Exploration of the effect of missing data on statistical analysis

Leo Watson, Nathalie Moon

UNIVERSITY OF TORONTO

ABSTRACT

Analysis of **missing data mechanisms** and **modern approaches to handling missing data**. **Designing R simulations** to investigate hypotheses about imputation technique.

INTRODUCTION

Motivations

- Interested in what scenarios different imputation techniques should be used to reduce runtime without sacrificing bias, error, and other performance measures.
- Determine the types of missing data in the real world

Definitions

Missing Data Mechanisms

MCAR: When probability of missingness for data points in a dataset is constant.

- Each student's mark is stored in a spreadsheet by the instructor but following a computer update 10% of the data is deleted at random.

MAR: When probability of missingness is dependent on some observed variable of the dataset.

- Most students joined a class from day 1, but some students joined late from the waitlist due to capacity restrictions. 10% of students who joined on time had a missing submission for the first problem set, while 30% of students who joined late missed the first problem set.

MNAR: When probability of missingness is dependent on the true value of the data point which we don't know for all subjects.

• Due to a system failure, the instructor loses all the students' marks. Left with no choice, the instructor requests the students to calculate and share their true final marks to the instructor. If they don't, the instructor will input that they got a B.

– If a student's true mark is an A, they are 90% likely to state their true mark. – If a student's true mark is a B, they are 70% likely to state their true mark. – If a student's true mark is a C, they are 50% likely to state their true mark.

Imputation Techniques

Listwise Deletion: Eliminates all observations containing ANY missing values in variables of interest

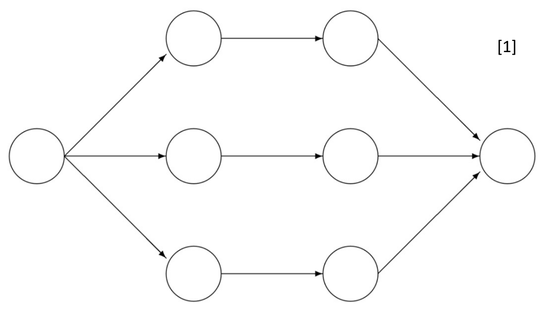
Y	X_1	X_2	X_3
4	0.2	1.2	20
3	NA	1.2	21
2	0.3	1.1	16
2	0.4	1.1	17
1	0.5	2	18
2	0.4	2.1	18
NA	0.2	1.4	19
2	0.1	1.2	22
2	0.1	NA	NA

→

Y	X_1	X_2	X_3
4	0.2	1.2	20
2	0.3	1.1	16
2	0.4	1.1	17
1	0.5	2	18
2	0.4	2.1	18
2	0.1	1.2	22
—	—	—	—
—	—	—	—
—	—	—	—

Multiple Imputation:

1. Takes incomplete dataset and creates multiple copies of it.
2. Impute incomplete columns with plausible values through an iterative predictive method for each copy
3. Obtain estimate for parameter of interest for each copy
4. Pool estimators together to create a single pooled estimate.



[1]

Incomplete data Imputed data Analysis results Pooled result

CONCLUSION

If in doubt, choosing multiple imputation for your dataset is generally safest approach.

- Depending on the missinginess mechanism underlying your data, the quality of your imputations will vary significantly (*investigation 1*).
- Moreover, there are situations where listwise deletion is orders of magnitude faster and provides unbiased regression coefficients (*investigation 2*).

There are far more imputation methods than just the two discussed here; it is vital to deliberately consider which imputation method is best for your dataset when performing statistical analyses.

References & Acknowledgment

- **Textbook, paper (TODO)**
- **Thank you to Professor Nathalie Moon for her expertise and guidance**
- **Github link for full code**

INVESTIGATIONS

(1) Multiple imputation under varying levels of MCAR, MAR, MNAR

```
MCAR.create.data <- function(beta = 1, sigma2 = 1, n = 200,
                             run = 1) {
  set.seed(seed = run)
  x <- rnorm(n)
  y <- beta * x + rnorm(n, sd = sqrt(sigma2))
  cbind(x = x, y = y)
}
MCAR.make.missing <- function(data, p = 0.5){
  rx <- rbinom(nrow(data), 1, p)
  data[rx == 0, "x"] <- NA
  data
}
MCAR.test.impute <- function(data) {
  imp <- mice(data, print = FALSE)
  fit <- with(imp, lm(y ~ x))
  tab <- summary(pool(fit), "all", conf.int = TRUE)
  as.numeric(tab[2, c("estimate", "2.5 %", "97.5 %")])
}
MCAR.simulate <- function(runs = 10) {
  res <- array(NA, dim = c(1, runs, 3))
  dimnames(res) <- list(c("MCAR"),
                        as.character(1:runs),
                        c("estimate", "2.5 %","97.5 %"))
  for(run in 1:runs) {
    data <- MCAR.create.data(run = run)
    data <- MCAR.make.missing(data)
    res[1, run, ] <- MCAR.test.impute(data)
  }
  res
}
```

Simulate determining $\beta_1 = 1$

1. MCAR: $y_i = x_i\beta_{1,} + \epsilon_i$
2. MAR: $y_i = x_{1,i}\beta_1 + x_{2,i}\beta_2 + \epsilon_i$
3. MNAR: $y_i = x_i\beta_{1,} + \epsilon_i$

```
apply(MAR.res, c(1, 3), mean, na.rm = TRUE)

##              estimate     2.5 %   97.5 %
## No_miss     1.0003582 0.9807380 1.019978
MAR.res <- simulate(100)
## lightMAR    0.9768534 0.9228824 1.030824
## moderateMAR 0.9798801 0.9323819 1.027378
## extremeMAR  0.9841179 0.9433072 1.024929

true <- 1
RB <- rowMeans(MAR.res[,, "estimate"]) - true
PB <- 100 * abs((rowMeans(MAR.res[,, "estimate"]) - true)/ true)
CR <- rowMeans(MAR.res[,, "2.5 %"] < true & true < MAR.res[,, "97.5 %"])
AW <- rowMeans(MAR.res[,, "97.5 %"] - MAR.res[,, "2.5 %"])
RMSE <- sqrt(rowMeans((MAR.res[,, "estimate"] - true)^2))
data.frame(RB, PB, CR, AW, RMSE)

##                       RB         PB   CR         AW         RMSE
## No_miss      0.0003582023 0.03582023 0.95 0.03924041 0.009852852
## MCAR        -0.0220908076 2.20908076 0.97 0.10170455 0.026032486
## lightMAR    -0.0231466261 2.31466261 0.91 0.10794194 0.028181202
## moderateMAR -0.0201198748 2.01198748 0.91 0.09499644 0.026311825
## extremeMAR  -0.0158820761 1.58820761 0.90 0.08162145 0.023626457
```

	Estimate	PB	CR	AW
MCAR	0.9779	2.209	0.97	0.102
MAR-light	0.9768	2.315	0.91	0.108
MAR-moderate	0.9799	2.011	0.91	0.095
MAR-heavy	0.9841	1.588	0.90	0.082
MNAR-light	1.0174	1.740	0.96	0.306
MNAR-moderate	1.0262	2.615	0.95	0.331
MNAR-heavy	1.0485	4.853	0.88	0.388

(2) When Listwise Deletion Outperforms Multiple Imputation

Case 1: Missing Data only in Response *Y*

Determining $\beta_1 = 1, \beta_2 = 2, \beta_3 = 3, \beta_4 = 4$ in model $y_i = x_{1,i}\beta_1 + x_{2,i}\beta_2 + x_{3,i}\beta_3 + x_{4,i}\beta_4 + \epsilon_i$

	Y	X_1	X_2	X_3	X_4
1	*NA*	-0.96193342	0.726838902	0.61735005	-1.29134932
2	*NA*	-0.29252572	-0.809440902	-0.40507751	2.63504537
3	26.986093	0.25878822	0.267085116	1.05310376	0.48707229
4	*NA*	-1.15213189	-1.737263711	0.60228425	0.85389232
5	24.321907	0.19578283	-1.411425136	1.01746118	1.08844271

Multiple Imputation

```
##                       RB         PB   CR         AW         RMSE
## Intercept -0.004632882 0.02316441 1.00 0.5360155 0.0955964
## X_1       -0.063330448 6.33304482 0.96 0.5788813 0.1180301
## X_2       -0.085072564 4.25362822 0.97 0.5297375 0.1353696
## X_3       -0.202638162 6.75460540 0.64 0.5033990 0.2373411
## X_4       -0.103835898 2.59589744 0.90 0.4892921 0.1475618
times <- res_MI2[[2]]
mean(times)

## [1] 0.07354221
```

Listwise Deletion

```
##                       RB         PB   CR         AW         RMSE
## Intercept 0.01412967 0.07064837 1.00 0.3649604 0.04946551
## X_1       0.02335495 2.33549497 1.00 0.3977020 0.05633452
## X_2       0.06959586 3.47979321 0.98 0.3617599 0.08869656
## X_3       0.02198526 0.73284197 1.00 0.3417499 0.04601087
## X_4       0.03357291 0.83932276 1.00 0.3353844 0.05205810
times_LD <- result_LD[[2]]
mean(times_LD)

## [1] 0.007305567
```

Case 2: Missing Data independent of response *Y*

Determining $\beta_1 = 1, \beta_2 = 2, \beta_3 = 3, \beta_4 = 4$ in model $y_i = x_{1,i}\beta_1 + x_{2,i}\beta_2 + x_{3,i}\beta_3 + x_{4,i}\beta_4 + \epsilon_i$

	Y	X_1	X_2	X_3	X_4
95	18.06782258	0.558486426	0.134447661	-1.10891000	0.14510663
96	27.50100053	*NA*	0.128855402	0.57562029	1.08715025
97	21.65300743	-0.461644730	0.099078487	-0.93590256	1.29675557
98	11.33108355	0.716707476	0.082965734	*NA*	-1.52315577
99	12.44660145	-1.015847465	0.045010598	-0.43610692	-1.22499116
100	21.17633208	*NA*	0.004398704	*NA*	*NA*
101	12.46882475	*NA*	0.002131860	*NA*	*NA*
102	23.94489455	*NA*	-0.005344028	*NA*	*NA*
103	18.02430387	*NA*	-0.008309014	*NA*	*NA*
104	33.60898116	*NA*	-0.013399523	*NA*	*NA*

Multiple Imputation

```
##                       RB         PB   CR         AW         RMSE
## Intercept  0.13268692 0.6634346 0.95 1.1691412 0.2328769
## X_1        0.09318012 9.3180118 0.98 0.7916511 0.1608131
## X_2       -0.02529328 1.2646638 0.97 1.1818267 0.1804150
## X_3        0.07662874 2.5542914 0.97 0.6610958 0.1331688
## X_4        0.04258306 1.0645764 0.98 0.6218411 0.1099079
times <- res_MI2[[2]]
mean(times)

## [1] 0.191239
```

Listwise Deletion

```
##                       RB         PB   CR         AW         RMSE
## Intercept  0.11429526 0.5714763 0.98 1.0689402 0.21563689
## X_1        0.06477740 6.4777396 1.00 0.6403192 0.13364648
## X_2       -0.03368863 1.6844316 1.00 1.0811044 0.17563507
## X_3        0.01466790 0.4889301 1.00 0.5801619 0.09709261
## X_4       -0.01747858 0.4369644 1.00 0.5405804 0.08590829
times_LD <- result_LD[[2]]
mean(times_LD)

## [1] 0.007702417
```

Case 3: Logistic regression model & probability to be missing depends *only* on *Y*

Simulate determining $\beta_1 = 1, \beta_2 = 2$ in logistic regression model. MNAR missingness where *Y = 0* observations have greater missingness probability for predictors than *Y = 1* cases

Multiple Imputation

```
##                       RB         PB   CR         AW         RMSE
## Intercept 0.07608593        Inf 0.99 0.8060835 0.1511027
## x1        0.11169267 11.16927 0.98 1.4481633 0.2403467
## x2        0.21261059 10.63053 0.98 1.7733321 0.3379104
times <- res_MI2[[2]]
mean(times)

## [1] 0.2123698
```

	Y	x1	x2
98	1	1.10992029	0.45623640
99	1	-0.49929202	*NA*
100	1	0.09458353	0.88425082
101	1	*NA*	1.50390061
102	1	-1.25127136	0.84573154
103	0	*NA*	*NA*
104	0	1.71506499	-0.47624689
105	0	0.46091621	-0.78860284
106	0	*NA*	*NA*
107	0	*NA*	*NA*

Listwise Deletion

```
##                       RB         PB   CR         AW         RMSE
## Intercept 0.92154738        Inf 0.00 0.5545185 0.9290863
## x1        0.14804061 14.804061 0.96 0.6491966 0.1979282
## x2        0.05219365  2.609682 1.00 0.8593025 0.1655004
times_LD <- result_LD[[2]]
mean(times_LD)

## [1] 0.007046299
```

Investigation (2) Summarized Results:

	Listwise Del (s)	Multiple Imp (s)	Rate
Case 1	0.00731	0.07354	10x faster
Case 2	0.00770	0.19124	~25x faster
Case 3	0.00705	0.21237	~30x faster

Exploration of the effect of missing data on statistical analysis

Leo Watson, Nathalie Moon

UNIVERSITY OF TORONTO

ABSTRACT

Analysis of **missing data mechanisms** and **modern approaches to handling missing data**. **Designing R simulations** to investigate hypotheses about imputation technique.

INTRODUCTION

Motivations

- Interested in what scenarios different imputation techniques should be used to reduce runtime without sacrificing bias, error, and other performance measures.
- Determine the types of missing data in the real world

Definitions

Missing Data Mechanisms

MCAR: When probability of missingness for data points in a dataset is constant.

- Each student's mark is stored in a spreadsheet by the instructor but following a computer update 10% of the data is deleted at random.

MAR: When probability of missingness is dependent on some observed variable of the dataset.

- Most students joined a class from day 1, but some students joined late from the waitlist due to capacity restrictions. 10% of students who joined on time had a missing submission for the first problem set, while 30% of students who joined late missed the first problem set.

MNAR: When probability of missingness is dependent on the true value of the data point which we don't know for all subjects.

• Due to a system failure, the instructor loses all the students' marks. Left with no choice, the instructor requests the students to calculate and share their true final marks to the instructor. If they don't, the instructor will input that they got a B.

– If a student's true mark is an A, they are 90% likely to state their true mark. – If a student's true mark is a B, they are 70% likely to state their true mark. – If a student's true mark is a C, they are 50% likely to state their true mark.

Imputation Techniques

Listwise Deletion: Eliminates all observations containing ANY missing values in variables of interest

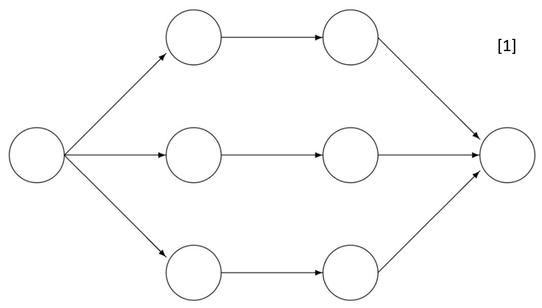
Y	X_1	X_2	X_3
4	0.2	1.2	20
3	NA	1.2	21
2	0.3	1.1	16
2	0.4	1.1	17
1	0.5	2	18
2	0.4	2.1	18
NA	0.2	1.4	19
2	0.1	1.2	22
2	0.1	NA	NA

→

Y	X_1	X_2	X_3
4	0.2	1.2	20
2	0.3	1.1	16
2	0.4	1.1	17
1	0.5	2	18
2	0.4	2.1	18
2	0.1	1.2	22
—	—	—	—
—	—	—	—
—	—	—	—

Multiple Imputation:

1. Takes incomplete dataset and creates multiple copies of it.
2. Impute incomplete columns with plausible values through an iterative predictive method for each copy
3. Obtain estimate for parameter of interest for each copy
4. Pool estimators together to create a single pooled estimate.



[1]

INVESTIGATIONS

1) Comparing multiple imputation under varying degrees of MCAR, MAR, MNAR

Simulation

```
MCAR.create.data <- function(beta = 1, sigma2 = 1, n = 200,
                             run = 1) {
  set.seed(seed = run)
  x <- rnorm(n)
  y <- beta * x + rnorm(n, sd = sqrt(sigma2))
  cbind(x = x, y = y)
}
MCAR.make.missing <- function(data, p = 0.5){
  rx <- rbinom(nrow(data), 1, p)
  data[rx == 0, "x"] <- NA
  data
}
MCAR.test.impute <- function(data) {
  imp <- mice(data, print = FALSE)
  fit <- with(imp, lm(y ~ x))
  tab <- summary(pool(fit), "all", conf.int = TRUE)
  as.numeric(tab[2, c("estimate", "2.5 %", "97.5 %")])
}
MCAR.simulate <- function(runs = 10) {
  res <- array(NA, dim = c(1, runs, 3))
  dimnames(res) <- list(c("MCAR"),
                        as.character(1:runs),
                        c("estimate", "2.5 %","97.5 %"))
  for(run in 1:runs) {
    data <- MCAR.create.data(run = run)
    data <- MCAR.make.missing(data)
    res[1, run, ] <- MCAR.test.impute(data)
  }
  res
}
```

Simulate determining $\beta_1 = 1$

1. MCAR: $y_i = x_i\beta_{1,} + \epsilon_i$
2. MAR: $y_i = x_{1,i}\beta_1 + x_{2,i}\beta_2 + \epsilon_i$
3. MNAR: $y_i = x_i\beta_{1,} + \epsilon_i$

```
MAR.res <- simulate(100)
apply(MAR.res, c(1, 3), mean, na.rm = TRUE)
```

```
##               estimate     2.5 %   97.5 %
## No_miss      1.0003582 0.9807380 1.019978
## MCAR         0.9779092 0.9270569 1.028761
## lightMAR     0.9768534 0.9228824 1.030824
## moderateMAR  0.9798801 0.9323819 1.027378
## extremeMAR   0.9841179 0.9433072 1.024929
```

```
true <- 1
RB <- rowMeans(MAR.res[,, "estimate"]) - true
PB <- 100 * abs((rowMeans(MAR.res[,, "estimate"]) - true)/ true)
CR <- rowMeans(MAR.res[,, "2.5 %"] < true & true < MAR.res[,, "97.5 %"])
AW <- rowMeans(MAR.res[,, "97.5 %"] - MAR.res[,, "2.5 %"])
RMSE <- sqrt(rowMeans((MAR.res[,, "estimate"] - true)^2))
data.frame(RB, PB, CR, AW, RMSE)
```

```
##                       RB         PB   CR         AW         RMSE
## No_miss      0.0003582023 0.03582023 0.95 0.03924041 0.009852852
## MCAR        -0.0220908076 2.20908076 0.97 0.10170455 0.026032486
## lightMAR    -0.0231466261 2.31466261 0.91 0.10794194 0.028181202
## moderateMAR -0.0201198748 2.01198748 0.91 0.09499644 0.026311825
## extremeMAR  -0.0158820761 1.58820761 0.90 0.08162145 0.023626457
```

	Estimate	PB	CR	AW
MCAR	0.9779	2.209	0.97	0.102
MAR-light	0.9768	2.315	0.91	0.108
MAR-moderate	0.9799	2.011	0.91	0.095
MAR-heavy	0.9841	1.588	0.90	0.082
MNAR-light	1.0174	1.740	0.96	0.306
MNAR-moderate	1.0262	2.615	0.95	0.331
MNAR-heavy	1.0485	4.853	0.88	0.388

2) When Listwise Deletion Outperforms Multiple Imputation

Hypothesis 2a:
Missing Data only in Response *Y*

Simulation

Results

Hypothesis 2b:
Probability of missingness doesn't depend on *Y*

Simulation

Results

Hypothesis 2c:
Data follows Logistic Regression, probability of missingness depends only on *Y*

Simulation

Results

CONCLUSION

References

```r
MCAR.create.data <- function(beta = 1, sigma2 = 1, n = 200,
                             run = 1) {
  set.seed(seed = run)
  x <- rnorm(n)
  y <- beta * x + rnorm(n, sd = sqrt(sigma2))
  cbind(x = x, y = y)
}
```

```r
MCAR.make.missing <- function(data, p = 0.5){
  rx <- rbinom(nrow(data), 1, p)
  data[rx == 0, "x"] <- NA
  data
}
```

```r
MCAR.test.impute <- function(data) {
  imp <- mice(data, print = FALSE)
  fit <- with(imp, lm(y ~ x))
  tab <- summary(pool(fit), "all", conf.int = TRUE)
  as.numeric(tab[2, c("estimate", "2.5 %", "97.5 %")])
}
```

```r
MCAR.simulate <- function(runs = 10) {
  res <- array(NA, dim = c(1, runs, 3))
  dimnames(res) <- list(c("MCAR"),
                        as.character(1:runs),
                        c("estimate", "2.5 %","97.5 %"))
  for(run in 1:runs) {
    data <- MCAR.create.data(run = run)
    data <- MCAR.make.missing(data)
    res[1, run, ] <- MCAR.test.impute(data)
  }
  res
}
```

	Estimate	PB	CR	AW
MCAR	0.9779	2.209	0.97	0.102
MAR-light	0.9768	2.315	0.91	0.108
MAR-moderate	0.9799	2.011	0.91	0.095
MAR-heavy	0.9841	1.588	0.90	0.082
MNAR-light	1.0174	1.740	0.96	0.306
MNAR-moderate	1.0262	2.615	0.95	0.331
MNAR-heavy	1.0485	4.853	0.88	0.388

```
result_LD <- simulate_LD()
```

```
# Obtain confidence intervals & estimates for all coefficients, intercept.
apply(result_LD[[1]], c(1, 3), mean, na.rm = TRUE)
```

```
##             estimate       2.5%      97.5%
## Intercept 20.014130 19.8316495 20.1966106
## X_1        1.023355  0.8246039  1.2222206
## X_2        2.069696  1.8887159  2.2504476
## X_3        3.021985  2.8511103  3.1928860
## X_4        4.033573  3.8658807  4.2017265
```

```
# Mean time for 100 instances of LD
times_LD <- result_LD[[2]]
mean(times_LD)
```

```
## [1] 0.007305567
```

```
# Mean time for the multiple imputation instances
times <- res_MI2[[2]]
mean(times)
```

```
## [1] 0.07354221
```

```
## X_3      0.021985l6 0.7328419l 1.00 0.3417499 0.0460108l
## X_4      0.0335729l 0.8393227l 1.00 0.3353844 0.0520581l
```

```
# Evaluating imputation method performance for estimating
# all parameters of interest.
res <- res_MI2[[1]]
true <- c(20, 1, 2, 3, 4)
RB <- rowMeans(res[, "estimate"]) - true
PB <- 100 * abs((rowMeans(res[, "estimate"]) - true)/ true)
CR <- rowMeans(res[, "2.5%"] < true & true < res[, "97.5%"])
AW <- rowMeans(res[, "97.5%"] - res[, "2.5%"])
RMSE <- sqrt(rowMeans((res[, "estimate"] - true)^2))
data.frame(RB, PB, CR, AW, RMSE)
```

```
##                RB        PB   CR        AW      RMSE
## Intercept -0.0046328l2 0.0231644l 1.00 0.5360155 0.0955964
## X_1       -0.0633304l8 6.3330448 0.96 0.5788813 0.1180301
## X_2       -0.0850725l4 4.2536282l 0.97 0.5297375 0.1353696
## X_3       -0.2026381l2 6.7546054l 0.64 0.5033990 0.2373411
## X_4       -0.1038358l98 2.5958974l4 0.90 0.4892921 0.1475618
```

```
create.data <- function(alpha = 20, beta_1 = 1, beta_2 = 2, beta_3 = 3, beta_4 = 4,
                        sigma2 = 1, n = 50, run = 1) {
  set.seed(seed = run)
  x_1 <- rnorm(n)
  x_2 <- rnorm(n)
  x_3 <- rnorm(n)
  x_4 <- rnorm(n)
  y <- beta_1 * x_1 + beta_2 * x_2 + beta_3 * x_3 + beta_4 * x_4 + alpha +
    rnorm(n, sd = sqrt(sigma2))
  cbind("Y" = y, "X_1"= x_1, "X_2" = x_2, "X_3" = x_3, "X_4" = x_4)
}
MCAR.make.missing <- function(data, p = 0.5){
  rx <- rbinom(nrow(data), 1, p)
  data[rx == 0, "Y"] <- NA
  data
}
```

```
sim_dataset <- as.data.frame(create.data(n = 200))
for (run in 1:runs){
  # Note that time is only measured for the MI/imp steps
  # (i.e. filtering, predicting)
  missingness_sim_dataset <- MCAR.make.missing(sim_dataset, p = 0.5)
  start_time <- Sys.time()
  imp_MI <- mice(missingness_sim_dataset, print = FALSE)
```

```
fit <- with(imp_MI, lm(Y ~ X_1 + X_2 + X_3 + X_4))
end_time <- Sys.time()
tab <- summary(pool(fit), "all", conf.int = TRUE)
res[1, run, ] <- as.numeric(tab[1, c("estimate", "2.5 %", "97.5 %")])
res[2, run, ] <- as.numeric(tab[2, c("estimate", "2.5 %", "97.5 %")])
res[3, run, ] <- as.numeric(tab[3, c("estimate", "2.5 %", "97.5 %")])
res[4, run, ] <- as.numeric(tab[4, c("estimate", "2.5 %", "97.5 %")])
res[5, run, ] <- as.numeric(tab[5, c("estimate", "2.5 %", "97.5 %")])

times[run, 1] <- as.numeric(end_time - start_time)
}
list(res, times)
```