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离散数学HW14. 1. 由于 $b>a$

因此存在双射函数

$$f:[0,1]\to[a,b],\ f(x)=(b-a)x+a$$

因此 $[0,1]\approx[a,b]$

2. 集合 $A_1=\{x \mid \cancel{x\in N\wedge}(\exists t)(t\in N\wedge x=t^2)\}$

$A_2=\{x \mid \cancel{x\in N\wedge}(\exists t)(t\in N\wedge x=2t)\}$

$A_3=\{x \mid \cancel{x\in N\wedge}(\exists t)(t\in N\wedge x=3t)\}$

3. (1) $k^m\le(2^k)^m=2^{k\cdot m}=2^m\le k^m$

$\therefore k^m=2^m$

(2) 由对称性，不妨设 $k\le l$

则 $k^m\le l^m\le(k^l)^m=k^{l\cdot m}=k^m$

$\therefore k^m=l^m$

4. 对整数坐标点，可与自然数进行一一配对

$$\begin{tikzcd}
\cdots & (-2,-2) \arrow[r] & (-2,-1) \arrow[r] & (-2,0) \arrow[r] & (-2,1) \arrow[r] & (-2,2) \arrow[r] & \cdots \\
\cdots & (-1,-2) \arrow[u] & (-1,-1) \arrow[r] & (-1,0) \arrow[r] & (-1,1) \arrow[r] & (-1,2) \arrow[d] & \cdots \\
\cdots & (0,-2) \arrow[u] & (0,-1) \arrow[u] & (0,0) \arrow[r] & (0,1) \arrow[d] & (0,2) \arrow[d] & \cdots \\
\cdots & (1,-2) \arrow[u] & (1,-1) \arrow[u] & (1,0) \arrow[l] & (1,1) \arrow[l] & (1,2) \arrow[d] & \cdots \\
\cdots & (2,-2) \arrow[u] & (2,-1) \arrow[l] & (2,0) \arrow[l] & (2,1) \arrow[l] & (2,2) \arrow[l] & \cdots
\end{tikzcd}$$

因此为可数集

6. $R\approx R-N$. 因为存在双射函数.

$$f:R\to R-N$$

$$f(x)=\begin{cases}\dfrac{2x+1}{2} & \text{当 } x\in N\\[2mm] \dfrac{2x+1}{2^{r+1}} & \text{当 } x=\dfrac{2t+1}{2^r},\ t=0,1,2,\cdots,\ r=1,2,3,\cdots\\[2mm] x & \text{当 } x \text{ 取其他值}\end{cases}$$