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With_Gensyms Package

With_Gensyms Package

A Maxima Package
for version 0.1.0, 2016-10-05

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This manual documents the `with_gensyms` package (version 0.1.0, 2016-10-05), a user-level package for the Maxima computer algebra system.

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Table of Contents

1	WITH_GENSYMS	1
1.1	Introduction to WITH_GENSYMS	1
1.2	Examples	1
1.3	Explanation	2
1.4	Definitions for SYMBOLS.LISP	2
1.5	Definitions for WITH_GENSYMS	3
1.6	Acknowledgements	4
Appendix A	Code Listings	5
A.1	SYMBOLS.LISP	5
A.2	WITH_GENSYMS.MAC	6
A.3	RTEST_WITH_GENSYMS.MAC	8
A.4	Regression Tests	11
Appendix B	Function and Variable index.....	17
Appendix C	GNU Free Documentation License.....	18

(%06) 2

1.3 Explanation

A simple cure to fix the problem encountered with `f` is to give the parameter `x` a more unique name. Something like

```
f(my_local_variable_x_121423412,n) := my_local_variable_x_121423412[n];
```

There are two obvious flaws here: 1. the code is almost unreadable; and 2. there is nothing to prevent someone else (you, at another time) from choosing the same variable name.

The `WITH_GENSYMS` package overcomes both problems by having Maxima automatically rewrite the code using gensyms, which are symbols (roughly, variable names) which are guaranteed to have no name clashes.

1.4 Definitions for SYMBOLS.LISP

`maxima-symbol-p (x)` [System Function]

Returns `true` if the symbol-name of `x` has more than 2 characters or `x` has a non-empty property list. Used to populate *symbols* when package is initially loaded.

`symbols` [Variable]

A Lisp hashtable used to look-up Maxima symbols. This is initialized when `symbols.lisp` is loaded, with the symbol table existing at that time. A user may add to this table. See `[add_maxima_symbol]`, page 2.

`add_maxima_symbol (x,[s])` [Function]

The default value of the hashtable `s` is *symbols*. If `x` is a symbol, add it to `s`; otherwise, if `x` is a list, iterate over it.

`maxima_symbolp (x)` [Function]

Return `true` if `x` is a key in *symbols* or if `x` is a function name listed in *functions*.

`remove_maxima_symbols (x)` [Function]

Iterate over the list `x` and remove each entry that satisfies `maxima_symbolp`.

`delete_maxima_symbols (x)` [Function]

If `x` is a symbol, remove it from *symbols*. If `x` is a Maxima list, map over it.

`maxima_symbols ()` [Function]

Creates a Maxima list of the current keys in *symbols*.

`wg_gensymize (x)` [Function]

Returns a gensymized version of the symbol held by `x`, with the Maxima property `gensym`. By default, the printed representation of the gensym is not valid Maxima syntax.

```
(%i1) load(with_gensyms)$
(%i2) wg_gensymize(x);
(%o2)          $x
(%i3) apply('properties,[%]);
(%o3)          [database info, kind($x, gensym)]
If x already has the gensym property, return x.
(%i4) wg_gensymize(%o2);
(%o4)          $x
(%i5) is(% = %o2);
(%o5)          true
```

The display property of the returned gensym is governed by the variable `wg_reversealias`.

```
(%i6) wg_gensymize(y), wg_reversealias:false;
```

```
(%o6)                                y
(%i7) lisp_print(%);
(%o7) #:Y_631
```

1.5 Definitions for WITH_GENSYMS

wg_reversealias [User Option]

See the entry for [wg-gensymize], page 2.

gensym [Feature]

A property of a symbol. Used by **wg-gensymize** to ensure idempotency of that function. See the entry for [wg-gensymize], page 2.

wg_make_binding (undef) [Function]

Creates a function of two variables, *x* and *y*, where *x* is an assignment *a*:*b*. If *y* = **undef**, then return *b*; otherwise return the binding *b*:*y*.

```
(%i8) map(wg_make_binding(undef), '[a:b,a:b], [undef,3]);
(%o8) [b, b : 3]
```

wg_check_op (oper,expr) [Function]

If *expr* is an atom or the operand of *expr* is not *oper*, signal an error; otherwise return *expr*.

wg_atom_or_quote (expr) [Function]

If *expr* is an atom or the operand is **'**, return **true**; else return **false**.

with_gensyms (bindings, [body]) [Macro]

Replace the variables in *bindings* with gensyms, and substitute these into *body*.

```
(%i1) load(with_gensyms)$
(%i2) with_gensyms([x:1, y:2],
                  ['[x,y], [x,y]]);
(%o2) [[x, y], [1, 2]]
(%i3) with_gensyms([x,n], f(x) := a+x[n]);
(%o3) f(x) := a + x
      $n
```

Here is an implementation of a **lambda** function using **with_gensyms**:

```
(%i4) wg_lambda1(vars,[body]) ::> buildq([vars:vars,body:body],
    with_gensyms(
        vars,
        lambda(vars,
            splice(body)))) $
(%i5) wg_lambda1([x,y], if a=1 then x+1 else if a=2 then y+2 else a);
(%o5) lambda([x, y], if a = 1 then 1 + x
    else (if a = 2 then 2 + y else a))
(%i6) apply(%o5, '[u,v]);
(%o6) a
(%i7) apply(%o5, '[u,v]), a=1;
(%o7) u + 1
(%i8) apply(%o5, '[u,v]), a=2;
(%o8) v + 2
```

wg_listofvars (expr,[listvars]) [Function]

Extract the list of variables appearing in *expr*, after removing constants (such as **%e**, **%pi**, etc.) and variables listed in **symbols**. Dummy variables (such as **%r1**, etc.) are included.

If *listvars* contains *listconstvars*, then constants are included.

```
(%i9) declare( $\pi$ , constant) $
(%i10) f(x,y) := for i from 1 thru 10 do x+i* $\pi$ *y $
(%i11) wg_listofvars(%i10);
(%o11) [i, x, y]
(%i12) wg_listofvars(%i10, 'listconstvars);
(%o12) [ $\pi$ , i, x, y]
```

wg_funargs (*expr*) [Function]

If *expr* is an atom, apply **wg_funargs** to the lefthand side of the function definition of *expr*; otherwise, return the result of **wg_listofvars**(*expr*, 'listconstvars).

```
(%i13) wg_funargs(wg_listofvars);
(%o13) [$expr, $listvars]
(%i14) wg_funargs(lhs(%i10));
(%o14) [x, y]
```

wg_lambda (*vars*, [*body*]) [Macro]

Create a gensymized lambda function.

```
(%i1) load(with_gensyms) $
(%i2) wg_lambda([x,y], if a=1 then x+1 else if a=2 then y+2 else a);
(%o2) lambda([$x, $y], if a = 1 then 1 + $x
                      else (if a = 2 then 2 + $y else a))
(%i3) apply(%o2, '[u,v]);
(%o3) a
(%i4) apply(%o2, '[u,v]), a=1;
(%o4) u + 1
(%i5) apply(%o2, '[u,v]), a=2;
(%o5) v + 2
```

See [with_gensyms], page 3.

wg_block (*bindings*, [*body*]) [Macro]

Create a gensymized block. An alias for **with_gensyms**.

```
(%i6) macroexpand(
      wg_block([x:b,y:4],
      if a=1 then x+1 else if a=2 then y+2 else a));
(%o6) block([$x : b, $y : 4],
      if a = 1 then $x + 1 else (if a = 2 then $y + 2 else a))
(%i7) macroexpand(
      with_gensyms([x:b,y:4],
      if a=1 then x+1 else if a=2 then y+2 else a));
(%o7) block([$x : b, $y : 4],
      if a = 1 then $x + 1 else (if a = 2 then $y + 2 else a))
```

: [Macro]

Function definition operator. See Section 1.2 [Examples], page 1.

1.6 Acknowledgements

Thanks to Robert Dodier. His code for **blex**, written in Lisp, inspired the composition of **with_gensyms** in Maxima and its extension to this package.

Appendix A Code Listings

A.1 SYMBOLS.LISP

```

;;/* -*- Mode: lisp -*- */
;;
;; $Id:$
;;
;; Author: Leo Butler (l_butler@users.sourceforge.net)
;;
;; This file is Maxima/Lisp code (http://maxima.sourceforge.net/)
;;
;; It is free software; you can redistribute it and/or modify
;; it under the terms of the GNU General Public License as published by
;; the Free Software Foundation; either version 3 of the License, or (at your
;; option) any later version.
;;
;; This software is distributed in the hope that it will be useful, but
;; WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY
;; or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public
;; License for more details.
;;
;; You should have received a copy of the GNU General Public License
;; along with this file. If not, see http://www.gnu.org/licenses/.
;;
;; Time-stamp: <2016-09-29 09:15:56>

(in-package :maxima)

(defun maxima-symbol-p (x)
  (let ((n (symbol-name x)))
    (or (and (> (length n) 2)
            (or (char= (char n 0) #\$) (char= (char n 0) #\%))) ;; single character symbols like $A should be
            (cdr (mfuncall '$properties x)))) ;; only symbols beginning with $ or %
    ;; any symbol with a non-trivial property li

(declare (special $symbols))
(defmvar $wg_reversealias t)

(defun $add_maxima_symbol (x &optional (s $symbols))
  (assert (or (symbolp x) ($listp x)))
  (cond ((symbolp x)
        (setf (gethash x s) t))
        (($listp x)
        (dolist (e (cdr x))
          ($add_maxima_symbol e s)))
        (t nil))
  '$done)

(defmvar $symbols
  (let ((s (make-hash-table :test #'eq)))
    (do-symbols (x)
      (if (maxima-symbol-p x) ($add_maxima_symbol x s)))
    s))

(defun $maxima_symbolp (x)
  (or (gethash x $symbols nil)

```

```

(member x (mapcar #'caar (cdr $functions)) :test #'eq)))

(defun $remove_maxima_symbols (x)
  (assert (listp x))
  (remove-if #'$maxima_symbolp x))

(defun $delete_maxima_symbols (x)
  "Deletes 'x' from the hash-table of symbols '$symbols'. The input
may be a symbol or mlist of symbols."
  (assert (or ($listp x) (symbolp x)))
  (cond ((symbolp x)
         (remhash x $symbols))
        (($listp x)
         (dolist (e (cdr x)) (remhash e $symbols)))
        (t ;; never get here
         nil))
  '$done)

(defun $maxima_symbols ()
  (let (s)
    (maphash (lambda (k v)
                (declare (ignore v))
                (push k s)) $symbols)
    (cons '(mlist simp) s)))

(defun $wg_gensymize (x)
  (declare (special $wg_reversealias))
  (assert (symbolp x))
  ;; make function idempotent by returning a symbol produced by $wg_gensymize
  (cond ((mfuncall '$featurep x '$gensym)
         x)
        (t
         (let ((w (gensym (format nil "%~a_" (stripdollar x)))))
           (setf (get w 'reversealias) (if $wg_reversealias (make-symbol (format nil "%~a" x)) x))
           (mfuncall '$declare w '$gensym
                     w))))))

;; Local Variables:
;; time-stamp-format:  "%:y-%02m-%02d %02H:%02M:%02S"
;; End:
;;/* end of symbols.lisp */

```

A.2 WITH_GENSYMS.MAC

```

/* -*- Mode:  maxima; Package:  MAXIMA -*- */
/*
 * $Id:$
 *
 * Author:  Leo Butler (l_butler@users.sourceforge.net)
 *
 * This file is Maxima code (http://maxima.sourceforge.net/)
 *
 * It is free software; you can redistribute it and/or modify
 * it under the terms of the GNU General Public License as published by
 * the Free Software Foundation; either version 3 of the License, or (at your
 * option) any later version.
 *
 * This software is distributed in the hope that it will be useful, but

```

```

* WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY
* or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public
* License for more details.
*
* You should have received a copy of the GNU General Public License
* along with this file. If not, see http://www.gnu.org/licenses/.
*/

define_constant('const,val):=(
  buildq([const:const,val:val],
    block([err],
      err:lambda([y],error('const," is a constant.")),
      if apply(constantp,['const])
      then apply(err,[const]),
      define_variable(const,val,any_check),
      qput(const,err,value_check),
      declare(const,constant))) ,ev(%%),const)$
if symbolp(wg_version)
then (define_constant(wg_version,"0.1.0"),
  define_constant(wg_last_updated,
    substring("Time-stamp: <2016-10-05 12:11:46>",14,
      14+10)))$

load("symbols.lisp");
declare(gensym,feature) $

wg_make_binding(undef):=buildq([undef:undef],
  lambda([x,y],
    if is(y = undef) then rhs(x)
    else funmake(":",[rhs(x),y])))$
wg_check_op(oper,expr):=if atom(expr) or not is(op(expr) = oper)
then error("~a: expected operand ~a.",expr,oper) else expr$
wg_atom_or_quote(expr):=is(atom(expr) or op(expr) = "'")$

(with_gensyms(bindings,[body]):=block([vals,undef,simp:false],
  vals:map(lambda([x],
    if wg_atom_or_quote(x) then undef
    else rhs(wg_check_op(":",x))),bindings),
  bindings:map(lambda([x],if atom(x) then x else lhs(x)),bindings),
  bindings:map(lambda([x],funmake("=", [x,wg_gensymize(x)])),bindings),
  buildq([body:psubst(bindings,body),bindings:bindings,
    v:map(wg_make_binding(undef),bindings,vals)],
    block(v,splice(body))))
/* bootstrap with_gensyms by using it to define itself! */
apply(with_gensyms,['([bindings,body,vals,undef,x,v]),%%]))$

wg_listofvars(expr,[listvars]):=block(
  [v,listconstvars:false,listdummyvars:true],
  if member('listconstvars,listvars) then listconstvars:true,
  v:unique(listofvars(expr)),
  for r in v do if not atom(r) then v:delete(r,v),
  v:remove_maxima_symbols(v),v)$

wg_funargs(expr):=if atom(expr)
then (errcatch(wg_funargs(first(apply('fundef,[expr])))),
  if %% = [] then [] else first(%))
else wg_listofvars(expr,'listconstvars)$

```

```

/* substitutes for := and ::= */

kill(":>","::>") $
infix(":>",180,20) $
infix("::>",180,20) $
(x ::> y)::=block([simp:false],
  buildq([v:unique(append(wg_listofvars(x),wg_funargs(x),wg_listofvars(y))),
    x:x,y:y],with_gensyms(v,x::=y)))$
(x :> y) ::> block([simp:false],
  buildq([v:unique(append(wg_listofvars(x),wg_funargs(x),
    wg_listofvars(y))),x:x,y:y],
    with_gensyms(v,x:=y)))$

(wg_redefun(fun,[vars]):=block([fd],
  fd:if atom(fun) then apply('fundef,[fun])
  else (if member(op(fun),[":=","::="]) then fun
  else error(
    "argument must be function name or definition")),
  if vars = [] then vars:wg_listofvars(fd),
  apply('with_gensyms,[flatten(append(args(lhs(fd)),vars)),fd])),
/* bootstrap wg_redefun by using it to define itself! */
wg_redefun('wg_redefun,'fd))$
map(wg_redefun,
  '([wg_make_binding,wg_check_op,wg_atom_or_quote,":>",wg_listofvars,
    wg_funargs]))$

wg_lambda(vars,[body]) ::> buildq([vars:vars,body:body],
  with_gensyms(vars,lambda(vars,splice(body))))$

alias(wg_block,with_gensyms) $

/*
Local Variables:
time-stamp-format:  "%:y-%02m-%02d %02H:%02M:%02S"
End:
*/
/* end of with_gensyms.mac */
1;

```

A.3 RTEST_WITH_GENSYMS.MAC

```

/* -*- Mode: maxima; Package: MAXIMA -*- */
/*
* $Id:$
*
* Author:  Leo Butler (l_butler@users.sourceforge.net)
*
* This file is Maxima code (http://maxima.sourceforge.net/)
*
* It is free software; you can redistribute it and/or modify
* it under the terms of the GNU General Public License as published by
* the Free Software Foundation; either version 3 of the License, or (at your
* option) any later version.
*
* This software is distributed in the hope that it will be useful, but
* WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY
* or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public

```

```

* License for more details.
*
* You should have received a copy of the GNU General Public License
* along with this file.  If not, see http://www.gnu.org/licenses/.
*
* Time-stamp:  <2016-07-15 11:40:07>
*/

(load("with_gensyms.mac"), 'done) $
done $

maxima_symbolp(a);
false $

maxima_symbolp(b);
false $

maxima_symbolp(n);
false $

maxima_symbolp(true);
true $

maxima_symbolp(false);
true $

maxima_symbolp(lambda);
true $

maxima_symbolp(block);
true $

maxima_symbolp(?great);
true $

maxima_symbolp(?x);
false $

with_gensyms([x:1,y:'b],
  x+y);
1+'b $

with_gensyms([c:makelist(i,i,1,4)],
  length(c));
4 $

with_gensyms([a:'a],a);
a $

with_gensyms([a,b,c],
  a:1, b:2,
  with_gensyms([a:a,b:b,c],
    a+b));
3 $

(wg_redefun(g(x) := x), g(x));
'x$

```

```

(f(x) :> 0*x, simp:false, atom(last(fundef(f))));
false;

simp:true;
true;

kill(myf,g,f);
done $

/* check idempotency */
block([g,h,partition_bag],
  local(partition_bag),
  g : partition_bag(L,p) :> block([t:[],f:[]], if listp(L) then (map(lambda([x], apply('push,[x, if p(x)=true
  h : subst([":=">"],g), h : ev(h,nouns),
  is(h = g));
true $

length(wg_listofvars(fundef(wg_listofvars))));
4;

wg_listofvars([a,b,c,%pi,block,x]);
[a,b,c,x];

length(wg_funargs(wg_listofvars));
2;

length(wg_funargs(first(fundef(wg_listofvars))));
2;

/* declared constants are discarded by wg_listofvars, but included by wg_funargs
   system constants like %pi are discarded because they are in $symbols */
block([a,b,f], local(a,f), declare(a,constant),
  [wg_listofvars([a,b,f,%e]), wg_funargs(f(a,b,%pi))]);
[[b,f],[a,b]];

map(maxima_symbolp, [%e,quit,constant,maxima_symbolp]);
[true,true,true,false];

(add_maxima_symbol(maxima_symbolp), map(maxima_symbolp, [%e,quit,constant,maxima_symbolp]));
[true,true,true,true];

remove_maxima_symbols([%e,quit,constant,maxima_symbolp]);
[];

(delete_maxima_symbols(maxima_symbolp), map(maxima_symbolp, [%e,quit,constant,maxima_symbolp]));
[true,true,true,false];

remove_maxima_symbols([%e,quit,constant,maxima_symbolp]);
[maxima_symbolp];

/*
Local Variables:
time-stamp-format:  "%:y-%02m-%02d %02H:%02M:%02S"
End:
*/

/* end of rtest_with_gensyms.mac */

```

A.4 Regression Tests

```
batch: write error log to #<output stream rtest_with_gensyms.ERR>
***** Problem 1 (line 25) *****
```

```
Input:
(load(with_gensyms.mac), 'done')
```

```
Result:
done
```

```
... Which was correct.
```

```
***** Problem 2 (line 28) *****
```

```
Input:
maxima_symbolp(a)
```

```
Result:
false
```

```
... Which was correct.
```

```
***** Problem 3 (line 31) *****
```

```
Input:
maxima_symbolp(b)
```

```
Result:
false
```

```
... Which was correct.
```

```
***** Problem 4 (line 34) *****
```

```
Input:
maxima_symbolp(n)
```

```
Result:
false
```

```
... Which was correct.
```

```
***** Problem 5 (line 37) *****
```

```
Input:
maxima_symbolp(true)
```

```
Result:
true
```

```
... Which was correct.
```

```
***** Problem 6 (line 40) *****
```

```
Input:
maxima_symbolp(false)
```



```
Result:
true
```

```
... Which was correct.
```

```
***** Problem 7 (line 43) *****
```

```
Input:
maxima_symbolp(lambda)
```

```
Result:
true
```

```
... Which was correct.
```

```
***** Problem 8 (line 46) *****
```

```
Input:
maxima_symbolp(block)
```

```
Result:
true
```

```
... Which was correct.
```

```
***** Problem 9 (line 49) *****
```

```
Input:
maxima_symbolp(great)
```

```
Result:
true
```

```
... Which was correct.
```

```
***** Problem 10 (line 52) *****
```

```
Input:
maxima_symbolp(x)
```

```
Result:
false
```

```
... Which was correct.
```

```
***** Problem 11 (line 56) *****
```

```
Input:
wg_block([x : 1, y : 'b], x + y)
```

```
Result:
b + 1
```

```
... Which was correct.
```

```
***** Problem 12 (line 60) *****
```

```
Input:
wg_block([c : makelist(i, i, 1, 4)], length(c))
```

Result:

4

... Which was correct.

***** Problem 13 (line 63) *****

Input:

wg_block([a : 'a'], a)

Result:

a

... Which was correct.

***** Problem 14 (line 69) *****

Input:

wg_block([a, b, c], a : 1, b : 2, wg_block([a : a, b : b, c], a + b))

Result:

3

... Which was correct.

***** Problem 15 (line 72) *****

Input:

(wg_redefun(g(x) := x), g(x))

Result:

x

... Which was correct.

***** Problem 16 (line 75) *****

Input:

(f(x) :> 0 x, simp : false, atom(last(fundef(f))))

Result:

false

... Which was correct.

***** Problem 17 (line 78) *****

Input:

simp : true

Result:

true

... Which was correct.

***** Problem 18 (line 81) *****

```
Input:
kill(myf, g, f)
```

```
Result:
done
```

```
... Which was correct.
```

```
***** Problem 19 (line 89) *****
```

```
Input:
block([g, h, partition_bag], local(partition_bag),
g : partition_bag(L, p) :> block([t : [], f : []],
if listp(L) then (map(lambda([x], apply('push,
[x, if p(x) = true then 't else 'f])), L), map(reverse, [t, f]))
else (if not atom(L) then partition_bag(flatten(subst([matrix = [, { = [],
args(L))), p))), h : subst([:= = :>], g), h : ev(h, nouns), is(h = g))
```

```
Result:
true
```

```
... Which was correct.
```

```
***** Problem 20 (line 92) *****
```

```
Input:
length(wg_listofvars(fundef(wg_listofvars)))
```

```
Result:
4
```

```
... Which was correct.
```

```
***** Problem 21 (line 95) *****
```

```
Input:
wg_listofvars([a, b, c, %pi, block, x])
```

```
Result:
[a, b, c, x]
```

```
... Which was correct.
```

```
***** Problem 22 (line 98) *****
```

```
Input:
length(wg_funargs(wg_listofvars))
```

```
Result:
2
```

```
... Which was correct.
```

```
***** Problem 23 (line 101) *****
```

```
Input:
length(wg_funargs(first(fundef(wg_listofvars))))
```

... Which was correct.

***** Problem 29 (line 122) *****

Input:

```
remove_maxima_symbols([%e, quit, constant, maxima_symbolp])
```

Result:

```
[maxima_symbolp]
```

... Which was correct.

29/29 tests passed

Appendix B Function and Variable index

:		R	
:>	4	remove_maxima_symbols	2
A		S	
add_maxima_symbol	2	symbols	2
D		W	
delete_maxima_symbols	2	wg_atom_or_quote	3
G		wg_block	4
gensym	3	wg_check_op	3
M		wg_funargs	4
maxima-symbol-p	2	wg_gensymize	2
maxima_symbolp	2	wg_lambda	3, 4
maxima_symbols	2	wg_listofvars	3
G		wg_make_binding	3
		wg_reversealias	3
		with_gensyms	3
		S	
		symbols	2
		W	
gensym	3	wg_reversealias	3

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