2023010747 刘一[illegible] 计32

线代作业2.

习题8. 1. $f(x)=x^5-x^3+3x^2-1$

$g(x)=x^3-3x+2$

$$
\begin{array}{r|llll|l}
x^3-3x+2 & x^5 & -x^3 & +3x^2 & -1 & x^2+2=q(x) \\
 & x^5 & -3x^3 & +2x^2 & & \\
\hline
 & & 2x^3 & +x^2 & -1 & \\
 & & 2x^3 & & -6x+4 & \\
\hline
 & & & & x^2+6x-5=r(x) &
\end{array}
$$

故商式为 x^2+2，余式为 x^2+6x-5

2.
$$
\begin{array}{r|l|l}
x^2-2ax+2 & x^4+3x^2+ax+b & x^2+2ax+(1+4a)=q(x) \\
 & x^4-2ax^3+2x^2 & \\
\hline
 & 2ax^3+x^2+ax+b & \\
 & 2ax^3-4a^2x^2+4ax & \\
\hline
 & (1+4a^2)x^2-3ax+b & \\
 & (1+4a^2)x^2-2a(1+4a^2)x+2+8a^2 & \\
\hline
 & (-a+8a^3)x+b-(2+8a^2)=r(x) &
\end{array}
$$

若 $q(x)$ 整除 $f(x)$.

则 $r(x)=0 \Rightarrow \begin{cases} -a+8a^3=0 \\ b-2-8a^2=0 \end{cases} \Rightarrow \begin{cases} a=0 \\ b=2 \end{cases}$ 或 $\begin{cases} a=\pm\frac{\sqrt{2}}{4} \\ b=3 \end{cases}$

4. $f(x)\mid g(x)$ 且 $g(x)\mid f(x)$

设 $f(x)=g(x)\cdot h(x)$. $g(x)=f(x)\cdot r(x)$

故 $f(x)=f(x)\cdot h(x)\cdot r(x)$

$\Rightarrow \deg(h(x)\cdot r(x))=0 \Rightarrow \deg(h(x))=\deg(r(x))=0$

故 $h(x)$. $r(x)$ 均为常数. 又 $f(x)$ 和 $g(x)$ 均不为0 (否则0为除数)

故 $c=h(x)$ 是非零常数.

6. 对 $\forall h(x)$. $r(x)$. 若 $f(x)\mid h(x)$. $f(x)\mid r(x)$.

设 $h(x)=f(x)\cdot h_1(x)$. $r(x)=f(x)\cdot r_1(x)$

故 $u_1(x)h(x)=u_1(x)f(x)h_1(x)$.

$u_2(x)r(x)=u_2(x)f(x)\cdot r_1(x)$

$u_1(x)h(x)+u_2(x)r(x)=(u_1(x)h_1(x)+u_2(x)r_1(x))\cdot f(x)$

$\therefore f(x)\mid u_1(x)h(x)+u_2(x)r(x)$

回到原题. $s=1,2$ 时成立. 若 $s=k$ 时成立 $s=k+1$ 时

有 $f(x)\mid (u_1(x)g_1(x)+\cdots+u_k(x)g_k(x))+u_{k+1}g_{k+1}(x)$ 归纳成立.

因此 $\forall s\in Z^+$, 有 $f(x)\mid u_1(x)g_1(x)+\cdots+u_s(x)g_s(x)$

11.
$$
\begin{array}{r|rrrrr}
2 & 1 & -2 & 0 & 0 & 3 \\
 & & 2 & 0 & 0 & 0 \\
\hline
 & 1 & 0 & 0 & 0 & 3
\end{array}
\qquad
\begin{array}{r|rrr}
2 & 1 & 2 & 4 \\
 & & 2 & 8 \\
\hline
 & 1 & 4 & 12
\end{array}
$$

$f(x)=x^3(x-2)+3$ $\qquad$ $x^2+2x+4=(x+4)(x-2)+12$

$$
\begin{array}{r|rrrr}
2 & 1 & 0 & 0 & 0 \\
 & & 2 & 4 & 8 \\
\hline
 & 1 & 2 & 4 & 8
\end{array}
\qquad
\begin{array}{r|rr}
2 & 1 & 4 \\
 & & 2 \\
\hline
 & 1 & 6
\end{array}
$$

$x^3=(x^2+2x+4)(x-2)+8$ $\qquad$ $x+4=(x-2)+6$

$\therefore f(x)=((((x-2)+6)(x-2)+12)(x-2)+8)(x-2)+3$

$=(x-2)^4+6(x-2)^3+12(x-2)^2+8(x-2)+3$

2023010747 刘一铭 计32

若 $f(x), g(x), h(x)$ 均不等于0

13. 设 $\deg(f(x)) = c$

$\deg(g(x)) = a$ 则 $a, b, c \in N$

$\deg(h(x)) = b$

则 $f^2(x) = x g^2(x) + x h^2(x) \Rightarrow 2c = \max\{2a+1, 2b+1\}$ 矛盾.

故 $f(x), g(x), h(x)$ 至少有一个为0

若 $f(x)=0$, 则 $0 = x g^2(x) + x h^2(x)$ 令 $x>0$, 知 $xg^2(x) \geq 0$, $xh^2(x) \geq 0$

故对 $x>0$, 均有 $g(x)=h(x)=0$. 因此 $g(x), h(x)$ 均有无穷多个根, 根数量大于次数.

$\therefore g(x)=h(x)=0$

若 $f(x) \neq 0$, 不失一般性, 设 $h(x)=0$.

$f^2(x) = x g^2(x)$, 若 $g(x) \neq 0$, 设 $\deg(f(x)) = c$, $\deg(g(x)) = a$

$\therefore 2c = 2a+1$ 矛盾.

故 $g(x)=0$.

$\therefore f(x) = x g^2(x) + x h^2(x) = 0$ 矛盾

综上知 $f(x)=g(x)=h(x)=0$

15. 设 $h(x) = f(x) - g(x)$.

则 $\deg(h(x)) \leq \max\{f(x), g(x)\} \leq n$

又 $h(x)$ 有 $n+1$ 个根 α_i, $i=1,2,\cdots,n+1$.

因此 $h(x) \equiv 0$

故 $f(x) = g(x)$

16.

$f(x)$	$g(x)$
$x^5 - x^4 + 2x^3 + x^2 + 3$	$x^4 + x^2 + x - 1$
$x^5 + x^3 + x^2 - x$	$x^4 - x^3 - x - 3$
$-x^4 + x^3 + x + 3$	$x^3 + x^2 + 2x + 2$
$-x^4 - x^3 - 2x^2 - 2x$	$x^3 + x^2 + \frac{3}{2}x + \frac{3}{2}$
$2x^3 + 2x^2 + 3x + 3$	$\frac{1}{2}x + \frac{1}{2}$
$2x^3 + 2x^2$	
$3x + 3$	
$3x + 3$	
0	

因此 $(f(x), g(x)) = x + 1$