logistic_regression

2022-07-31

2) Complete data model is logistic regression & probability to be missing depends ## only on Y

WTS: If probability to be missing depends ONLY on Y and NOT on X, and missing

data is confined to either Y or X (will confine to just X in our case), then

LD is more efficient (and generates unbiased regression coefficients)

```
create.data <- function(beta_0 = 0, beta_1 = 1, beta_2 = 2, n = 200){
# Data. Given this.
x1 = rnorm(n)           # some continuous variables
x2 = rnorm(n)
# defining z to be this. Coeffcents are unknown. They are estimated using MLE as part of
# logistic regression.
z = beta_0 + beta_1*x1 + beta_2*x2        # linear combination with a bias. Here,
# B_0 = 1, B_1 = 2. The coefficeints represent change in log odds. I.e. if
# x2 increases by 2, log odds increase by 2, i.e. odds of a "1" if x2 increases
# by 2 is exp(2) = 7.38 higher than original x2.
pr = 1/(1+exp(-z))          # pass through an inv-logit (sigmoid) function to
                            # to constrain to [0,1]; represents odds of event
                            # occuring
Y = rbinom(n,1,pr)
data = as.data.frame(cbind(Y, x1, x2))

}
```

relative to multiple imputation.

Add Missingness

```
# probability of missingness is greater for x1/x2 if Y == 1.
MNAR.make.missing <- function(data, prob_missing_larger = 0.2,
                              prob_missing_smaller = 0.6){
  # Setting up the randomness categories for missingness in x1, x2,
  higher <- data %>% filter(Y == 1) %>% select(x1)
  rx1_larger <- rbinom(nrow(higher), 1, prob_missing_larger)
  rx1_smaller <- rbinom(nrow(data) - nrow(higher), 1, prob_missing_smaller)
  rx2_larger <- rbinom(nrow(higher), 1, prob_missing_larger)
  rx2_smaller <- rbinom(nrow(data) - nrow(higher), 1, prob_missing_smaller)
  rx1 <- c(rx1_larger, rx1_smaller)
  rx2 <- c(rx2_larger, rx2_smaller)
  data <- data %>%
```

```
    arrange(desc(Y)) %>%
    cbind(rx1, rx2)
  # Implementing the missingness in x1, X_3, X_4
  data <- data %>% mutate(x1 = case_when(rx1 == 1 ~ as.numeric(NA),
                    rx1 == 0 ~ as.numeric(data$x1))) %>%
    mutate(x2 = case_when(rx2 == 1 ~ as.numeric(NA),
                    rx2 == 0 ~ as.numeric(data$x2)))
  # Remove setup variables
  data <- select(data, -c(rx1, rx2))
  data
}
```

Multiple Imputation

```
# Simulate multiple imputation, obtaining estimates and 95% confidence interval.
simulate_MI2 <- function(runs = 100) {
  res <- array(NA, dim = c(3, runs, 3))
  times <- array(NA, dim = c(runs, 1, 1))
  dimnames(res) <- list(c("Intercept", "x1", "x2"),
                        as.character(1:runs), c("estimate", "2.5%", "97.5%"))
  sim_dataset <- as.data.frame(create.data(n = 1000))
  for (run in 1:runs){
      # Note that time is only measured for the MI/imp steps
      # (i.e. filtering, predicting)
    missingness_sim_dataset <- MNAR.make.missing(sim_dataset, 0.2, 0.8)
    start_time <- Sys.time()
    imp_MI <- mice(missingness_sim_dataset, print = FALSE)
    fit <- with(imp_MI, glm(Y ~ x1 + x2, family = "binomial"))
    end_time <- Sys.time()
    tab <- summary(pool(fit), "all", conf.int = TRUE)
    res[1, run, ] <- as.numeric(tab[1, c("estimate", "2.5 %", "97.5 %")])
    res[2, run, ] <- as.numeric(tab[2, c("estimate", "2.5 %", "97.5 %")])
    res[3, run, ] <- as.numeric(tab[3, c("estimate", "2.5 %", "97.5 %")])

    times[run, 1, 1] <- as.numeric(end_time - start_time)
  }
  list(res, times)
}
```

```
# Run 100 iterations
res_MI2 <- simulate_MI2(100)
```

```
# Obtain confidence intervals & estimates for all coefficients, intercept.
apply(res_MI2[[1]], c(1, 3), mean, na.rm = TRUE)
```

```
##                estimate       2.5%     97.5%
## Intercept -0.01784054 -0.5152294 0.4795483
## x1         1.05183641  0.1689295 1.9347433
## x2         1.93094267  0.9808541 2.8810312
```

```
# Mean time for the multiple imputation instances
times <- res_MI2[[2]]
mean(times)
```

```
## [1] 0.2079214
```

```r
# Evaluating imputation method performance for estimating
# all parameters of interest.
res <- res_MI2[[1]]
true <- c(0, 1, 2)
RB <- rowMeans(res[,, "estimate"]) - true
PB <- 100 * abs((rowMeans(res[,, "estimate"]) - true)/ true)
CR <- rowMeans(res[,, "2.5%"] < true & true < res[,, "97.5%"])
AW <- rowMeans(res[,, "97.5%"] - res[,, "2.5%"])
RMSE <- sqrt(rowMeans((res[,, "estimate"] - true)^2))
data.frame(RB, PB, CR, AW, RMSE)
```

```
##                    RB       PB   CR        AW      RMSE
## Intercept -0.01784054      Inf 0.99 0.9947777 0.1377022
## x1         0.05183641 5.183641 0.98 1.7658137 0.2301167
## x2        -0.06905733 3.452867 0.98 1.9001771 0.2666542
```

Listwise Deletion

```r
# Simulate listwise deletion, obtaining estimates and 95% confidence interval.

simulate_LD <- function(runs = 100){
  res <- array(NA, dim = c(3, runs, 3))
  dimnames(res) <- list(c("Intercept", "x1", "x2"),
                        as.character(1:runs), c("estimate", "2.5%", "97.5%"))
  times <- array(NA, dim = c(runs, 1, 1))
  sim_dataset <- as.data.frame(create.data(n = 1000))
  # Note that time is only measured for the LD/imp steps (i.e. filtering, predicting)
  for (run in 1:runs){
    missingness_sim_dataset <- MNAR.make.missing(sim_dataset, 0.2, 0.5)
    start_time <- Sys.time()
    filtered_sim_dataset <- missingness_sim_dataset %>%
      select(Y, x1, x2) %>%
      filter(!is.na(x1), !is.na(x2))

    fit <- with(filtered_sim_dataset, glm(Y ~ x1 + x2, family = "binomial"))
    end_time <- Sys.time()
    times[run, 1, 1] <- as.numeric(end_time - start_time)
    # loop over each variable. Note we do the imputation just ONCE b/c LD is
    # deterministic.
    for (var in 1:3){
      edges <- as.numeric((confint.default(fit))[var,])
      estimate <- as.numeric(fit$coefficients)[var]
      interval <- c(estimate, edges)
      res[var, run, ] <- interval
      }

  }
  list(res, times)
}
```

```r
result_LD <- simulate_LD(100)
```

```r
# Obtain confidence intervals & estimates for all coefficients, intercept.
apply(result_LD[[1]], c(1, 3), mean, na.rm = TRUE)
```

```
##           estimate      2.5%    97.5%
## Intercept 1.015762 0.7280626 1.303461
## x1        1.072069 0.7599650 1.384174
## x2        2.120931 1.6736838 2.568178
```

```
# Evaluating imputation method performance for estimating
# all parameters of interest.
res <- result_LD[[1]]
true <- c(0, 1, 2)
RB <- rowMeans(res[,, "estimate"]) - true
PB <- 100 * abs((rowMeans(res[,, "estimate"]) - true)/ true)
CR <- rowMeans(res[,, "2.5%"] < true & true < res[,, "97.5%"])
AW <- rowMeans(res[,, "97.5%"] - res[,, "2.5%"])
RMSE <- sqrt(rowMeans((res[,, "estimate"] - true)^2))
data.frame(RB, PB, CR, AW, RMSE)
```

```
##                   RB       PB   CR        AW      RMSE
## Intercept 1.01576158      Inf 0.00 0.5753979 1.0218548
## x1        0.07206925 7.206925 0.98 0.6242086 0.1545905
## x2        0.12093088 6.046544 1.00 0.8944942 0.2048534
```

```
# Mean time for 100 instances of LD
times_LD <- result_LD[[2]]
mean(times_LD)
```

```
## [1] 0.007288096
```