HW9 2023010747

1. $P(\phi)=\{\phi\}$

$PP(\phi)=P(\{\phi\})=\{\phi,\{\phi\}\}$

$PPP(\phi)=P(\{\phi,\{\phi\}\})=\{\phi,\{\phi\},\{\{\phi\}\},\{\phi,\{\phi\}\}\}$

(1) $\cup\{PPP(\phi),PP(\phi),P(\phi),\phi\}=\{\phi,\{\phi\},\{\{\phi\}\},\{\phi,\{\phi\}\}\}$

(2) $\cap\{PPP(\phi),PP(\phi),P(\phi)\}=\{\phi\}$

2. $A=\{\{\phi\},\{\{\phi\}\}\}$

(1) $P(A)=\{\phi,\{\{\phi\}\},\{\{\{\phi\}\}\},\{\{\phi\},\{\{\phi\}\}\}\}$

$\cup P(A)=\{\{\phi\},\{\{\phi\}\}\}$

(2) $\cup A=\{\phi,\{\phi\}\}$

$P(\cup A)=\{\phi,\{\phi\},\{\{\phi\}\},\{\phi,\{\phi\}\}\}$

3. (1) $(A-B)-C=(A\cap -B)\cap -C=A\cap -B\cap -C$

$(A-C)-(B-C)=(A\cap -C)\cap -(B\cap -C)=A\cap -C\cap(-B\cup C)=(A\cap -C\cap -B)\cup(A\cap -C\cap C)=(A\cap -B\cap -C)\cup\phi=A\cap -B\cap -C$

$\therefore (A-B)-C=(A-C)-(B-C)$

(2) $A\oplus B=\phi \Rightarrow A\oplus(A\oplus B)=A\oplus\phi\Rightarrow B=A$ $\quad A=B\Rightarrow A\oplus B=(A-B)\cup(B-A)=\phi\cup\phi=\phi$

(3) $A\cap B=\phi\Leftrightarrow \neg(\exists x)(x\in A\wedge x\in B)$

$\Leftrightarrow(\forall x)(\neg(x\in A)\vee\neg(x\in B))$

$\Leftrightarrow(\forall x)((x\in A)\rightarrow\neg(x\in B))$

$\Leftrightarrow A\subseteq -B$

$\therefore A\cap B=\phi\Leftrightarrow A\subseteq -B\Leftrightarrow B\subseteq -A$

$A\cap B=\phi\Leftrightarrow\neg(\exists x)(x\in A\wedge x\in B)$

$\Leftrightarrow(\forall x)(\neg(x\in A)\vee\neg(x\in B))$

$\Leftrightarrow(\forall x)(\neg(x\in B)\vee\neg(x\in A))$

$\Leftrightarrow(\forall x)((x\in B)\rightarrow\neg(x\in A))$

$\Leftrightarrow B\subseteq -A$

4. (1) $A-B=A\cap -B$

$B=B\cap B=(A-B)\cap B=A\cap -B\cap B=A\cap\phi=\phi\Rightarrow A=A-B=B=\phi$

$\therefore A=B=\phi$

(2) $$\left.\begin{aligned}&A-B=B-A\Rightarrow A-B=B\cap -A\Rightarrow B\cup(A-B)=B\cup(B\cap -A)\Rightarrow B\cup A=B\Rightarrow A\subseteq B\\&A-B=B-A\Rightarrow A\cap -B=B-A\Rightarrow A\cup(A\cap -B)=A\cup(B-A)\Rightarrow B\cup A=A\Rightarrow B\subseteq A\end{aligned}\right\}\Rightarrow A=B$$

(3) $$\left.\begin{aligned}&A\cap B=A\cup B\Rightarrow(A\cap B)\cup A=A\cup B\cup A\Rightarrow B=A\cup B\Rightarrow A\subseteq B\\&A\cap B=A\cup B\Rightarrow(A\cap B)\cup B=A\cup B\cup B\Rightarrow A=A\cup B\Rightarrow B\subseteq A\end{aligned}\right\}\Rightarrow A=B$$

(4) $A\oplus B=A\Rightarrow A\oplus(A\oplus B)=A\oplus A\Rightarrow B=\phi$ ，$B=\phi$时 $A\oplus\phi=A$

$\therefore B=\phi$. A任意

5. (1) $(A-B)\cup(A-C)=A\Leftrightarrow A-(B\cap C)=A\Leftrightarrow A\cap-(B\cap C)=A\Leftrightarrow A\subseteq-(B\cap C)$

(2) $(A-B)\oplus(A-C)=\phi\Leftrightarrow(A\cap -B)\oplus(A\cap -C)=\phi\Leftrightarrow((A\cap -B)-(A\cap -C))\cup((A\cap -C)-(A\cap -B))=\phi$

$\Leftrightarrow(A\cap -B\cap(-A\cup C))\cup(A\cap -C\cap(-A\cup B))=\phi\Leftrightarrow\begin{cases}A\cap -B\cap(-A\cup C)=\phi\Leftrightarrow A\cap -B\cap C=\phi\\A\cap -C\cap(-A\cup B)=\phi\Leftrightarrow A\cap -C\cap B=\phi\end{cases}$

或

$((A\cap -B)-(A\cap -C))\cup((A\cap -C)-(A\cap -B))=\phi$

$\left\{\begin{aligned}&\Leftrightarrow(A\cap -B)-(A\cap -C)=\phi\Leftrightarrow A\cap -B\subseteq A\cap -C\\&\Leftrightarrow(A\cap -C)-(A\cap -B)=\phi\Leftrightarrow A\cap -C\subseteq A\cap -B\end{aligned}\right\}\Leftrightarrow A\cap -B=A\cap -C\Leftrightarrow A-B=A-C$

6. (1) $A\times B=\phi\Rightarrow\{\langle x,y\rangle\mid x\in A,y\in B\}=\phi\Rightarrow A=\phi\vee B=\phi$

(2) 可能 若$A=A\times A$

则$|A|=|A\times A|$. 设A有n个元素. 又$|A\times A|\geq n^2$

故$n\geq n^2\Rightarrow n\leq 1$. 若$n=1$. 设$A=\{a\}$. $A\times A=\{\langle a,a\rangle\}=\{\{a\}\}\neq A$.

因此$n=0$. $A=\phi$. 此时$A\times A=\phi=A$

7. 能被2整除的数个数为$[\frac{250}{2}]=125$

能被3整除的数个数为$[\frac{250}{3}]=83$

能被5整除的数个数为$[\frac{250}{5}]=50$

能被2.3整除的数个数为$[\frac{250}{6}]=41$

能被2.5整除的数个数为$[\frac{250}{10}]=25$

能被3.5整除的数个数为$[\frac{250}{15}]=16$

能被2.3.5整除的数个数为$[\frac{250}{30}]=8$

因此，由容斥原理知能被2.3.5任何一个整除的数个数为$125+83+50-41-25-16+8=184$