

Algoritmos e Estruturas de Dados

First assignment: Merkle-Hellman cryptosystem

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Leonardo dos Santos Flório, 103360, 33.3%

Gabriel Hall Abreu, 102851, 33.3%

Diogo Alves e Silva, 103925, 33.3%

Problema Proposto

O problema que nos foi proposto baseia-se numa simplificação do cryptosystem de Merkle e Hellman. Ao invés de utilizar um problema knapsack como no original, este é substituído por um subset-sum.

O nosso objetivo consiste então em resolver os vários subset-sum que nos foram dados.

Para tal precisamos de computar as possíveis somas e verificar se alguma corresponde ao resultado desejado. Não é necessário, no entanto, verificar todas as somas, sendo até preferível não fazer mais do que as estritamente necessárias de modo a tornar o programa mais eficiente tanto em memória como em processamento.

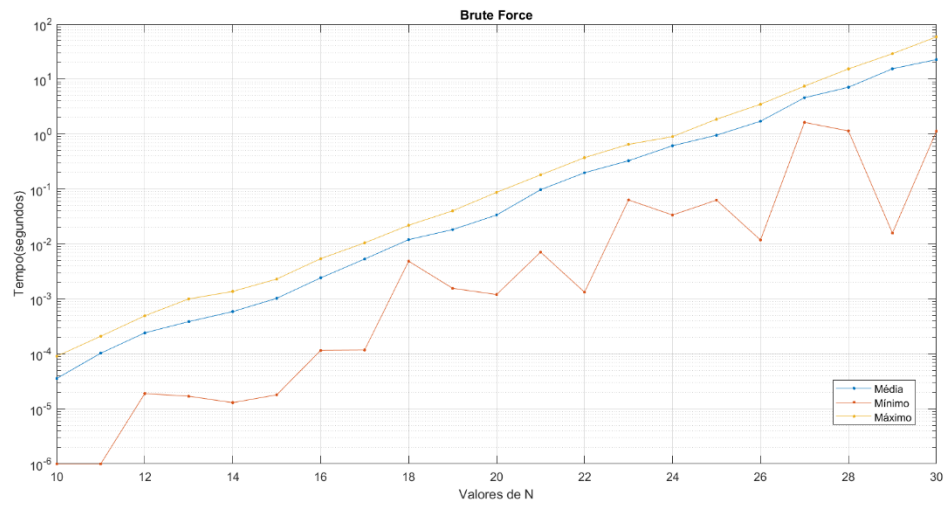
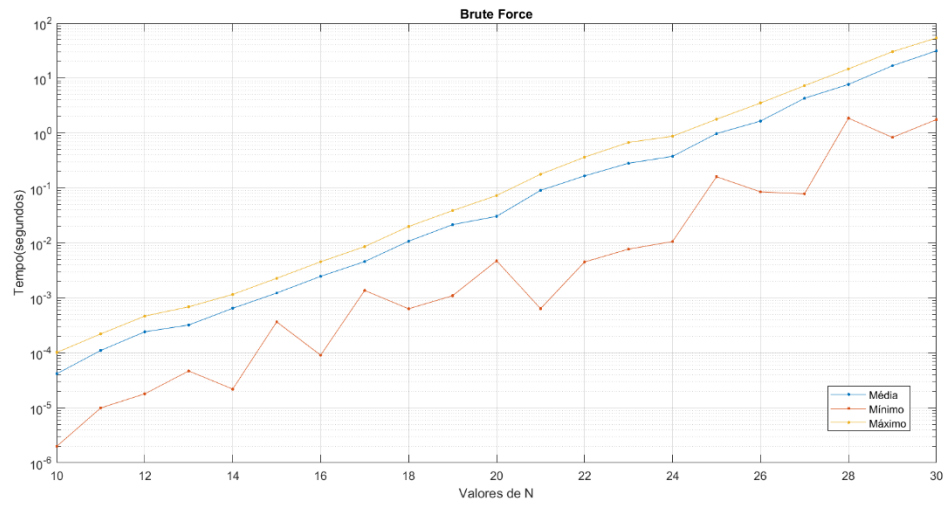
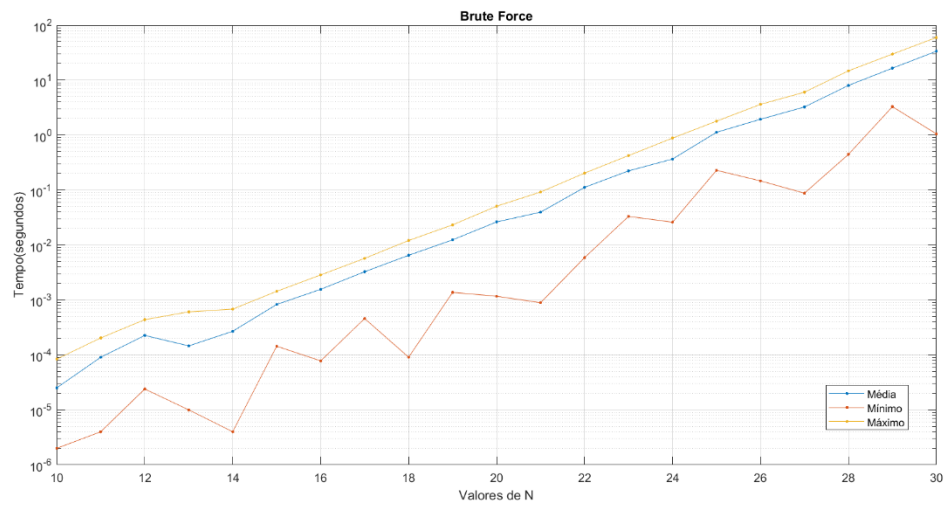
Nós demonstramos isto através do desenvolvimento de 4 algoritmos diferentes, sendo que todos eles obtêm os resultados desejados, mas em tempos de execução distintos.

Brute Force

As primeiras 2 soluções desenvolvidas foram através da técnica de brute force a qual consiste em ir tentando todas as somas possíveis até chegar ao resultado intencionado ou retornar 0, como indicação de que não há solução possível.

A primeira é a função "bruteForceV2", na qual iteramos por todos os elementos de p e tentamos todas as combinações possíveis de soma até esse elemento, $p[index]$. Para cada elemento é selecionado, para a soma com ele, o primeiro que ainda não tenha sido selecionado, até este ser o imediatamente anterior ao primeiro elemento da soma. Quando isso acontece passa a selecionar, sucessivamente, um terceiro elemento, o primeiro que ainda não tenha sido selecionado, para se somar aos restantes dois. Mais uma vez quando for o imediatamente anterior aos restantes passa a selecionar um quarto elemento para repetir o processo, e assim sucessivamente até se terem realizado todas as somas até ao elemento principal da iteração.

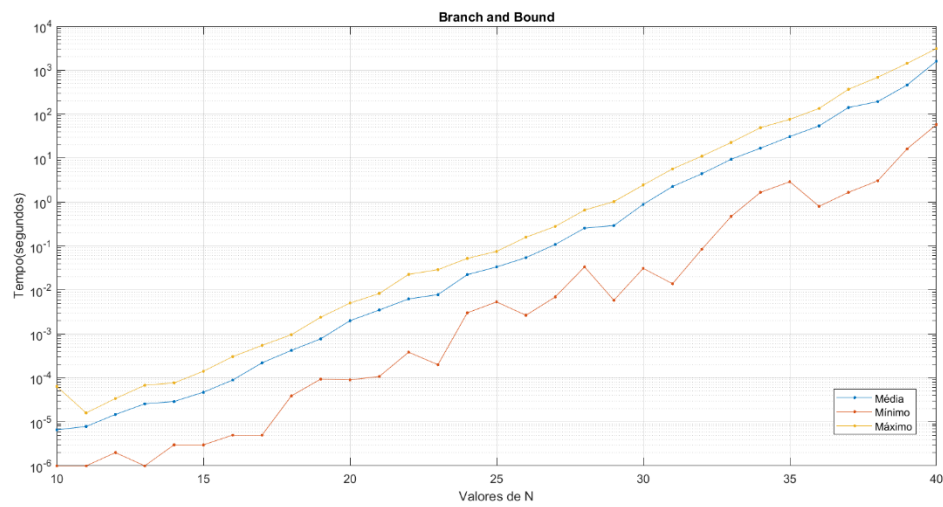
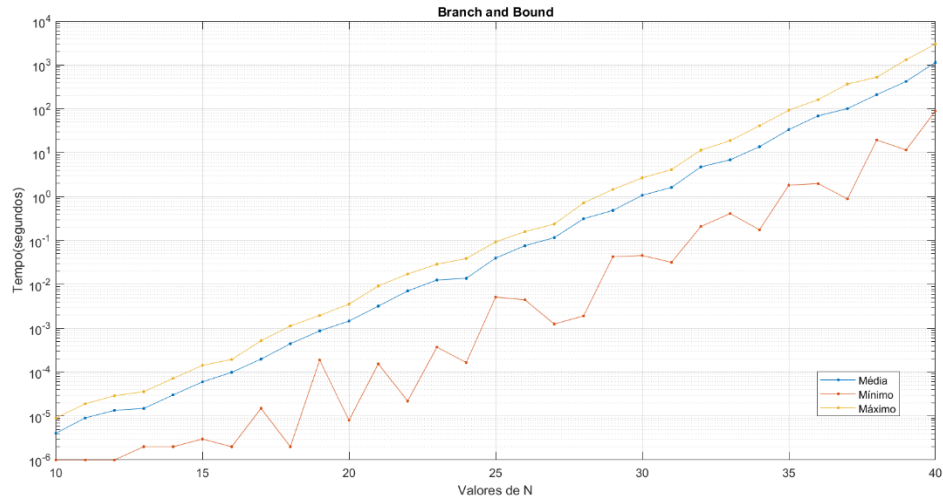
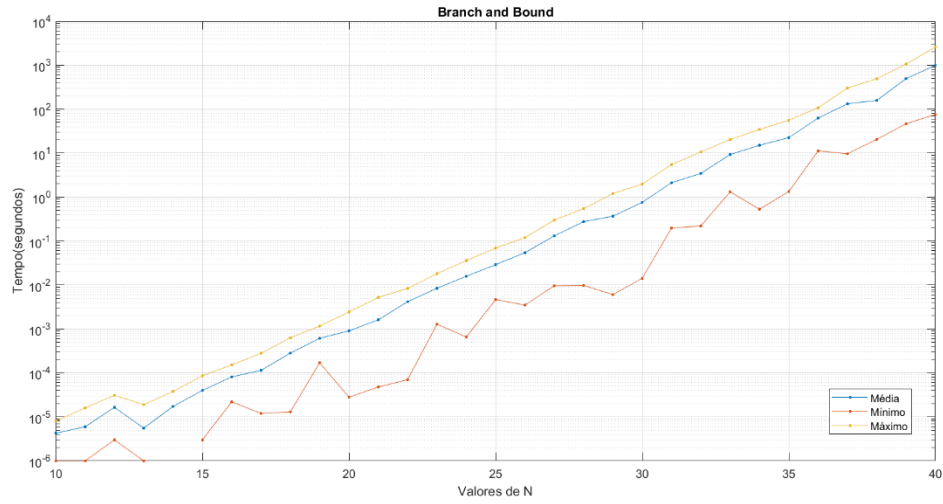
Todas as somas são testadas antes de passar para a próxima para não fazer mais do que as necessárias.



Branch and Bound

A segunda é usando a função “branchAndBound”, na qual iteramos por todos os elementos de p e tentamos todas as combinações possíveis de soma a começar nesse elemento, $p[index]$. Isto é possível de forma recursiva, ao chamarmos a mesma função com o valor de $current_index$ incrementado e com a valor de $partial_sum$ atualizado. Ao incrementamos o $current_index$, não repetimos combinações nem consideramos combinações inválidas, como a utilização duplicada de um elemento de p numa mesma soma.

Por outro lado, atualizamos o $partial_sum$ com os valores dos elementos de p selecionados para o cálculo da soma, verificando em todas as chamadas de função se essa soma é a desejada pelo problema. Todos os elementos considerados em cada simulação da soma ($partial_sum$) são identificados em $result$, ao alterarmos o valor de $result$ correspondente ao elemento para 1 ($result[index] = 1$, uma vez que existe um mapeamento direto entre as posições dos valores de p e de $result$). Aqui, 1 representa a presença do elemento na soma parcial e 0 a sua ausência. Por pre-definição, os elementos de $result$ começam todos em 0. Sempre que um elemento deixa de ser considerado na soma parcial, o seu valor em $result$ passa também novamente a 0 ($result[index] = 0$).



Horowitz and Shahni

A terceira e quarta soluções encontradas foram através da técnica de "meet in the middle", no qual fizemos todas as somas e iteramos da menor até a meio, e da maior até meio somando a menor com a maior até encontrar a soma esperada ou chegar até meio, concluindo que não há resposta.

Para começar precisávamos de todas as somas possíveis ordenadas e divididas em 2 arrays, pelo que fizemos a função "HorowitzShahniSums" para tratar disso mesmo.

Esta começa por iterar pelos elementos de `p[]` e adiciona-os ao array `p1[]` até chegar a meio e a `p2[]` a partir daí.

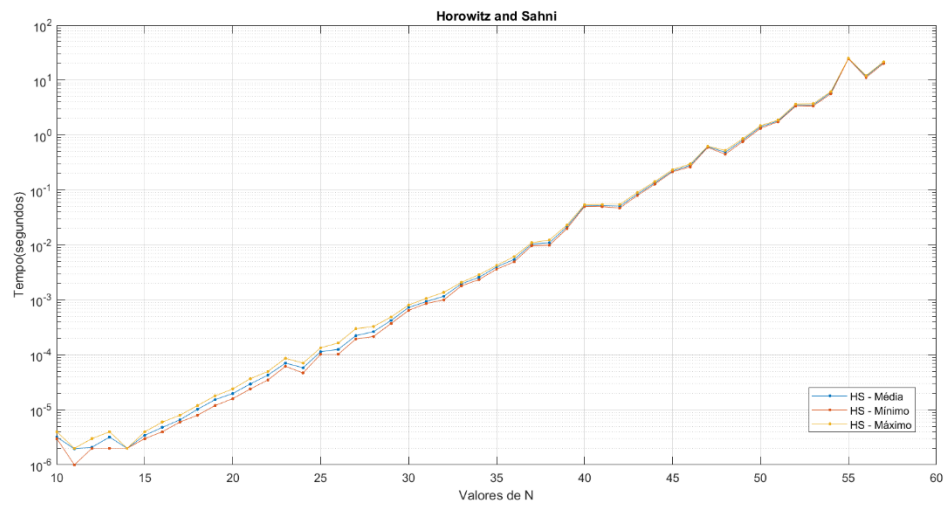
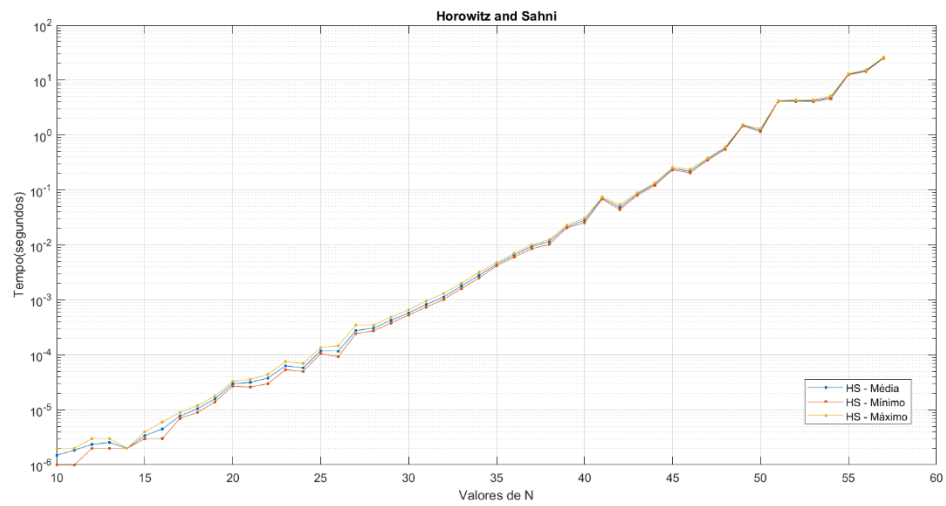
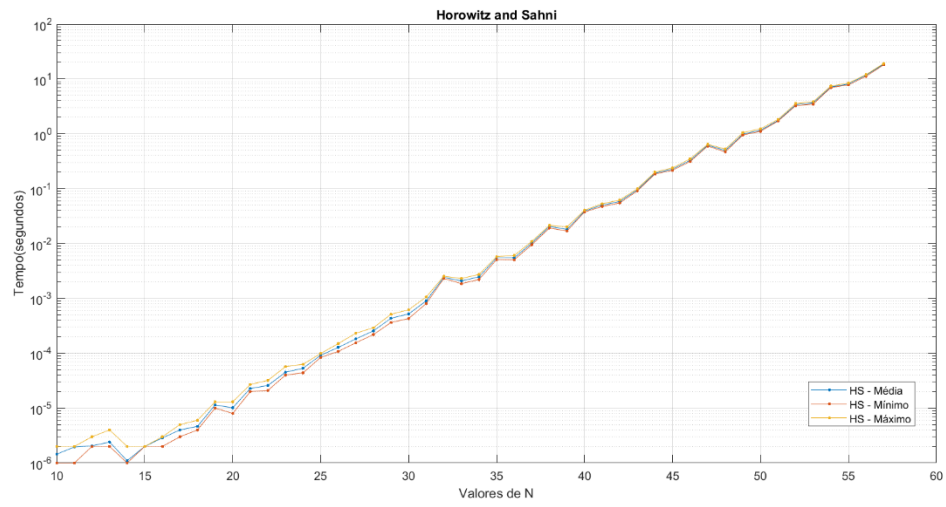
De seguida itera por `p1[]` e `p2[]` para fazer todas as somas possíveis de cada array e armazena-as em `s1[]` e `s2[]`.

Mais tarde, quando obtivermos a resposta, vamos precisar dos indexes dos elementos, e como `s1[]` e `s2[]` vão ser ordenados de forma crescente duplicámos os arrays (`duplicated_s1` e `duplicated_s2`).

Assim que gerar todas as somas de cada array vai então ordenar os arrays através da técnica de quick sort (através da função "quickSort").

Com as todas as somas feitas o script vai então chamar a função "HorowitzShahni" que vai procurar pela soma que dá o resultado desejado. Com recurso a um loop e dois counters itera pelo array `s1[]` do menor elemento até ao maior e pelo `s2[]` do maior até ao menor. Para cada iteração testa se a soma é a resposta certa e caso seja vai iterar pelos array que não foram ordenados (`duplicated_s1` e `duplicated_s2`) até encontrar as soluções para poder saber os seus indexes, visto que estes, em binário, representam a resposta ao problema `result[]`.

Caso chegue a meio sem nenhuma resposta retorna 0.

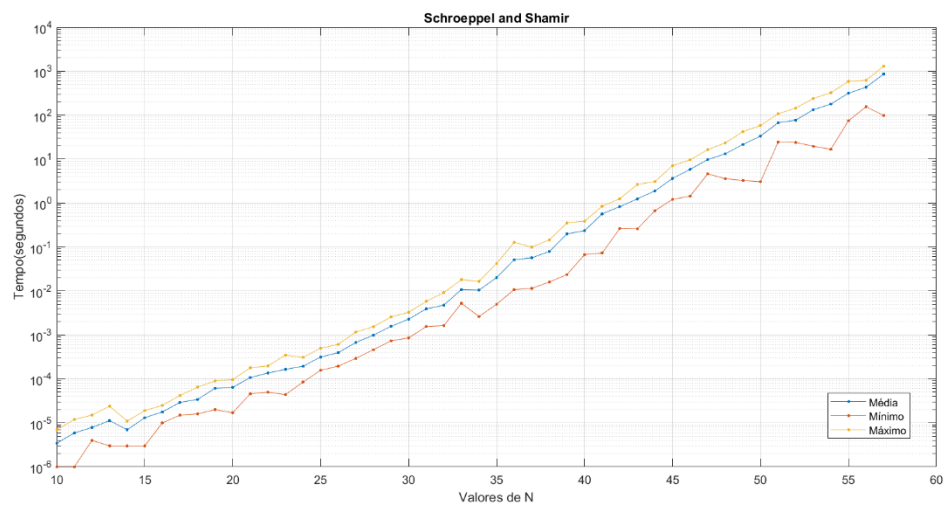
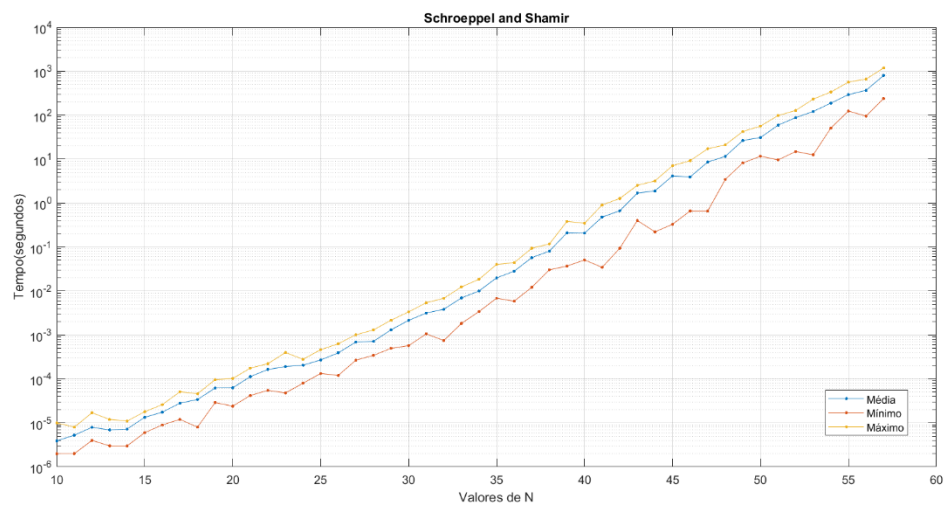
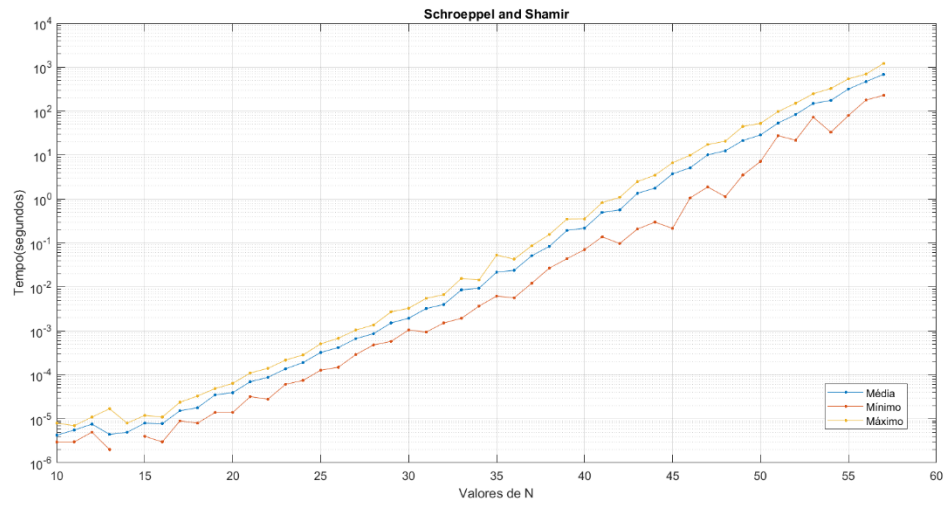


Schroppel and Shamir

A última solução foi através da técnica de Schroppel e Shamir a qual se baseia na anterior, mas utilizando menos memória através da implementação de heaps.

Começa por dividir $p[]$ em 4 arrays e gerar todas as somas de cada um. Itera depois pelos menores dois arrays, testando se a soma de um elemento de cada dá o resultado desejado e caso não dê adiciona essa soma a um heap "minheap". Repete o processo para os dois maiores arrays se necessário.

Caso ainda não tenha encontrado a solução compara a soma do primeiro elemento de cada heap com a soma desejada, terminando o loop se encontrar a soma certa, ou então, remove o menor ou o maior elemento do seu heap caso a soma seja menor ou o maior, respetivamente e repete o loop, até encontra a solução ou acabar os elementos de um dos heaps.



Código

```
//
// AED, November 2021
//
// Solution of the first practical assignment (subset sum problem)
//
// Leonardo dos Santos Flório, 103360
//
// Gabriel Hall Abreu, 102851
//
// Diogo Alves e Silva, 103925
//

#if __STDC_VERSION__ < 199901L
#error "This code must be compiled in c99 mode or later (-std=c99)" // to handle the unsigned long long data type
#endif
#ifndef STUDENT_H_FILE
#define STUDENT_H_FILE "103360.h"
#define STUDENT_H_FILE "102851.h"
#define STUDENT_H_FILE "103925.h"
#endif

//
// include files
//

#include <stdio.h>
#include <stdlib.h>
#include "elapsed_time.h"
#include STUDENT_H_FILE

//
// functions
//

//
// First approach to the problem
//

char bruteForceV1(int n, const integer_t p[n], integer_t desired_sum)
{
    for (int comb = 0; comb < (1 << n); ++comb) // (1 << n) == pow(2,n)
    {
        integer_t test_sum = 0;

        for (int bit = 0; bit < n; bit++)
        {
            if (comb & (1 << bit))
            {
                test_sum += p[bit];
            }
        }

        if (test_sum == desired_sum)
        {
            return 1;
        }
    }
    return 0;
}

char bruteForceV2(int n, const integer_t p[n], integer_t desired_sum, int result[])
{
    for (int comb = 0; comb < (1 << n); ++comb)
    {
        integer_t test_sum = 0;
```

```
for (int bit = 0; bit < n; bit++)
{
    if (comb & (1 << bit))
    {
        result[bit] = 1;
        test_sum += p[bit];
    }
    else
    {
        result[bit] = 0;
    }
}

if (test_sum == desired_sum)
{
    return 1;
}
return 0;
}

//
// Second approach to the problem
//

char bruteForceRecursiveV1(int n, integer_t p[n], integer_t desired_sum, int current_index, integer_t partial_sum)
{
    if (partial_sum == desired_sum)
        return 1;

    if (current_index == n)
        return 0;

    for (int index = current_index; index < n; index++)
    {
        if (bruteForceRecursiveV1(n, p, desired_sum, ++current_index, partial_sum + p[index]))
            return 1;
    }
    return 0;
}

char bruteForceRecursiveV2(int n, integer_t p[n], integer_t desired_sum, int current_index, integer_t partial_sum, int result[])
{
    if (partial_sum == desired_sum)
        return 1;

    if (current_index == n)
        return 0;

    for (int index = current_index; index < n; index++)
    {
        result[index] = 1;
        if (bruteForceRecursiveV2(n, p, desired_sum, ++current_index, partial_sum + p[index], result))
            return 1;
        result[index] = 0;
    }
    return 0;
}

char branchAndBound(int n, integer_t p[n], integer_t desired_sum, int current_index, integer_t partial_sum, int result[])
{
    if (partial_sum == desired_sum)
        return 1;

    if (current_index == n)
        return 0;

    if (partial_sum > desired_sum)
        return 0;

    for (int index = current_index; index < n; index++)
    {
        result[index] = 1;
        if (branchAndBound(n, p, desired_sum, ++current_index, partial_sum + p[index], result))
```

```
        return 1;
        result[index] = 0;
    }
    return 0;
}

//
// Third approach to the problem
//

//
// HorowitzSahni starts here
//

//
// Implementing quick sort function
//

void swap(integer_t *a, integer_t *b)
{
    integer_t t = *a;
    *a = *b;
    *b = t;
}

/* This function takes last element as pivot, places
the pivot element at its correct position in sorted
array, and places all smaller (smaller than pivot)
to left of pivot and all greater elements to right
of pivot */

integer_t partition(integer_t arr[], integer_t low, integer_t high)
{
    integer_t pivot = arr[high]; // pivot
    integer_t i = (low - 1); // Index of smaller element and indicates the right position of pivot found so far

    for (integer_t j = low; j <= high - 1; j++)
    {
        // If current element is smaller than the pivot
        if (arr[j] < pivot)
        {
            i++; // increment index of smaller element
            swap(&arr[i], &arr[j]);
        }
    }

    swap(&arr[i + 1], &arr[high]);
    return (i + 1);
}

/* The main function that implements QuickSort
arr[] --> Array to be sorted,
low --> Starting index,
high --> Ending index */

void quickSort(integer_t arr[], integer_t low, integer_t high)
{
    if (low < high)
    {
        /* pi is partitioning index, arr[pi] is now
        at right place */
        integer_t pi = partition(arr, low, high);

        // Separately sort elements before
        // partition and after partition
        quickSort(arr, low, pi - 1);
        quickSort(arr, pi + 1, high);
    }
}

//
// Function to transform decimal into binary
// n = number | i = binary length -> it will return in reverse order
//
```

```

char decToBinary(integer_t n, integer_t c, int result[], int index)
{
    for (integer_t i = index; i < c + index; i++)
    {
        integer_t k = n >> (i - index);
        if (k & 1)
        {
            result[i] = 1;
        }
        else
        {
            result[i] = 0;
        }
    }
    return 0;
}

//
// Function to make Subset Sums
//

void HorowitzSahniSums(int n, integer_t total1, integer_t total2, const integer_t p[n], integer_t s1[], integer_t s2[], integer_t duplicated_s1[],
integer_t duplicated_s2[])
{
    // Calculating subset sums and duplicating arrays -> Need to find another way to save sums indexes
    // When converting the index to binary and inverting order I will have the result

    integer_t p1[n % 2 + n / 2];
    integer_t p2[n / 2];
    int c = 0;
    for (int j = 0; j < n; j++)
    {
        if (j < (n % 2 + n / 2))
        {
            p1[j] = p[j];
        }
        else
        {
            p2[c] = p[j];
            c++;
        }
    }

    integer_t i = 0;
    while (i < total1)
    {
        integer_t sum1 = 0;
        integer_t sum2 = 0;
        for (int j = 0; j < n; j++)
        {
            if (i & (1ull << j))
            {
                sum1 += p1[j];
                if (i < total2)
                    sum2 += p2[j];
            }
        }
        s1[i] = sum1;
        duplicated_s1[i] = sum1;

        if (i < total2)
        {
            s2[i] = sum2;
            duplicated_s2[i] = sum2;
        }
        i++;
    }

    // Sorting with quick sort method
    quickSort(s1, 0, total1 - 1);
    quickSort(s2, 0, total2 - 1);
}

char HorowitzSahni(integer_t total1, integer_t total2, int lengthP1, int lengthP2, const integer_t s1[], const integer_t s2[], const integer_t
duplicated_s1[], const integer_t duplicated_s2[], integer_t desired_sum, int result[])

```

```

{
    integer_t k = 0;
    integer_t j = total2 - 1;
    while (k < total1 && j >= 0)
    {
        if (s1[k] + s2[j] == desired_sum)
        {
            // finding indexes
            integer_t count = 0;
            while (count < total1)
            {
                if (duplicated_s1[count] == s1[k])
                {
                    decToBinary(count, lengthP1, result, 0);
                    break;
                }
                count++;
            }
            count = 0;
            while (count < total2)
            {
                if (duplicated_s2[count] == s2[j])
                {
                    decToBinary(count, lengthP2, result, lengthP1);
                    break;
                }
                count++;
            }
            return 1;
        }
        else if (s1[k] + s2[j] < desired_sum)
        {
            k++;
        }
        else if (s1[k] + s2[j] > desired_sum)
        {
            j--;
        }
    }
    return 0;
}

//
// Fourth approach to the problem
//

//
// SchroeppeShamirs starts here
//

typedef struct Heap Heap;
struct Heap
{
    // type = 0 (Heap) || type = 1 (MaxHeap)
    char type;
    //
    integer_t *arr;
    // Current Size of the Heap
    integer_t size;
    // Maximum capacity of the heap
    integer_t capacity;
};

integer_t parent(integer_t i)
{
    // Get the index of the parent
    return (i - 1) / 2;
}

integer_t left_child(integer_t i)
{
    return (2 * i + 1);
}

integer_t right_child(integer_t i)

```



```
{
    return (2 * i + 2);
}

Heap *init_heap(integer_t capacity)
{
    Heap *heap = (Heap *)calloc(1ull, sizeof(Heap));
    heap->arr = (integer_t *)calloc(capacity, sizeof(integer_t));
    heap->capacity = capacity;
    heap->size = 0;
    return heap;
}

Heap *insert_heap(Heap *heap, integer_t element, char type)
{
    if (type == 0)
    {
        // Inserts an element to the min heap
        // We first add it to the bottom (last level)
        // of the tree, and keep swapping with it's parent
        // if it is lesser than it. We keep doing that until
        // we reach the root node. So, we will have inserted the
        // element in it's proper position to preserve the min heap property
        if (heap->size == heap->capacity)
        {
            fprintf(stderr, "Cannot insert %llu. Heap is already full!\n", element);
            return heap;
        }
        // We can add it. Increase the size and add it to the end
        heap->size++;
        heap->arr[heap->size - 1ull] = element;

        // Keep swapping until we reach the root
        integer_t curr = heap->size - 1ull;
        // As long as you aren't in the root node, and while the
        // parent of the last element is greater than it
        while (curr > 0 && heap->arr[parent(curr)] > heap->arr[curr])
        {
            // Swap
            integer_t temp = heap->arr[parent(curr)];
            heap->arr[parent(curr)] = heap->arr[curr];
            heap->arr[curr] = temp;
            // Update the current index of element
            curr = parent(curr);
        }
        return heap;
    }
    else if (type == 1)
    {
        // Inserts an element to the max heap
        // We first add it to the bottom (last level)
        // of the tree, and keep swapping with it's parent
        // if it is grater than it. We keep doing that until
        // we reach the root node. So, we will have inserted the
        // element in it's proper position to preserve the max heap property
        if (heap->size == heap->capacity)
        {
            fprintf(stderr, "Cannot insert %llu. Heap is already full!\n", element);
            return heap;
        }
        // We can add it. Increase the size and add it to the end
        heap->size++;
        heap->arr[heap->size - 1ull] = element;

        // Keep swapping until we reach the root
        integer_t curr = heap->size - 1ull;
        // As long as you aren't in the root node, and while the
        // parent of the last element is lesser than it
        while (curr > 0 && heap->arr[parent(curr)] < heap->arr[curr])
        {
            // Swap
            integer_t temp = heap->arr[parent(curr)];
            heap->arr[parent(curr)] = heap->arr[curr];
            heap->arr[curr] = temp;
            // Update the current index of element
```

```
    curr = parent(curr);
}
return heap;
}
return heap;
}

Heap *heapify(Heap *heap, integer_t index, char type)
{
    switch (type)
    {
        case 0:
            // Rearranges the heap as to maintain
            // the min-heap property
            if (heap->size <= 1ull)
                return heap;

            integer_t left = left_child(index);
            integer_t right = right_child(index);

            // Variable to get the smallest element of the subtree
            // of an element an index
            integer_t smallest = index;

            // If the left child is smaller than this element, it is
            // the smallest
            if (left < heap->size && heap->arr[left] < heap->arr[index])
                smallest = left;

            // Similarly for the right, but we are updating the smallest element
            // so that it will definitely give the least element of the subtree
            if (right < heap->size && heap->arr[right] < heap->arr[smallest])
                smallest = right;

            // Now if the current element is not the smallest,
            // swap with the current element. The min heap property
            // is now satisfied for this subtree. We now need to
            // recursively keep doing this until we reach the root node,
            // the point at which there will be no change!
            if (smallest != index)
            {
                integer_t temp = heap->arr[index];
                heap->arr[index] = heap->arr[smallest];
                heap->arr[smallest] = temp;
                heap = heapify(heap, smallest, 0);
            }

            return heap;
        case 1:
            // Rearranges the heap as to maintain
            // the max-heap property
            if (heap->size <= 1ull)
                return heap;

            left = left_child(index);
            right = right_child(index);

            // Variable to get the greatest element of the subtree
            // of an element an index
            integer_t greatest = index;

            // If the left child is greatest than this element, it is
            // the greatest
            if (left < heap->size && heap->arr[left] > heap->arr[index])
                greatest = left;

            // Similarly for the right, but we are updating the greatest element
            // so that it will definitely give the least element of the subtree
            if (right < heap->size && heap->arr[right] > heap->arr[greatest])
                greatest = right;

            // Now if the current element is not the greatest,
            // swap with the current element. The max heap property
            // is now satisfied for this subtree. We now need to
            // recursively keep doing this until we reach the root node,
```

```

        // the point at which there will be no change!
        if (greatest != index)
        {
            integer_t temp = heap->arr[index];
            heap->arr[index] = heap->arr[greatest];
            heap->arr[greatest] = temp;
            heap = heapify(heap, greatest, 1);
        }

        return heap;
    }
    return heap;
}

Heap *delete_minimum(Heap *heap)
{
    // Deletes the minimum element, at the root
    if (!heap || heap->size == 0)
        return heap;

    integer_t size = heap->size;
    integer_t last_element = heap->arr[size - 1];

    // Update root value with the last element
    heap->arr[0] = last_element;

    // Now remove the last element, by decreasing the size
    heap->size--;
    size--;

    // We need to call heapify(), to maintain the min-heap
    // property
    heap = heapify(heap, 0, 0);
    return heap;
}

Heap *delete_maximum(Heap *heap)
{
    // Deletes the maximum element, at the root
    if (!heap || heap->size == 0)
        return heap;

    integer_t size = heap->size;
    integer_t last_element = heap->arr[size - 1];

    // Update root value with the last element
    heap->arr[0] = last_element;

    // Now remove the last element, by decreasing the size
    heap->size--;
    size--;

    // We need to call heapify(), to maintain the max-heap
    // property
    heap = heapify(heap, 0, 1);
    return heap;
}

void free_heap(Heap *heap)
{
    if (!heap)
        return;
    free(heap->arr);
    free(heap);
}

void SchroeppeShamirSums(int n, const integer_t p[n], integer_t lp1, integer_t lp2, integer_t lp3, integer_t lp4, integer_t L1[], integer_t L2[],
integer_t R1[], integer_t R2[])
{
    // Divide P into 4 nearly equal parts
    integer_t p1[lp1];
    integer_t p2[lp2];
    integer_t p3[lp3];
    integer_t p4[lp4];

```

```

integer_t c = 0;
for (integer_t i = 0; i < lp1; i++)
{
    p1[c] = p[i];
    c++;
}
c = 0;
for (integer_t i = lp1; i < lp1 + lp2; i++)
{
    p2[c] = p[i];
    c++;
}
c = 0;
for (integer_t i = lp1 + lp2; i < lp1 + lp2 + lp3; i++)
{
    p3[c] = p[i];
    c++;
}
c = 0;
for (integer_t i = lp1 + lp2 + lp3; i < lp1 + lp2 + lp3 + lp4; i++)
{
    p4[c] = p[i];
    c++;
}

integer_t i = 0;
while (i < (1ull << lp1))
{
    integer_t sum1 = 0;
    integer_t sum2 = 0;
    integer_t sum3 = 0;
    integer_t sum4 = 0;
    for (integer_t j = 0; j < n; j++)
    {
        if (i & (1ull << j))
        {
            sum1 += p1[j];
            if (i < (1 << lp2))
            {
                sum2 += p2[j];
            }
            if (i < (1 << lp3))
            {
                sum3 += p3[j];
            }
            if (i < (1 << lp4))
            {
                sum4 += p4[j];
            }
        }
    }
    if (i < (1 << lp1))
    {
        L1[i] = sum1;
    }
    if (i < (1 << lp2))
    {
        L2[i] = sum2;
    }
    if (i < (1 << lp3))
    {
        R1[i] = sum3;
    }
    if (i < (1 << lp4))
    {
        R2[i] = sum4;
    }
    i++;
}
}

char SchroeppeShamir(integer_t lp1, integer_t lp2, integer_t lp3, integer_t lp4, const integer_t L1[], const integer_t L2[], const integer_t R1[],
const integer_t R2[], Heap *minheap, Heap *maxheap, integer_t desired_sum)
{
    integer_t i = 0, j;

```

```

while (i < 1ull << lp1)
{
    j = 0;
    while (j < 1ull << lp2)
    {
        if (L1[i] + L2[j] == desired_sum)
        {
            return 1;
        }
        if (L1[i] + L2[j] < desired_sum)
        {
            insert_heap(minheap, L1[i] + L2[j], 0);
        }
        j++;
    }
    i++;
}
i = 0;
while (i < 1ull << lp3)
{
    j = 0;
    while (j < 1ull << lp4)
    {
        if (R1[i] + R2[j] == desired_sum)
        {
            return 1;
        }
        if (R1[i] + R2[j] < desired_sum)
        {
            insert_heap(maxheap, R1[i] + R2[j], 1);
        }
        j++;
    }
    i++;
}
integer_t counter = 0;
while (1)
{
    if (minheap->arr[0] + maxheap->arr[0] == desired_sum)
    {
        return 1;
    }
    else if (minheap->arr[0] + maxheap->arr[0] > desired_sum)
    {
        delete_maximum(maxheap);
    }
    else if (minheap->arr[0] + maxheap->arr[0] < desired_sum)
    {
        delete_minimum(minheap);
    }
    if (counter > (minheap->capacity * maxheap->capacity))
    {
        break;
    }
    counter++;
}
return 0;
}

//
// main program
//

int main(void)
{
    /*
    fprintf(stderr, "Program configuration:\n");
    fprintf(stderr, "  min_n ..... %d\n", min_n);
    fprintf(stderr, "  max_n ..... %d\n", max_n);
    fprintf(stderr, "  n_sums ..... %d\n", n_sums);
    fprintf(stderr, "  n_problems .. %d\n", n_problems);
    fprintf(stderr, "  integer_t ... %d bits\n", 8 * (int)sizeof(integer_t));
    */
    FILE *fp1 = fopen("data.txt", "w");
    FILE *fp2 = fopen("results.txt", "w");

```

```

for (int i = 0; i < 41; i++)
{
    int n = all_subset_sum_problems[i].n; // The value of n
    integer_t *p = all_subset_sum_problems[i].p; // The weights

    char found;
    int result[n]; // Array with the result
    for (int index = 0; index < n; index++)
        result[index] = 0;

    // Making Subset Sums for HorowitzSahni()
    double timeSumsH1 = cpu_time();
    int lengthP1 = n % 2 + n / 2;
    int lengthP2 = n / 2;
    integer_t total1 = (1 << lengthP1); // s1 length = (2^lengthP1)
    integer_t total2 = (1 << lengthP2); // s2 length = (2^lengthP2)
    integer_t *s1, *s2, *duplicated_s1, *duplicated_s2;
    s1 = (integer_t *)malloc(total1 * sizeof(integer_t));
    s2 = (integer_t *)malloc(total2 * sizeof(integer_t));
    duplicated_s1 = (integer_t *)malloc(total1 * sizeof(integer_t));
    duplicated_s2 = (integer_t *)malloc(total2 * sizeof(integer_t));

    HorowitzSahniSums(n, total1, total2, p, s1, s2, duplicated_s1, duplicated_s2);
    double timeSumsH2 = cpu_time();
    double timeSumsH = (timeSumsH2 - timeSumsH1)/(double)20;

    // Making Subset Sums for SchroeppeShamir()
    double timeSumsS1 = cpu_time();
    int len1 = n % 2 + n / 2;
    int len2 = n / 2;
    integer_t lp1 = len1 % 2 + len1 / 2;
    integer_t lp2 = len1 / 2;
    integer_t lp3 = len2 % 2 + len2 / 2;
    integer_t lp4 = len2 / 2;
    // Declaring Sum of all p L1 & L2 for minheap and R1 & R2 for max heap size = 2^lp
    integer_t L1[(1 << lp1)];
    integer_t L2[(1 << lp2)];
    integer_t R1[(1 << lp3)];
    integer_t R2[(1 << lp4)];

    SchroeppeShamirSums(n, p, lp1, lp2, lp3, lp4, L1, L2, R1, R2);
    integer_t min_size = 1ull << len1;
    integer_t max_size = 1ull << len2;
    double timeSumsS2 = cpu_time();
    double timeSumsS = (timeSumsS2 - timeSumsS1)/(double)20;

    for (int k = 0; k < n_sums; k++)
    {
        integer_t desired_sum = all_subset_sum_problems[i].sums[k]; // The desire_sum

        //
        // bruteForce() ---> id = 0
        //

        if (n <= 30)
        {
            double t1 = cpu_time();
            found = bruteForceV2(n, p, desired_sum, result);
            double t2 = cpu_time();
            fprintf(fp1, "%d %d %d %d %f\n", 0, n, k, found, t2 - t1);
        }

        //
        // branchAndBound() ---> id = 1
        //

        if (n <= 40)
        {
            double t1 = cpu_time();
            found = branchAndBound (n, p, desired_sum, 0, 0, result);
            double t2 = cpu_time();
            fprintf(fp1, "%d %d %d %d %f\n", 1, n, k, found, t2 - t1);
        }
    }
}

```

```
//
// HorowitzSahni() ---> id = 2
//

double timeAlgorithm1 = cpu_time();
found = HorowitzSahni(total1, total2, lengthP1, lengthP2, s1, s2, duplicated_s1, duplicated_s2, desired_sum, result);
double timeAlgorithm2 = cpu_time();
fprintf(fp1, "%d %d %d %d %f\n", 2, n, k, found, (timeAlgorithm2 - timeAlgorithm1) + timeSumsH);

//
// SchroeppeShamir() ---> id = 3
//

timeAlgorithm1 = cpu_time();
Heap *minheap = init_heap(min_size);
Heap *maxheap = init_heap(max_size);
found = SchroeppeShamir(lp1, lp2, lp3, lp4, L1, L2, R1, R2, minheap, maxheap, desired_sum);
timeAlgorithm2 = cpu_time();
fprintf(fp1, "%d %d %d %d %f\n", 3, n, k, found, (timeAlgorithm2 - timeAlgorithm1) + timeSumsS);
free_heap(minheap);
free_heap(maxheap);

// Write results to the file
fprintf(fp2, "For n: %d | IdSum = %d | Found: %d | Result: ", n, k, found);
for (int j = 0; j < n; j++)
    fprintf(fp2, "%d", result[j]);
fprintf(fp2, "\n");
}
free(s1);
free(s2);
free(duplicated_s1);
free(duplicated_s2);
}
fclose(fp1);
fclose(fp2);
return 0;
}
```

Código Matlab dos Gráficos

```
M = readmatrix("data_102851.txt");

timingsM0 = reshape(M(M(:, 1) == 0, 5), 20, []);
minM0 = min(timingsM0);
maxM0 = max(timingsM0);
averageM0 = mean(timingsM0);

timingsM1 = reshape(M(M(:, 1) == 1, 5), 20, []);
minM1 = min(timingsM1);
maxM1 = max(timingsM1);
averageM1 = mean(timingsM1);

timingsM2 = reshape(M(M(:, 1) == 2, 5), 20, []);
minM2 = min(timingsM2);
maxM2 = max(timingsM2);
averageM2 = mean(timingsM2);

timingsM3 = reshape(M(M(:, 1) == 3, 5), 20, []);
minM3 = min(timingsM3);
maxM3 = max(timingsM3);
averageM3 = mean(timingsM3);

figure(1);
semilogy((1:length(averageM0)) + 9, averageM0, ".-", 'DisplayName', "Média"); hold on;
semilogy((1:length(minM0)) + 9, minM0, ".-", 'DisplayName', "Mínimo"); hold on;
semilogy((1:length(maxM0)) + 9, maxM0, ".-", 'DisplayName', "Máximo"); hold on;
xlabel("Valores de N");
ylabel("Tempo(segundos)");
title("Brute Force");
grid on;
legend;

figure(2);
semilogy((1:length(averageM1)) + 9, averageM1, ".-", 'DisplayName', "Média"); hold on;
semilogy((1:length(minM1)) + 9, minM1, ".-", 'DisplayName', "Mínimo"); hold on;
semilogy((1:length(maxM1)) + 9, maxM1, ".-", 'DisplayName', "Máximo"); hold on;
xlabel("Valores de N");
ylabel("Tempo(segundos)");
grid on;
title("Branch and Bound");
legend;

figure(3);
semilogy((1:length(averageM2)) + 9, averageM2, ".-", 'DisplayName', "HS - Média"); hold on;
semilogy((1:length(minM2)) + 9, minM2, ".-", 'DisplayName', "HS - Mínimo"); hold on;
semilogy((1:length(maxM2)) + 9, maxM2, ".-", 'DisplayName', "HS - Máximo"); hold on;
xlabel("Valores de N");
ylabel("Tempo(segundos)");
grid on;
title("Horowitz and Sahni");
legend;

figure(4);
semilogy((1:length(averageM3)) + 9, averageM3, ".-", 'DisplayName', "Média"); hold on;
semilogy((1:length(minM3)) + 9, minM3, ".-", 'DisplayName', "Mínimo"); hold on;
semilogy((1:length(maxM3)) + 9, maxM3, ".-", 'DisplayName', "Máximo"); hold on;
```



```
xlabel("Valores de N");  
ylabel("Tempo(segundos)");  
grid on;  
title("Schroeppel and Shamir");  
legend;
```

```
figure(5);  
semilogy((1:length(averageM0)) + 9, averageM0, ".-", 'DisplayName', "BF - Média");  
hold on;  
semilogy((1:length(averageM1)) + 9, averageM1, ".-", 'DisplayName', "BB - Média");  
hold on;  
semilogy((1:length(averageM2)) + 9, averageM2, ".-", 'DisplayName', "HS - Média");  
hold on;  
semilogy((1:length(averageM3)) + 9, averageM3, ".-", 'DisplayName', "SS - Média");  
hold on;  
xlabel("Valores de N");  
ylabel("Tempo(segundos)");  
grid on;  
legend;
```

```
figure(6);  
semilogy((1:length(averageM0)) + 9, averageM0, ".-", 'DisplayName', "BF - Média");  
hold on;  
semilogy((1:length(minM0)) + 9, minM0, ".-", 'DisplayName', "BF - Mínimo"); hold on;  
semilogy((1:length(maxM0)) + 9, maxM0, ".-", 'DisplayName', "BF - Máximo"); hold on;  
  
semilogy((1:length(averageM1)) + 9, averageM1, ".-", 'DisplayName', "BB - Média");  
hold on;  
semilogy((1:length(minM1)) + 9, minM1, ".-", 'DisplayName', "BB - Mínimo"); hold on;  
semilogy((1:length(maxM1)) + 9, maxM1, ".-", 'DisplayName', "BB - Máximo"); hold on;  
  
semilogy((1:length(averageM2)) + 9, averageM2, ".-", 'DisplayName', "HS - Média");  
hold on;  
semilogy((1:length(minM2)) + 9, minM2, ".-", 'DisplayName', "HS - Mínimo"); hold on;  
semilogy((1:length(maxM2)) + 9, maxM2, ".-", 'DisplayName', "HS - Máximo"); hold on;  
  
semilogy((1:length(averageM3)) + 9, averageM3, ".-", 'DisplayName', "SS - Média");  
hold on;  
semilogy((1:length(minM3)) + 9, minM3, ".-", 'DisplayName', "SS - Mínimo"); hold on;  
semilogy((1:length(maxM3)) + 9, maxM3, ".-", 'DisplayName', "SS - Máximo"); hold on;  
xlabel("Valores de N");  
ylabel("Tempo(segundos)");  
grid on;  
legend;
```

Resultados

103360	102851	103925
1101100001	1111100110	0100001100
0100110001	0010101111	0010111101
0110010000	1100011010	0110100110
0101111011	0011010101	0011110111
1100001010	1100111110	1001001101
1001101110	0110011001	1110011100
1010111001	1011010100	0010000011
0101010010	1011000111	1010011101
1101110100	1010000001	0010110100
0100011000	1110100010	1011001010
0110110000	1001101011	1111110011
1110000100	0110010000	1001111110
0001110001	0010110011	0011111101
0111111000	1100011011	1101000010
0010110110	1011100001	0000001000
1000100010	1001101010	1000000000
0011110101	0111010100	1111000111
1110010001	0001011101	1111011111
0111111110	1010111111	0110010000
0010001110	1111010101	1110001010
00011110000	01011101101	0110111000
00000110101	11011100101	11101001101
00010001010	01100111010	00100111110
10000010101	00101000100	00111100100
11100000101	10000001110	01011000101
11010101010	00100111111	01010000011
11111111011	00001000001	10100110100
01110000001	10101101010	00000110100
01011101110	00000010111	10111110111
01110001011	10111001000	10010000000
10000001000	01100110010	01110011010
01001010000	10101101111	11111110001
01000101000	00001001101	10000001111
01001110001	10001001000	10000100110
11110101111	11110001100	00110011000
01100110010	01101111101	01010110111
11100111000	01110101101	11111111110
10001100101	00011000110	01011101110
01101111001	11010011001	10101110011
01010101110	01011100011	01011011110
011011011010	111111000110	100100001100
100100111010	101110101000	100011001110
111000011100	001111010101	011011000001
001111010011	001111011111	011001010000
001001110000	110001101000	000100111000
101011010101	101000100101	010101111011
001011111110	110000110011	010111101000
001101011100	100010001111	000101000100
010010010111	100001101001	111001100001
011111101101	010001010111	011110001100
001100100100	001001110110	000111101011
110100110101	111101001100	001000111110
000111101111	101011011001	101110101101
101110100011	110010100100	011111111100
100111001001	111101100011	110000110001
001011101100	011011000001	010100110011
100101000001	001111010000	110100110010
001111101010	001111010111	111100011011
011100110010	101010011010	010101010111
110000001011	000001010100	110111110110
1110111000101	1010000100110	0010101000011
1101101011101	1101010110101	1010101100101
1010011000010	1110110010011	0101111000010
0010100000010	1000110110100	0000100000110
1100110111011	0110000000100	1101111010001
1001111100000	0101111000001	

111011111110	0110001011001	0100100110110
0000000111001	0101011010011	0101001010010
0000100010101	1000011110011	1110110110000
0001011001000	1010010111011	0000000000100
1101111000100	0100110010110	1010010011000
0011010001000	1101110110101	1011111100000
1011101001000	1011001101000	1110001010011
1111001011001	0000111001101	0010111111010
1110010011110	1111000111000	1111001110110
0000001010110	1111000101111	0001000111110
1001100000010	1010011010110	0010011100111
1000010001010	1101011010100	1001110101000
1110101100100	0001010011101	0000011111101
1011001001000	0011000001010	1100111001001
00100010111011	11000001001000	00010110111011
00011001011000	00010101100100	10110011110100
11011011001001	01001111000111	11010000101100
01101001001000	10001000111001	10000000100110
01011110010100	10101001100101	10110001011010
10111001000110	10110010101101	00001110000100
01101100100110	00111000100101	01011011001010
01111010001100	11001110001111	01010011111111
01010001100111	01100011110011	10001110111001
11111010100000	11111001000110	11010001110010
00111010111000	10000010100000	01100100101000
11001110000000	00100000110001	10110011000000
10001010101000	10111111010111	00110010101010
00011011001110	01111011000010	11000111000001
00110001001111	01001110111000	01000100110000
01110000000100	00110101010001	11101100011000
00100011011101	01000101010100	00100010111111
11101011101111	00010101101011	11100011111001
00100100110001	10001110110101	01001101101011
01101001010100	11000010110011	10100111111111
101111010000011	000111011010111	111011100101011
001101101000001	011100010111010	101111011011010
000100111101101	111010000001011	111100001100101
011011011111011	101011101001001	000001111000110
011111000011010	0110010111110110	101110110010101
001000110100100	100011011101010	100111111001000
011011000111011	111110111111010	011001111100000
011110011111100	101011110010100	000101010110110
000001000111111	001101111000101	100001010000101
001111110111111	011011000010111	110010111110100
100010011111100	000010000100010	101101111111000
101111110011000	100110011010011	001001000001110
100001011011011	100001011010111	101110110000000
111101101010110	010000010100010	110000100111110
101000111011110	100110110001100	000000010011000
110001100101100	011110101100100	010110001100101
110111110101011	000101110100110	101101100010100
001010011000111	101110100010110	111011011101001
011110101100111	111110000010101	011101000111011
000010101011000	100011100100110	110001100100000
1000110010110010	100010010011110	1011100100110010
0111101010111011	1000010010100001	0000111101011100
0110000111010010	1110100010100000	1110100011011111
0101011011101000	1000100010100111	1111101010011100
0011001101101011	1111000011111110	1011001100000111
1000011101110001	0001110101100001	1010001000001110
0010110000110111	0100110110100110	1110111010100000
1001101010000001	0011101111100010	1110010101111100
1011010101010100	1110010110110011	1010001001001101
1000001011011101	0100100001100111	0000010010100011
1100001010001011	0111000101101011	0000001101001111
0001111000110100	0111100110101010	1011110001110000
0011000111010101	0100011000110010	0010100011101010
1000111111111000	1110111000110100	1111000001110100
1010010110110000	0111011110001001	1101110101100110
0010000000100111	1011001100001100	1100001100100100
0001110001100011	0111010010111101	0101111101000001
0100100110101111	1000001000100101	0111110101100110
0101000011100000	0011100110000101	1001111001001010
1100110100000011	0110111110010000	1110010101110011
01011000000101100	01100110001000100	01000111101000000



01010100100000001	10001111011001011	11100100111110101
01110101011000100	11001101001100001	00101100101011010
10000100000110101	11101100110101100	001001011011000010
01111110110111110	00010101000111110	01110111110000110
01111110011100011	11111001111101010	10111011101011110
11110011100011101	10101011000110010	11101000100111011
01001100010011101	10101110101010001	10010111011001110
11101110101110111	01110100000011010	00000110001011011
10011000110011101	10011111110000110	01111110011001111
11001001101100110	10001001110111110	11101011000010100
00010101001001010	01001011110100001	01100101100001010
11000110000000110	00011011110010001	01101110001000101
01000101100001111	01111100111100101	01001010111111110
01010010001100010	00010110000001110	00111100000111110
00100111010010011	00110000010010101	11100111100111100
00100101110110110	01100100101000100	00011000000001011
11011011010111110	10100100100100101	11001110011110100
10011101100000011	10100110000100110	11111101111000011
11100001111001000	11000011101111100	10111000000010010
100101110110001101	000111010110100001	111011011101100001
100000111011010110	110000110001010111	101110101100000010
01010000000101111	010111110010111010	010010001011100010
111100100110111010	000111010100101001	011001110010110010
110111111110000000	011101001001001001	001110000011111110
001100101011001101	011001101010010111	001111001001111001
101010001000111010	100010001100110011	010111000000000001
000100010000111110	100111100100001011	010110000011010001
100110000110010100	110100110000010101	000100001101001011
0001001111111101000	111111111011110110	100100101110011111
101100100000011111	110101100010100100	101011000101001011
100110110010101011	111010000000010000	101000000011101010
100101010010010110	011000110100011000	111000100010010011
101001001111110100	000110001111110111	001111000111101100
000110010111011101	001100111110110101	011010101101001110
011010011101000110	110110000111001100	101110110011110001
111110011011100001	0000110110111010000	110110110001111110
111001010110101111	100111010111101001	101001101010101110
01011111111101111	011001010011010011	111010001100101011
000010010011000010	010011010111010000	011001011001001110
0001110100110111100	1000111111110110101	1001111100000111010
1010011101000001001	1100011001000100111	0101110111000110010
0100000100010110010	0000010011010000101	1100001110001010101
1101011111100011110	0000010101101011001	1010011111100110010
1000000110001010010	0001000001100011010	1101010010110101100
0110111001101100011	0100010101000100001	0010000101111111110
1100110110101100010	0000011010000011010	1110100001000000111
1101010111101011010	0100101110000111110	0011010011001100100
0010001011101011011	0111001000110001110	1011101111111010000
0100010110011111001	1000001111110001101	1100101111101100110
0001000010011010011	1001101010100111001	1100000000110011011
0110111101001010011	1100001110101011100	0001010000010010000
0110011101001100110	0110010111100101110	0111011010001011010
0100001110001110000	1101100111100010011	0111011110111110110
1101010011001110010	1011001001101100000	1100110110001111100
0110011111010011110	0110110101000100011	1001011110101010101
0001101001000001101	0111111010111101010	1101100001011000111
0101000010011110110	1000101011110010110	0101001111001101000
0011000000111010111	0111100100101010010	0001100010001010100
0110010101011001100	0111110101001000110	1000000011101110001
00110100110011100010	00100011110110100100	11100101101110101000
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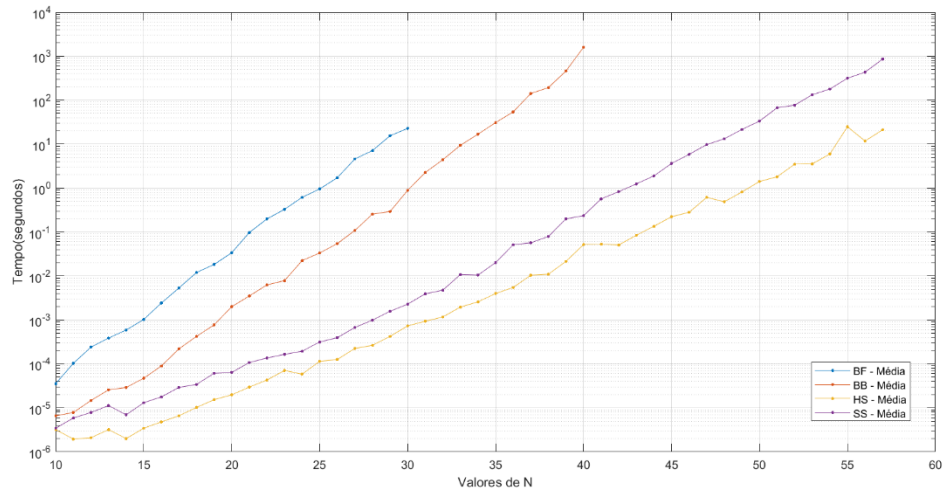
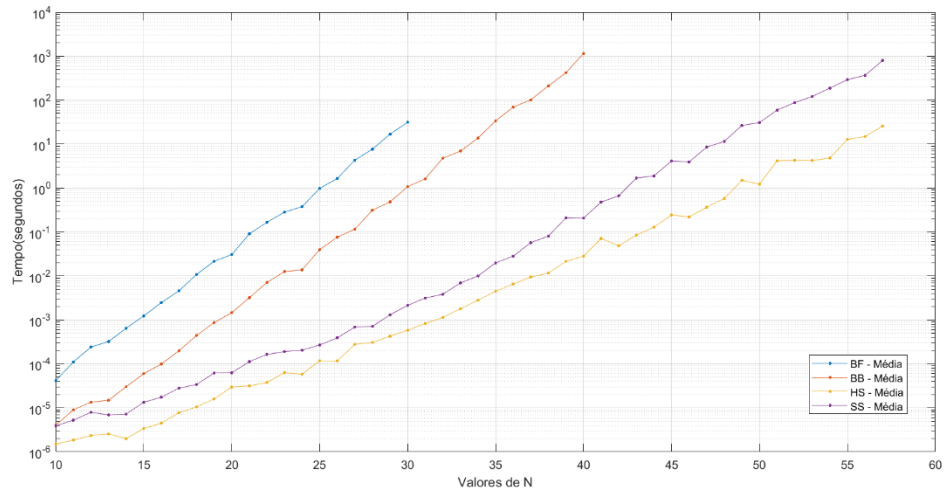
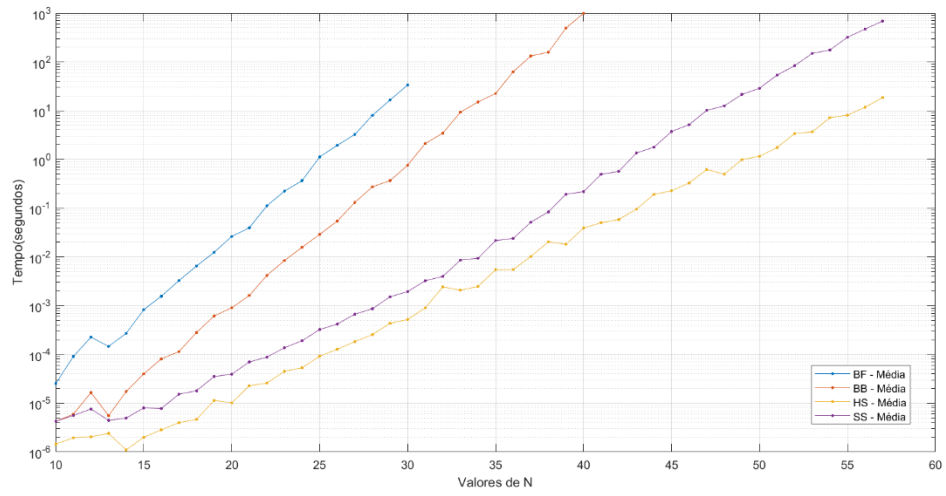
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Conclusões

Observando os gráficos torna-se claro que a técnica de bruteForce é pouco consistente, visto depender muito da tentativa em que encontra a solução e bastante ineficiente, sendo inviável para problemas muito grandes. Mesmo a segunda função "branch and Bound", que traz algumas otimizações a técnica, continua a ter os mesmos problemas, o que a demonstra consideravelmente pior que a técnica de meet in the middle.

Seria uma opção a considerar, apenas em programas destinados a resolver problemas pequenos, devido à maior facilidade de implementação.

A melhor opção para o problema que nos foi proposto seria a terceira função, a técnica de Horowitz e Sahni a qual se provou ser a mais rápida de todas.



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