24 - 677
Fall 2020
Mid-term Exam
10/22/20
Time: 24 Hours

Name: ______________________

Andrew id: ______________________

Print your initials on each page that has your answers

This exam contains 20 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use your equation sheet and calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	10	
6	10	
7	20	
Total:	100	

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1. Please state whether each of the following statement is **True** or **False**. Explanation is not required.

   (a) (3 points) The system $y(t) = t^2 u(t-1)$ is linear

   (b) (3 points) The following system is controllable

   $$\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -3 \end{bmatrix} u(t)$$

   (c) (3 points) The system given in (b) is stabilizable

   (d) (3 points) Assume that $\dot{x}(t) = Ax(t)$ is an asymptotically stable continuous-time LTI system. Assuming A^{-1} exists, the system $\dot{x}(t) = A^{-1}x(t)$ is asymptotically stable

   (e) (3 points) The continuous time system

   $$\dot{y}(t) = -ay(t) + u(t) \text{ s.t. } a < 0,\ y(0) = 0,\ t \in \mathbb{R}$$

   is BIBO stable

2. Consider a model of fisheries management. State x_1 is the population level of a prey species, x_2 is the population level of a predator species, and x_3 is the effort expended by humans in fishing the predator species. The model is

$$\begin{aligned} \dot{x}_1 &= (r_1 - x_2)x_1 \\ \dot{x}_2 &= (r_2 - x_3)x_2 \\ \dot{x}_3 &= u \\ y &= x_2 \end{aligned}$$

where u is the input, y is the measurement of the predator species, and $r_1 > 0$ and $r_2 > 0$

(a) (5 points) Find the equilibrium point if the prey species population is known to be $\bar{x}_1 > 0$ i.e. at $x_1 = \bar{x}_1 > 0$

(b) (5 points) Linearize the model using the equilibrium point from (a)

(c) (5 points) Find the transfer function of the linearized state model from (b)

3. (15 points) For the following dynamical system

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

compute $x(0)$ when $u(t) = 0$ and $x(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

4. (15 points) An engineer would like to deploy an autonomous communications balloon to provide internet connectivity to a particular geographical region. The balloon can control its altitude (a) by changing its buoyancy, but doesn't have any engines. In order to move horizontally (horizontal position p), the balloon drifts on air currents.

   Given the buoyancy control u, the balloon's dynamics are described as

$$\begin{bmatrix} \dot{p} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} -20 & 20 \\ -20 & 20 \end{bmatrix} \begin{bmatrix} p \\ a \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u$$

   Check the stability of the balloon system (stable i.s.L or asymptotic stability or none)

5. Consider the following nonlinear system

$$\dot{x}_1 = -\frac{x_2}{1+x_1^2} - 2x_1$$
$$\dot{x}_2 = \frac{x_1}{1+x_1^2}$$

   (a) (5 points) Using the candidate Lyapunov function $V(x) = x_1^2 + x_2^2$, find the stability of the system at the equilibrium point

   (b) (5 points) Linearize the system about the equilibrium point and find the stability of the linearized system using Lyapunov indirect method

6. (10 points) Find the minimal realization for

$$G(s) = \begin{bmatrix} \frac{-2s-20}{s+11} \\ \\ \frac{s+20}{3s+33} \end{bmatrix}$$

7. Consider you are invited as a control engineering consultant to investigate a critical safety issue for an airplane company. You are provided with an approximate linear model of the lateral dynamics of the aircraft which has the state and control vectors

$$x = \begin{bmatrix} p & r & \beta & \phi \end{bmatrix}^T \quad \text{and} \quad u = \begin{bmatrix} \delta_a & \delta_r \end{bmatrix}^T$$

where p and r are incremental roll and yaw rates, β is an incremental sideslip angle, and ϕ is an incremental roll angle. The control inputs are the incremental changes in the aileron angle δ_a and in the rudder angle δ_r, respectively. In a consistent set of units, the linearized model is given as $\dot{x} = Ax + Bu$ with

$$\mathbf{A} = \begin{bmatrix} -10 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 10 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(a) (5 points) Is the aircraft locally asymptotically stable? Is it locally stable i.s.L.?

(b) (5 points) Is the aircraft controllable with just δ_r?

(c) (5 points) Suppose a malfunction prevents manipulation of the aileron angle δ_r, is it possible to control the aircraft using only the rudder angle δ_a?

(d) (5 points) If the only output is a measurement of the roll rate p (provided by a rate gyro), is the system observable?

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