HW10 2023010747

Problem 10.1

1. $e_i^T H_n^2 e_j = e_i^T H_n H_n e_j = e_i^T H_n^T H_n e_j = (H_n e_i)^T H_n e_j$

if $i=j$. then $(H_n e_i)^T H_n e_j = n$

if $i \neq j$. since all columns are mutually orthogonal. thus $(H_n e_i)^T H_n e_j = 0$

2. $tr(H_n) = tr(H_{\frac{n}{2}}) + tr(-H_{\frac{n}{2}}) = 0$

so $\lambda_1 + \lambda_2 + \cdots + \lambda_n = 0$

$H_n' = \frac{H_n}{\sqrt{n}}$ then $(H_n')^2 = I$ $\quad H_n' v = \lambda v \Leftrightarrow v = \lambda H_n' v \Leftrightarrow v = \lambda^2 v \Leftrightarrow (\lambda^2 - 1) v = 0$

so eigenvalues of H_n' must be ± 1

so eigenvalues of H_n must be $\pm\sqrt{n}$

since $\lambda_1 + \cdots + \lambda_n = 0$

so $\frac{n}{2}$ eigenvalues are $\sqrt{n}$

$\frac{n}{2}$ eigenvalues are $-\sqrt{n}$

3. $(\frac{H_2}{\sqrt{2}})^2 = I$. so it's a reflection, eigenvectors can be $\begin{bmatrix} \sqrt{2}+1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1-\sqrt{2} \\ 1 \end{bmatrix}$

4. $H_n v = \lambda v \qquad H_{2n} v' = \sqrt{2}\lambda v'$

$$H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & -H_n \end{bmatrix}$$

$$\begin{bmatrix} H_n & H_n \\ H_n & -H_n \end{bmatrix}\begin{bmatrix} v \\ (\sqrt{2}-1)v \end{bmatrix} = \begin{bmatrix} \sqrt{2} H_n v \\ (2-\sqrt{2}) H_n v \end{bmatrix} = \sqrt{2}\lambda \begin{bmatrix} v \\ (\sqrt{2}-1)v \end{bmatrix}$$ thus we can pick $v' = \begin{bmatrix} v \\ (\sqrt{2}-1)v \end{bmatrix}$ as an eigenvector

Problem 10.2

1. $A = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix} \qquad H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

$$AH_4 = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 5 & 5 \\ 1 & -5 & 5 & -5 \\ 1 & 5 & -5 & -5 \\ 1 & -5 & -5 & 5 \end{bmatrix}$$

2. $H_4^2 = 4I \Rightarrow H_4^{-1} = \frac{1}{4} H_4$

$$H_4^{-1} A H_4 = \frac{1}{4} H_4 A H_4 = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 & 5 & 5 \\ 1 & -5 & 5 & -5 \\ 1 & 5 & -5 & -5 \\ 1 & -5 & -5 & 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 20 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

3. $\det(A) = \det(H_4^{-1} A H_4) = \det(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}) = 125$

$$A^{-1} = H_4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}^{-1} H_4^{-1} = \frac{1}{4} H_4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{1}{5} \end{bmatrix} H_4 = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{1}{5} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} & -\frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} \\ \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} \frac{8}{5} & \frac{4}{5} & \frac{4}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{8}{5} & \frac{4}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{4}{5} & \frac{8}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{4}{5} & \frac{4}{5} & \frac{8}{5} \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

Problem 10.3

1. $AF_4 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} = \begin{bmatrix} 10 & -2-2i & -2 & -2+2i \\ 10 & 2-2i & 2 & 2+2i \\ 10 & 2+2i & -2 & 2-2i \\ 10 & -2+2i & 2 & -2-2i \end{bmatrix}$

2. $X = F_4 \qquad F_4^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$

$$D = F_4^{-1} A F_4 = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}\begin{bmatrix} 10 & -2-2i & -2 & -2+2i \\ 10 & 2-2i & 2 & 2+2i \\ 10 & 2+2i & -2 & 2-2i \\ 10 & -2+2i & 2 & -2-2i \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 40 & 0 & 0 & 0 \\ 0 & -8-8i & 0 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & 0 & -8+8i \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & -2-2i & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2+2i \end{bmatrix}$$

3. $\det(A) = \det(D) = -20 \cdot (4+4) = -160$

$$A^{-1} = F_4 D^{-1} F_4^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}\begin{bmatrix} \frac{1}{10} & 0 & 0 & 0 \\ 0 & \frac{i-1}{4} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{i+1}{4} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}\begin{bmatrix} \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{i-1}{4} & \frac{-1+i}{4} & \frac{1-i}{4} & \frac{-1-i}{4} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{-1-i}{4} & \frac{1-i}{4} & \frac{1+i}{4} & \frac{-1+i}{4} \end{bmatrix} = \begin{bmatrix} -\frac{9}{40} & \frac{11}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{40} & -\frac{9}{40} & \frac{11}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{1}{40} & -\frac{9}{40} & \frac{11}{40} \\ \frac{11}{40} & \frac{1}{40} & \frac{1}{40} & -\frac{9}{40} \end{bmatrix}$$

Problem 10.4 all rows add up to 1

so 1 is an eigenvalue and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector

since second column is half of the first column. so 0 is an eigenvalue and $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ is an eigenvector

the trace is 0. so the last eigenvalue is -1. $Ker(\begin{bmatrix} 111 & 55 & -164 \\ 42 & 22 & -62 \\ 88 & 44 & -130 \end{bmatrix}) = Ker(\begin{bmatrix} 33 \\ -1 \\ 22 \end{bmatrix})$

$$\text{so } \begin{bmatrix} 110 & 55 & -164 \\ 42 & 21 & -62 \\ 88 & 44 & -131 \end{bmatrix}^{2017} = \begin{bmatrix} 1 & 1 & 33 \\ 1 & -2 & -1 \\ 1 & 0 & 22 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}^{2017}\begin{bmatrix} 1 & 1 & 33 \\ 1 & -2 & -1 \\ 1 & 0 & 22 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 1 & 33 \\ 1 & -2 & -1 \\ 1 & 0 & 22 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}^{2017}\begin{bmatrix} 1 & 1 & 33 \\ 1 & -2 & -1 \\ 1 & 0 & 22 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 110 & 55 & -164 \\ 42 & 21 & -62 \\ 88 & 44 & -131 \end{bmatrix}$$

Problem 10.5 1. $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}^{1024} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}^{1024}\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2^{1024} & 0 \\ 0 & 5^{1024} \end{bmatrix}\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \frac{2^{1024}}{3} & -\frac{2^{1024}}{3} \\ \frac{5^{1024}}{3} & \frac{2 \cdot 5^{1024}}{3} \end{bmatrix} = \begin{bmatrix} \frac{2^{1025}+5^{1024}}{3} & \frac{2(5^{1024}-2^{1024})}{3} \\ \frac{5^{1024}-2^{1024}}{3} & \frac{2\cdot 5^{1024}+2^{1024}}{3} \end{bmatrix}$

since $\frac{1}{3}(5^{1024}-2^{1024}) > 10^{700}$

so all entries of $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}^{1024}$ are larger than 10^{700}

2. $\begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix}^{1024} = \begin{bmatrix} \frac{3+i}{5} & \frac{3-i}{5} \\ -1 & -1 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}^{1024}\begin{bmatrix} \frac{3+i}{5} & \frac{3-i}{5} \\ -1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{3+i}{5} & \frac{3-i}{5} \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{3+i}{5} & \frac{3-i}{5} \\ -1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} -5 & -7 \\ 3 & 4 \end{bmatrix}^{1024} = \begin{bmatrix} \frac{-9+\sqrt{3}i}{6} & \frac{-9-\sqrt{3}i}{6} \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{\frac{2\pi}{3}i} & 0 \\ 0 & e^{\frac{4\pi}{3}i} \end{bmatrix}^{1024}\begin{bmatrix} \frac{-9+\sqrt{3}i}{6} & \frac{-9-\sqrt{3}i}{6} \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{-9+\sqrt{3}i}{6} & \frac{-9-\sqrt{3}i}{6} \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{\frac{2\pi}{3}i} & 0 \\ 0 & e^{\frac{4\pi}{3}i} \end{bmatrix}\begin{bmatrix} \frac{-9+\sqrt{3}i}{6} & \frac{-9-\sqrt{3}i}{6} \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -5 & -7 \\ 3 & 4 \end{bmatrix}$

Problem 10.6 1. this is Jordan Block J_{10}.

so the eigenvalue is 10. its algebraic multiplicity is 3 and geometric multiplicity is 1

algebraic multiplicity > geometric multiplicity so it's not diagonalizable

2. $\begin{bmatrix} 10 & 1 & 0 \\ 0 & 10.001 & 1 \\ 0 & 0 & 10.002 \end{bmatrix}$

the eigenvalues are 10, 10.001, 10.002

10 : its algebraic multiplicity is 1 and geometric multiplicity is 1

10.001 : its algebraic multiplicity is 1 and geometric multiplicity is 1

10.002 : its algebraic multiplicity is 1 and geometric multiplicity is 1

geometric multiplicities add up to 3 so it's diagonalizable

3. the eigenvalue is 0 its algebraic multiplicity is 4 and geometric multiplicity is 1 ($\ker(-A) = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right)$)

algebraic multiplicity > geometric multiplicity so it's not diagonalizable

4. the eigenvalue is $0.1, -0.1, 0.1i, -0.1i$ 0.1: its algebraic multiplicity is 1 and geometric multiplicity is 1

-0.1 : its algebraic multiplicity is 1 and geometric multiplicity is 1 $0.1i$: its algebraic multiplicity is 1 and geometric multiplicity is 1

$-0.1i$: its algebraic multiplicity is 1 and geometric multiplicity is 1 algebraic multiplicity = geometric multiplicity so it's diagonalizable

Problem 10.7 1. $\begin{bmatrix} a & b & \cdots & b \\ b & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & b \\ b & \cdots & b & a \end{bmatrix} = aI + b\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix} - I$

$\dim\left(\ker\left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix}\right)\right) = n-1$. so $\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix}$ has eigenvalue 0 with algebraic multiplicity $n-1$

$\text{tr}\left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix}\right) = n$. so it also has eigenvalue n with algebraic multiplicity 1

$\left(\dim \ker\left(\begin{bmatrix} b & \cdots & b \\ \vdots & \cdots & \vdots \\ b & \cdots & b \end{bmatrix}\right)\right)$

$\begin{bmatrix} a & b & \cdots & b \\ b & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & b \\ b & \cdots & b & a \end{bmatrix}$ has eigenvalue $a-b$ with algebraic multiplicity $n-1$ and geometric multiplicity $n-1$, eigenvectors are $\begin{bmatrix} a_1 \\ \vdots \\ a_{n-1} \\ -a_1-a_2-\cdots-a_{n-1} \end{bmatrix}$ $(\forall a_i \neq 0)$

has eigenvalue $a+(n-1)b$ with algebraic multiplicity 1 and geometric multiplicity 1, eigenvectors are $\begin{bmatrix} t \\ t \\ \vdots \\ t \end{bmatrix}$ $(t \neq 0)$

$A = \begin{bmatrix} a & b & \cdots & b \\ b & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & b \\ b & \cdots & b & a \end{bmatrix}$ can be diagonalized. $\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ -1 & 0 & 0 & & 0 & 1 \\ 0 & -1 & 0 & & \vdots & 1 \\ \vdots & 0 & -1 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & & 0 & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}\begin{bmatrix} a-b & & & \\ & a-b & & \\ & & \ddots & \\ & & & a-b & \\ & & & & a+(n-1)b \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ -1 & 0 & 0 & & 0 & 1 \\ 0 & -1 & 0 & & \vdots & 1 \\ \vdots & 0 & -1 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & & 0 & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}^{-1}$

2. $A = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$ has eigenvalue -1 with algebraic multiplicity 2 and geometric multiplicity 1 eigenvectors are $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

A can't be diagonalized. its Schur decomposition is $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Problem 10.8 1. $A^2 = I$ $(A-I)(A+I) = 0$

for v, $v_1 = \frac{I-A}{2}v$ $v_2 = \frac{I+A}{2}v$ so $v_1 \in \ker(A+I)$, $v_2 \in \ker(A-I)$ $v_1 + v_2 = Iv = v$

so for every $v \in \mathbb{C}^n$, $v \in \ker(A+I) + \ker(A-I)$

besides, $\ker(A+I) + \ker(A-I) \subseteq \mathbb{C}^n$

So $\ker(A+I) + \ker(A-I) = \mathbb{C}^n$

$\lambda^2 = 1 \Rightarrow \lambda = \pm 1$ $\ker(A+I) + \ker(A-I) = \mathbb{C}^n$ so A is diagonalizable

2. $A^2 = A$ $A(A-I) = 0$

for v, $v_1 = (I-A)v$, $v_2 = Av$ so $v_1 \in \ker(A)$, $v_2 \in \ker(A-I)$ $v_1 + v_2 = Iv = v$

so for every $v \in \mathbb{C}^n$, $v \in \ker(A) + \ker(A-I)$

besides, $\ker(A) + \ker(A-I) \subseteq \mathbb{C}^n$

So $\ker(A) + \ker(A-I) = \mathbb{C}^n$

$\lambda^2=\lambda \Rightarrow \lambda=0,1$ $Ker(A)+Ker(A-I)=\mathbb{C}^n$ so A is diagonalizable

3. $a=\frac{1}{2}$. $b=\frac{1}{2}$, $c=-1$ and we have $v=\frac{1}{2}A(A-I)v+\frac{1}{2}A(A+I)v-(A-I)(A+I)v$

4. $A^3=A$ $A(A+I)(A-I)=0$

for v. $v_1=-(A+I)(A-I)v$ $v_2=\frac{1}{2}A(A-I)v$ $v_3=\frac{1}{2}A(A+I)v$ so $v_1\in Ker(A)$, $v_2\in Ker(A+I)$. $v_3\in Ker(A-I)$ $v_1+v_2+v_3=Iv=v$

so for every $v\in\mathbb{C}^n$, $v\in Ker(A)+Ker(A+I)+Ker(v-I)$

besides. $Ker(A)+Ker(A+I)+Ker(v-I)\subseteq\mathbb{C}^n$

so $Ker(A)+Ker(A+I)+Ker(v-I)=\mathbb{C}^n$

$\lambda^3=\lambda \Rightarrow \lambda=0,\pm1$, $Ker(A)+Ker(A+I)+Ker(v-I)=\mathbb{C}^n$ so A is diagonalizable