BIOS 622 Homework 2

February 4, 2025

   Please answer each question precisely and completely. It is permissible to discuss each other, but all of your work must be your own.

1. Weibull distribution has a survival function $S(t) = e^{-\lambda t^\gamma}$ where λ is the scale parameter and γ is the shape parameter. (1) Derive $f(t)$, $h(t)$ and $H(t)$. (2) It is helpful to plot $\log t$ versus $\log\{-log(S(t)\}$ and you should see a straight line indeed if the data are from a Weibull distribution. Prove the slope $= \gamma$ (shape parameter) and intercept $= \log\lambda$ (where λ is scale parameter).

2. Use R to make the same figure as on Slide 34 from the course notes (Weibull distribution with scale $\lambda = 1$ and shape $\gamma = 1.5$). You may submit your R codes and figure with separate files.

3. A random variable T has a log-normal distribution if $X = log(T)$ has a normal p.d.f:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

(1) Prove T has the p.d.f:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma t} e^{-\frac{(log(t)-\mu)^2}{2\sigma^2}}.$$

(2) Plot $h(t)$ over the time interval (0, 4) in R (see below). Does the hazard function $h(t)$ monotonically increase or decrease when t increases?

```
  > time <- seq(0, 4, by=0.05)
> logft1 <- dlnorm(time, meanlog=0, sdlog=1, log=FALSE)
> logSt1 <- (1- plnorm(time, meanlog=0, sdlog=1, lower.tail=TRUE, log.p=FALSE))
> loght1 <- logft1/logSt1
> plot(time, loght1, type="l",col=1, ylab="h(t)", xlab="Time (t)") ## h(t)
> #plot(time, logSt1)    #### survival curve
```