I am a member of team 3.

Our team name is MechThree

Exercise 1:

First let's get the eigenvalues of $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ let $\det(A-\lambda I)=0$, we have

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \qquad \therefore (1-\lambda)^2 \cdot (-\lambda) = 0$$

$$\therefore \lambda_1 = \lambda_2 = 1 \quad \lambda_3 = 0.$$

In order to calculate A^{10}, we need to find the value of $\beta_2, \beta_1, \beta_0$ in the equation $\lambda^{10} = \beta_2\lambda^2 + \beta_1\lambda + \beta_0$

let $\lambda=1$ and $\lambda=0$ we have.

$$\begin{cases} 1 = \beta_2 + \beta_1 + \beta_0 & ① \\ 0 = \beta_0 & ② \end{cases}$$

$\because 10\lambda^9 = 2\beta_2\lambda + \beta_1$

$\therefore$ let $\lambda = 1$ we have

$$10 = 2\beta_2 + \beta_1 \quad ③$$

According to ① ② ③, We have $\begin{cases} \beta_0 = 0 \\ \beta_1 = -8 \\ \beta_2 = 9 \end{cases}$

$$\therefore \boxed{A^{10} = \beta_2 A^2 + \beta_1 A + \beta_0 I = 9A^2 - 8A = 9\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} - 8\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 9 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$$

As for e^{At}, let $e^{\lambda t} = \alpha_2\lambda^2 + \alpha_1\lambda + \alpha_0$

Let $\lambda=1$ and $\lambda=0$, we have

$$\begin{cases} e^t = \alpha_2 + \alpha_1 + \alpha_0 & ④ \\ 1 = \alpha_0 & ⑤ \end{cases}$$

$\because te^{\lambda t} = 2\alpha_2\lambda + \alpha_1$

$\therefore te^t = 2\alpha_2 + \alpha_1 \quad ⑥.$

$\therefore$ According to ④ ⑤ ⑥, we can get

$$\begin{cases} \alpha_0 = 1 \\ \alpha_1 = 2e^t - 2 - te^t. \\ \alpha_2 = te^t - e^t + 1. \end{cases}$$

$$\therefore \boxed{e^{At}} = \alpha_2 A^2 + \alpha_1 A + \alpha_0 I$$

$$= \boxed{\begin{bmatrix} e^t & e^t-1 & te^t - e^t + 1 \\ 0 & 1 & e^t - 1 \\ 0 & 0 & e^t \end{bmatrix}}$$

Exercise 2:

$\because \frac{dx_1}{dt} = -\alpha x_1 + u$

$\frac{dx_2}{dt} = \alpha x_1 - \beta x_2$

$\Rightarrow \dot{x} = \begin{pmatrix} -\alpha & 0 \\ \alpha & -\beta \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$

$\because \alpha = 0.1 \quad \beta = 0.2$

$\therefore \dot{x} = \begin{pmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$

Now, we need to calculate $x(5)$.

$\because x(t) = e^{A(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} Bu \, d\tau.$

and $x_1(0) = 2 \quad x_2(0) = 1, \quad u = 1$

$\therefore x(5) = e^{5A} x(0) + \int_0^5 e^{A(5-\tau)} B \, d\tau.$

As for e^{At}:

$\because A = \begin{pmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{pmatrix}$ Let $\det(A - \lambda I) = 0$

$\therefore \lambda_1 = -0.2 \quad \lambda_2 = -0.1.$

Let $e^{\lambda t} = \beta_1 \lambda + \beta_0$, then we have

$\left. \begin{aligned} e^{-0.2t} &= -0.2\beta_1 + \beta_0 \\ e^{-0.1t} &= -0.1\beta_1 + \beta_0 \end{aligned} \right\} \Rightarrow \begin{cases} \beta_0 = 2e^{-0.1t} - e^{-0.2t} \\ \beta_1 = 10(e^{-0.1t} - e^{-0.2t}) \end{cases}$

$\therefore e^{At} = \beta_1 A + \beta_0 I = \begin{pmatrix} e^{-0.1t} & 0 \\ e^{-0.1t} - e^{-0.2t} & e^{-0.2t} \end{pmatrix}$

$\therefore e^{5A} = \begin{pmatrix} e^{-0.5} & 0 \\ e^{-0.5} - e^{-1} & e^{-1} \end{pmatrix}$

$e^{A(5-\tau)} = \begin{pmatrix} e^{0.1\tau-0.5} & 0 \\ e^{0.1\tau-0.5} - e^{0.2\tau-1} & e^{0.2\tau-1} \end{pmatrix}$

$\therefore \int_0^5 e^{A(5-\tau)} B \, d\tau = \int_0^5 \begin{pmatrix} e^{0.1\tau-0.5} \\ e^{0.1\tau-0.5} - e^{0.2\tau-1} \end{pmatrix} d\tau = \begin{pmatrix} 10e^{0.1\tau-0.5} \\ 10e^{0.1\tau-0.5} - 5e^{0.2\tau-1} \end{pmatrix} \Bigg|_{\tau=0}^{\tau=5}$

$= \begin{pmatrix} 10(1-e^{-0.5}) \\ 5 + 5e^{-1} - 10e^{-0.5} \end{pmatrix}$

$$\therefore X(5) = \begin{pmatrix} e^{-0.5} & 0 \\ e^{-0.5}-e^{-1} & e^{-1} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 10-10e^{-0.5} \\ 5+5e^{-1}-10e^{-0.5} \end{pmatrix}$$

$$= \begin{pmatrix} 10-8e^{-0.5} \\ 5+4e^{-1}-8e^{-0.5} \end{pmatrix}$$

$$\therefore X_1(5) = 10-8e^{-0.5} = 5.1478\,m$$

$$X_2(5) = 5+4e^{-1}-8e^{-0.5} = 1.6193\,m$$

$\therefore$ The water level in tank 1 will be 5.1478m after 5s.

The water level in tank 2 will be 1.6193m after 5s.

Exercise 3. (Here I will use m_i and q_i to represent the algebraic and geometric multiplicities of λ_i.)

(1) $A_1 = \begin{pmatrix} 1 & 4 & 8 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Let $\det(A_1 - \lambda I) = 0$, then we have $(1-\lambda)(2-\lambda)(3-\lambda) = 0$.

$\therefore \lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 3$. and $m_1 = m_2 = m_3 = 1$.

Since. $r(A_1 - \lambda_1 I) = 2 \quad r(A_1 - \lambda_2 I) = 2 \quad r(A_1 - \lambda_3 I) = 2$,

$q_1 = q_2 = q_3 = 1$

$\therefore$ The jordan form of A_1, $J = \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{pmatrix}$ and $J_1 = 1 \quad J_2 = 2 \quad J_3 = 3$

$\therefore J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(2) $A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{pmatrix}$ Let $\det(A_2 - \lambda I) = 0$ then we have $(\lambda+1)(\lambda^2 + 2\lambda + 2) = 0$

$\therefore \lambda_1 = -1 \quad \lambda_2 = -1+i \quad \lambda_3 = -1-i$ and $m_1 = m_2 = m_3 = 1$.

Since $r(A_2 - \lambda_1 I) = 2 \quad r(A_2 - \lambda_2 I) = 2 \quad r(A_2 - \lambda_3 I) = 2$.

$q_1 = q_2 = q_3 = 1$

$\therefore$ The jordan form of A_2, $J = \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{pmatrix}$ and $J_1 = -1 \quad J_2 = -1-i \quad J_3 = -1+i$

$\therefore J = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1-i & 0 \\ 0 & 0 & -1+i \end{pmatrix}$

(3) $A_3 = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ Let $\det(A_3 - \lambda I) = 0$ then we have $(\lambda-1)^2(\lambda-2) = 0$

$\therefore \lambda_1 = 1 \quad \lambda_2 = 2 \quad \therefore m_1 = 2 \quad m_2 = 1$.

Since $r(A_3 - \lambda_1 I) = 1 \quad r(A_3 - \lambda_2 I) = 2$

$\therefore q_1 = 2 \quad q_2 = 1$

$\therefore$ The jordan form of A_3. $J = \begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix}$ and $J_1 = \begin{pmatrix} J_{11} & 0 \\ 0 & J_{12} \end{pmatrix}$ $J_2 = 2$

Since $J_{11} = 1 \quad J_{12} = 1 \quad \therefore J_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \therefore J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(4) $A_4 = \begin{pmatrix} 0 & 4 & 3 \\ 0 & 20 & 16 \\ 0 & -25 & -16 \end{pmatrix}$ Let $\det(A_4 - \lambda I) = 0$, then we have $\lambda^3 = 0$

$\therefore \lambda_1 = 0$ and $m_1 = 3$. Since $r(A_4 - \lambda_1 I) = 2 \quad \therefore q_1 = 1$.

$\therefore$ The jordan form of A_4, $J = J_1$ and $J_1 = J_{11}$.

$\because J_{11} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \therefore J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

Exercise 4:

1): Here we have $A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = [2, 3]$ $D = 0$.

First, Let's get e^{At}. Let $\det(A - \lambda I) = 0$, then we have $\lambda_1 = -1+i$ $\lambda_2 = -1-i$

Let $e^{\lambda t} = \beta_1 \lambda + \beta_0$, then we have $\begin{cases} e^{(-1+i)t} = \beta_1(-1+i) + \beta_0 \\ e^{(-1-i)t} = \beta_1(-1-i) + \beta_0 \end{cases}$

$$\therefore \begin{cases} \beta_1 = \dfrac{e^{(-1-i)t} - e^{(-1+i)t}}{2} i \\ \beta_0 = \dfrac{(i+1) e^{(-1-i)t} + (1-i) e^{(-1+i)t}}{2} \end{cases}$$

$$\therefore e^{At} = \beta_1 A + \beta_0 = \begin{pmatrix} 0 & \beta_1 \\ -2\beta_1 & -2\beta_1 \end{pmatrix} + \begin{pmatrix} \beta_0 & 0 \\ 0 & \beta_0 \end{pmatrix} = \begin{pmatrix} \beta_0 & \beta_1 \\ -2\beta_1 & -2\beta_1 + \beta_0 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{(i+1) e^{(-1-i)t} + (1-i) e^{(-1+i)t}}{2}, & \dfrac{e^{(-1-i)t} - e^{(-1+i)t}}{2} i \\ (e^{(-1+i)t} - e^{(-1-i)t}) i, & \dfrac{(1-i) e^{(-1-i)t} + (1+i) e^{(-1+i)t}}{2} \end{pmatrix}$$

$$\because y(t) = C e^{A(t-t_0)} x(t_0) + C \int_{t_0}^{t} e^{A(t-\tau)} B u(\tau) d\tau + D u(t)$$

$$\therefore y(5) = [2,3] e^{A5} x(0) + [2,3] \int_0^5 e^{A(5-\tau)} \begin{bmatrix} 1 \\ 1 \end{bmatrix} d\tau$$

$$\because x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore y(5) = [2,3] \int_0^5 e^{A(5-\tau)} \begin{bmatrix} 1 \\ 1 \end{bmatrix} d\tau$$

$$= [2,3] \int_0^5 \begin{pmatrix} (\frac{1}{2}+i) e^{(-1-i)(5-\tau)} + (\frac{1}{2}-i) e^{(-1+i)(5-\tau)} \\ (\frac{1}{2} - \frac{3}{2} i) e^{(-1-i)(5-\tau)} + (\frac{1}{2} + \frac{3}{2} i) e^{(-1+i)(5-\tau)} \end{pmatrix} d\tau$$

$$= [2,3] \begin{pmatrix} \dfrac{-e^{-5}(3\cos(5) - 3e^5 + \sin(5))}{2} \\ e^{-5}(\cos(5) - e^5 + 2\sin 5) \end{pmatrix} = [2,3] \begin{pmatrix} 1.5004 \\ -1.0110 \end{pmatrix}$$

$$= -0.0323$$

2):

It is easy to know the discretized state space representation of this system is

$$x((k+1)T) = G(T)\,x(kT) + H(T)\,u(kT)$$

$$y(kT) = Cx(kT) + Du(kT).$$

Here $T = 1s$, $G(T) = e^{AT}$, $H(T) = \left(\int_0^T e^{A\tau} d\tau\right) B.$

According to question 1, we have.

$$G(1) = e^{A} = \begin{pmatrix} 0.5083 & , & 0.3096 \\ -0.6191 & , & -0.1108 \end{pmatrix}$$

$$H(1) = \int_0^1 e^{A\tau} d\tau \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{pmatrix} 1-\cos(1)e^{-1}, & \frac{-e^{-1}(\cos(1)-e+\sin(1))}{2} \\ e^{-1}(\cos(1)-e+\sin(1)), & e^{-1}\cdot\sin(1) \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1.0471 \\ -0.1821 \end{pmatrix}$$

$\therefore$ The discretized state space representation of this system is

$$x(k+1) = \begin{pmatrix} 0.5083 & , & 0.3096 \\ -0.6191 & , & -0.1108 \end{pmatrix} x(k) + \begin{pmatrix} 1.0471 \\ -0.1821 \end{pmatrix} u(k).$$

$$y(k) = (2, 3)\,x(k).$$

3):

$\because\ X(k) = G^k X(0) + \sum_{j=0}^{k-1} G^{k-j-1} H\, u(j)$, and $X(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $u(j) = 1$

$\therefore\ X(k) = \sum_{j=0}^{k-1} G^{k-j-1} H\, u(j) = \sum_{j=0}^{k-1} G^{k-j-1} H.$

$\therefore\ X(5) = \sum_{j=0}^{4} G^{k-j-1} H = \left(G^4 + G^3 + \cdots + G^0\right) H.$

$$= \left(\begin{pmatrix} -0.0258, & -0.0139 \\ 0.0277, & 0.0019 \end{pmatrix} + \begin{pmatrix} -0.0423, & 0.007 \\ -0.0141, & -0.0563 \end{pmatrix} + \begin{pmatrix} 0.0667, & 0.1231 \\ -0.2461, & -0.1744 \end{pmatrix} \right.$$

$$\left. + \begin{pmatrix} 0.5083, & 0.3046 \\ -0.6191, & -0.1108 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} 1.0471 \\ -0.1821 \end{pmatrix}$$

$$= \begin{pmatrix} 1.50179 \\ -1.01169 \end{pmatrix}$$

$\therefore\ y(5) = C X(5) = (2, 3) \begin{pmatrix} 1.50179 \\ -1.01169 \end{pmatrix} = -0.03151.$

In [1]:

```
# This is the code for Exercise 4 question 1.
import numpy as np
from scipy.signal import StateSpace, lsim, dlsim
from scipy.linalg import expm
import matplotlib.pyplot as plt

# Build the CT system.
A = np.asarray([[0., 1.],
                [-2., -2.]])
B = np.asarray([[1.],[1.]])
C = np.asarray([2.,3.])
D = np.asarray([0.])
t_CT = np.arange(0, 5.01, 0.01)
input_CT = np.ones(len(t_CT))
sys_CT = StateSpace(A, B, C, D)
_, y_CT, x_CT = lsim(sys_CT, input_CT, t_CT, X0=[0., 0.])
y5_CT = y_CT[-1]
print("In the CT system, y(5) is:", y5_CT)
```

```
In the CT system, y(5) is: -0.03230590469409034
```

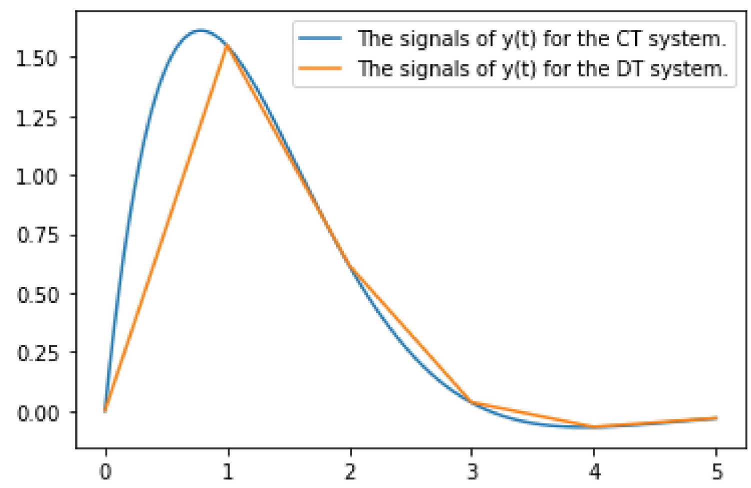
In [2]:

```
# This is the code for Exercise 4 question 2 and 3.
# Calculate the DT system.
G = expm(A);
H = np.asarray([[0.,0.],[0.,0.]])
step = np.arange(0,1.001,0.001)
for i in step:
    H += expm(A*i) * 0.001
H = np.dot(H, B)

# Build the DT system.
sys_DT = StateSpace(G, H, C, D, dt = 1)
print("The discretized state space representation of this system is:")
print(sys_DT)
t_DT = np.arange(0, 6, 1);
input_DT = np.ones(len(t_DT))
_, y_DT, x_DT = dlsim(sys_DT, input_DT, t_DT, x0=[0., 0.])
y5_DT = y_DT[-1,0]
print("In the DT system, y(5) is:", y5_DT)

# Plot the result of CT and DT system.
plt.plot(t_CT, y_CT, label = "The signals of y(t) for the CT system.")
plt.plot(t_DT, y_DT, label = "The signals of y(t) for the DT system.")
plt.legend()
plt.show()
```

```
The discretized state space representation of this system is:
StateSpaceDiscrete(
array([[ 0.50832599,  0.30955988],
       [-0.61911975, -0.11079377]]),
array([[ 1.0479797 ],
       [-0.18197878]]),
array([[2., 3.]]),
array([[0.]]),
dt: 1
)
In the DT system, y(5) is: -0.03150713422428986
```



In [ ]:

Exercise 5:

$\because F_{k+2} = F_{k+1} + F_k.$

$\therefore$ Let's define $F(k)$ as $\begin{pmatrix} F_k \\ F_{k+1} \end{pmatrix}$

Then it is easy to know

$$F(k+1) = \begin{pmatrix} F_{k+1} \\ F_{k+2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} F_k \\ F_{k+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} F(k)$$

$$\therefore F(19) = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^{19} F(0).$$

$$\Rightarrow \begin{pmatrix} F_{19} \\ F_{20} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^{19} \begin{pmatrix} F_0 \\ F_1 \end{pmatrix}.$$

Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$, then

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(\lambda - 1)\lambda - 1 = 0$$

$$\therefore \lambda_1 = \frac{1-\sqrt{5}}{2} \quad \lambda_2 = \frac{1+\sqrt{5}}{2}$$

When $\lambda_1 = \frac{1-\sqrt{5}}{2}$, we have $\begin{pmatrix} \frac{\sqrt{5}-1}{2} & 1 \\ 1 & \frac{\sqrt{5}+1}{2} \end{pmatrix} P_1 = 0$

$\therefore P_1$ could be $\begin{pmatrix} 1 \\ \frac{1-\sqrt{5}}{2} \end{pmatrix}$

When $\lambda_2 = \frac{1+\sqrt{5}}{2}$, we have $\begin{pmatrix} -\frac{\sqrt{5}+1}{2} & 1 \\ 1 & \frac{1-\sqrt{5}}{2} \end{pmatrix} P_2 = 0$

$\therefore P_2$ could be $\begin{pmatrix} 1 \\ \frac{\sqrt{5}+1}{2} \end{pmatrix}$

$$\therefore A(P_1, P_2) = (P_1, P_2) \begin{pmatrix} \frac{1-\sqrt{5}}{2}, & 0 \\ 0, & \frac{\sqrt{5}+1}{2} \end{pmatrix} \Rightarrow A = \begin{pmatrix} 1, & 1 \\ -\frac{\sqrt{5}-1}{2}, & \frac{\sqrt{5}+1}{2} \end{pmatrix} \begin{pmatrix} \frac{1-\sqrt{5}}{2}, & 0 \\ 0, & \frac{\sqrt{5}+1}{2} \end{pmatrix} \begin{pmatrix} 1, & 1 \\ \frac{1-\sqrt{5}}{2}, & \frac{\sqrt{5}+1}{2} \end{pmatrix}^{-1}$$

$$\therefore A = \begin{pmatrix} 1, & 1 \\ -\frac{\sqrt{5}-1}{2}, & \frac{\sqrt{5}+1}{2} \end{pmatrix} \begin{pmatrix} \frac{1-\sqrt{5}}{2}, & 0 \\ 0, & \frac{\sqrt{5}+1}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{5}+1}{2\sqrt{5}}, & -\frac{1}{\sqrt{5}} \\ \frac{\sqrt{5}-1}{2\sqrt{5}}, & \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$\boxed{\therefore F_{20} = 6765}$$

$$\therefore \begin{pmatrix} F_{19} \\ F_{20} \end{pmatrix} = \begin{pmatrix} 1, & 1 \\ -\frac{\sqrt{5}-1}{2}, & \frac{\sqrt{5}+1}{2} \end{pmatrix} \begin{pmatrix} \frac{1-\sqrt{5}}{2}, & 0 \\ 0, & \frac{\sqrt{5}+1}{2} \end{pmatrix}^{19} \begin{pmatrix} \frac{\sqrt{5}+1}{2\sqrt{5}}, & -\frac{1}{\sqrt{5}} \\ \frac{\sqrt{5}-1}{2\sqrt{5}}, & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} F_0 \\ F_1 \end{pmatrix} = \begin{pmatrix} 4181 \\ 6765 \end{pmatrix}$$

In [1]:

```
# This is the code I use for Exercise 5.
import numpy as np
from scipy.signal import StateSpace, dlsim
A = np.asarray([[0.,1.],
                [1.,1.]])
B = np.asarray([[0.],[0.]])
C = np.asarray([0.,0.])
D = np.asarray([0.])
plane_sys = StateSpace(A, B, C, D, dt = 1)
t = np.arange(0, 20, 1)
input = np.zeros(len(t))
_, y, x = dlsim(plane_sys, input, t, x0=[0, 1])
F20 = int(x[19, 1])
print("F20 is:", F20)
```

```
F20 is: 6765
```

In [ ]:

```
```