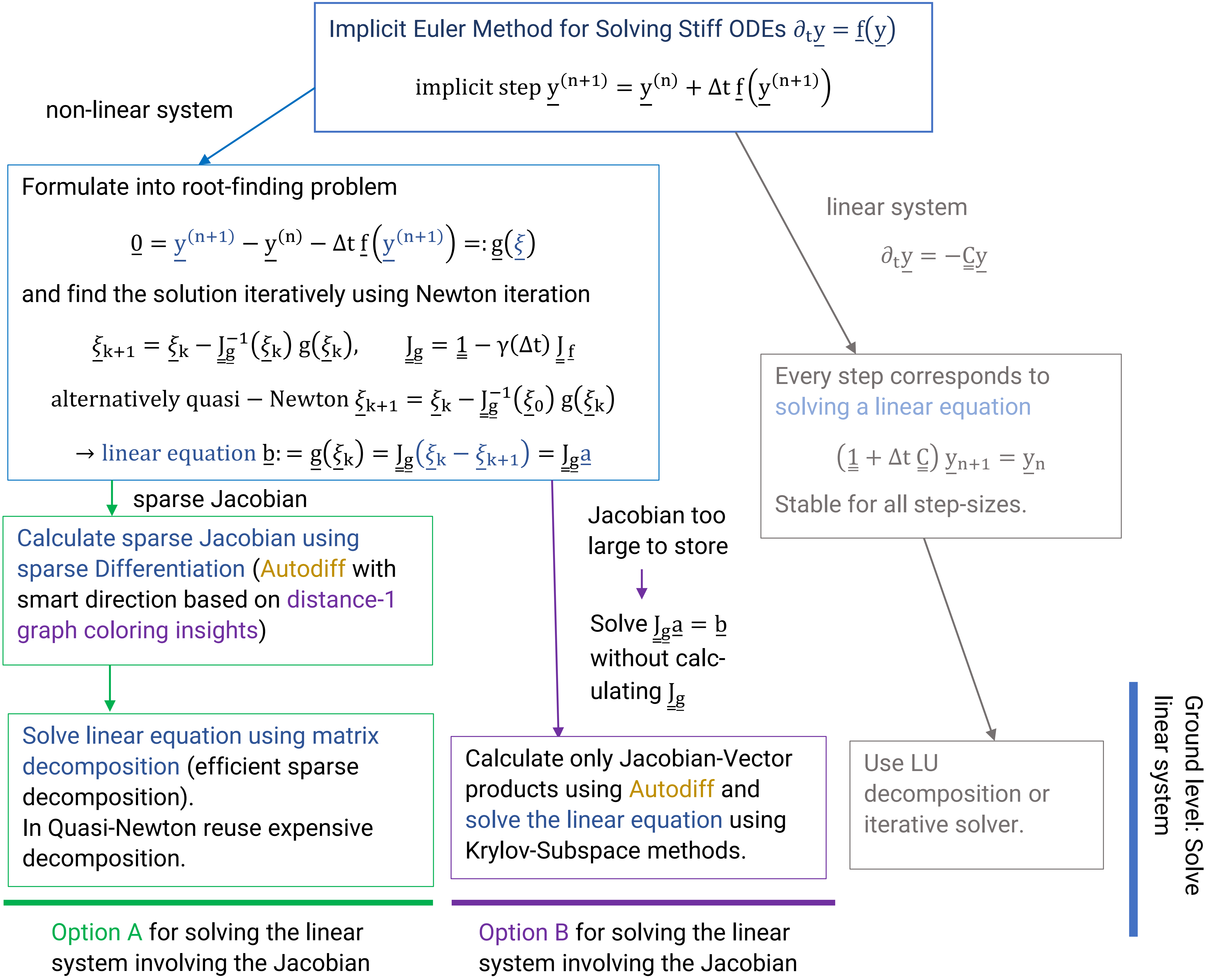

Implicit Euler Method for Solving Stiff ODEs $\partial_t \underline{y} = \underline{f}(\underline{y})$
implicit step $\underline{y}^{(n+1)} = \underline{y}^{(n)} + \Delta t\, \underline{f}\left(\underline{y}^{(n+1)}\right)$
non-linear system
Formulate into root-finding problem
$\underline{0} = \underline{y}^{(n+1)} - \underline{y}^{(n)} - \Delta t\, \underline{f}\left(\underline{y}^{(n+1)}\right) =: \underline{g}(\underline{\xi})$
and find the solution iteratively using Newton iteration
$\underline{\xi}_{k+1} = \underline{\xi}_k - \underline{\underline{J}}_g^{-1}(\underline{\xi}_k)\, \underline{g}(\underline{\xi}_k), \qquad \underline{\underline{J}}_g = \underline{\underline{1}} - \gamma(\Delta t)\, \underline{\underline{J}}_f$
alternatively quasi − Newton $\underline{\xi}_{k+1} = \underline{\xi}_k - \underline{\underline{J}}_g^{-1}(\underline{\xi}_0)\, \underline{g}(\underline{\xi}_k)$
→ linear equation $\underline{b} := \underline{g}(\underline{\xi}_k) = \underline{\underline{J}}_g(\underline{\xi}_k - \underline{\xi}_{k+1}) = \underline{\underline{J}}_g \underline{a}$
linear system
$\partial_t \underline{y} = -\underline{\underline{C}}\underline{y}$
Every step corresponds to solving a linear equation
$\left(\underline{\underline{1}} + \Delta t\, \underline{\underline{C}}\right) \underline{y}_{n+1} = \underline{y}_n$
Stable for all step-sizes.
sparse Jacobian
Calculate sparse Jacobian using sparse Differentiation (Autodiff with smart direction based on distance-1 graph coloring insights)
Jacobian too large to store
Solve $\underline{\underline{J}}_g \underline{a} = \underline{b}$ without calc-ulating $\underline{\underline{J}}_g$
Solve linear equation using matrix decomposition (efficient sparse decomposition).
In Quasi-Newton reuse expensive decomposition.
Calculate only Jacobian-Vector products using Autodiff and solve the linear equation using Krylov-Subspace methods.
Use LU decomposition or iterative solver.
Ground level: Solve linear system
Option A for solving the linear system involving the Jacobian
Option B for solving the linear system involving the Jacobian