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线性代数附加题2.

1. $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & x-8 \end{pmatrix}(\lambda I_3 - A_1)\begin{pmatrix} 1 & -2 & x \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & x-3 & 0 \\ 0 & 0 & (x-3)^2 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 7 & -2 \\ 119 & -70 & x-17 \end{pmatrix}(\lambda I_3 - A_2)\begin{pmatrix} 1 & 10 & 31+x \\ 0 & 17 & 34 \\ 0 & 0 & 119 \end{pmatrix} = 119\begin{pmatrix} 1 & 0 & 0 \\ 0 & x-3 & 0 \\ 0 & 0 & (x-3)^2 \end{pmatrix}$

因此$\lambda I_3 - A_1$与$\lambda I_3 - A_2$有相同的Smith标准形，故A_1、A_2相似

2. $A = cI_n$.

故$\lambda I_n - A = \begin{pmatrix} \lambda-c & & & \\ & \lambda-c & & \\ & & \ddots & \\ & & & \lambda-c \end{pmatrix}$ 的$n-1$阶子式行列式若不为0，则$\lambda-c$均在对角线上.

故 $D_{n-1}(\lambda) \,\Big|\, \underbrace{\begin{vmatrix} \lambda-c & & \\ & \ddots & \\ & & \lambda-c \end{vmatrix}}_{(n-1)\text{行}} \Rightarrow D_{n-1}(\lambda) \mid (\lambda-c)^{n-1}$.

又$D_{n-1}(\lambda)$为最大公因式，故$D_{n-1}(\lambda) = (\lambda-c)^{n-1}$为$n-1$次的

4. 因为$(f(\lambda), g(\lambda)) = 1$.

因此$f(\lambda)$与$g(\lambda)$无相同的根.

若有λ_1使$f(\lambda_1)=0$，则$g(\lambda_1)\neq 0$，$f(\lambda_1)g(\lambda_1)=0$

则$\begin{pmatrix} f(\lambda_1) & 0 \\ 0 & g(\lambda_1) \end{pmatrix}$与$\begin{pmatrix} 1 & 0 \\ 0 & f(\lambda_1)g(\lambda_1) \end{pmatrix}$秩均为1.

$f(\lambda_1)\neq 0$，$g(\lambda_1)=0$同理.

若λ_1有$f(\lambda_1)\neq 0$，$g(\lambda_1)\neq 0$，则$\begin{pmatrix} f(\lambda_1) & 0 \\ 0 & g(\lambda_1) \end{pmatrix}$与$\begin{pmatrix} 1 & 0 \\ 0 & f(\lambda_1)g(\lambda_1) \end{pmatrix}$秩均为2.

因此$\begin{pmatrix} f(\lambda) & 0 \\ 0 & g(\lambda) \end{pmatrix}$与$\begin{pmatrix} 1 & 0 \\ 0 & f(\lambda)g(\lambda) \end{pmatrix}$秩相同，因此相抵