

Final Exam Review

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April 25, 2018

Contents

- 1 Conditional Probability Models
- 2 Trees
- 3 Bagging and Random Forest
- 4 Multiclass

Conditional Probability Models

Conditional Exponential Distribution

- Input: x gives location and time
- Output: y gives waiting time for taxi pickup
- Exponential distributions are a natural candidate:

$$\text{ExpDists} = \{p_\lambda(y) = \lambda e^{-\lambda y} 1(y \in [0, \infty)) \mid \lambda \in (0, \infty)\}.$$

- For input x , we want to give back λ , the exponential distribution **parameter**.
- Let's make a generalized linear model.
- So we'll predict $x \mapsto f(w^T x)$ for some x .
- What can we use for f ?

Conditional Exponential Distribution

- Taking $w^T x \mapsto \exp(w^T x)$ does the trick. Maps into $(0, \infty)$.
- The likelihood for observation $y \geq 0$ for

$$p_\lambda(y) = \lambda e^{-\lambda y}$$

- For input x , predicted parameter is $\lambda = \exp(w^T x)$.
- Likelihood of $y | x$ is then

$$p_w(y | x) = \exp(w^T x) e^{-\exp(w^T x)y}$$

Conditional Exponential Distribution

- Log-likelihood of $y \mid x$ is then

$$\begin{aligned} p_w(y|x) &= \exp(w^T x) e^{-\exp(w^T x)y} \\ \implies \log p_w(y|x) &= w^T x - y \exp(w^T x) \end{aligned}$$

- Log-likelihood of $(x_1, y_1), \dots, (x_n, y_n)$ is

$$\sum_{i=1}^n [w^T x_i - y_i \exp(w^T x_i)]$$

- MLE is then

$$\hat{w}_{\text{MLE}} = \arg \max_{w \in \mathbb{R}^d} \sum_{i=1}^n [w^T x_i - y_i \exp(w^T x_i)]$$

Conditional Exponential Distribution with GBM?

- For linear version, we take parameter to be $f(w^T x)$.

$$\log p_w(y|x) = w^T x - y \exp(w^T x)$$

- Replace $w^T x$ by a general function $g(x)$ that we will learn with GBM.
- Log-likelihood objective function is

$$J(g) = \sum_{i=1}^n [g(x_i) - y_i \exp(g(x_i))].$$

(we want to maximize it)

- Differentiate w.r.t. $g(x_i)$... etc...

- doesn't depend on actual values of features – just ordering within each feature

- Know how to do gradient boosting with a new loss function and a black box regression algorithm.

Likelihood, MLE, Conditional Likelihood

- go through examples in slides (Poisson regression, Gaussian regression, binomial, multinomial)
- note that you can use these same likelihoods as losses for gradient boosting (take negative log of likelihood)

Trees

Bagging and Random Forest

- T/F: In bagging, as we increase the number of bootstrap samples, we expect that we will eventually overfit.
 - False.
- With bagging, how can we get an estimate of test performance while still using all our data for training?
 - “out-of-bag” error

- T/F: Random forest is **just** bagging with trees.
 - The tree growing algorithm is modified by features selecting randomly to consider for splitting
- T/F: Generating too many trees in a random forest will probably lead to overfitting.
 - False

Multiclass

Multiclass Classification

- Understand the key pieces
 - class sensitive loss function $\Delta(y, y')$
 - feature map: $\Psi: \mathcal{X} \times \mathcal{Y} \rightarrow \mathbf{R}^d$
 - linear score function $(x, y) \mapsto \langle w, \Psi(x, y) \rangle$, parameterized by $w \in \mathbf{R}^d$
 - final prediction function $x \mapsto \arg \max_{y \in \mathcal{Y}} \langle w, \Psi(x, y) \rangle$
 - loss functions:
 - multiclass hinge / SVM loss
 - multinomial regression loss