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微积分 习题1.7 1.(6)

$r_u' = (1, 2u, 3u^2)$

$r_v' = (1, 2v, 3v^2)$

当 $u=1, v=2$ 时 $r_u' = (1,2,3)$, $r_v' = (1,4,12)$

法向量 $\vec{n} \parallel r_u' \times r_v'$

故 $\vec{n} \parallel (12,-9,2)$. 法线为 $\frac{x-3}{12} = \frac{y-5}{-9} = \frac{z-9}{2}$

切平面方程为 $(x-3, y-5, z-9)\cdot\vec{n} = 0$, 即 $12x-9y+2z-9=0$

3. 在 $P_0(x_0, y_0, z_0)$ 处的切面为

$$\frac{\partial F}{\partial x}(P_0)(x-x_0) + \frac{\partial F}{\partial y}(P_0)(y-\overset{平}{y_0}) + \frac{\partial F}{\partial z}(P_0)(z-z_0) = 0$$

即为 $2x_0(x-x_0) + 4y_0(y-y_0) + 6z_0(z-z_0) = 0$

与 $x+4y+6z=0$ 平行

故 $\frac{2x_0}{1} = \frac{4y_0}{4} = \frac{6z_0}{6} = t_0$

又 $x_0^2 + 2y_0^2 + 3z_0^2 = 21$. 故 $\frac{t_0^2}{4} + 2t_0^2 + 3t_0^2 = 21 \Rightarrow t_0 = \pm 2$

因此切平面为 $x+4y+6z-21=0$ 或 $x+4y+6z+21=0$

5. $F(x) = x^2+y^2+z^2-6$

$G(x) = x+y+z$

切向量 $\vec{v} = \text{grad}\,F(1,-2,1) \times \text{grad}\,G(1,-2,1) = (2,-4,2)\times(1,1,1) = (-6,0,6)$

故切线为 $\begin{cases} x = -t+1 \\ y = -2 \\ z = t+1 \end{cases}$. 法面为 $-6(x-1)+6(z-1)=0$, 即 $x-z=0$

6. 切向量 $r'(t_0) = (-a\sin t_0, a\cos t_0, b)$

z 轴正方向向量为 $(0,0,1)$. 故切线与 z 轴正方向夹角为 $\arccos\frac{b}{\sqrt{a^2+b^2}}$

习题1.8 . 2. (1) $z = (1+(x-1))^{(y-1)+1}$

$$= e^{(1+(y-1))\ln(1+(x-1))}$$

$$= e^{(x-1)-\frac{(x-1)^2}{2}+(x-1)(y-1)+\frac{(x-1)^3}{3}-\frac{(x-1)^2(y-1)}{2}+o(\rho^3)}$$

将e进行展开.知三阶泰勒展开为 $z = 1+((x-1)+(x-1)(y-1)+\frac{(x-1)^3}{4}+\frac{(x-1)^2(y-1)}{2})+o(\rho^3)$

故 $(1.1)^{1.02} \approx 1+(0.1+0.002+0.00025+0.0001)$

≈ 1.102

(2) $z = \frac{\cos x}{\cos y}$ $\qquad \frac{\partial^2 z}{\partial x^2} = \frac{-\cos x}{\cos y}$

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$\frac{\partial z}{\partial y} = \frac{\cos x\sin y}{\cos^2 y}$ $\qquad \frac{\partial^2 z}{\partial y^2} = \frac{\cos x\cos^3 y + 2\cos y\sin^2 y\cos x}{\cos^4 y}$

故在 $(0,0)$ 处二阶Taylor多项式为 $z = 1-\frac{x^2}{2}+\frac{y^2}{2}$

(3) $z = e^{-x}\ln(1+y)$

$= (1-x+\frac{x^2}{2}+o(x^2))(y-\frac{y^2}{2}+o(y^2))$

故 $z = e^{-x}\ln(1+y)$ 在 $(0,0)$ 处的二阶Taylor多项式为 $z = y-xy-\frac{y^2}{2}$