# BIOS 622 Homework 6

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1. The data of time to death $(T_1, ..., T_n)$ are given as follows:

   - Steroid: 1, 1+(3), 4+, 5, 7, 8, 10, 10+, 12+, 16+(3)
   - Control: 1+, 2(3), 3(2), 3+, 5+(2), 16+(6)

   (a) This homework is based on Weibull distribution assumption as follows. Let $Z = 1$ for the steroid and $Z = 0$ for the control. Assume that survival time $T|Z$ is Weibull with scale parameter $\lambda_Z$ and shape parameter $\gamma$ (exponential if $\gamma = 1$) and the survival function is

   $$S_Z(t) = exp\{-\lambda_Z t^\gamma\}$$

   where

   $$\lambda_Z = \frac{1}{exp\{(\beta_0 + \beta_1 Z)\gamma\}} = exp\{(-\beta_0 - \beta_1 Z)\gamma\}$$

   Write down the corresponding hazard function $h_Z(t)$

   (b) Write down the likelihood function on $\beta_0$, $\beta_1$ and $\gamma$; Explain briefly how you might find the maximum-likelihood estimates of $\beta_0$, $\beta_1$ and $\gamma$ (do not attempt to find them)

   (c) Find estimates of $\beta_0$, $\beta_1$ and $\gamma$ using proc 'lifereg' in SAS (or 'survreg' in R). What is the p-value on $\beta_1$? Please state the null and alternative hypotheses. Is there a big difference on p-value, in comparison with log-rank test in (a)?

   (d) What is the predicted median survival time for each group under the Weibull assumption?

   (e) What is hazard ratio of treatment effect and its 95% CI (you may use the R package/library 'SurvRegCensCov') ?

   (f) A survival model is considered as an accelerated failure time (AFT) model if $S(t|Z = 1) = S(t/\phi|Z = 0)$ where $\phi^{-1}$ is accelerated factor (a constant). Is this Weibull model a AFT? If yes, what is the value of the accelerated factor $\phi^{-1}$ (hint: review course notes Page 137, 143 and 146).

   (g) Estimate the median follow-up time for each group using reverse Kaplan-Meier estimation.

2. (Bonus) Let $S(t)$ is the survival function of $T$, show that

   $$E(T) = \int S(t)dt.$$

   This implies that the mean survival time is the area under the survival function/curve (relating to the bonus question in homework 5: a non-parametric way to find mean survival time).