Recitation 3
Geometric Derivation of SVMs

Brett Bernstein

CDS at NYU

February 16, 2017
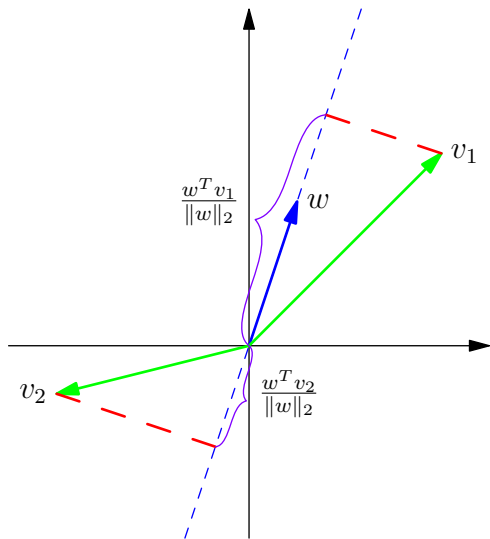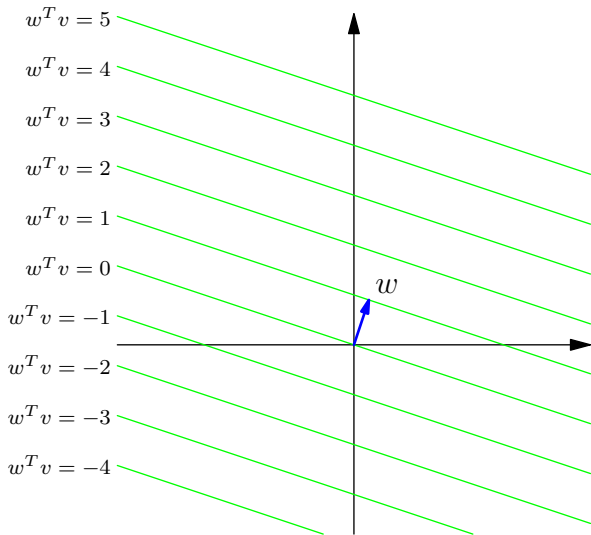
Intro Question

Question

You have been given a data set (x_i, y_i) for $i = 1, \ldots, n$ where $x_i \in \mathbb{R}^d$ and $y_i \in \{-1, 1\}$. Assume $w \in \mathbb{R}^d$ and $a \in \mathbb{R}$.

1. Suppose $y_i(w^T x_i + a) > 0$ for all i. Use a picture to explain what this means when $d = 2$.

2. Fix $M > 0$. Suppose $y_i(w^T x_i + a) \geq M$ for all i. Use a picture to explain what this means when $d = 2$.
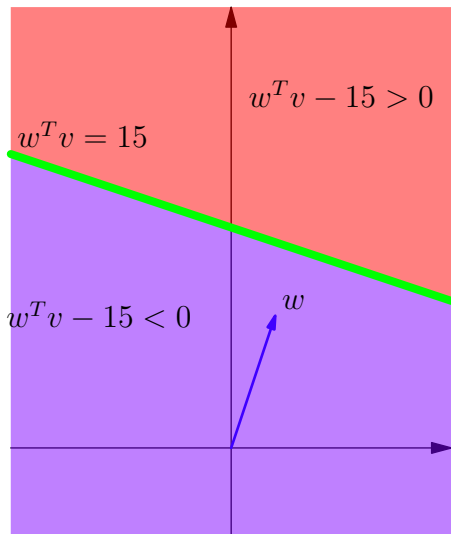
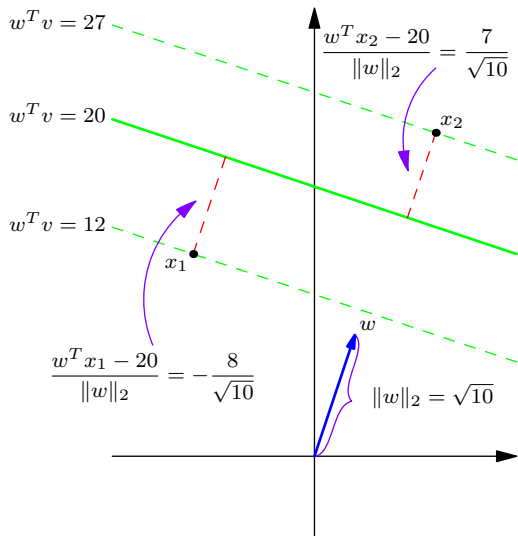# Component of v_1, v_2 in the direction w

Level Surfaces of $f(v) = w^T v$ with $\|w\|_2 = 1$
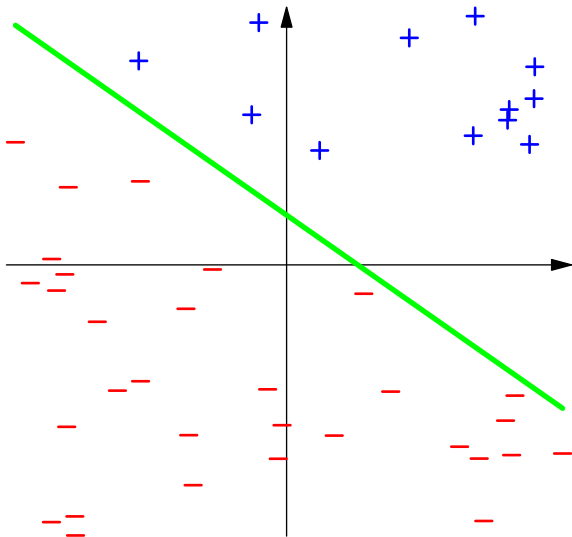
Sides of the Hyperplane $w^T v = 15$

$w^T v - 15 > 0$

$w^T v = 15$

$w^T v - 15 < 0$

w

Signed Distance from x_1, x_2 to Hyperplane $w^T v = 20$

$w^T v = 27$

$\dfrac{w^T x_2 - 20}{\|w\|_2} = \dfrac{7}{\sqrt{10}}$

$w^T v = 20$

x_2

$w^T v = 12$

x_1

$\dfrac{w^T x_1 - 20}{\|w\|_2} = -\dfrac{8}{\sqrt{10}}$

w

$\|w\|_2 = \sqrt{10}$
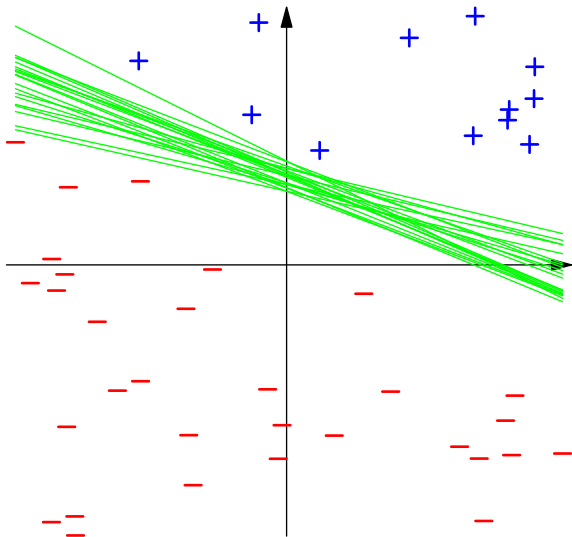
Linearly Separable

Definition

We say (x_i, y_i) for $i = 1, \ldots, n$ are *linearly separable* if there is a $w \in \mathbb{R}^d$ and $a \in \mathbb{R}$ such that $y_i(w^T x_i + a) > 0$ for all i. The set $\{v \in \mathbb{R}^d \mid w^T v + a = 0\}$ is called a *separating hyperplane*.
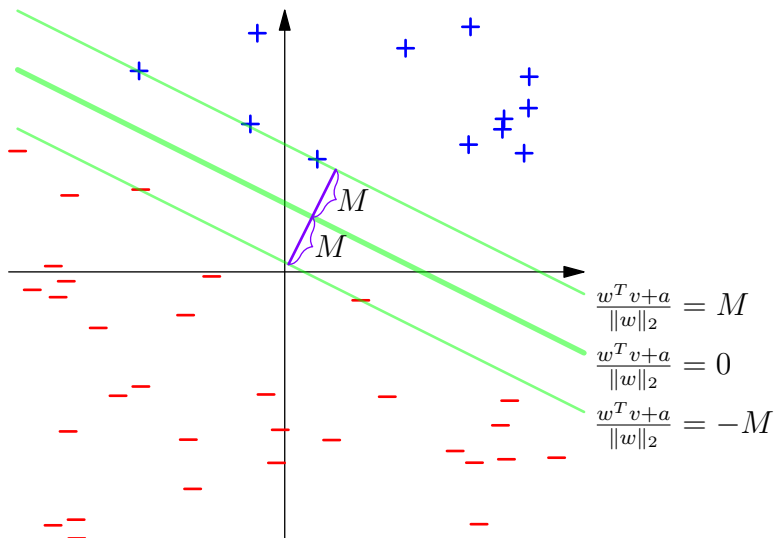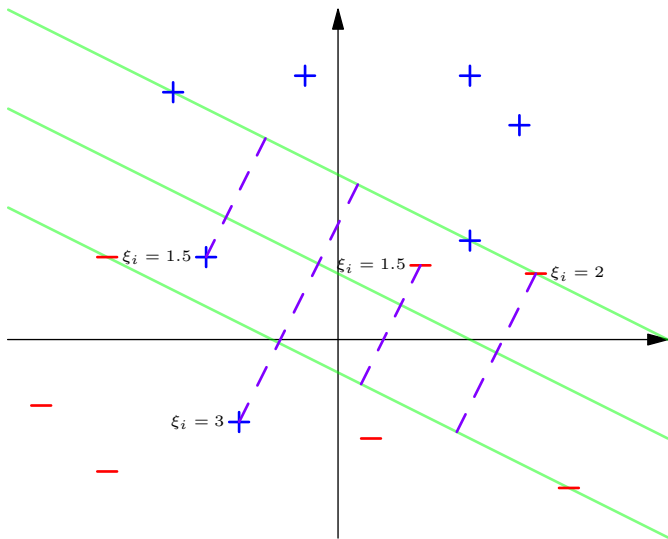
Linearly Separable Data

Many Separating Hyperplanes Exist

Maximum Margin Separating Hyperplane

$$\frac{w^T v + a}{\|w\|_2} = M$$

$$\frac{w^T v + a}{\|w\|_2} = 0$$

$$\frac{w^T v + a}{\|w\|_2} = -M$$

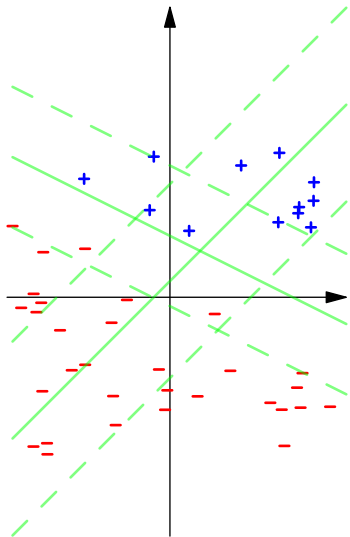# Soft Margin SVM (unlabeled points have $\xi_i = 0$)

Questions

Questions

1. If your data is linearly separable, which SVM (hard margin or soft margin) would you use?

2. Explain geometrically what the following optimization problem computes:

$$\begin{aligned}
&\text{minimize}_{w,a,\xi} && \tfrac{1}{n}\sum_{i=1}^{n}\xi_i \\
&\text{subject to} && y_i(w^T x_i + a) \geq 1 - \xi_i \quad \text{for all } i \\
& && \|w\|_2^2 \leq r^2 \\
& && \xi_i \geq 0 \quad \text{for all } i.
\end{aligned}$$

Optimize Over Cases Where Margin Is At Least $1/r$

Overfitting: Tight Margin With No Misclassifications

Almost no margin
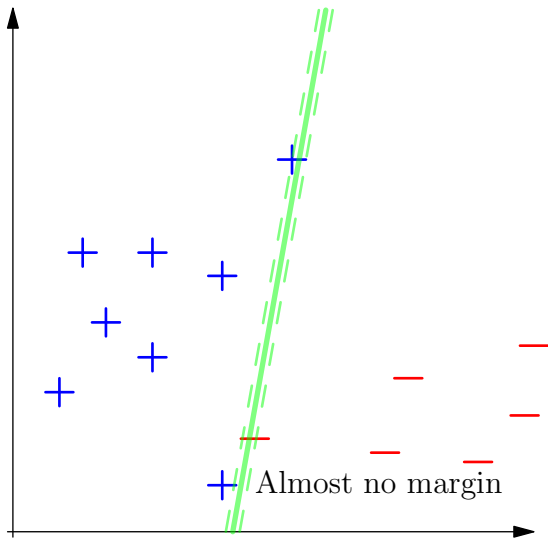
Training Error But Large Margin

Large margin