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微积分作业10.

习题3.5 1.(3) 将表面积分成两部分，对 $z\in[0,a]$ 和 $[a,2a]$ 分别求表面积

$z\in[0,a]$ 时 $z=\frac{x^2+y^2}{a}$. $S_1=\iint\limits_{x^2+y^2\le a^2}\sqrt{1+(\frac{2x}{a})^2+(\frac{2y}{a})^2}\,dxdy=\frac{1}{a}\iint\limits_{x^2+y^2\le a^2}\sqrt{a^2+4x^2+4y^2}\,dxdy$

$=\frac{1}{a}\iint\limits_{0\le\rho\le a}\sqrt{a^2+4\rho^2}\,\rho\,d\rho\,d\theta=\frac{2\pi}{a}\int_0^a\rho\sqrt{a^2+4\rho^2}\,d\rho=\frac{2\pi}{a}\cdot\frac{1}{12}(a^2+4\rho^2)^{\frac{3}{2}}\Big|_0^a=\frac{(5\sqrt{5}-1)\pi}{6}\cdot a^2$

$z\in[a,2a]$ 时 $S_2=\iint\limits_{x^2+y^2\le a^2}\sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}\,dxdy=\sqrt{2}\pi a^2$

因此表面积为 $\frac{6\sqrt{2}+5\sqrt{5}-1}{6}\cdot\pi a^2$

8. 上表面所受压力为 $\iint\limits_{x^2+y^2\le a^2}\frac{a}{\sqrt{a^2-x^2-y^2}}\delta\,(h-\sqrt{a^2-x^2-y^2})\cos\gamma\,dxdy$

$=\iint\limits_{x^2+y^2\le a^2}\delta(h-\sqrt{a^2-x^2-y^2})\,dxdy=-h\pi a^2\delta+\frac{2\pi\delta}{3}\sqrt{(a^2-r^2)^3}\Big|_0^a=\pi a^2\delta(h-\frac{2a}{3})$

上表面所受压力为 $\iint\limits_{x^2+y^2\le a^2}\delta(h+\sqrt{a^2-x^2-y^2})\,dxdy=\pi a^2\delta(h+\frac{2a}{3})$

习题4.3. 1.(4). $dS=\sqrt{1+(\frac{\partial z}{\partial x})^2+(\frac{\partial z}{\partial y})^2}\,dxdy=\sqrt{2}\,dxdy$

投影区域为 $x^2+(y-a)^2\le a^2$

故 $\iint\limits_S(xy+yz+zx)\,dS=\iint\limits_D\sqrt{2}\,(xy+(x+y)\sqrt{x^2+y^2})\,dxdy$

$x=\rho\cos\theta$. $y=\rho\sin\theta$

$dxdy=\rho\,d\rho\,d\theta$. $\iint\limits_{\substack{0\le\rho\le 2a\cos\theta\\ -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}}}\sqrt{2}\,(\rho^2\sin\theta\cos\theta+\rho^2(\sin\theta+\cos\theta))\,\rho\,d\rho\,d\theta=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta\int_0^{2a\cos\theta}\sqrt{2}\rho^3(\sin\theta\cos\theta+\sin\theta+\cos\theta)\,d\rho$

$=4\sqrt{2}a^4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos^5\theta\sin\theta+\cos^4\theta\sin\theta+\cos^5\theta)\,d\theta=4\sqrt{2}a^4\cdot2\cdot\frac{8}{15}=\frac{64\sqrt{2}}{15}a^4$

6. $\bar{z}=\frac{\iint_D z\sqrt{1+\frac{x^2}{z^2}+\frac{y^2}{z^2}}\,dxdy}{\iint_D\sqrt{1+\frac{x^2}{z^2}+\frac{y^2}{z^2}}\,dxdy}=\frac{\int_0^{\frac{\pi}{2}}d\varphi\int_0^{\frac{\pi}{2}}a\cos\theta\,a^2\sin\theta\,d\theta}{\frac{\pi a^2}{2}}=\frac{a}{2}$

由对称性知质心坐标为 $(\frac{a}{2},\frac{a}{2},\frac{a}{2})$

上半球面：对称性知质心在z轴上.

$\bar{z}=\frac{\int_0^{2\pi}d\varphi\int_0^{\frac{\pi}{2}}a\cos\theta\,a^2\sin\theta\,d\theta}{2\pi a^2}=\frac{a}{2}$. 因此质心坐标为 $(0,0,\frac{a}{2})$

10. 过 $P(x_0,y_0,z_0)$ 的切平面为 $\frac{2x_0}{a^2}(x-x_0)+\frac{2y_0}{b^2}(y-y_0)+\frac{2z_0}{c^2}(z-z_0)=0$，法向量为 $(\frac{2x_0}{a^2},\frac{2y_0}{b^2},\frac{2z_0}{c^2})$

因此距离 $\lambda(x,y,z)=\frac{2}{2\sqrt{\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}}}$

又 $dS=\frac{c^2}{|z|}\sqrt{\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}}\,dxdy$.

故 $\iint\limits_S\lambda(x,y,z)\,dS=\iint\limits_D\frac{c^2}{|z|}\,dxdy=2c\int_0^{2\pi}d\theta\int_0^1\frac{ab\rho}{\sqrt{1-\rho^2}}\,d\rho=4\pi abc$