Statistics II **Final Exam** 2022/6/20

總共三大題，滿分64，除以2即為期末考成績。本學期總分為110分。

答案以google表單回傳: https://forms.gle/aGiE2Bf9btdqUdQ9A

1. (16分) A research organization wishes to determine whether four brands of batteries for transistor radios perform equally well. Three batteries of each type were randomly selected and installed in the three test radios. The number of hours of use for each battery is given below.

	Brand			
Radio	**1**	**2**	**3**	**4**
A	25	27	20	28
B	29	38	24	37
C	21	28	16	19

Consider the three different radios as the blocking variable and carry out the ANOVA procedure to determine whether there is a significant difference in the mean useful life of the four types of batteries. Use $\alpha = 0.05$ and be sure to construct the ANOVA table.

Factorial Designs (for two-factor ANOVA)

SSA = $br\sum_{i=1}^{a}(\bar{x}_{i.} - \bar{\bar{x}})^2$ SSB = $ar\sum_{j=1}^{b}(\bar{x}_{.j} - \bar{\bar{x}})^2$

SSAB = $r\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{x}_{ij} - \bar{x}_{i.} - \bar{x}_{.j} - \bar{\bar{x}})^2$

SSE = $\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}(x_{ijk} - \bar{x}_{ij})^2$

SST = $\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}(x_{ijk} - \bar{\bar{x}})^2$

SST = SSA + SSB + SSAB +SSE

Randomized Block Designs (for one factor and one block)

SSTR = $b\sum_{j=1}^{k}(\bar{x}_{.j} - \bar{\bar{x}})^2$ SSBL = $k\sum_{i=1}^{b}(\bar{x}_{i.} - \bar{\bar{x}})^2$

SSE = $\sum_{i=1}^{b}\sum_{j=1}^{k}(x_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{\bar{x}})^2$

SST = $\sum_{i=1}^{b}\sum_{j=1}^{k}(x_{ij} - \bar{\bar{x}})^2$

SST = SSTR + SSBL + SSE

2. (28分) Jason believes that the sales of coffee at his coffee shop depend upon the weather. He has taken a sample of 6 days. Below you are given the results of the sample.

Cups of Coffee Sold	**350**	**200**	**210**	**100**	**60**	**40**
Temperature (in F)	**50**	**60**	**70**	**80**	**90**	**100**

(a) (2分) Which variable is the dependent variable? Which is the independent variable?
(b) (6分) Compute the least squares estimated regression equation.
(c) (4分) Explain the meaning of the coefficient b_1.
(d) (10分) Set up the ANOVA table and use the F test to determine whether or not the regression model is significant at $\alpha = 0.05$. State your conclusion.
(e) (6分) Is there a significant relationship between the sales of coffee and temperature? Use a *t*-test and a 0.05 level of significance. State your conclusion.

Simple Linear Regression Model

$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma^2), \quad i = 1, ..., n.$

Least Squares Estimators:

$$b_0 = \bar{y} - b_1 \bar{x},$$

$$b_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2},$$

Partitioning of Sum of Squares:

SST = $\sum (y_i - \bar{y})^2$,

SSR = $\sum (\hat{y}_i - \bar{y})^2$,

SSE = $\sum (y_i - \hat{y}_i)^2$.

SST = SSR + SSE

The test statistic $\dfrac{b_1 - \beta_1}{\sqrt{\dfrac{MSE}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}}$ has a *t* distribution with (n-2) degrees of freedom

3. (20分) A microcomputer manufacturer has developed a regression model relating his sales (Y in $10,000s) with three independent variables. The three independent variables are price per unit (Price in $100s), advertising (ADV in $1,000s) and the number of product lines (Lines). Part of the regression results is shown below.

Predictor	Coefficient	Standard Error	T	P-value
Constant	1.0211	22.8752	0.0446	0.96
Price	-0.1524	0.1411	-1.0801	0.30
ADV	0.8849	0.2886	3.0662	0.01
Lines	-0.1463	1.5340	-0.0953	0.92

Analysis of Variance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F
Regression	2708.61			
Error	2840.51	14		
Total				

(a) (3分) Write out the estimated regression equation for the relationship between variables.
(b) (3分) If the manufacturer has 10 product lines, advertising of $40,000, and the price per unit is $3,000, what is your estimate of their sales? **Give your answer in dollars.**
(c) (3分) At α = 0.05, test to see if there is a significant relationship between sales and unit price.
(d) (3分) At α = 0.05, test to see if there is a significant relationship between sales and the number of product lines.
(e) (5分) Is the regression model significant at α = 0.05? (Perform an F test.)
(f) (3分) Interpret the meaning of the regression coefficient of Price -0.1524.

本頁後面是兩個表格: F distribution & t distribution

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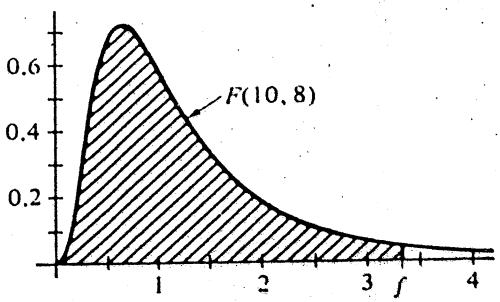
TABLE VII
The *F* Distribution

$$P(F \le f) = \int_0^f \frac{\Gamma[(r_1 + r_2)/2](r_1/r_2)^{r_1/2} w^{r_1/2-1}}{\Gamma(r_1/2)\Gamma(r_2/2)(1 + r_1 w/r_2)^{(r_1+r_2)/2}}\, dw$$



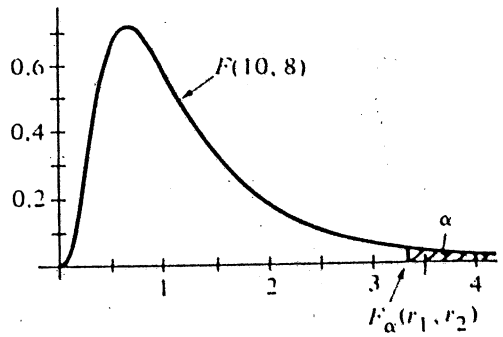


TABLE VII (continued)

$$P(F \le f) = \int_0^f \frac{\Gamma[(r_1 + r_2)/2](r_1/r_2)^{r_1/2} w^{r_1/2-1}}{\Gamma(r_1/2)\Gamma(r_2/2)(1 + r_1 w/r_2)^{(r_1+r_2)/2}}\, dw$$

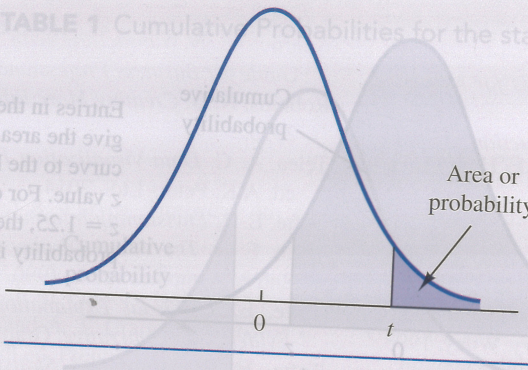
α	$P(F \le f)$	Den. d.f. r_2	Numerator Degrees of Freedom, r_1									
			1	2	3	4	5	6	7	8	9	10
0.05	0.95	1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
0.025	0.975		647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63
0.01	0.99		4052	4999.5	5403	5625	5764	5859	5928	5981	6022	6056
0.05	0.95	2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
0.025	0.975		38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
0.01	0.99		98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
0.05	0.95	3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
0.025	0.975		17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
0.01	0.99		34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
0.05	0.95	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
0.025	0.975		12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
0.01	0.99		21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
0.05	0.95	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
0.025	0.975		10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
0.01	0.99		16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
0.05	0.95	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
0.025	0.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
0.01	0.99		13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
0.05	0.95	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
0.025	0.975		8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
0.01	0.99		12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
0.05	0.95	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
0.025	0.975		7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
0.01	0.99		11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
0.05	0.95	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
0.025	0.975		7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
0.01	0.99		10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
0.05	0.95	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
0.025	0.975		6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72
0.01	0.99		10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
0.05	0.95	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
0.025	0.975		6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37
0.01	0.99		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
0.05	0.95	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
0.025	0.975		6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06
0.01	0.99		8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
0.05	0.95	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
0.025	0.975		5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77
0.01	0.99		8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
0.05	0.95	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
0.025	0.975		5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64
0.01	0.99		7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
0.05	0.95	30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
0.025	0.975		5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51
0.01	0.99		7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
0.05	0.95	40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
0.025	0.975		5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39
0.01	0.99		7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
0.05	0.95	60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
0.025	0.975		5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27
0.01	0.99		7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
0.05	0.95	120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
0.025	0.975		5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16
0.01	0.99		6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
0.05	0.95	∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83
0.025	0.975		5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05
0.01	0.99		6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

TABLE VII (*continued*)

$$P(F \le f) = \int_0^f \frac{\Gamma[(r_1 + r_2)/2](r_1/r_2)^{r_1/2} w^{r_1/2-1}}{\Gamma(r_1/2)\Gamma(r_2/2)(1 + r_1 w/r_2)^{(r_1+r_2)/2}}\, dw$$

α	$P(F \le f)$	Den. d.f. r_2	Numerator Degrees of Freedom, r_1								
			12	15	20	24	30	40	60	120	∞
0.05	0.95	1	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
0.025	0.975		976.71	984.87	993.10	997.25	1001.4	1005.6	1009.8	1014.0	1018.3
0.01	0.99		6106	6157	6209	6235	6261	6287	6313	6339	6366
0.05	0.95	2	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
0.025	0.975		39.42	39.43	39.45	39.46	39.47	39.47	39.48	39.49	39.50
0.01	0.99		99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
0.05	0.95	3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
0.025	0.975		14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
0.01	0.99		27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
0.05	0.95	4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
0.025	0.975		8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
0.01	0.99		14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
0.05	0.95	5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
0.025	0.975		6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
0.01	0.99		9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
0.05	0.95	6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
0.025	0.975		5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
0.01	0.99		7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
0.05	0.95	7	3.57	3.51	3.41	3.41	3.38	3.34	3.30	3.27	3.23
0.025	0.975		4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
0.01	0.99		6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
0.05	0.95	8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
0.025	0.975		4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
0.01	0.99		5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
0.05	0.95	9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
0.025	0.975		3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
0.01	0.99		5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
0.05	0.95	10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
0.025	0.975		3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
0.01	0.99		4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
0.05	0.95	12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
0.025	0.975		3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
0.01	0.99		4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
0.05	0.95	15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
0.025	0.975		2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
0.01	0.99		3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
0.05	0.95	20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
0.025	0.975		2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
0.01	0.99		3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
0.05	0.95	24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
0.025	0.975		2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
0.01	0.99		3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
0.05	0.95	30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
0.025	0.975		2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
0.01	0.99		2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
0.05	0.95	40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
0.025	0.975		2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
0.01	0.99		2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
0.05	0.95	60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
0.025	0.975		2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
0.01	0.99		2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
0.05	0.95	120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
0.025	0.975		2.05	1.95	1.82	1.76	1.69	1.61	1.53	1.43	1.31
0.01	0.99		2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
0.05	0.95	∞	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00
0.025	0.975		1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00
0.01	0.99		2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

TABLE 2 *t* Distribution



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756
30	.854	1.310	1.697	2.042	2.457	2.750
31	.853	1.309	1.696	2.040	2.453	2.744
32	.853	1.309	1.694	2.037	2.449	2.738
33	.853	1.308	1.692	2.035	2.445	2.733
34	.852	1.307	1.691	2.032	2.441	2.728