20230107247 刘一[illegible] 计32

习题1.5. 4. $\frac{dz}{dx}=\frac{dz}{du}\cdot\frac{du}{dx}+\frac{dz}{dv}\cdot\frac{dv}{dx}$

$=(\ln(u-v)+\frac{u}{u-v})(-e^{-x})+u\cdot\frac{1}{u-v}\cdot(-1)\cdot\frac{1}{x}$

$=\frac{\frac{e^{-x}}{\ln x-e^{-x}}-\ln(e^{-x}-\ln x)}{e^{x}}+\frac{1}{xe^{x}(\ln x-e^{-x})}$

5. $\frac{\partial u}{\partial r}=f'_x\cdot\cos\theta+f'_y\cdot\sin\theta$

$\frac{\partial u}{\partial\theta}=rf'_x(-\sin\theta)+rf'_y\cdot\cos\theta$

$\frac{\partial u}{\partial x}=f'_x\qquad\frac{\partial u}{\partial y}=f'_y$

$\therefore(\frac{\partial u}{\partial r})^2+(\frac{1}{r}\frac{\partial u}{\partial\theta})^2=f'^2_x(\sin^2\theta+\cos^2\theta)+f'^2_y(\sin^2\theta+\cos^2\theta)=f'^2_x+f'^2_y=(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2.$

8. $x=u$

$y=v-u$

$z=w-v$

$\frac{\partial z}{\partial x}=\frac{\partial w}{\partial x}-1=\frac{\partial w}{\partial u}\cdot1+\frac{\partial w}{\partial v}\cdot1-1=\frac{\partial w}{\partial u}+\frac{\partial w}{\partial v}-1$

$\frac{\partial z}{\partial y}=\frac{\partial w}{\partial y}-1=\frac{\partial w}{\partial u}\cdot0+\frac{\partial w}{\partial v}\cdot1-1=\frac{\partial w}{\partial v}-1$

$\frac{\partial^2 z}{\partial x^2}=\frac{\partial^2 w}{\partial u^2}+\frac{\partial^2 w}{\partial v\partial u}+\frac{\partial^2 w}{\partial u\partial v}+\frac{\partial^2 w}{\partial v^2}$

$\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 w}{\partial v^2}$

$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 w}{\partial u\partial v}+\frac{\partial^2 w}{\partial v^2}$

故原方程可化简为 $\frac{\partial^2 w}{\partial u^2}+\frac{\partial^2 w}{\partial v\partial u}-\frac{\partial^2 w}{\partial u\partial v}+\frac{\partial w}{\partial u}=0$

习题1.6 4. 对 $f(u^2-x^2, u^2-y^2, u^2-z^2)=0$

两边同时对 x, y, z 求偏导知

$f'_1(2u\cdot u'_x-2x)+f'_2\cdot2uu'_x+f'_3\cdot2uu'_x=0\Rightarrow(f'_1+f'_2+f'_3)\frac{u'_x}{x}=\frac{1}{u}f'_1$

$f'_1\cdot2uu'_y+f'_2(2uu'_y-2y)+f'_3\,2uu'_y=0\Rightarrow(f'_1+f'_2+f'_3)\frac{u'_y}{y}=\frac{1}{u}f'_2$

$f'_1\cdot2uu'_z+f'_2\,2uu'_z+f'_3(2uu_z-2z)=0\Rightarrow(f'_1+f'_2+f'_3)\frac{u'_z}{z}=\frac{1}{u}f'_3$

三式相加知 $\frac{1}{x}\frac{\partial u}{\partial x}+\frac{1}{y}\frac{\partial u}{\partial y}+\frac{1}{z}\frac{\partial u}{\partial z}=\frac{1}{u}$

5. 能 $\begin{cases}x=u+v\\y=u-v\\z=u^2v^2\end{cases}\Rightarrow\begin{cases}1=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}\\0=\frac{\partial u}{\partial x}-\frac{\partial v}{\partial x}\\0=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}\\1=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial y}\\\frac{\partial z}{\partial x}=2uv^2\cdot\frac{\partial u}{\partial x}+2vu^2\frac{\partial v}{\partial x}\\\frac{\partial z}{\partial y}=2uv^2\frac{\partial u}{\partial y}+2vu^2\cdot\frac{\partial v}{\partial y}\end{cases}\Rightarrow\begin{cases}\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}=\frac{1}{2}\\\frac{\partial z}{\partial x}=uv^2+vu^2=\frac{x(x^2-y^2)}{4}\\\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial y}=\frac{1}{2}\\\frac{\partial z}{\partial y}=uv^2-vu^2=\frac{y(y^2-x^2)}{4}\end{cases}$

7. 对等式两边同时对z求导. $\begin{cases}2x\frac{dx}{dz}+2y\frac{dy}{dz}=z\\\frac{dx}{dz}+\frac{dy}{dz}+1=0\end{cases}$ 代入 $\begin{cases}x=1\\y=-1\\z=2\end{cases}$ 得 $\begin{cases}\frac{dx}{dz}=0\\\frac{dy}{dz}=-1\end{cases}$

再对z求导知 $\begin{cases}2(\frac{dx}{dz})^2+2\cdot x\frac{d^2x}{dz^2}+2(\frac{dy}{dz})^2+2y\frac{d^2y}{dz^2}=1\\\frac{d^2x}{dz^2}+\frac{d^2y}{dz^2}=0\end{cases}$ 代入得 $\begin{cases}\frac{d^2x}{dz^2}=-\frac{1}{4}\\\frac{d^2y}{dz^2}=\frac{1}{4}\end{cases}$

10. (1) $\frac{\partial u}{\partial x}=2\xi\cdot\cos y\cdot e^x-2\eta\sin y\cdot e^x\quad\frac{\partial u}{\partial y}=2\xi e^x\cdot(-\sin y)-2\eta e^x\cos y\quad\frac{\partial v}{\partial x}=2\eta e^x\cos y+2\xi e^x\sin y\quad\frac{\partial v}{\partial y}=2\eta e^x(-\sin y)+2\xi e^x\cos y$

故Jacobi行列式 $\left.\frac{D(u,v)}{D(x,y)}\right|_{(1,0)}=\begin{vmatrix}2e^2&0\\0&2e^2\end{vmatrix}=4e^4>0.$

故能确定可微的 $(g\circ f)^{-1}$