

# Formalizing Rust's Type System

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# Date of the exam

Formalizing Rust's  
Type System

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The date of the exam is March 9th, 2022 (12:45).

If you cannot be available (another exam...), speak **now!**

This is the last session for this course.

Next week (March 2nd) is free time for reviewing!

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# Abstract from last weeks

Formalizing Rust's  
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During the last three weeks, we had an introduction to Rust programming

- Type system enforces: “mutation XOR aliasing”,
- Traits: an abstraction mechanism, sometimes at zero-cost.
- Unsafe blocks/functions: workaround strong static type-checking constraints,
- Encapsulation: clients can safely use libraries written with unsafe code.
- Interior mutability (the ability to mutate through shared borrows): a typical example of well-encapsulated unsafe code.
- Rust is a good fit for multithreading.
  - Protects against data races (with `Send` and `Sync`).
  - Provides abstractions for multithreading.

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I keep telling you Rust is type-safe.  
How can I be so sure?

I.e., is this something one can prove formally?

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# The problem we are trying to solve

Formalizing Rust's  
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“Well-typed Rust programs do not go wrong.”

Really?

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# The problem we are trying to solve

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“Well-typed Rust programs **not using unsafe** do not go wrong.”

Is that all?

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"Well-typed Rust programs **not using** `unsafe` do not go wrong."

A vast majority of real-life Rust programs (directly or indirectly) use unsafe code.  
Example: `Vec`, interior mutability (e.g., `Cell`), low-level optimizations, ...

We want an **extensible** theorem to prove those programs safe too.

- Safe pieces of code are safe thanks to **syntactic typing rules**.
- Unsafe pieces are safe thanks to a **specialized proof**.

Both kinds of proofs will be linked together thanks to a **logical relation**.

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## Logical relations for type safety

## The general approach

A type system is defined from:

- One (or several) **typing judgment(s)**  $\dots \vdash \dots$
- Syntactic **typing rules**.

Ex. for lambda-calculus: 
$$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2}$$

The logical relation approach to type soundness (i.e., semantic type soundness):

- 1 Define a **semantic typing judgment**  $\dots \models \dots$  from the operational semantics.
- 2 Prove the **fundamental theorem**: " $\dots \vdash \dots \Rightarrow \dots \models \dots$ "
- 3 Prove **adequacy** for the logical relation: " $\dots \models \dots \Rightarrow \text{safety}$ ".
  - "Semantically well-typed programs do not go wrong."
  - Usually easy from the definition of  $\dots \models \dots$ .

Why is this approach more extensible than subject reduction+progress?

# Extending semantic type safety

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The fundamental theorem: by induction over the typing tree using semantic typing rules:

$$\frac{\begin{array}{c} \dots \vdash \dots \\ \dots \vdash \dots \\ \vdots \qquad \dots \vdash \dots \\ \hline \dots \vdash \dots \end{array} \quad \dots \vdash \dots}{\dots \vdash \dots} \Rightarrow \frac{\begin{array}{c} \dots \models \dots \\ \dots \models \dots \\ \vdots \qquad \dots \models \dots \\ \hline \dots \models \dots \end{array} \quad \dots \models \dots}{\dots \models \dots}$$

# Extending semantic type safety

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The fundamental theorem: by induction over the typing tree using semantic typing rules:

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To prove  $\dots \models \dots$  for the whole program, we don't need  $\dots \vdash \dots$  everywhere.  
We can fill some "holes" using custom proofs of  $\dots \models \dots$ .

# Extending semantic type safety

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`Cell` is not syntactically well-typed in safe Rust: “ $\dots \not\vdash \text{Cell}::\text{get} : \dots$ ”.

But we can prove “ $\dots \models \text{Cell}::\text{get} : \dots$ ”.

Combining with an otherwise syntactically well-typed program:

$$\frac{\begin{array}{c} \dots \vdash \dots \\ \hline \dots \vdash \dots \\ \vdots \\ \dots \models \text{Cell}::\text{get} : \dots \\ \hline \dots \models \dots & \dots \vdash \dots \\ \hline \dots \models \dots \end{array}}{\dots \models \dots}$$

And we can conclude safety thanks to adequacy!

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# What's left to be done...

- Define formally the language and its syntactic type system.
- Define the logical relation and prove adequacy+fundamental theorem.
- Extend the logical relation for types like `Cell`...

That's a lot of work.

We will only see a **sketch here**.

The details are in a paper, and the accompanying Coq development:

*RustBelt: Securing the foundations of the Rust programming language. In POPL '18.*

To make sure nothing is wrong, this is **formalized with Coq**. Particularly **useful** to design and maintain the proof.

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# The language: $\lambda_{\text{Rust}}$

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Rust is **way too complex** to be formalized as-is.

Should we formalize MIR (recall: the language behind the borrow checker)?

It still has a lot of technicalities unrelated to the type system, refers to traits...

Instead, we formalize a **core language**.

It has most of the features of MIR relevant for type soundness, but **idealized** to make the theory simpler.

# The language: $\lambda_{\text{Rust}}$

Paths, elementary values

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$\text{Val} \ni v ::= \text{false} \mid \text{true} \mid z \mid \ell \mid \text{funrec } f(\bar{x}) \text{ ret } k := F$

$\text{Path} \ni p ::= x \mid p.n$

Elementary values that can be stored in local registers  $x, \dots$

Complex values (e.g., `struct`) are stored in memory, and accessed through pointers.

Note: function values take a continuation in parameter (language in CFG, see later).

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# The language: $\lambda_{\text{Rust}}$

Paths, elementary values

$Val \ni v ::= \text{false} \mid \text{true} \mid z \mid \ell \mid \text{funrec } f(\bar{x}) \text{ ret } k := F$

$Path \ni p ::= x \mid p.n$

Pointers to fields of complex values can be created with [paths](#).

$p.n$ : pointer to field at offset  $n$  of [struct](#) pointed to by  $p$ .

Operationally: just a pointer offset (i.e., an addition).

$$\begin{aligned} Instr \ni I ::= & v \mid p \mid p_1 + p_2 \mid p_1 - p_2 \mid p_1 \leq p_2 \mid p_1 == p_2 \\ & \mid \text{new}(n) \mid \text{delete}(n, p) \mid {}^*p \mid p_1 := p_2 \mid p_1 :=_n {}^*p_2 \\ & \mid \dots \end{aligned}$$

Instructions are the elementary operations of  $\lambda_{\text{Rust}}$ .

They take **paths** as operands.

# The language: $\lambda_{\text{Rust}}$

Instructions

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$$\begin{aligned} \text{Instr} \ni I ::= & v \mid p \mid p_1 + p_2 \mid p_1 - p_2 \mid p_1 \leq p_2 \mid p_1 == p_2 \\ & \mid \text{new}(n) \mid \text{delete}(n, p) \mid {}^*p \mid p_1 := p_2 \mid p_1 :=_n {}^*p_2 \\ & \mid \dots \end{aligned}$$

$\lambda_{\text{Rust}}$ 's instructions include values, paths, arithmetic operations...

# The language: $\lambda_{\text{Rust}}$

Instructions

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$\text{Instr} \ni I ::= v \mid p \mid p_1 + p_2 \mid p_1 - p_2 \mid p_1 \leq p_2 \mid p_1 == p_2$   
 $\quad \mid \text{new}(n) \mid \text{delete}(n, p) \mid {}^*p \mid p_1 := p_2 \mid p_1 :=_n {}^*p_2$   
 $\quad \mid \dots$

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... and instruction to allocate/free memory, and read/write it.

Read/write one word from/to a register:  ${}^*p$ ,  $p_1 := p_2$ .

- Operational semantics defined so that races  $\Rightarrow$  undefined behavior.

Copy a complex value (several words) from one memory location to another:  $p_1 :=_n {}^*p_2$ .

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# The language: $\lambda_{\text{Rust}}$

## Function bodies

$$\begin{aligned} \textit{FuncBody} \ni F ::= & \text{letcont } k(\bar{x}) := F_1 \text{ in } F_2 \mid \text{jump } k(\bar{x}) \\ & \mid \text{let } x = l \text{ in } F \mid \text{if } p \text{ then } F_1 \text{ else } F_2 \\ & \mid \text{call } f(\bar{x}) \text{ ret } k \\ & \mid \text{newlft;} F \mid \text{endlft;} F \\ & \mid \dots \end{aligned}$$

**Function bodies** combine instructions to build functions.

Code is written in CPS: a way to model code written as a CFG.

We can declare a new continuation (i.e., label in the CFG), and jump to it.

# The language: $\lambda_{\text{Rust}}$

## Function bodies

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Basic function bodies:

- An instruction (+ binding the result to a variable).
- If-then-else.
- Calling a function (passing a continuation).

# The language: $\lambda_{\text{Rust}}$

## Function bodies

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$$\begin{aligned} \mathit{FuncBody} \ni F ::= & \text{letcont } k(\bar{x}) := F_1 \text{ in } F_2 \mid \text{jump } k(\bar{x}) \\ & \mid \text{let } x = l \text{ in } F \mid \text{if } p \text{ then } F_1 \text{ else } F_2 \\ & \mid \text{call } f(\bar{x}) \text{ ret } k \\ & \mid \text{newlft;} F \mid \text{endlft;} F \\ & \mid \dots \end{aligned}$$

Ghost instructions for creating/ending a lifetime.

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$$\begin{aligned} Instr \ni I ::= & \dots \\ & | p \stackrel{\text{inj } i}{\equiv} () \mid p_1 \stackrel{\text{inj } i}{\equiv} p_2 \mid p_1 \stackrel{\text{inj } i}{\equiv}_n * p_2 \\ & | \dots \end{aligned}$$

$$\begin{aligned} FuncBody \ni F ::= & \dots \\ & | \text{case } *p \text{ of } \bar{F} \end{aligned}$$

λ<sub>Rust</sub> has a notion of sum types, stored in memory, with a tag in the first word.

There are special instructions to write such values together with the tag.

And a case statement for “pattern matching” the in-memory tag.

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$$\begin{aligned} \text{Type } \exists \tau ::= & \text{bool} \mid \text{int} \mid \text{own } \tau \mid \&_{\text{mut}}^{\kappa} \tau \mid \&_{\text{shr}}^{\kappa} \tau \mid \not\in n \\ & \mid \Pi \bar{\tau} \mid \Sigma \bar{\tau} \mid \forall \bar{\alpha}. \text{fn}(F : E; \bar{\tau}) \rightarrow \tau \mid T \mid \mu T. \tau \\ & \mid \dots \end{aligned}$$

Types for integer and Booleans.

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*Type*  $\exists \tau ::= \mathbf{bool} \mid \mathbf{int} \mid \mathbf{own} \, \tau \mid \&_{\mathbf{mut}}^{\kappa} \tau \mid \&_{\mathbf{shr}}^{\kappa} \tau \mid \not{\in} n$   
 $\mid \Pi \bar{\tau} \mid \Sigma \bar{\tau} \mid \forall \bar{\alpha}. \mathbf{fn}(F : E; \bar{\tau}) \rightarrow \tau \mid T \mid \mu T. \tau$   
 $\mid \dots$

Types for pointers: **Box** is written **own**  $\tau$ .

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$Type \exists \tau ::= \mathbf{bool} \mid \mathbf{int} \mid \mathbf{own} \tau \mid \&_{\mathbf{mut}}^{\kappa} \tau \mid \&_{\mathbf{shr}}^{\kappa} \tau \mid \textcolor{brown}{\&}_n$   
 $\mid \Pi \bar{\tau} \mid \Sigma \bar{\tau} \mid \forall \bar{\alpha}. \mathbf{fn}(F : E; \bar{\tau}) \rightarrow \tau \mid T \mid \mu T. \tau$   
 $\mid \dots$

Type of “uninitialized memory”.

- When memory is just initialized, or when non-**Copy** values are moved.
- Used to replace the “initializedness” analysis of the borrow checker.

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$$\begin{aligned} \text{Type } \exists \tau ::= & \text{bool} \mid \text{int} \mid \text{own } \tau \mid \&_{\text{mut}}^{\kappa} \tau \mid \&_{\text{shr}}^{\kappa} \tau \mid \not{\downarrow} n \\ & \mid \Pi \bar{\tau} \mid \Sigma \bar{\tau} \mid \forall \bar{\alpha}. \text{fn}(\mathcal{F} : E; \bar{\tau}) \rightarrow \tau \mid T \mid \mu T. \tau \\ & \mid \dots \end{aligned}$$

Complex types: sum, products.

Used to model `struct`, `enum` and tuples.

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*Type*  $\exists \tau ::= \mathbf{bool} \mid \mathbf{int} \mid \mathbf{own} \tau \mid \&_{\mathbf{mut}}^{\kappa} \tau \mid \&_{\mathbf{shr}}^{\kappa} \tau \mid \not\vdash n$   
 $\mid \Pi \bar{\tau} \mid \Sigma \bar{\tau} \mid \forall \bar{\alpha}. \mathbf{fn}(F : E; \bar{\tau}) \rightarrow \tau \mid T \mid \mu T. \tau$   
 $\mid \dots$

Type of function pointers, guarded equirecursive types...

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 $\mid \Pi \bar{\tau} \mid \Sigma \bar{\tau} \mid \forall \bar{\alpha}. \mathbf{fn}(F : E; \bar{\tau}) \rightarrow \tau \mid T \mid \mu T. \tau$   
 $\mid \dots$

Extensible for types providing safe abstraction over **unsafe**!

# Typing judgments

There are two main typing judgments:

- One for instructions:  $E; L \mid T_1 \vdash I \dashv x. T_2$
- One for function bodies:  $E; L \mid K; T \vdash F$

They depend on four different kinds of contexts:

- $T$  is the **typing context**. Elements:
  - $p \triangleleft \tau$ : path  $p$  contains an elementary value of type  $\tau$ .
  - $p \triangleleft^{\dagger\kappa} \tau$ : same, but **frozen** until lifetime  $\kappa$  ends.
  - **Substructural**: elements cannot be duplicated (ownership tracking!).
- $E$  and  $L$  are **lifetime contexts**. They contain:
  - information on **lifetime inclusion**, and
  - information on **local lifetimes**: which are they, and when they can be ended.
- $K$  is the **continuation context**.
  - Describes the continuations we can jump to, and the required contexts.
  - Elements:  $k \triangleleft \text{cont}(L; \bar{x}. T)$ .  
“We can jump to  $k$ , passing parameters  $\bar{x}$ , if the contexts contain  $L$  and  $T$ .”

# Typing judgments

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# Typing judgments

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"We can jump to  $k$ , passing parameters  $\bar{x}$ , if the contexts contain  $L$  and  $T$ ."

# Typing judgments

There are two main typing judgments:

- One for instructions:  $E; L \mid T_1 \vdash I \dashv x. T_2$
- One for function bodies:  $E; L \mid K; T \vdash F$

The typing context for instructions has two typing contexts **an input typing context  $T_1$**  and **an output typing context  $T_2$** .

The instruction  $I$  **consumes  $T_1$**  and **produces  $T_2$** .

$x. T_2$  binds a special variable  $x$ : to give a type to the result of the instruction.

Function bodies never return (CPS). They only consume  $T$ .

# Typing rules

## Typing instructions

Selected rules:

$$E; L \mid \emptyset \vdash z \dashv x. x \triangleleft \mathbf{int}$$

$$E; L \mid p_1 \triangleleft \mathbf{int}, p_2 \triangleleft \mathbf{int} \vdash p_1 \leq p_2 \dashv x. x \triangleleft \mathbf{bool}$$

$$E; L \mid \emptyset \vdash \mathbf{new}(n) \dashv x. x \triangleleft \mathbf{own} \not\in n$$

$$\frac{n = \text{size}(\tau)}{E; L \mid p \triangleleft \mathbf{own} \tau \vdash \mathbf{delete}(n, p) \dashv \emptyset}$$

$$\tau \text{ copy} \quad \text{size}(\tau) = 1$$

$$\frac{}{E; L \mid p \triangleleft \mathbf{own} \tau \vdash {}^* p \dashv x. p \triangleleft \mathbf{own} \tau, x \triangleleft \tau}$$

$$\text{size}(\tau) = 1$$

$$\frac{}{E; L \mid p \triangleleft \mathbf{own} \tau \vdash {}^* p \dashv x. p \triangleleft \mathbf{own} \not\in 1, x \triangleleft \tau}$$

$$E; L \vdash \kappa \text{ alive}$$

$$\frac{}{E; L \mid p_1 \triangleleft \&_{\mathbf{mut}}^\kappa \tau, p_2 \triangleleft \tau \vdash p_1 := p_2 \dashv p_1 \triangleleft \&_{\mathbf{mut}}^\kappa \tau}$$

# Typing rules

## Typing function bodies

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Selected rules:

$$\frac{E; L \mid T_1 \vdash I \dashv x. T_2 \quad E; L \mid K; T_2, T \vdash F}{E; L \mid K; T_1, T \vdash \text{let } x = I \text{ in } F}$$

$$\frac{k \lhd \mathbf{cont}(L; \bar{x}. T) \in K}{E; L \mid K; T[\bar{y}/\bar{x}] \vdash \text{jump } k(\bar{y})}$$

$$\frac{E; L \vdash T \xrightarrow{\text{ctx}} T' \quad E; L \mid K; T' \vdash F}{E; L \mid K; T \vdash F}$$

Helper judgment for transforming a typing context:  $E; L \vdash T \xrightarrow{\text{ctx}} T'$ .

# Typing rules

Transforming typing environments

Some transformations can happen on environments before typing a piece of code:

$$\frac{\tau \text{ copy}}{E; L \vdash p \triangleleft \tau \xrightarrow{\text{ctx}} p \triangleleft \tau, p \triangleleft \tau}$$

$$E; L \vdash p \triangleleft \&_{\mu}^{\kappa} (\tau_1 \times \tau_2) \xrightarrow{\text{ctx}} p.0 \triangleleft \&_{\mu}^{\kappa} \tau_1, p.\text{size}(\tau_1) \triangleleft \&_{\mu}^{\kappa} \tau_2$$

$$\frac{E; L \vdash \kappa \text{ alive}}{E; L \vdash p \triangleleft \&_{\text{mut}}^{\kappa} \tau \xrightarrow{\text{ctx}} p \triangleleft \&_{\text{shr}}^{\kappa} \tau} \quad E; L \vdash p \triangleleft \text{own}_n \tau \xrightarrow{\text{ctx}} p \triangleleft \&_{\text{mut}}^{\kappa} \tau, p \triangleleft^{\dagger \kappa} \text{own}_n \tau$$

$$\frac{E; L \vdash \kappa' \sqsubseteq \kappa}{E; L \vdash p \triangleleft \&_{\text{mut}}^{\kappa} \tau \xrightarrow{\text{ctx}} p \triangleleft \&_{\text{mut}}^{\kappa'} \tau, p \triangleleft^{\dagger \kappa'} \&_{\text{mut}}^{\kappa} \tau}$$

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# Recap

We have presented **syntactic typing judgments**  $E; L \mid T_1 \vdash I \dashv x. T_2$  and  $E; L \mid K; T \vdash F$ .

We need to:

- Give them **semantic counterparts**  $E; L \mid T_1 \models I \doteq x. T_2$  and  $E; L \mid K; T \models F$
- Prove adequacy: if  $\emptyset \mid \emptyset; \emptyset \mid \emptyset \models f \doteq x. x \triangleleft \mathbf{fn}() \rightarrow \Pi[]$ , then  $f$  cannot execute to a stuck state.
- Prove the **semantic typing rules**: same as syntactic rules, but replacing  $\vdash$  with  $\models$ .
- Extend the proof to types defined using **unsafe**.

The main difficulty is to find the definitions of the semantic judgments.

Once defined, their properties follow “naturally” (somewhat). In this course, we will **not give the proofs** and focus on the definitions.

Let's define a model for semantic judgments.

To do that, we need a semantic interpretation of types:  $[\![\tau]\!]$ .

Problem: we need to speak ownership. Any ideas how to models this?

# Separation Logic

# to the Rescue!



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# Separation Logic to the Rescue!

## Extension of Hoare logic (O'Hearn-Reynolds-..., 1999)

- For reasoning about pointer-manipulating programs

## Major influence on many verification & analysis tools

- e.g. Infer, VeriFast, Viper, Bedrock, jStar, ...

## Separation logic = Ownership logic

- Perfect fit for modeling Rust's ownership types!

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We use **Iris**, a modern and expressive separation logic.

It features:

- standard separation logic mechanisms,
- support for concurrency,
- built-in support for step-indexing (equirecursive types...),
- a powerful mechanism to create new mechanisms of ownership (borrows, ...).

# Semantic interpretation of types

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We define an ownership predicate  $[\![\tau]\!].\text{own}(t, \bar{v})$  in separation logic for every type  $\tau$ .

- It represents what it means in separation logic that  $\bar{v}$  is an object of type  $\tau$  in thread  $t$ .
- It not only states what  $\bar{v}$  should look like, it also describes the resources behind  $\tau$ .

Note: it depends on the thread  $t$ : the interpretation of some types depend on it!  
Why is this useful?

# Semantic interpretation of types

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We define an ownership predicate  $[\![\tau]\!].\text{own}(t, \bar{v})$  in separation logic for every type  $\tau$ .

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Note: it depends on the thread  $t$ : the interpretation of some types depend on it!  
Why is this useful?

Semantic interpretation of `Send`: types for which  $[\![\tau]\!].\text{own}(t, \bar{v})$  do not depend on  $t$ .  
⇒ The interpretation will still be valid in another thread.

# Interpreting semantic environments

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We directly lift semantic types to semantic environments using the separating conjunction:

$$\begin{aligned} \llbracket p \triangleleft \tau, T \rrbracket &:= \llbracket \tau \rrbracket.\text{own}(t, p) * \llbracket T \rrbracket \\ \llbracket \emptyset \rrbracket &:= \text{True} \end{aligned}$$

Meaning: every element in the typing context correspond to a distinct piece of ownership.

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# Semantic typing judgments

$$E; L \mid T_1 \models I \models x. T_2 :=$$
$$\forall t. \{[E] * [L] * [Na : t] * [T_1](t)\} \mid \{v. [L] * [Na : t] * [T_2]_{[x \leftarrow v]}(t)\}$$

Semantic typing judgment for function bodies is similar.

(I have omitted the definition of  $[E]$  and  $[L]$  – not important for understanding.)

We use a separation logic **Hoare triple**.

- All we say is that  $I$  behaves well when placed in an environment described by the contexts, and return an environment described by the output contexts.

$[Na : t]$  is a special resource owned by one thread only.

Non-thread-safe types need  $[Na : t]$ . Thus their interpretation depends on  $t$ .

# Semantic typing judgments

$$E; L \mid T_1 \models I \models x. T_2 :=$$

$$\forall t. \{[E] * [L] * [Na : t] * [T_1](t)\} \mid \{v. [L] * [Na : t] * [T_2]_{[x \leftarrow v]}(t)\}$$

Semantic typing judgment for function bodies is similar.

Adequacy follows from the soundness of the separation logic.

- If we have  $\emptyset \mid \emptyset; \emptyset \mid \emptyset \models f \models x. x \triangleleft \mathbf{fn}() \rightarrow \Pi[]$ , then we have a Hoare triple with a trivial precondition. Thus  $f$  is safe.

Proving the semantic typing rules amounts to proving a Hoare triple.

- We can use the rules of the program logic, just like when proving programs.
- Proof of the fundamental theorem: translating a typing derivation to a Hoare logic derivation.

# Semantic typing judgments

$$E; L \mid T_1 \models I \models x. T_2 :=$$

$$\forall t. \{[E] * [L] * [Na : t] * [T_1](t)\} \mid \{v. [L] * [Na : t] * [T_2]_{[x \leftarrow v]}(t)\}$$

Semantic typing

Adequacy follows

Remaining problem: define semantic interpretation of types  
 $[\tau].\text{own}(t, \bar{v})$ .

- If we have  $\emptyset \mid \emptyset; \emptyset \mid \emptyset \models f \models x. x \triangleleft \text{fn}() \rightarrow \Pi$ , then we have a Hoare triple with a trivial precondition. Thus  $f$  is safe.

Proving the semantic typing rules amounts to proving a Hoare triple.

- We can use the rules of the program logic, just like when proving programs.
- Proof of the fundamental theorem: translating a typing derivation to a Hoare logic derivation.

# Basic types

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$[\![\text{bool}]\!].\text{own}(t, \bar{v}) := ?$

$[\![\text{int}]\!].\text{own}(t, \bar{v}) := ?$

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`[[bool]].own(t, v) := v = true ∨ v = false`

`[[int]].own(t, v) := ∃z ∈ ℤ. v = z`

So far, so good...

# Products

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$$[\![\tau_1 \times \tau_2]\!].\text{own}(t, \bar{v}) := ?$$

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$$[\![\tau_1 \times \tau_2]\!].\text{own}(t, \bar{v}) := \exists \bar{v}_1 \bar{v}_2. \bar{v} = \bar{v}_1 \bar{v}_2 * [\![\tau_1]\!].\text{own}(t, \bar{v}_1) * [\![\tau_2]\!].\text{own}(t, \bar{v}_2)$$

Okay, that seems to work...

# Fully owned pointer

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Recall: **own**  $\tau$  is  $\lambda_{\text{Rust}}$  equivalent of `Box<T>`.

$$[\![\mathbf{own} \, \tau]\!].\text{own}(t, \bar{v}) := ?$$

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# Fully owned pointer

Recall: **own**  $\tau$  is  $\lambda_{\text{Rust}}$  equivalent of `Box<T>`.

$$\llbracket \mathbf{own} \, \tau \rrbracket.\text{own}(t, \bar{v}) := \exists \ell \bar{w}. \bar{v} = \ell * \ell \mapsto \bar{w} * \triangleright \llbracket \tau \rrbracket.\text{own}(t, \bar{w})$$

We claim the ownership of memory at  $\ell$  containing  $\bar{w}$ , but also the ownership related to  $\tau$ , recursively.

Important remark:  $\mapsto \bar{w}$  and  $\llbracket \tau \rrbracket.\text{own}(t, \bar{w})$  are **separated**.

If `x: Box<T>`, then nothing in `*x` can point back to  $w$ .

A form of non-aliasing.

# Unique borrows

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$$[\![\&_{\mathbf{mut}}^{\kappa} \tau]\!].\text{own}(t, \bar{v}) := ?$$

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# Unique borrows

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$$[\![\&_{\mathbf{mut}}^{\kappa} \tau]\!].\text{own}(t, \bar{v}) := ?$$

Problem: we do not claim full ownership.

We claim ownership up to  $\kappa$ . We can use it only when  $\kappa$  is alive.

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# The lifetime logic

We extend the separation logic with new connectives:

- $\&_{\text{full}}^{\kappa} P$ : ownership of  $P$  when  $\kappa$  is still alive.
- $[\kappa]$ : resource witnessing that  $\kappa$  is still alive.
- $[\dagger\kappa]$ : assertion witnessing that  $\kappa$  is dead.
  - **persistent** assertion: not a resource, duplicable.

And prove a few rules (using Iris mechanisms):

$$\text{True} \Rightarrow \exists \kappa. [\kappa] * ([\kappa] \Rightarrow [\dagger\kappa]) \quad \triangleright P \Rightarrow \&_{\text{full}}^{\kappa} P * ([\dagger\kappa] \Rightarrow \triangleright P)$$

$$\&_{\text{full}}^{\kappa} P * [\kappa] \Rightarrow \triangleright P * (\triangleright P \Rightarrow \&_{\text{full}}^{\kappa} P * [\kappa])$$

( $\Rightarrow$  and  $\Rightarrow$  are kinds of implication in Iris separation logic.)

# Modeling unique borrows

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$$[\![\&_{\mathbf{mut}}^{\kappa} \tau]\!].\text{own}(t, \bar{v}) := ?$$

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# Modeling unique borrows

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$$[\![\&_{\mathbf{mut}}^{\kappa} \tau]\!].\text{own}(t, \bar{v}) := \exists \ell. \bar{v} = \ell * \&_{\text{full}}^{\kappa} (\exists \bar{w}. \ell \mapsto w * [\![\tau]\!].\text{own}(t, \bar{w}))$$

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# Modeling unique borrows

$$[\![\&_{\mathbf{mut}}^{\kappa} \tau]\!].\text{own}(t, \bar{v}) := \exists \ell. \bar{v} = \ell * \&_{\text{full}}^{\kappa} (\exists \bar{w}. \ell \mapsto w * [\![\tau]\!].\text{own}(t, \bar{w}))$$

Creating a borrow:

- We use  $\triangleright P \not\models \&_{\text{full}}^{\kappa} P * ([\dagger \kappa] \not\models \triangleright P)$ .
- We place  $[\dagger \kappa] \not\models \triangleright P$  in the interpretation of the frozen typing context element:

$$[\![p \triangleleft^{\dagger \kappa} \tau]\!] := [\dagger \kappa] \not\models [\![\tau]\!].\text{own}(t, p)$$

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# Modeling unique borrows

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$$[\![\&_{\mathbf{mut}}^{\kappa} \tau]\!].\text{own}(t, \bar{v}) := \exists \ell. \bar{v} = \ell * \&_{\text{full}}^{\kappa} (\exists \bar{w}. \ell \mapsto w * [\![\tau]\!].\text{own}(t, \bar{w}))$$

Using a borrow:

- After getting a lifetime token  $[\kappa]$ , we temporarily “open”  $\&_{\text{full}}^{\kappa} P$  using the rule:

$$\&_{\text{full}}^{\kappa} P * [\kappa] \not\equiv \triangleright P * (\triangleright P \not\equiv \&_{\text{full}}^{\kappa} P * [\kappa])$$

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# Modeling unique borrows

$$[\![\&_{\mathbf{mut}}^{\kappa} \tau]\!].\text{own}(t, \bar{v}) := \exists \ell. \bar{v} = \ell * \&_{\mathbf{full}}^{\kappa} (\exists \bar{w}. \ell \mapsto w * [\![\tau]\!].\text{own}(t, \bar{w}))$$

Reborrowing:

- Recall the syntactic rule:

$$\frac{E; L \vdash \kappa' \sqsubseteq \kappa}{E; L \vdash p \triangleleft \&_{\mathbf{mut}}^{\kappa} \tau \xrightarrow{\mathsf{ctx}} p \triangleleft \&_{\mathbf{mut}}^{\kappa'} \tau, p \triangleleft^{\dagger \kappa'} \&_{\mathbf{mut}}^{\kappa} \tau}$$

We use a lifetime logic rule for reborrowing:

$$\kappa' \sqsubseteq \kappa * \&_{\mathbf{full}}^{\kappa} P \Rightarrow \&_{\mathbf{full}}^{\kappa'} P * ([\dagger \kappa'] \Rightarrow \&_{\mathbf{full}}^{\kappa} P)$$

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# The plan

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We have seen how to prove the syntactic type system sound.

How do we extend it to, say, `Cell`?

The plan:

- 1 Define  $\llbracket \text{cell}(\tau) \rrbracket$
- 2 Prove the semantic typing judgments on functions on cell (`new`, `from_inner`, `get`, `set`, ...).

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# Example, for `Cell::new`

Example, for `Cell::new`:

```
impl<T> Cell<T>{
    fn new(x: T) -> Cell<T> { <code> }
}
```

Where `<code>` is essentially `return x`.

Unfolding the definitions of the semantic judgment, we should have (roughly):

$$\{\llbracket \tau \rrbracket.\text{own}(x, t)\} \text{ <code> } \{v. \llbracket \mathbf{cell}(\tau) \rrbracket.\text{own}(v, t)\}$$

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# Example, for `Cell::new`

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```
impl<T> Cell<T>{
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```

Where `<code>` is essentially `return x`.

Unfolding the definitions of the semantic judgment, we should have (roughly):

$$\{\llbracket \tau \rrbracket.\text{own}(x, t)\} \text{ <code> } \{v. \llbracket \mathbf{cell}(\tau) \rrbracket.\text{own}(v, t)\}$$

And similarly for `Cell::into_inner`...

Example, for `Cell::new`

```
impl<T> Cell<T>{  
    fn new(x: T) ->  
}
```

Where `<code>` is:

Unfolding the def.

We can go from  $[\tau].\text{own}(v, t)$  to  $[\text{cell}(\tau)].\text{own}(v, t)$  and back, without changing anything in the memory state...

⇒ We really don't have a choice, we must use:

$$[\text{cell}(\tau)].\text{own}(v, t) := [\tau].\text{own}(v, t)$$

But... does that mean that semantically  $\text{cell}(\tau)$  and  $\tau$  are equivalent?

No!

And similarly for `Cell::into_inner`...

# Sharing predicates

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Even though we want  $\llbracket \text{cell}(\tau) \rrbracket.\text{own}(v, t)$  and  $\llbracket \tau \rrbracket.\text{own}(v, t)$  to be equivalent,  
we do **not want**  $\llbracket \&_{\text{shr}}^{\kappa} \text{cell}(\tau) \rrbracket.\text{own}(v, t)$  and  $\llbracket \&_{\text{shr}}^{\kappa} \tau \rrbracket.\text{own}(v, t)$  to be equivalent

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# Sharing predicates

Even though we want  $[\![\text{cell}(\tau)]].\text{own}(v, t)$  and  $[\![\tau]\].\text{own}(v, t)$  to be equivalent, we do **not want**  $[\!&_{\text{shr}}^{\kappa} \text{cell}(\tau)].\text{own}(v, t)$  and  $[\!&_{\text{shr}}^{\kappa} \tau]\].\text{own}(v, t)$  to be equivalent

- The ownership predicate of  $\&_{\text{shr}}^{\kappa} \tau$  is not defined from the ownership predicate of  $\tau$ .
- Each type has its **own definition** of  $[\!&_{\text{shr}}^{\kappa} \tau]\].\text{own}(v, t)$ .
  - We call this  $[\![\tau]\].\text{shr}(\kappa, t, \ell)$ , and define:

$$[\!&_{\text{shr}}^{\kappa} \tau]\].\text{own}(t, \bar{v}) := \exists \ell. \bar{v} = \ell * [\![\tau]\].\text{shr}(\kappa, t, \ell)$$

- $[\![\tau]\].\text{shr}(\kappa, t, \ell)$  is the **sharing predicate** of  $\tau$ . It describes the sharing protocol of  $\tau$ .

# Sharing predicates

Even though we want  $[\![\text{cell}(\tau)]].\text{own}(v, t)$  and  $[\![\tau]\].\text{own}(v, t)$  to be equivalent, we do **not want**  $[\!&_{\text{shr}}^{\kappa} \text{cell}(\tau)].\text{own}(v, t)$  and  $[\!&_{\text{shr}}^{\kappa} \tau]\].\text{own}(v, t)$  to be equivalent

- The ownership predicate of  $\&_{\text{shr}}^{\kappa} \tau$  is not defined from the ownership predicate of  $\tau$ .
- Each type has its **own definition** of  $[\!&_{\text{shr}}^{\kappa} \tau]\].\text{own}(v, t)$ .
  - We call this  $[\![\tau]\].\text{shr}(\kappa, t, \ell)$ , and define:

$$[\!&_{\text{shr}}^{\kappa} \tau]\].\text{own}(t, \bar{v}) := \exists \ell. \bar{v} = \ell * [\![\tau]\].\text{shr}(\kappa, t, \ell)$$

- $[\![\tau]\].\text{shr}(\kappa, t, \ell)$  is the **sharing predicate** of  $\tau$ . It describes the sharing protocol of  $\tau$ .

Recall: **Send** types are those for which  $[\![\tau]\].\text{own}(t, \bar{v})$  do not depend on thread  $t$ . How should we semantically interpret **Sync**?

# Sharing predicates

Even though we want  $\llbracket \mathbf{cell}(\tau) \rrbracket.\text{own}(v, t)$  and  $\llbracket \tau \rrbracket.\text{own}(v, t)$  to be equivalent, we do **not want**  $\llbracket \&_{\text{shr}}^{\kappa} \mathbf{cell}(\tau) \rrbracket.\text{own}(v, t)$  and  $\llbracket \&_{\text{shr}}^{\kappa} \tau \rrbracket.\text{own}(v, t)$  to be equivalent

- The ownership predicate of  $\&_{\text{shr}}^{\kappa} \tau$  is not defined from the ownership predicate of  $\tau$ .
- Each type has its **own definition** of  $\llbracket \&_{\text{shr}}^{\kappa} \tau \rrbracket.\text{own}(v, t)$ .
  - We call this  $\llbracket \tau \rrbracket.\text{shr}(\kappa, t, \ell)$ , and define:

$$\llbracket \&_{\text{shr}}^{\kappa} \tau \rrbracket.\text{own}(t, \bar{v}) := \exists \ell. \bar{v} = \ell * \llbracket \tau \rrbracket.\text{shr}(\kappa, t, \ell)$$

- $\llbracket \tau \rrbracket.\text{shr}(\kappa, t, \ell)$  is the **sharing predicate** of  $\tau$ . It describes the sharing protocol of  $\tau$ .

Recall: **Send** types are those for which  $\llbracket \tau \rrbracket.\text{own}(t, \bar{v})$  do not depend on thread  $t$ .

**Sync** types are those for which  $\llbracket \tau \rrbracket.\text{shr}(\kappa, t, \ell)$  do not depend on thread  $t$ .

- This perfectly matches  $\mathbf{T}: \text{Sync} \Leftrightarrow \&\mathbf{T}: \text{Send}$ .

# Sharing predicates

Formalizing Rust's  
Type System

Jacques-Henri  
Jourdan

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We will not give sharing predicates for every types we have seen...

The idea is to define the **sharing protocol** during  $\kappa$  for each of these types..

- **[[int]].shr( $\kappa, t, \ell$ )** and **[[bool]].shr( $\kappa, t, \ell$ )** only allow read access during  $\kappa$ .
- **[[cell( $\tau$ )].shr( $\kappa, t, \ell$ )** allow reading and writing during  $\kappa$ :
  - but only in thread  $t$  (token  $[\text{Na} : t]$  required),
  - without extracting ownership.
- ...

# Conclusion

Formalizing Rust's  
Type System

Jacques-Henri  
Jourdan

I hope I gave you a taste of what it is to define a logical relation for a large language!

As you have seen, this is quite technical.

Formalizing in Coq not only improves trust. It also **helps** to manage the various details.

We can add many extensions to this proof. Examples:

- Add other unsafe libraries (**RefCell**, **Rc**, **Mutex**, **Arc**, ...).
- Support for weak memory model.
- Add a specification part to the judgments to make it possible to compute specifications to programs while typing...

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# Last words

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Did you learn things in this course?

Please comment in the course evaluation. We would be happy to improve it!

Reminder: no course next week (March, 2nd), exam on March, 9th.

Good luck for your exams, and I wish you all the best for the rest of your studies!