$\text{eq1.006} := A^c \partial_c Q^d_{ab} A^a A^b - Q^a_{ce} A^c A^e Q^d_{ab} A^b - Q^b_{ec} A^e A^c Q^d_{ab} A^a$$

$$\text{eq1.007} := A^c \partial_c Q^d_{ab} A^a A^b - Q^a_{ce} A^c A^e Q^d_{ab} A^b - Q^b_{ec} A^e A^c Q^d_{ab} A^a$$

$$\text{eq1.008} := A^a A^b A^c \partial_c Q^d_{ab} - A^b A^c A^e Q^a_{ce} Q^d_{ab} - A^a A^c A^e Q^b_{ec} Q^d_{ab}$$

$$\text{eq1.009} := A^a A^b A^c \partial_c Q^d_{ab} - A^a A^b A^c Q^e_{bc} Q^d_{ea} - A^a A^b A^c Q^e_{cb} Q^d_{ae}$$

$$\text{eq1.010} := A^a A^b A^c \partial_a Q^d_{bc} - 2A^a A^b A^c Q^d_{ae} Q^e_{bc}$$

$$\begin{aligned}
\text{eq2.000} &:= A^c \partial_c (A^a A^b A^f \partial_a Q^d_{bf} - 2A^a A^b A^f Q^d_{ae} Q^e_{bf}) \\
\text{eq2.001} &:= A^c \partial_c (A^a A^b A^f \partial_a Q^d_{bf}) - 2A^c \partial_c (A^a A^b A^f Q^d_{ae} Q^e_{bf}) \\
\text{eq2.002} &:= A^c \partial_c (A^a A^b A^f \partial_a Q^d_{bf}) - 2A^c \partial_c (A^a A^b A^f Q^d_{ae} Q^e_{bf}) \\
\text{eq2.003} &:= A^c (\partial_c A^a A^b A^f \partial_a Q^d_{bf} + A^a \partial_c A^b A^f \partial_a Q^d_{bf} + A^a A^b \partial_c A^f \partial_a Q^d_{bf} + A^a A^b A^f \partial_{ca} Q^d_{bf}) \\
&\quad - 2A^c (\partial_c A^a A^b A^f Q^d_{ae} Q^e_{bf} + A^a \partial_c A^b A^f Q^d_{ae} Q^e_{bf} + A^a A^b \partial_c A^f Q^d_{ae} Q^e_{bf} + A^a A^b A^f \partial_c Q^d_{ae} Q^e_{bf} + A^a A^b A^f Q^d_{ae} \partial_c Q^e_{bf}) \\
\text{eq2.004} &:= A^c \partial_c A^a A^b A^f \partial_a Q^d_{bf} + A^c A^a \partial_c A^b A^f \partial_a Q^d_{bf} + A^c A^a A^b \partial_c A^f \partial_a Q^d_{bf} + A^c A^a A^b A^f \partial_{ca} Q^d_{bf} - 2A^c \partial_c A^a A^b A^f Q^d_{ae} Q^e_{bf} \\
&\quad - 2A^c A^a \partial_c A^b A^f Q^d_{ae} Q^e_{bf} - 2A^c A^a A^b \partial_c A^f Q^d_{ae} Q^e_{bf} - 2A^c A^a A^b A^f \partial_c Q^d_{ae} Q^e_{bf} - 2A^c A^a A^b A^f Q^d_{ae} \partial_c Q^e_{bf} \\
\text{eq2.005} &:= -Q^a_{ce} A^c A^e A^b A^f \partial_a Q^d_{bf} - Q^b_{ec} A^e A^c A^a A^f \partial_a Q^d_{bf} - Q^f_{ce} A^c A^e A^a A^b \partial_a Q^d_{bf} + A^c A^a A^b A^f \partial_{ca} Q^d_{bf} + 2Q^a_{cg} A^c A^g A^b A^f Q^d_{ae} Q^e_{bf} \\
&\quad + 2Q^b_{gc} A^g A^c A^a A^f Q^d_{ae} Q^e_{bf} + 2Q^f_{cg} A^c A^g A^a A^b Q^d_{ae} Q^e_{bf} - 2A^c A^a A^b A^f \partial_c Q^d_{ae} Q^e_{bf} - 2A^c A^a A^b A^f Q^d_{ae} \partial_c Q^e_{bf} \\
\text{eq2.006} &:= -Q^a_{ce} A^c A^e A^b A^f \partial_a Q^d_{bf} - Q^b_{ec} A^e A^c A^a A^f \partial_a Q^d_{bf} - Q^f_{ce} A^c A^e A^a A^b \partial_a Q^d_{bf} + A^c A^a A^b A^f \partial_{ca} Q^d_{bf} + 2Q^a_{cg} A^c A^g A^b A^f Q^d_{ae} Q^e_{bf} \\
&\quad + 2Q^b_{gc} A^g A^c A^a A^f Q^d_{ae} Q^e_{bf} + 2Q^f_{cg} A^c A^g A^a A^b Q^d_{ae} Q^e_{bf} - 2A^c A^a A^b A^f \partial_c Q^d_{ae} Q^e_{bf} - 2A^c A^a A^b A^f Q^d_{ae} \partial_c Q^e_{bf} \\
\text{eq2.007} &:= A^c A^a A^b A^f \partial_{ca} Q^d_{bf} - Q^a_{ce} A^c A^e A^b A^f \partial_a Q^d_{bf} - Q^b_{ec} A^e A^c A^a A^f \partial_a Q^d_{bf} - Q^f_{ce} A^c A^e A^a A^b \partial_a Q^d_{bf} - 2A^c A^a A^b A^f \partial_c Q^d_{ae} Q^e_{bf} \\
&\quad - 2A^c A^a A^b A^f Q^d_{ae} \partial_c Q^e_{bf} + 2Q^a_{cg} A^c A^g A^b A^f Q^d_{ae} Q^e_{bf} + 2Q^b_{gc} A^g A^c A^a A^f Q^d_{ae} Q^e_{bf} + 2Q^f_{cg} A^c A^g A^a A^b Q^d_{ae} Q^e_{bf} \\
\text{eq2.008} &:= A^a A^b A^c A^f \partial_{ca} Q^d_{bf} - A^b A^c A^e A^f Q^a_{ce} \partial_a Q^d_{bf} - A^a A^c A^e A^f Q^b_{ec} \partial_a Q^d_{bf} - A^a A^b A^c A^e Q^f_{ce} \partial_a Q^d_{bf} - 2A^a A^b A^c A^f Q^e_{bf} \partial_c Q^d_{ae} \\
&\quad - 2A^a A^b A^c A^f Q^d_{ae} \partial_c Q^e_{bf} + 2A^b A^c A^f A^g Q^a_{cg} Q^d_{ae} Q^e_{bf} + 2A^a A^c A^f A^g Q^b_{gc} Q^d_{ae} Q^e_{bf} + 2A^a A^b A^c A^g Q^d_{ae} Q^e_{bf} Q^f_{cg} \\
\text{eq2.009} &:= A^a A^b A^c A^e \partial_{ca} Q^d_{be} - A^a A^b A^c A^e Q^f_{bc} \partial_f Q^d_{ae} - A^a A^b A^c A^e Q^f_{cb} \partial_a Q^d_{fe} - A^a A^b A^c A^e Q^f_{ce} \partial_a Q^d_{bf} - 2A^a A^b A^c A^e Q^f_{be} \partial_c Q^d_{af} \\
&\quad - 2A^a A^b A^c A^e Q^d_{af} \partial_c Q^f_{be} + 2A^a A^b A^c A^e Q^f_{be} Q^d_{fg} Q^g_{ac} + 2A^a A^b A^c A^e Q^f_{eb} Q^d_{ag} Q^g_{fc} + 2A^a A^b A^c A^e Q^d_{af} Q^f_{bg} Q^g_{ce} \\
\text{eq2.010} &:= A^a A^b A^c A^e \partial_{ab} Q^d_{ce} - A^a A^b A^c A^e Q^f_{ab} \partial_f Q^d_{ce} - 4A^a A^b A^c A^e Q^f_{ab} \partial_c Q^d_{ef} \\
&\quad - 2A^a A^b A^c A^e Q^d_{af} \partial_b Q^f_{ce} + 2A^a A^b A^c A^e Q^d_{fg} Q^f_{ab} Q^g_{ce} + 4A^a A^b A^c A^e Q^d_{af} Q^f_{bg} Q^g_{ce}
\end{aligned}$$



$$\begin{aligned}
\text{eq3.010} := & A^a A^b A^c A^e A^f \partial_{abc} Q^d_{ef} - A^a A^b A^c A^e A^f \partial_g Q^d_{ab} \partial_c Q^g_{ef} - 6 A^a A^b A^c A^e A^f \partial_a Q^d_{bg} \partial_c Q^g_{ef} - 3 A^a A^b A^c A^e A^f Q^g_{ab} \partial_{cg} Q^d_{ef} \\
& - 6 A^a A^b A^c A^e A^f Q^g_{ab} \partial_{ce} Q^d_{fg} - 2 A^a A^b A^c A^e A^f Q^d_{ag} \partial_{bc} Q^g_{ef} + 2 A^a A^b A^c A^e A^f Q^g_{ab} Q^h_{cg} \partial_h Q^d_{ef} + 6 A^a A^b A^c A^e A^f Q^g_{ab} Q^h_{ce} \partial_g Q^d_{fh} \\
& + 12 A^a A^b A^c A^e A^f Q^g_{ab} Q^h_{cg} \partial_e Q^d_{fh} + 6 A^a A^b A^c A^e A^f Q^g_{ab} Q^h_{ce} \partial_f Q^d_{gh} + 6 A^a A^b A^c A^e A^f Q^d_{gh} Q^g_{ab} \partial_c Q^h_{ef} \\
& + 2 A^a A^b A^c A^e A^f Q^d_{ag} Q^h_{bc} \partial_h Q^g_{ef} + 8 A^a A^b A^c A^e A^f Q^d_{ag} Q^h_{bc} \partial_e Q^g_{fh} + 4 A^a A^b A^c A^e A^f Q^d_{ag} Q^g_{bh} \partial_c Q^h_{ef} \\
& - 12 A^a A^b A^c A^e A^f Q^d_{gh} Q^g_{ab} Q^h_{ci} Q^i_{ef} - 4 A^a A^b A^c A^e A^f Q^d_{ag} Q^g_{hi} Q^h_{bc} Q^i_{ef} - 8 A^a A^b A^c A^e A^f Q^d_{ag} Q^g_{bh} Q^h_{ci} Q^i_{ef}
\end{aligned}$$

$$\begin{aligned}
\text{eq4.010} := & A^a A^b A^c A^e A^f A^g \partial_{abce} Q^d_{fg} - 4A^a A^b A^c A^e A^f A^g \partial_a Q^h_{bc} \partial_{eh} Q^d_{fg} - A^a A^b A^c A^e A^f A^g \partial_h Q^d_{ab} \partial_{ce} Q^h_{fg} - 12A^a A^b A^c A^e A^f A^g \partial_a Q^h_{bc} \partial_{ef} Q^d_{gh} \\
& - 8A^a A^b A^c A^e A^f A^g \partial_a Q^d_{bh} \partial_{ce} Q^h_{fg} - 6A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_{ceh} Q^d_{fg} - 8A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_{cef} Q^d_{gh} \\
& + 8A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_i Q^d_{ch} \partial_e Q^i_{fg} + A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_i Q^d_{ce} \partial_h Q^i_{fg} + 4A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_i Q^d_{ce} \partial_f Q^i_{gh} \\
& + 12A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_h Q^d_{ci} \partial_e Q^i_{fg} + 24A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_c Q^d_{hi} \partial_e Q^i_{fg} + 8A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_c Q^d_{ei} \partial_h Q^i_{fg} \\
& + 32A^a A^b A^c A^e A^f A^g Q^h_{ab} \partial_c Q^d_{ei} \partial_f Q^i_{gh} - 2A^a A^b A^c A^e A^f A^g Q^d_{ah} \partial_{bce} Q^h_{fg} + 2A^a A^b A^c A^e A^f A^g Q^h_{ai} \partial_h Q^d_{bc} \partial_e Q^i_{fg} \\
& + 16A^a A^b A^c A^e A^f A^g Q^h_{ai} \partial_b Q^d_{ch} \partial_e Q^i_{fg} + 6A^a A^b A^c A^e A^f A^g Q^d_{hi} \partial_a Q^h_{bc} \partial_e Q^i_{fg} + 2A^a A^b A^c A^e A^f A^g Q^d_{ah} \partial_b Q^i_{ce} \partial_i Q^h_{fg} \\
& + 12A^a A^b A^c A^e A^f A^g Q^d_{ah} \partial_b Q^h_{ci} \partial_e Q^i_{fg} + 8A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ch} \partial_{ei} Q^d_{fg} + 3A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} \partial_{hi} Q^d_{fg} \\
& + 24A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} \partial_{fh} Q^d_{gi} + 24A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ch} \partial_{ef} Q^d_{gi} + 12A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} \partial_{fg} Q^d_{hi} \\
& + 8A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{ab} \partial_{ce} Q^i_{fg} + 6A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^i_{bc} \partial_{ei} Q^h_{fg} + 12A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^i_{bc} \partial_{ef} Q^h_{gi} \\
& + 4A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{bi} \partial_{ce} Q^i_{fg} - 4A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ch} Q^j_{ei} \partial_j Q^d_{fg} - 2A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} Q^j_{hi} \partial_j Q^d_{fg} \\
& - 16A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} Q^j_{fh} \partial_j Q^d_{gi} - 24A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} Q^j_{fh} \partial_i Q^d_{gj} - 12A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} Q^j_{fg} \partial_h Q^d_{ij} \\
& - 32A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ch} Q^j_{ei} \partial_f Q^d_{gj} - 16A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} Q^j_{hi} \partial_f Q^d_{gj} - 48A^a A^b A^c A^e A^f A^g Q^h_{ab} Q^i_{ce} Q^j_{fh} \partial_g Q^d_{ij} \\
& - 24A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{aj} Q^j_{bc} \partial_e Q^i_{fg} - 8A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{ab} Q^j_{ce} \partial_j Q^i_{fg} - 32A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{ab} Q^j_{ce} \partial_f Q^i_{gj} \\
& - 4A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^i_{bc} Q^j_{ei} \partial_j Q^h_{fg} - 12A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^i_{bc} Q^j_{ef} \partial_i Q^h_{gj} - 24A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^i_{bc} Q^j_{ei} \partial_f Q^h_{gj} \\
& - 12A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^i_{bc} Q^j_{ef} \partial_g Q^h_{ij} - 16A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{ab} Q^i_{cj} \partial_e Q^j_{fg} - 12A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{ij} Q^i_{bc} \partial_e Q^j_{fg} \\
& - 4A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{bi} Q^j_{ce} \partial_j Q^i_{fg} - 16A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{bi} Q^j_{ce} \partial_f Q^i_{gj} - 8A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{bi} Q^i_{cj} \partial_e Q^j_{fg} \\
& + 24A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{aj} Q^i_{bk} Q^j_{ce} Q^k_{fg} + 16A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{ab} Q^i_{jk} Q^j_{ce} Q^k_{fg} + 32A^a A^b A^c A^e A^f A^g Q^d_{hi} Q^h_{ab} Q^i_{cj} Q^j_{ek} Q^k_{fg} \\
& + 24A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{ij} Q^i_{bc} Q^j_{ek} Q^k_{fg} + 8A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{bi} Q^i_{jk} Q^j_{ce} Q^k_{fg} + 16A^a A^b A^c A^e A^f A^g Q^d_{ah} Q^h_{bi} Q^i_{cj} Q^j_{ek} Q^k_{fg}
\end{aligned}$$

```

def reformat (obj):
    bah := @(obj).
    distribute      (bah)
    bah = product_sort (bah)
    rename_dummies  (bah)
    canonicalise     (bah)
    factor_out       (bah,$A^{a?}$)
    substitute       (bah,$A^{a}->y^{a}$)
    substitute       (bah,$Q^{a}_{b\ c}->\Gamma^{a}_{b\ c}$)
    ans := @(bah).
    return ans

eq0 = reformat(eq0)  # cdb (eq0.100,eq0)
eq1 = reformat(eq1)  # cdb (eq1.100,eq1)
eq2 = reformat(eq2)  # cdb (eq2.100,eq2)
eq3 = reformat(eq3)  # cdb (eq3.100,eq3)
eq4 = reformat(eq4)  # cdb (eq4.100,eq4)

checkpoint.append (eq0)
checkpoint.append (eq1)
checkpoint.append (eq2)
checkpoint.append (eq3)
checkpoint.append (eq4)

```

## Convert from local RNC coords (y) to generic (x)

$$x^a = x_i^a + \overset{0}{x}^a - \overset{1}{x}^a - \overset{2}{x}^a - \overset{3}{x}^a - \overset{4}{x}^a - \overset{5}{x}^a$$

$$\overset{0}{x}^a = y^a$$

$$2! \overset{1}{x}^a = y^a y^b \Gamma_{ab}^d$$

$$3! \overset{2}{x}^a = y^a y^b y^c (\partial_a \Gamma_{bc}^d - 2\Gamma_{ae}^d \Gamma_{bc}^e)$$

$$4! \overset{3}{x}^a = y^a y^b y^c y^e (\partial_{ab} \Gamma_{ce}^d - \Gamma_{ab}^f \partial_f \Gamma_{ce}^d - 4\Gamma_{ab}^f \partial_c \Gamma_{ef}^d - 2\Gamma_{af}^d \partial_b \Gamma_{ce}^f + 2\Gamma_{fg}^d \Gamma_{ab}^f \Gamma_{ce}^g + 4\Gamma_{af}^d \Gamma_{bg}^f \Gamma_{ce}^g)$$

$$5! \overset{4}{x}^a = y^a y^b y^c y^e y^f (\partial_{abc} \Gamma_{ef}^d - \partial_g \Gamma_{ab}^d \partial_c \Gamma_{ef}^g - 6\partial_a \Gamma_{bg}^d \partial_c \Gamma_{ef}^g - 3\Gamma_{ab}^g \partial_{cg} \Gamma_{ef}^d - 6\Gamma_{ab}^g \partial_{ce} \Gamma_{fg}^d - 2\Gamma_{ag}^d \partial_{bc} \Gamma_{ef}^g + 2\Gamma_{ab}^g \Gamma_{cg}^h \partial_h \Gamma_{ef}^d + 6\Gamma_{ab}^g \Gamma_{ce}^h \partial_g \Gamma_{fh}^d \\ + 12\Gamma_{ab}^g \Gamma_{cg}^h \partial_e \Gamma_{fh}^d + 6\Gamma_{ab}^g \Gamma_{ce}^h \partial_f \Gamma_{gh}^d + 6\Gamma_{gh}^d \Gamma_{ab}^g \partial_c \Gamma_{ef}^h + 2\Gamma_{ag}^d \Gamma_{bc}^h \partial_h \Gamma_{ef}^g + 8\Gamma_{ag}^d \Gamma_{bc}^h \partial_e \Gamma_{fh}^g + 4\Gamma_{ag}^d \Gamma_{bh}^g \partial_c \Gamma_{ef}^h - 12\Gamma_{gh}^d \Gamma_{ab}^g \Gamma_{ci}^h \Gamma_{ef}^i \\ - 4\Gamma_{ag}^d \Gamma_{hi}^g \Gamma_{bc}^h \Gamma_{ef}^i - 8\Gamma_{ag}^d \Gamma_{bh}^g \Gamma_{ci}^h \Gamma_{ef}^i)$$

$$6! \overset{5}{x}^a = y^a y^b y^c y^e y^f y^g (\partial_{abce} \Gamma_{fg}^d - 4\partial_a \Gamma_{bc}^h \partial_{eh} \Gamma_{fg}^d - \partial_h \Gamma_{ab}^d \partial_{ce} \Gamma_{fg}^h - 12\partial_a \Gamma_{bc}^h \partial_{ef} \Gamma_{gh}^d - 8\partial_a \Gamma_{bh}^d \partial_{ce} \Gamma_{fg}^h - 6\Gamma_{ab}^h \partial_{ceh} \Gamma_{fg}^d - 8\Gamma_{ab}^h \partial_{cef} \Gamma_{gh}^d \\ + 8\Gamma_{ab}^h \partial_i \Gamma_{ch}^d \partial_e \Gamma_{fg}^i + \Gamma_{ab}^h \partial_i \Gamma_{ce}^d \partial_h \Gamma_{fg}^i + 4\Gamma_{ab}^h \partial_i \Gamma_{ce}^d \partial_f \Gamma_{gh}^i + 12\Gamma_{ab}^h \partial_h \Gamma_{ci}^d \partial_e \Gamma_{fg}^i + 24\Gamma_{ab}^h \partial_c \Gamma_{hi}^d \partial_e \Gamma_{fg}^i + 8\Gamma_{ab}^h \partial_c \Gamma_{ei}^d \partial_h \Gamma_{fg}^i \\ + 32\Gamma_{ab}^h \partial_c \Gamma_{ei}^d \partial_f \Gamma_{gh}^i - 2\Gamma_{ah}^d \partial_{bce} \Gamma_{fg}^h + 2\Gamma_{ai}^h \partial_h \Gamma_{bc}^d \partial_e \Gamma_{fg}^i + 16\Gamma_{ai}^h \partial_b \Gamma_{ch}^d \partial_e \Gamma_{fg}^i + 6\Gamma_{hi}^d \partial_a \Gamma_{bc}^h \partial_e \Gamma_{fg}^i + 2\Gamma_{ah}^d \partial_b \Gamma_{ce}^h \partial_i \Gamma_{fg}^h \\ + 12\Gamma_{ah}^d \partial_b \Gamma_{ci}^h \partial_e \Gamma_{fg}^i + 8\Gamma_{ab}^h \Gamma_{ch}^i \partial_{ei} \Gamma_{fg}^d + 3\Gamma_{ab}^h \Gamma_{ce}^i \partial_{hi} \Gamma_{fg}^d + 24\Gamma_{ab}^h \Gamma_{ce}^i \partial_{fh} \Gamma_{gi}^d + 24\Gamma_{ab}^h \Gamma_{ch}^i \partial_{ef} \Gamma_{gi}^d + 12\Gamma_{ab}^h \Gamma_{ce}^i \partial_{fg} \Gamma_{hi}^d \\ + 8\Gamma_{hi}^d \Gamma_{ab}^h \partial_{ce} \Gamma_{fg}^i + 6\Gamma_{ah}^d \Gamma_{bc}^i \partial_{ei} \Gamma_{fg}^h + 12\Gamma_{ah}^d \Gamma_{bc}^i \partial_{ef} \Gamma_{gi}^h + 4\Gamma_{ah}^d \Gamma_{bi}^h \partial_{ce} \Gamma_{fg}^i - 4\Gamma_{ab}^h \Gamma_{ch}^i \Gamma_{ei}^j \partial_j \Gamma_{fg}^d - 2\Gamma_{ab}^h \Gamma_{ce}^i \Gamma_{hi}^j \partial_j \Gamma_{fg}^d \\ - 16\Gamma_{ab}^h \Gamma_{ce}^i \Gamma_{fh}^j \partial_j \Gamma_{gi}^d - 24\Gamma_{ab}^h \Gamma_{ce}^i \Gamma_{fh}^j \partial_i \Gamma_{gj}^d - 12\Gamma_{ab}^h \Gamma_{ce}^i \Gamma_{fg}^j \partial_h \Gamma_{ij}^d - 32\Gamma_{ab}^h \Gamma_{ch}^i \Gamma_{ei}^j \partial_f \Gamma_{gj}^d - 16\Gamma_{ab}^h \Gamma_{ce}^i \Gamma_{hi}^j \partial_f \Gamma_{gj}^d \\ - 48\Gamma_{ab}^h \Gamma_{ce}^i \Gamma_{fh}^j \partial_g \Gamma_{ij}^d - 24\Gamma_{hi}^d \Gamma_{aj}^h \Gamma_{bc}^j \partial_e \Gamma_{fg}^i - 8\Gamma_{hi}^d \Gamma_{ab}^h \Gamma_{ce}^j \partial_j \Gamma_{fg}^i - 32\Gamma_{hi}^d \Gamma_{ab}^h \Gamma_{ce}^j \partial_f \Gamma_{gj}^i - 4\Gamma_{ah}^d \Gamma_{bc}^i \Gamma_{ei}^j \partial_j \Gamma_{fg}^h \\ - 12\Gamma_{ah}^d \Gamma_{bc}^i \Gamma_{ef}^j \partial_i \Gamma_{gj}^h - 24\Gamma_{ah}^d \Gamma_{bc}^i \Gamma_{ei}^j \partial_f \Gamma_{gj}^h - 12\Gamma_{ah}^d \Gamma_{bc}^i \Gamma_{ef}^j \partial_g \Gamma_{ij}^h - 16\Gamma_{hi}^d \Gamma_{ab}^h \Gamma_{cj}^i \partial_e \Gamma_{fg}^j - 12\Gamma_{ah}^d \Gamma_{ij}^h \Gamma_{bc}^i \partial_e \Gamma_{fg}^j \\ - 4\Gamma_{ah}^d \Gamma_{bi}^h \Gamma_{ce}^j \partial_j \Gamma_{fg}^i - 16\Gamma_{ah}^d \Gamma_{bi}^h \Gamma_{ce}^j \partial_f \Gamma_{gj}^i - 8\Gamma_{ah}^d \Gamma_{bi}^h \Gamma_{cj}^i \partial_e \Gamma_{fg}^j + 24\Gamma_{hi}^d \Gamma_{aj}^h \Gamma_{bk}^i \Gamma_{ce}^j \Gamma_{fg}^k + 16\Gamma_{hi}^d \Gamma_{ab}^h \Gamma_{jk}^i \Gamma_{ce}^j \Gamma_{fg}^k \\ + 32\Gamma_{hi}^d \Gamma_{ab}^h \Gamma_{cj}^i \Gamma_{ek}^j \Gamma_{fg}^k + 24\Gamma_{ah}^d \Gamma_{ij}^h \Gamma_{bc}^i \Gamma_{ek}^j \Gamma_{fg}^k + 8\Gamma_{ah}^d \Gamma_{bi}^h \Gamma_{jk}^i \Gamma_{ce}^j \Gamma_{fg}^k + 16\Gamma_{ah}^d \Gamma_{bi}^h \Gamma_{cj}^i \Gamma_{ek}^j \Gamma_{fg}^k)$$