Exercise Model Linearization:

$\because S_1 = \begin{pmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{pmatrix}$ $\therefore$ the equivalent point will be $\dot{y}=0$

$\ddot{y} = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right)$

$\dot{\psi}=0$

$\ddot{\psi} = \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right)=0$

$\therefore \dot{y}=0 \quad \dot{\psi}=0 \quad \delta = 0$

Let $f_1(y, \dot{y}, \psi, \dot{\psi}, \delta, F) = \ddot{y} = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right)$

$\therefore \frac{\partial f_1}{\partial y} = 0 \quad \frac{\partial f_1}{\partial \dot{y}} = \frac{-2C_\alpha(\cos\delta+1)}{m\dot{x}} = \frac{-4C_\alpha}{m\dot{x}} \quad \frac{\partial f_1}{\partial \psi} = 0$

$\frac{\partial f_1}{\partial \dot{\psi}} = -\dot{x} + \frac{2C_\alpha}{m}\left(-\frac{l_f}{\dot{x}}\cos\delta + \frac{l_r}{\dot{x}}\right) = -\dot{x} + \frac{2C_\alpha}{m\dot{x}}(-l_f + l_r)$

$\frac{\partial f_1}{\partial \delta} = \frac{2C_\alpha}{m}\cos\delta - \frac{2C_\alpha}{m}\sin\delta\,\delta = \frac{2C_\alpha}{m}$

$\frac{\partial f_1}{\partial F} = 0$

Let $f_2 = (y, \dot{y}, \psi, \dot{\psi}, \delta, F) = \ddot{\psi} = \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) + \frac{2l_r C_\alpha}{I_z}\left(\frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right)$

$\frac{\partial f_2}{\partial y} = 0 \quad \frac{\partial f_2}{\partial \dot{y}} = \frac{2C_\alpha}{I_z}(-l_f + l_r) \quad \frac{\partial f_2}{\partial \psi} = 0$

$\frac{\partial f_2}{\partial \dot{\psi}} = -\frac{2C_\alpha(l_f^2 + l_r^2)}{I_z\dot{x}} \quad \frac{\partial f_2}{\partial \delta} = \frac{2l_f C_\alpha}{I_z} \quad \frac{\partial f_2}{\partial F} = 0$

$$\therefore A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_\alpha}{m\dot{x}}(-l_f+l_r) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha}{I_z\dot{x}}(-l_f+l_r) & 0 & \frac{-2C_\alpha(l_f^2+l_r^2)}{I_z\dot{x}} \end{bmatrix} \quad B_1 = \begin{pmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{pmatrix}$$

$\because m = 1888.6\,kg \quad l_r = 1.39\,m \quad l_f = 1.55\,m \quad C_\alpha = 20000\,N \quad I_z = 25854\,kg\,m^2$

$$\therefore \dot{S}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-42.36}{\dot{x}} & 0 & \frac{-3.399}{\dot{x}} - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-0.298}{\dot{x}} & 0 & \frac{-6.706}{\dot{x}} \end{pmatrix} S_1 + \begin{pmatrix} 0 & 0 \\ 21.18 & 0 \\ 0 & 0 \\ 2.398 & 0 \end{pmatrix} u$$

For S_2, we have.

$$\dot{S_2} = (x, \dot{x}, \delta, F) = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \dot{\phi} \dot{y} + \frac{1}{m}(F - fmg) \end{pmatrix}$$

$$A_2 = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \dot{x}} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \dot{x}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \frac{\partial f_1}{\partial \delta} & \frac{\partial f_1}{\partial F} \\ \frac{\partial f_2}{\partial \delta} & \frac{\partial f_2}{\partial F} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}$$

$\because m = 1888.6\ kg$

$$\therefore \dot{S_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} S_2 + \begin{bmatrix} 0 & 0 \\ 0 & 0.000529 \end{bmatrix} u.$$

This is the result of my controller.