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**With\_Gensyms Package**



# With\_Gensyms Package

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A Maxima Package  
for version 0.1.0, 2016-07-14

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This manual documents the `with_gensyms` package (version 0.1.0, 2016-07-14), a user-level package for the Maxima computer algebra system.

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# 1 WITH\_GENSYMS

## 1.1 Introduction to WITH\_GENSYMS

WITH\_GENSYMS is a Maxima package that provides a user-level drop-in replacement for the function definition operator `:=`, and the macro definition operator `::=`.

In addition, it offers a macro, `with_gensyms`, as a general-purpose tool to re-write Maxima code using anonymous variable names, or gensyms.

## 1.2 Examples

Here is a sample of WITH\_GENSYMS. Consider the following example where Maxima's scoping conventions introduce a difficult to understand bug.

```
(%i1) f(x,n) := x[n]          /* x is a local parameter to f */;
(%o1)          f(x, n) := x
                      n

(%i2) f(h,1);
(%o2)          h
                1

(%i3) f(x,3);
(%o3)          x
                3

(%i4) f(x,2);
(%o4)          x
                2

(%i5) x[2] : 2          /* x is now an undeclared array */;
(%o5)          2

(%i6) f(h,1);
(%o6)          x
                1

(%i7) f(x,3);
(%o7)          x
                3

(%i8) f(x,2);
(%o8)          2
```

The discrepancy in behavior is due to the creation of the undeclared array `x` in %i5. This introduces a global property on the symbol `x` that frustrates the expected behavior of `f`.

```
(%i1) load(with_gensyms)$
(%i2) x[2] : 2          /* x is now an undeclared array */;
(%o2)          2

(%i3) f(x,n) :> x[n]    /* :> function definition operator */;
(%o3)          f($x, $n) := $x
                      $n

(%i4) f(h,1);
(%o4)          h
                1

(%i5) f(x,3);
(%o5)          x
                3

(%i6) f(x,2);
(%o6)          2
```

### 1.3 Explanation

A simple cure to fix the problem encountered with `f` is to give the parameter `x` a more unique name. Something like

```
f(my_local_variable_x_121423412,n) := my_local_variable_x_121423412[n];
```

There are two obvious flaws here: 1. the code is almost unreadable; and 2. there is nothing to prevent someone else (you, at another time) from choosing the same variable name.

The `WITH_GENSYMS` package overcomes both problems by having Maxima automatically rewrite the code using gensyms, which are symbols (roughly, variable names) which are guaranteed to have no name clashes.

### 1.4 Definitions for SYMBOLS.LISP

**maxima-symbol-p** (*x*) [System Function]

Returns `true` if the symbol-name of *x* has more than 2 characters or *x* has a non-empty property list. Used to populate *symbols* when package is initially loaded.

**symbols** [Variable]

A Lisp hashtable used to look-up Maxima symbols. This is initialized when `symbols.lisp` is loaded, with the symbol table existing at that time. A user may add to this table. See `[add_maxima_symbol]`, page 2.

**add\_maxima\_symbol** (*x*, [*s*]) [Function]

The default value of the hashtable *s* is *symbols*. If *x* is a symbol, add it to *s*; otherwise, if *x* is a list, iterate over it.

**maxima\_symbolp** (*x*) [Function]

Return `true` if *x* is a key in *symbols* or if *x* is a function name listed in *functions*.

**remove\_maxima\_symbols** (*x*) [Function]

Iterate over the list *x* and remove each entry that satisfies `maxima_symbolp`.

**delete\_maxima\_symbols** (*x*) [Function]

If *x* is a symbol, remove it from *symbols*. If *x* is a Maxima list, map over it.

**maxima\_symbols** () [Function]

Creates a Maxima list of the current keys in *symbols*.

**wg\_gensymize** (*x*) [Function]

Returns a gensymized version of the symbol held by *x*, with the Maxima property `gensym`. By default, the printed representation of the gensym is not valid Maxima syntax.

```
(%i1) load(with_gensyms)$
(%i2) wg_gensymize(x);
(%o2)                               $x
(%i3) apply('properties, [%]);
(%o3) [database info, kind($x, gensym)]
```

If *x* already has the `gensym` property, return *x*.

```
(%i4) wg_gensymize(%o2);
(%o4)                               $x
(%i5) is(% = %o2);
(%o5)                               true
```

The display property of the returned gensym is governed by the variable `wg_reversealias`.

```
(%i6) wg_gensymize(y), wg_reversealias:false;
```

```
(%o6)                                     y
(%i7) lisp_print(%);
(%o7) #:%Y_690
```

## 1.5 Definitions for WITH\_GENSYMS

**wg\_reversealias** [User Option]

See the entry for [wg-gensymize], page 2.

**gensym** [Feature]

A property of a symbol. Used by **wg-gensymize** to ensure idempotency of that function. See the entry for [wg-gensymize], page 2.

**wg\_make\_binding (undef)** [Function]

Creates a function of two variables, *x* and *y*, where *x* is an assignment *a:b*. If *y* = **undef**, then return *b*; otherwise return the binding *b:y*.

```
(%i8) map(wg_make_binding(undef), '[a:b,a:b], [undef,3]);
(%o8) [b, b : 3]
```

**wg\_check\_op (oper,expr)** [Function]

If *expr* is an atom or the operand of *expr* is not *oper*, signal an error; otherwise return *expr*.

**wg\_atom\_or\_quote (expr)** [Function]

If *expr* is an atom or the operand is **'**, return **true**; else return **false**.

**with\_gensyms (bindings, [body])** [Macro]

Replace the variables in *bindings* with gensyms, and substitute these into *body*.

```
(%i1) load(with_gensyms)$
(%i2) with_gensyms([x:1, y:2],
                  ['[x,y], [x,y]]);
(%o2) [[$x, $y], [1, 2]]
(%i3) with_gensyms([x,n], f(x) := a+x[n]);
(%o3) f($x) := a + $x
      $n
```

Here is an implementation of a **lambda** function using **with\_gensyms**:

```
(%i4) wg_lambda1(vars,[body]) ::> buildq([vars:vars,body:body],
                                          with_gensyms(
                                            vars,
                                            lambda(vars,
                                                  splice(body)))) $
(%i5) wg_lambda1([x,y], if a=1 then x+1 else if a=2 then y+2 else a);
(%o5) lambda([$x, $y], if a = 1 then 1 + $x
                      else (if a = 2 then 2 + $y else a))
(%i6) apply(%o5,'[u,v]);
(%o6) a
(%i7) apply(%o5,'[u,v]), a=1;
(%o7) u + 1
(%i8) apply(%o5,'[u,v]), a=2;
(%o8) v + 2
```

**wg\_listofvars (expr,[listvars])** [Function]

Extract the list of variables appearing in *expr*, after removing constants (such as **%e**, **%pi**, etc.) and variables listed in **symbols**. Dummy variables (such as **%r1**, etc.) are included.



If *listvars* contains *listconstvars*, then constants are included.

```
(%i9) declare( $\pi$ , constant) $
(%i10) f(x,y) := for i from 1 thru 10 do x+i* $\pi$ *y $
(%i11) wg_listofvars(%i10);
(%o11) [i, x, y]
(%i12) wg_listofvars(%i10, 'listconstvars);
(%o12) [ $\pi$ , i, x, y]
```

**wg\_funargs** (*expr*) [Function]

If *expr* is an atom, apply **wg\_funargs** to the lefthand side of the function definition of *expr*; otherwise, return the result of **wg\_listofvars**(*expr*, 'listconstvars).

```
(%i13) wg_funargs(wg_listofvars);
(%o13) [$expr, $listvars]
(%i14) wg_funargs(lhs(%i10));
(%o14) [x, y]
```

**wg\_lambda** (*vars*, [*body*]) [Macro]

Create a gensymized lambda function.

```
(%i1) load(with_gensyms) $
(%i2) wg_lambda([x,y], if a=1 then x+1 else if a=2 then y+2 else a);
(%o2) lambda([$x, $y], if a = 1 then 1 + $x
                     else (if a = 2 then 2 + $y else a))
(%i3) apply(%o2, ' [u,v]);
(%o3) a
(%i4) apply(%o2, ' [u,v]), a=1;
(%o4) u + 1
(%i5) apply(%o2, ' [u,v]), a=2;
(%o5) v + 2
```

See [with\_gensyms], page 3.

**wg\_block** (*bindings*, [*body*]) [Macro]

Create a gensymized block. An alias for **with\_gensyms**.

```
(%i6) macroexpand(
      wg_block([x:b,y:4],
        if a=1 then x+1 else if a=2 then y+2 else a));
(%o6) block([$x : b, $y : 4],
  if a = 1 then $x + 1 else (if a = 2 then $y + 2 else a))
(%i7) macroexpand(
      with_gensyms([x:b,y:4],
        if a=1 then x+1 else if a=2 then y+2 else a));
(%o7) block([$x : b, $y : 4],
  if a = 1 then $x + 1 else (if a = 2 then $y + 2 else a))
```

**:** [Macro]

Function definition operator. See Section 1.2 [Examples], page 1.

## 1.6 Acknowledgements

Thanks to Robert Dodier. His code for **blex**, written in Lisp, inspired the composition of **with\_gensyms** in Maxima and its extension to this package.

## Appendix A Code Listings

### A.1 SYMBOLS.LISP

```

;;/* -*- Mode: lisp -*- */
;;
;; $Id:$
;;
;; Author: Leo Butler (l_butler@users.sourceforge.net)
;;
;; This file is Maxima/Lisp code (http://maxima.sourceforge.net/)
;;
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;;
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;; along with this file. If not, see http://www.gnu.org/licenses/.
;;
;; Time-stamp: <2016-07-15 11:41:17>

(in-package :maxima)

(defun maxima-symbol-p (x)
  (let ((n (symbol-name x)))
    (or (and (> (length n) 2)
            (or (char= (char n 0) #\$) (char= (char n 0) #\%))) ;; single character symbols like $A should be
            (cdr (mfuncall 'properties x))))) ;; only symbols beginning with $ or %
    ;; any symbol with a non-trivial property li

(declare (special $symbols))

(defun $add_maxima_symbol (x &optional (s $symbols))
  (assert (or (symbolp x) ($listp x)))
  (cond ((symbolp x)
        (setf (gethash x s) t))
        (($listp x)
        (dolist (e (cdr x))
          ($add_maxima_symbol e s)))
        (t nil))
  '$done)

(defmvar $symbols
  (let ((s (make-hash-table :test #'eq)))
    (do-symbols (x)
      (if (maxima-symbol-p x) ($add_maxima_symbol x s)))
    s))

(defun $maxima_symbolp (x)
  (or (gethash x $symbols nil)
      (member x (mapcar #'caar (cdr $functions)) :test #'eq)))

```

```

(defun $remove_maxima_symbols (x)
  (assert (listp x))
  (remove-if #'$maxima_symbolp x))

(defun $delete_maxima_symbols (x)
  "Deletes 'x' from the hash-table of symbols '$symbols'. The input
  may be a symbol or mlist of symbols."
  (assert (or ($listp x) (symbolp x)))
  (cond ((symbolp x)
        (remhash x $symbols))
        (($listp x)
         (dolist (e (cdr x)) (remhash e $symbols)))
        (t ;; never get here
         nil))
  '$done)

(defun $maxima_symbols ()
  (let (s)
    (maphash (lambda (k v)
              (declare (ignore v))
              (push k s)) $symbols)
    (cons '(mlist simp) s)))

(defun $wg_gensymize (x)
  (declare (special $wg_reversealias))
  (assert (symbolp x))
  ;; make function idempotent by returning a symbol produced by $wg_gensymize
  (cond ((mfuncall '$featurep x '$gensym)
        x)
        (t
         (let ((w (gensym (format nil "%~a_" (stripdollar x)))))
           (setf (get w 'reversealias) (if $wg_reversealias (make-symbol (format nil "%~a" x)) x))
           (mfuncall '$declare w '$gensym
                     w))))))

;; Local Variables:
;; time-stamp-format:  "%:y-%02m-%02d %02H:%02M:%02S"
;; End:
;;/* end of symbols.lisp */

```

## A.2 WITH\_GENSYMS.MAC

```

/* -*- Mode: maxima; Package: MAXIMA -*- */
/*
 * $Id:$
 *
 * Author: Leo Butler (l_butler@users.sourceforge.net)
 *
 * This file is Maxima code (http://maxima.sourceforge.net/)
 *
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 * the Free Software Foundation; either version 3 of the License, or (at your
 * option) any later version.
 *
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```

```

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* License for more details.
*
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* along with this file. If not, see http://www.gnu.org/licenses/.
*/

define_constant('const,val):=(
  buildq([const:const,val:val],
    block([err],
      err:lambda([y],error('const," is a constant.")),
      if apply(constantp,['const])
      then apply(err,[const]),
      define_variable(const,val,any_check),
      qput(const,err,value_check),
      declare(const,constant))),ev(%%),const)$

if symbolp(wg_version) then (
define_constant(wg_version, "0.1.0"),
define_constant(wg_last_updated, substring("Time-stamp:  <2016-07-14 23:16:11>",14,14+10)))$

wg_reversealias : true $

load("symbols.lisp");
declare(gensym,feature) $

wg_make_binding(undef) := buildq([undef:undef],lambda([x,y],if is(y=undef) then rhs(x) else funmake(":",[rhs(x)])),
wg_check_op(oper,expr) := if atom(expr) or not(is(op(expr)=oper)) then error("~a: expected operand ~a.",expr,oper),
wg_atom_or_quote(expr) := is(atom(expr) or op(expr)="'");

(
  with_gensyms(bindings,[body]) ::= block([vals,undef,simp:false],
    vals:map(lambda([x],if wg_atom_or_quote(x) then undef else rhs(wg_check_op(":",x))),bindings),
    bindings:map(lambda([x],if atom(x) then x else lhs(x)),bindings),
    bindings:map(lambda([x],funmake("=", [x,wg_gensymize(x)])),bindings),
    buildq([body:psubst(bindings,body),bindings:bindings,v:map(wg_make_binding(undef),bindings,vals)],
      block(v,splice(body))))),
/* bootstrap with_gensyms by using it to define itself! */
apply(with_gensyms,['[bindings,body,vals,undef,x,v],%]))$

wg_listofvars(expr,[listvars]) := block([v,listconstvars:false,listdummyvars:true],
  if member('listconstvars,listvars) then listconstvars:true,
  v:unique(listofvars(expr)),
  for r in v do if not(atom(r)) then v:delete(r,v),
  v:remove_maxima_symbols(v),
  v)$
wg_funargs(expr) := if atom(expr) then (errcatch(wg_funargs(first(apply('fundef,[expr])))), if %%=[] then [] else %)
/* substitutes for := and ::= */

kill(":>","::>") $
infix(":>",180,20) $
infix("::>",180,20) $
(x ::> y) ::= block([simp:false], buildq([v:unique(append(wg_listofvars(x),wg_funargs(x),wg_listofvars(y))),x:x],
(x :> y) ::> block([simp:false], buildq([v:unique(append(wg_listofvars(x),wg_funargs(x),wg_listofvars(y))),x:x],
(

```

```

wg_redefun(fun,[vars]) := block([fd],
  fd:if atom(fun) then apply('fundef,[fun]) else if member(op(fun),[":=", "::="]) then fun else error("argument
  if vars=[] then vars:wg_listofvars(fd),
  apply('with_gensyms,[flatten(append(args(lhs(fd)),vars)),fd])),
/* bootstrap wg_redefun by using it to define itself! */
wg_redefun('wg_redefun,'fd));
map(wg_redefun,'[wg_make_binding,wg_check_op,wg_atom_or_quote,"::>",wg_listofvars,wg_funargs]);

wg_lambda(vars,[body]) ::> buildq([vars:vars,body:body], with_gensyms(vars,lambda(vars,splice(body)))) $
alias(wg_block,with_gensyms) $

/*
Local Variables:
time-stamp-format:  "%:y-%02m-%02d %02H:%02M:%02S"
End:
*/
/* end of with_gensyms.mac */
1;

```

### A.3 RTEST\_WITH\_GENSYMS.MAC

```

/* -*- Mode: maxima; Package: MAXIMA -*- */
/*
* $Id:$
*
* Author:  Leo Butler (l_butler@users.sourceforge.net)
*
* This file is Maxima code (http://maxima.sourceforge.net/)
*
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*
* Time-stamp:  <2016-07-15 11:40:07>
*/

(load("with_gensyms.mac"), 'done) $
done $

maxima_symbolp(a);
false $

maxima_symbolp(b);
false $

maxima_symbolp(n);
false $

```

```

maxima_symbolp(true);
true $

maxima_symbolp(false);
true $

maxima_symbolp(lambda);
true $

maxima_symbolp(block);
true $

maxima_symbolp(?great);
true $

maxima_symbolp(?x);
false $

with_gensyms([x:1,y:'b],
  x+y);
1+'b $

with_gensyms([c:makelist(i,i,1,4)],
  length(c));
4 $

with_gensyms([a:'a],a);
a $

with_gensyms([a,b,c],
  a:1, b:2,
  with_gensyms([a:a,b:b,c],
    a+b));
3 $

(wg_redefun(g(x) := x), g(x));
'x$

(f(x) := 0*x, simp:false, atom(last(fundef(f))));
false;

simp:true;
true;

kill(myf,g,f);
done $

/* check idempotency */
block([g,h,partition_bag],
  local(partition_bag),
  g : partition_bag(L,p) :=> block([t:[],f:[]], if listp(L) then (map(lambda([x], apply('push,[x, if p(x)=true
  h : subst([":=">"],g), h : ev(h,nouns),
  is(h = g));
true $

length(wg_listofvars(fundef(wg_listofvars)));
4;

```

```

wg_listofvars([a,b,c,%pi,block,x]);
[a,b,c,x];

length(wg_funargs(wg_listofvars));
2;

length(wg_funargs(first(fundef(wg_listofvars))));
2;

/* declared constants are discarded by wg_listofvars, but included by wg_funargs
   system constants like %pi are discarded because they are in $symbols */
block([a,b,f], local(a,f), declare(a,constant),
      [wg_listofvars([a,b,f,%e]), wg_funargs(f(a,b,%pi))]);
[[b,f],[a,b]];

map(maxima_symbolp, [%e,quit,constant,maxima_symbolp]);
[true,true,true,false];

(add_maxima_symbol(maxima_symbolp), map(maxima_symbolp, [%e,quit,constant,maxima_symbolp]));
[true,true,true,true];

remove_maxima_symbols([%e,quit,constant,maxima_symbolp]);
[];

(delete_maxima_symbols(maxima_symbolp), map(maxima_symbolp, [%e,quit,constant,maxima_symbolp]));
[true,true,true,false];

remove_maxima_symbols([%e,quit,constant,maxima_symbolp]);
[maxima_symbolp];

/*
Local Variables:
time-stamp-format:  "%:y-%02m-%02d %02H:%02M:%02S"
End:
*/

/* end of rtest_with_gensyms.mac */

```

## Appendix B Function and Variable index

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