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微积分作业7.

3.2.4. 若存在 $f(x,y)>0$.

由于 $f(x,y)$ 连续. 故存在 D' 区域使在 D' 上 $f(x,y)>0$

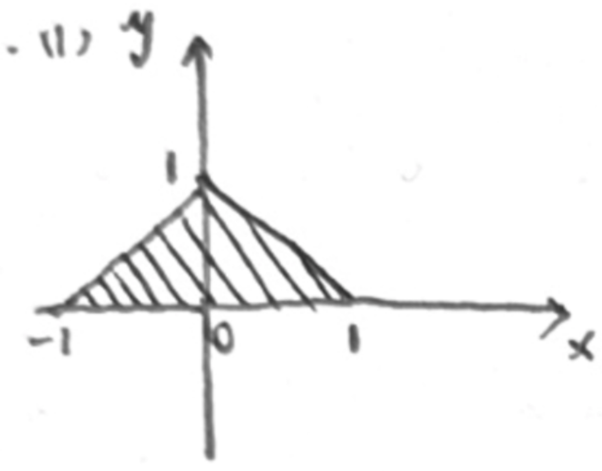
因此 $\iint\limits_{D'} f(x,y)\,dx\,dy>0$

又 $f(x,y)\geq 0$. 故 $\iint\limits_{D-D'} f(x,y)\,dx\,dy\geq 0$

故 $\iint\limits_{D} f(x,y)\,dx\,dy>0$ 矛盾.

因此 $f(x,y)=0\quad \forall (x,y)\in D$

3.3.5.(1)

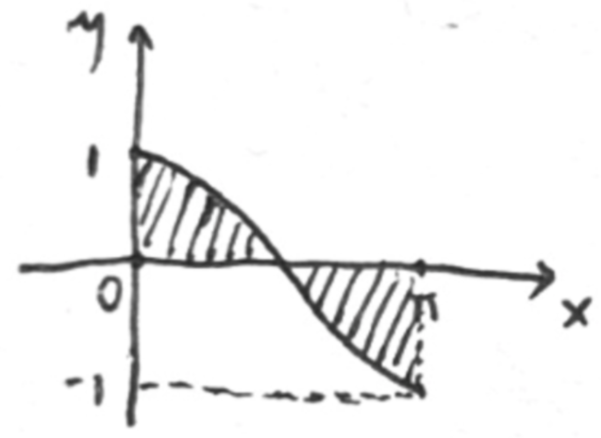


$$\int_{-1}^{0}dx\int_{0}^{1+x}f(x,y)\,dy+\int_{0}^{1}dx\int_{0}^{1-x}f(x,y)\,dy$$

$$=\int_{0}^{1}dy\int_{y-1}^{0}f(x,y)\,dx+\int_{0}^{1}dy\int_{0}^{1-y}f(x,y)\,dx$$

$$=\int_{0}^{1}dy\int_{y-1}^{1-y}f(x,y)\,dx$$

(5)



$$\int_{0}^{\pi}dx\int_{0}^{\cos x}f(x,y)\,dy=-\int_{-1}^{0}dy\int_{\arccos y}^{\pi}f(x,y)\,dx+\int_{0}^{1}dy\int_{0}^{\arccos y}f(x,y)\,dx$$

6.(2) $\iint\limits_{D}\frac{1}{\sqrt{2a-x}}\,dx\,dy=\int_{0}^{a}dy\int_{a-\sqrt{2ay-y^2}}^{a}\frac{1}{\sqrt{2a-x}}\,dx$

$$=\int_{0}^{a}-2\sqrt{2a-x}\,\Big|_{a-\sqrt{2ay-y^2}}^{a}\,dy$$

$$=-2\int_{0}^{a}\sqrt{2a-a}\,dy+2\int_{0}^{a}\sqrt{a+\sqrt{2ay-y^2}}\,dy$$

令 $y=a+a\sin t$. $t\in[-\frac{\pi}{2},0]$

$$\int_{0}^{a}\sqrt{2a-a}\,dy=a\sqrt{a}$$

$$\int_{0}^{a}\sqrt{a+\sqrt{2ay-y^2}}\,dy=\int_{-\frac{\pi}{2}}^{0}\sqrt{2a}\cos\frac{t}{2}\,dt\,a\cos t\,dt=a\sqrt{2a}\,(\sin\frac{x}{2}+\frac{1}{3}\sin\frac{3x}{2})\Big|_{-\frac{\pi}{2}}^{0}=\frac{4a\sqrt{a}}{3}$$

故原式 $=\frac{2a\sqrt{a}}{3}$

6.(7) $\iint\limits_{D}\cos(x+y)\,dx\,dy=\int_{0}^{\pi}dx\int_{0}^{\pi}\cos(x+y)\,dy=\int_{0}^{\pi}\sin(x+y)\Big|_{0}^{\pi}dx=\int_{0}^{\pi}-2\sin x\,dx=2\cos x\Big|_{0}^{\pi}=-4$

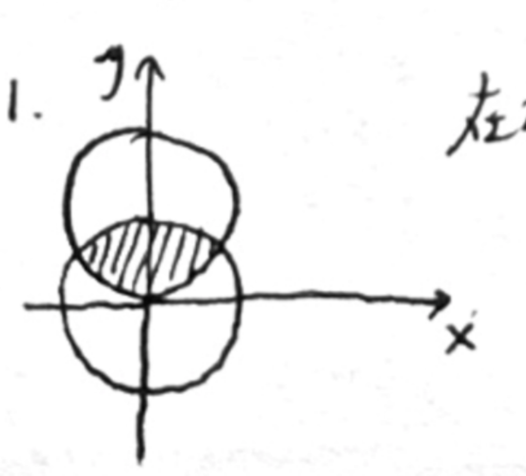
(9) $x=x(t)$ 在 $[0,2\pi]$ ↑ $\exists$ 反函数 $t=t(x)$. $y=y(t(x))$ D 为 $\{(x,y)\mid 0\leq x\leq 2\pi a,\ 0\leq y\leq y(t(x))\}$

$$\iint\limits_{D}y^2\,dx\,dy=\int_{0}^{2\pi a}dx\int_{0}^{y(t(x))}y^2\,dy=\int_{0}^{2\pi a}\frac{1}{3}y^3(t(x))\,dx=\int_{0}^{2\pi}\frac{1}{3}y^3(t)\,a(1-\cos t)\,dt$$

$$=\frac{a^4}{3}\int_{0}^{2\pi}(1-\cos t)^4\,dt$$

$$=\frac{35}{12}\pi a^4$$

11.



在极坐标下积分为

$$2\left(\int_{0}^{\frac{\pi}{6}}d\theta\int_{0}^{2\sin\theta}f(\rho\cos\theta,\rho\sin\theta)\rho\,d\rho+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}d\theta\int_{0}^{1}f(\rho\cos\theta,\rho\sin\theta)\rho\,d\rho\right)$$