Crypto Engineering 2025–2026

Security definitions & proof methods

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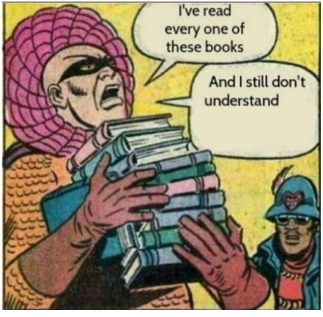
Some references



- Framework of this course:
  The Joy of Cryptography, Mike Rosulek
  `https://joyofcryptography.com/`
- *Introduction to Modern Cryptography*, Jonathan Katz & Yehuda Lindell
- *Foundation of Cryptography*, Oded Goldreich

Symmetric cryptography

With me:

- 5 CMs, 4 TDs (3h with computers)
- Symmetric cryptography, in particular:
  - Symmetric encryption & block ciphers
  - Authentication (MAC)
  - Hash functions & specificity of password hashing

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https://www.youtube.com/watch?v=a_HIHG5Nvpk
(slightly improved)

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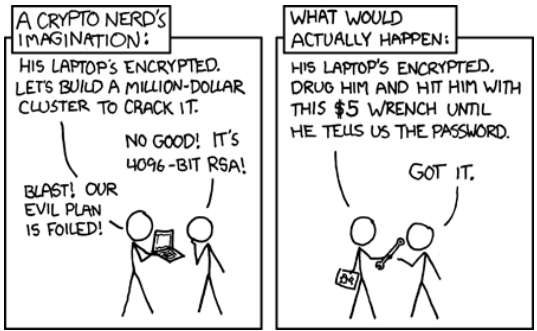
- Understand the **implications** of these models?



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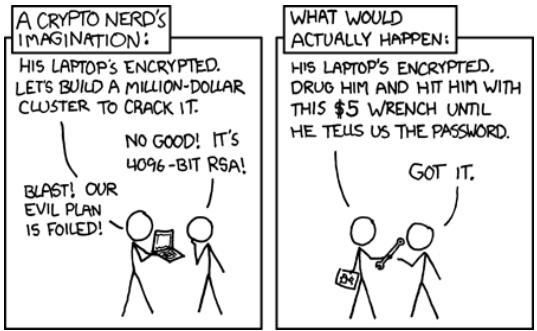
- How to **formally write security proofs**?
- Understand the **implications** of these models?
- Things you should **NEVER** do!!



Associated moodle course

https://moodle.caseine.org/course/view.php?id=1342

Notations

Notation	Meaning
$x \xleftarrow{\$} X$	x is obtained by sampling an element uniformly at random from the set X
$y \leftarrow A(x)$	If A is a (probabilistic) algorithm or a distribution, we run A on input x and store the result in x
$x \stackrel{?}{=} y$	Returns 1 (true) if x equals y, 0 (false) otherwise
$\mathrm{negl}(\lambda)$	An arbitrary function f that is negligible (= smaller than any inverse polynomial), i.e. $\forall c \in \mathbb{N}, \lim_{\lambda \to \infty} \lambda^c f(\lambda) = 0$
$\mathrm{poly}(\lambda)$	Any function f smaller than some polynomial, i.e. $\exists c \in N, N \in N, \forall \lambda > N, f(\lambda) \leq \lambda^c$

Which functions are negligible?

- A $f(\lambda) = \frac{1}{2^\lambda}$
- B $f(\lambda) = \frac{1}{\lambda^{1000}}$
- C $f(\lambda) = 2^{-\log \lambda}$

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NB: $\mathsf{negl}(\lambda) + \mathsf{negl}(\lambda) = \mathsf{negl}(\lambda)$, $\mathsf{negl}(\lambda) \times \mathsf{negl}(\lambda) = \mathsf{negl}(\lambda)$, $\mathsf{poly}(\lambda)\mathsf{negl}(\lambda) = \mathsf{negl}(\lambda)$

Symmetric vs asymmetric cryptography

Symmetric vs asymmetric cryptography

Symmetric encryption

Both parties share the same secret

$\neq$

Asymmetric encryption

One party has an extra secret information (**trapdoor** that can be used to invert a function easily)

Symmetric vs asymmetric cryptography

private key

Symmetric encryption

Both parties share the same secret

≠

public key

Asymmetric encryption

One party has an extra secret information (**trapdoor** that can be used to invert a function easily)

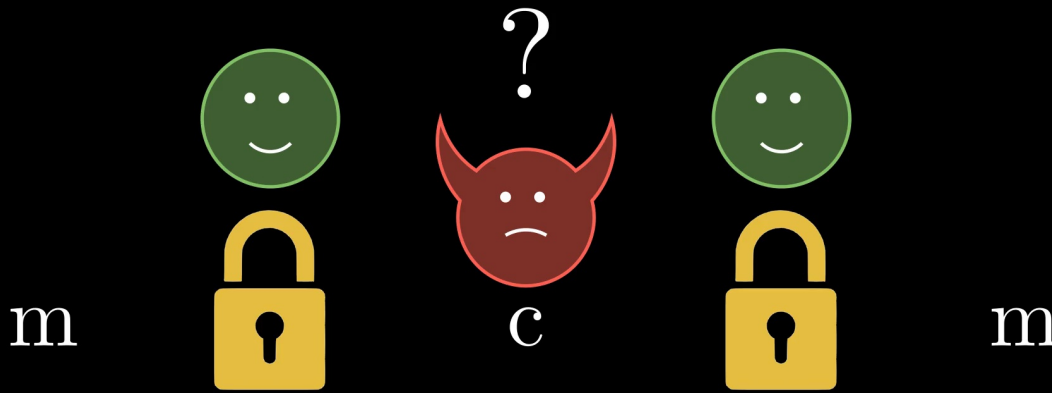


m

m

c


?
m
c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Symmetric vs asymmetric cryptography

Asymmetric encryption	Symmetric encryption
😄 No need to share secrets (e.g. internet)	😰 Need to share secrets
😰 Stronger assumptions… factoring, LWE… (functions highly structured)	😄 Weaker assumptions (less structure)
😰 Less efficient	😄 More efficient
😰 No statistical security	😄 Statistical security possible (but impractical)

⇒ Hybrid systems: **combine both** = best of both world (efficient + no secret to distribute)

Security models

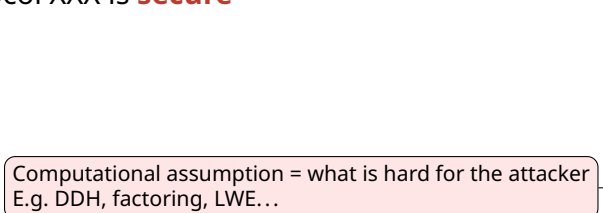
When designing a crypto system, we want to say:

“The protocol XXX is **secure**

Security models

When designing a crypto system, we want to say:

“The protocol XXX is **secure** assuming YYY is hard.



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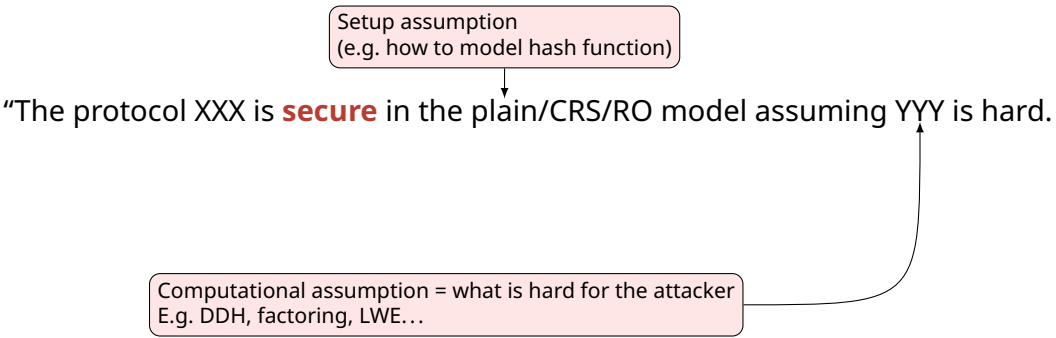

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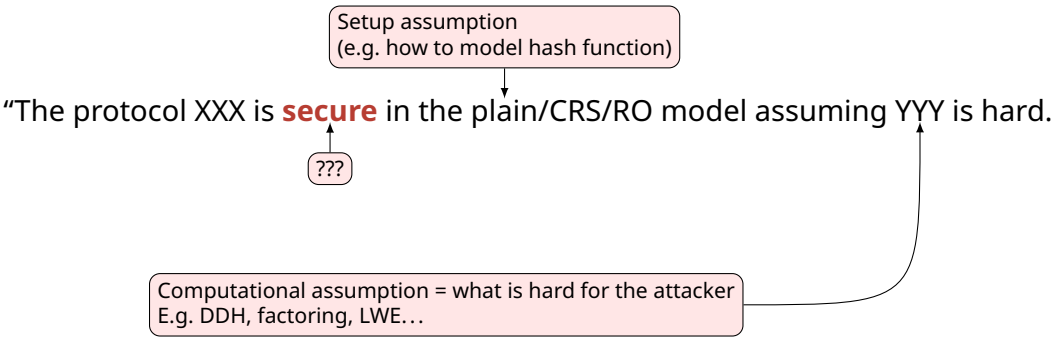

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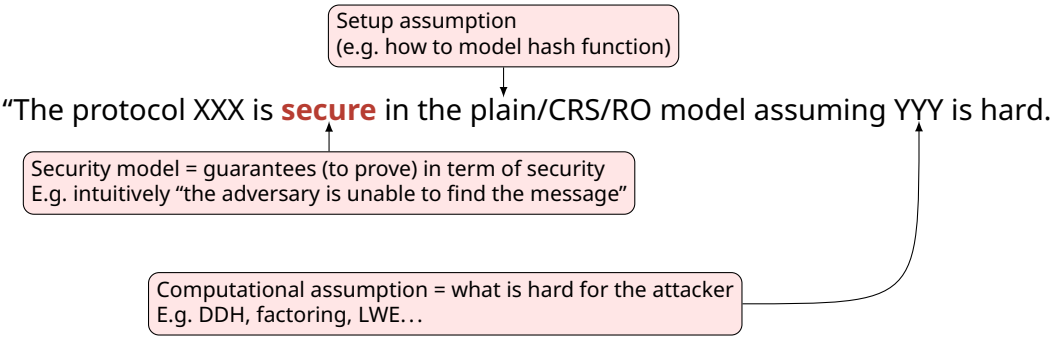
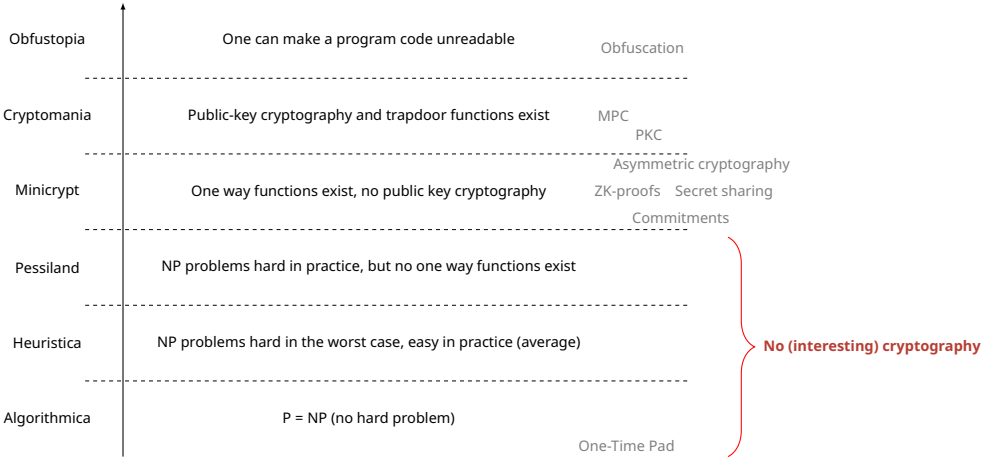


Hardness assumptions: Impagliazzo's worlds



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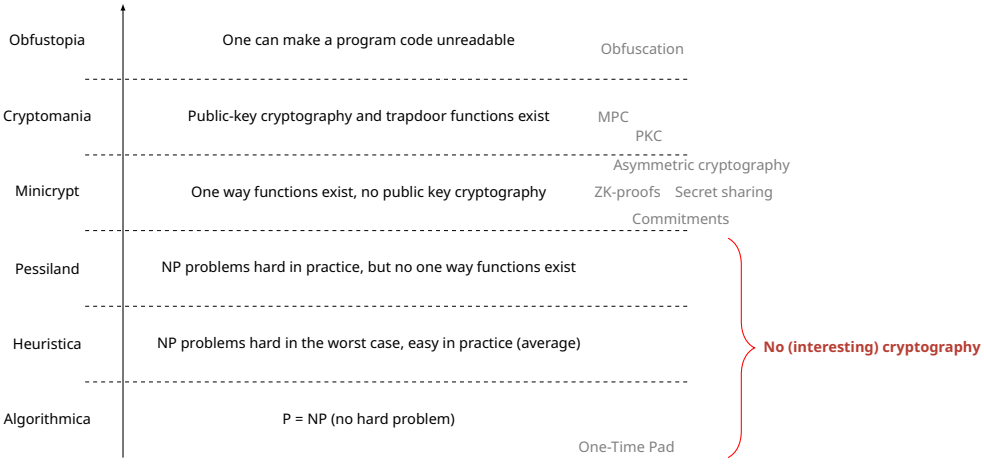

Big question (harder than $P = NP$): in which world are we?

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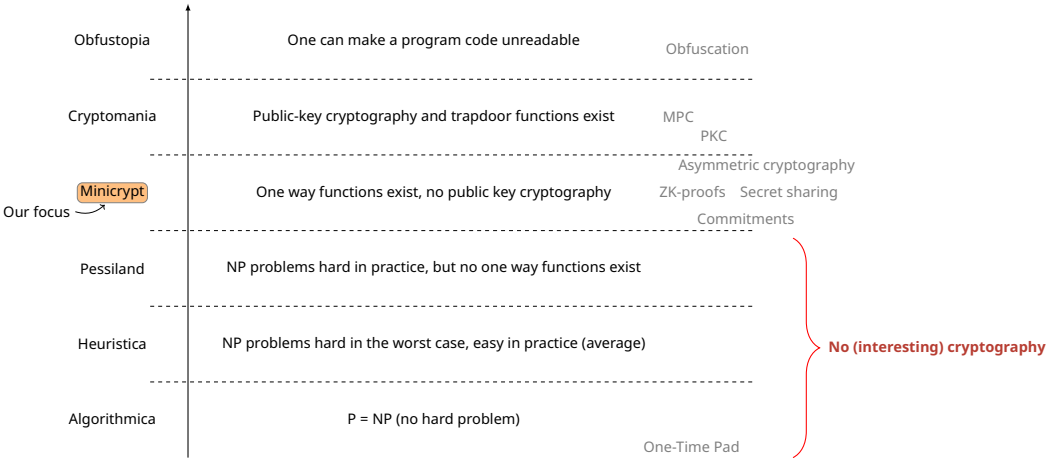
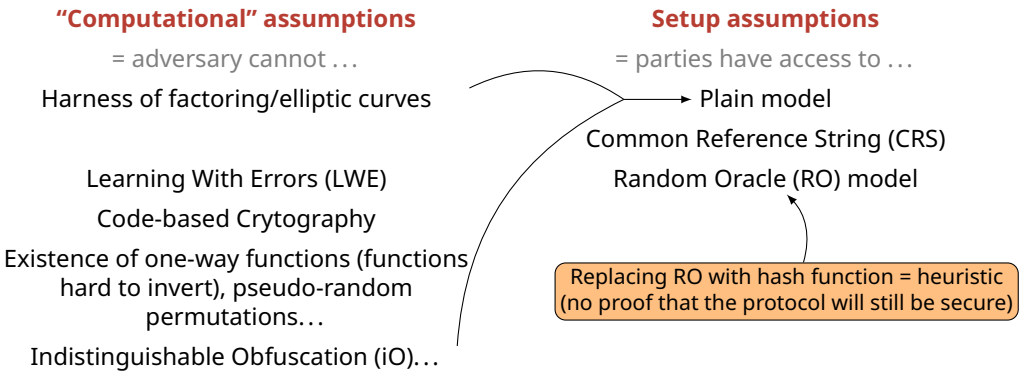


Big question (harder than $P = NP$): in which world are we?

No absolute security

Since we don't know in which world we are = **no unconditional security** (except One-Time Pad) $\Rightarrow$ always rely on some **assumptions**:



Important to **clearly state them** and understand their implications!

No absolute security

Since we don't know in which world we are = **no unconditional security** (except One-Time Pad) ⇒ always rely on some **assumptions**:

"Computational" assumptions

= adversary cannot …

Harness of factoring/elliptic curves

(broken against quantum computers)

Learning With Errors (LWE)

Code-based Crytography

Existence of one-way functions (functions hard to invert), pseudo-random permutations…

Indistinguishable Obfuscation (iO)…

Setup assumptions

= parties have access to …

Plain model

Common Reference String (CRS)

Random Oracle (RO) model

Replacing RO with hash function = heuristic (no proof that the protocol will still be secure)

Important to **clearly state them** and understand their implications!

Kerckhoff's principle

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The adversaries knows all details of the protocol (but cannot know directly the values sampled while running the protocol)

Security models

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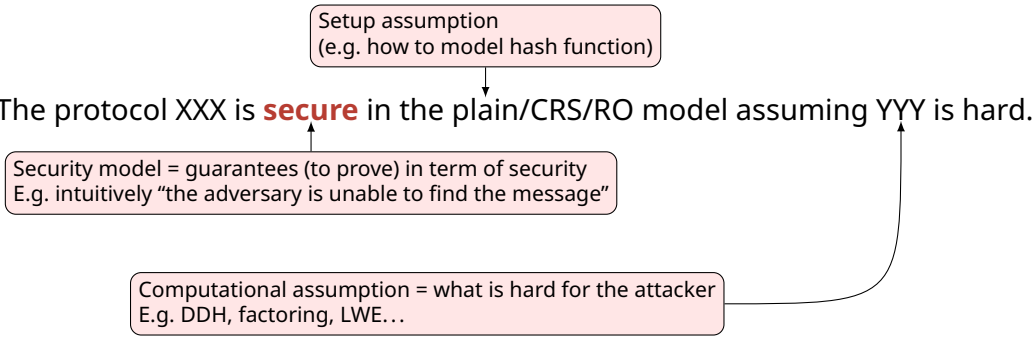

Security models

Easy to intuitively say what we expect, **hard to find a good security model** that captures all possible unwanted behaviors:

E.g. for encryption:

Attempt 1: “Given an encryption of m, an adversary should not be able to recover m”. Is this a good security definition? (if not, find a scenario where this could go wrong)

A Yes

B No

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A Yes 

B No  Recovering 3/4 of the message is already a big issue! E.g. $m =$ `"?????????????, hence we attack tomorrow"`

Security models

Attempt 2: “Given an encryption of m, an adversary should not be able to recover any bit of m”. Is this a good security definition? (if not, find a scenario where this could go wrong)

A Yes

B No

Security models

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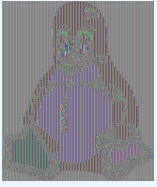
?

A Yes ✗

B No ✓ Knowing which groups of bits are different already leaks a lot:



NEVER DO THIS

AN ENCRYPTION MUST ALWAYS BE NON-DETERMINISTIC!!!

https://pieropan.ca/portfolio/a-ne-pas-faire-a-la-maison/

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AN ENCRYPTION MUST ALWAYS BE NON-DETERMINISTIC!!!

NEVER USE A HOME-MADE ENCRYPTION, IT WILL BE INSECURE!!!

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Better solution

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Better to say that it is **unable to distinguish** between $\mathrm{Enc}_k(m_0)$ and $\mathrm{Enc}_k(m_1)$ where m_0 and m_1 are chosen by the adversary.

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Better to say that it is **unable to distinguish** between $\mathrm{Enc}_k(m_0)$ and $\mathrm{Enc}_k(m_1)$ where m_0 and m_1 are chosen by the adversary.

Does this definition implies that an adversary can't learn any information XXX about m given $\mathrm{Enc}_k(m)$?

A. Yes ✓ Idea: pick m_0 and m_1 with different XXX value, learn XXX from $\mathrm{Enc}_k(m_b)$, deduce b

B. No

How to formalize this intuition?

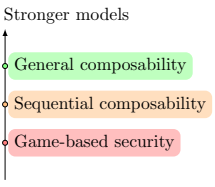
Formalism

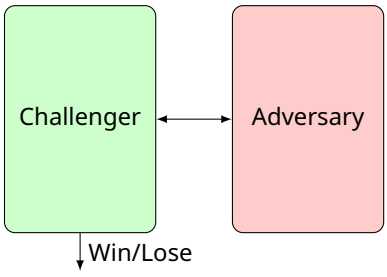
Security models

So how to define a secure protocol/encryption? ⇒ There is not one, but **multiple** definitions of security (with different guarantees)

3 **classes** of security models:

1: Game-based security = Fix a **challenger**:





Secure if for any adversary, **the probability of winning is "low"**
(might be $1/2 + \mathrm{negl}(\lambda)$ or $0 + \mathrm{negl}(\lambda)$ depending on the game)

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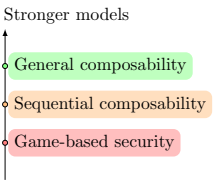

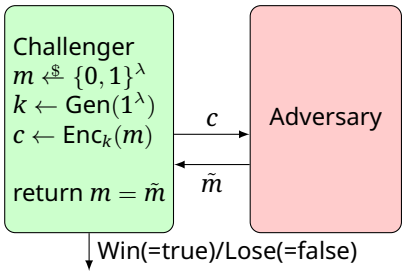

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Q: Is this challenger corresponding to the "don't learn m" (A) or "learn no bit about m" (B) security notion?

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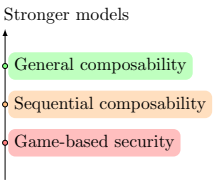

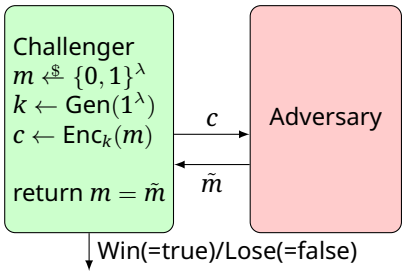

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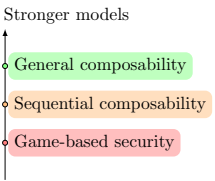
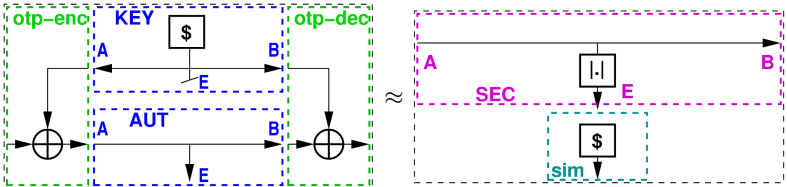


2 & 3: Composable frameworks = security based on a **simulator** that translates attacks on the real protocol to attacks on a **functionality** (trusted party) in an ideal world, supposed to be secure by definition:

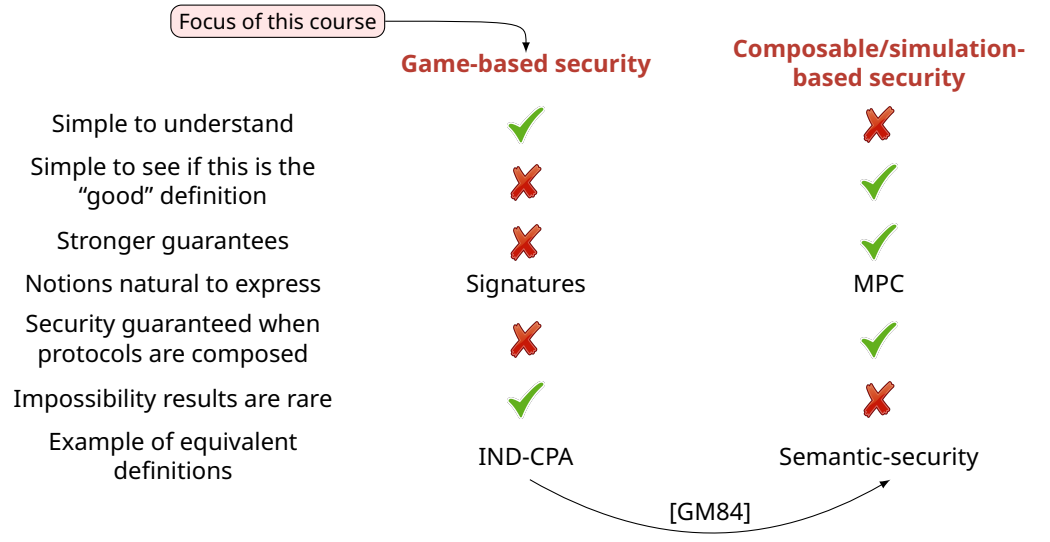


Main frameworks: standalone security (sequential), Universal Composability [Can10], Abstract Crytography [MR11,M12] (general)

Security frameworks: comparison

	Game-based security	Composable/simulation-based security
Simple to understand	✓	✗
Simple to see if this is the "good" definition	✗	✓
Stronger guarantees	✗	✓
Notions natural to express	Signatures	MPC
Security guaranteed when protocols are composed	✗	✓
Impossibility results are rare	✓	✗
Example of equivalent definitions	IND-CPA	Semantic-security

Security frameworks: comparison



Game-based security

The challenger models what the adversary is allowed to do and what is considered to be “bad” in term of security:

- Which message/function can the adversary read/call?
- Passive (= eavedropper) or active adversary (= man in the middle)?
- Blackbox or with physical access to a device?
  - Side channel attacks (= record electric consumption, noise…)
  - Fault attacks (e.g. shooting magnetic waves to disturb a circuit…)
- What must be kept secret? (based on the return value of the challenger)

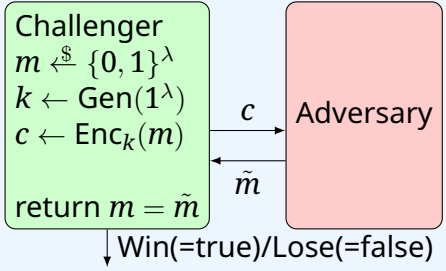
Questions

This models:

A. a passive adversary,

B. an active one?

?



Questions

Consider the following challenger, and assume that for any adversary $\mathcal{A}$, the probability of winning this game is negligible. Let $\mathcal{A}$ be an adversary, then:

- A The probability to **completely** recover a **random** message given its cipher is negligible
- B The probability to recover the **first half** of a **random** message given its cipher is negligible
- C The probability to recover the **first half** of **any** message given its cipher is negligible

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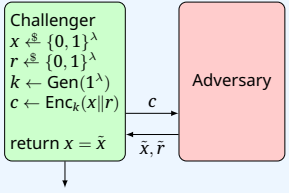

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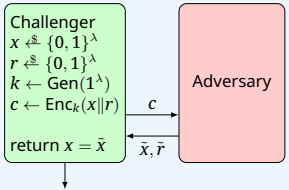
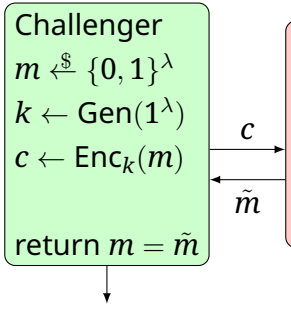


Equivalent notations/formulations



$$= \Pr_{\substack{m \xleftarrow{\$} \{0,1\}^\lambda \\ k \leftarrow \mathsf{Gen}(1^\lambda) \\ c \leftarrow \mathsf{Enc}_k(m) \\ \tilde{m} \leftarrow \mathcal{A}(c)}} [m = \tilde{m}]$$

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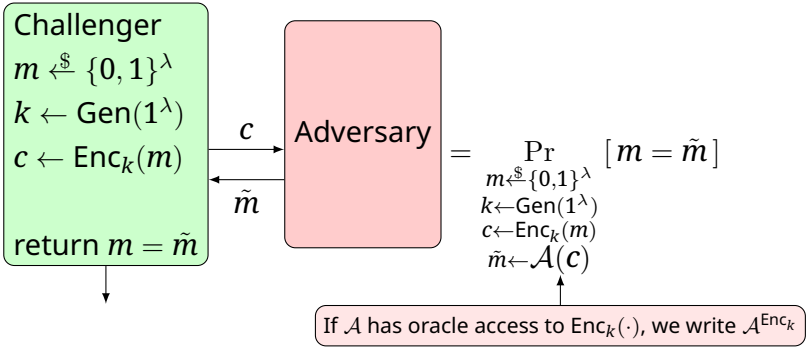

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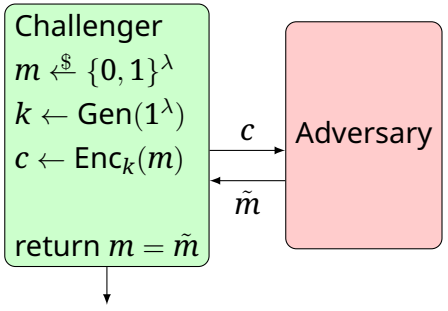
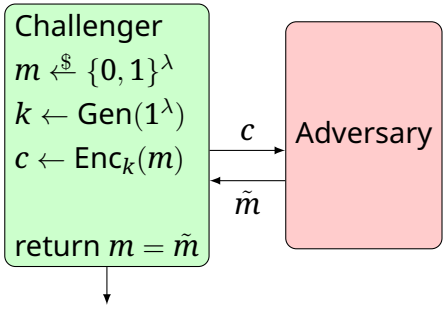


$$= \Pr_{\substack{m \xleftarrow{\$} \{0,1\}^\lambda \\ k \leftarrow \mathsf{Gen}(1^\lambda) \\ c \leftarrow \mathsf{Enc}_k(m) \\ \tilde{m} \leftarrow \mathcal{A}(c)}} [m = \tilde{m}] = \Pr_{\substack{m \xleftarrow{\$} \{0,1\}^\lambda \\ \tilde{m} \leftarrow \mathcal{A} \diamond \mathcal{L}_m}} [m = \tilde{m}]$$

$\mathcal{L}_m$
$k \leftarrow \mathsf{Gen}(1^\lambda)$
$c \leftarrow \mathsf{Enc}_k(m)$
GETC():
return c

$\mathcal{A} \diamond \mathcal{L}$ means that $\mathcal{A}$ has oracle access to $\mathcal{L}$ (called library), like $\mathcal{A}^{\mathcal{L}}$ but this notation is used in *Joy of cryptography* and is practical when chaining multiple libraries.

Equivalent notations/formulations



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GETC():
return c

Verbose, hard to manipulate formally

More standard but often harder to manipulate and check

From *Joy of cryptography*: **easier** to re-use and write/check proofs (explicit dependency, small reductions easy to check)

But **fundamentally the same**, just different presentations!

Exercice library evaluation

We consider the following libraries:

$\mathcal{A}_1$
$r_1 \leftarrow \mathsf{RAND}(6)$
return $r_1 \stackrel{?}{=} 4$

$\mathcal{L}_1$
$\mathsf{RAND}(n)$:
$r \xleftarrow{\$} \mathbb{Z}_{\lceil n/2 \rceil}$
return $2r$

What is the value of $\Pr[\mathcal{A}_1 \diamond \mathcal{L}_1 = 1]$[a]?

A. 0
B. 1/6
C. 1/3
D. 1

[a] From now on, we define true $= 1$ and false $= 0$.

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Exercice well-defined libraries

We consider the following libraries: What is the value of $\Pr[\mathcal{A}_1 \diamond \mathcal{L}_1 = 1]$?

- A 0
- B 1/2
- C 1
- D This is not defined for one reason
- E This is not defined for two reasons

$\mathcal{A}_1$
$a \coloneqq 46$
return sample() $\stackrel{?}{=} c$

$\mathcal{L}_1$
$c \coloneqq 51$
$d \coloneqq 5$
sample():
return $a + d$

Exercice well-defined libraries

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- A 0
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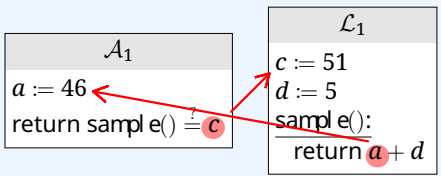
- D This is not defined for one reason
- E This is not defined for two reasons ✓



Game-based security: power of the adversary

We can also model the power of an adversary (typically modeled as a Turing machine) in the quantification of the adversary:

- "For any **unbounded** $\mathcal{A}$, the probability of winning is low" = statistical/information theoretic security
- "For any **polynomially** bounded adversary $\mathcal{A}$, the probability of winning is low" = computational security

If the running time of $\mathcal{A}(n)$ is $\sqrt{n}$, is $\mathcal{A}$ polynomial?

A. Yes

B. No

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If the running time of $\mathcal{A}(n)$ is $\sqrt{n}$, is $\mathcal{A}$ polynomial?

A. Yes ✗

B. No ✓ It must run in polynomial time in the **length** ($\log(n)$) of the input (otherwise factoring is efficient!)

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Search vs decision

Definition of “low” = depends on the challenger, but typically we have 2 cases:

- **Search problem**: adversary needs to find a **bit-string** (e.g. “decrypt this message”): low $= \mathsf{negl}(\lambda)$
- **Decision problem**: adversary needs to find a **single bit** b (e.g. “is this an encryption of m_0 or m_1?”): low $= 1/2 + \mathsf{negl}(\lambda)$

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Focus

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Focus

Definition (interchangeability)

Two libraries $\mathcal{L}_0$ and $\mathcal{L}_1$ are *interchangeable* (or *equal*), written $\mathcal{L}_0 \equiv \mathcal{L}_1$, if for any adversary $\mathcal{A}$,

$$\Pr\left[\mathcal{A} \diamond \mathcal{L}_{\mathbf{0}} = 1\right] = \Pr\left[\mathcal{A} \diamond \mathcal{L}_{\mathbf{1}} = 1\right]$$

Practice time

Caseine: faire le quiz “Distinguer des librairies”

Goal

Sometimes we need a relaxed version when adversaries are computationally bounded:

Definition (advantage and indistinguishability)

We say that two libraries $\mathcal{L}_0$ and $\mathcal{L}_1$ are **indistinguishable** (denoted $\mathcal{L}_0 \approx \mathcal{L}_1$) if for any computationally bounded adversary (polynomial time) $\mathcal{A}$, **the advantage** $\mathsf{Adv}_{\mathcal{A}}(\lambda)$ of $\mathcal{A}$ is negligible, with:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) \coloneqq \left| \Pr\left[\mathcal{A}(1^\lambda) \diamond \mathcal{L}_0 = 1 \right] - \Pr\left[\mathcal{A}(1^\lambda) \diamond \mathcal{L}_1 = 1 \right] \right| \leq \mathsf{negl}(\lambda)$$

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Asymptotic notion!

IND-CPA

We finally have all the tools to define security of encryption!

Antoine Daniel will finally be able to define security of an encryption scheme

IND-CPA

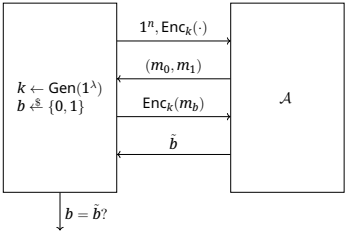
Definition (IND-CPA)

An encryption scheme $\Sigma = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ has indistinguishable security against *chosen-plaintext attacks* (IND-CPA security) if:

$\mathcal{L}^{\Sigma}_{\text{cpa-L}}$		$\mathcal{L}^{\Sigma}_{\text{cpa-R}}$
$k \leftarrow \mathsf{Gen}(1^\lambda)$ EAVESDROP$(m_L, m_R \in \mathcal{M})$: return $\mathsf{Enc}_k(m_L)$	$\approx$	$k \leftarrow \mathsf{Gen}(1^\lambda)$ EAVESDROP$(m_L, m_R \in \mathcal{M})$: return $\mathsf{Enc}_k(m_R)$

Various definitions of IND-CPA

You might see this other **equivalent** definition of IND-CPA:



vs

$\mathcal{L}^{\Sigma}_{\text{cpa-L}}$

$k \leftarrow \mathsf{Gen}(1^\lambda)$

$\mathsf{EAVESDROP}(m_L, m_R \in \mathcal{M})$:

return $\mathsf{Enc}_k(m_L)$

$\approx$

$\mathcal{L}^{\Sigma}_{\text{cpa-R}}$

$k \leftarrow \mathsf{Gen}(1^\lambda)$

$\mathsf{EAVESDROP}(m_L, m_R \in \mathcal{M})$:

return $\mathsf{Enc}_k(m_R)$

- Instead of b, when $b = 0$ we play $\mathcal{L}^{\Sigma}_{\text{cpa-L}}$ otherwise $\mathcal{L}^{\Sigma}_{\text{cpa-R}}$.
- In our definition, no access to oracle $\mathsf{Enc}_k(\cdot)$, but we can **simulate it** by calling $\mathsf{EAVESDROP}(m, m)$ (same message twice).
- In our definition, no restriction on the number of allowed calls to $\mathsf{EAVESDROP}$ (= stronger notion, while in the other we have a single message $Enc_k(m_b)$). But equivalent (advantage is multiplied by the maximum number of queries done by $\mathcal{A}$, but still negligible): proof via a sequence of **hybrids on the number of queries**.

How to prove INsecurity?

How to prove INsecurity

To prove **in**security for a decision game between $\mathcal{L}_0$ and $\mathcal{L}_1$:

1. exhibits a given attacker $\mathcal{A}$
2. compute $\varepsilon = |\Pr[\mathcal{A} \diamond \mathcal{L}_0 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1]|$
3. show that $\exists c \in \mathbb{N}$ s.t. ε is greater than $\frac{1}{\lambda^c}$

How to prove INsecurity

We consider the encryption scheme $\mathsf{Gen}(1^\lambda) \coloneqq \textbf{return } 0$ and $\mathsf{Enc}_k(m) \coloneqq m \oplus 1 \ldots 1$. Is this scheme IND-CPA secure, and if not, which attacker can distinguish these two libraries, and with which advantage ?

?

$$\boxed{\begin{array}{l}\mathcal{L}^{\Sigma}_{\text{cpa-L}} \\ \hline k \leftarrow \mathsf{Gen}(1^\lambda) \\ \underline{\textsc{Eavesdrop}(m_L, m_R \in \mathcal{M}):} \\ \quad \text{return } \mathsf{Enc}_k(m_L)\end{array}} \overset{?}{\approx} \boxed{\begin{array}{l}\mathcal{L}^{\Sigma}_{\text{cpa-R}} \\ \hline k \leftarrow \mathsf{Gen}(1^\lambda) \\ \underline{\textsc{Eavesdrop}(m_L, m_R \in \mathcal{M}):} \\ \quad \text{return } \mathsf{Enc}_k(m_R)\end{array}} \tag{1}$$

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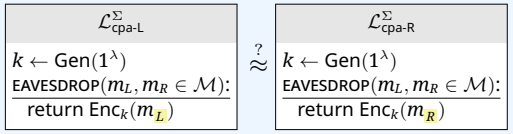

(1)

?

1. $\mathcal{A}$: $c \coloneqq$ EAVESDROP(0^λ); return $c \oplus 1\ldots1 \overset{?}{=} 0^\lambda$, advantage 0 (A), $1/2$ (B), $1/2 - \frac{1}{2^\lambda}$ (C) or 1 (D)
2. $\mathcal{A}$: $c \coloneqq$ EAVESDROP(0^λ); return $c \oplus c \overset{?}{=} 0^\lambda$, advantage 0 (E), $1/2$ (F), $1/2 - \frac{1}{2^\lambda}$ (G) or $1 - \frac{1}{2^\lambda}$ (H)

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(1)

?

1. | $\mathcal{A}$ |
   |---|
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   , advantage 0 (A), $1/2$ (B), $1/2 - \frac{1}{2^\lambda}$ (C) or 1 (D ✓)

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   |---|
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How to prove INsecurity

Which attacker can distinguish these two libraries, and with which advantage?

?

$\mathcal{L}^{\Sigma}_{\text{ots\$-real}}$

$\textsc{ctxt}(m \in \{0,1\}^{\lambda})$:
$k \leftarrow \{0,1\}^{\lambda}$ // Σ.KeyGen
$c := k \,\&\, m$ // Σ.Enc
return c

$\mathcal{L}^{\Sigma}_{\text{ots\$-rand}}$

$\textsc{ctxt}(m \in \{0,1\}^{\lambda})$:
$c \leftarrow \{0,1\}^{\lambda}$ // $\Sigma.C$
return c

How to prove INsecurity

Which attacker can distinguish these two libraries, and with which advantage?

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1. $\mathcal{A}$: $c := \textsc{ctxt}(0^{\lambda})$; return $c = 0^{\lambda}$, advantage $1/4$ (A), $1/2$ (B), $1/2 - \frac{1}{2^{\lambda}}$ (C) or $1 - \frac{1}{2^{\lambda}}$ (D)
2. $\mathcal{A}$: $c := \textsc{ctxt}(1^{\lambda})$; return $c = 0^{\lambda}$, advantage $1/4$ (E), $1/2$ (F), $1/2 - \frac{1}{2^{\lambda}}$ (G) or $1 - \frac{1}{2^{\lambda}}$ (H)

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Practice

See previous exercise in Caseine for more examples

Concrete vs asymptotic cryptography

Asymptotic vs actual security

In theoretical analysis, security is asymptotic. In practice: **How to choose λ ?**
Typically:

A. Study the best known attacks, **count the number of operations T** and the advantage ε (trade-off time/precision), consider that the actual number of operations is roughly[1] T/ε.
$\Rightarrow$ this protocol has $\log(T/\varepsilon)$-bits of security.

B. Realize that:
   - 2^{40} operations is really easy to do (small raspberry pi cluster)
   - 2^{60} operations doable with large CPU/GPU cluster
   - 2^{80} operations doable with an ASIC cluster (bitcoin mining)
   - 2^{128} operations = **very hard** (next slide)

[1] More details in [Watanabe, Yasunaga 2021] and [Micciancio, Walter 2018].

How big is 2^{128}?

Say that:

- problem is parallelizable
- you can access all 500 best super-computers $=$ 10 000 000 000 GFLOPS (FLOPS = floating point operations per second)

Then, you need in total:

$$\frac{2^{128}}{10 \times 10^9 \times 10^9 \times 3600 \times 24 \times 365} \approx \boxed{1\ 000\ 000\ 000\ 000 \text{ years}}$$

(roughly $4\times$ age of earth)

How to write security proofs

Basic properties

Properties (also hold when replacing $\approx$ with $\equiv$)

- **Transitivity**: $(\mathcal{L}_0 \approx \mathcal{L}_1) \wedge (\mathcal{L}_1 \approx \mathcal{L}_2) \Rightarrow \mathcal{L}_0 \approx \mathcal{L}_2$
- **Chaining**: $(\mathcal{L}_0 \approx \mathcal{L}_1) \Rightarrow ((\mathcal{L} \diamond \mathcal{L}_0) \approx (\mathcal{L} \diamond \mathcal{L}_1))$

Preuves: exercice

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Proof transitivity (basically triangle inequality): We assume $\mathcal{L}_0 \approx \mathcal{L}_1 \wedge \mathcal{L}_1 \approx \mathcal{L}_2$. Let $\mathcal{A}$ run in polynomial time. Then by definition:

$$|\Pr[\mathcal{A} \diamond \mathcal{L}_0 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1]| \leq \mathsf{negl}(\lambda) \wedge |\Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_2 = 1]| \leq \mathsf{negl}(\lambda)$$

But

$$\begin{aligned}
&|\Pr[\mathcal{A} \diamond \mathcal{L}_0 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1]| \\
&= |\Pr[\mathcal{A} \diamond \mathcal{L}_0 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1] + \Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_2 = 1]| \\
&\leq |\Pr[\mathcal{A} \diamond \mathcal{L}_0 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1]| + |\Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_2 = 1]| \\
&\leq \mathsf{negl}(\lambda) + \mathsf{negl}(\lambda) \leq \mathsf{negl}(\lambda)
\end{aligned}$$

Basic properties

Properties (also hold when replacing $\approx$ with $\equiv$)

- **Transitivity**: $(\mathcal{L}_0 \approx \mathcal{L}_1) \wedge (\mathcal{L}_1 \approx \mathcal{L}_2) \Rightarrow \mathcal{L}_0 \approx \mathcal{L}_2$
- **Chaining**: $(\mathcal{L}_0 \approx \mathcal{L}_1) \Rightarrow ((\mathcal{L} \diamond \mathcal{L}_0) \approx (\mathcal{L} \diamond \mathcal{L}_1))$

Proof chaining: We assume that $\mathcal{L}_0 \approx \mathcal{L}_1$. Let $\mathcal{A}$ run in poly time. We want to show $(\mathcal{L} \diamond \mathcal{L}_0) \approx (\mathcal{L} \diamond \mathcal{L}_1)$:

$$
\begin{aligned}
& |\Pr[\mathcal{A} \diamond (\mathcal{L} \diamond \mathcal{L}_0) = 1] - \Pr[\mathcal{A} \diamond (\mathcal{L} \diamond \mathcal{L}_2) = 1]| \\
\mathcal{A}' \coloneqq \mathcal{A} \diamond \mathcal{L} \quad & = |\Pr[(\mathcal{A} \diamond \mathcal{L}) \diamond \mathcal{L}_0 = 1] - \Pr[(\mathcal{A} \diamond \mathcal{L}) \diamond \mathcal{L}_1 = 1]| \\
& \stackrel{\downarrow}{=} |\Pr[\mathcal{A}' \diamond \mathcal{L}_0 = 1] - \Pr[\mathcal{A}' \diamond \mathcal{L}_1 = 1]|
\end{aligned}
$$

since $\mathcal{A}$ runs in poly time, so does $\mathcal{A}'$. Hence using $\mathcal{L}_0 \approx \mathcal{L}_1$ the above is $\mathrm{negl}(\lambda)$. □

Methods

Six main methods:

1. **Hybrid games** : Decompose into a sequence of hybrid games (to make methods 2 – 6 easier)
2. **Probabilities** : Explicitly compute the probability, and show equality or bound the statistical distance (statistical security only)
3. **Equality** : Show that the two games are trivially doing exactly the same thing (variant of 2)
   (e.g. code simply externalized to a sub-library, code that is simply inlined…)
4. **Reduction** : show that if we can distinguish them, then $\mathcal{A}$ can be used to break a hard problem (factor numbers…)
5. **Theorem/assumption** : use a theorem already seen in the course or an assumption
6. **Chaining** : prove $\mathcal{L}_1 \approx \mathcal{L}_2$, then $\mathcal{A} \diamond \mathcal{L}_1 \approx \mathcal{A} \diamond \mathcal{L}_2$

We detail methods 1,2,3,4 now (5 & 6 trivial).

Hybrid games

Proof = sequence of **hybrid** games:

Hybrid games

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Hybrid games

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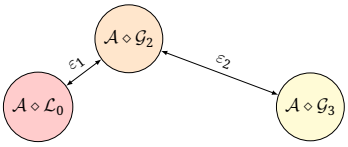
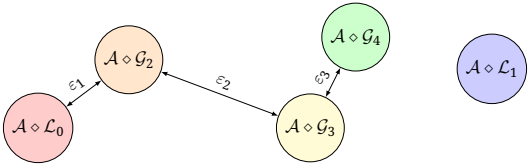


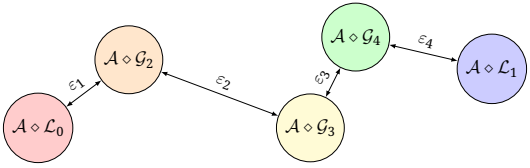
Hybrid games

Proof = sequence of **hybrid** games:



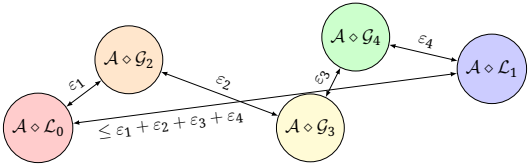
Hybrid games

Proof = sequence of **hybrid** games:



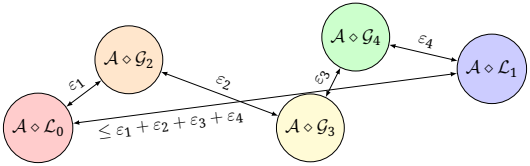
Hybrid games

Proof = sequence of **hybrid** games:



Hybrid games

Proof = sequence of **hybrid** games:



By transitivity, if $\mathcal{L}_0 \approx \mathcal{G}_2 \approx \mathcal{G}_3 \approx \mathcal{G}_4 \approx \mathcal{L}_1$, then $\mathcal{L}_0 \approx \mathcal{L}_1$.

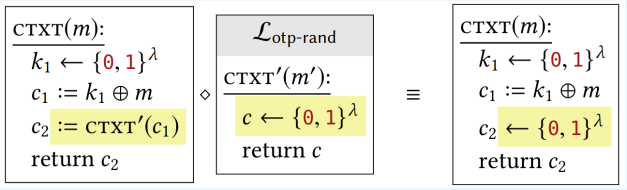
Equality

Just realize two libraries are trivially **doing the exact same thing** (e.g. move a call in a sub-library or inline a sub-library in a code)
WARNING: Make sure variables are always well defined, with no naming collision and well **scoped** (a sub-library cannot refer to a variable of a parent library)

Equality

Are these two libraries equal?

?



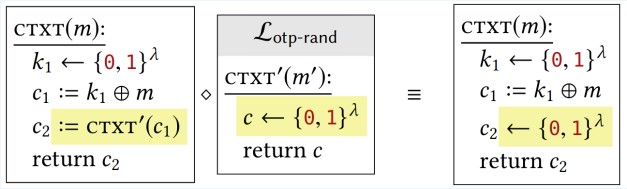
A Yes

B No

Equality

Are these two libraries equal?

?



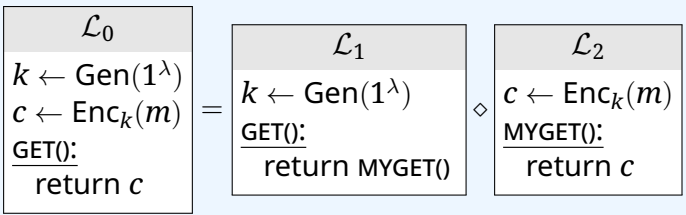
A Yes ✓ Variable are well scoped, inlined a sub-library

B No ✗

Equality

Are these two libraries equal?

?



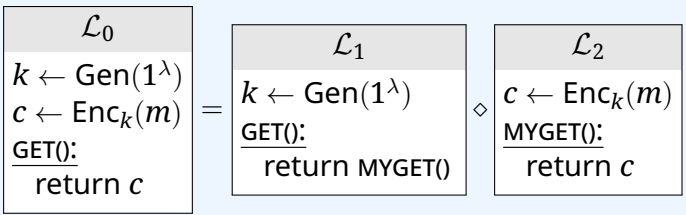
A. Yes

B. No

Equality

Are these two libraries equal?

?



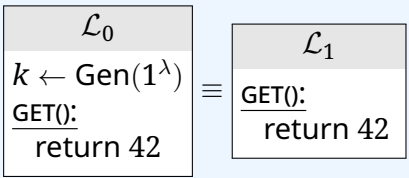
A. Yes ✗

B. No ✓ k is not defined in $\mathcal{L}_2$

Equality

Are these two libraries equal?

?



A. Yes<2> ✓ k is never used, safe to remove it

B. No<2> ✗

Method: compute probabilities

Theorem (One-time-pad uniform ciphertext)

$$\begin{array}{|c|}\hline \mathcal{L}_{\text{otp-real}} \\ \hline \underline{\mathsf{OTENC}(m \in \{0,1\}^\lambda):} \\ k \xleftarrow{\$} \{0,1\}^\lambda \\ \text{return } k \oplus m \\ \hline \end{array} \equiv \begin{array}{|c|}\hline \mathcal{L}_{\text{otp-rand}} \\ \hline \underline{\mathsf{OTENC}(m \in \{0,1\}^\lambda):} \\ c \xleftarrow{\$} \{0,1\}^\lambda \\ \text{return } c \\ \hline \end{array}$$

Proof Let $m, \tilde{c} \in \{0,1\}^\lambda$. In $\mathcal{L}_{\text{otp-rand}}$, $\Pr[\mathsf{OTENC}(m) = \tilde{c}] = \frac{1}{2^\lambda}$ (uniform sampling). In $\mathcal{L}_{\text{otp-real}}$:

$$\Pr[\mathsf{OTENC}(m) = \tilde{c}] = \Pr\left[k \oplus m = \tilde{c} \mid k \xleftarrow{\$} \{0,1\}^\lambda\right] = \Pr\left[\tilde{c} \oplus m = k \mid k \xleftarrow{\$} \{0,1\}^\lambda\right]$$

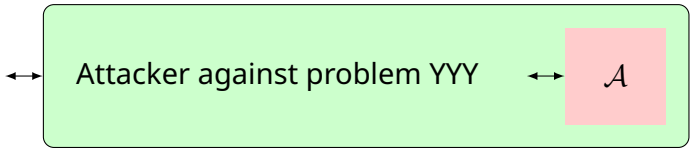
$$= \Pr\left[C = k \mid k \xleftarrow{\$} \{0,1\}^\lambda\right] = \frac{1}{2^\lambda} = \Pr[\mathsf{OTENC}(m) = \tilde{c}]$$

where $C := \tilde{c} \oplus m$. Henc $\mathcal{L}_{\text{otp-real}} = \mathcal{L}_{\text{otp-rand}}$ □

Method: reduction

All the above methods = interchangeability (statistical indistinguishability). What about **computational** indistinguishability? Either directly an assumption that the two libraries are hard to distinguish (possibly need an hybrid sequence first), otherwise:

Reduction!

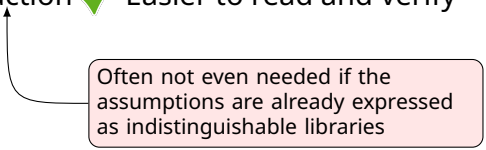


Idea: to prove $\mathcal{L}_0 \approx \mathcal{L}_1$, assume $\mathcal{L}_0 \not\approx \mathcal{L}_1$, i.e. $\exists$ polynomial adversary $\mathcal{A}$ s.t. $|\Pr[\mathcal{A} \diamond \mathcal{L}_0 = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_1 = 1]|$. **Use $\mathcal{A}$ as a subroutine to break a hard problem (compute explicitly the success probability)** $\Rightarrow$ contradiction!

Method: reduction

Option 1: single huge reduction: ✗ hard to write and read

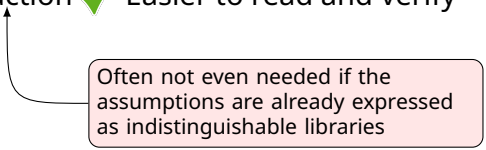
Option 2: hybrids + small reduction ✓ Easier to read and verify



Method: reduction

Option 1: single huge reduction: ✗ hard to write and read

Option 2: hybrids + small reduction ✓ Easier to read and verify



Some useful theorems

Bad event lemma

Bad event lemma

Let $\mathcal{L}_{\text{left}}$ and $\mathcal{L}_{\text{right}}$ be two libraries that define a variable named bad, that is initialized to 0. If $\mathcal{L}_{\text{left}}$ and $\mathcal{L}_{\text{right}}$ have identical code except for code blocks reachable only when bad $= 1$ (e.g. guarded with an "if bad $= 1$" statement), then:

$$|\Pr[\mathcal{A} \diamond \mathcal{L}_{\text{left}} = 1] - \Pr[\mathcal{A} \diamond \mathcal{L}_{\text{right}} = 1]| \leq \Pr[\mathcal{A} \diamond \mathcal{L}_{\text{left}} \text{ sets bad } = 1] \tag{2}$$

Proof: Define A_{left} the event "$\mathcal{A} \diamond \mathcal{L}_{\text{left}} = 1$", A_{right} the event "$\mathcal{A} \diamond \mathcal{L}_{\text{right}} = 1$", B_{left} the event $\mathcal{A} \diamond \mathcal{L}_{\text{left}}$ sets bad $= 1$, and B_{right} the event $\mathcal{A} \diamond \mathcal{L}_{\text{left}}$ sets bad $= 1$, and $\bar{\cdot}$ is the negation of event $\cdot$.

$$\begin{aligned}
&|\Pr[A_{\text{left}}] - \Pr[A_{\text{right}}]| = |\Pr[B_{\text{left}}]\Pr[A_{\text{left}} \mid B_{\text{left}}] + \Pr[\bar{B}_{\text{left}}]\Pr[A_{\text{left}} \mid \bar{B}_{\text{left}}] \\
&- \Pr[B_{\text{right}}]\Pr[A_{\text{right}} \mid B_{\text{right}}] - \Pr[\bar{B}_{\text{right}}]\Pr[A_{\text{right}} \mid \bar{B}_{\text{right}}]| \\
&\leq \Pr[\bar{B}_{\text{left}}] \underbrace{|\Pr[A_{\text{left}} \mid \bar{B}_{\text{left}}] - \Pr[A_{\text{right}} \mid \bar{B}_{\text{right}}]|}_{=0 \text{ (same code when bad is 0)}} + \Pr[B_{\text{left}}] \underbrace{|\Pr[A_{\text{left}} \mid B_{\text{left}}] - \Pr[A_{\text{right}} \mid B_{\text{right}}]|}_{\leq 1} \\
&\leq \Pr[B_{\text{left}}]
\end{aligned}$$

Triangle ineq. & $\Pr[B_{\text{left}}] = \Pr[B_{\text{right}}]$ (identical code before setting bad))

□

Application bad event lemma

We want to show that $\mathcal{L}_{\text{left}}$ / PREDICT(x): / $s \xleftarrow{\$} \{0,1\}^\lambda$ / return $x \stackrel{?}{=} s$ $\approx$ $\mathcal{L}_{\text{right}}$ / PREDICT(x): / return false. A student already wrote these two hybrid games: $\mathcal{G}_1$ and $\mathcal{G}_2$. How can you finish the proof?

$\mathcal{G}_1$:

```
bad := 0
PREDICT(x):
  s ←$ {0,1}^λ
  if x ?= s:
    bad := 1
  return false
```

$\mathcal{G}_2$:

```
bad := 0
PREDICT(x):
  s ←$ {0,1}^λ
  if x ?= s:
    bad := 1
    return true
  return false
```

A $\mathcal{L}_{\text{left}} = \mathcal{G}_1 \approx \mathcal{G}_2 = \mathcal{L}_{\text{right}}$

B $\mathcal{L}_{\text{left}} \approx \mathcal{G}_1 = \mathcal{G}_2 \approx \mathcal{L}_{\text{right}}$

C $\mathcal{L}_{\text{left}} = \mathcal{G}_2 \approx \mathcal{G}_1 = \mathcal{L}_{\text{right}}$

D $\mathcal{L}_{\text{left}} \approx \mathcal{G}_2 = \mathcal{G}_1 \approx \mathcal{L}_{\text{right}}$

Application bad event lemma

We want to show that $\mathcal{L}_{\text{left}}$: PREDICT$(x)$: $s \xleftarrow{\$} \{0,1\}^\lambda$; return $x \stackrel{?}{=} s$ $\approx$ $\mathcal{L}_{\text{right}}$: PREDICT$(x)$: return false. A student already wrote these two hybrid games: $\mathcal{G}_1$: bad $\coloneqq 0$; PREDICT(x): $s \xleftarrow{\$} \{0,1\}^\lambda$; if $x \stackrel{?}{=} s$: bad $\coloneqq 1$; return false and $\mathcal{G}_2$: bad $\coloneqq 0$; PREDICT(x): $s \xleftarrow{\$} \{0,1\}^\lambda$; if $x \stackrel{?}{=} s$: bad $\coloneqq 1$; return true; return false. How can you finish the proof?

?

- A $\mathcal{L}_{\text{left}} = \mathcal{G}_1 \approx \mathcal{G}_2 = \mathcal{L}_{\text{right}}$
- B $\mathcal{L}_{\text{left}} \approx \mathcal{G}_1 = \mathcal{G}_2 \approx \mathcal{L}_{\text{right}}$
- C $\mathcal{L}_{\text{left}} = \mathcal{G}_2 \approx \mathcal{G}_1 = \mathcal{L}_{\text{right}}$ ✓ We use the bad event lemma to show $\mathcal{G}_2 \approx \mathcal{G}_1$ ($\Pr[\text{bad} = 1] = \frac{1}{2^\lambda} = \text{negl}(\lambda)$)
- D $\mathcal{L}_{\text{left}} \approx \mathcal{G}_2 = \mathcal{G}_1 \approx \mathcal{L}_{\text{right}}$

Conclusion

Conclusion

- Can't dissociate cryptography from the **security models** and **proofs**
- **Lot's of parameters** to consider ((un)bounded), computational assumptions, setup assumptions, asymptotic/concrete, security model…
- For us: prove security of protocol = **show that two libraries are indistinguishable**
- One example is the **IND-CPA** security property
- We saw a list of **methods to write security proofs**
- Conversely, **to prove the insecurity** of a protocol we must exhibit an efficient (=polynomial) distinguisher that can distinguish the libraries with a non-negligible advantage