SURVEYING

for CIVIL and GEODETIC Licensure Exam

CONTENTS:

Plane Surveying, Higher Surveying, Mine Surveying, Hydrographic Surveying, Topographic Surveying, Astronomy, Simple Curves, Compound Curves, Spiral Curves, Reversed Curves, Parabolic Curves, Sight Distance, Earthworks, Mass Diagram, Highway Engineering, Transportation Engineering, Traffic Engineering.

SURVEYING
for CIVIL and GEODETIC Licensure Exam

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5. Imperfection of observation
6. Variations in temperature
7. Variations in tension

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1. Adding or dropping a full tape length.
2. Adding a cm., usually in measuring the fractional part of tape length at the end of the line.
3. Recording numbers incorrectly, example 78 is read as 87.
4. Reading wrong meter mark.

Correction Applied for Measurement of Distances:

1. ***Temperature Correction:*** *(To be added or subtracted)*

$$\pm C_t = K(T_2 - T_1)\,L_1$$

K = 0.00000645 ft. per degree F.
K = 0.0000116 m. per degree C.
T_1 = temp. when the length of tape is L_1
T_2 = temp. during measurement

2. ***Pull Correction:***
(To be added or subtracted)

$$\pm C_p = \frac{(P_2 - P_1)\,L_1}{AE}$$

P_2 = actual pull during measurement
P_1 = applied pull when the length of tape is L_1
A = Cross-sectional area of tape
E = Modulus of elasticity of tape

3. ***Sag Correction:***
(To be subtracted only)

$$-C_s = \frac{w^2 L^3}{24\,P^2}$$

w = weight of tape in plf. or kg.m.
L = unsupported length of tape
p = actual pull or tension applied

4. ***Slope Correction:***
(To be subtracted only)

$$-C_s = \frac{h^2}{2\,S}$$

$H = S - C_s$
H = horizontal distance or corrected distance
S = inclined distance
h = difference in elevation at the end of the tape

TAPE CORRECTION

5. ***Sea Level Correction:***

Reduction factor = $1 - \frac{h}{R}$

$$B' = B\left(1 - \frac{h}{R}\right)$$

B = horizontal distance corrected for temperature, sag and pull.
B' = sealevel distance
h = average altitude or observation
R = Radius of curvature

6. ***Normal Tension:***

It is the tension which is applied to a tape supported over two supports which balances the correction due to pull and due to sag. The application of the tensile force increases the length of the tape whereas the sag decreases its length, the normal tension neutralizes both corrections, therefore no correction is necessary.

$$P_N = \frac{0.204\, W \sqrt{AE}}{\sqrt{P_N - P_1}}$$

P = applied normal tension
P_1 = tension at which the tape is standardized
W = total weight of tape
A = cross-sectional area of tape
E = modulus of elasticity of tape

Tape too long:

a) Add correction when measuring distances
b) Subtract correction when laying out distances

Tape too short:

a) Subtract correction when measuring distances
b) Add correction when laying out distances

Two methods of erecting perpendicular to a given line

1. ***The 3:4:5 Method:***

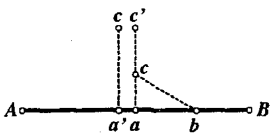
To erect a perpendicular to the line AB, from a given point C a point a on line AB is assumed to be on the perpendicular and a pin is set at a. With sides a multiple of 3, 4 and 5 m. such as 24.32 and 40 m., a right triangle abc is constructed as follows: A pin is set on line AB at b, 32 m. from a. The zero end of the tape is fixed with a pin at a, and the 100 m. end at b. The zero end of the tape is fixed with a pin at a, and the 100 m. end at b. The head chainman moves to c and holds the 24 m. and the 60 m. marks of the tape in one hand, with the tape between these marks laid out so as to avoid kinking. He then sets a pin at c. The rear chairman moves from a to b as necessary to check the position of the tape at these points as c is established. He then sights along ac to C' beside C, usually C to the aC' is measured, and the foot a of the perpendicular aC' is moved along the line AB by an equal amount, to the point a'. If the trial perpendicular aC' fails to include the point C by several feet, the process is repeated for a', the new point, otherwise the location of a' may be assumed as correct.

2. ***The Chord Bisection Method:***

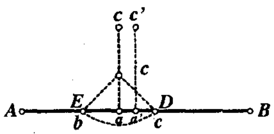
To erect a perpendicular to the line AB, from a given point C, the position of the perpendicular is estimated, and a pin is set at d on this estimated perpendicular, somewhat less than one tape length from the line AB. With d as center and the length of tape as radius, the head chainman describe the arc ED of a circle, setting pins at the intersections b and c of the arc with the line AB. The rear chainman stationed at A or B determines the location of the intersections b and c on line. The point a is established midway between b and c.

TAPE CORRECTION

The line ad is prolonged to C' besides C, and the point a is moved if necessary as described for the 3:4:5 method.



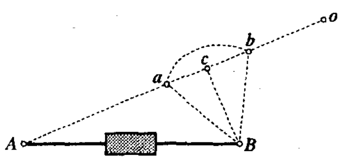
3:4:5 Method



Chord Bisection Method

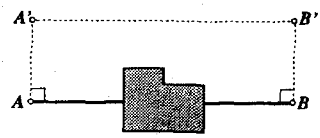
Measuring Obstructed Distancesby use of Tape

1. ***Swing offsets:***



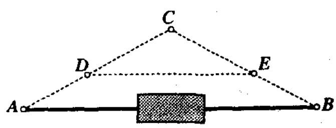
To find the distance AB by the swing offset method, the head chainman attached the end of the tape to one end of the line as at B and describes an arc with center B and radius 100 m. The rear chainman stationed at A lines in the end of the tape with some distant object as O and directs the setting of pins at points a and b where the end of the tape crossed line AO. A point C midway between a b lies on the perpendicular CB. A pin is set at C, and the distances BC and CA are measured to obtain the necessary data for computing the length of AB.

2. ***Parallel lines:***



If the necessary distance from the line AB is short, perpendicular AA' = BB' are erected by either using 3:4:5 method of the chord bi-section method to clear the obstacle. The line A'B' is then chained, and its length is taken as that of AB.

3. ***Similar Triangles:***



Let C be a point from A and B are visible. AC and BC are measured. CD and CE bears to CB: that is CD/CA = CE/CB. It will generally be convenient to make this a simple ratio such as 1/2 of 1/3. The triangles ACB and DCE are similar. DE is measured and AB is computed.

Problem 1:

A line was determined to be 2395.25 m. when measured with a 30 m. steel tape supported throughout its length under a pull of 4 kg at a mean temperature of 35°C. Tape used is of standard length at 20°C under a pull of 5 kg. Cross-sectional area of tape is 0.03 sq.cm., Coefficient of thermal expansion is 0.0000116/°C, Modulus of elasticity of tape is 2×10^6 kg/cm^2.

① Determine the error of the tape due to change in temperature.
② Determine the error due to tension.
③ Determine the corrected length of the line.

TAPE CORRECTION

Solution:

① *Temp. correction:*

$C_t = KL\,(T - T_s)$

$C_t = 0.0000116\,(2395.25)(35 - 20)$

$C_t = \boldsymbol{+\,0.4168\ m.}$

② *Tension correction:*

$C_p = \dfrac{(P - P_s)\,L}{AE}$

$C_p = \dfrac{(4 - 5)(2395.25)}{0.03\,(2)\,10^6}$

$C_p = \boldsymbol{-\,0.0399\ m.}$

③ *Corrected length:*

$L = 2395.25 + 0.4168 - 0.0399$

$L = \boldsymbol{2395.6269\ m.}$

Problem 2:

A 50 m. tape was standardized and was found to be 0.0042 m. too long than the standard length at an observed temperature of 58°C and a pull of 15 kilos. The same tape was used to measure a certain distance and was recorded to be 673.92 m. long at an observed temp. of 68°C and a pull of 15 kilos. Coefficient of linear expansion is 0.0000116 m/°C.

① Determine the standard temperature.
② Determine the total correction.
③ Determine the true length of the line.

Solution:

① *Standard temperature:*

$C_T = K\,(T_2 - T_1)\,L_1$

$+\,0.0042 = 0.0000116\,(58 - T_1)(50)$

$+\,0.0042 = 0.03364 - 0.00058\,T_1$

$T_1 = \boldsymbol{50.76°C}$ *(standard temp.)*

② *Total correction:*

$C_T = K\,(T - T_1)\,L_1$

$C_T = 0.000016\,(68 - 50.76)(50)$

$C_T = 0.01$ *(tape is too long)*

$\textit{Total correction} = \dfrac{673.92\,(0.01)}{50}$

$\textit{Total correction} = \boldsymbol{0.1348\ m.}$

$E = C N$

$= C\,\dfrac{MD}{Lo}$

$= 0.01\left(\dfrac{673.92}{50}\right)$

$= 0.1348\text{m}$

③ *True length of the line:*

Corrected hor. distance = 673.92 + 0.1348

Corrected hor. distance = ***674.055 m.***

3. CE Board May 1991

A 50 m. steel tape was standardized and suppported throughout its whole length and found to be 0.00205 m. longer at an observed temperature of 31.8°C and a pull of 10 kilos. This tape was used to measure a line which was found to be 662.702 m. at an average temperature of 24.6°C using the same pull. Use coefficient of expansion of 0.0000116 m. per degree centigrade.

① Compute the standard temp.
② Compute the total temp. correction.
③ Compute the correct length of the line.

Solution:

① *Standard temperature:*

$C_T = K\,(T_2 - T_s)\,L$

$+0.00205 = 0.0000116(31.8 - T_s)(50)$

$31.8 - T_s = 3.53$

$T_s = \boldsymbol{28.27°C}$ *(standard temp.)*

② *Total correction:*

$C_T = K\,(T_2 - T_s)\,L$

$C_T = 0.0000116(24.6 - 28.27)(50)$

$C_T = 0.00213$ *(too short)*

$\textit{Total correction} = \dfrac{0.00213(662.702)}{50}$

$\textit{Total correction} = \boldsymbol{0.02823\ m.}$

③ *Corrected length of line:*

Corrected horizontal distance

= 662.702 - 0.02823

= ***662.67377 m.***

TAPE CORRECTION

Problem 4:

A line is recorded as 472.90 m. long. It is measured with a 0.65 kg. tape which is 30.005 m. long at 20°C under a 50 N pull supported at both ends. During measurement, the temperature is 5°C and the tape is suspended under a 75 N pull. The line is measured on 3% grade. E = 200 GPa, cross-sectional area of tape is 3 mm² and the coefficient of linear expansion is 0.0000116 m/°C.

① Compute the actual length of tape during measurement.
② Compute the total error to be corrected for the inclined distance.
③ What is the true horizontal distance?

Solution:

① *Actual length:*

$C_T = K(T_2 - T_1)L_1$

$C_T = 0.0000116(5 - 20)(30.005)$

$C_T = -0.00522$ mm

$C_P = \dfrac{(P_2 - P_1)L_1}{AE}$

$C_P = \dfrac{(75 - 50)(30.005)}{3(200 \times 10^3)}$

$C_P = +0.00125$ m

$C_S = \dfrac{w^2L^3}{24P^2}$

$w = \dfrac{0.65 \times 9.81}{30}$

$C_S = \dfrac{(0.65 \times 9.81)^2(30)^3}{(30)^2(24)(75)^2}$

$C_S = -0.00904$ m

Total correction $= -0.00522 + 0.00125 - 0.00904$

~~*Total correction* = - 0.00904~~

Total correction $= -0.00522 + 0.00125 - 0.00904$

Total correction $= -0.013$ m

Actual length of tape during measurement
$= 30.005 - 0.013$
$= 29.992$ *m.*

② *Total error:*

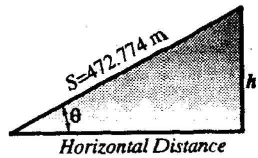
Too short by $= 30 - 29.992$

Too short by $= 0.008$ m

Total error $= \dfrac{472.90\,(0.008)}{30}$

Total error = 0.126 m. (to be subtracted)

③ *True horizontal distance:*



Corrected inclined distance
$= 472.90 - 0.126$
$= 472.774$ m

$\sin\theta = \dfrac{h}{472.774}$

$h = 472.774\,(0.03)$

$h = 14.183$ m

Correction for slope $= \dfrac{(14.183)^2}{2\,(472.774)}$

Correction for slope $= 0.213$ m

Corrected horizontal distance
$= 472.774 - 0.213$
$= 472.561$ *m.*

Problem 5:

A 30 m. steel tape is 2 mm too long at 20°C with a pull of 55 N. A rectangle is measured with this tape. The sides are recorded as 144.95 m. and 113.00 m. The average temperature during the measurement is 30°C with a pull of 55 N. Use coefficient of expansion of steel tape as a 0.0000116 /°C.

① Compute the actual length of tape during measurement.
② What is the true area.
③ What is the error in area in sq.m.

TAPE CORRECTION

Solution:

① *Actual length:*

$C_T = K(T_2 - T_1) L_1$

$C_T = 0.000016(30 - 20)(30.002)$

$C_T = 0.00348$ m *(too long)*

Actual length of tape during measurement

$= 30.002 + 0.00348$

$= \mathbf{30.00548\ m.}$

② *True area:*

Therefore the tape is 0.00548 m too long

For the 144.95 m side:

$$\text{Total error} = \frac{144.95(0.00548)}{30} = 0.26\text{ m.}$$

True length = 144.95 + 0.026

True length = 144,976 m

For the 113 m side:

$$\text{Total error} = \frac{113(0.00548)}{30} = 0.021$$

True length = 113 + 0.021

True length = 113.021 m

True area = (144.976)(113.021)

True area = ***16,385.33 m²***

③ *Error in area:*

Erroneous area = (144.95)(113)

Erroneous area = 16,379.35 m^2

Error in area = 16,385.33 - 16,379.35

Error in area = ***5.982 m²***

Problem 6:

A baseline was measured using a 100 m. tape which is standardized at 15°C with a standard pull of 10 kg. The recorded distance was found out to be 430.60 meters. At the time of measurement the temperature was 20°C and the pull exerted was 16 kg. The weight of one cubic cm of steel is 7.86 gr. weight of tape is 2.67 kg. $E = 2 \times 10^6$ kg/cm², $K = 7 \times 10^{-7}$ m/°C.

① Determine the cross sectional area of the tape.
② Compute the total correction.
③ Compute the true length of the base line.

Solution:

① *Cross-sectional area:*

$$\frac{A(100)(100)(7.86)}{1000} = 2.67$$

Vol

$A = \mathbf{0.034\ sq.m.}$ $W_{Tape} = \gamma_{steel}(AL)$

② *Total correction:*

$C_T = K(T_2 - T_1) L$

$C_T = 7 \times 10^{-7}(20 - 15)(10\cancel{0})$ m

$C_T = 0.00035$ m.

Pull correction:

$$C_P = \frac{(P_2 - P_1) L}{AE}$$

$$C_P = \frac{(16 - 10)(100)}{0.034\,(2)\,10^6}$$

$C_P = 0.009$ m

Correction = 0.00035 + 0.009

Correction = 0.00935 m

$$\text{No. of tape lengths} = \frac{430.6}{100} = 4.306$$

Total correction = 4.306 (0.00935)

Total correction = ***0.0403 m***

③ *True length of baseline:*

True length of baseline = 430.60 + 0.0403

True length of baseline = ***430.6403 m***

Problem 7:

A 30 m. steel tape weighing 1.45 kg is of standard length under a pull of 5 kg supported for full length. The tape was used in measuring a line 938.55 m. long on a smooth level ground under a steady pull of 10 kg. Assuming $E = 2 \times 10^6$ kg/cm² and the unit weight of steel to be 7.9×10^{-3} kg/cm³.

① Determine the cross-sectional area of the tape in cm².
② Determine the correction for increase in tension.
③ Determine the correct length of the line measured.

Solution:

① *Cross-sectional area of tape:*

$w = AL\,Y_s$

$1.45 = A\,(3000)(7.9) \times 10^{-3}$

$A = \mathbf{0.061\ cm^2}$

② *Pull Correction:*

$$C_p = \frac{(P - P_s)\,L}{AE}$$

$$C_p = \frac{(10 - 5)(30)}{0.061\,(2 \times 10^6)}$$

$C_p = +\,0.00123$

$$\text{Total Correction} = \frac{0.00123\,(938.55)}{30}$$

Total Correction = **+ 0.038 m.**

③ *Correct length of line:*

Corrected length = 938.55 + 0.038

Corrected length = **938.588 m.**

Problem 8:

A steel tape with a coefficient of linear expansion of 0.0000116 per degree centigrate is known to be 50 m. long at 20°C. The tape was used to measure a line which was found to be 532.28 meters long when the temperature was 35°C. Determine the following:

① Temperature correction per tape length.
② Temperature correction for the measured line.
③ Corrected length of the line.

Solution:

① *Temperature correction per tape length:*

$C_T = K\,(T - T_s)\,L$

$C_T = 0.0000116\,(35 - 20)(50)$

$C_T = \mathbf{0.0087\ m.}$ *too long*

② *Temperature correction for the measured line:*

E = CL

$$\text{Total correction} = \frac{532.28\,(0.0087)}{50} \qquad C\left(\frac{MD}{LD}\right)$$

Total correction = **+ 0.0926 m.**

③ *Corrected length of line:*

Corrected length = 532.28 + 0.0926

Corrected length = **532.3726 m.**

Problem 9:

A civil engineer used a 30 m tape in measuring an inclined distance. The measured length on the slope was recorded to be 459.20 m long. The difference in elevation between the initial point and the end point was found to be 1.25 m. The 30 m tape is of standard length at a temperature of 10°C and a pull of 50 N. During measurement the temperature reading was 15°C and the tape was supported at both ends with an applied pull of 75 N. The cross-sectional area of the tape is 6.50 mm² and the modulus of elasticity is 200 GPa. The tape has a mass of 0.075 kg/m. K = 0.0000116 m/c°.

① Determine the total correction per tape length.
② Determine the correction for slope.
③ Determine the horizontal distance.

Solution:

① *Total correction per tape length:*

$C_T = K\,(T_2 - T_1)\,L_1$

$C_T = 0.0000116\,(15 - 10)(30)$

$C_T = +\,0.00174$ m

Pull correction:

$$C_p = \frac{(P_2 - P_1)\,L_1}{AE}$$

$$C_p = \frac{(75 - 50)(30)}{6.50\,(200 \times 10^3)}$$

$C_p = +\,0.00058$ m

Sag Correction:

$$C_S = \frac{w^2 L^2}{24\,P^2}$$

$$C_S = \frac{(0.075 \times 9.81)^2\,(30)^3}{24\,(75)^2}$$

$C_S = 0.10827$ m

Total correction per tape length:

$C = 0.00174 + 0.00058 - 0.10827$

$C = \mathbf{-\,0.10595\ m}$

② *Correction for slope:*

$$\text{Total correction} = \frac{459.20\,(-0.10595)}{30}$$

Total correction = - 1.622

Correct slope distance = 459.20 - 1.622
Correct slope distance = 457.578 m

$$C_S = \frac{h^2}{2\,S}$$

$$C_S = \frac{(1.25)^2}{2\,(457.578)}$$

$$C_S = \mathbf{0.002}$$

③ *Horizontal distance:*
Corrected horizontal distance
= 457.578 - 0.002
= ***457.576 m***

Problem 10:

A steel tape is 100 m. long at a temp. of 20°C and a pull of 10 kg. It was used to measure a distance of 624.95 m. at a temp. of 32°C with an applied pull of 15 kg. during measurement with the tape supported at both ends. Coefficient of thermal expansion is 0.0000116 m°c and a modulus of elasticity of 2×10^6 kg/cm^2. Weight of tape is 0.04 kg/m. and a cross sectional area of 0.06 cm^2.

① Compute the sag correction.
② Compute the total correction for tension, sag and temperature.
③ Compute the corrected length of the line by applying the combined corrections for tension, sag and temperature.

Solution:

① *Sag correction:*

$$Cs_1 = \frac{w^2\,L^3}{24\,P^2}$$

$$Cs_1 = \frac{(0.04)^2\,(100)^3}{24\,(15)^2} = 0.296 \text{ m.}$$

1.85

$$Cs_2 = \frac{(0.04)^2\,(24.95)^3}{24\,(15)^2} = 0.005$$

Total sag correction = 6(0.296) + 0.005
Total sag correction = ***1.781*** *(too short)*

② *Total correction:*
Temp. corrections:

$$C_T = KL\,(T - T_1)$$

$$C_T = 0.0000116\,(624.95)(32 - 20)$$

$$C_T = +\,0.087 \quad \textit{(too long)}$$

Pull correction:

$$C_p = \frac{(P_2 - P_1)\,L}{AE}$$

$$C_p = \frac{(15 - 10)\,(624.95)}{0.06\,(2)\,10^6}$$

$$C_p = +\,0.026 \quad \textit{(too long)}$$

Total correction = + 0.087 - 1.781 + 0.026
Total correction = ***- 1.668 m.***

③ *Corrected length of the line:*
Corrected length = 624.95 - 1.668
Corrected length = ***623.282 m.***

Problem 11:

A civil engineer used a 100 m tape which is of standard length at 32°C in measuring a certain distance and found out that the length of tape have different lengths at different tensions were applied as shown. K = 0.0000116 m/c°

Length of tape	Tension Applied
99.986 m	10 kg
99.992 m	14 kg
100.003 m	20 kg

① What tension must be applied to the tape at a temp. of 32°C so that it would be of standard length?
② What tension must be applied to the tape at a temp. of 40.6°C so that it would be of standard length?
③ What tension must be applied to the tape at a temp. of 30°C so that it would be of standard length?

TAPE CORRECTION

Solution:

① *Tension applied at 32˚.*

$$\left.\begin{array}{l} 0.008\left\{\begin{array}{l}99.992\\ \\ 100.00\end{array}\right. \\ \\ \quad 100.003\end{array}\right\} 0.011 \qquad \left.\begin{array}{l} x\left\{\begin{array}{l}14\\ \\ \ \end{array}\right. \\ \\ \quad 20\end{array}\right\} 6$$

$$\frac{x}{6} = \frac{0.008}{0.011}$$

$x = 4.36$ kg.

Tension applied = 14 + 4.36

Tension applied = ***18.36 kg***

② *Tension applied at 40.6˚.*

Temperature correction:

$C_T = K\,(T_2 - T_1)\,L$

$C_T = 0.0000116\,(40.6 - 32)\,100$

$C_T = 0.00998$

Actual length of tape at 40.6˚C	Tension
99.986 + 0.00998 = 99.99598	10 kg
99.992 + 0.00998 = 100.00198	14 kg
100.003 + 0.00998 = 100.01298	20 kg

$$0.006\left\{\begin{array}{l} \left.\begin{array}{l}99.99598\\ \\ 100.0000\end{array}\right\} 0.00402 \\ \\ 100.00198\end{array}\right. \qquad \left.\begin{array}{l} x\left\{\begin{array}{l}10\\ \\ \ \end{array}\right. \\ \\ \quad 14\end{array}\right\} 4$$

$$\frac{x}{4} = \frac{0.00402}{0.006}$$

$x = 2.68$ kg

Tension to be applied = 10 + 2.68

Tension to be applied = ***12.68 kg***

③ *Tension applied at 30˚C:*

Temperature correction:

$C_T = K\,(T_2 - T_1)\,L$

$C_T = 0.0000116\,(30 - 32)\,100$

$C_T = -\,0.00232$

Actual length of tape at 40.6˚C	Tension
99.986 - 0.00232 = 99.98368	10 kg
99.992 - 0.00232 = 99.98968	14 kg
100.003 - 0.00232 = 100.00068	20 kg

$$\left.\begin{array}{l} 0.01032\left\{\begin{array}{l}99.98968\\ \\ 100.0000\end{array}\right. \\ \\ \quad 100.00068\end{array}\right\} 0.011 \qquad \left.\begin{array}{l} x\left\{\begin{array}{l}14\\ \\ \ \end{array}\right. \\ \\ \quad 20\end{array}\right\} 6$$

$$\frac{x}{6} = \frac{0.01032}{0.011}$$

$x = 5.63$ kg

Tension applied = 14 + 5.63

Tension applied = ***19.63 kg***

Problem 12:

A 50 m. tape of standard length has a weight of 0.05 kg/m, with a cross sectional area of 0.04 sq.cm. It has a modulus of elasticity of 2.10×10^6 kg/cm^2. The tape is of standard length under a pull of 5.5 kg when supported throughout its length and a temp of 20˚C. This tape was used to measure a distance between A and B and was recorded to be 458.65 m. long. At the time of measurement the pull applied was 8 kg. with the tape supported only at its end points and the temperature observed was 18˚C. Assuming coefficient of linear expansion of the tape is 0.0000116 m/˚C.

① Compute the correction due to the applied pull of 8 kg.
② Compute the correction due to weight of tape.
③ Compute the true length of the measured line AB due to the combined effects of tension, sag and temperature.

TAPE CORRECTION

Solution:

① *Pull correction:*

$$C_P = \frac{(P_2 - P_1)\,L_1}{AE}$$

$$C_P = \frac{(8 - 5.5)(458.650)}{0.04\,(2.10 \times 10^6)} = \boldsymbol{+0.014\ m.}$$

② *Correction due to weight of tape:*

$$C_s = \frac{w^2 L^3}{24\,p^2}$$

458.65 / 50 = 9.173

$$C_s = \frac{(0.05)^2\,(50)^3\,(9)}{24\,(8)^2} + \frac{(0.05)^2\,(8.65)^3}{24\,(8)^2}$$

C_s = **- 1.832 m.** *(always negative)*

③ *True length of measured line AB:*

$$C_T = K\,(T_2 - T_1)\,L$$

$$C_T = 0.0000116\,(18 - 20)(458.65)$$

$$C_T = -\,0.011\ \text{m.}$$

Total correction = 0.014 - 1.832 - 0.011
Total correction = - 1.829 m.
True length AB = 458.65 - 1.829
True length AB = **456.821 m.**

---

13. CE Board Nov. 1998

A line 100 m. long was paced by a surveyor for four times with the following data. 142, 145, 145.5 and 146. Then another line was paced for four times again with the following results, 893,893.5, 891 and 895.5.

① Determine the pace factor.
② Determine number of paces for the new line.
③ Determine the distance of the new line.

Solution:

① *Pace factor:*

$$\text{No. of paces} = \frac{142 + 145 + 145.5 + 146}{5}$$

No. of paces = 144.625

MODE 31

100 ÷ x̄

$$\text{Pace factor} = \frac{100}{144.625} = \boldsymbol{0.691}$$

② *Number of paces for the new line:*

$$\text{No. of paces} = \frac{893 = 893.5 + 891 + 895.5}{5}$$

No. of paces = **893.25** = x̄

③ *Distance of the new line:*
Distance of new line = 893.25 (0.691)
Distance of new line = **617.236 m.**

Problem 14:

① This sides of a square lot having an area of 2.25 hectares were measured using a 100 m. tape that was 0.04 m. too short. Compute the error in the area in sq.m.

② The correct distance between two point is 220.45 m. Using a 100 m. tape that is "x" m. too long, the length to be laid on the ground should be 220.406 m. What is the value of "x"?

③ The distance from D to E, as measured, is 165.2 m. If the 50 m. tape used is 0.01 m. too short, what is the correct distance in m.?

Solution:

MODE 32

x	y
0	0
100²	2.25

ŷ(100-0.04)

① *Error in area:*

$$\frac{(99.96)^2}{A} = \frac{(100^2)}{2.25}$$

A = 2.2482 hectares

Error in area = 2.25 - 2.2482 = 2.2482
Error in area = 0.0018 hectares
Error in area = 0.0018 x 10000 m²
Error in area = **18 sq.m.**

Note: 1 hectare = 1000 sq.m.

② *Value of x:*

$$\frac{220.45}{100}\,x = 220.45 - 220.406$$

x = **0.02 m.**

③ *Corrected distance:*

$$\text{Correct distance} = 165.2 - \frac{165.2}{50}\,(0.01)$$

Correct distance = **165.167 m.**

TAPE CORRECTION

Problem 15:

① Compute the normal tension which will be applied to a tape supported over two supports in order to make the tape equal to its nominal length when supported only at ends points. The steel tape is 30 m. long and weighs 0.84 kg when supported throughout its length under a standard pull of 5.6 kg, with the modulus of elasticity is 2×10^6 kg/cm^2 and area of 0.06 cm^2.

② A steel tape is 30 m. long under a standard pull of 6 kg. with a constant cross-sectional area of 0.05 cm^2. If the normal tension applied to make the tape equal to its nominal length when supported only at the end points, that is the effect of sag will be eliminated by the elongation of the tape due to the application of this load is equal to 16 kg., determine the unit weight of the tape. Modulus of elasticity of tape is 2×10^6 kg/cm^2.

③ Under a standard pull of 8 kg, the steel tape is 40 m. long. A normal tension of 18 kg makes the elongation of the tape offset the effect of sag. If the tape weighs 0.025 kg/m, and E = 2×10^6 kg/cm^2, determine its cross sectional area in sq.cm.

Solution:

① *Normal tension:*

$$P_N = \frac{0.204\sqrt{AE}}{\sqrt{P_N - P_1}}$$

$$P_N = \frac{0.204(0.84)\sqrt{0.06(2)\,10^6}}{\sqrt{P_N - P_1}}$$

$$P_N = \frac{59.3608}{\sqrt{P_N - 5.6}}$$

By trial and error:

$P_N = 17.33$ kg

$$P_N = \frac{59.3608}{\sqrt{17.33 - 5.6}}$$

$P_N = \mathbf{17.33\ kg}$

② *Unit weight of tape:*

$$P_N = \frac{0.204\,w\sqrt{AE}}{\sqrt{P_N - P_s}}$$

$$16 = \frac{0.204\,w\sqrt{0.05(2)\,10^6}}{\sqrt{16 - 6}}$$

$w = 0.784$ kg

$$w = \frac{0.784}{30} = \mathbf{0.026\ kg/m}$$

③ *Cross sectional area:*

$$P_N = \frac{0.204\,w\sqrt{AE}}{\sqrt{P_N - P_s}}$$

$$18 = \frac{0.204(0.0025)(40)\sqrt{AE}}{\sqrt{18 - 8}}$$

$\sqrt{AE} = 279.02$

$AE = 77854.67$

$$A = \frac{77854.67}{2 \times 10^6} = \mathbf{0.039\ cm^2}$$

Problem 16:

① Determine the length of the line in meters if there were 3 tallies, 8 pins and the last pin was 9 m. from the end of the line. The tape used was 50 m. long.

② A line was measured with a 50 m. tape and found to be 100 m. long. If was discovered that the first pin was stuck 30 cm. to the left of the line and the second pin 30 cm. to the right. Find the error in the measurement in cm?

③ A line was measured with a 50 m. tape and recorded 100 m. long. While measuring the first pin was stuck 20 cm to the right of the line and the second pin 40 cm. to the left. Find the correct length of the line.

Solution:

① *Length of the line:*

$L = 3(10)(50) + 8(50) + 9$

$L = \mathbf{1.909\ m}$

② *Error in the measurement:*

$$Error = \frac{h^2}{2s}$$

$$Error = \frac{(0.30)^2}{2(50)} + \frac{(0.60)^2}{2(50)}$$

$Error = 0.0045$ m. = ***0.45 cm.***

③ *Correct length of the line:*

$$Error = \frac{(0.20)^2}{2(50)} + \frac{(0.60)^2}{2(50)}$$

$Error = 0.004$ m.

Correct length of line = 100 - 0.004

Correct length of line = ***99.996 m.***

Problem 17:

① A line was measured to have 5 tallies, 6 marking pins and 63.5 links. How long is the line in ft.?

② A line was measured witha 50 m. tape. There were 2 tallies, 8 pins, and the distance from the last pin to the end of the line was 2.25 m. Find the length of the line in meters?

③ A distance was measured and was recorded to have a value equivalent to 8 perch, 6 rods and 45 vara. Compute the total distance in meters.

Solution:

① *Distance of line:*

Note: 1 tally = 10 pins
1 link = 1 ft
1 pin = 100 links

$L = 5\,(10)(100) + 6\,(100) + 63.5$

$L = 5663.5$ *ft*

② *Length of the line:*

Note: 1 tally = 10 pins
1 pin = 1 chain

2 (10)(50) + 8 (50) + 2.25 = ***1402.25 m.***

③ *Total distance:*

Note: 1 perch = 1 rod = 16.5 feet
1 vara = 33 inches

$$Total\ distance = 8\,(16.5) + 6\,(16.5) + \frac{45\,(33)}{12}$$

Total distance = 354.75 ft.

Total distance = ***108.16 m.***

Problem 18:

① A 100 m. tape is 12 mm. wide and 0.80 mm. thick. If the tape is correct under a pull of 54 N, compute the error made by using a pull of 68 N. E = 200,000 MPa.

② The length of a series of lines is found to be 3427.62 m. in the forward direction and 3427.84 m. in the reversed direction. What is the ratio of the error?

③ A substense bar is mounted at a certain distance from the instrument and the angle subtended by the bar is 0'04'. Compute the horizontal distance from the instrument station to the location of the subtense bar.

Solution:

① *Error made by using a pull of 68 N:*

$$C_p = \frac{(P_2 - P_1)\,L}{AE} = \frac{(68 - 54)\,100}{12\,(0.8)(200000)}$$

$C_p = 0.0007$

$$Error = \frac{0.0007}{100} \times 100$$

Error = ***0.0007%***

② *Ratio of the error:*

$$Average\ length = \frac{3427.62 + 3427.84}{2}$$

Average length = 3427.73

$$Ratio\ of\ error = \frac{3427.84 - 3427.62}{3427.73}$$

$$Ratio\ of\ error = \frac{0.22}{3427.73} = \frac{1}{15581}$$

③ *Horizontal distance:*

$$\tan 0.2' = \frac{1}{H}$$

H = ***1,718.87 m.***

Note: Subtense bar is standard
to be 2 m. long

ERRORS AND MISTAKES

ERRORS

is defined as the difference between the true value and the measured value of a quantity.

MISTAKES

are inaccuracies in measurements which occur because some aspect of a surveying operation is performed by the Geodetic Engineer with carelessness, poor judgment and improper execution.

Types of Errors:

1. Systematic Error
2. Accidental Error

Sources of Errors:

1. Instrumental Error
2. Natural Error
3. Personal Error

PROBABILITY

is define as the number of times something will probably occur over the range of possible occurrences.

1. ***Probable Error a single observation:***

$$E = 0.6745\sqrt{\frac{\Sigma V^2}{n-1}}$$

Where E = probable error
ΣV^2 = sum of the squares of the residuals
n = number of observation

2. ***Probable Error of the Mean:***

$$E_m = 0.6745\sqrt{\frac{\Sigma V^2}{n(n-1)}}$$

$$E_m = \frac{E}{\sqrt{n}}$$

$X\sigma n-1$
$\sqrt{\sigma n-1}$

3. ***Standard deviation:***

$$\text{Standard deviation} = \sqrt{\frac{\Sigma V^2}{(n-1)}}$$

4. ***Standard error:***

$$\text{Standard error} = \frac{\text{Standard deviation}}{\sqrt{n}}$$

Adjustments of Weighted Observations

1. The weights are inversely proportional to the square of the corresponding probabl errors.

$$W_1 = \frac{K}{E_1^2} \qquad W_2 = \frac{K}{E_2^2} \qquad W_3 = \frac{K}{E_3^2}$$

$$W_1E_1^2 = W_2E_2^2 = W_3E_3^2$$

$$\frac{W_1}{W_2} = \frac{E_2^2}{E_1^2} \qquad \frac{W_1}{W_3} = \frac{E_3^2}{E_1^2}$$

2. The weights are also proportional to the number of observations.

3. Errors are directly proportional to the square roots of distances.

Problem 19:

The following data observed are the difference in between BM_1 and BM_2 by running a line of levels over four different routes.

Route	Diff. in Elevations	Probable Error
1	340.22	±02
2	340.30	±04
3	340.26	±06
4	340.32	±08

① What is the weight of route 2 assuming weight of route 1 is equal to 1.
② Determine the most probable value of diff. in elevation.
③ If the elevation of BM_1 is 650.42 m, what is the elevation of BM_2 assuming it is higher than BM_1.

Solution:

① *Weight of route 2:*

The weights are invserseley proportional to the square of the corresponding probable errors.

$$W_1 = \frac{K}{(2)^2} \qquad W_2 = \frac{K}{(4)^2}$$

$$W_3 = \frac{K}{(6)^2} \qquad W_4 = \frac{K}{(8)^2}$$

$$4\,W_1 = 16\,W_2 = 36\,W_3 = 64\,W_4$$

$$W_1 = 4\,W_2 = 9\,W_3 = 16\,W_4$$

Assume $W1 = 1$

$$W_2 = \frac{1}{4}$$

$$W_2 = \mathbf{0.25}$$

$$W_3 = \frac{1}{9}$$

$$W_3 = 0.111$$

$$W_4 = \frac{1}{16}$$

$$W_4 = 0.0625$$

② *Most probable value of diff. in elevation:*

Route	Diff. in Elev.	Weight
1	340.22	1
2	340.30	0.25
3	340.26	0.1111
4	340.32	0.0625
		Sum = 1.4236

Weighted Observation

340.22 (1)	=	340.220
340.30 (0.25)	=	85.075
340.26 (0.1111)	=	37.803
340.32 (0.0625)	=	21.270
Sum	=	484.368

$$\textit{Most Probable Value} = \frac{484.368}{1.4236} = \frac{\Sigma WX}{\Sigma W}$$

$$\textit{Most Probable Value} = \mathbf{340.242}$$

③ *Elev. of BM_2:*

Elev. = 650.42 + 340.242

Elev. = ***990.662 m.***

Problem 20:

The following data shows the difference in elevation between A and B.

Trial	Diff. in Elevation	No. of Measurements
1	520.14 m	1
2	520.20 m	3
3	520.18 m	6
4	520.24 m	8

① Compute the probable weight of trial 3.
② Determine the most probable diff. in elevation.
③ Compute the elevation of B if elevation of A is 1000 with B higher than A.

ERRORS AND MISTAKES

Solution:

① *Weight of trial 3:*

The weights are also proportional to the number of observation.

Weight of trial 3 = ***6***.

② *Probable diff. in elevation:*

MODE 32

Distance (x)	Weight (y)
520.14	1
520.20	3
520.18	**6**
520.24	8
	Sum = 18

Weighted Values

520.14 (1)	=	520.14
520.20 (3)	=	1560.60
520.18 (6)	=	3121.08
520.21 (8)	=	4161.92
Sum	=	9363.74

$$\textit{Probable value of diff. in elev.} = \frac{9363.74}{18} \quad \frac{\Sigma xy}{\Sigma y}$$

Probable value of diff. in elev. = ***520.208***

③ *Elevation of B:*

Elev. of B = 1000 + 520.208

Elev. of B = ***1520.208 m.***

Problem 21:

From the measured values of distance AB, the following trials were recorded.

Trials (x)	Distance (y)
1	120.68
2	120.84
3	120.76
4	120.64

① Find the probable error.
② Find the standard deviation.
③ Find the standard error.

Solution: MODE 33

① *Probable error:*

Mean value

$$\bar{y} = \frac{120.68 + 120.84 + 120.76 + 120.64}{4}$$

Mean value = 120.73

Residual V	V^2
120.68 - 120.73 = - 0.05	0.0025
120.84 - 120.73 = +0.11	0.0121
120.76 - 120.73 = +0.03	0.0009
120.64 - 120.73 = - 0.09	0.0081
	$\Sigma V^2 = 0.0236$

$$\textit{Probable error} = 0.6745\sqrt{\frac{\Sigma V^2}{n(n-1)}} \quad \frac{0.6745\, y\sigma n-1}{2}$$

$$\textit{Probable error} = 0.6745\sqrt{\frac{0.0236}{4(3)}}$$

Probable error = ***± 0.0299***

② *Standard deviation:*

$$\textit{Standard deviation} = \sqrt{\frac{\Sigma V^2}{(n-1)}}$$

$$\textit{Standard deviation} = \sqrt{\frac{0.0236}{3}}$$

Standard deviation = ***± 0.0887*** $= y\sigma n-1$

③ *Standrad error:*

$$\textit{Standard error} = \frac{\textit{Standard deviation}}{\sqrt{n}}$$

$$\textit{Standard error} = \frac{\pm 0.0887}{\sqrt{4}}$$

Standard error = ***± 0.0443*** $= y\sigma n-1 \div \sqrt{(n)}$

Problem 22:

Three independent line of levels are run from BM_1 to BM_2. Route A is 6 km. long, route B is 4 km. long and route C is 8 km. By route A, BM_2 is 82.27 m. above BM_1 by route B, BM_2 is 82.40 m. above BM_1 and by route C, BM_2 is 82.10 m. above BM_1. The elevation of BM_1 is 86.42.

① Using the weighted mean values, what is the weight of route B.
② What is the probable value of the weighted mean.
③ What is the elevation of BM_2.

Solution:

① *Weight of route A:*

ROUTE	DISTANCE	DIFF. IN ELEV.
A	6	82.27
B	4	82.40
C.	8	82.10

$\frac{1}{6}\ \frac{1}{4}\ \frac{1}{8}$ LCD = 24

$E = \sqrt{D}$

Weight computations

$W = \frac{K}{(\sqrt{D})^2} = \frac{K}{D}$

A $W_1 = \frac{24}{6} = 4$

B $W_2 = \frac{24}{4} = 6$

C $W_3 = \frac{24}{8} = 3$

Weight of B = ***6***

② *Probable value of weighted mean:*

82.27(4) = 329.08
82.40(6) = 494.40
82.10(3) = 246.30
1069.78

Probable value of the weighted mean

$= \frac{1069.78}{13}$

= ***82.29***

③ *Elevation of BM_2:*

BM_2 = 82.46 + 82.29

BM_2 = ***168.71 m.***

Problem 23:

The observed angles of a triangle are as follows: A = 34°20'36" B = 49°16'34" C = 96°22'41"

① Determine the most probable value of angle C.
② Determine the most probable value of angle A.
③ Determine the most probable value of angle B.

Solution:

① *Probable value of angle C:*

Sum of all angles = 180°

34°20'36" + 49°16'34" + 96°22'41"

= 179°59'51"

Error = 180° - 179°59'51" = 09" *(too small)*

Correction $= \frac{9}{3}$

Correction = 3"

Probable value of angle C = 96°22'41" + 3"

Probable value of angle C = ***96°22'44"***

② *Probable value of angle A:*

Probable value of angle A = 34°20'36" + 3"

Probable value of angle A = ***34°20'39"***

③ *Probable value of angle B:*

Probable value of angle B = 49°16'34" + 3"

Probable value of angle B = ***49°16'37"***

Problem 24:

The weight of an angle is assumed to be proportional to the number of times it has been repeated. Five angles in a five sided figure are measured with the following results.

Angle	Observed Value	No. of Repetitions
A	86°15'20"	6
B	134°44'35"	2
C	75°48'50"	~~6~~ 2
D	167°02'05"	6
E	76°08'50"	4

① Compute the adjusted value of angle D.
② Compute the adjusted value of angle B.
③ Compute the adjusted value of angle E.

Solution:

ANGLE	OBSERVED VALUE	WEIGHT
A	86°15.20"	$\frac{1}{6} = 0.167$
B	134°44'35"	$\frac{1}{2} = 0.50$
C	75°48'50"	$\frac{1}{2} = 0.50$
D	167°02.05"	$\frac{1}{6} = 0.167$
E	76°08'50"	$\frac{1}{4} = 0.25$
Sum =	539°59'40"	1.584

CORRECTION	ADJUSTED ANGLES
$\frac{0.167\,(20)}{1.584} = 2.11"$	86°15'22.11"
$\frac{0.50\,(20)}{1.584} = 6.31"$	134°44'41.31"
$\frac{0.50\,(20)}{1.584} = 6.31"$	75°48'56.31"
$\frac{0.167\,(20)}{1.584} = 2.11"$	167°02'07.11"
$\frac{0.25\,(20)}{1.584} = 3.16"$	76°08'53.16"
Sum = 20"	540°00'00"

Sum = (*n* - 2) 180 = (5 - 2) 180 = 540°
Error = 540° - 539°59'40" = 20"

① *Adjusted value of angle D:*
Adjusted value of angle D = ***167°02'07.11"***

② *Adjusted value of angle B:*
Adjusted value of angle B = ***134°44'41.31"***

③ *Adjusted value of angle E:*
Adjusted value of angle E = ***76°08'53.16"***

25. CE Board May 2003

The following interior angles of a triangle traverse were measured with the same precision.

Angle	Value (Degrees)	No. of Measurements
A	41°	5
B	77°	6
C	63°	2

① Determine the most probable value of angle A.
② Determine the most probable value of angle B.
③ Determine the most probable value of angle C.

Solution:

① *Probable value of angle A:*
A + B + C = 41 + 77 + 63 = 181°
Error = 181° - 180° = 01°
Error = 60 mins.
LCD of 5, 6 and 2 is 30

Sta.	Weight	Correction
A	LCD $\frac{30}{5} = 6$	Σw $\frac{6}{26}(60) = 13.84'$
B	$\frac{30}{6} = 5$	$\frac{5}{26}(60) = 11.54'$
C	$\frac{30}{2} = \frac{15}{26}$	$\frac{15}{26}(60) = \frac{34.62'}{60'}$

Corrected value of A = 41° - 13.84'
Corrected value of A = ***40°46.16'***

② *Probable value of angle B:*
Corrected value of B = 77° - 11.54'
Corrected value of B = ***76°48.46'***

③ *Probable value of angle C:*
Corrected value of C = 63° - 34.62'
Corrected value of C = ***62°25.38'***

26. CE Board May 1999

The following interior angles of a triangle traverse were measured with the same precision.

Station	Value (Degrees)	No. of Measurements
A	39°	3
B	65°	4
C	75°	2

① Determine the most probable value of angle A.
② Determine the most probable value of angle B.
③ Determine the most probable value of angle C.

Solution:

① *Probable value of angle A:*
$A + B + C = 39 + 65 + 75 = 179°$
Error = 180° - 179° = 01°
Error = 60 mins.
LCD of 3, 4 and 2 is 12

LCD
ΣW

Sta.	Weight	Correction
A	$\frac{12}{3} = 4$	$\frac{4}{13}(60) = 18.46$
B	$\frac{12}{4} = 3$	$\frac{3}{13}(60) = 13.85$
C	$\frac{12}{2} = 6$	$\frac{6}{13}(60) = 27.69$
	13	60

Corrected value of A = 39° + 18.46'
Corrected value of A = ***39°18.46'***

② *Probable value of angle B:*
Corrected value of B = 65° + 13.85'
Corrected value of B = ***65°13.85'***

③ *Probable value of angle C:*
Corrected value of C = 75° + 27.69'
Corrected value of C = ***75°27.69'***

Problem 27:

A civil engineer measures the distance of points A and B and the following values were recorded in a series of measurements.

Trials	No. of Measurements
1	200.58
2	200.40
3	200.38
4	200.46

① Determine the average value (mean).
② Determine the probable error of mean.
③ Determine the precision of the measurements.

Solution:

① *Average mean value:*
Average value (mean)

$$= \frac{200.58 + 200.40 + 200.38 + 200.46}{4}$$

Average value (mean) = ***200.455***

② *Probable error of the mean:*

Length	V	V^2
200.58	200.58 - 200.455 = +0.125	0.015625
200.40	200.40 - 200.455 = - 0.055	0.003025
200.38	200.38 - 200.455 = - 0.075	0.005625
200.46	200.46 - 200.455 = +0.005	0.000025
		$\sum V^2 = 0.0243$

ERRORS AND MISTAKES

P.E. = 0.6745 $\frac{X\sigma n-1}{2}$

$$\text{Probable error of mean} = 0.6745\sqrt{\frac{\Sigma V^2}{n(n-1)}}$$

$$P.E. = 0.6745\sqrt{\frac{0.0243}{4(3)}}$$

$$P.E. = \pm 0.03$$

③ *Precision of the measurements:*

$$\text{Precision} = \frac{0.03}{200.455} = \frac{P.E}{\bar{X}}$$

$$\text{Precision} = \frac{1}{6681.83}$$

$$\text{Precision} = \frac{\mathbf{1}}{\mathbf{6682}}$$

#21 Repeated

Problem 28:

From the measured values of distance AB, the following trials were recorded.

TRIALS	DISTANCE
1	120.68
2	120.84
3	120.76
4	120.64

① Find the probable error.
② Find the standard deviation.
③ Find the standard error.

Solution:

① *Probable error.*

Mean value

$$= \frac{120.68 + 120.84 + 120.76 + 120.64}{4}$$

Mean value = 120.73

Residual V	V^2
120.68 - 120.73 = - 0.05	0.0025
120.84 - 120.73 = +0.11	0.0121
120.76 - 120.73 = +0.03	0.0009
120.64 - 120.73 = - 0.09	0.0081
	$\Sigma V^2 = 0.0236$

$$\text{Probable error} = 0.6745\sqrt{\frac{\Sigma V^2}{n(n-1)}}$$

$$\text{Probable error} = 0.6745\sqrt{\frac{0.0236}{4(3)}}$$

$$\text{Probable error} = \pm \mathbf{0.0299}$$

② *Standard deviation:*

$$\text{Standard deviation} = \sqrt{\frac{\Sigma V^2}{(n-1)}}$$

$$\text{Standard deviation} = \sqrt{\frac{0.0236}{3}}$$

$$\text{Standard deviation} = \pm \mathbf{0.0887}$$

③ *Standard error.*

$$\text{Standard error} = \frac{\text{Standard deviation}}{\sqrt{n}}$$

$$\text{Standard error} = \frac{\pm 0.0887}{\sqrt{4}} = \pm \mathbf{0.0443}$$

Problem 29:

The following data are the observed elevation of a point by running a line of levels over four different routes.

ROUTE	ELEVATIONS	PROBABLE ERRORS
1	340.22	± 02
2	340.30	± 04
3	340.26	± 06
4	340.32	± 08

① What is the weight of route 3 assuming the weight of route 1 equal to 1.
② What is the sum of the weighted observation.
③ What is the most probable value of the elevation.

Solution:

① *Weight of route 3:*

The weights are inversely proportional to the square of the corresponding probable errors.

$$W_1 = \frac{K}{(2)^2} \qquad W_2 = \frac{K}{(4)^2}$$

; k = 4

$$W_3 = \frac{K}{(6)^2} \qquad W_4 = \frac{K}{(8)^2}$$

$$4W_1 = 16W_2 = 36W_3 = 64W_4$$
$$W_1 = 4W_2 = 9W_3 = 16W_4$$

If $W_1 = 1$ $\qquad W_3 = \frac{1}{9} = 0.1111$

$W_2 = \frac{1}{4} = 0.25 \qquad W_4 = \frac{1}{16} = 0.0625$

Therefore the weight of route 3 = ***0.1111.***

② *Sum of the weighted observation:*

ROUTE	DIFFERENCE IN ELEVATION	WEIGHT
1	340.22	1
2	340.30	0.25
3	340.26	0.1111
4	340.32	0.0625
	Sum =	1.4236

Σy

WEIGHTED OBSERVATION		
240.22 (1)	=	240.220
340.30 (0.25)	=	85.075
340.36(0.1111)	=	37.803
340.32 (0.0625)	=	21.270
Sum	=	484.368

Σxy

The sum of weighted observation ***= 484.368***

③ *Most probable value of the elevation:*

$$\text{Most Probable Value} = \frac{484.368}{1.4236} = \frac{\Sigma xy}{\Sigma y}$$

Most Probable Value = ***340.424***

340.292

Problem 30:

A baseline measured with an invar tape, and with a steel tape as follows:

Invar tape	Steel tape
571.185	571.193
571.186	571.190
571.179	571.185
571.180	571.189
571.183	571.182

① What are the most probable value under each set.
② What are the probable errors under each set.
③ What is the most probable value of the two sets.
④ What is the probable error of the general mean.

Solution:

① *Probable value under each set:*
Most probable value using the Invar tape in measurements:

$$= \frac{571.185 + 571.186 + 571.179 + 571.180 + 571.183}{5}$$

$\bar{x}$ = ***571.183*** ≒ A

Most probable value using the Steel tape in measurements:

$$= \frac{571.193 + 571.190 + 571.185 + 571.189 + 571.182}{5}$$

$\bar{y}$ = ***571.188***

② *Probable Errors under each set:*
Probable error using Invar tape:

$\bar{x}$ ≒ A

Invar tape	Residual (V)	V^2
571.185 - 571.183 =	0.002	0.000004
571.186 - 571.183 =	0.003	0.000009
571.179 - 571.183 =	- 0.004	0.000016
571.180 - 571.183 =	- 0.003	0.000009
571.183 - 571.183 =	0	0.000000
		$\sum V^2 = 0.000038$

$$PE = 0.6745\sqrt{\frac{\sum V^2}{n(n-1)}}$$

$$PE = \sqrt{\frac{0.000038}{5(4)}} = \pm 0.00093$$

ERRORS AND MISTAKES

Probable error using Steel tape:

Steel tape	Residual (V)	V^2
571.193 - 571.188	0.005	0.000025
571.190 - 571.188	0.002	0.000004
571.185 - 571.188	- 0.003	0.000009
571.189 - 571.188	+0.001	0.000001
571.182 - 571.188	- 0.006	0.000036
		$\sum V^2 = 0.000075$

$$PE = 0.6745\sqrt{\frac{\sum V^2}{n(n-1)}}$$

$$PE = 0.6745\sqrt{\frac{0.000075}{5(4)}}$$

$PE = \pm\mathbf{0.00131}$

③ *Most probable value of the two sets:*

	Probable value	Probable error
x	571.183	$E_1 = 0.00093$
y	571.188	$E_2 = 0.00131$

$$W_1 = \frac{K}{E^2} \qquad W_2 = \frac{K}{E_2^2}$$; k = 0.00131²

$W_1E_1^2 = W_2E_2^2$

$W_1\,(0.00093)^2 = W_2\,(0.00131)^2$

Ass. $W_2 = 1$

$W_1\,(0.00093)^2 = 1\,(0.00131)^2$

$W_1 = 1.98$

0.00131² / 0.00093²

	Weight	Wt. x value
	$W_1 = 1.98$	1130.94
	$W_2 = 1.00$	571.188
Sum =	$W = 2.98$ Σy	1702.18 Σy

$$\textit{Most probable value of the two sets} = \frac{1702.128}{2.98}$$

Most probable value of the two sets = ***571.184***

④ *Probable error of the general mean:*

$$W = \frac{K}{E^2}$$

$$W_1 = \frac{K}{E_1^2}$$

$$E^2 = \frac{W_1E_1^2}{W}$$

$$E^2 = \frac{W_2E_2^2}{W}$$

$WE^2 = W_1E_1^2$

$WE^2 = W_2E_2^2$

$$E^2 = \frac{W_1E_1^2}{W}$$

$$E^2 = \frac{1.98(0.00093)^2}{2.98}$$

$E = \pm 0.00076$ *(probable error of mean)*

$$E^2 = \frac{W_2E_2^2}{W}$$

$$E^2 = \frac{1.00(0.00131)^2}{2.98}$$

$E = \pm\mathbf{0.00076}$

Problem 31:

Three trials of the measured angle between two points *x* and *y* were observed and the following data were recorded.

Trials	Measured Angle	Time
1	40°31'	9:00:00
2	40°34'	9:02:00
3	40°36'	9:04:30

① Find the probable error.
② Find the standard deviation.
③ Find the standard error.

Solution:

① *Probable error:*

$$\textit{Mean value} = \frac{40°31' + 40°34' + 40°36'}{3}$$

Mean value = 40°33.7'

Residual	V	V^2
40°31' - 40°33.7' =	2.7	7.29
40°34' - 40°33.7' =	+0.3	0.09
40°36' - 40°33.7' =	+2.3	5.29
		$\sum V^2 = 12.67$

$$\text{Probable error} = 0.6745\sqrt{\frac{\Sigma V^2}{n(n-1)}}$$

$$\text{Probable error} = 0.6745\sqrt{\frac{12.67}{3(3-1)}}$$

$$\text{Probable error} = \pm\ \mathbf{0.98}$$

② *Standard deviation:*

$$\text{Standard deviation} = \sqrt{\frac{\Sigma V^2}{n-1}}$$

$$\text{Standard deviation} = \sqrt{\frac{12.67}{2}}$$

$$\text{Standard deviation} = \pm\ \mathbf{2.52}$$

③ *Standard error:*

$$\text{Standard error} = \frac{2.52}{\sqrt{3}}$$

$$\text{Standard error} = \pm\ \mathbf{1.453}$$

Problem 32:

The distance BC was measured 3 times are recorded as follows:

Trial	Distance (meters)
1	141.60
2	141.80
3	141.70

① Determine the probable error.
② Determine the standard error.
③ Determine the precision of the measurements.

Solution:

Average value (mean)

$$= \frac{141.60 + 141.80 + 141.70}{3}$$

Average value (mean) = 141.70

V	V^2
141.60 - 141.70 = - 0.10	+0.01
141.80 - 141.70 = +0.10	+0.01
141.70 - 141.70 = 0	0
	$\Sigma V^2 = 0.02$

① *Probable error:*

$$\text{Probable error} = 0.6745\sqrt{\frac{\Sigma V^2}{n(n-1)}}$$

$$\text{Probable error} = 0.6745\sqrt{\frac{0.02}{3(2)}} = \pm\ \mathbf{0.039}$$

② *Standard error:*

$$\text{Standard deviation} = \sqrt{\frac{\Sigma V^2}{n-1}}$$

$$\text{Standard deviation} = \sqrt{\frac{0.02}{2}} = 0.10$$

$$\text{Standard error} = \frac{\text{Standard deviation}}{\sqrt{n}}$$

$$\text{Standard error} = \frac{0.10}{\sqrt{3}} = \pm\ \mathbf{0.577}$$

③ *Precision:*

$$\text{Precision} = \frac{0.039}{141.70}$$

$$\text{Precision} = \frac{\mathbf{1}}{\mathbf{3633}}$$

Problem 33:

The observed interior angles of a triangle and their corresponding number of times measured are as follows:

Angle	No. of measurement
A = 39'	2
B = 65'	3
C = 75'	4

① Find the probable value of angle A.
② Find the probable value of angle B.
③ Find the probable value of angle C.

Solution:

Error = 180 - (39' + 65' + 75')

Error = 01'

Error = 60' *(too small)*

		Weight
A	39°	$\frac{12}{2} = 6$
B	65°	$\frac{12}{3} = 4$
C	75°	$\frac{12}{4} = 3$
		13

Correction	Corrected Angles
$\frac{6}{13}(60) = 27.69'$	**39° 27' 41"**
$\frac{4}{13}(60) = 18.46'$	**65°18' 28"**
$\frac{3}{13}(60) = 13.85'$	**75° 13' 51"**
60'	

① *Probable value of angle A:*
Probable value of angle A = **39° 27' 41"**

② *Probable value of angle B:*
Probable value of angle B = **65°18' 28"**

③ *Probable value of angle C:*
Probable value of angle C = **75° 13' 51"**

Problem 34:

The interior angles of a quadrilateral are as follows:

Angles	Value	No. of Measurements
A	92°	2
B	88°	4
C	71°	3
D	110°	6

① Compute the corrected value of angle A.
② Compute the corrected value of angle B.
③ Compute the corrected value of angle C.

Solution:

① *Corrected angle A:*

Angles	Value	Weight	Corrections
A	92°	12/6 = 6	6/15 (60) = 24'
B	88°	12/4 = 3	3/15 (60) = 12'
C	71°	12/3 = 4	4/15 (60) = 16'
D	110°	12/6 = 2	2/15 (60) = 8'
		15	60'

Error = (92 + 88 + 71 + 110) - 360
Error = 01° = **60'** *(too big)*
Corrected angle A = 92° - 24'
Corrected angle A = **91°36'**

② *Corrected angle B:*
Corrected angle B = 88° - 12'
Corrected angle B = **87°48'**

③ *Corrected angle C:*
Corrected angle C = 71° - 16'
Corrected angle C = **70° 44'**

Problem 35:

From the following measured interior angles of a five sided figure, compute the following:

① Probable value of angle A.
② Probable value of angle C.
③ Probable value of angle D.

Station	Value of Angles	No. of Measurements
A	110°	2
B	98°	3
C	108°	4
D	120°	6
E	105°	4

Solution:

① *Probable value of angle A:*
Sum of interior angles = (*n* - 2)180
Sum of interior angles = (5 - 2)(180)
Sum of interior angles = 540°
Sum = 110° + 98° + 108° + 120° + 105°
Sum = 541°
Error = 01° or 60' *(to be subtracted)*

ERRORS AND MISTAKES

Sta.	Angles	Weight
A	110°	$\frac{12}{2}=6$
B	98°	$\frac{12}{3}=4$
C	108°	$\frac{12}{4}=3$
D	120°	$\frac{12}{6}=2$
E	105°	$\frac{12}{4}=3$
		18

Correction	Corrected Angles
$\frac{6}{18}(60)=20'$	109° 40' 00"
$\frac{4}{18}(60)=13.33'$	97° 46' 40.2"
$\frac{3}{18}(60)=10'$	107° 50' 00"
$\frac{2}{18}(60)=6.67'$	119° 53' 19.8"
$\frac{3}{18}(60)=10'$	104° 50' 00"
60'	540° 00' 00"

Probable value of angle A = ***109°40' 00"***

② *Probable value of angle C =* ***107°50' 00"***

③ *Probable value of angle D =* ***119°53'19.8"***

Problem 36:

Measured from point A, angles BAC, CAD and BAD were recorded as follows:

Angle	Value	No. of Measurements
BAC	28°34' 00"	2
CAD	61°15' 00"	2
BAD	89°49' 40"	4

① Compute the most probable value of angle BAC.
② Compute the most probable value of angle BAD.
③ Compute the most probable value of angle CAD.

Solution:

28°34' + 61°15' = 89°49'

Error = 40"

Angle	Weight
BAC	$\frac{1}{2}=0.5$
BAD	$\frac{1}{4}=0.25$
CAD	$\frac{1}{2}=0.5$
	1.25

Correction	Corrected Values
$\frac{40\,(0.5)}{1.25}=16"$	***28°34'16"***
$\frac{40\,(0.25)}{1.25}=-8"$	***89°49'32"***
$\frac{40\,(0.5)}{1.25}=16"$	***61°15'16"***

① *Probable value of angle BAC =* ***28°34'16"***

② *Probable value of angle BAD =* ***89°49'32"***

③ *Probable value of angle CAD =* ***61°15'16"***

Problem 37:

Lines of levels between B and C are run over four different routes. B is at elevation 825 m. and is higher than C.

Route	Distance (km)	Difference in Elevation (m)
1	2	0.86
2	6	0.69
3	4	0.75
4	8	1.02

① Determine the weight of route number 2.
② Determine the most probable difference in elevation.
③ Determine the most probable elevation of C in meters.

ERRORS AND MISTAKES

Solution:

① *Weight of route no. 2:*

$w_1D_1 = w_2D_2 = w_3D_3 = w_4D_4$

Assume:

$W_1 = 6$

$6(2) = w_2(6)$

$w_2 = 2$

$2(6) = w_3(4)$

$w_3 = 3$

$3(4) = w_4(8)$

$w_4 = 1.5$

Weight of route 2 = ***2***

② *Probable difference in elev:*

Route	Weight	Wt. x Diff in elevation	
1	6	6(0.86)	= 5.16
2	2	2(0.69)	= 1.38
3	3	3(0.75)	= 2.25
4	1.5	1.5(1.02)	= 1.53
	12.5		10.32

Probable diff. in elev. $= \dfrac{10.32}{12.5}$

Probable diff. in elev. = ***0.826***

③ *Probable elevation of C:*

Probable elevation of C = 825 + 0.82

Probable elevation of C = ***825.82***

Problem 38:

Lines of levels are run from BM_1 to BM_2 over three different routes. If the elevation of BM_1 is 100 m. above the sea level.

Route	Length	(Diff. In Elev.) Between Bm_1 & Bm_2
A	10	632.81
B	16	632.67
C	40	633.30

① Determine the probable weight of route B.
② Determine the most probable difference in elev. of BM_1 and BM_2.
③ Determine the most probable elevation of BM_2. BM_1 is lower than BM_2.

Solution:

Determine first the weight of each route

$\dfrac{1}{10} \quad \dfrac{1}{16} \quad \dfrac{1}{40}$

To find the weight, divide the L.C.D. by its distance.

① *Probable weight of route B:*

ROUTE	LENGTH	WEIGHT
A	10	$\frac{160}{10} = 16$
B	16	$\frac{160}{16} = \mathbf{10}$
C	40	$\frac{160}{40} = 4$
		Sum = 30

Weight of route B = ***10***

② *Probable difference in elevation:*

16(632.81) = 10124.96
10(632.67) = 6326.78
4(633.30) = 2533.28
18984.86

Most probable diff. in elev. $= \dfrac{18984.86}{30}$

Most probable diff. in elev. = ***632.83***

② *Probable elevation of BM_2:*

Probable elevation of BM_2

= 100 + 632.83

= ***732.83 m.*** *above sea level*

Problem 39:

① The base and altitude of a triangular lot were measured to have certain probable errors of 314.60 ± 0.16 and 92.60 ± 0.14, compute the probable error of the resulting computation.

② The following sides of a rectangle and its probable errors are 120.40 ± 0.04 and 360.50 ± 0.08 respectively. Compute the probable errors of the sum of the sides (perimeter) of the rectangle.

③ The probable error of the mean of 6 observation is 0.043 and the most probable value of the measurement is 860 m. Compute the relative precision.

Solution:

① *Probable error of the resulting computation:*

$P.E. = \sqrt{(b\,E_h)^2 + (h\,E_b)^2}$

$b = 314.60$

$h = 92.60$

$E_b = +0.16$

$E_h = 0.14$

$P.E. = \sqrt{[314.6(0.14)]^2 + [92.60(0.16)]^2}$

$P.E. = \mathbf{+46.47}$

② *Probable error of the sum of the sides:*

$P.E. = \sqrt{(PE_1)^2 + (PE_2)^2 + (PE_3)^2 + (PE_4)^2}$

$P.E. = \sqrt{(0.04)^2 + (0.08)^2 + (0.04)^2 + (0.08)^2}$

$P.E. = \mathbf{+0.126}$

③ *Relative precision:*

$$\text{Relative precision} = \frac{0.043}{860}$$

$$\text{Relative precision} = \frac{\mathbf{1}}{\mathbf{2000}}$$

Problem 40:

From the following data of a precise leveling from BM_1 to BM_2, compute the following:

① Compute the probable weighted mean value of the difference in elevation.
② Compute the standard deviation.
③ Compute the probable elevation BM_2 assuming it is lower than BM_1 whose elevation is 212.4 m.

LINE	DIFF. IN ELEV	WEIGHT
1	41.16	6
2	41.20	4
3	41.12	3

Solution:

② *Probable weighted mean:*

Line	Diff. in Elev.	Weight
1	41.16	6
2	41.20	4
3	41.12	3
		13

Wt. x Diff. in Elev.	V	WV²
6(41.16) = 246.96	0	0
4(41.20) = 164.80	+0.04	0.0064
3(41.12) = 123.36	- 0.04	0.0048
535.12		0.0112

$V_1 = 41.16 - 41.16 = 0$

$V_2 = 41.20 - 41.16 = +0.04$

$V_3 = 41.12 - 41.16 = -0.04$

$$\text{Weighted mean} = \frac{535.12}{13} = \mathbf{41.16}$$

② *Standard deviation:*

$$\text{Standard deviation} = \sqrt{\frac{\Sigma WV^2}{\Sigma W(n-1)}}$$

$$\text{Standard deviation} = \sqrt{\frac{0.30112}{13(3-1)}}$$

Standard deviation = **±0.021**

③ *Elevation BM_2:*

Elevation $BM_2 = 212.40 - 41.16$

Elevation $BM_2 = \mathbf{171.24}$

Problem 41:

Three level lines established on three different routes to established bench marks, the result of which is as follows:

Route 1: Elevation of BM_1 is 30.162 m. BM_2 has an elevation which is 68.258 m. above BM_1 and BM_3 is 75.442 m. above BM_2. Distance of route from BM_1 to BM_2 is 3 km. and from BM_2 to BM_3 is 7 km.

Route 2: BM_3 is 143.62 m. above BM_1. Distance of route from BM_1 to BM_3 is 6 km. long.

Route 3: BM_3 is 143.58 m. above BM_1. Distance of route from BM_1 to BM_3 is 15 km. long.

① Compute the weighted difference in elevation between BM_1 & BM_3.
② What is the elevation of BM_3.
③ What is the adjusted elevation of BM_2.

Solution:

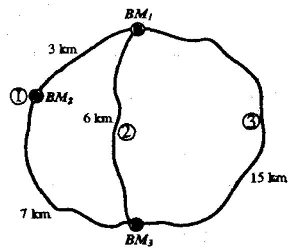
① *Weighted difference in elevation between BM_1 & BM_3:*

Route	Distance	Diff. in elev.
1	10 km.	68.258 + 75.442 = 143.70
2	6 km.	143.62
3	15 km.	143.58

$w_1 D_1 = w_2 D_2 = w_3 D_3$
$w_1 (10) = 6w_2 = 15w_3$

Ass: $w_2 = 5$
$w_1 (10) = 6(5)$
$w_1 = 3$
$3(10) = 15\ w_3$
$w_3 = 2$

Diff. in elev.	Weight	Wt. x Diff. in Elev.
143.70	3	3(143.7) = 431.10
143.62	5	5(143.62) = 718.10
143.58	2	2(143.58) = 287.16
	10	1436.36



Weighted diff. in elev. $= \dfrac{1436.36}{10}$
Weighted diff. in elev. = ***143.636***

② *Elevation of BM_3:*
Elev. of BM_3 = 143.636 + 30.162
Elev. of BM_3 = ***173.798 m.***

③ *Adjusted elevation of BM_2:*
Total Correction for route 1
= 143.70 - 143.636
= 0.064 m.

Correction for BM_2 $= \dfrac{3}{10}(0.064)$
Correction for BM_2 = 0.0192
Correction for diff. in elev. of BM_1 and BM_2
= 68.258 - 0.0192
= 68.2388 m.

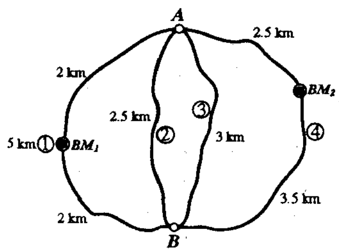
Adj. Elev. of BM_2 = 30.162 + 68.2388
Adj. Elev. of BM_2 = ***98.4008 m.***

Problem 42:

From starting point A, elevation 340.85 m., the elevation of a second point B is found, the route, distance and elevation of B being respectively as follows: Route 1 - 4 km, 364.84 m., Route 2 - 2.5 km., 364.20 m. Route 3 - 3 km., 365.01 m., Route 4 - 6 km., 364.37 m. Midway along route 1, BM_1 is located with an elevation of 351.29. Along route 4, 2.5 km. from A towards B, BM_2 is located, with an elevation of 349.86 m.

① Compute the weighted elevation of B.
② Compute the error in elevation of B using route 4.
③ Compute the adjusted elevation of BM_2 using route 4.

Solution:
① *Weighted elevation of B:*



Route	Distance	Elevation of B
1	4 km.	364.84 m
2	2.5 km.	364.20 m.
3	3 km.	365.01 m.
4	6 km.	364.37 m.

$w_1 = \frac{k}{D_1}$ $\quad w_2 = \frac{k}{D_2}$

$w_3 = \frac{k}{D_3}$ $\quad w_4 = \frac{k}{D_4}$

$w_1 D_1 = w_2 D_2 = w_3 D_3 = w_4 D_4$

Ass: $w_1 = 1.0$
$(1)(4) = 2.5(w_2)$
$w_2 = 1.6$
$w_1 D_1 = w_3 D_3$
$(1)(4) = w_3 (3)$
$w_3 = 1.33$
$w_1 D_1 = w_4 D_4$
$(1)(4) = w_4 (6)$
$w_4 = 0.67$

Wt.	Wt. x Elev.	
1.0	1(364.84)	= 364.84
1.6	1.6(364.20)	= 582.72
1.33	1.33(365.01)	= 485.13
0.67	0.67(364.37)	= 244.13
4.60		1677.15

Weighted elev. of B:

$$Elev.\ B = \frac{1677.15}{4.60} = \mathbf{364.60}$$

② *Error in elevation of B using route 4:*
Error in route 4 = 364.60 - 364.37
*Error in route 4 = **0.23 m.***

③ *Adjusted elevation of BM_2 using route 4:*

$$Correction\ for\ BM_2 = \frac{2.5}{6}(0.23)$$

Correction for BM_2 = 0.096 m.
Corrected Elev. of BM_2 = 349.86 + 0.96
*Corrected Elev. of BM_2 = **349.956***

Problem 43:

An angle was carefully measured 10 times with an optical theodolite by observers A and B on two separate days. The calculated results are as follows:

Observer A	Observer B
Mean = 42°16'25"	Mean = 42°16'20"
Em = ±3.2"	Em = ±1.6"

① Compute the probable weight of A assuming the weight of B equal to 1.
② What is the sum of the weight of A and B.
③ What is the most probable value of the angle.

Solution:
① ***Probable weight of A:***

$$w_1 = \frac{k}{E_1^2}$$

$w_1 E_1^2 = w_2 E_2^2$
$w_1 (3.2)^2 = w_2 (1.6)^2$

Assume $w_2 = 1$

$$w_1 = \frac{(1.6)^2}{(3.2)^2}$$

$w_1 = \mathbf{0.25}$

Observer	Angle	Error
A	42°16'25"	±3.2"
B	42°16'20"	±1.6"

Weight	Wt. x Angle
0.25	25(0.25) = 6.25
1.00	20(1) = 20.0
1.25	26.25

ERRORS AND MISTAKES

② *Sum of the weight of A and B:*
Sum of the weight of A and B = 1 + 0.25
Sum of the weight of A and B = ***1.25***

③ *Most probable value of the angle:*

Best value of the angle = $42°16' + \frac{26.25}{1.25}$

Best value of the angle = ***42°16'21"***

Problem 44:

① A line measures 6846.34 m. at elevation 993.9 m. The average radius of curvature in the area is 6400 km. Compute the sea level distance.

② The ground distance as corrected for temp., sag and pull correction is 10000 m. If the sea level reduction factor is 0.9998756 and the grid scale factor is 0.9999000, compute the grid distance of the same line.

③ The grid area of a parcel of land is 25425 sq.m. If the sea level reduction factor is 0.9998756 and the grid scale factor is 0.9999, determine the actual ground area.

Solution:

① *Sealevel distance:*

Reduction factor = $1 - \frac{h}{R}$

Reduction factor = $1 - \frac{993.9}{6400000}$

Reduction factor = 0.99984
Sea level distance = 6846.34 (0.99984)
Sea level distance = ***6845.24 m.***

② *Grid distance:*
Combination factor
= 0.9998756(0.9999000)
= 0.9997756

Grid distance = 10000(0.9997756)
Grid distance = ***9997.756 m.***

③ *Actual ground area:*
Combined factor = 0.9998756 (0.9999)
Combined factor = 0.999775612

Actual ground area = $\frac{25425}{(0.999775612)^2}$

Actual ground area = ***25436.41 sq.m.***

Problem 45:

① The difference of elevation between two points was determined by trigonometric leveling. The slope distance was measured electronically and was found to be 1486.72 m. and the zenith distance was 83°14'20". Calculate the difference in elevation between the two points.

② The geodetic length of a line on the earth's surface is found to be 5280 m. and its grid distance is equal to 5279.67 m. Compute the scale factor used.

③ The corrected field distance on the surface of the earth was found to be 3296.43 m. If the elevation factor is 0.9999642 and a scale factor of 0.9999424, compute the grid distance.

Solution:

① *Difference in elevation:*
Vertical angle = 90° - 83°14'20"
Vertical angle = 6°45'40"
Diff. in elevation = 1486.72 Sin 6°45'40"
Diff. in elevation = ***175.03 m.***

② *Scale factor:*
Grid distance = *Geodetic length x scale factor*

Scale factor = $\frac{5279.67}{5280}$

Scale factor = ***0.9999375***

③ *Grid factor:*
Grid factor = *elevation factor x scale factor*
Grid factor = 0.9999642(0.9999424)
Grid factor = 0.9999066
Grid distance = 3296.43(0.9999066)
Grid distance = ***3296.12 m.***

LEVELING

Adjustments of Dumpy Level:

1. *Adjustment of Level Tube:*
   To make the axis of the level tube perpendicular to the vertical axis.

2. *Adjustment of Horizontal Cross-Hair:*
   To make the horizontal cross-hair lie in a plane perpendicular to the vertical axis.

3. *Adjustment of the Line of Sight:*
   To make the line of sight parallel to the axis of level tube.

Adjustments of Wye Level:

1. *Adjustments of Level Tube:*
   To make the axis of level tube lie in the same plane with the axis of the wyes.

2. *Adjustment of Level Tube:*
   To make the axis of the level tube parallel to the axis of wye.

3. *Adjustment of Horizontal Cross-Hair:*
   To make the horizontal cross-hair lie in a plane perpendicular to the vertical axis.

4. *Adjustment of Line of Sight:*
   To make the line of sight coincide with the axis of the wyes.

5. *Adjustment of Level Tube:*
   To make the axis of level tube perpendicular to the vertical axis.

Two distinct types of Engineer's Level

1. Dumpy Level
2. Wye Level

Main differences between Dumpy and Wye Level

1. In the dumpy level, the bubble tube is attached to the level bar while in the wye level it is attached to the telescope.

2. In the dumpy level, the bubble tube can be adjusted in a vertical plane only, while in the wye level it may be adjusted vertically and laterally.

3. In the dumpy level, the telescope is rigidly fastened to the level bar and can not be removed there from, while in the wye level, the telescope rests in Y-shaped supports which permit it to be removed and reversed end for end or revolved about the axis of collars.

4. The dumpy is more rigidly constructed that the wye that it has fewer adjustments. However, to adjust a dumpy level would require at least two men whereas a wye level may be adjusted by only one man.

5. In the dumpy the supports are not adjustable, while in the wye, one end of the level bar may be adjusted vertically.

LEVELING

Methods of Eliminating Errors in Leveling

1. Imperfect adjustment of instrument:
   This could be eliminated by adjusting the instrument or by balancing the sum of foresight and backsight distances.

2. Rod not of standard length:
   This could be eliminated by standardizing the rod and apply corrections same as for tape.

3. Parallax:
   This could be eliminated by focusing carefully.

4. Bubble not centered at instant of sighting:
   This could be eliminated by checking the bubble before making each sight.

5. Rod not held plumb:
   This could be eliminated by waving the rod or using rod level.

6. Faulty of reading the rod:
   This could be eliminated by checking each rod reading before recording.

7. Faulty turning point:
   This could be eliminated by choosing definite and stable points.

8. Variation of temperature:
   This could be eliminated by protecting the level from the sun while making observations.

9. Earth's curvature:
   This could be eliminated by balancing each backsight and foresight distance, or apply the computed correction.

10. Atmospheric refraction:
    This could be eliminated by balancing each backsight and foresight distance, also take short sights well above ground and take backsight and foresight readings quick succession.

11. Settlement of tripod or turning points:
    This could be eliminated by choosing stable locations, and taking backsight and foresight readings in quick succession.

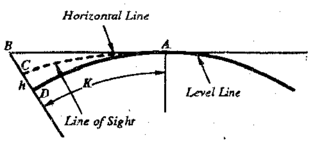
Common Errors in Leveling

1. Imperfect adjustment of instrument
2. Parallax
3. Earth's curvature
4. Atmospheric refraction
5. Variation in temperature
6. Rod not standard length
7. Expansion or contraction of rod
8. Rod not held plumb
9. Faulty turning points
10. Settlement of tripod or turning points
11. Bubble not exactly centered at the instant of sighting
12. Inability of observer to read the rod exactly.

Common Mistakes in Leveling

1. Confusion of numbers in reading and recording.
2. Recording B.S. on the F.S. column and vice-versa.
3. Faulty additions and subtractions.
4. Rod not held on the same point for both B.S. and F.S.
5. Wrong reading of the vernier when the target rod is used.
6. Not having target set properly when the long rod is used.

Earths Curvature and Atmospheric Refraction



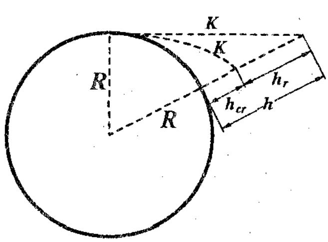
Horizontal Line = a straight line tangent to a level surface.

Level Surface = a curved surface every element of which is normal to the plumb line.

Level line = a line in a level surface.

From the figure shown, an object actually at C would appear to be at B, due to atmospheric refraction, wherein the rays of light transmitted along the surface of the earth is bent downward slightly. The value of h represents the effect of earths curvature and atmospheric refraction and has the following values.

DERIVATION OF CURVATURE and REFRACTION CORRECTION



$K^2 + R^2 = (R + h)^2$

$K^2 + R^2 = R^2 + 2\,Rh + h^2$

Since h is so small, h^2 is negligible

R = radius of earth

R = 6400 km.

$h = \dfrac{K^2}{2R}$

$h = \dfrac{K^2\,(1000)}{2\,(6400)}$

$h = 0.078\,K^2$

$h_r = \dfrac{1h}{7}$

$h_r = \dfrac{1\,(0.078\,K^2)}{7}$

$h_r = 0.011\,K^2$

$h_{cr} = h - h_r$

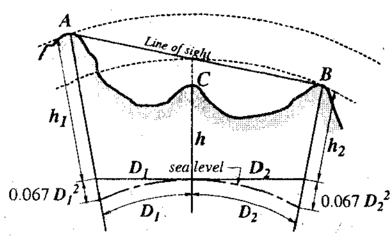
$h_{cr} = 0.078\,K^2 - 0.011\,K^2$

$$\boldsymbol{h_{cr} = 0.067\,K^2}$$

h_{cr} = in meters

K = in thousand of meters

Derivation:



Conditions:

h = height in m. of the line of sight, at the intervening hill C, above sea level.

h_1 = height in m. of the station occupied A, above sea level.

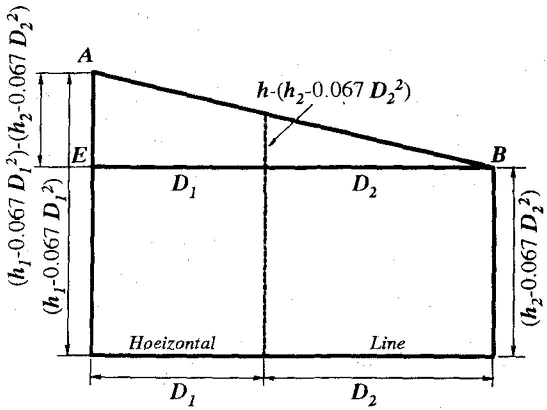
h_2 = height in m. of the station observed B, above sea level.

D_1 = distance in miles of the intervening hill C from A.

D_2 = distance in miles of the intervening hill C from B.

LEVELING

Since h_1, h, and h_2 are vertical heights, and considering the effects of curvature and refraction at A and B, as reckoned from a tangent (horizontal) line at sea level vertically below C, the figure can be reconstructed in its plane sense.



In triangle ABE, by proportion:

$$\frac{(h_1 - 0.067D_1^2) - (h_2 - 0.067D_2^2)}{D_1 + D_2} = \frac{h - (h_2 - 0.067D_2^2)}{D_2}$$

$$\frac{h_1 - h_2 - 0.067\,(D_1^2 - D_2^2)}{D_1 + D_2} = \frac{h - h_2 + 0.067D^2}{D_2}$$

$$h - h_2 + 0.067D_2^2 = \frac{D_2}{D_1 + D_2}\left[h_1 - h_2 - 0.067(D_1 + D_2)(D_1 - D_2)\right]$$

$$h = h_2 - 0.067D_2^2 + \frac{D_2}{D_1 + D_2}(h_1 - h_2) - 0.067D_2\,(D_1 - D_2)$$

$$h = h_2 - 0.067D_2^2 + \frac{D_2}{D_1 + D_2}(h_1 - h_2) - 0.067D_1D_2 + 0.067D_2^2$$

$$\boldsymbol{h = h_2 + \frac{D_2}{D_1 + D_2}(h_1 - h_2) - 0.067D_1D_2}$$

Problem 46:

From the given data of a differential leveling as shown in the tabulation:

STA.	B.S.	F.S.	ELEV.
1	5.87		392.25
2	7.03	6.29	
3	3.48	6.25	
4	7.25	7.08	
5	10.19	5.57	
6	9.29	4.45	
7		4.94	

① Find the diff. in elevation of station 7 and station 5.
② Find the diff. in elevation of station 7 and station 4.
③ Find the elevation of station 3.

Solution:

Note: H.I. = Elev. + B.S.
Elev. = H.I. - F.S.

Sta.	B.S.	H.I.	F.S.	Elev.
1	5.87	398.12		392.25
2	7.03	398.86	6.29	391.83
3	3.48	396.09	6.25	392.61
4	7.25	396.26	7.08	389.01
5	10.19	400.88	5.57	390.69
6	9.29	405.72	4.45	396.43
7			4.94	400.78

$\sum BS = 43.11$ $\sum FS = 34.58$

Arithmetic check:
$\sum BS - \sum FS = 43.11 - 34.58 = 8.53$
$400.78 - 392.25 = 8.53$

① *Diff. in elevation of station 7 and station 5:*
Diff. in elevation of station 7 and 5
= 400.78 - 390.69
= ***10.09 m.***

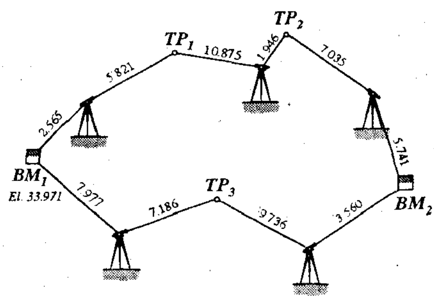
② *Diff. in elevation of station 7 and station 4:*
Diff. in elevation of station 7 and 4
= 400.78 - 389.01
= ***11.77***

③ *Elevation of station 3* = ***392.61***

Problem 47:

In the plan below shows a differential leveling from bench mark to another bench mark, along each line represents a sight in the actual rod reading. The direction of the field work is indicated by the number of turning points.

① Compute the elevation of TP_2.
② Compute the elevation of BM_2.
③ Compute the elevation of TP_3.



Solution:

Sta.	B.S.	H.I.	F.S.
BM_1	2.565	36.536	
TP_1	10.875	41.59	5.821
TP_2	7.035	46.679	1.946
BM_2	3.560	44.498	5.741
TP_3	7.186	41.948	9.736
BM_1			7.977

Sta.	Elev.	Remarks
BM_1	33.971	Bench Mark No.1
TP_1	30.715	Turning point
TP_2	39.644	Turning point
BM_2	40.938	Bench Mark No. 2
TP_3	34.762	Turning point
BM_1	33.971	Bench Mark No. 1

Arithmetical check:

$\sum F.S. = 5.821 + 1.946 + 5.741$
$+ 9.736 + 7.977$
$\sum F.S. = 31.221$
$\sum B.S. = 2.565 + 10.875 + 7.035$
$+ 3.560 + 7.186$
$\sum B.S. = 31.221$
$\sum F.S. - B.S. = 31.221 - 31.221 = 0$
$33.971 - 33.971 = 0$

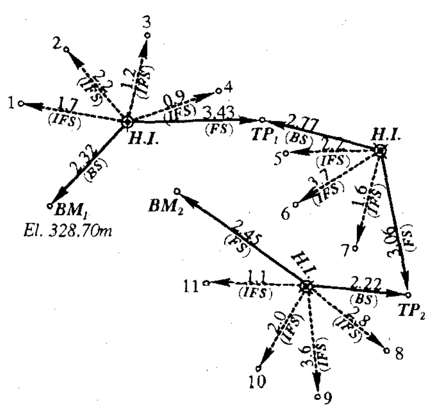
① *Elevation of TP_2:*
*Elevation of TP_2 = **39.644 m.***

② *Elevation of BM_2:*
*Elevation of BM_2 = **40.938 m.***

③ *Elevation of TP_3:*
*Elevation of TP_3 = **34.762 m.***

Problem 48:

The figure shows a schematic arrangement of a profile level route from BM_1 and BM_2. The values indicated represent backsight, foresight, and intermediate foresight reading taken on stations along the route. Elevation of BM_1 = 328.70 m.



① Find the difference in elevation between stations 5 and 9.
② Find the elevation of TP_2.
③ Find the elevation of BM_2.

Solution:

STA	BS	HI	FS	IFS	ELEV
BM_1	2.32	331.02			328.70
1				1.7	329.32
2				2.2	328.82
3				1.2	329.82
4				0.9	330.12
TP_1	2.77	330.36	3.43		327.59
5				2.2	328.16
6				3.7	326.66
7				1.6	328.76
TP_2	2.22	329.52	3.06		327.30
8				2.8	326.72
9				3.6	325.92
10				2.0	327.52
11				1.1	328.42
BM_2			2.45		327.07
	7.31		8.94		

Arithmetic check:
8.94 - 7.31 = 1.63
328.70 - 327.07 = 1.63

① *Difference in elevation between stations 5 and 9:*
= 328.16 - 325.92
= **2.24 m.**

② *Elevation of TP_2:*
*Elevation of TP_2 = **327.30 m.***

② *Elevation of BM_2:*
*Elevation of BM_2 = **327.07 m.***

Problem 49:

From the given profile leveling notes:

STA	BS	FS	IFS	ELEV
BM_1	0.95			225.50
1			3	
2			2.3	
TP_1	3.13	0.64		
3			2.7	
4			2.8	
5			3.1	
6			0.5	
7			0.8	
TP_2	2.16	1.28		
8			0.9	
9			1.2	
10			1.7	
11			2.8	
TP_3	0.82	2.37		
TP_4	1.35	3.50		
12			3.0	
BM_2		1.24		

① What is the difference in elevation between station 5 and 2.
② Compute the elevation of TP_2.
③ Compute the elevation of BM_2.

Solution:

STA	BS	HI	FS	IFS	ELEV
BM_1	0.95	226.45			225.50
1				3.0	223.50
2				2.3	224.2
TP_1	3.13	228.94	0.64		225.81
3				2.7	226.2
4				2.8	226.1
5				3.1	225.8
6				0.5	228.4
7				0.8	228.1
TP_2	2.16	229.82	1.28		227.66
8				0.9	228.9
9				1.2	228.6
10				1.7	228.1
11				2.8	227.0
TP_3	0.82	228.27	2.37		227.45
TP_4	1.35	226.12	3.50		224.88
12				3.0	223.1
BM_2			1.24		224.88
	8.41		9.03		

Arithmetic check:
9.03 - 8.41 = 0.62
226.12 - 225.50 = 0.62

① *Difference in elevation between station 5 and 2:*
= 225.8 - 224.2
= ***1.6 m.***

② *Elevation of TP_2:*
Elevation of TP_2 = ***227.66 m.***

③ *Elevation of BM_2:*
Elevation of BM_2 = ***224.88 m.***

Problem 50:

Arrange the following description in the form of profile level notes complete to elevation. A level is set up and a reading of 2.995 m. is taken on a bench mark the elevation of which is 12.135 m. At the beginning of the line to be profiled, the rod reading is 2.625 m. 30 m. from the beginning, it is 1.617 m. at 60 m., it is 0.702 m. at 66 m. and 81 m., the rod readings are 1.281 m. and 0.762 m., respectively. On a rock that is not on line, the rod reading is 0.555 m. The level is then removed ahead, set up and a rod reading of 1.952 m. is observed, the rod still being held on the rock. The readings along the profile are then resumed: 90 m. from the beginning of the line, the rod reading is 1.159 m., 120 m. from the beginning of the line rod reading is 1.434 m., finally 150 m. from the beginning of the line the rod reading is 2.196 m.

① Compute the elevation at the point 60 m. from the beginning of the line.
② Compute the elevation of the turning point.
③ Compute the difference in elevation at a point 150 m. and 81 m. from the beginning of the line.

LEVELING

Solution:

STA	BS	HI	FS	IFS	ELEV
BM	2.995	15.130			12.135
0				2.625	12.505
30				1.617	13.513
60				0.702	14.428
66				1.281	13.849
81				0.762	14.368
TP	1.952	16.527	0.555		14.575
90				1.159	15.368
120				1.434	15.093
150				2.196	14.331
	4.947		0.555		

Arithmetic check:
4.947 - 0.555 = 4.392
16.527 - 12.135 = 4.392

① *Elevation at point 60 m. from the beginning of the line:*
= ***14.428 m.***

② *Elevation of the turning point =* ***14.575 m.***

③ *Difference in elevation at point 150 m. and 81 m. from the beginning of the line:*
= 14.368 - 14.331
= ***0.037 m.***

Problem 51:

Data shown is obtained from a double rodded line of levels of a certain cross-section of the proposed Manila-Bataan Road.

STA.	B.S.	F.S.	ELEV.
BM_1	9.08		749.06
BM_1	9.08		
TP_1 -L	12.24	3.73	
TP_1-H	10.10	1.60	
TP_2 -L	11.04	2.21	
TP_2-H	9.92	1.08	
TP_3 -L	1.75	9.84	
TP_3-H	0.55	8.62	
BM_2		11.27	
BM_2		11.27	

① Find the diff. in elevation between TP_1 and TP_3.
② Find the elevation of BM_2.
③ What is the difference in elevation between BM_1 and TP_2.

Solution:

STA	BS	HI	FS	ELEV
BM_1	9.08	758.14		749.06
BM_1	9.08	758.14		
TP_1 -L	12.24	766.65	3.73	754.41
TP_1 -R	10.10	766.64	1.60	756.54
TP_2 -L	11.04	775.48	2.21	764.44
TP_2 -H	9.92	775.48	1.08	765.56
TP_3 -L	1.75	767.39	9.84	765.64
TP_3 -H	0.55	767.41	8.62	766.86
BM_2			11.27	756.12
BM_2			11.27	756.14
	63.76		49.62	

Arithmetic check:
63.76 - 49.62 = 14.14

$$\frac{14.14}{2} = 7.07$$

Ave. elev. of BM_2

$$= \frac{756.12 + 756.14}{2}$$

= 756.13
756.13 - 749.06 = 7.07

① *Diff. in elevation between* TP_1 *and* TP_3:

$$\text{Elev. of } TP_1 = \frac{754.41 + 756.54}{2}$$

Elev. of TP_1 = 755.475

$$\text{Elev. of } TP_3 = \frac{765.64 + 766.86}{2}$$

Elev. of TP_3 = 766.250
Diff. in elevation = 766.250 - 755.475
Diff. in elevation = ***10.775 m.***

② *Elevation of* BM_2:

$$= \frac{756.12 + 756.14}{2}$$

= ***756.13 m.***

③ *Difference in elevation betwwen BM_1 and TP_2:*

$$Elev.\ of\ TP_2 = \frac{764.44 + 765.56}{2}$$

Elev. of TP_2 = 765.00
Diff. in elevation = 765 - 749.06
Diff. in elevation = ***15.94 m.***

Problem 52:

The following shows a tabulated data of leveling notes using rise and fall method.

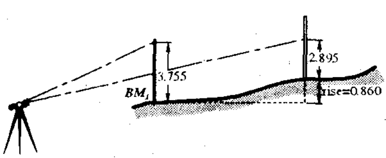
ROD READINGS			Reduced	Rod
BS	IFS	FS	Level	Station
3.755			346.75	BM_1
	2.895			1
	1.742			2
	1.683			3
	2.729			4
		2.057		BM_2

① Compute the rise or fall at station 2.
② Compute the radius level at station 3.
③ Compute the radius level of BM_2.

Solution:

① *Rise or fall at station 2:*

ROD READINGS			
STA.	B.S.	I.F.S.	F.S.
BM_1	3.755		
1		2.895	
2		1.742	
3		1.683	
4		2.729	
BM_2			2.057
	3.755		2.057



STA.	Rise	Fall	Reduce Level
BM_1			346.75
1	+0.860		347.61
2	+1.153		348.763
3	+0.059		348.822
4		- 1.046	347.776
BM_2	+0.672		348.448
	2.744	1.046	

Rise = 3.755 - 2.895
Rise = 0.860 m.
Rise = 2.895 - 1.742
Rise = 1.153 m.
Rise = 1.742 - 1.683
Rise = 0.059 m.
Fall = 1.683 - 2.729
Fall = 1.046 m.
Rise = 2.729 - 2.057
Rise = 0.672 m.
Rise at station 2 = ***1.153 m.***

② *Reduced elevation at station 3:*
Reduced elevation at station 1
= 346.75 + 0.86
= 347.61 m.
Reduced elevation at station 2
= 347.61 + 1.153
= 348.763 m.
Reduced elevation at station 3
= 348.763 + 0.059
= ***348.822 m.***

③ *Reduced Level of BM_2:*
Reduced Level of station 4
= 348.822 - 1.046
= 347.776 m.
Reduced Level of BM_2
= 347.776 + 0.672
= ***348.448 m.***

Arithmetic check:
$\sum BS - \sum FS = 3.755 - 2.057 = 1.698$ (check)
$\sum Rise = 0.860 + 1.153 + 0.059 + 0.672$
$\sum Rise = 2.744$
$\sum Fall = 1.046$
$\sum Rise - \sum Fall = 2.744 - 1.046 = 1.698$ (check)

Problem 53:

A reciprocal leveling is observed across a wide river and the reciprocal level readings were taken between points A and B as follows. With instrument set up near A, the rod readings on A are 2.283 and 2.285 m. The reciprocal level reading on the opposite side of the river at point B are 3.618, 3.619, 3.621 and 3.622 m. With the instrument set up near B, the rod readings on B are 4.478 and 4.476 m. and the rod readings on the opposite side of the river at point A, the rod readings are 3.143, 3.140, 3.146 and 3.144 m.

① Compute the difference in elevation between A and B with the instrument set up near A.
② What is the true difference in elevation between A and B.
③ If the elevation at A is 300 m., what is the elevation of B.

Solution:

① *Difference in elevation between A and B with instrument set up near A:*
With instrument near A:
Mean rod reading on A.

$$A_m = \frac{2.283 + 2.285}{2} = 2.284 \text{ m.}$$

Mean rod reading on B:

$$B_m = \frac{3.618 + 3.619 + 3.621 + 3.622}{4}$$

$B_m = 3.62$ m.

Diff. in elevation between A and B
= 2.284 - 3.62
= ***- 1.336 m.***

② *True diff. in elevation between A and B:*
With instrument near B:
Mean rod reading on A:

$$A_m = \frac{3.143 + 3.140 + 3.146 + 3.144}{4}$$

$A_m = 3.143$

Mean rod reading on B:

$$B_m = \frac{4.478 + 4.476}{2} = 4.477$$

Diff. in elevation between A and B
= 3.143 - 4.477
= - 1.334 m.

True difference in elevation A and B

$$= \frac{(-1.336) + (-1.334)}{2}$$

= ***1.335 m.***

③ *Elevation of B:*
B = 300 - 1.335
B = ***298.665 m.***

Problem 54:

In leveling across a wide river on Pampanga, a reciprocal level readings were taken between points B and C as shown in the tabulation.

Instrument set up near B			Instrument set up near C		
STA obs.	BS	FS	STA obs.	BS	FS
B	2.283		C		2.478
B	2.284		C		2.480
B	2.286		C		2.476
B	2.283		C		2.478
C		1.675	B	3.143	
C		1.674	B	3.140	
C		1.677	B	3.145	
C		1.674	B	3.142	
C		1.677	B	3.143	
C		1.678	B	3.146	

① Compute the difference in elevation between B and C with instrument set up near B.
② Compute the true difference in elevation between B and C.
③ If the elevation of B is 346.50 m., compute the elevation of C.

Solution:

① *Difference in elevation between B and C with instrument set up near B:*

Mean rod reading on B:

$$B_m = \frac{2.283 + 2.284 + 2.286 + 2.283}{4}$$

$B_m = 2.284$ m.

Mean rod reading on C:

$$C_m = \frac{1.675 + 1.674 + 1.677 + 1.674 + 1.677 + 1.678}{6}$$

$C_m = 1.676$ m.

Diff. in elevation between B and C with instrument set up near B

= 2.284 - 1.676

= ***+0.608 m.***

② *True diff. in elevation between B and C:*

Mean rod reading on C:

$$C_m = \frac{2.478 + 2.480 + 2.476 + 2.478}{4}$$

$C_m = 2.478$ m.

Mean rod reading on B:

$$B_m = \frac{3.143 + 3.140 + 3.145 + 3.142 + 3.143 + 3.146}{6}$$

$B_m = 3.143$ m.

Diff. in elevation between B and C

= 3.143 - 2.478

= 0.665 m.

True difference in elevation

$$= \frac{0.608 + 0.665}{2}$$

= ***0.6365 m.***

③ *Elevation of C:*

Elevation of C = 346.50 + 0.6365

Elevation of C = ***347.1365 m.***

Problem 55:

A line of levels 10 km. long was run over soft ground. Starting from BM_1 with elevation 22.5 meters. The elevation of BM_2 was computed to be 17.25 m. It was found out however that the level settles 5 mm between the instant of every backsight reading, the rod settles 2 mm if the backsight and foresight distance have an average 100 m. Find the correct elevation of BM_2.

① Find the error due to settlement of level.
② Determine the error due to settlement of rod.
③ Compute the corrected elevation of BM_2.

Solution:

① *Error due to settlement of level:*

$$\text{No. of set ups} = \frac{10000}{100 + 100}$$

No. of set ups = 50

Error due to settlement of level

= 50(0.005)

= ***0.25 m.***

② *Error due to settlement of rod:*

$$\text{No. of turning points} = \frac{10000}{100 + 100} - 1$$

No. of turning points = 49

Error due to settlement of rod

= 49(0.002)

= ***0.098 m.***

③ *Corrected elevation of BM_2:*

Total error = 0.25 + 0.098

Total error = 0.348 m.

Corrected elevation of BM_2

= 17.25 - 0.348

= ***16.902 m.***

Problem 56:

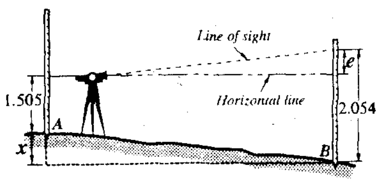
In the two peg-test method of a dumpy level the following observations were taken.

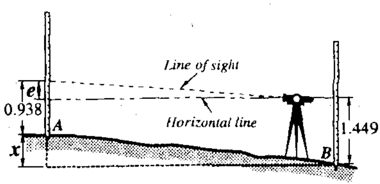
	Instrument set up near A	Instrument set up near B
Rod reading on A	1.505 m.	0.938 m.
Rod reading on B	2.054 m.	1.449 m.

① What is the difference in elevation between A and B?
② If the line of sight is not in adjustment, determine the correct rod reading on A with the instrument still set up at B.
③ Determine the error in the line of sight.

Solution:

① *Diff. in elevation between A and B:*





$1.505 + x = 2.054 - e$

$x + e = 0.549$

$x + 0.938 - e = 1.449$

$x - e = 0.511$

$\underline{x + e = 0.549}$

$2x = 1.06$

$x = \mathbf{0.53\ m.}$ *(diff. in elevation)*

② *Rod reading on A with instrument near B:*

$x + e = 0.549$
$e = 0.549 - 0.53$
$e = 0.019$

Rod reading on A = 0.938 - 0.019
Rod reading on A = ***0.919 m.***

③ *Error in line of sight:*
= ***0.019 m.***

Problem 57:

In a two peg test using model Wild NA2 dumpy level, the following observations were taken

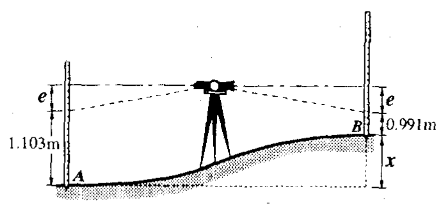
	Instrument at C	Instrument at D
Rod reading on A	1.103	0.568
Rod reading on B	0.991	0.289

Point C is equidistant from A and B and D is 12 m. from A and 72 m. from B.

① What is the true difference in elevation between A and B?
② With the level in the same position at D, to what rod reading on B should the line of sight be adjusted.
③ What is the corresponding rod reading on A for a horizontal line of sight with instrument still at D?

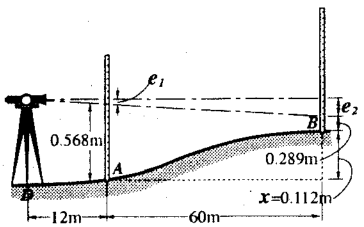
Solution:

① *True diff. in elevation between A and B:*



$1.103 + e = 0.991 + e + x$
$x = \mathbf{0.112\ m.}$

② *Rod reading on B with level at D:*



$0.568 + e_1 = e_2 + 0.289 + 0.112$
$0.568 + e_1 = e_2 + 0.401$
$e_2 - e_1 = 0.167$
$\frac{e_1}{12} = \frac{e_2}{72}$
$e_2 = 6e_1$
$6e_1 - e_1 = 0.167$
$5e_1 = 0.167$
$e_1 = 0.0334$ m.
$e_2 = 6(0.0334)$
$e_2 = 0.2004$

Rod reading on B = 0.289 + 0.2004
Rod reading on B = ***0.4894 m.***

③ *Rod reading on A to have a horizontal line of sight with instrument still at D:*
Rod reading on A = 0.568 + e_1
Rod reading on A = 0.568 + 0.0334
Rod reading on A = ***0.6014 m.***

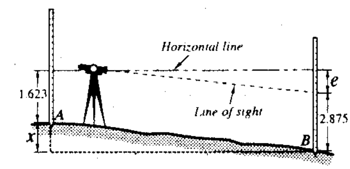
Problem 58:

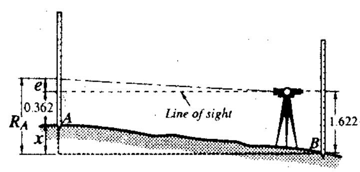
In a topographic survey undertaken by Karman Surv. Corp. before any leveling is conducted, the engineers usually check whether the engineers level is in perfect adjustment. A two peg test is used to check whether the line of sight is in perfect adjustment and the following rod readings are taken, with instrument set up near A, backsight on A is 1.623 m. and foresight reading on B is 2.875 m. with the instrument set up near B, backsight on B is 1.622 m. and a foresight on A is 0.362 m.

① Compute the difference in elevation between A and B.
② What should be the correct rod reaoing on A to give a level line of sight with the instrument still set up at B?
③ What should have been the reading on B with the instrument at A to give a level line of sight?

Solution:

① *Diff. in elevation between A and B:*





$1.623 + x = e + 2.875$
$e + 0.362 + x = 1.622$
$x - e = 1.252$
$x + e = 1.26$
$2x = 2.512$
$x = \mathbf{1.256}$ *(diff. in elev. bet. A and B)*

② *Rod reading on A to give a level line of sight with instrument at B:*
$R_A = e + 0.362$
$x + e = 1.26$
$1.256 + e = 1.26$
$e = 0.004$
$R_A = 0.004 + 0.362$
$R_A = \mathbf{0.366\ m.}$

③ *Rod reading on B with the instrument at A to give a level line of sight:*
$R_B = 2.875 + e$
$R_B = 2.875 + 0.004$
$R_B = \mathbf{2.879\ m.}$

Problem 59:

In the two peg-test of a dumpy level using alternate method, the foll. observations were taken.

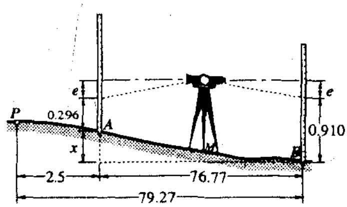
	Instrument set up near M	Instrument set up near P
Rod reading on point A	0.296 m.	1.563 m.
Rod reading on point B	0.910 m.	2.140 m.

Point M is equidistant from both A and B, while P is 2.50 m. away from A along the extension of line AB and 79.27 m. from B.

① Determine the true difference in elevation between A and B.
② Determine the error in the rod reading at B with the instrument still at P.
③ Determine the correct reading on rod B for a horizontal line of sight with the instrument still at P.

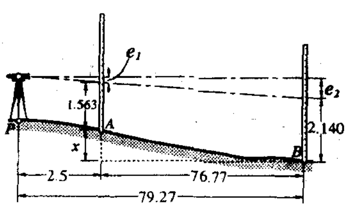
Solution:

① *Difference in elevation between A and B:*



$x + 0.296 + e = 0.910 + e$
$x = 0.910 - 0.296$
$x = \mathbf{0.614\ m.}$

② *Error in rod reading at B with instrument still at P:*



$x + 1.563 + e_1 = e_2 + 2.140$
$0.614 + 1.563 + e_1 = 2.140 + e_2$
$0.037 + e_1 = e_2$
$\frac{e_1}{2.5} = \frac{e_2}{79.27}$
$e_1 = 0.0315\ e_2$
$0.037 + 0.0315\ e_2 = e_2$
$e_2 = \mathbf{0.038\ m.}$

③ *Rod reading on B for a horizontal line of sight with level at P:*

$R_B = 2.140 + e_2$
$R_B = 2.140 + 0.038$
$R_B = \mathbf{2.178\ m.}$

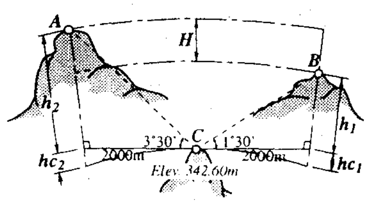
Problem 60:

A trigonometric leveling conducted by Jereza Surveying Company, the two points A and B of a certain rough terrain are each distance 2000 m. from a third point C, from which the measured vertical angles to A is + 3°30' and to B is + 1°30'. Elevation at C is known to be 342.60 m. above sea level. Point C is in between A and B.

① Compute the difference in elevation between A and B considering the effect of the earth's curvature and refraction.
② Compute the difference in elevation between B and C.
③ Compute the elevation of A.

Solution:

① *Diff. in elevation between A and B:*



$\tan 3°30' = \dfrac{h_2}{2000}$
$h_2 = 122.33$ m.
$hc_2 = 0.067\,(2)^2$
$hc_2 = 0.268$ m.
$h_1 = 2000 \tan 1°30'$
$h_1 = 52.37$
$hc_1 = 0.067\,(2)^2$
$hc_1 = 0.268$ m.
$H + h_1 + hc_1 = h_2 + hc_2$
$H + 52.37 + 0.268 = 122.33 + 0.268$
$H = \mathbf{69.96\ m.}$

② *Difference in elevation between B and C:*
$= h_1 + hc_1$
$= 52.37 + 0.268$
$= \mathbf{52.638\ m.}$

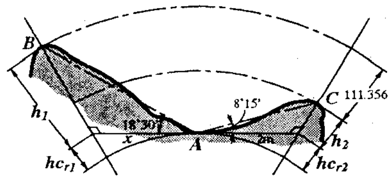
③ *Elev. of A:*
Elev. A = 342.60 + 122.33 + 0.268
Elev. A = **465.20 m.**

Problem 61:

Considering the effects of curvature and refraction, the difference in elevation of points B and C is found out to be 111.356 m. From point A in between B and C is the angle of elevation of B and C are 18°30' respectively.

① If C is 2000 m. from A, how far is B from A?
② If the elevation of A is equal to 200 m., find the elevation of B.
③ Find also the elevation of C.

Solution:
① *Distance of B from A:*



$hc_{r_1} = 0.067\,x^2$
$hc_{r_2} = 0.067\,(2)^2$
$hc_{r_2} = 0.268$ m.
$h_1 = x \tan 18°30'$
$h_1 = 0.3346x$ km.
$h_1 = 334.6x$ m.
$h_2 = 2000 \tan 8°15'$
$h_2 = 289.99$ m.
$h_1 + hc_{r_1} = 111.356 + h_2 + hc_{r_2}$
$334.6x + 0.067x^2$
$= 111.356 + 289.99 + 0.268$
$x^2 + 4994x - 5994.24 = 0$
$x = 1.2$ km.
$x = \mathbf{1200\ m.}$

② *Elevation of B:*
Elev. of B = Elev. A + h_1 + hc_{r_1}
Elev. of B = 200 + 334.6(1.200) + 0.067(1.2)^2
Elev. of B = **601.62 m.**

③ *Elevation of C:*
Elev. C = Elev. B - 111.356
Elev. C = 601.62 - 111.356
Elev. C = **490.264 m.**

Problem 62:

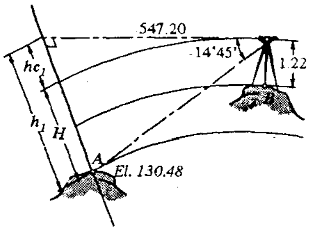
A is a point having an elevation of 130.48 m. above datum, and B and C are points of unknown elevation, B is in between A and C. By means of an instrument set 1.22 m. above B, vertical angles are observed, that to A being 14°45' and that to C being + 8°32'. The horizontal distance AB is 547.20 and the horizontal distance BC is 923.25 m. Making due allowance for earth's curvature and atmospheric refraction.

① Compute the difference in elevation between A and B.
② Determine the difference in elevation between B and C.
③ Determine the elevation of C.

LEVELING

Solution:

① *Diff. in elevation of A and B:*



$\tan 14°45' = \dfrac{h_1}{547.20}$

$h_1 = 144.07$ m.

$hc_1 = 0.067\ K_1^2$

$hc_1 = 0.067(0.5472)^2$

$hc_1 = 0.02$

$H = 144.07 - 0.02$

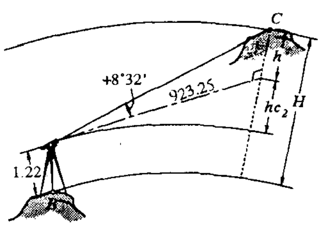
$H = 144.05$ m.

Diff. in elevation of A and B

$= 144.05 - 1.22$

$=$ ***142.83 m.***

② *Diff. in elevation between B and C:*



$h = 923.25 \tan 8°32'$

$h = 138.53$

$hc_2 = 0.067(0.92325)^2$

$hc_2 = 0.057$

$H = 138.53 + 0.057 + 1.12$

$H =$ ***139.81***

③ *Elev. of C:*

Elev. of C $= 130.48 + 142.83 + 139.81$

Elev. of C $=$ ***413.12 m.***

Problem 63:

A man's eyes 1.75 m. above sea level can barely see the top of a lighthouse which is at a certain distance away from a man.

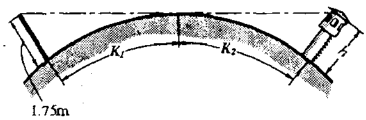
① What is the elevation of the top of the lighthouse above sea level if the lighthouse is 20 km. away from the man.

② How far is the lighthouse from the man in meters if the top of the lighthouse is 14.86 m. above sea level.

③ What is the height of the tower at a distance 20 km. away from the man that will just be visible without the line of sight approaching nearer than 1.75 m. to the water.

Solution:

① *Elevation of the top of the lighthouse:*



$1.75 = 0.067\ K_1^2$

$K_1 = 5.11$ km.

$h = 0.067\ K_2^2$

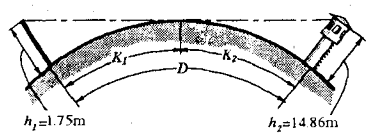
$K_2 = 20 - 5.11$

$K_2 = 14.89$ km.

$h = 0.067\ (14.89)^2$

$h =$ ***14.86 m.*** *above sea level*

② *Distance from lighthouse from the man:*



$h_1 = 0.067\ K_1^2$

$1.75 = 0.067\ K_1^2$

$K_1 = 5.11$

$h_2 = 0.067\ K_2^2$

$14.86 = 0.067\ K_2^2$

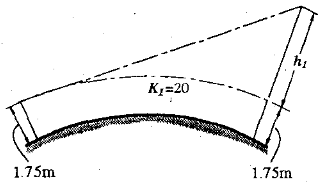
$K_2 = 14.89$ km.

$D = K_1 + K_2$

$D = 5.11 + 14.89$

$D =$ ***20 km.***

③ *Height of tower at a distance of 20 km. away from the man:*



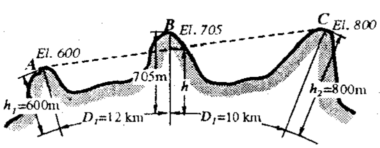
$h_1 = 0.067\,(20)^2$
$h_1 = 26.8$ m.
$H = h_1 + 1.75$
$H = 26.8 + 1.75$
$H = \mathbf{28.55}$ ***meters***

Problem 64:

Two hills A and C have elevations of 600 m. and 800 m. respectively. In between A and C is another hill B which has an elevation of 705 m. and is located at 12 km. from A and 10 km. from C.

① Determine the clearance or obstruction of the line of sight at hill B if the observer is at A so that C will be visible from A.
② If C is not visible from A, what height of tower must be constructed at C so that it could be visible from A with the line of sight having a clearance of 2 m. above hill B.
③ What height of equal towers at A and C must be constructed in order that A, B and C will be intervisible.

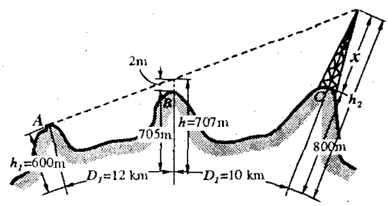
Solution:
① *Obstruction of the line of sight at hill B:*



$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$
$$h = 800 + \frac{10\,(600 - 800)}{12 + 10} - 0.067\,(12)(10)$$
$$h = 701.05$$

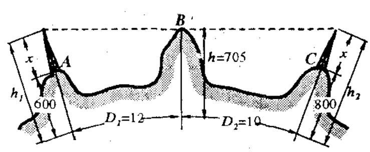
Obstruction = 705 - 701.05
Obstruction = ***3.95 m.***

② *Height of tower at C so that it could be visible from A with a 2 m. clearance above hill B:*



$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$
$$707 = (800 + x) + \frac{10\,[600 - (800 + x)]}{12 + 10} - 0.067\,(12)(10)$$
$$707 = 800 + x - 90.91 - 0.4545x - 8.04$$
$$x = \mathbf{10.91\ m.}$$

③ *Height of equal towers at A and C so that it will be intervisible:*



$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$
$$705 = (800 + x) + \frac{10\,[(600 + x) - (800 + x)]}{12 + 10} - 0.067\,(12)(10)$$
$$705 = 800 + x - 90.91 - 8.04$$
$$x = \mathbf{3.95\ m.}$$

65. CE Board Nov. 2004

Given:

Station	Elevation (m)	Distance (km)
Alpha	680 m.	Alpha to Bravo=12km
Bravo	645 m.	Bravo to Charlie=15km
Charlie	620 m.	

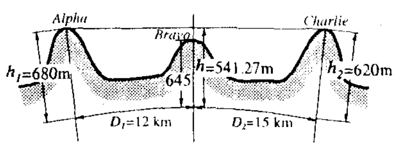
① Compute the elevation of the line of sight at station Bravo with the instrument placed at station Alpha such that station Charlie would be visible from station Alpha considering the effect of curvature and refraction correction.

② Assuming that station Bravo will obstruct the line of sight from station Alpha while observing station Charlie and a 4 m. tower is constructed on top of station Bravo. Compute the height of equal towers at station Alpha and station Charlie in order that both three stations as observed from station Alpha will still be intervisible.

③ Without constructing any tower at station Bravo, what height of tower must be constructed at station Charlie so that both station Bravo and Charlie would be visible from station Alpha.

Solution:

① *Elevation of line of sight at station Bravo:*

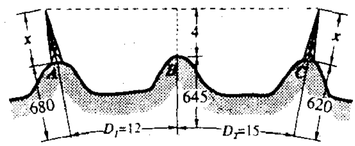


$$h = h_2 + \frac{D_2}{D_1 + D_2}(h_1 - h_2) - 0.067\, D_1 D_2$$

$$h = 620 + \frac{15}{12 + 15}(680 - 620) - 0.067\,(12)(15)$$

$$h = \boldsymbol{641.27\ m.}$$

② *Equal height of towers at Alpha and Charlie:*



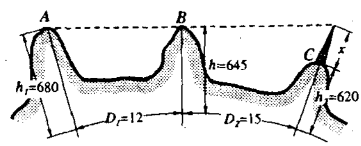
$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\, D_1 D_2$$

$$649 = (620 + x) + \frac{15\,[(680 + x) - (620 + x)]}{12 + 15} - 0.067\,(12)(15)$$

$$649 = 620 + x + 0.556(60) - 12.06$$

$$x = \boldsymbol{7.7\ m.}$$

③ *Height of tower at station Charlie:*



$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\, D_1 D_2$$

$$645 = (620 + x) + \frac{15}{12 + 15}[(680) - (620 + x)] - 0.067\,(12)(15)$$

$$645 = 620 + x + 0.556(60 - x) - 12.06$$

$$37.06 = x + 33.36 - 0.556x$$

$$x = \boldsymbol{8.33\ m.}$$

Problem 66:

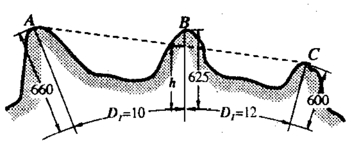
Three hills A, B and C has elevations of 660 m., 625 m. and 600 m. respectively. B is in between A and C and is 10 km. from A and 12 km. from C.

① Considering the effect of curvature and refraction correction, what is the clearance or obstruction of the line of sight at B considering that C is visible from A.

② If a 5 m. tower is erected on top of B, what would be the height of equal towers to be erected at A and C in order that A, B and C will be intervisible.
③ What should be the height of tower to be erected at C so that B and C will be intervisible from A

Solution:

① *Clearance or obstruction of the line:*



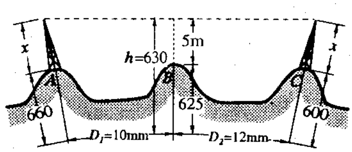
$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$

$$h = 600 + \frac{12\,(660 - 600)}{10 + 12} - 0.067\,(10)(12)$$

$$h = 624.69 < 625$$

Obstruction = 625 - 624.69
Obstruction = ***0.31 m.***

② *Equal height of towers at A and C:*



$$h = 625 + 5$$

$$h = 630$$

$$h_1 = 660 + x$$

$$h_2 = 600 + x$$

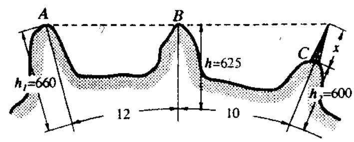
$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$

$$630 = 600 + x + \frac{12\,[(660 + x) - (600 + x)]}{(10 + 12)} - 0.067\,(10)(12)$$

$$30 = x + \frac{12(60)}{22} - 0.067\,(10)(12)$$

x = ***5.31 m.***

③ *Height of tower at C:*



$$h_2 = 600 + x$$

$$h = 625$$

$$h_1 = 660$$

$$h = h_2 + \frac{D_2\,(h_1 - h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$

$$625 = 600 + x + \frac{12\,[(660) - (600 + x)]}{10 + 12} - 0.067\,(10)912)$$

$$25 = x + \frac{12\,(60 - x)}{22} - 8.04$$

$$33.04 = x + 32.73 - 0.545\,x$$

x = ***0.68 m.***

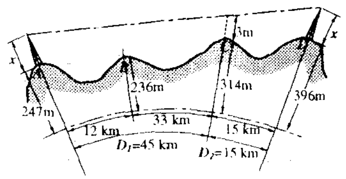
Problem 67:

Four hills A, B, C and D are in straight line. The elevations are A = 247 m, B = 236 m, C = 314 m and D = 396 m. respectively. The distances of B, C and D from A are 12 km, 45 km and 60 km. respectively. Considering the effect of curvature and refraction of the earth.

① Compute the height of equal towers on A and D to sight over B and C with a 3 m. clearance.
② Compute the elevation of the line of sight at B with the installation of the equal heights of tower at A and D.
③ Compute the height of tower at A with a clearance of 3 m. at C so that D will be visible from A, if the height of tower at D is 2 m.

Solution:

① *Equal heights of tower at A and D:*



Considering hilss A, C and D

$h_1 = 247 + x$

$h_2 = 396 + x$

$h = 314 + 3$

$h = 317$

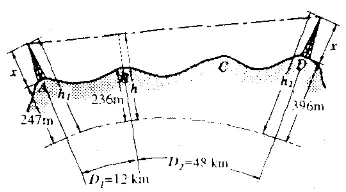
$$h = h_2 + \frac{D_2\,(h_1 + h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$

$$317 = 396 + x + \frac{15\,[(247 + x) - (396 + x)]}{45 + 15} - 0.067\,(45)(15)$$

$317 = 396 + x - 37.25 - 45.225$

$x =$ ***3.475 m.***

② *Elevation of line of sight at B:*



$h_1 = 247 + 3.475$

$h_1 = 250.475$

$h_2 = 396 + 3.475$

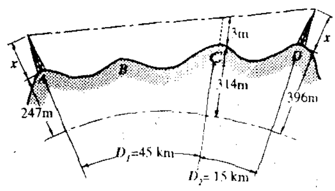
$h_2 = 399.475$

$$h = h_2 + \frac{D_2\,(h_1 + h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$

$$h = 399.475 + \frac{48\,(250.475 - 399.475)}{12 + 48} - 0.067\,(12)(48)$$

$h =$ ***241.633 m.*** > 236 m.

③ *Height of tower at A:*



$h_1 = 247 + x$

$h_2 = 396 + 2$

$h_2 = 398$

$h = 314 + 3$

$h = 317$

$$h = h_2 + \frac{D_2\,(h_1 + h_2)}{D_1 + D_2} - 0.067\,D_1D_2$$

$$317 = 398 + \frac{15\,[247 + x - 398]}{45 + 15} - 0.067\,(45)(15)$$

$317 = 378 + 0.25x - 37.75 - 45.225$

$x =$ ***7.9 m.***

Problem 68:

Considering curvature and refraction correction of the earth surface.

① The F.S. reading on the rod at point B is 1.86 m. The correction for curvature only is 0.048 m. If H.I. = 238.17 m. and the corrected elevation of B is 236.35 m., what is the correction for refraction only?

② At point B, the F.S. reading is 2.23 m. The corrected elevation of B is 144.86 m., considering refraction and curvature. If H.I. = 147.063 m. and the correction for refraction is 0.005, what is the correction for curvature?

③ Considering curvature and refraction, the corrected elevation of point C is 311.85 m. The F.S. reading on the rod at C is 2.16 m. The correction for curvature is 0.046 while that for refraction is 0.004. Determine H.I.

Solution:

① *Correction for curvature only:*
Corrected F.S. = 238.17 - 236.35
Corrected F.S. = 1.82 m.
Error in F.S. reading = 1.86 - 1.82
Error in F.S. reading = 0.04
Curvature and refraction correction = 0.04
Curvature and refraction = curvature - refraction
0.04 = 0.048 - x
x = ***0.008*** *refraction correction*

② *Curvature correction:*
F.S. = 147.063 - 144.86
F.S. = 2.203
Curvature and refraction correction
= 2.23 - 2.203
= 0.027
Curvature and refraction = Curvature - refraction correction
0.027 = *Curvature* - 0.005
Curvature = ***0.032 m.***

③ *Value of H.I.*
Curvature and refraction correction
= 0.046 - 0.004
= 0.042
Corrected F.S. = 2.16 - 0.042
Corrected F.S. = 2.118 m.
H.I. = Elev. + Corrected F.S.
H.I. = 311.85 + 2.118
H.I. = ***313.968 m.***

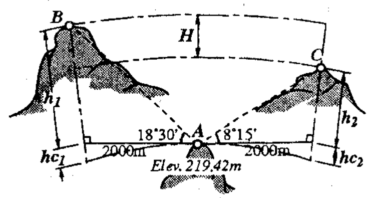
Problem 69:

From point A in between B and C, the angles of elevation of B and C are 18°30' and 8°15' respectively. Point C is 2000 m. from A and B is 1200 m. from A. Elevation of A is 219.42 m. above sea level.

① Compute the difference in elevation between B and C, considering the effect of the earths curvature and refraction.
② Compute the difference in elevation between A and C.
③ Compute the elevation of B.

Solution:

① *Diff. in elevation between B and C:*



$hc_1 = 0.067\ (1.2)^2$
$hc_1 = 0.09648$ m.
$hc_2 = 0.067\ (2)^2$
$hc_2 = 0.268$
$h_1 = 1200 \tan 18°30'$
$h_1 = 401.51$ m.
$h_2 = 2000 \tan 8°15'$
$h_2 = 289.99$ m.
$H = (h_1 + hc_1) - (h_2 - hc_2)$
$H = (401.51 + 0.09648) - (289.99 + 0.268)$
H = ***111.348 m.***

② *Diff. in elev. between A and C:*
Diff. in elev. = $h_2 + hc_2$
Diff. in elev. = 289.99 + 0.268
Diff. in elev. = ***290.258 m.***

③ *Elevation of B:*
Elev. B = 219.42 + 401.51 + 0.09648
Elev. B = ***621.026 m.***

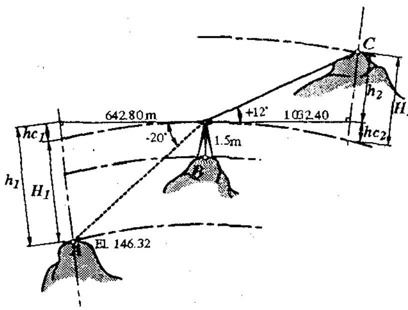
Problem 70:

A transit is set up at point B which is between A and B. The vertical angle observed towards A is known to be - 20° and that of C is +12°. The horizontal distance between A and B is 642.80 m. and that of B and C is 1032.40 m. The height of instrument is 1.5 m. above B with A having an elevation of 146.32 m. Considering the effect of curvature and refraction correction.

① Compute the difference in elevation between A and B.
② Compute the difference in elevation between A and C.
③ Compute the elevation of B.

Solution:

① *Diff. in elevation between A and B;*



$hc_1 = 0.067\,(0.64280)^2$
$hc_1 = 0.028$
$\tan 20^\circ = \dfrac{h_1}{642.80}$
$h_1 = 233.96$ m.
$H_1 = 233.96$ m. $- 0.028$
$H_1 = 233.932$

Diff. in elevation between A and B
$= 233.932 - 1.55$
$=$ ***232.382 m.***

② *Diff. in elevation between A and C:*
$hc_2 = 0.067\,(1.0324)^2$
$hc_2 = 0.071$
$h_2 = 1032.4 \tan 12^\circ$
$h_2 = 219.44$ m.
$H_2 = 219.44 + 0.071$
$H_2 = 219.511$ m.

Diff. in elevation between A and C
$= H_1 + H_2$
$= 233.932 + 219.511$
$=$ ***453.443 m.***

③ *Elev. of B:*
Elev. of B $= 146.32 + 233.932 - 1.55$
Elev. of B $=$ ***378.702 m.***

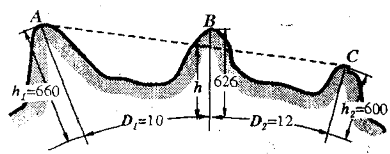
Problem 71:

Mirador hill with an elevation of 626 m. is on a line between Aurora hill whose elevation is 660 m. and Cathedral hill having an elevation of 600 m. Distance of Mirador hill from Aurora hill is 10 km and distance of Mirador hill from Cathedral hill is 12 km. Considering curvature and refraction correction.

① Compute the obstruction of the line of sight at Mirador hill when observing Cathedral hill from Aurora hill.
② What would be the height of equal towers to e erected at Aurora hill and Cathedral hill so that Cathedral hill, Aurora hill and Mirador hill will be intervisible with a 4 m. tower erected at the top of Mirador hill?
③ If no tower will be erected at Aurora hill and Mirador hill, what would be the height of tower to be erected at Cathedral hill so that Mirador and Cathedral hill will be intervisible from Aurora hill.

Solution:

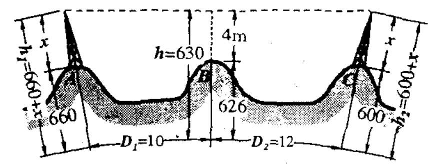
① *Obstruction of line of sight at Mirador hill:*



$$h = h_2 + \frac{(D_2)(h_1 - h_2) - 0.067\,D_1D_2}{D_1 + D_2}$$
$$h = 600 + \frac{12(660 - 600) - 0.067\,(10)(12)}{10 + 12}$$
$h = 624.69$ m.

Obstruction $= 626 - 624.69$
Obstruction $=$ ***1.31 m.***

② *Equal heights of towers at Aurora and Cathedral hills:*

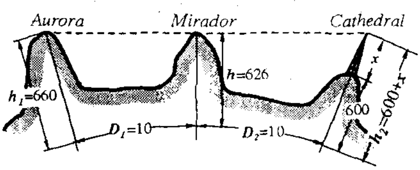


$$h = h_2 + \frac{D_2\,(h_1 - h_2) - 0.067\,D_1 D_2}{D_1 + D_2}$$

$$630 = 600 + x + \frac{12\,[(660 + x) - (600 + x)] - 0.067\,(10)(12)}{10 + 12}$$

$x = \mathbf{5.31\ m.}$

③ *Height of tower at Cathedral hill:*



$$h = h_2 + \frac{D_2\,(h_1 - h_2) - 0.067\,D_1 D_2}{D_1 + D_2}$$

$$626 = 600 + x + \frac{12\,[660 - (600 = x)] - 0.067(10)(12)}{10 + 12}$$

$$34.04 = x + \frac{12\,(60 - x)}{22}$$

$748.88 = 22x + 720 - 12x$

$10x = 28.88$

$x = \mathbf{2.89\ m.}$

Problem 72:

A line of levels is run from BM_1 to BM_2 which is 12 km long. Elevation of BM_1 was found out to be 100 m. and that of BM_2 is 125.382 m. Backsight and foresight distances were 150 m. and 100 m. respectively.

① Determine the corrected elevation of BM_2 considering the effect of curvature and refraction correction.

② If during the leveling process the line of sight is inclined downward by 0.004 m. in a distance of 10 m, what would be the corrected elevation of BM_2?

③ If the average backsight reading is 3.4 m. and every time it is taken, the rod is inclined to the side from the vertical by 4°, what should be the corrected elevation of BM_2?

Solution:

① *Corrected elevation of BM_2 considering curvature and refraction correction.*

$hc_1 = 0.067\,(0.15)^2$

$hc_1 = 0.00151$

$hc_2 = 0.067\,(0.100)^2$

$hc_2 = 0.00067$

Error per set up = 0.00151 - 0.00067

Error per set up = 0.00084

$$\text{No. of set ups} = \frac{12000}{150 + 100} = 48$$

Total error = 48(0.00084)

Total error = 0.04032

Corrected elevation of BM_2

= 125.382 - 0.04032

= **125.34168 m.**

② *Corrected elevation of BM_2 if the line of sight is inclined downward by 0.004 m. every 10 m:*

$$\frac{h_1}{150} = \frac{0.004}{10}$$

$h_1 = 0.06$

$$\frac{h_2}{100} = \frac{0.004}{10}$$

$h_2 = 0.04$

Error per set up = 0.06 - 0.04
Error per set up = 0.02
Total error = 0.02(48)
Total error = 0.96 m.
Corrected elev. of BM_2
= 125.382 + 0.96
= ***126.342 m.***

③ *Corrected elev. of* BM_2 *if the rod is inclined by 4° from the vertical:*

Error in reading per set up
= 3.4 - 3.4 Cos 4°
= 0.0083 m.

Total error = 48(0.0083)
Total error = 0.3984 m.
Corrected elev. of BM_2
= 125.382 - 0.3984
= ***124.9836 m.***

Problem 73:

A line of levels 9.36 km is run to check the elevation of BM_2 which has been found to be 31.388 meters, with BM_1 of elevation at sea level (reference datum). backsight and foresight distances are consistently 110 m. and 70 m. respectively.

① Determine the corrected elevation of BM_2 considering the effect of curvature and refraction correction.
② if the level used is out of adjustment so that when the bubble was centered the line of sight was inclined 0.003 m. upward in a distance of 20 m. Determine the corrected elevation of BM_2.
③ If at every turning points the rod settles about 0.004 m., determine the corrected elevation of BM_2.

Solution:

① *Corrected elevation of* BM_2 *due to curvature and refraction correction:*

$h_1 = 0.067\,(0.110)^2$
$h_1 = 0.0008107$
$h_2 = 0.067\,(0.070)^2$
$h_2 = 0.0003283$

Error per set up = 0.0008107 - 0.0003283
Error per set up = 0.0004824 m.

$\text{No. of set ups} = \dfrac{9360}{110 + 70} = 52$

Total error = 52 (0.0004824)
Total error = 0.0251 m.
Corrected elevation of BM_2
= 31.388 - 0.0251
= ***31.3629 m.***

② *Corrected elev. of* BM_2 *due to line of sight inclined upward by 0.003 m. every 25 m:*

Diff. in distance per set up = 110 - 70
Diff. in distance per set up = 40 m.

$\dfrac{x}{40} = \dfrac{0.003}{20}$

$x = 0.006$ m.

$\text{No. of set ups} = \dfrac{9360}{110 + 70} = 52$

Total error = 52(0.006)
Total error = 0.312 m.
Corrected elevation of BM_2
= 31.388 - 0.312
= ***31.076 m.***

③ *Corrected elevation of* BM_2 *due to rod settlement:*

$\text{No. of turning points} = \dfrac{9360}{110 + 70} - 1 = 51$

Total error = 0.004 (51)
Total error = 0.204 m.
Corrected elevation of BM_2
= 31.388 - 0.204
= ***31.184 m.***

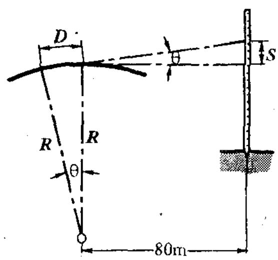
Problem 74:

Using an engineer's level, the reading on a rod 80 m. away was observed to be 2.81 m. The bubble was leveled thru 5 spaces on the level tube and the rod reading increased to 2.874 m.

① Determine the angle that the bubble on the tube was deviated due to an increase in the rod reading by moving the telescope upward in seconds of arc.
② Determine the angular value of one space of the tube, in seconds of arc.
③ Determine the radius of curvature of the level tube if one space on the tube is 0.60 mm long.

Solution:

① *Angle the bubble on the tube was deviated due to an increase in the rod reading:*



$S = 2.874 - 2.81 = 0.064$

$\tan\theta = \dfrac{0.064}{80}$

$\theta''\,(0.000005) = \dfrac{0.064}{80}$

$\theta'' = \mathbf{160''}$

② *Angular value of one space:*

$\phi = \dfrac{160}{5} = \mathbf{32''}$

③ *Radius of curvature:*

$\dfrac{D}{R} = \dfrac{S}{L}$

$D = 0.6(5)$

$D = 3 \text{ mm} = 0.003 \text{ m.}$

$\dfrac{0.003}{R} = \dfrac{0.064}{80}$

$R = \mathbf{3.75\ m.}$

Problem 75:

From the given data of a differential leveling shown in the tabulation.

Station	Distance (km)	Observed Elevation
BM_1	0	100.00 m.
BM_2	4	121.42 m.
BM_3	6	131.64 m.
BM_1	10	100.15 m.

① Compute the correction to be applied to the elevation of BM_2.
② Compute the corrected elevation of BM_2.
③ Compute the corrected elevation of BM_3.

Solution:

① *Correction to be applied to BM_2:*

Station	Distance (km)	Observed Elevation
BM_1	0	100.00 m.
BM_2	4	121.42 m.
BM_3	6	131.64 m.
BM_1	10	100.15 m.

Error of closure = 100.15 - 100

Error of closure = 0.15 m.

$\dfrac{C_1}{0.15} = \dfrac{4}{10}$

$C_1 = \mathbf{0.06}$ *Correction to be applied to BM_2*

② *Corrected elevation of BM_2:*

Corrected elevation = 121.42 - 0.06

Corrected elevation = **121.36 m.**

③ *Corrected elevation of BM_3:*

$\dfrac{\text{Correction}}{0.15} = \dfrac{6}{10}$

Correction = 0.09

Corrected elev. = 131.64 - 0.09

Corrected elev. = **131.55 m.**

Barometric Leveling

Problem 76:

The elevation of the base at station A is 1584 m. above sea level. The barometric reading at station A was 65.53 cm. of Hg. at the instant when the barometric reading at station B which is higher than A was 69.39 cm. of Hg. The temp. at the time of observation at A was 8°C while that at B was 22°C. Time of observation on both station is 10:20 A.M.

Temperature	Correction Factor
12.5°C	+ 0.0048
15.0°C	+ 0.0253
17.5°C	+ 0.0351

① Compute the uncorrected difference in elevation between A and B.
② Determine the correction for the difference in elevation.
③ Compute the adjusted elevation of station B.

Solution:

① *Uncorrected difference in elevation between A and B:*

$$z = 19122 \log \frac{76}{h_1} - 19122 \log \frac{76}{h_2}$$

$$z = 19122 \log \frac{76}{65.53} - 19122 \log \frac{76}{69.39}$$

$$z = 1230.95 - 755.64$$

$$z = \mathbf{475.31\ m.}$$

② *Correction for the difference in elevation:*

$$\text{Mean temp.} = \frac{8 + 22}{2}$$

Mean temp. = 15°C

Correction for diff. in elevation

= 0.0253 (475.31)

= ***12.03 m.***

③ *Adjusted elevation of B:*
Corrected diff. in elev. = 475.31 + 12.03
Corrected diff. in elev. = 487.34 m.
Adjusted elev. of B = 1584 + 487.34
Adjusted elev. of B = ***2071.34 m.***

Problem 77:

Given below are the corresponding barometric readings at two given places. The elevation of Mount Mayon is 1200 m. above sea level. Mount Mayon is lower than Mount Apo.

Station	Time	Barometric Reading (cm of Hg)	Air Temp.
Mt. Apo	9:25	74.73	8.3°C
Mt. Mayon	10:30	68.96	1.1°C
Mt. Apo	10:57	74.57	6.1°C

① Compute the barometric reading at Mount Apo at 10:30 A.M.
② Compute the difference in elevation between Mount Apo and Mount Mayon.
③ What is the elevation of Mount Apo above sea level.

Solution:

① *Barometric reading at Mount Apo at 10:30 AM:*

Station	Time	Barometric Reading (cm of Hg)	Air Temp.
Mt. Apo	9:25	74.73	8.3°C
Mt. Mayon	10:30	68.96	1.1°C
Mt. Apo	10:57	74.57	6.1°C

9:25 — 74.73
10:30 — 65 min. — x
10:57 — 74.57
92 min — 0.16

Diff. in time = 10:57 - 9:25
Diff. in time = 1:32 hrs. = 92 min.
Diff. in time = 10:30 - 9:25
Diff. in time = 1:05 hrs. = 65 min.
Diff. in reading = 74.73 - 74.57
Diff. in reading = 0.16 cm.

By ratio and proportion

$$\frac{65}{92} = \frac{x}{0.16}$$

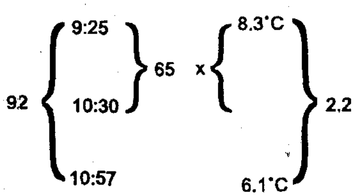
$x = 0.11$ cm.

Barometric reading of Mount Apo at 10:30
= 74.73 - 0.11
= ***74.62 cm.***

② *Difference in elevation between Mount Apo and Mount Mayon:*

$$H = 18336.6\,(\log h_1 - \log h_2)\left[1 + \frac{T_1 + T_2}{500}\right]$$

h_1 = 74.62 cm. (barometric reading of Mount Apo at 10:30)
h_2 = 68.96 cm. (barometric reading of Mount Mayon at 10:30)
T_1 = temp. at Mount Apo at 10:30
T_2 = temp. at Mount Mayon at 10:30



$$\frac{92}{65} = \frac{2.2}{x}$$

$x = 1.6$°C
$T_1 = 8.3 - 1.6 = 6.7$°C
$T_2 = 1.1$°C

$$H = 18336.6\,(\log 74.62 - \log 68.96)\left[1 + \frac{(6.7 + 1.1)}{500}\right]$$

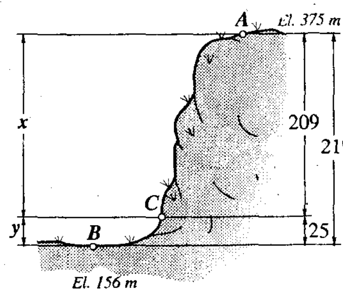
H = ***637.97 m.***

③ *Elevation of Mount Apo:*
Elev. = 1200 + 637.97
Elev. = ***1837.97 m.***

Problem 78:

The elevation of the upper base A is 375 m. while that of the lower base at B, the elevation is 156 m. At a given instant three altimeter readings indicate that the difference in elevation of an intermediate point C from the upper base A is 209 m. and the difference in elevation from the lower base B to point C is 25 m. Find the true elevation of point C in between A and B.

Solution:



x = *true difference in elevation between A and C*

By ratio and proportion:

$$\frac{x}{209} = \frac{219}{234}$$

$x = 195.60$

Elevation of C = 375 - 195.60
Elevation of C = 179.40 m.
$y = 219 - 195.60$
$y = 23.40$ m.

Elevation of C = 156 + 23.40
Elevation of C = ***179.40 m.***

COMPASS SURVEYING

Surveyor's Compass - an instrument for determining the horizontal direction of a line with reference to the direction of the magnetic needle.

Essential features of compass:

1. ***Compass box*** = with a circle graduated from 0° to 90° in both directions from the N. and S. points and usually having the E and W points interchanged.
2. ***Sight Vanes*** - which defines the line of sight in the direction of the SN points of the compass box.
3. ***Magnetic needle*** - has the property of pointing a fixed direction namely, the magnetic meridian.

Kinds of compass:

1. ***Pocket compass*** - which is generally held in the hand when bearings are observed; used on reconnaissance or other rough surveys.
2. ***Surveyor's compass*** - which is mounted usually on a light tripod, or sometimes on a Jacob's staff (a point stick about 1.5 m. long).
3. ***Transit compass*** - a compass box similar to the surveyor's compass, mounted on the upper or vernier plate of the engineer's transit.

Sources of errors in compass work:

1. Needle bent - if the needle is not perfectly straight, a constant error is introduced in all observed bearings. The needle can be corrected by using pliers.

2. Pivot bent - if the point of the pivot supporting the needle is not at the center of the graduated circle, there is introduced a variable systematic error, the magnitude of which depends on the direction in which the compass is sighted. The instrument can be corrected by bending the pivot until the end readings of the needle are 180° apart for any direction of pointing.

3. Plane of sight not vertical or graduated circle not horizontal.

4. Sluggish

5. Reading the needle

6. Magnetic variations

Advantages of a compass:

1. Compass is light and portable and it requires less time for setting up, sighting and reading.

2. An error in the direction of one line does not necessarily affect other lines of the survey.

3. The compass is especially adopted to running straight lines through woods and other places where obstacles are likely to interfere with the line of sight.

COMPASS SURVEYING

Disadvantages of a compass:

1. The compass reading is not very accurate.
2. The needle is unreliable especially with the presence of local attractions, such as electric wires, metals, magnets that may render it practically useless.

Magnetic declination - the angle that a magnetic meridian makes with the true meridian.

Magnetic dip - the vertical angle which the magnetic needle makes with the horizontal due to uneven magnetic attraction from the magnetic poles.

Isogonic lines - an imaginary lines passing through places having the same magnetic declination.

Isoclinic lines - an imaginary line passing through points having the same magnetic dip.

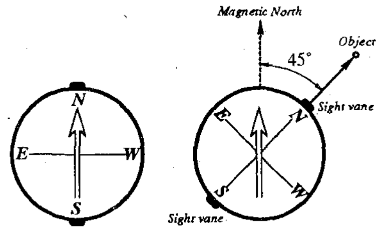
Agonic lines - imaginary line passing through places having a zero declination.

Adjustment of a closed compass traverse:

When the compass traverse forms a closed figure, the interior angle at each station is computed from the observed bearings at that particular point, the computed value which is free from local attraction. The sum of the interior angles of a closed polygon must be equal to (n - 2) 180° in which n is the number of sides of the polygon. Since the error of observing a bearing is accidental, it is assumed to be distributed equally at each interior angle. The bearings are then adjusted from a line whose observed bearing is to be correct using the adjusted values of each interior angle.

Why is the East and West points of a compass interchanged?

From the figure shows a compass having a NS and EW calibration. In using a compass, always sight the object with the north end of the compass and the compass needle when pivoted and brought to rest gives the magnetic bearing.



Let us say an object on the right side is observed, sight this object with the north end of the compass. The needle at this instant will point steadily on the magnetic north, so a reading could now be obtained as shown as NE.

Problem 79:

① The observed compass bearing of a line in 1981 was S. 37°30' E. and the magnetic declination of the place then was known to be 3°10' W. It has also discovered that during the observation local attraction of the place at that moment of 5°E existed. Find the true azimuth of the line.

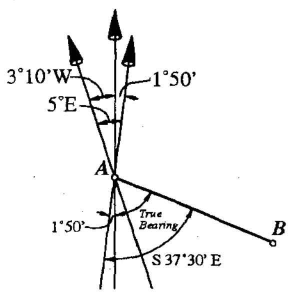
② The bearing of a line from A to B was measured as S. 16°30' W. It was found that there was local attraction at both A and B and therefore a forward and a backward bearing were taken between A and a point C at which there was no local attraction. If the bearing of AC was S.30°10' E. and that of CA was N. 28°20' W., what is the corrected bearing of AB?

COMPASS SURVEYING

③ In a particular year, the magnetic declination was 1°10' E and the magnetic bearing of line DE was N. 16°30' W. If the secular variation per year is 3' E., determine the magnetic bearing of line DE 5 years later?

Solution:

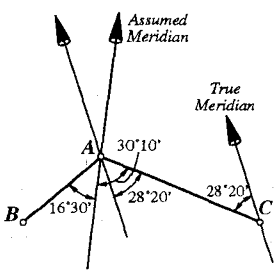
① *True azimuth of the line:*



True bearing = S 37°30' E - 1°50'
True bearing = S 35°40' E

True azimuth = ***324°20'***

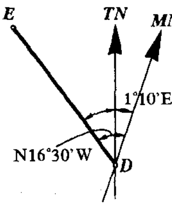
② *Corrected bearing of AB:*

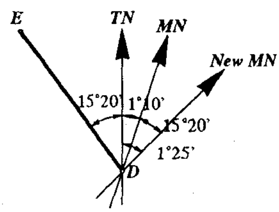


Angle at A = 16°30' + 30°10'
Angle at A = 46°40'

Bearing of AB = 46°40' - 28°20'
Bearing of AB = ***S 18°20' W***

③ *Magnetic bearing of line DE:*





True bearing DE = 16°30' - 1°10'
True bearing DE = N 15°20' W

Magnetic bearing of DE
= 15°20' + 1°25'
= ***N 16°45' W***

Problem 80:

A field is in the form of a regular pentagon. The direction of the bounding sides were surveyed with an assumed meridian 5° to the right of the true north and south meridian. As surveyed with an assumed meridian, the bearing of one side AB is N. 33°20' W.

① Compute the true bearing of line BC.
② Compute the true azimuth of line CD.
③ Compute the true bearing of line AE.

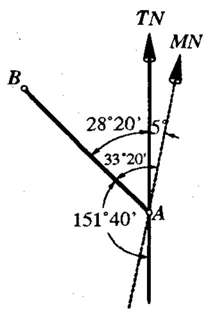
Solution:

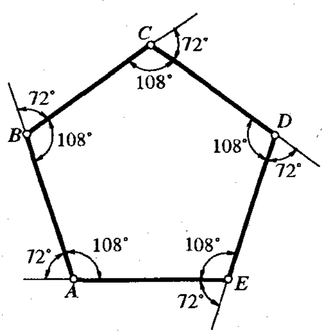
Sum of all interior angles of a closed polygon.

$(n-2)\,180 = (5-2)\,180 = 540^\circ$

Value of each interior angle $= \dfrac{540}{5}$

Value of each interior angle = 108





LINES	BEARING	AZIMUTH
AB	N. 28°20'W	151°40'
BC	N. 43°40' E	223°40'
CD	S. 64°20' E	295°40'
DE	S. 7°40' W	7°40'
AE	S. 79°40' W	79°40'
AB	N. 28°20' W	151°40'

① True bearing of line BC = ***N. 43°40' E***

② True azimuth of line CD = ***295°40'***

③ True bearing of line AE = ***S. 79°40' W***

Problem 81:

The following bearings taken on a closed compass. Assuming the observed bearing of line AB to be correct.

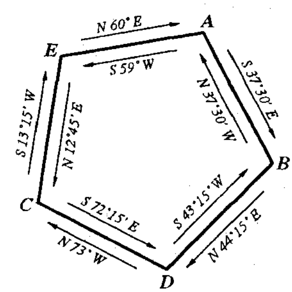
Line	Forward Bearing	Backward Bearing
AB	S. 37°30' E.	N. 37°30' W.
BC	S. 43°15' W.	N. 44°15' E.
CD	N. 73°00' W.	S. 72°15' E.
DE	N. 12°45' E.	S. 13°15' W.
EA	N. 60°00' E.	S. 59°00' W.

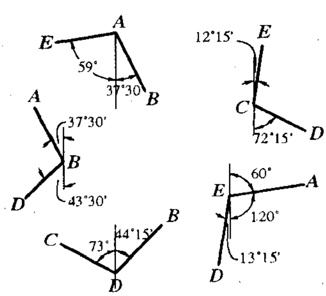
① Compute the bearing of line BC.

② Compute the bearing of line CD.

③ Compute the bearing of line DE.

Solution:





COMPASS SURVEYING

Point	Interior angles	
A	59°00' + 37°30'	= 96°30'
B	180° - (37°30' + 43°15')	= 99°15'
C	73°00' + 44°15'	= 117°15'
D	180° - (12°45' + 72°15')	= 95°00'
E	120° + 13°15'	= 133°15'
		541°15'

Sum of interior angles

= (n - 2) 180 = (5 - 2)(180) = 540°

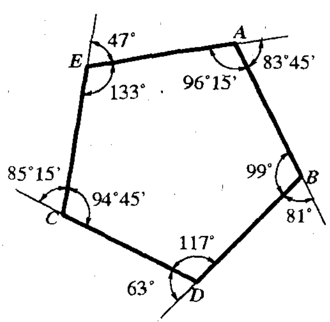
Error = 541°15' - 540°00'

Error = 1°15' (too big)

Error per station = $\frac{75'}{5}$ = 15'

POINTS	CORRECTED
A	96°15'
B	99°00'
C	117°00'
D	94°45'
E	133°00'

INTERIOR ANGLE



LINES	BEARING	AZIMUTH
AB	S. 37°30' E	322°30'
BC	S. 43°30' W	43°30'
CD	N. 73°30' W	106°30'
DE	N. 11°45' E	191°45'
AB	S. 37°30' E	322°30'

LINES	AZIMUTH
AB	322°30'
	+ 81°00'
	403°30'

(since there is no azimuth greater than 360°, subtract 360°)

	403°30'
	- 360°00'
BC	43°30'
	+ 53°00'
CD	106°30'
	+ 85°15'
DE	191°45'
	45°00'
EA	238°45'
	83°45'
AE	322°30'

① *Bearing of line BC* = ***S. 43°30' W***

② *Bearing of line CD* = ***N. 73°30' W***

③ *Bearing of line DE* = ***N. 11°45' E***

Problem 82:

The interior angles of a five side traverse are as follows:

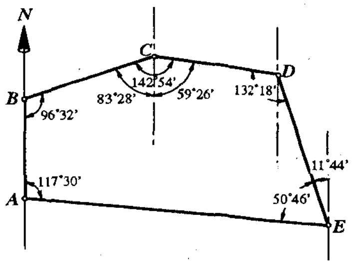
A = 117°30'   C = 142°54'
B = 96°32'   D = 132°18'

The angle E is not measured assumed AB due north.

① Compute the deflection angle at C.
② Compute the bearing of line DE.
③ Compute the bearing of line AE.

Solution:

① *Deflection angle at C:*



$C = 180° - 142°54'$
$C = \mathbf{37°06'\ R}$

② *Bearing of line DE:*
$AB = 180° - 96°32'$
$AB = N\ 83°28'\ E$
$CD = 142°54' - 83°28'$
$CD = S\ 59°26'\ E$
$DE = 180° - 59°26'$
$DE = 120°34'$
$DE = 132°18' - 120°34'$
$DE = \mathbf{S\ 11°44'\ E}$

③ *Bearing of line AE:*
$EA = 11°44' + 50°46'$
$EA = \mathbf{N\ 62°30'\ W}$

Problem 83:

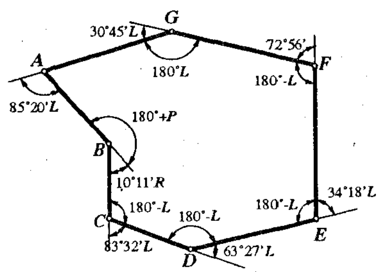
Given the following deflection angles of a closed traverse. Assume bearing of line AB is S. 40° E.

Station	Deflection Angles
A	85°20' L
B	10°11' R
C	83°32' L
D	63°27' L
E	34°18' L
F	72°56' L
G	30°45' L

① Compute the total error of the deflection angle.
② Compute the bearing of line DE.
③ Compute the bearing of line GA.

Solution:

Station	Interior Angles
A	180 - L
B	180 +R
C	180 - L
D	180 - L
E	180 - L
F	180 - L
G	180 - L



Sum of interior angles
$= 1080 - \sum L + 180 + \sum R$
$= 1260 - \sum L + \sum R$

Sum of interior angles $= (7 - 2)\ 180$
Sum of interior angles $= 900$
$900 = 1260 - \sum L + \sum R$
$360 = \sum L - \sum L$

Therefore the difference of the sum of deflection angles is always 360°

$\sum L = 50°20 + 83°32 + 63°27$
$\quad + 34°18' + 72°56' + 30°45'$
$\sum L = 370°18'\ L$
$\sum R = 10°11'\ R$
$\sum L - \sum R = 360$
$370°18' - 10°11' = 360°07'$

① *Total error of the deflection angle:*
Error $= 360°07' - 360°$
Error $=$ **07'** *too big*

② *Bearing of line DE:*

Points	Corrected	Reflection	Angle
A	85°20' L	- 01	85°19' L
B	10°32' R	+01	10°12' R
C	83°32' L	- 01	83°31' L
D	63°27' L	- 01	63°26' L
E	34°18' L	- 01	34°17' L
F	72°56' L	- 01	72°55' L
G	30°45' L	- 01	30°44' L

Check:

$\sum L = 85°10' + 93°31' + 63°26'$

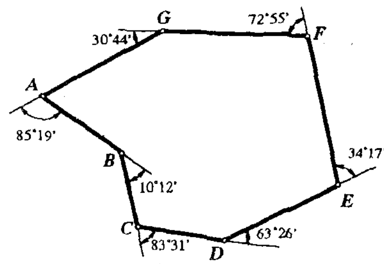
$+ 34°17' + 72°55' + 30°44'$

$\sum L = 10°12'$

$\sum L - \sum R = 370°12' - 10°12' = 360$ *(check)*

LINES	AZIMUTH
AB	320°
BC	320° - 10°12' = 330°12'
CD	330°12' - 83°31' = 246°41'
DE	246°41' - 63°26' = 183°15'
EF	183°15' - 34°17' = 148°58'
FG	148°58' - 72°55' = 76°03'
GA	76°03' - 30°44' = 45°19'
AB	45°19' + 180° + (180 - 85°19') = 320°00'

LINES	BEARING	AZIMUTH
AB	S. 40°E	320°00'
BC	S. 29°48' E	330°12'
CD	N. 66°41' E	246°41'
DE	N. 3°15' E	183°15'
EF	N. 31°02' W	148°58'
FG	S. 76°03' W	76°03'
GA	S. 45°19' W	45°19'
AB	S. 40° E	320°00'



Bearing of line DE = ***N. 3°15' E***

③ *Bearing of line GA =* ***S. 45°19' W***

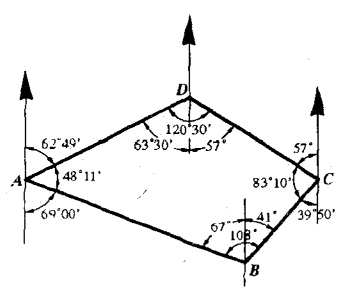
Problem 84:

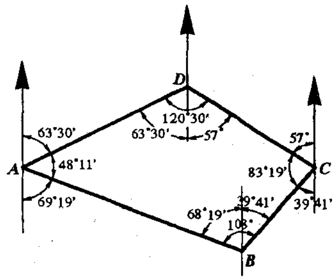
An engineers notebook gives the observed magnetic bearings of the following traverse.

Compass at	Line	Bearing
A	AD	N. 62°49' E.
	AB	S. 69°00' E.
B	BA	N. 67°00' W.
	BC	N.41°00' E.
C	CB	S. 39°50' W.
	CD	N. 57°00' W.
D	DC	S. 57°00' E.
	DA	S. 63°30' W.

① Compute the local attraction at A.
② Compute the local attraction at B.
③ Compute the local attraction at C.

Solution:





LINES	CORRECTED BEARING	LOCAL ATTRACTION
AB	S. 68°19' E	A = 0°41' E
BC	N. 39°41' E	B = 1°19' W
CD	N. 57°00' W	C = 09' W
DA	S. 63°30' N	D = 0

Check:
48°11' + 108° + 83°19' + 120°30' = 360°00'

① *Lcal attraction at A:*
Lcal attraction at A = 69° - 68°19'
Lcal attraction at A = ***0°41' E***

② *Local attraction at B:*
Local attraction at B = 68°19' - 67°
Local attraction at B = ***1°19' W***

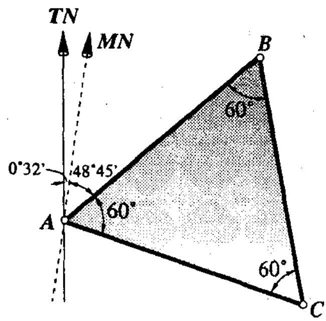
③ *Local attraction at C:*
Local attraction at C = 39°50' - 39°41'
Local attraction at C = ***09' W***

Problem 85:

The side AB of an equilateral field ABC with an area of 692.80 sq.m. has a magnetic bearing of N 48°45' E in 1930 when the magnetic declination was 0°52' E. Assume B and C is on the north east side.

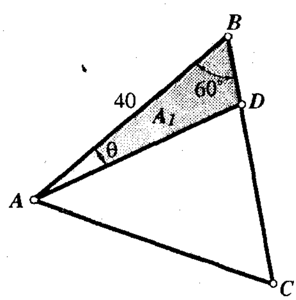
① Find the true bearing of AB.
② Find the length of AD with point D on the line BC and making the area of the triangle ABD one third of the whole area.
③ Compute the bearing of line AD.

Solution:



① *True bearing of AB:*
True bearing of AB = 48°45' + 0°52'
True bearing of AB = ***N 49°37' E***

② *Length of AD:*
Since the triangle is equilateral, it is also equiangular.



$AB = BC = CA$

$$Area = \frac{(AB)(AC)}{2}\,\text{Sin } 60°$$

$$69280 = \frac{(AB)^2\,\text{Sin } 60°}{2}$$

$AB = 40$ m.

$$A_1 = \frac{1}{3}(692.80)$$

$A_1 = 230.93$

$$A_1 = \frac{40\,(x)\,\text{Sin } 60°}{2}$$

$x = 13.3$ m.

$(AD)^2 = (40)^2 + (13.3)^2 - 2(40)(13.3)\,\text{Cos } 60°$

$(AD)^2 = 1245$

$AD =$ ***36.3 m.***

COMPASS SURVEYING

③ *Bearing of line AD:*

$$\frac{\text{Sin } ø}{13.3} = \frac{\text{Sin } 60°}{36.3}$$

$ø = 20°08'$

Bearing of AD = 49°37' + 20'08'

Bearing of AD = ***N 69°45' E***

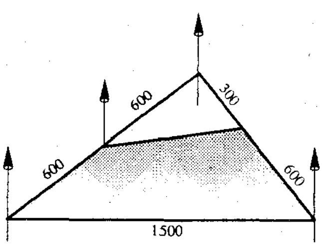
Problem 86:

A triangular lot has for one of its boundaries a line 1500 m. long which runs due East from A. The eastern boundary is 900 m. long and the western boundary 1200 m. long. A straight line cuts the western boundary at the middle point D and meets the easterly boundary E, 600 m. from the SE corner B.

① Find the bearing of line ED.
② Find the bearing of line BE.
③ Find the bearing of line DA.

Solution:

① *Bearing of line ED:*



Angle ACB is a right angle having the ratio of its side as 3:4:5.

$$\text{Sin } A = \frac{900}{1500}$$

$A = 36°52'12"$

$B = 90° - 36°52'12" = 53°07'48"$

$$\text{Cot } E = \frac{300}{600}$$

$E = 63°26'04"$

$D = 90° - 63°26'04" = 26°33'56"$

DE Sin 63°26'04 = 600

$DE = 670.83$

Bearing of line ED
= 180° - (36°52'12" + 63°26'04")
= ***S 79°41'44" W***

② *Bearing of line BE:*
= 90° - 53°07'48"
= ***N 36°52'12" W***

③ *Bearing of line DA:*
= 79°41'44" - 26°33'56"
= ***S 53°07'48" W***

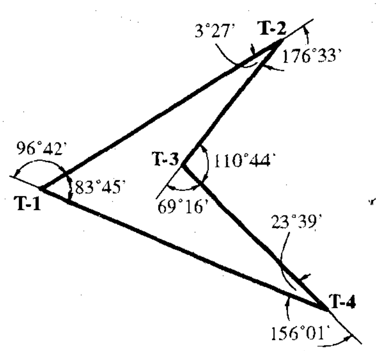
Problem 87:

In the deflection angle traverse with a transit survey data below. Assume deflection $T_1 T_2 T_3$ and bearing $T_1 T_2$ is correct.

Sta. Occ.	Sta. Obs.	Magnetic Bearing	Dist.	Deflection Angles
	T_1	N.54°00'W	85.26	
	T_3	N.30°00'W		
T_4				156°00' R
	T_4	S.30°15'E	83.44	
	T_2	N.39°00' E		
T_3				69°16' L
	T_3	S.39°00'W	83.22	
	T_1	S.42°25'W		
T_2				176°33' R
	T_2	N.42°25'E	118.38	
	T_4	S.54°00'E		
T_1				96°42' R

① Find the bearing of line $T_2 - T_3$.
② Find the bearing of line $T_3 - T_4$.
③ Find the bearing of line $T_4 - T_1$.

Solution:



$\sum$ Def. < S to the right

R 96°42'

R 176°33'

R 156°00'

429°15'

$\sum R = 429°15'$

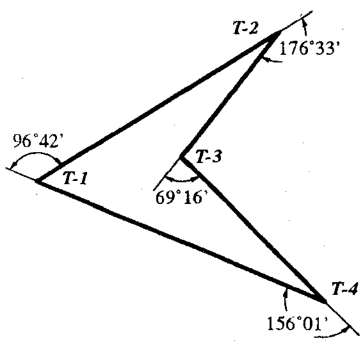
$\sum L = 69°16'$

359°59' *Error 01' too small*

$\sum$Def. < S to the left

$\sum L = 69°16'$

Correction is applied only at T-4 = 156°01'



Lines .	Azimuth	Back Azimuth	Bearing	Distances
1 - 2	225°25'	42°25'	N. 42°25' E	118.38
2 - 3	38°58'	218°58'	S. 38°58' W	83.22
3 - 4	329°42'	149°42'	S. 30°18' E	83.44
4 - 1	125°43'	305°43'	N. 54°17' w	85.26
1 → 2	222°25'	42°25'	N. 42°25' E	

① *Bearing of line T_2 - T_3:*
*= **S. 38°58' W***

② *Bearing of line T_3 - T_4:*
*= **S. 30°18' E***

③ *Bearing of line T_4 - T_1:*
*= **N. 54°17' w***

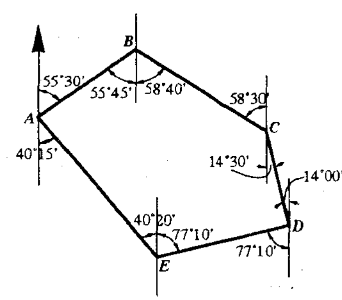
Problem 88:

The observed magnetic bearings (forward and back) of a closed compass traverse are as follows:

AB, Forward = N. 55°30' E.,
BA, Back = S. 55°45' W.,
BC, Forward = S. 58∆40' E.
CB, Back = N. 58°30' W.,
CD, Forward = S. 14°30' E.,
DC, Back = N. 14°00' W.,
DE, Forward = S. 77°10' W.,
ED, Back = N. 77°10' E.,
EA, Forward = N. 40°20' W.,
AE, Back = S. 40°15' E.

① Compute the mis-closure of the given traverse in degrees.
② Compute the adjusted interior angle at station C.
③ Compute the adjusted forward bearing of line CD.

Solution:



Interior ∠s:

∠B = 55°45' + 58°40' = 114°25'

∠C = 180 + 14°30' - 58°30' = 136°08'

∠D = 180 - 14°00' - 77°10' = 88°50'

∠E = 40°20' + 77°10' = 117°30'

∠A = 180 - 40°15' - 55°30' = 84°15'

541°00'

COMPASS SURVEYING

① *Error of misclosure:*

Error of misclosure = 541° - 540°

Error of misclosure = ***1°00'***

② *Adjusted interior angle at station C:*

Correction per interior angle

$$= \frac{1°00'}{5} = 12'$$

Corrected interior ∠s:

∠B = 114°25' - 0°12' = 114°13'

∠C = 136°08' - 0°12' = 135°56'

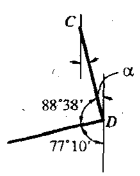
∠D = 88°50' - 0°12' = 88°38'

∠E = 117°30' - 0°12' = 117°18'

∠A = 84°15' - 0°12' = 84°03'

Angle at station C = ***135°56'***

③ *Adjusted forward bearing of line CD:*



α = 180 - 77°10' - 88°38'

α = 14°12'

Bearing of line CD = ***S 14°12' E***

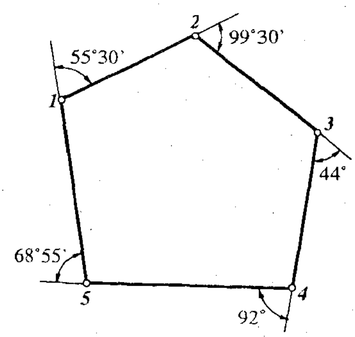
Problem 89:

Below is a transit and tape survey notes of a lot.

① Compute the angular error of deflection angle.
② Compute the linear error of closure of the traverse.
③ Compute the area in square meters using DMD method.

Station	Deflection Angle	Line	Bearing	Distance
1	55° - 30' R	1 - 2	N 10 E	650
2	99° - 30' R	2 - 3		895
3	44° - 00' R	3 - 4		315
4	92° - 00' R	4 - 5		875
5	68° - 55' R	5 - 1		410

Solution:



① *Error of deflection angle:*

$\sum R = 55°30' + 99°30' + 44°00' + 92°00' + 68°55'$

$\sum R = 359°55'$

$\sum L = 0$

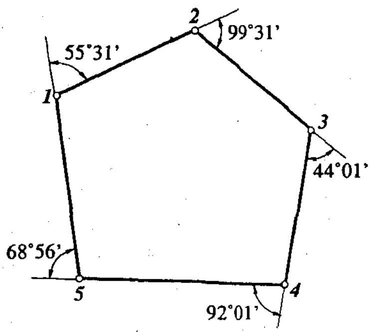
$\sum R - \sum L = 359°55'$

$\sum R - \sum L = 360°$

Error = ***05'*** *(to be added)*

Distribute the error equally at each station = 01'

STATION	CORRECTED
1	55°31' R
2	99°31' R
3	44°01' R
4	92°01' R
5	68°56' R



LINES	BEARING	AZIMUTH
1 - 2	N 10° E	190°00'
2 - 3	S 70°20' E	289°31'
3 - 4	S 26°28' E	333°32'
4 - 5	S 65°32' W	65°32'
5 - 1	N 45°30' W	134°30'

LINES	Distance	LAT	DEP
1 - 2	650	+640.13	+112.87
2 - 3	895	- 299.05	+843.56
3 - 4	315	- 281.98	+140.40
4 - 5	872	- 362.44	- 796.41
5 - 1	410	+287.37	- 292.43
		+927.50	+1096.83
		- 943.47	- 1088.84
		- 15.97	+ 7.99

② *Linear error of closure:*

Linear error of closure $= \sqrt{(15.97)^2 + (7.99)^2}$
Linear error of closure = ***17.86***

③ *Area by DMD method:*
Balance the traverse using transit rule.
Arithmetical sum of latitudes
= 972.50 + 343.47
= 1870.97

Arithmetical sum of departures
= 1096 + 1088.84
= 2185.67

Correction in latitude:

$$\frac{C_1}{15.97} = \frac{64.13}{1870.97}$$

C_1 = 0.00854 (640.13) = 5.47
C_2 = 0.00854 (299.05) = 2.55
C_3 = 0.00854 (281.98) = 2.41
C_4 = 0.00854 (362.44) = 3.10
C_5 = 0.00854 (287.37) = 2.44

15.97

Correction in departure:

$$\frac{C_1}{7.99} = \frac{112.87}{2185.67}$$

C_1 = 0.00366 (112.87) = 0.41
C_2 = 0.00366 (843.56) = 3.09
C_3 = 0.00366 (140.40) = 0.51
C_4 = 0.00366 (796.41) = 2.91
C_5 = 0.00854 (292.43) = 1.07

7.99

	(uncorrected)		(corrected)	
Lines	LAT	DEP	LAT	DEP
	+5.47	- 0.41		
1 - 2	+640.13	+112.87	+645.60	+112.46
	- 2.55	- 3.09		
2 - 3	- 239.05	+843.56	- 296.50	+840.47
	- 2.41	- 0.51		
3 - 4	- 281.98	+140.40	- 279.57	+139.89
	- 3.10	+2.91		
4 - 5	- 362.44	- 786.41	- 359.34	- 799.32
	+2.44	+1.07		
5 - 1	+287.37	- 292.43	+289.81	- 293.50

LINE	LAT	DMD	DOUBLE AREA
1 - 2	+645.60	+112.46	+72604.18
2 - 3	- 296.50	+1065.39	- 315888.14
3 - 4	- 279.57	+2054.75	- 571930.33
4 - 5	- 359.34	+1386.32	- 498160.23
5 - 1	+289.81	+293.50	+85059.24

$2A$ = 1228315.28
A = ***614157.64 m²***

ERRORS IN TRANSIT WORK

Errors in Transit Work

Transit - it is an instrument of designed primarily for measuring horizontal and vertical angle.

Types of Transit:

1. ***Engineer's transit*** - a transit provided with vertical circle and a long level tube on its telescope.
2. ***Plain transit*** - a transit without a vertical circle and telescope level.
3. ***City transit*** - a transit without a compass and having a U-shaped one piece standard.
4. ***Mining transit*** - a transit provided with an auxiliary telescope, a reflector for illuminating the cross hairs and a diagonal prismatic eyepiece for upward sighting, 60° above the horizon.
5. ***Theodolite*** - a transit designed for surveying of high precision.
6. ***Geodimeter*** - a transit which can measure distances using the principles of the speed of light.

Three principal subsidivions of a transit and parts under each subdivision:

1. ***Upper plate:***
   a. Telescope and telescope level
   b. Telescope standard
   c. Telescope clamp and tangent screw
   d. Vertical circle and vertical vernier
   e. Plate levels, compass box, upper tangent screw
   f. Vernier and inner spindle
2. ***Lower plate:***
   a. Outer plate
   b. Lower clamp
   c. Outer spindle
3. ***Leveling plate group:***
   a. Lower clamp and tangent screw
   b. Leveling screws
   c. Leveling head
   d. Foot plate

Line of collimation - a line segment joining the intersection of the cross hairs and the optical center of the objective lens when in proper adjustment.

Line of sight - the line joining the intersection of the cross hairs and the optical center of the objective lens, regardless of whether it is in adjustment or not. When in adjustment, the line of sight and the line of collimation can be termed either of the other.

Focusing - consists in the adjustment of the eyepiece and the objective so that the cross hairs and the image can be seen clearly at the same time.

Adjustments of the Transit:

1. The adjustment of the plate bubble
2. The adjustment of the vertical cross hair
3. The adjustment of the line of sight
4. The adjustment of the standards
5. The adjustment of the telescope bubble
6. The adjustment of the vertical vernier

Four adjustments of the transit which is not ordinarily performed:

7. To make the line of sight as defined by the horizontal hair coincide with the optical axis.
8. To make the axis of the objective slide perpendicular to the horizontal axis.
9. To center the eyepiece slide.
10. To make the axis of the striding level parallel to the horizontal axis.

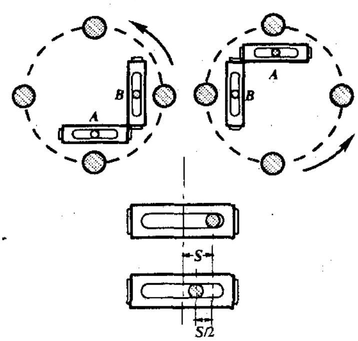
ERRORS IN TRANSIT WORK

1. ***Adjustment of the Plate Bubble:***

Object: To make the axis of the plate level lie in a plane perpendicular to the vertical axis.

Test: Rotate the instrument about the vertical axis until each level tube is parallel to a pair of opposite leveling screws. Center the bubbles by means of the leveling screws. Rotate the transit end for end about the vertical axis. If the bubble remains on the center, then the axis of the plate level tube is perpendicular to the vertical axis.

Correction: If the bubbles become displaced, bring them halfway back by means of the adjusting screws. Level the instrument again and repeat the test to verify the results.

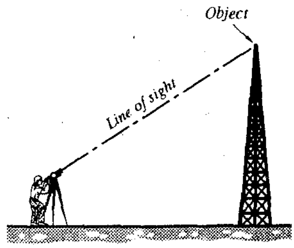


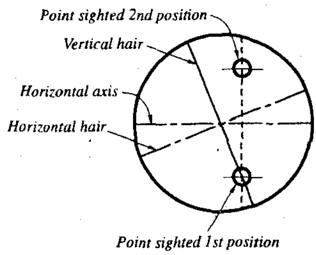
2. ***Adjustment of the vertical cross hair:***

Object: To make the vertical cross hair in a plane perpendicular to the horizontal axis.

Test: Slight the vertical cross hair on a well defined point not less than 60 m. away. With both horizontal motions of the instrument clamped, swing the telescope through a small vertical angle, so that the point traverses the length of the vertical cross hair. If the point appears to move continuously on the hair, then the cross hair is in adjustment.

Correction: If the point appears to depart from the cross hair, loosen the two adjacent capstan screws and rotate the cross hair ring in the telescope tube until the point traverses the entire length of the hair. Tighten the same screws.





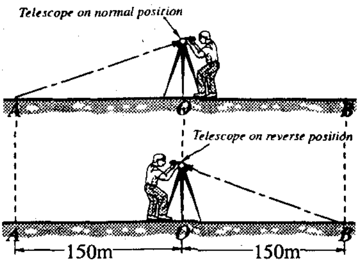
3. ***Adjustment of the line of sight:***

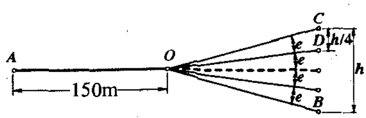
Object: To make the line of sight perpendicular to the horizontal axis.

Test: Level the instrument. Sight on the point A about 150 m. away, with the telescope on the normal position. With both horizontal motions of the instrument clamped, plunge the telescope and set another point B on the line of sight and about the same distance away on the opposite side of the transit. Unclamp the upper motion, rotate the instrument about the vertical axis, and again sight at A with the telescope inverted. Clamp the upper motion. Plunge the telescope as before, if B is on the line of sight, the desired relation exist.

ERRORS IN TRANSIT WORK

Correction: If the line of sight does not fall on B set a point C on the line of sight beside B. Marked a point D, 1/4 of the distance from C to B, and adjust the cross hair ring by means of the two opposite horizontal screws until the line of sight passes through D. The point sighted should be at the same elevation as the station occupied by the transit.

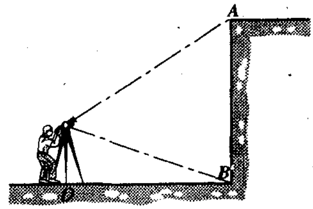


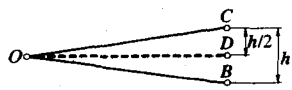


4. ***Adjustment of standards:***

Object: To make the horizontal axis perpendicular to t he vertical axis.

Test: Set up the transit near a building or other object on which is some well-defined point A at a certain vertical angle. Level the instrument very carefully thus making the vertical axis truly vertical. Sight at the high point A and with the horizontal motions clamped depress the telescope and set a point B on the ground below A. Plunge the telescope, rotate the instrument end for end about the vertical axis, and again sight on A. Depress the telescope as before, it the line of sight falls on B, then the desired relation exist.

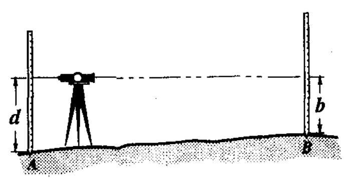


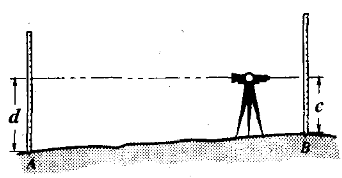


Correction: If the line of sight does not fall on B, set a point C, on the line of sight beside B. A point D, halfway between B and C, will lie in the same vertical plane with the height point A. Sight on D, elevate the telescope until the line of sight is beside A, loosen the crews of the bearing cap, and raise or lower the adjustable end of the horizontal axis until the line of sight is in the same vertical plane with A.

5. ***Adjustment of the telescope bubble:***

Object: To make the axis of the telescope level parallel to the line of sight.



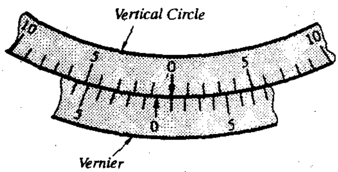


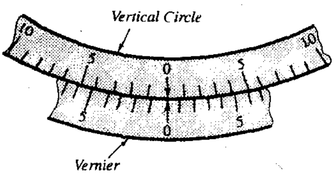
ERRORS IN TRANSIT WORK

Test & Correction: Use the two-Peg Test method. Select two point A and B say, 60 m. apart. Set up the transit close to A so that when the rod is held upon it, the eyepiece will be about a quarter of an inch from the rod. Look through the telescope with the wrong end - to at the rod and find the rod reading at the cross hair if visible. If not take the reading by means of a pencil point opposite the center of field of view. Turn the telescope toward B and take a rod reading on it. Subtract one reading from the other to secure the apparent difference in elevation between the two pegs. The transit is then taken to B and the operation is repeated. The mean of the two apparent difference in elevation is the true difference in elevation between the two pegs. The rod reading on A with the instrument still at B, is then computed. With the computed value for the rod reading at A known, the end of the telescope bubble tube is raised or lowered by means of the adjusting screws until the telescope bubble is centered.

6. ***Adjustment of the vertical circle and vernier.***

Object: To make the vernier read zero when the telescope bubble is centered.





Test: Level the instrument first by means of the plate levels and then by means of the telescope bubble, center the telescope bubble carefully and observe if the vernier reads zero. If not proceed as follows.

Correction: Slightly loosen the capstan screws holding the vernier and shift the vernier lightly by tapping lightly with a pencil until the zeros coincide.

Sources of Errors in Transit Work

1. Non-adjustment, eccentricity of circle, and errors of graduation.
2. Changes due to temperature and wind.
3. Uneven setting of tripod
4. Poor focusing (parallax)
5. Inaccurate setting over a point
6. Irregular refraction of atmosphere

Common Mistakes in Transit Work

1. Reading in the wrong direction from the index in a double vernier.
2. Reading the vernier opposite the one which was set.
3. Reading the circle wrongly that is reading 59° to 60°.

ERRORS IN TRANSIT WORK

Non-Adjustment of Transit

A) Error of line of sight: Line of sight not perpendicular to the horizontal axis.

X Cos h Sin E = X Sin e

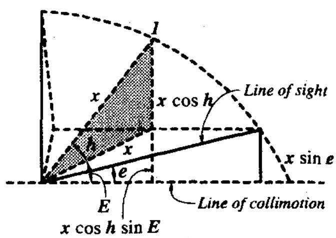
$$\text{Sin E} = \frac{\text{Sin e}}{\text{Cos h}}$$

Sin E = Sin 2 Sec h

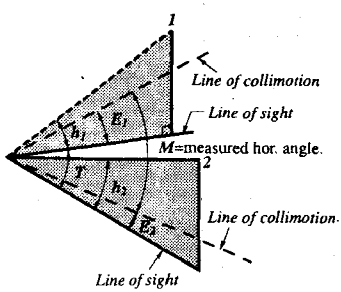
For small angle, Sin E = E

Sin e = e

E = e Sec h



1. Line of sight deflected to the right of line of collimation. (clockwise)



T = true horizontal angle

$T - E_2 = M - E_1$

$T = M + E_2 - E_1$

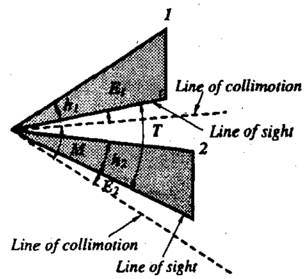
$T = M + (e \sec h_2 - e \sec h_1)$

$T = M + E (\sec h_2 - \sec h_1)$

$T = M + E'$

where $E' = e (\sec h_2 - \sec h_1)$

2. Line of sight deflected to the left of collimation. Angle measurement clockwise.



$M - E_2 = T - E_1$

$T = M - E_2 + E_1$

$T = M - (E_2 - E_1)$

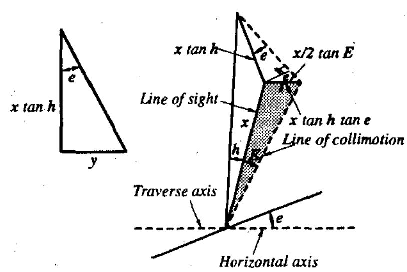
$T = M - E (\sec h_2 - \sec h_1)$

$T = M - E'$

1) When $h_1 = h_2$, there is no error.

2) When one angle is depression and the other is angle of elevation having numerically equal values, there is no error.

B) Error of traverse axis of the telescope is not horizontal or horizontal axis not perpendicular to the vertical axis.



Tan E = X tan h tan e

Tan E = tan h tan e

E = e tan h

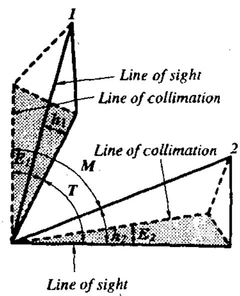
For small angles,

tan E = E

tan e = e

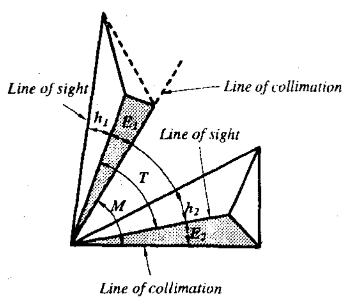
ERRORS IN TRANSIT WORK

1) Left end of transverese axis higher. Angle measurement clocwise.



$T - E_2 = M - E_1$
$T = M + E_2 - E_1$
$T = M + (e \sec h_2 - \tan h_1)$
$Tan = M + E'$
$E' = E_2 - E_1$

2) Right end of transverse axis higher.



Measuring Angles by Repetition:

To measure an angle by repetition means to measure it several times, allowing the vernier to remain clamped at each time at the previous reading instead of setting it back at zero when sighting at the backsight.

The first measurement is made in exactly the same manner as that described for a single angle. Then, do not touch the upper clamp or upper tangent screw, but loosen the lower clamp turn the telescope back to the first object and set exactly on it by means of the lower clamp and tangent screw. The circle now reads, not 0°, but the first single angle. Next loosen the upper clamp, turn the telescope to the second object and set exactly on it by the use of the upper clamp and its tangent screw. The index of the vernier now points to the double angle on the horizontal circle. Half the angle now read is the improved value of the required angle. If the process is repeated and a third angle is mechanically added to the last reading, the circle reading is divided by three and still more exact values of the angle is obtained. Six readings are usually the greatest number of times taken with the telescope in one position.

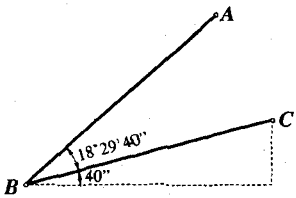
Laying Off An Angle By Repetition:

To lay off an angle of 18°30'20" with a transit to the nearest min. first lay off an angle of 18°30' by a single setting and establish a temporary stake. Measure this angle that has just been laid off by repetition. Assume that repetition determines 18°29'40" as the value of the angle to the temporary stake. A new stake must then be set a short perpendicular distance called an offset from the temporary stake, by 40".

ERRORS IN TRANSIT WORK

Consequently if the temporary stake is 600 m. from the transit it would be necessary to set the final stake.

$\frac{600\,(0.0003)(40)}{60}$ = 0.12 m. from the temporary stake.

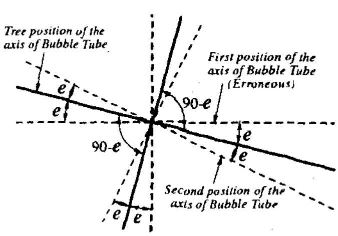


ANGLES BY REPETITION

PRINCIPLE OF REVERSION

(As applied to the Adjustment of Bubble Tube)

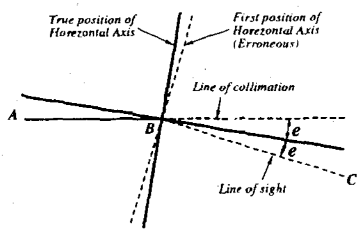
Let us say that there is an error of the axis of the bubble tube fro its position by an amount "e". If the telescope is rotated at 180°, the position of the axis of the bubble tube is now doubled as shown in the figure, with reference to its original, position in order to adjust the bubble just move it at half this value.



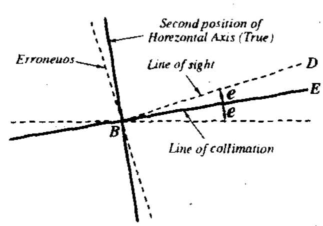
Total error from first to second position is 2e. Therefore to place the axis of the bubble tube to its true position, move by an amount "e".

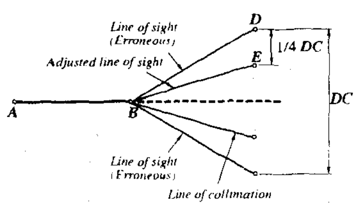
Error in Line of Sight:

The transit is set up at B, with the telescope in normal position and a backsight at A was taken. Assume that the true position of the line of sight (line of collimation) is deflected by an amount "e" as shown. When the telescope was plunged and point C was sighted, the line of sight is now deflected by an amount equal to 2e from the prolongation of line AB. The telescope is then rotated at 180° about its vertical axis and point A is again sighted but this time the telescope is in inverted position.



The telescope is again plunged and point D is established on the ground. Point D is erroneous by an amount 2e from the prolongation of line AB. The line of sight is adjusted by an amount e, backwards that is determine first the location of E, that is DE = 1/4 CD.





ERRORS IN TRANSIT WORK

VERNIERS

A vernier is a device for measuring the fractional part of one of the smallest divisions of a graduated scale more accurately than can be estimated by eye. The amount by which the smallest division on the vernier differs from the smallest division on the vernier differs from the smallest division on graduated scale determines the least count of the vernier.

Least Count:

$$L = \frac{S}{N}$$

where L = least count

S = smallest division on scale

N = Number of divisions on the vernier

Types of Verniers:

1. ***Direct vernier*** - is one in which the smallest division on the vernier is shorter than the smallest division on the scale.

2. ***Retrograde vernier*** - the division in the vernier is longer than the division of the scale.

3. ***Folded vernier*** - is a direct vernier; it is used where a double vernier would be too long as to make it impracticable.

Problem 90:

The single direct vernier of the horizontal circle of a transit is actually a graduated arc of 19°40' and the least count is 20".

① What is the smallest division of the circle?
② How many divisions are there on the vernier?

Solution:

① *Number of divisions on the vernier:*

Nv = number of divisions on the vernier

Ns = number of divisions of the scale

Lv = least count of vernier

Ls = least reading of circle

L = length of vernier

For retrograde vernier,

$Nv = Ns - 1$

For direct vernier,

$Nv = Ns + 1$

$L = 19°40' = 1180'$

$Lv = 20'' = \frac{1}{3}$

Equation ①

$L = Ns\ Ls$

$1180 = Ns\ Ls$

Equation ②

$$Lv = \frac{Ls}{Nv}$$

$$Ls = \frac{Nv}{3}$$

From Equation ②

$$Ls = \frac{Ns + 1}{3}$$

From Equation ①

$$Ls = \frac{1180}{Ns}$$

ERRORS IN TRANSIT WORK

Solving Equations ① & ②

$$\frac{Ns+1}{3} = \frac{1180}{Ns}$$

$$Ns^2 + Ns = 3540$$

$$Ns = \frac{-1 \pm \sqrt{(1)^2 - 4\,(-3540)}}{2}$$

Ns = 59 divisions

$Nv = Ns + 1$

$Nv = 59 + 1$

Nv = ***60** divisions on the vernier*

② *Least reading of circle:*

$$Ls = \frac{Nv}{3}$$

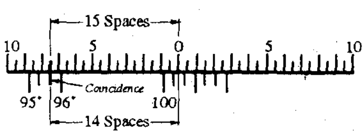
$$Ls = \frac{60}{3}$$

Ls = ***20'** smallest division of the circle*

Problem 91:

Design a folded vernier to read 30" with a least reading of 20' on the scale. Illustrate a reading of 100°32'30".

Solution:



$$L = \frac{S}{N}$$

$$30 = \frac{20\,(60)}{N}$$

N = 40 spaces in the vernier

$N - 1$ = 39 spaces in the scale

14 spaces = 14 (20) = 280'

= 4°40'

Reading on scale coincide is

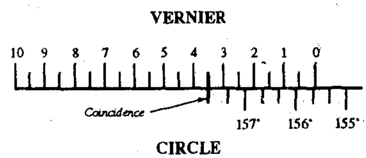
100°20' - 4°40' = 95°40'

Folded vernier with a reading of 100°32'30"

Problem 92:

Design a direct vernier for a circle graduated in degrees and thirds of a degree so that the least reading in the vernier is 30 seconds. Illustrate a reading of 155°43'30" clockwise. Determine the reading of the coincide in the circle.

Solution:



L = 30"

$$S = \frac{60}{3} = 20'$$

$$L = \frac{S}{N}$$

$$30 = \frac{20\,(60)}{N}$$

N = 40 divisions on the vernier which is equivalent to 39 divisions on the scale.

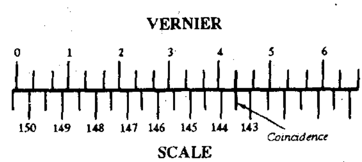
Since there are 7 spaces on the vernier, only 6 spaces on the circle will give us the coincide reading. Coincide reading on the scale = 155°40'

6 (20) = 157°40'.

Problem 93:

Design a retrograde vernier for a vernier having a lest reading of 20 sec. and a least reading in the circle of 30 min. Indicate a reading of 150°34'20". Give the reading of the coincide in the scale.

Solution:



$L = \frac{S}{N}$

$20 = \frac{30\,(60)}{N}$

N = 90 spaces on the vernier which is equivalent to 91 spaces on the scale.

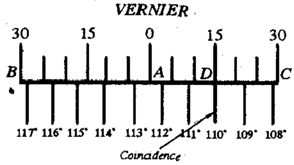
There are 13 spaces on the vernier, therefore 14 spaces on the scale must be laid out to determine the coincide.

Therefore the reading on scale
= 150°30' - 7° = 143°30'
, 14 spaces = 7°.
; 1 spaces = 30 min.

Problem 94:

Compute the vernier and scale coincidence of a folded vernier reading 112°45' clockwise such that there shall be 12 divisions in the vernier with a circle reading in degrees.

Solution:



$L = \frac{S}{N}$

$L = \frac{1°\,(60)}{12}$

$L = 5'$

Number of divisions on the SCALE
Scale or circle is 12 - 1 = 11 division
Vernier coincidence

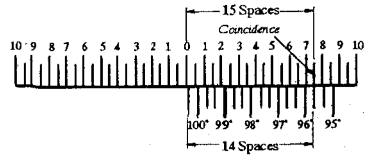
$= \frac{45}{5}$ = 9 division marks on the vernier

Start counting from A to B, then proceed from C, the coincidence is at D which is 3 divisions from C. Scale coincidence is 112°- 2 (1) = 110°.

Problem 95:

Design a folded vernier to read 30" with a least reading of 20' on the scale. Illustrate a reading of 100°32' in clockwise direction.

Solution:



$L = \frac{S}{N}$

$30 = \frac{20\,(60)}{N}$

N = 40 division in the vernier

N - 1 = 39 spaces in the scale

Diff. in reading = 100°32'30" - 100°20"

Diff. in reading = 12'30".

12'30" - 25 spaces in the vernier.

Start counting from A to B, then from C to D. From A to D in the vernier there are 15 spaces, therefore it is equivalent to 14 spaces in the scale.

Therefore the reading in the scale for the location of the coincidence
= 100°20' - 4°40' = 95°40'.

14 spaces on the scale = 4°40'.

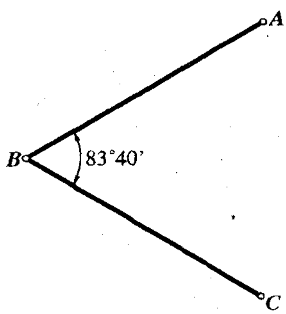
ERRORS IN TRANSIT WORK

Problem 96:

Sta. Occ.	Sta. Obs.	Tel.	Repetitions	Vernier (A)	(B)	Mean
B	A	D	0	0-00	180°01'	00-00-30"
	C	D	1	83°40'		
	C	R	6	142°02'	322°03'	142°02'30"
	A	R	6	0°01'	180°02'	00°01'30"

Determine the true value of angle A B C.

Solution:



Mean values

First reading = $\dfrac{00\text{-}00 + 00\text{-}01}{2}$

First reading = 00'30"
Third reading = 142°02'30"
Fourth reading = 00-01'-30"

Take the mean of the first and fourth reading:
00-00-30
00-01-30'
00-02'-00

Mean = 00-01' (too big)

Correction of third reading
= 142°02'30" - 00°01'00"
= 142°01'30"

Divide the corrected 3rd reading by 6

$\dfrac{142°01'30"}{6} = 23°40'15"$

Since the 2nd reading is 83°40' add multiple of 60°, 120°, 180°, 240°

True horizontal angle = 60° + 23°40'15"
True horizontal angle = **83°40'15"**

Problem 97:

In determining the horizontal angle between two points A and B, the transit is mounted at T - 1. The field notes were recorded as shown:

Sta. Occ.	Sta. Obs.	Azimuth	Vertical Angle
	B	260°40'30"	60°
	C	180°30'20"	45°

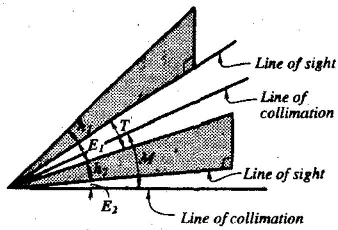
Check on the adjustment of the instrument reveals the following errors. The line of sight with the telescope on the normal position is deflected 30" to the left of its correct position and the horizontal axis (right end lower) makes an angle of 15" with the true horizontal.

① Compute the correction due to line of sight not perpendicular to the horizontal axis.
② Compute the correction due to the horizontal axis not perpendicular to the vertical axis.
③ Compute the corrected horizontal angle between A and B.

ERRORS IN TRANSIT WORK

Solution:

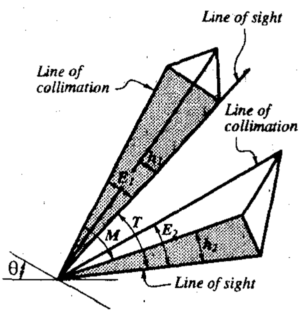
① *Correction due to line of sight:*



$E' = e\,(\sec h_2 - \sec h_1)$
$E' = 30\,(\sec 60° - \sec 45°)$
$E' = 20\,(2 - 1.414)$
$E' = 30\,(0.586)$
$E' = \mathbf{17.58''}$

② *Correction due to the horizontal axis:*
$E' = e\,(\tan h_2 - \tan h_1)$
$E' = 15\,(\tan 60° - \tan 45°)$
$E' = 15\,(1.932 - 1.0)$
$E' = \mathbf{10.98''}$

③ *Corrected horizontal angle:*



$T - E_1 = M - E_2$
$T = M - (E_2 + E_1)$
$T = M - (E_2 - E_1)$
$T = M - E$
$T - E_2 = M - E_1$
$T = M + E_2 - E_1$
$T = M + E$

Corrected horizontal angle
$= 80°10'10' - 17.58'' + 10.98''$
$= \mathbf{80°10'3.4''}$

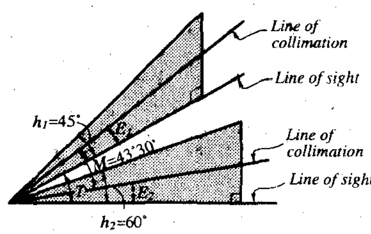
Problem 98:

A civil engineer desires to determine the azimuth of line AB. With a transit at station A, he sights point C which is on the left position of point B and measured a vertical angle at C to be 45°. He then turns the instrument in clockwise direction and sight at point B. The measured horizontal angle CAB is 43°30' and the vertical angle reading at B was 60°. The line of sight with the telescope on the normal position is deflected 03' to the right of its correct position.

① Compute the error due to line of sight not perpendicular to the horizontal axis.
② Compute the corrected horizontal angle between B and C.
③ If the azimuth of line AC is 210°30', compute the azimuth of line AB.

Solution:

① *Error due to line of sight not perpendicular to the horizontal axis:*



$E = e\,(\sec h_2 - \sec h_1)$
$E = 03'\,(\sec 60° - \sec 45°)$
$E = \mathbf{1.758'}$

② Corrected horizontal angle:
$T = M + E$
$T = 43°30' + 1.758'$
$T = 43°31.76'$
$T = \mathbf{43°31'46''}$

③ *Azimuth of AB:*
Azimuth of AB $= 210°30' + 43°31'46''$
Azimuth of AB $= \mathbf{254°01'46''}$

ERRORS IN TRANSIT WORK

Problem 99:

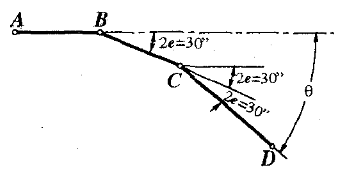
Maladjustment of the transit is such that the line of sight with the telescope in normal position, is deflected "e" seconds to the left of its correct position or not perpendicular to the horizontal axis. This causes an error of 8.79" in the measured horizontal angle when the vertical angle to the first point is 45° and that of the second point is 60°.

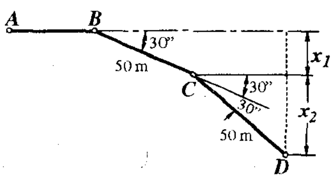
① What is the value of "e" in seconds.

② If this transit is used to layout a straight line by prolonging a line AB by setting up the transit at succeeding points A, B and C and plunging the telescope. If the procedure were such that each backsight were taken with the telescope at normal position, what would be the angular error in the segment CD.

③ What is the offset distance from the true prolongation of line AB from point D if AB = BC = CD = 50 m.

Solution:





① *Value of "e":*

$E = e\,(\sec h_2 - \sec h_1)$

$8.79'' = e\,(\sec 60^\circ - \sec 45^\circ)$

$e'' = \mathbf{15''}$

② *Angular error in segment CD:*

$\theta = 30 + 30$

$\theta = 60''$

$\theta = \mathbf{01'}$

③ *Offset distance:*

Offset distance = $x_1 + x_2$

Offset distance = 50 Sin 30" + 50 Sin 60"

Offset distance = ***0.218 m.***

Problem 100:

What error would be introduced if the measured horizontal angle if through non-adjustment, the horizontal axis were inclined 05' with the horizontal.

① With one sight at the same elevation as the transit and the other sight at an elevation 45°.

② Both sights are 45°.

③ One sight is + 45° and the other is - 45°.

Solution:

① *Error with one sight at the same elevation:*

$E = e\,(\tan h_2 - \tan h_1)$

$E = 0.05\,(\tan 45^\circ - \tan 0^\circ)$

$E = \mathbf{05'}$

② *Error with both sights are 45°.*

$E = e\,(\tan h_2 - \tan h_1)$

$E = 05\,(\tan 45^\circ - \tan 45^\circ)$

$E = \mathbf{0}$

③ *Error with one sight is +45° and the other is -45°:*

$E = 05\,[\tan 45^\circ - \tan(-45)]$

$E = 05(1 + 1)$

$E = \mathbf{10'}$

ERRORS IN TRANSIT WORK

Problem 101:

The horizontal angle between two points measured clockwise is 150°20'20". The angle of elevation of the first point is 42°30' while that of the second is 63°58'. The instrument was then tested for errors of collimation and for probable inclination of the transverse axis. It was found out after measurement that the instrument has an error in the line of sight which is deflected to the right of the line of collimation by an amount equal to 15". In the latter case, striding level was used to check the inclination of the transverse axis and was found that the right end is lower than the left end of the transverse axis in terms of 2 divisions. The angular value of one division is 10".

① Compute the error due to line of sight deflected to the right.
② Compute the error due to transverse axis with left end higher.
③ Compute the correct horizontal angle between the two points.

Solution:

① *Error due to line of sight deflected to the right:*

$E_1 = e\,(\sec h_2 - \sec h_1)$

$E_1 = 15''\,(\sec 63°58' - \sec 42°30')$

$E_1 = \mathbf{13.83''}$

② *Eror due to transverse axis with left end higher:*

$E_2 = e\,(\tan h_2 - \tan h_1)$

$e = 2\,(10)$

$e = 20''$

$E_2 = 20\,(\tan 63°58' - \tan 42°30')$

$E_2 = \mathbf{22.62''}$

③ *Corrected horizontal angle:*

$T = M + E_1 + E_2$

$T = 150°20'20'' + 13.83'' + 22.62''$

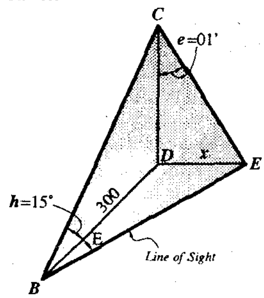
$T = \mathbf{150°20'56.45''}$

Problem 102:

In prolonging a straight line the transit is set at B, a backsight is taken at A, and the telescope is plunged to C, 300 m. in advance of B. If the vertical axis were inclined 0' with the true vertical in a vertical plane making 90° with the direction of the line, what would be the linear error in the located position of C.

① If A and B are at the same elevation, but the vertical angle from B to C is + 15°?
② If A, B and C are all at the same elevation?
③ If the vertical angle from B to A and from B to C is + 15°?

Solution:



① *Linear error if A and B are at the same elevation:*

$E = e \tan h$

$E = 01 \tan 15°$

$E = 0.268'$

$\tan E = \dfrac{x}{300}$

$x = 300 \tan 0.268'$

Note: $\tan 1' = 0.0003$

$x = \mathbf{0.024\ m.}$ *linear offset*

② *Linear error if A, B and C are all at the same elevation:*

= ***There is no error***

③ *Linear error if the vertical angle from B to A and from B to c is +15°:*

The error is doubled

$E = 0.268\,(2) = 0.536'$

$x = 300 \tan 0.536'$

$x = 300\,(0.0003)(0.536)$

$x = \mathbf{0.048\ m.}$

ERRORS IN TRANSIT WORK

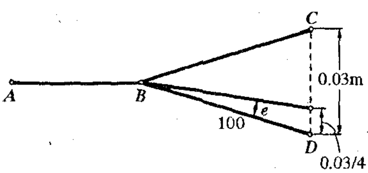
Problem 103:

The horizontal angle between two points measured clockwise is 179°20'20". The angle of elevation of the first point is 42°30' while that of the second is 63°58'. The instrument was then tested for errors of collimation and for the probable inclination of the transverse axis. In the former case the displacement of the second point established on the foreward side of the transit is 3 cm. to the right of the first point. These points are 100 m. from the transit station. In the latter case a striding level was used to check the inclination of the transverse axis and was found lower than the left end of the transverse axis in terms of 2 divisions. The angular value of one division is 10 seconds.

① Compute the error of collimation.
② Compute the error in the transverse axis.
③ Compute the horizontal angle between the two points.

Solution:

① *Error of collimation:*



$$\tan e = \frac{0.03/4}{100}$$

$e = 15.47"$

Error of collimation:

$E_1 = e\,(\sec h_2 - \sec h_1)$

$E_1 = 15.47\,(\sec 63°58' - \sec 42°30')$

$E_1 =$ **14.27"** *(is added if the line of sight is to the right of the line of collimation)*

② *Error in the transverse axis:*

$E_2 = e\,(\tan h_2 - \tan h_1)$

$E_2 = 20"\,(\tan 63°58' - \tan 42°30')$

$E_2 =$ **22.62"** *(is added if the left end is higher than the right end)*

③ *Horizontal angle:*

$H = 179°20'20" + E_1 + E_2$

$H = 179°20'20" + 14.27" + 22.62"$

$H =$ ***179°20'56.89"***

Problem 104:

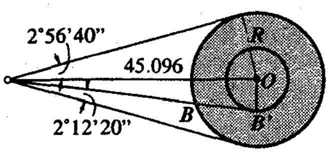
The following measurement were taken to check the perpendicularity of a tapering factory chimney of circular cross-section. Point A was established on the ground about 45 m. from the base of the chimney and the shortest dimension to the base measured carefully and found to be 45.096 m. The instrument was set up at A and sighted so as to bisect the top of the chimney; the telescope was then lowered and a point B set on the circumference at the base, which was level with the telescope. The instrument was reversed and the sighting repeated, and the instrument being in adjustment, the line of sight again fell at B. With the transit at A, the angle was measured from B to a line tangent to the right side of the chimney at the base and found by repetition to be 2°12'20". Similarly, angle was measured from AB to the extreme left side of the chimney and found to be 2°58'40".

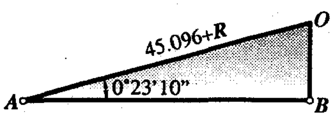
① What is the radius of the chimney?
② How much is the chimney out of plumb at the top in a direction at right angles to AB?
③ How far is the center of chimney from the point of observation.

ERRORS IN TRANSIT WORK

Solution:

① *Radius of the chimney:*





Mean value of the angle

$$= \frac{2°58'40" + 2°12'20"}{2}$$

$$= 2°35'30"$$

Angle AOB' = 2°35'30" - 2°12'20"

Angle AOB' = 0°23'10"

$$\text{Sin } 2°35'30" = \frac{R}{45.096 + R}$$

$$0.04522 = \frac{R}{45.096 + R}$$

$45.096\,(0.04522) + (0.04522)\,R = R$

$0.95478\,R = 45.096\,(0.04522)$

$R = \mathbf{2.136}$

② *Direction at right angles to AB:*

OB' = (45.096 + 2.136) Sin 0°23'10"

OB' = ***0.318 m.***

③ *Distance of the center of chimney from the point of observation:*

AO = 46.096 + *R*

AO = 46.096 + 2.136

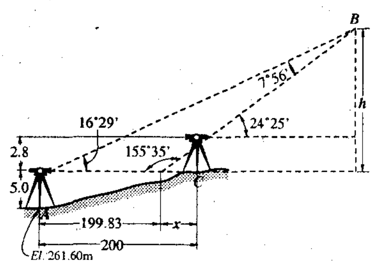
AO = ***48.232 m.***

Problem 105:

In order to determine the elevation of a point B on top of a cliff, a transit having an index error of + 05' was set over a point A, the elevation of which was known to be 261.60 m. and the height of the traverse axis of the instrument was 5 m. and when the horizontal wire was brought on point B, the vertical limb read + 16°34'. A point C was next located at a horizontal distance of 200 m. from a point A. Then the transit was next set up at point C and the vertical limb reading was found to be + 24°30'. Finally, with the telescope horizontal, the reading on the leveling rod on the point A was 7.8 m.

① Find the elevation of the H.I. of the instrument at A.
② Find the elevation of the H.I. of the instrument at C.
③ Find the elevation of B.

Solution:



① *Elevation of the H.I. of the instrument at A:*

= 261.60 + 5

= ***266.60***

② *Elevation of the H.I. of the instrument at C:*

= 261.60 + 5 + 2.8

= ***269.4***

ERRORS IN TRANSIT WORK

③ *Elevation of B:*

$$\tan 24°25' = \frac{2.8}{x}$$

$x = 6.17$ m.

$200 - x = 193.83$ m.

$$\frac{DB}{\text{Sin } 155°35'} = \frac{193.83}{\text{Sin } 7°56'}$$

$DB = 580.52$ ft.

$h = 580.52$ Stin $16°29'$

$h = 164.71$ m.

Elevation of B = 261.60 + 5 + 164.71

Elevation of B = ***431.21 m.***

Problem 106:

① The horizontal axis of a transit was inclined at 4' with the horizontal due to non-adjustment. The first sight had a vertical angle of 50°, the next had - 30°. Determine the error in the measured horizontal angle.

② A transit is set up at B and a backsight at A. By double reversal two points C and D at a distance equal to 0.145 m. were established. If BC = 250 m. and BD = 250 m. (app.), how much is the angular error of the line of sight from true position.

③ In testing for the magnifying power of a level telescope, a transit is set up and the angle between two very far points which are very near each other has been found to be 5°15'. The level telescope whose magnifying power is desired is placed in front of the transit telescope with its objective close to the objective end of the transit telescope. Again the same angle is measured thru the two telescope and found to be 09'. What is the magnifying power of the level telescope?

Solution:

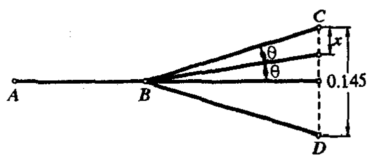
① *Error in horizontal angle:*

$E = e\,(\tan h_2 - \tan h_1)$

$E = 04'\,[\tan 50° - \tan(-30°)]$

$E =$ ***7'4.6"***

② *Angular error of line:*



$$x = \frac{1}{4}(0.145)$$

$x = 0.03625$

$$\text{Sin } \theta = \tan \theta = \frac{0.03625}{250}$$

$\theta = 30"$

Angular error = 2(30")

Angular error = ***60"***

③ *Magnifying power:*

$$M.P. = \frac{5°15'}{09'}$$

$$M.P. = \frac{315}{9}$$

MP = ***35 diameters***

TRIANGULATION

Triangulation - a method for extending horizontal control for topographic and similar surveys which require observations of triangular figures whose angles are measured and whose sides are determined by trigonometric computations.

Four common geometric figures used in triangulation:

1. Chain of single and independent triangles.
2. Chain of quadrilaterals formed with overlapping triangles.
3. Chain of polygons or central-point figures.
4. Chain of polygons each with an extra diagonal.

Approximate method of adjusting the angles and sides of triangulation systems.

1. Station Adjustment
2. Figure Adjustment

Two methods of adjustment of Quadrilateral

1. Angle Condition Equations
2. Side Condition Equations

Station Adjustment and Figure Adjustment

1. Sum of angles about a station = 360°
2. Sum of three angles in each triangle = 180°

Problem 107:

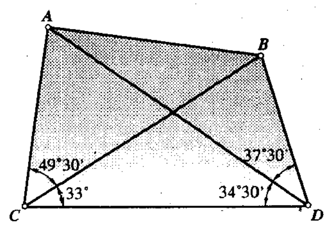
In the quadrilateral ABCD, the following angles are measured.

Angle BCD = 33°
Angle ADC = 34°30'
Angle ACB = 49°30'
Angle ADB = 37°30'

① Find the angle CAD.
② Find the angle BAD.
③ Find the angle ABC.

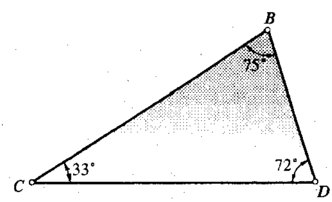
Solution:

① *Angle CAD:*



Angle CAD = 180° - 49°30' - 33° - 34°30'
Angle CAD = **63°**

② *Angle BAD:*
Consider triangle BCD:



Assume CD = 1

$$\frac{BC}{\text{Sin } 72^\circ} = \frac{1}{\text{Sin } 75^\circ}$$

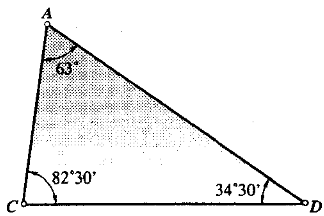
BC = 0.985

$$\frac{BD}{\text{Sin } 33^\circ} = \frac{1}{\text{Sin } 75^\circ}$$

BD = 0.564

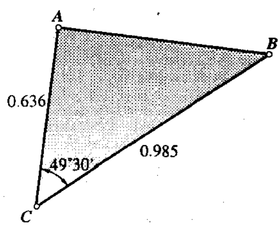
TRIANGULATION

Consider triangle ADC:



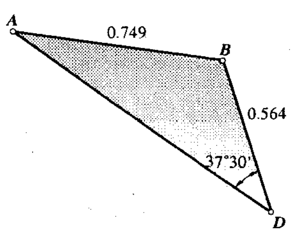
$$\frac{AC}{\text{Sin } 34°30'} = \frac{1}{\text{Sin } 63°}$$
$AC = 0.636$

Consider triangle ABC:



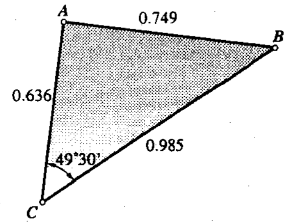
Using Cosine Law:
$$(AB)^2 = (0.636)^2 + (0.985)^2 - 2\,(0.636)(0.985)\text{ Cos } 49°30'$$
$AB = 0.749$

Consider triangle ABD:



Using Sine Law:
$$\frac{0.564}{\text{Sin } A} = \frac{0.749}{\text{Sin } 37°30'}$$
$A = 27°17'$
Angle BAD = ***27°17'***

③ *Angle ABC:*



$$\frac{0.636}{\text{Sin } B} = \frac{0.749}{\text{Sin } 49°30'}$$
$B = 40°13'$
Angle ABC = ***40°13'***

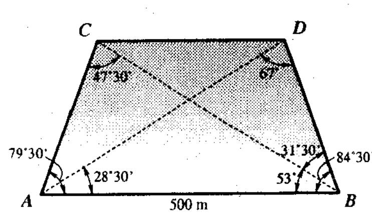
Problem 108:

From two inaccessible but intervissiible points A and B, the angles to two triangulation stations C and D were observed as follows: Line AB = 500 m. long.

Angle CAB = 79°30'
Angle DAB = 28°30'
Angle DBC = 31°30'
Angle DBA = 84°30'

① Find the distance BC.
② Find the distance BD.
③ Find the distance CD.

Solution:
① *Distance BC:*



$$\frac{BC}{\text{Sin } 79°30'} = \frac{500}{\text{Sin } 47°30'}$$
BC = ***666.81 m.***

② *Distance BD:*

$$\frac{BD}{\text{Sin } 28°30'} = \frac{500}{\text{Sin } 67°}$$

BD = ***259.18 m.***

③ *Distance CD:*

$$(CD)^2 = (BC)^2 + (BD)^2 - 2(BC)(BD) \text{ Cos } 31°30'$$

$$(CD)^2 = (666.81)^2 + (259.18)^2 - 2(666.81)(259.18) \text{ Cos } 31°30'$$

CD = ***465.94 m.***

Problem 109

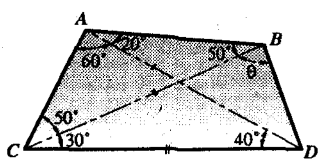
Triangulation stations A, B, C and D has the following observation angles.

Angle	Values
ACB	50°
BAD	20°
BCD	30°
ABC	50°
CAD	60°

① Find the angle CBD.
② Find the angle BDA.
③ Find the angle BDC.

Solution:

① *Angle CBD:*



Assume AC = 1.0

$$\frac{1}{\text{Sin } 50°} = \frac{BC}{\text{Sin } 80°}$$

BC = 1.2856

$$\frac{CD}{\text{Sin } 60°} = \frac{1}{\text{Sin } 40°}$$

CD = 1.3473

$$\frac{AD}{\text{Sin } 80°} = \frac{1}{\text{Sin } 40°}$$

AD = 1.532

$$\frac{AB}{\text{Sin } 50°} = \frac{1}{\text{Sin } 50°}$$

AB = 1.0

$$(BD)^2 = (AB)^2 + (AD)^2 - 2(AB)(AD) \text{ Cos } 20°$$

$$(BD)^2 = (1)^2 + (1.532)^2 - 2(1)(1.532) \text{ Cos } 20°$$

BD = 0.684

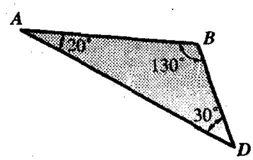
$$\frac{AD}{\text{Sin } (50 + \theta)} = \frac{BD}{\text{Sin } 20°}$$

$$\frac{1.532}{\text{Sin } (50 + \theta)} = \frac{0.684}{\text{Sin } 20°}$$

$50 + \theta = 130°$

θ = ***80°*** *(angle CBD)*

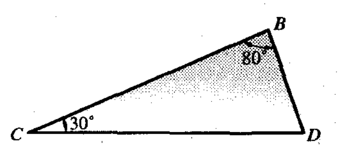
② *Angle BDA:*



BDA = 180 - (20 + 130)

BDA = ***30°***

③ *Angle BDC:*



BDC = 180 - (30 + 80)

BDC = ***70°***

TRIANGULATION

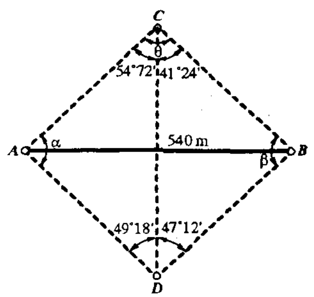
Problem 110

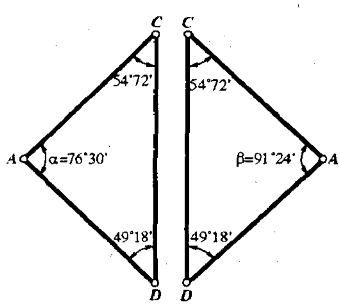
Two stations A and B are 540 m. apart. From the following triangulation stations C and D on opposite sides of AB, the following angles were observed.

Angle ACD = 54°12'
Angle DCB = 41°24'
Angle ADC = 49°18'
Angle BDC = 47°12'

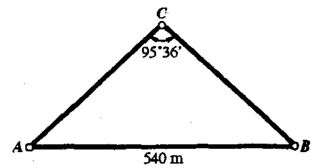
① Find the distance BC.
② Find the distance CD.
③ Find the distance AC.

Solution:





① *Distance BC:*



$\alpha = 180° - 54°12' - 49°18'$

$\alpha = 76°30'$

$\beta = 180° - 41°24' - 47°12'$

$\beta = 91°24'$

$\phi = 54°12' + 41°24'$

$\phi = 95°36'$

Using Cosine Law
Considering triangle ABC:

$(AB)^2 = (BC)^2 + (AC)^2 - 2(BC)(AC) \cos 95°36'$

$(540)^2 = (BC)^2 + (1.062\,BC)^2 - 2(BC)(1.062\,BC) \cos 95°36'$

$BC = \mathbf{353.38\ m.}$

② *Distance CD:*
Using Sine Law
Considering triangle ADC:

$$\frac{CD}{\sin 76°30'} = \frac{AC}{\sin 49°18'}$$

$CD = 1.283\,AC$

Considering triangle CDB:

$$\frac{CD}{\sin 91°24'} = \frac{BC}{\sin 47°12'}$$

$CD = 1.362\,BC$

$CD = 1.362\,(353.38)$

$CD = \mathbf{481.30\ m.}$

③ *Distance AC:*

$1.283\,AC = 1.362\,BC$

$AC = 1.062\,BC$

$AC = 1.062\,(353.38)$

$AC = \mathbf{375.38\ m.}$

Problem 111:

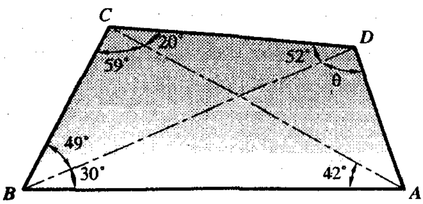
The observed angles of a quadrilateral after station and side adjustments are given in the accompanying tabulation:

Angles	Observed Values	Angles	Observed Values
DBA	30°	BCA	59°
CBD	49°	DCA	20°

① Compute the angle BDA.
② Compute the angle DAC.
③ Compute the angle DAB.

Solution:

① *Angle BDA:*



Assume BC = 1.0

$$\frac{AC}{\text{Sin } 79°} = \frac{1.0}{\text{Sin } 42°}$$

AC = 1.467

$$\frac{CD}{\text{Sin } 49°} = \frac{1}{\text{Sin } 52°}$$

CD = 0.958

Using Cosine Law:

$$(AD)^2 = (0.958)^2 + (1.467)^2 - (0.958)(1.467)\text{ Cos } 20°$$

AD = 0.655 m.

Using Sine Law:

$$\frac{AC}{\text{Sin }(52 + \theta)} = \frac{AD}{\text{Sin } 20°}$$

$$\frac{1.467}{\text{Sin }(52 + \theta)} = \frac{0.655}{\text{Sin } 20°}$$

$52 + \theta = 50°$ or $130°$

$52 + \theta = 130°$

$\theta = 78°$

Angle BDA = ***78°***

② *Angle DAC:*

Angle DAC + 42° + 30° + 78° = 180

Angle DAC = ***30°***

③ *Angle DAB:*

Angle DAB = 42° + 30°

Angle DAB = ***72°***

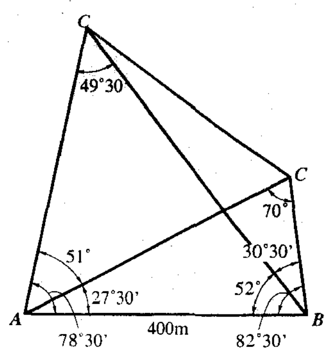
Problem 112:

The baseline AB of a triangulation system is equal to 400 m. long. Stations C and D are other points of the triangulation system. The angles observed from A and B are as follows. Angle DAB = 27°30', angle CAB = 78°30', angle CBA = 52° and angle DBC = 30°30'.

① Compute the distance CB.
② Compute the distance DB.
③ Compute the distnace DC.

Solution:.

① *Distance CB:*



$$\frac{400}{\text{Sin } 49°30'} = \frac{CB}{\text{Sin } 78°30'}$$

CB = ***515.47 m.***

② *Distance DB:*

$$\frac{DB}{\text{Sin } 27°30'} = \frac{400}{\text{Sin } 70°}$$

DB = ***196.55 m.***

TRIANGULATION

③ *Distance DC:*

$(DC)^2 = (515.47)^2 + (196.55)^2 - 2(515.47)(196.55)\cos 30°30'$

DC = ***360.21 m.***

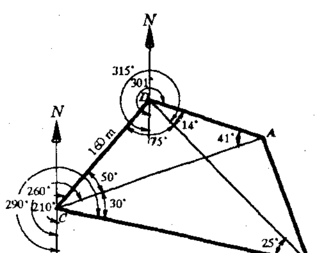
Problem 113:

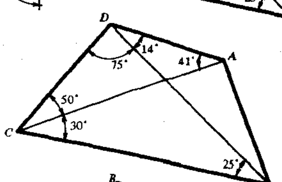
A and B are two points located on each bank of a river and near the abutments of a proposed bridge. To determine its distance, a baseline CD 180 m. long was established on one bank of the river and the transit was set up at stations C and D and the azimuth were taken as follows:

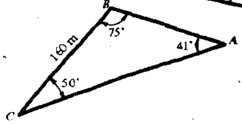
LINE	AZIMUTH
C-D	210°00'
C-A	260°00'
C-B	290°00'
D-A	301°00'
D-B	315°00'

① Find the distance AD.
② Find the distance BD.
③ Find the distance AB.

Solution:





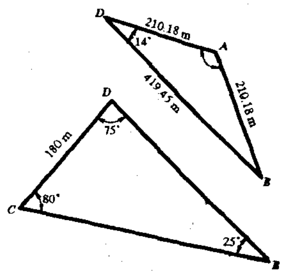


① *Distance AD:*

Consider triangle CDA:

$$\frac{AD}{\sin 50°} = \frac{180}{\sin 41°}$$

AD = ***210.18 m.***



② *Distance BD:*

Considering triangle CDB:

Using Sine Law

$$\frac{180}{\sin 25°} = \frac{BD}{\sin 80°}$$

BD = ***419.45 m.***

③ *Distance AB:*

Consider triangle ABD:

Using Cosine Law

$(AB)^2 = (210.18)^2 + (419.45)^2 - 2(210.18)(419.45)\cos 14°$

AB = ***221.43 m.***

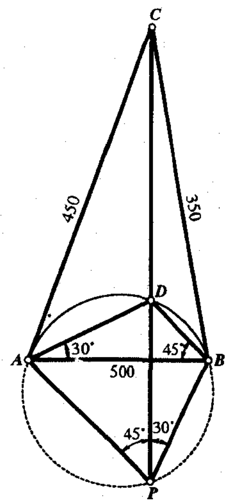
Problem 114:

In a topographic survey, three triangulation stations A, B and C are sighted from a point P. The distance between the stations are AB = 500 m., BC = 350 m. and CA = 450 m. At P, the angle subtending AC is 45° while for BC is 30°. AC is due North.

① Find the distance CD.
② Find the distance PA.
③ Find the azimuth of PA.

Solution:

① *Distance CD:*



From the triangle ABD:

$$\frac{AD}{\text{Sin } 45°} = \frac{500}{\text{Sin } 105°}$$

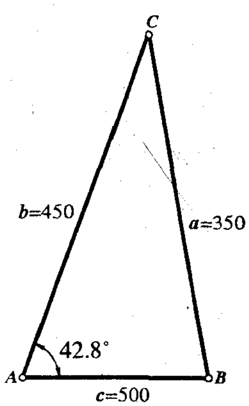
$$AD = \frac{500 \text{ Sin } 45°}{\text{Sin } 105°}$$

$AD' = 366$ m.

$$\frac{BD}{\text{Sin } 30°} = \frac{500}{\text{Sin } 75°}$$

$$BD = \frac{500 \text{ Sin } 30°}{\text{Sin } 75°}$$

$BD = 259$ m.



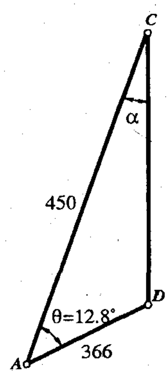
Taking triangle ABC:

$a = 350 \quad b = 450 \quad c = 500$

Using Cosine Law:

$(350)^2 = (450)^2 + (500)^2 - 2(450)(500) \text{ Cos } A$

$A = 42.8°$



From triangle ACD:

$ø = 42.8° - 30°$

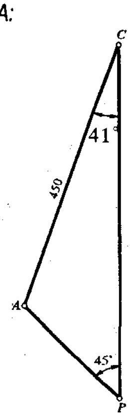
$ø = 12.8°$

Using Cosine Law:

$(CD)^2 = (450)^2 + (366)^2$
$\quad - 2(450)(366) \text{ Cos } 12.8°$

$CD =$ ***123.65 m.***

② *Distance PA:*



$$\frac{123.65}{\text{Sin } 12.8°} = \frac{366}{\text{Sin } \alpha}$$

$$\text{Sin } \alpha = \frac{366 \text{ Sin } 12.8°}{123.65}$$

$\alpha = 41°$

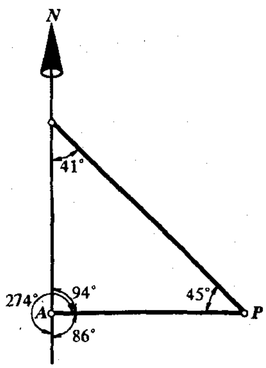
TRIANGULATION

$$\frac{450}{\text{Sin } 45^\circ} = \frac{PA}{\text{Sin } 41^\circ}$$

$$PA = \frac{450 \text{ Sin } 41^\circ}{\text{Sin } 45^\circ}$$

$PA = \mathbf{417.55\ mm}$

③ *Azimuth of PA:*



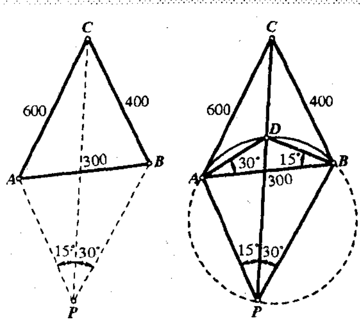
Azimuth of AP = 274°
Azimuth of PA = 274° - 180°
Azimuth of PA = **94°**

Problem 115:

From the figure shown,

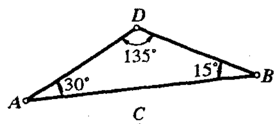
AC = 600 m. APC = 15°
BC = 400 m. CPB = 30°
AB = 300 m.

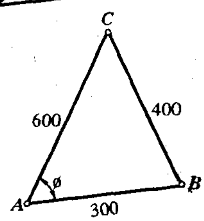
① Find the distance CD.
② Find the distance AP.
③ Find the distance of BP.

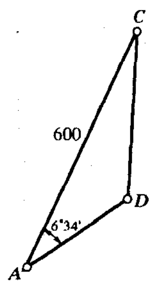


Solution:

① *Distance CD:*







Construct a circle passing through A, B and P.

Considering triangle ABD:

Using Sine Law

$$\frac{AD}{\text{Sin } 15^\circ} = \frac{300}{\text{Sin } 135^\circ}$$

$AD = 109.81$ m.

Considering triangle ACB:

Using Cosine Law

$(400)^2 = (600)^2 + (300)^2 - 2(600)(300) \text{ Cos } ø$

$360000 \text{ Cos } ø = (600)^2 + (300)^2 - (400)^2$

$ø = 36.34^\circ$

$ø = 36^\circ 20'$

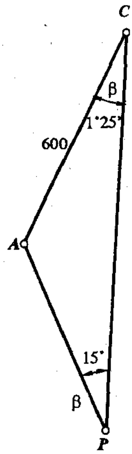
Considering triangle ACD:

Using Cosine Law:

$(CD)^2 = (600)^2 + (109.81)^2$
$\quad - 2 (600)(109.81) \text{ Cos } 6.34^\circ$

$CD = \mathbf{491.01\ m.}$

② *Distance AP:*



Using Sine Law

$$\frac{\text{Sin } \beta}{109.81} = \frac{\text{Sin } 6.34°}{491.01}$$

$\beta = 1.42°$

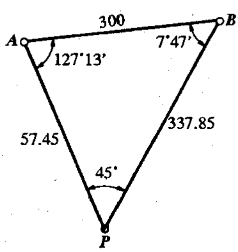
$\beta = 1°25'$

Considering triangle ACP:
Using Sine Law

$$\frac{600}{\text{Sine } 15°} = \frac{AP}{\text{Sine } 1°25'}$$

AP = ***57.45 m.***

③ *Azimuth of BP:*



Considering triangle ABP:

$$\frac{\text{Sin B}}{57.45} = \frac{\text{Sin } 45°}{300}$$

$B = 7°47'$

$A = 180° - 45° - 7°47'$

$A = 127°13'$

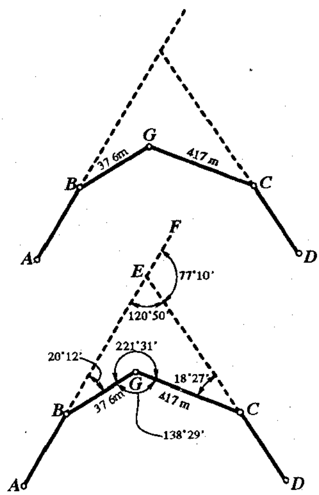
$$\frac{BP}{\text{Sine } 127°13'} = \frac{300}{\text{Sin } 45°}$$

BP = ***337.85 m.***

Problem 116:

Two lines AB and CD as shown in the figure, intersect at some in accessible point E. The points B and C are not visible from each other but both can be seen from the point G. If the angle EBG = 20°12', ECG = 15°27'. BGC = 138°29', and the distances BG and CG are 376 and 417 meters respectively.

① Determine the angle of intersection FEC.
② Determine the distance BC.
③ Determine the distance EC.



Solution:

① *Angle of intersection FEC:*
Angle FEC = ***77°10'***

② *Distance BC:*

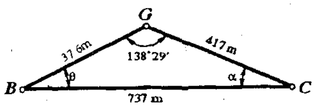
$(BC)^2 = (376)^2 + (417)^2 - 2(376)(417) \text{ Cos } 138°29'$

$(BC)^2 = 137500 + 16900 + 236000$

BC = ***737 m.***

TRIANGULATION

③ *Distance EC:*



$$\frac{\text{Sin } \phi}{417} = \frac{\sin 138°29'}{737}$$

$$\text{Sin } \phi = \frac{417 \text{ Sin } 41°31'}{737}$$

$$\phi = 21°36'$$

$$\alpha + \phi + 138°29' = 180;$$

$$\alpha = 19°55'$$

$$\frac{BE}{\text{Sin } 53°22'} = \frac{737}{\text{Sin } 102°50'}$$

$$BE = \frac{737 \text{ Sin } 35°22'}{\text{Sin } 77°10'}$$

$$BE = 435 \text{ m.}$$

$$\frac{CE}{\text{Sin } 41°48'} = \frac{737}{\text{Sin } 102°50'}$$

$$CE = \frac{737 \text{ Sin } 41°48'}{\text{Sin } 77°10'}$$

$CE =$ ***505 m.***

Problem 117:

From the figure shown:

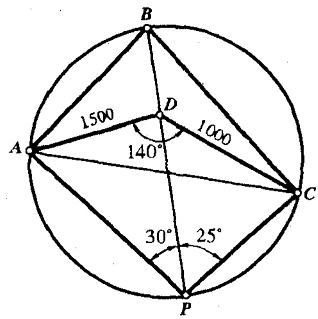
Angle APB = 30°00'
Angle BPC = 25°00'
Angle ADC = 140°00'

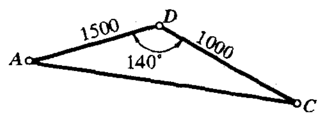
AD = 1500 m.
CD = 1000 m.

① Find the distance BD.
② Find the distance AP.
③ Find the distance PC.

Solution:

① *Distance BD:*





Considering triangle ADC:
Using Sine Law

$$\frac{\text{Sin } A}{1000} = \frac{\text{Sin } 140°}{2355.45}$$

$$A = 15°50'$$

$$C = 180° - 140° - 15°50'$$

$$C = 24°10'$$

Considering ABC:
Using Sine Law

$$\frac{AB}{\text{Sin } 30°} = \frac{2355.45}{\text{Sin } 125°}$$

$$AB = 1437.74 \text{ m.}$$

$$\frac{BC}{\text{Sin } 25°} = \frac{2355.45}{\text{Sin } 125°}$$

$$BC = 1215.23 \text{ m.}$$

Considering triangle ABD:

$$\phi = 25° - 15°50'$$

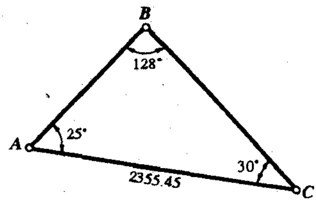
$$\phi = 9°10'$$

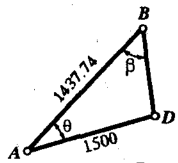
Using Cosine Law

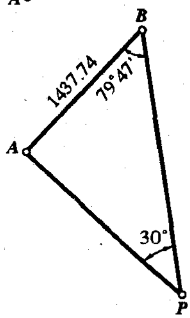
$$(BD)^2 = (1500)^2 + (1437.74)^2 - 2(1500)(1437.74) \text{ Cos } 9°10'$$

$BD =$ ***242.90 m.***

② *Distance AP:*







Using Sine Law

$$\frac{\text{Sin } B}{1500} = \frac{\text{Sin } 9°10'}{242.90}$$

$B = 70°47'$

Using Cosine Law

$$(AC)^2 = (1500)^2 + (1000)^2 - 2(1500)(1000)\text{ Cos } 140°$$

$AC = 2355.45$ m.

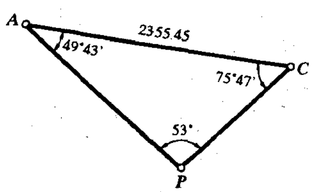
Consider triangle ABP:

Using Sine Law

$$\frac{AP}{\text{Sin } 79°47'} = \frac{1437.74}{\text{Sin } 30°}$$

$AP =$ ***2829.86 m.***

③ *Distance PC:*



Consider triangle APC:

$$\frac{\text{Sin } ACP}{2829.86} = \frac{\text{Sin } 55°}{2355.45}$$

Angle $ACP = 79°47'$

Angle $CAP = 180° - 55° - 79°47'$

Angle $CAP = 45°43'$

Using Sine Law:

$$\frac{PC}{\text{Sin } 45°43'} = \frac{2355.45}{\text{Sin } 55°}$$

$PC =$ ***2058.66 m.***

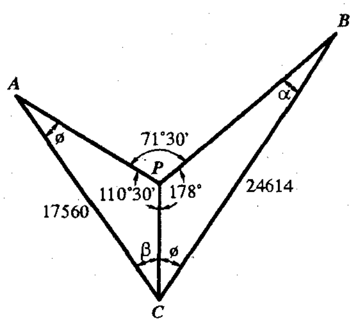
Problem 118:

In measuring the angles at a triangulation station C, it was necessary to set the transit over another point P at a distance of 7.46 m. from C. The angle measured at P from C to station A was 110°30' and the angle between stations A and B measured from P is found to be 71°30'. Distance AC is 17560 m., and BC = 24614 m. long.

① Reduce the angle to center C, that is solve for angle ACB.
② Compute the distance AP.
③ Compute the distance PB.

Solution:

① *Angle ACB:*



Angle $CPB = 360° - 71°30' - 10°30'$

Angle $CPB = 178°$

Using Sine Law:

$$\frac{7.46}{\text{Sin } ø} = \frac{17560}{\text{Sin } 110°30'}$$

$ø = 01' 22"$

$$\frac{7.46}{\text{Sin } \alpha} = \frac{24614}{\text{Sin } 178°}$$

$\alpha = 00' 2.18"$

$ß = 180° - 110°30' - 01'22"$

$ß = 69°31' 22"$

$ø = 180° - 178° - 0' 2.18"$

$ø = 1°59' 57.82"$

Angle ACB $= 69°31' 22" + 1°59' 57.82"$

Angle ACB $=$ ***71°31' 19.82"***

② *Distance AP:*

$$\frac{AP}{\text{Sin } 69°31' 22"} = \frac{17560}{\text{Sin } 110°30'}$$

AP = ***17562.61 m.***

③ *Distance PB:*

$$\frac{PB}{\text{Sin } 1°59' 57.82"} = \frac{24614}{\text{Sin } 178°}$$

PB = 24606.55 m.

Problem 119:

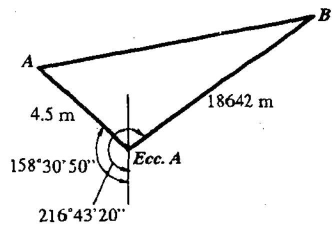
In a triangulation an eccentric station (Ecc. A) is occupied instead of the true station A. Observations are then made to true station A and to station B. The observation are as follows:

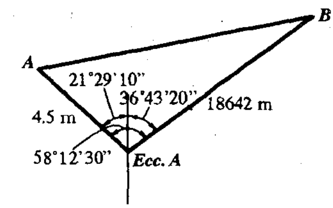
LINE	AZIMUTH	DISTANCE
Ecc. A to True A	158°30' 50"	4.50 m.
Ecc. A to B	216°43' 20"	18642.00 m.

① Find the distance AB.
② Find the angle BA Ecc. A.
③ Compute the azimuth of AB.

Solution:

① *Distance AB:*



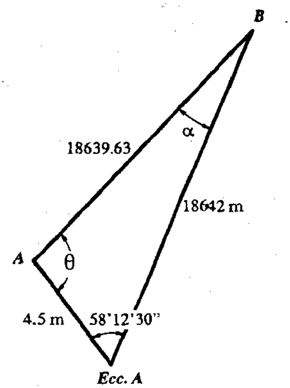


Using Cosine Law:

$(AB)^2 = (4.50)^2 + (18642)^2$
$\quad - 2(4.50)(18642) \text{ Cos } 58°12' 30"$

AB = ***18639.63 m.***

② *Angle BA Ecc. A:*

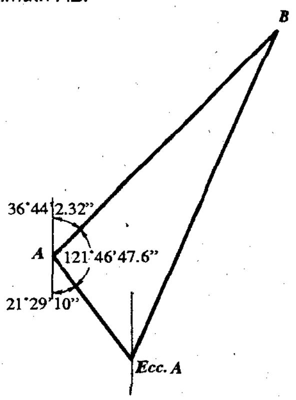


Using Sine Law:

$$\frac{18642}{\text{Sin } \theta} = \frac{18639.63}{\text{Sin } 158°12' 30"}$$

$\theta =$ ***121°46' 47.6"***

③ *Azimuth AB:*



Azimuth AB = 180° + 36°44' 2.32"
Azimuth AB = ***216°44' 2.32"***

Problem 120:

Given the three point resection with the following data:

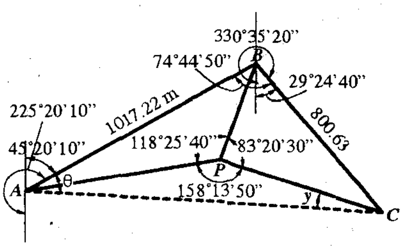
LINE	AZIMUTH	DISTANCE
A - B	225°20' 10"	1017.22 m.
B - C	330°35' 20"	800.63 m.

The instrument is at P, south west of B with angle APB = 118°25' 40" and angle BPC = 83°20' 30".

① Compute the angle PCA.
② Find the angle PCA.
③ Find the distance BP.

Solution:

① *Angle PCA:*

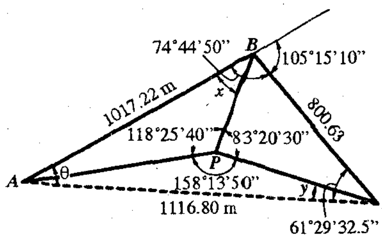


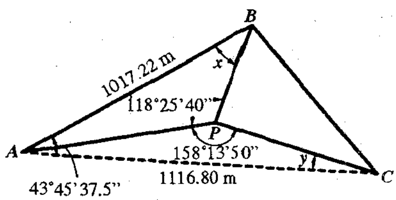
Using Cosine Law:
$(AC)^2 = (1017)^2 + (800.63)^2$
$- 2(1017.22)(800.63) \text{ Cos } 74°44' 50"$
$AC = 1116.80$ m.

Using Sine Law:

$$\frac{800.63}{\text{Sin } \theta} = \frac{1116.80}{\text{Sin } 74°44' 50"}$$

$\theta = 43°45' 37.25"$





Angle BCA = 180° - 43°45' 37.5" - 74°44' 50"
Angle BAC = 61°29' 32.5"
$x + 118°25' 40" + 158°13' 50" + y$
$+ 43°45' 37.5" = 360$
$x + y = 39°34' 52.5"$

$$\frac{AP}{\text{Sin } x} = \frac{1017.22}{\text{Sin } 118°25' 40"}$$

AP = 1156.70 Sin x

$$\frac{AP}{\text{Sin } y} = \frac{116.80}{\text{Sin } 158°13' 50"}$$

$AP = 3011.28 \text{ Sin } y$

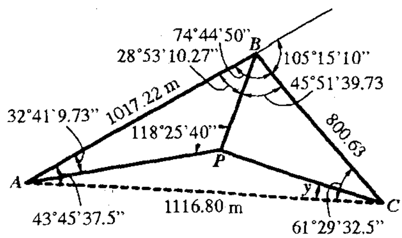
$1156.70 \text{ Sin } x = 3011.28 \text{ Sin } y$
$\text{Sin } x = 2.603 \text{ Sin } y$
$x = 39°34' 52.5" - y$
$\text{Sin } (39°34' 52.5" - y) = 2.603 \text{ Sin } y$
$\text{Sin } 39°34' 52.5" \text{ Cos } y - \text{Sin } y \text{ Cos } 39°34' 52.5"$
$= 2.603 \text{ Sin } y$
$0.63717 \text{ Cos } y - 0.77072 \text{ Sin } y = 2.603 \text{ Sin } y$
$0.63717 \text{ Cos } y = 3.37372 \text{ Sin } y$

$\tan y = \dfrac{0.63717}{3.37372}$

$y = 10°41'42.23"$

Angle PCA = ***10°41'42.23"***

② *Azimuth of BP:*



$x + y = 39°34'52.5"$

$x = 39°34'52.5" - 10°41'42.23"$

$x = 28°53'10.27"$

Angle BAP = 180° - 28°53'10.27" - 118°25'40"

Angle BAP = 32°41'9.73"

Azimuth of BP = 225°20'10" + 105°15'10" + 45°51'39.73"

Azimuth of BP = 376°26'59.7"

Azimuth of BP = ***16°26'59.7"***

③ *Distance BP:*

$$\frac{BP}{\sin 32°41'9.73"} = \frac{1017.22}{\sin 118°25'40"}$$

BP = ***624.66 m.***

Problem 121:

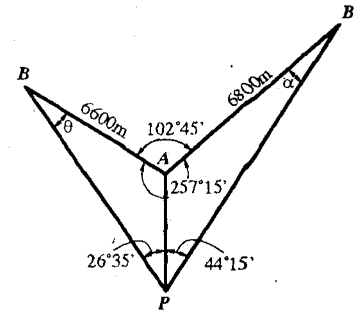
In the figure shown, stations A, B and C are observed from P whose known data are:

Angle BAC	=	102°45'
Angle BPA	=	26°35'
Angle APC	=	44°15'
Distance AC	=	6800 m.
Distance AB	=	6600 m.

① Find angle ABP.
② Find angle ACP.
③ Find distance BP.

Solution:

① *Angle ABP:*



$\theta + \beta + 257°15' + 26°35' + 44°15' = 360°$

$\theta + \beta = 31°55'$

Using Sine Law:

$$\frac{AP}{\sin\theta} = \frac{6600}{\sin 26°35'}$$

$AP = 1474.64 \sin\theta$

$$\frac{\sin AP}{\sin\beta} = \frac{6800}{\sin 44°15'}$$

$AP = 9745.05 \sin\beta$

$1474.64 \sin\theta = 9745.05 \sin\beta$

$\sin\theta = 0.6607 \sin\beta$

$\theta = 31°55' - \beta$

$\sin(31°55' - \beta) = 0.6607 \sin\beta$

$\sin 31°55' \cos\beta - \cos 31°55' \sin\beta = 0.6607 \sin\beta$

$1.50952 \sin\beta = 0.52869 \cos\beta$

$\beta = 19°18'$

$\theta = 31°55' - 19°18'$

$\theta = 12°37'$

Angle ABP = ***12°37'***

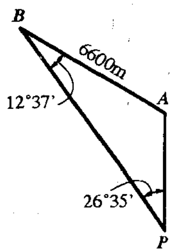
② *Angle ACP:*

Angle ACP = ß

Angle ACP = ***19°18'***

TRIANGULATION

③ ***Distance BP:***



Angle BAP = 180° - 12°37' - 26°35'
Angle BAP = 140°48'

$$\frac{BP}{\sin 140°48'} = \frac{6600}{\sin 26°35'}$$

BP = ***9321.57 m.***

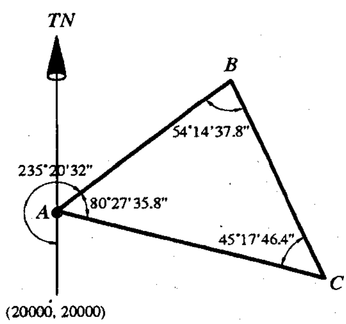
Problem 122:

Station C is located by intersection from triangulation points B and A for which the coordinates of corner A as 20000 Northings and 20000 Eastings. The distance and azimuth from north of the line A to B are 895.86 m. and 235°20' 32" respectively. The measured horizontal angles are angle BAC = 80°27' 35.8" and angle CBA = 54°14' 37.8".

① Compute the distance BC.
② Compute the distance AC.
③ Compute the coordinates of corner C by intersection.

Solution:

① *Distance BC:*



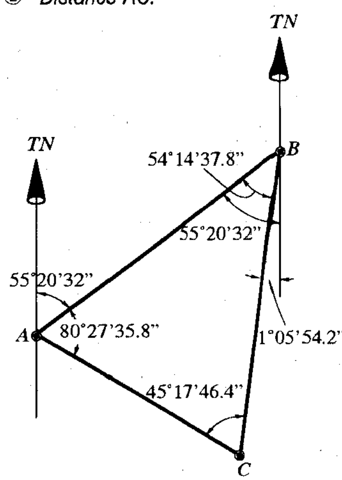
Angle BCA = 180° - 80°27' 35.8" - 54°14' 37.8"
Angle BCA = 45°17' 46.4"

Using Sine Law:

$$\frac{895.86}{\sin 45°17'46.4"} = \frac{BC}{\sin 80°27'35.8"}$$

BC = ***1243.01 m.***

② *Distance AC:*



Using Sine Law:

$$\frac{AC}{\sin 54°14'37.8"} = \frac{895.86}{\sin 45°17'46.4"}$$

AC = ***1022.86 m.***

③ Coordinates of corner C:

STA.	LINE	BEARING	DISTANCE
A	AB	N. 55°20'32" E.	895.86
B	BC	S. 1°05'54.2" W.	1243.01

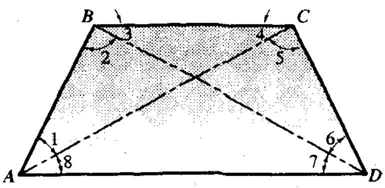
STA.	LAT	DEP
A	20000.00	20000.00
	+ 736.90	+ 509.45
B	20736.90	20509.45
	- 23.83	- 1242.78
C	20713.07	19266.67

Coordinates of C = ***20713.07 N., 19266.67 E.***

TRIANGULATION

Adjustment of a Quadrilateral

A. ***Angle condition equations.***



1. $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$
2. $\angle 3 + \angle 4 + \angle 5 + \angle 6 = 180^\circ$
3. $\angle 1 + \angle 2 + \angle 7 + \angle 8 = 180^\circ$
4. $\angle 5 + \angle 6 + \angle 7 + \angle 8 = 180^\circ$
5. $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 180^\circ$
6. $\angle 1 + \angle 2 = \angle 5 + \angle 6$
7. $\angle 3 + \angle 4 = \angle 7 + \angle 8$

B. ***Side Consition equations.***

$$\frac{\sin\angle 1 \sin\angle 3 \sin\angle 5 \sin\angle 7}{\sin\angle 2 \sin\angle 4 \sin\angle 6 \sin\angle 8} = 1$$

$$\frac{AB}{\sin\angle 4} = \frac{BC}{\sin\angle 1}$$

$$BC = \frac{AB\sin\angle 1}{\sin\angle 4}$$

$$\frac{BC}{\sin\angle 6} = \frac{CD}{\sin\angle 3}$$

$$BC = \frac{CD\sin\angle 6}{\sin\angle 3}$$

$$\frac{AB\sin\angle 1}{\sin\angle 4} = \frac{CD\sin\angle 6}{\sin\angle 3}$$

$$\frac{AB}{\sin\angle 7} = \frac{AD}{\sin\angle 2}$$

$$AD = \frac{AB\sin\angle 2}{\sin\angle 7}$$

$$\frac{AD}{\sin\angle 5} = \frac{CD}{\sin\angle 8}$$

$$AD = \frac{CD\sin\angle 5}{\sin\angle 8}$$

$$\frac{AB\sin\angle 2}{\sin\angle 7} = \frac{CD\sin\angle 5}{\sin\angle 8}$$

$$CD = \frac{AB\sin\angle 1 \sin\angle 3}{\sin\angle 4 \sin\angle 6}$$

$$\frac{AB\sin\angle 2}{\sin\angle 7} = \frac{AB\sin\angle 1 \sin\angle 3 \sin\angle 5}{\sin\angle 4 \sin\angle 8 \sin\angle 6}$$

$$\frac{\sin\angle 1 \sin\angle 3 \sin\angle 5 \sin\angle 7}{\sin\angle 2 \sin\angle 4 \sin\angle 6 \sin\angle 8} = 1$$

Strength of Figure:

In a triangulation system, to be able to adopt the best shaped triangulation network it is necessary to apply a criterion of strength to the different figures that may be formed such an index or criterion is known as the strength of figure and is given in the equation:

$$R = \frac{D - C}{D}\sum(\Delta_A^2 + \Delta_A\Delta_B + \Delta_B^2)$$

$$F = \frac{D - C}{D}$$

where

R = relative strength of figure

D = number of directions observed (forward and backward) not including the fixed or known side of a given figure.

C = number of geometric conditions to be satisfied in a given figure.

F = a factor for computing the strength of figure and is equal to $\frac{D - C}{D}$

Δ_A , Δ_B = tabular difference for 1 second, expressed in units of the 6th decimal place corresponding to the distance angles A and B of a triangle.

$\sum(\Delta_A^2 + \Delta_A\Delta_B + \Delta_B^2)$ = summation of values for particular chain of triangles through which computation is carried from the known line to the line required.

C = (n' - s' + 1) + (n - 2s + 3)

n' = number of lines observed in both directions, including the known side of the given figure.

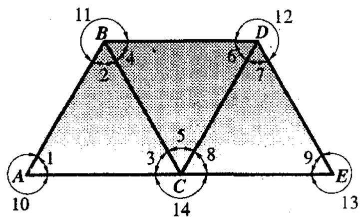
s' = number of occupied stations

n = total number of lines in the figure including known line.

s = total number of stations

Problem 123:

From the chain of triangles as shown in the figure, perform the station and figure adjustment using approximate method.



Tabulated data:

Angle	Observed Value	Angle	Observed Value
1	58°25'15"	8	63°10'08"
2	59°10'05"	9	45°10'20"
3	62°25'10"	10	301°34'49"
4	60°29'10"	11	240°21'00"
5	59°25'10"	12	228°14'51"
6	60°05'10"	13	314°49'42"
7	71°40'02"	14	174°59'24"

① Compute the corrected value of angle 3.
② Compute the corrected value of angle 6.
③ Compute the corrected value of angle 9.

Solution:

① *Corrected value of angle 3:*
Station Adjustment:
Station A:
58°25'15" + 301°34'49" = 360°00'04"
Error = 04'

$$Correction = \frac{04'}{2} = 02'$$

Adjusted angle:
Angle 1 = 58°25'15" - 02" = 58°25'13"
Angle 10 = 301°34'49" - 02" = 301°34'47"
360°00'00"

Station B:
Angle 2 = 59°10'05"
Angle 4 = 60°29'10"
Angle 11 = 240°21'00"
360°00'15"

$$Error = \frac{15"}{3} = 05"$$

Adjusted angle 2 = 59°10'00"
Adjusted angle 4 = 60°29'05"
Adjusted angle 11 = 240°20'55"
360°00'00"

Station C:
Angle 3 = 62°25'10"
Angle 5 = 59°25'10"
Angle 8 = 63°10'08"
Angle 14 = 174°59'24"
359°59'52"

Error = 08"

$$Correction = \frac{08'}{4} = 02"$$

Adjusted angle 3 = 62°25'12"
Adjusted angle 5 = 59°25'12"
Adjusted angle 8 = 63°10'10"
Adjusted angle 14 = 174°59'26"
360°00'00"

Station D:
Angle 6 = 60°05'10"
Angle 7 = 71°40'20"
Angle 12 = 228°14'52"
360°00'03"

Error = 03"

$$Correction = \frac{03"}{3} = 01"$$

Adjusted angle 6 = 60°05'09"
Adjusted angle 7 = 71°40'01"
Adjusted angle 12 = 228°14'50"
360°00'00"

Station E:
Angle 9 = 45°10'20"
Angle 13 = 314°49'42"
360°00'02"

Error = 02"

TRIANGULATION

$$Correction = \frac{02''}{2} = 01''$$

Adjusted angle 9 = 45°10'19"
Adjusted angle 13 = 314°49'41"
360°00'00"

Figure Adjustment
Considering triangle ABC
Angle 1 = 58°25'13"
Angle 2 = 59°10'00"
Angle 3 = 62°25'12"
180°00'25"
Error = 25"

Adjusted Angle 1 = 58°25'05" - 08" = 58°25'05"
Adjusted Angle 2 = 59°10'00" - 08" = 59°09'52"
Adjusted Angle 3 = 62°25'12" - 09" = 62°25'03"
180°00'00"

Corrected value of angle 3 = ***62°25'03"***

② *Corrected value of angle 6:*
Considering triangle BCD
Angle 4 = 60°29'05"
Angle 5 = 59°25'12"
Angle 6 = 60°05'09"
179°59'26"

Error = 34"

Adjusted angle 4 = 60°29'05" + 12" = 60°29'17"
Adjusted angle 5 = 59°25'12" + 11" = 59°25'23"
Adjusted angle 6 = 60°05'09" + 11" = 60°05'20"
180°00'00"

Corrected angle 6 = ***60°05'20"***

③ *Corrected value of angle 9:*
Considering triangle CDE:
Angle 7 = 71°40'01"
Angle 8 = 63°10'10"
Angle 9 = 45°10'19"
180°00'30"

Error = 30"

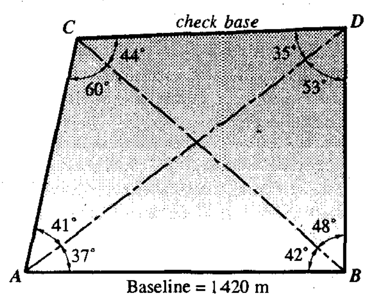
$$Correction = \frac{30''}{3} = 10''$$

Adjusted angle 7 = 71°39'51"
Adjusted angle 8 = 63°10'00"
Adjusted angle 9 = 45°10'09"
180°00'00"

Corrected angle 9 = ***45°10'09"***

Problem 124:

From the given quadrilateral, all stations were occupied and all lines are observed in both directions.



① Compute strength of figure constant F.
② Compute the relative strength of the quadrilateral ABCD.
③ Compute the length of check base CD.

Solution:

① *Constant F:*

$$\text{Constant } F = \frac{D - C}{D}$$

D = 10 (no. of directions observed forward and backward not including Ab)

$C = (n' - s' + 1) + (n - 2s + 3)$

n' = no. of lines observed in both directions.

$n' = 6$

s' = no. of occupied stations

$s' = 4$

n = total no. of lines in the figure including known lines

$n = 6$

s = total no. of stations

$s = 4$

$C = (6 - 4 + 1) + [6 - 2(4) + 3]$

$C = 3 + 1$

$C = 4$

$$F = \frac{D - C}{D}$$

$$F = \frac{10 - 4}{10}$$

$F = \mathbf{0.60}$

② *Strength of figure which gives the strongest route:*
Consider triangle ABC dand ACD with AC as common side.

$$\frac{AC}{\text{Sin } 42^\circ} = \frac{AB}{\text{Sin } 60^\circ}$$

$$AC = \frac{AB \text{ Sin } 42^\circ}{\text{Sin } 60^\circ}$$

$$\frac{CD}{\text{Sin } 41^\circ} = \frac{AC}{\text{Sin } 35^\circ}$$

$$CD = \frac{AC \text{ Sin } 41^\circ}{\text{Sin } 35^\circ}$$

$$CD = \frac{AB \text{ Sin } 42^\circ \text{ Sin } 41^\circ}{\text{Sin } 60^\circ \text{ Sin } 35^\circ}$$

Distance angles are 42° and 60° for triangle ABC

log Sine 42°00'00"	9.825510895
log Sine 42°00'00"	9.825513234
	2339

$\Delta_A = 2.339$

log Sine 60°00'00"	9.937530632
log Sine 60°00'01"	9.937531847
	1215

$\Delta_B = 1.215$

$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2)$
$= (2.339)^2 + 2.339(1.215) + (1.215)^2$
$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 9.789$

Distance angles for triangle ACD are 41° and 35°

log Sin 41°00'00" =	9.816942917
log Sin 41°00'01" =	9.816945339
	2422

log Sin 35°00'00" =	9.758591301
log Sin 35°00'01" =	9.758594308
	3007

$\Delta_A = 2.422$
$\Delta_B = 3.007$
$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2)$
$= (2.422)^2 + (2.422)(3.007) + (3.007)^2$
$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 22.19$
$\Sigma\,(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 9.789 + 22.19$
$\Sigma\,(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 31.979$

$$R = \left(\frac{D - C}{C}\right) \Sigma\,(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2)$$

$R = 0.60(31.979)$
$R = 19.19$

Consider triangle ABD and ACD: with AD as common sides.

$$\frac{AD}{\text{Sin } 90^\circ} = \frac{AB}{\text{Sin } 53^\circ}$$

$$AD = \frac{AB \text{ Sin } 90^\circ}{\text{Sin } 53^\circ}$$

$$\frac{CD}{\text{Sin } 40^\circ} = \frac{AD}{\text{Sin } 104^\circ}$$

$$CD = \frac{AD \text{ Sin } 40^\circ}{\text{Sin } 104^\circ}$$

$$CD = \frac{AB \text{ Sin } 90^\circ \text{ Sin } 40^\circ}{\text{Sin } 53^\circ \text{ Sin } 104^\circ}$$

Distance angles are 53° and 90° for ABD

log Sin 53°00'00" =	9.902348617
log Sin 53°00'01" =	9.902350203
	1586

$\Delta_A = 1.586$

log Sin 90°00'00" = 0
log Sin 90°00'01" = 0
$\Delta_B = 0$

$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = (1.586)^2 + 0 + 0$
$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 2.51$

Distance angles are 41° and 104° for triangle ACD

log Sin 41°00'00" =	9.816942917
log Sin 41°00'01" =	9.816945339
	2422

$\Delta_A = 2.422$

log Sin 104°00'00" =	9.986904119
log Sin 104°00'01" =	9.986903594
	525

$\Delta_B = 0.525$

$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2)$
$= (2.422)^2 + 2.422(0.525) + (0.525)^2$
$(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 7.41$

$\Sigma\,(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 2.51 + 7.41$
$\Sigma\,(\Delta_A{}^2 + \Delta_A\,\Delta_B + \Delta_B{}^2) = 9.92$

TRIANGULATION

$$R = \left(\frac{D-C}{C}\right) \Sigma (\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$$
$R = 0.60(9.92)$
$R = 5.952$

Considering triangle ABC and BCD with BC as common side:

$$\frac{BC}{\text{Sin } 78^\circ} = \frac{AB}{\text{Sin } 60^\circ}$$
$$BC = \frac{AB \text{ Sin } 78^\circ}{\text{Sin } 60^\circ}$$
$$\frac{CD}{\text{Sin } 48^\circ} = \frac{BC}{\text{Sin } 88^\circ}$$
$$CD = \frac{BC \text{ Sin } 48^\circ}{\text{Sin } 88^\circ}$$
$$CD = \frac{AB \text{ Sin } 78^\circ \text{ Sin } 48^\circ}{\text{Sin } 60^\circ \text{ Sin } 88^\circ}$$

The distance angles are 60° and 78° for triangle ABC and 48° and 88° for BCD

log Sin 60°00'00" = 9.937530632
log Sin 60°00'01" = 9.937531847
1215

$\Delta_A = 1.215$

log Sin 78°00'00" = 9.990404394
log Sin 78°00'01" = 9.990404842
448

$\Delta_B = 0.448$

$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$
$= (1.215)^2 + (1.215)(0.448) + (0.448)^2$
$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 2.22$

log Sin 48°00'00" = 9.871073458
log Sin 48°00'01" = 9.871075354
1896

$\Delta_A = 1.896$

log Sin 88°00'00" = 9.999735359
log Sin 88°00'01" = 9.999735432
073

$\Delta_B = 0.073$

$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$
$= (1.896)^2 + (1.896)(0.073) + (0.073)^2$
$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 3.74$
$\Sigma (\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 2.22 + 3.74$
$\Sigma (\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 5.96$

$$R = \left(\frac{D-C}{C}\right) \Sigma (\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$$
$R = 0.60(5.96)$
$R = 3.58$

Consider triangles ABD and BCD with BD as common side.

$$\frac{BD}{\text{Sin } 37^\circ} = \frac{AB}{\text{Sin } 53^\circ}$$
$$BD = \frac{AB \text{ Sin } 37^\circ}{\text{Sin } 53^\circ}$$
$$\frac{CD}{\text{Sin } 48^\circ} = \frac{BD}{\text{Sin } 44^\circ}$$
$$CD = \frac{BD \text{ Sin } 48^\circ}{\text{Sin } 44^\circ}$$
$$CD = \frac{AB \text{ Sin } 37^\circ \text{ Sin } 48^\circ}{\text{Sin } 53^\circ \text{ Sin } 54^\circ}$$

Distance angles of triangle ABD are 37° and 53° and for triangle BCD are 44° and 48°

log Sin 37°00'00" = 9.779463025
log Sin 37°00'01" = 9.779465819
2794

$\Delta_A = 2.794$

log Sin 53°00'00" = 9.902348617
log Sin 53°00'01" = 9.902350203
1586

$\Delta_B = 1.586$

$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$
$= (2.794)^2 + (2.794)(1.586) + (1.586)^2$
$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 14.75$

log Sin 44°00'00" = 9.841771273
log Sin 44°00'01" = 9.841773454
2181

$\Delta_A = 2.181$

log Sin 48°00'00" = 9.871073458
log Sin 48°00'01" = 9.871075354
1896

$\Delta_B = 1.896$

$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$
$= (2.181)^2 + (2.181)(1.896) + (1.896)^2$
$(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 12.49$
$\Sigma (\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 14.75 + 12.49$
$\Sigma (\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 27.24$

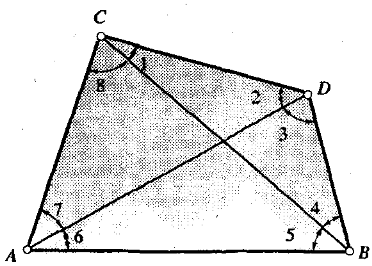
$$R = \left(\frac{D-C}{C}\right) \Sigma (\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$$
$R = 0.60(27.24)$
$R = 16.34$

*Relative strength of the quadrilateral R = **3.58** (smallest value)*

③ *Length of check base CD:*
$$CD = \frac{AB \sin 78° \sin 48°}{\sin 60° \sin 88°}$$
$$CD = \frac{1420 \sin 78° \sin 48°}{\sin 60° \sin 88°}$$
$CD =$ ***1192.61 m.***

Problem 125:

From the following quadrilateral with a corresponding angles tabulated shown.



① Compute the adjusted value of angle 4 by applying the angle condition only.
② Compute the adjusted value of angle 7 by applying the angle condition only.
③ Compute the strength of figure factor.

Solution:

① *Adjusted value of angle 4:*

	ADJUST A	ADJUST B	ADJUST C
∠1 =	23°44' 38"	23°44' 37"	23°44' 35"
∠2 =	42°19' 09"	42°19' 08"	42°19' 06"
∠3 =	44°52' 01"	44°52' 00"	44°52' 00"
∠4 =	69°04' 21"	69°04' 20"	69°04' 19"
∠5 =	39°37' 48"	39°37' 47"	39°37' 49"
∠6 =	26°25' 51"	26°25' 50"	26°25' 52"
∠7 =	75°12' 14"	75°12' 13"	75°12' 13"
∠8 =	38°44' 06"	38°44' 05"	38°44' 06"
Sum =	360°00' 08"	360°00' 00"	360°00' 00

Error = 8"

$$Correction = \frac{8}{8}$$
Correction = 1" *(sub)*
∠1 + ∠2 = ∠5 + ∠6

23°44' 37"	39°37' 47"
42°19' 08"	26°25' 50"
66°03' 45"	66°03' 37"

Error = 45 - 37
Error = 8"

$$Correction = \frac{8}{4}$$
Correction = 2" (subtract from ∠1 and ∠2 and add to ∠5 and ∠6)

∠3 + ∠4 = ∠7 + ∠8

44°52' 00"	75°12' 13"
69°04' 20"	38°44' 05"
113°56' 20"	113°56' 18"

Error = 20 - 18 = 2"
Correction = Add 1" to ∠8 and subtract 1" from ∠4

TRIANGULATION

Check:

∠1	23°44' 35"	∠8	38°44' 06"
∠8	38°44' 06"	∠7	75°12' 13"
∠2	42°19' 06"	∠6	26°25' 22"
∠7	75°12' 13"	∠8	39°37' 49"
	180°00' 00"		180°00' 00"

∠1	23°44' 35"	∠3	44°52' 00"
∠2	42°19' 06'	∠4	69°04' 19"
∠3	44°52' 00"	∠5	39°37' 49"
∠4	69°04' 19"	∠6	26°25' 52"
	180°00' 00"		180°00' 00"

*Angle 4 = **69°04' 19"***

② *Angle 7 = **75°12' 13"***

③ *Strength of figure factor:*

$D = 10$

$n' = 6$

$n = 6$

$s = 4$

$s' = 4$

$C = (n' - s + 4) + (n - 2s + 3)$

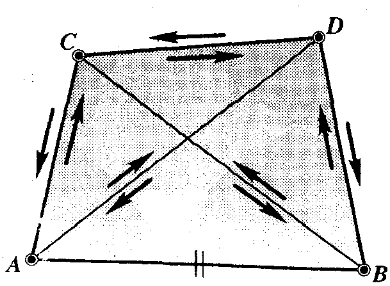
$C = (6 - 4 + 4) + (6 - 8 + 3)$

$C = 4$

$$F = \frac{D - C}{D}$$

$$F = \frac{10 - 4}{10}$$

$F = \mathbf{0.60}$ *(Strength of figure factor)*



Problem 126

Given the quadrilateral shown which has been adjusted using angle condition. It is required to adjust the angles using the side condition.

① Compute the adjusted angle 3.
② Compute the adjusted angle 5.
③ Compute the adjusted angle 8.

∠1 = 39°37' 49"

∠2 = 26°25' 52"

∠3 = 75°12' 13"

∠4 = 38°44' 06"

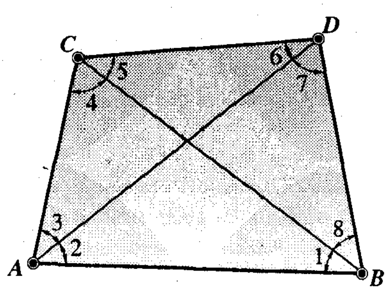
∠5 = 23°44' 35"

∠6 = 42°19' 06"

∠7 = 44°52' 00"

∠8 = 69°04' 19"

Sum = 360°00' 00"



Solution:

① *Adjusted angle 3:*

$$\frac{\sin \angle 2 \sin \angle 4 \sin \angle 6 \sin \angle 8}{\sin \angle 1 \sin \angle 3 \sin \angle 5 \sin \angle 7} = 1$$

log Sin 26°25' 52"	=	9.64847855
log Sin 38°44' 06"	=	9.796379535
log Sin 42°19' 06"	=	9.82817581
log Sin 69°04' 19"	=	9.970360677
		9.243394570

		Diff. in 1"
log Sin 26°25' 52"	= 9.64847855	4.2
log Sin 26°25' 53"	= 9.648482785	
log Sin 38°44' 06"	= 9.796379535	2.62
log Sin 38°44' 07"	= 9.79638216	
log Sin 42°19' 06"	= 9.82817581	2.31
log Sin 42°19' 07"	= 9.828178122	
log Sin 69°04' 19"	= 9.970360677	0.81
log Sin 69°04' 20"	= 9.970361483	
		9.94

log Sin 39°37' 49" = 9.804705675
log Sin 75°12' 13" = 9.985354379
log Sin 23°44' 35" = 9.604912331
log Sin 44°52' 00" = 9.848471997
9.243444381

		Diff. in 1"
log Sin 39°37' 49"	= 9.804705675	2.54
log Sin 39°37' 50"	= 9.804708217	
log Sin 75°12' 13"	= 9.985354379	0.56
log Sin 75°12' 14"	= 9.985354935	
log Sin 23°44' 35"	= 9.604912331	4.79
log Sin 23°44' 36"	= 9.604917117	
log Sin 44°52' 00"	= 9.848471997	2.12
log Sin 44°52' 01"	= 9.848474112	
		10.01

Subtract: 9.243394570 - smaller
9.243444381 - bigger
0.000049811

Add: 9.94 + 10.01 = 19.95

Difference = 49.81

$$ø = \frac{49.81}{8}$$
$ø = 6.23$
$$ß = \frac{19.95}{8}$$
$ß = 2.49$
$$\text{Correction} = \frac{6.23}{2.49}$$
Correction = 2.5" say 2"

Add 2" to all angles in the numerator: (smaller)
Subtract 2" to all angles in the denominator: (bigger)

- ∠1	=	39°37' 49"	- 02"	39°37' 47"
+ ∠2	=	26°25' 52"	+ 02"	26°25' 54"
- ∠3	=	75°12' 13"	- 02"	75°12' 11"
+ ∠4	=	38°44' 06"	+ 02"	38°44' 08"
- ∠5	=	23°44' 35"	- 02"	23°44' 33"
+ ∠6	=	42°19' 06"	+ 02"	42°19' 08"
- ∠7	=	44°52' 00"	- 02"	44°51' 58"
+ ∠8	=	69°04' 19"	+ 02"	69°04' 21"
				360°00' - 00"

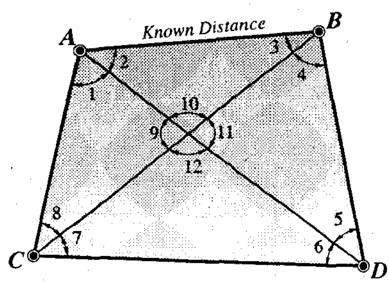
*Adjusted angle 3 = **75°12' 11"***

② *Adjusted angle 5 = **23°44' 33"***

③ *Adjusted angle 8 = **69°04' 21"***

Problem 127:

In the figure shows a quadrilateral with their corresponding angular measurements designated.



① Which of the following equation does not satisfy the figure shown.

a) ∠2 + ∠3 = ∠7 + ∠6
b) ∠1 + ∠8 = ∠4 + ∠5
c) ∠1 + ∠2 + ∠3 + ∠4 = 180°
d) ∠1 + ∠8 + ∠6 + ∠7 = 180°

TRIANGULATION

② Which of the equation is correct, in side condition:

a) $\dfrac{\sin\angle 1\ \sin\angle 3\ \sin\angle 5\ \sin\angle 7}{\sin\angle 8\ \sin\angle 2\ \sin\angle 4\ \sin\angle 6}=0$

b) $\dfrac{\sin\angle 2\ \sin\angle 4\ \sin\angle 6\ \sin\angle 8}{\sin\angle 3\ \sin\angle 5\ \sin\angle 7\ \sin\angle 1}=1$

c) $\dfrac{\sin\angle 1\ \sin\angle 3}{\sin\angle 2\ \sin\angle 4}=\dfrac{\sin\angle 5\ \sin\angle 1}{\sin\angle 6\ \sin\angle 8}$

d) $\dfrac{\sin\angle 2\ \sin\angle 4}{\sin\angle 6\ \sin\angle 8}=\dfrac{\sin\angle 1\ \sin\angle 3}{\sin\angle 5\ \sin\angle 7}$

③ What will be the factor (F) in solving the strength of figure.

a) 0.80 c) 0.90
b) 0.60 d) 0.40

$F=\dfrac{D-C}{D}$

D = no. of directions observed (forward and backwards) not including the fixed or unknown side of a given figure.

C = no. of geometric conditions to be satisfied in a given figure.

$C=(n'-s'+1)+(n-2s+3)$

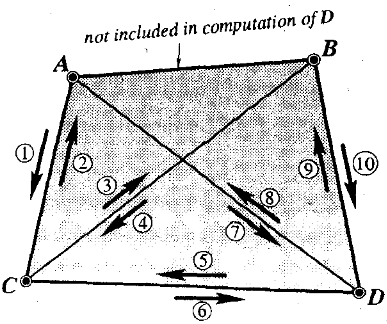
n' = no. of lines observed in both directions, including the fixed or known side of a given figure.

n = total number of lines in the figure including fixed or known line.

s = total number of stations

No. of directions observed (forward and backwards) not including the known side of

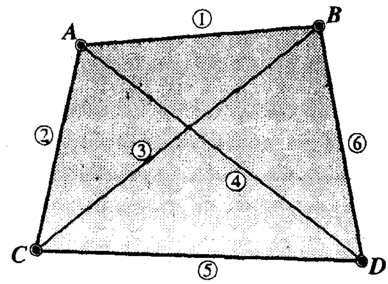
$D=10$



Values of D

n = total number of lines in figure, including the known side

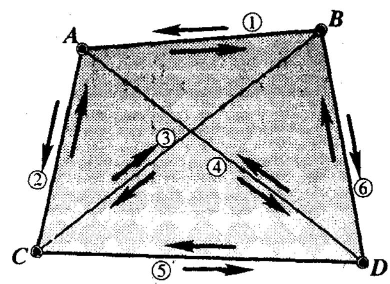
$n=6$



Values of n

n' = no. of lines observed in both directions including the known side

$n'=6$



Values of n'

S = total no. of stations

$S=4$

$C=(n'-s'+1)+(n-2s+3)$

$C=(6-4+1)+[6-2(4)+3]$

$C=3+1$

$C=4$

$F=\dfrac{D-C}{D}$

$F=\dfrac{10-4}{10}$

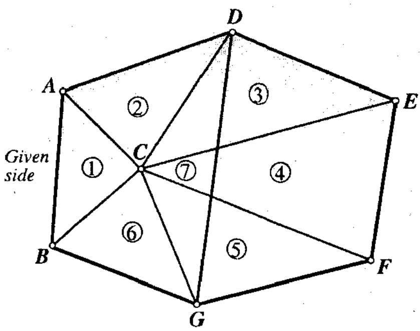
$F=0.60$

Answer:

① c
② b
③ b

Problem 128:

From the figure shown.



① Assuming that all stations are occupied and all lines are observed, what will be the number of geometric condition C? where $C = (n' - s' - 1) + (n - 2s + 3)$

② What will be the factor (F) in computing the strength of figure $R = F(\Delta_A^2 + \Delta_A\Delta_B + \Delta_B^2)$

③ If the value of $(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2) = 5.02$, what will be the strength of figure R.

Solution:

① *Value of C:*

n' = no. of lines observed in both directions including known side

$n' = 13$

n = total no. of lines in figure uncluding known side

$n = 13$

s' = no. of occupied stations

$s' = 7$

s = total no. of stations

$s = 7$

$C = (n' - s' + 1) + (n - 2s + 3)$

$C = (13 - 7 + 1) + [13 - 2(7) + 3]$

$C = 7 + 2$

$C = \mathbf{9}$

② *Fraction F:*

$$F = \frac{D - C}{D}$$

D = no. of directions observed(forward and backward) not including the known side.

$D = 24$

$$F = \frac{D - C}{D} = \frac{24 - 9}{24}$$

$F = \mathbf{0.625}$

③ *Strength of figure R:*

$R = F(\Delta_A^2 + \Delta_A \Delta_B + \Delta_B^2)$

$R = 0.625(5.02)$

$R = \mathbf{3.14}$

Spherical Excess

Problem 129:

The interior angles of a spherical triangle ABC and side AB were measured as follows:

A = 61°45'20"
B = 56°10'30"
C = 62°04'11"
AB = 35965.47 m.

Assume radius of earth to be 6372 km.

① Compute the adjusted value of angle A by distributing the spherical excess and the remaining error equally.

② Compute the adjusted value of angle B by distributing the spherical excess and the remaining error equally.

③ Compute the adjusted value of angle C by distributing the spherical excess and the remaining error equally.

Solution:

$$e = \frac{A}{R^2 \text{ Sin } 01''}$$

$$A = \frac{bc \text{ Sin } A}{2}$$

$$\frac{b}{\text{Sin } 56°10'30''} = \frac{35965.47}{\text{Sin } 62°04'11''}$$

$b = 33814.89$

SPHERICAL EXCESS

$$A = \frac{bc \sin A}{2}$$

$$A = \frac{33814.89\,(35965.47)\sin 61°45'20''}{2}$$

$$A = 535683650.2\ m^2$$

$$e'' = \frac{A}{R^2 \sin 01''}$$

$$e'' = \frac{535683650.2}{(6372000)^2 \sin 01''}$$

$$\varepsilon'' = 2.72''$$

61°45'20"
56°10'30"
62°04'11"

180°-00'01"
180°-00'02.72"

Error = 1.72"

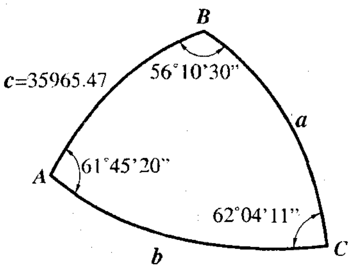
$$1\text{st } Corr. = \frac{1.72}{3}$$

1st *Corr.* = 0.573 (added)

$$2\text{nd } Corr. = \frac{2.72}{3}$$

2nd *Corr.* = 0.907 (to be subtracted)

61°45'20" + 0.573" - 0.907 = 61°45' 19.676"
56°10'30" + 0.573" - 0.907 = 56°10' 29.676"
62°04'11" + 0.573" - 0.907 = 62°04' 10.676"
180°00' 00"



① *Adjusted value of angle A* = ***61°45'19.676"***

② *Adjusted value of angle B* = ***56°10'29.676"***

③ *Adjusted value of angle C* = ***62°04'10.676"***

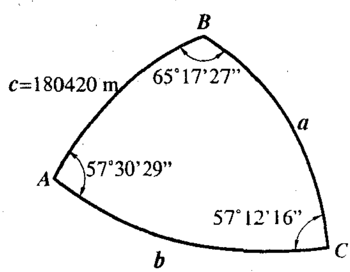
Problem 130:

The interior angles in triangle ABC are A = 57°30' 29", B = 65°17' 27" and C = 57°12' 16". The distance from A to B is equal to 180,420 m. Assuming the average radius of curvature is 6400 km.

① Compute the area of the triangle.
② Compute the second term of the spherical excess.
③ Compute the total spherical excess.

Solution:

① *Area of triangle:*



Using Sine Law:

$$\frac{b}{\sin 65°17'27''} = \frac{180420}{\sin 57°12'16''}$$

$b = 194978.94$ m.

$$Area = \frac{bc \sin A}{2}$$

$$Area = \frac{194978.94\,(180420)\sin 57°30'29''}{2}$$

Area = ***14836 x 10⁶ m²***

② Second term of the spherical excess:
When the sides of the triangle are over 100 miles (160,000 m.) use the accurate formula for spherical excess.

$$e'' = \frac{Area}{R^2 \sin 01''}\left[1 + \frac{a^2 + b^2 + c^2}{24\,R^2}\right]$$

The second term is

$$\frac{Area}{R^2 \sin 01''}\left(\frac{a^2 + b^2 + c^2}{24\,R^2}\right)$$

$$\frac{a}{\text{Sin } 57°30'29"} = \frac{180420}{\text{Sin } 57°12'16"}$$

$a = 181033.49$ m.

Second term:

$a^2 = 32773 \times 10^6$

$b^2 = 38017 \times 10^6$

$c^2 = 32551 \times 10^6$

$R^2 = 40960000 \times 10^6$

$Area = 14836 \times 10^6$

$$2nd\ term = \frac{Area}{R^2 \text{ Sin } 01"} \left(\frac{a^2+b^2+c^2}{24\,R^2}\right)$$

$$2nd\ term = \frac{14836 \times 10^6}{40960000 \times 10^6 \text{ Sin } 01"} \left[\frac{(32773+38017+32551)\,10^6}{24\,(40960000)\,10^6}\right]$$

$$2nd\ term = 74.7106" \left(\frac{103341}{24(40960000)}\right)$$

$2nd\ term = \mathbf{0.00785"}$

③ *Total spherical excess:*

$$e" = \frac{Area}{R^2 \text{ Sin } 01"}\left[1 + \frac{a^2+b^2+c^2}{24\,R^2}\right]$$

$e" = 74.7106 + 0.00785$

$e" = \mathbf{74.71845"}$

Problem 131:

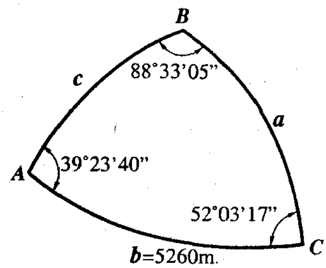
Given the following data of spherical triangle as follows:

A = 39°23'40"
B = 88°33'05"
C = 52°03'17"

Distance AC = 5260 m.
Latitude of the center of triangle = 14°30'
log m = 1.40658 - 10
Compute the adjusted value of angle B.

① Compute the adjusted value of angle A by distributing the spherical excess and the remaining error equally.
② Compute the adjusted value of angle B by distributing the spherical excess and the remaining error equally.
③ Compute the adjusted value of angle C by distributing the spherical excess and the remaining error equally.

Solution:



$$\frac{C}{\text{Sin } 52°03'17"} = \frac{5260}{\text{Sin } 88°33'05"}$$

$C = 4149.35$ m.

$\log m = 1.40658 - 10$

$m = 2.55023 \times 10^{-9}$

$e" = m\,bc \text{ Sin } A$

$e" = 2.55023 \times 10^{-9}\,(5260)(4149.35) \text{ Sin } 39°23'40$

$e" = 0.035"$

A = 39°23'40"
B = 88°33'05"
C = <u>52°03'17"</u>
180°- 00'02"
<u>180°- 00 - 00.035"</u>
Error = 1.965

$$First\ Correction: \frac{1.965}{3} = 0.655"$$

$$Second\ Correction: \frac{0.035}{3} = 0.012$$

39°23'40" - 0.655" - 0.012" = 39°23' 39.333"
88°33'05" - 0.655" - 0.012" = 88°33' 4.333"
52°03'17" - 0.655 - 0.012" = <u>52°03' 16.333"</u>
180°00' 00"

① *Adjusted value of angle A* = **39°23'39.333"**

② *Adjusted value of angle B* = **88°33' 4.333"**

③ *Adjusted value of angle C* = **52°03'16.333"**

SPHERICAL EXCESS

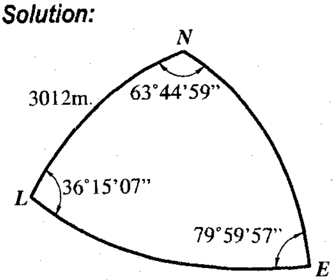
Problem 132:

Given the following data of spherical triangle as follows:

Ans. e" = 0.012"   Angle N = 63°44'59"
∠N = 63°44'58"   Angle E = 79°59'57"
Angle L = 36°15'07"
Distance LN = 3012 m.
Latitude of center of triangle
= 14°25'39.02"N
R = 6378160 m. radius of earth at latitude 14°25'39.02" N
N = 6376032 m

① Compute the adjusted value of angle E by distributing the spherical excess and the remaining error equally.
② Compute the adjusted value of angle N by distributing the spherical excess and the remaining error equally.
③ Compute the adjusted value of angle L by distributing the spherical excess and the remaining error equally.

Solution:



$$e'' = \frac{A}{R^2 \text{ Sin } 01''}$$

$$\text{Arc } 1'' = \frac{\pi}{180\ (3600)}$$

$$A = \frac{bc \text{ Sin A}}{2} \quad \textit{(area of triangle)}$$

$$e = m\, bc \text{ Sin } A$$

$$m = \frac{1}{2 \text{ R N Arc } 1''}$$

$$m = \frac{1}{2(6378160)(6376032)\ \frac{\pi}{180(3600)}}$$

$$m = 2.536 \times 10^{-9}$$

$$\log m = 1.40415 - 10$$

$$\frac{b}{\text{Sin } B} = \frac{c}{\text{Sin C}}$$

$$\frac{b}{\text{Sin } 63°44'59''} = \frac{3012}{\text{Sin } 79°59'57''}$$

$$b = 2743.05 \text{ m.}$$

❶ $e'' = \dfrac{A}{R^2 \text{ Sin } 01''}$ *if R is known.*

❷ $e'' = m\, bc \text{ Sin A}$ *if no radius is given*

$$m = \frac{1}{2 \text{ R N Arc } 1''}$$

$$\text{Arc } 1'' = \frac{\pi}{180(3600)}$$

$$N = 6376032 \text{ m.}$$

At a given latitude

$e'' = m\, bc \text{ Sin } A$

$e'' = 2.536 \times 10^{-9}\,(2743.05)(3012) \text{ Sin } 36°15'07''$

$e'' = \mathbf{0.012''}$ *(spherical excess)*

79°59'27"
63°44'59"
<u>36°15'07"</u>
180°- 00'- 03"
<u>180°- 00'- 0.012"</u>
Error = 2.988" *(error of spherical triangle)*

Correction for each angle

$$= \frac{2.988}{3}$$

= 0.966" *(First Correction)*

Second Correction

$$= \frac{0.012}{3}$$

= 0.004" *(spherical excess subtracted from each angle)*

79°59'57" - 0.996" - 0.004" = 79°59'56"
63°44'59" - 0.996" - 0.004" = 63°44'58"
36°15'07" - 0.995" - 0.004" = <u>36°15'06"</u>
180°00' 00"

① *Adjusted value of angle E* = ***79°59'56"***

② *Adjusted value of angle N* = ***63°44'58"***

③ *Adjusted value of angle L* = ***36°15'06"***

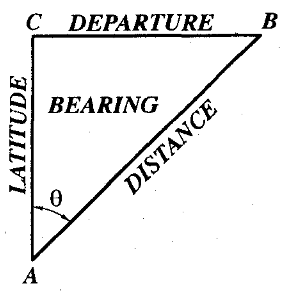
AREA OF CLOSED TRAVERSE

AREA OF CLOSED TRAVERSE

LATITUDE and DEPARTURE

Latitude of any line - is the projection on a north and south lines. It may be called as north or positive latitude and south or negative latitude.

Departure of any line - is the projection on the east and west line. West departure is sometimes called negative departure and East departure is sometimes called positive departure.



Line AB has its latitude AC and departure BC. The angle ø is the bearing of the line AB.

BC = AB Sin ø

Departure = Distance x Sin Bearing

AC = AB Cos ø

Latitude = Distance x Cos Bearing

$$\text{Dist} = \frac{\text{Latitude}}{\text{Cos Bearing}}$$

$$\text{Dist} = \frac{\text{Departure}}{\text{Sin Bearing}}$$

Error of Closure

In any closed traversed, there is always an error. No survey is geometrically perfect, until proper adjustment are made. For a closed traversed, the sum of the north and south latitudes should always be zero.

BALANCING A SURVEY

1. ***Compass rule*** - the correction to be applied to the latitude or departure of any course is to the total correction in latitude or departure as the length of the course is to the length of the traverse.
2. ***Transit rule*** - the correction to be applied to the latitude or departure of any course is to the total correction in latitude or departure as the latitude or departure of that course is to the arithmetical sum of all the latitudes or departures in the traverse without regards to sign.

$$\text{Error of closure} = \sqrt{\sum L^2 + \sum D^2}$$

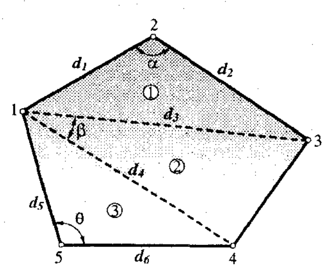
$$\text{Relative error} = \frac{\text{Error of closure}}{\text{Perimeter of all courses}}$$

$\sum L$ = error in latitude

$\sum D$ = error in departure

Computation of Areas

1. *Area by Triangle Method*



AREA OF CLOSED TRAVERSE

$$A = A_1 + A_2 + A_3$$

$$A_1 = \frac{d_1 d_2 \sin \alpha}{2}$$

$$A_2 = \frac{d_3 d_4 \sin \beta}{2}$$

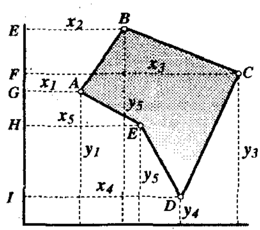
$$A_3 = \frac{d_5 d_6 \sin \phi}{2}$$

2. Area by Rule of Thumb Method

$$2A = [y_1 (x_1 - x_2) + y_2 (x_1 - x_3) + y_3 (x_2 - x_4) + y_4 (x_3 - x_5) + y_5 (x_4 - x_1)]$$

3. Area by coordinates

Area ABCDE = Area BCEF + Area CDIF
- Area EDIH - Area AEGH - Area ABGE



$$A = \frac{(x_2 + x_3)(y_2 - y_3)}{2} + \frac{(x_3 + x_4)(y_3 - y_4)}{2} - \frac{(x_4 + x_5)(y_5 - y_4)}{2} - \frac{(x_5 + x_1)(y_1 - y_5)}{2} - \frac{(x_1 + x_2)(y_2 - y_1)}{2}$$

Simplifying this relation:

① $2A = -[y_1 (x_5 - x_2) + Y_2 (x_1 - x_3) + y_3 (x_2 - x_4) + y_4 (x_4 - x_5) + y_5 (x_4 - x_1)]$

or

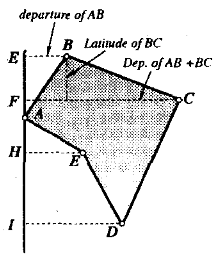
② $2A = y_1x_1 + y_3y_2 + x_4y_3 + x_5y_4 + x_1y_5 - x_1y_2 - x_2y_3 - y_3y_4 - x_4y_5 - x_5y_1$

or

③ $2A = \left[\frac{x_1}{y_1}\frac{x_2}{y_2}\frac{x_3}{y_3}\frac{x_4}{y_4}\frac{x_5}{y_5}\frac{x_1}{y_1}\right]$

4. Area by Double Meridian Distance

Double Meridian Distance of line BC is the sum of meridian distances of the two extremeties.



D.M.D. of BC = EB + FC
Latitude of BC = EF

Area of BCFE:

$$A = \frac{(EB + FC)\,EF}{2}$$

$2A = (EB + FC)\,EF$
$2A = (D.M.D.)\,\text{Latitude}$
Double Area = Double Meridian Distance x Latitude

Computation of D.M.D.

1. D.M.D. of the first course is equal to the departure of that course.
2. D.M.D. of any other course is equal to the DMD of the preceding course, plus the departure of the preceding course plus the departure of the course itself.
3. D.M.D. of the last course is numerically equal to the departure of the last course but opposite in sign.

AREA OF CLOSED TRAVERSE

Computing Area by D.M.D. Method:

1. Compute the latitudes and departures of all courses.
2. Compute the error of closure in latitudes and departures.
3. Balance the latitudes and departures by applying either transit rule or compass rule.
4. Compute for the D.M.D. of all courses.
5. Compute the double areas by multiplying each D.M.D. by the corresponding latitude.
6. Determine the algebraic sum of the double areas.
7. Divide the algebraic sum of the double area to obtain the area of the whole tract.

Double Area = D.M.D. x Latitude

5. *Area by: Double Parallel Distance*

Computation of D.P.D.

1. D.P.D. of the first course is equal to the latitude of that course.
2. D.P.D. of any other course is equal to the D.P.D. of the preceding course, plus the latitude of the preceding course, plus the latitude of the course itself.
3. D.P.D. of the last course is numerically equal to the latitude of the last course but opposite in sign.

Double Area = Double Parallel Distance x Departure

Example:

Area by Double Meridian Distance

Lines	LAT.	DEP.	DMD	Double Area
1 - 2	+60	- 30	- 30	- 30(60) = - 1800
2 - 3	- 20	+20	- 40	- 40(- 20) = +800
3 - 4	- 80	+60	+40	+40(- 80) = - 3200
4 - 1	+40	- 50	+50	+50(40) = +2000

$2A = -2200$

$A = -1100$ m^2

Area by Double Parallel Distance

Lines	LAT.	DEP.	DPD	Double Area
1 - 2	+60	- 30	+60	60(- 30) = - 1800
2 - 3	- 20	+20	+100	100(20) = +2000
3 - 4	- 80	+60	0	0(60) = 0
4 - 1	+40	- 50	- 40	- 40(- 50) = +2000

$2A = -2200$

$A = 1100$ m^2

Problem 133:

From the field notes of a closed traverse fhown below, adjust the traverse.

LINES	BEARING	DISTANCES
AB	Due North	400.00 m.
BC	N 45° E	800.00 m.
CD	S 60° E	700.00 m.
DE	S 20° W	600.00 m.
EA	S 86°59' W	966.34 m.

① Compute the correction of latitude on line CD using transit rule.
② Compute the linear error of closure.
③ Compute the relative error or precision.

AREA OF CLOSED TRAVERSE

Solution:

Lines	Bearing	Distances	LAT	DEP
AB	Due North	400.00 m	+400.00	0
BC	N 45° E	800.00 m	+565.69	+565.69
CD	S 60° E	700.00 m	- 350.00	+606.22
DE	S 20° W	600.00 m	- 563.82	- 205.21
EA	S 86°59' W	966.34 m	- 50.86	- 965.00

Perimeter = 3466.34 +1.01 +1.7

400 + 565.69 + 350 + 563.82 + 50.86 = 1930.37

① *Correction of latitude on line CD using transit rule:*

$$\frac{C_{CD}}{1.01} = \frac{350.00}{1930.37}$$

$$C_{CD} = \mathbf{0.18}$$

② *Linear error of closure:*

$$LEC = \sqrt{(1.01)^2 + (1.7)^2}$$

$$LEC = \mathbf{1.97740}$$

③ *Relative error or precision:*

$$\text{Relative error} = \frac{1.97740}{3466.34}$$

$$\text{Relative error} = \frac{\mathbf{1}}{\mathbf{1753}}$$

Problem 134:

A parcel of land has been surveyed in the field and the lengths and bearings of the various sides are shown.

LINES	BEARING	DISTANCES
AB	N. 53°27' E.	59.82 m.
BC	S. 66°54' E.	70.38 m.
CD	S. 29°08' W.	76.62 m.
DA	N. 52°00' W.	95.75 m.

① Compute the error of closure for the traverse shown.
② What is the precesion of linear measurement of this traverse.
③ What is the total area included within the traverse in acres.

Solution:

① *Error of closure:*

Lines	Bearing	Distances	LAT	DEP
AB	N.53°37'E.	59.82 m	+35.62	+48.06
BC	S.66°54'E.	70.38 m	- 27.61	+64.74
CD	S.29°08'W.	76.62 m.	- 66.93	- 37.30
DA	N.52°00'W.	95.75 m	+58.95	- 75.45
		302.57	+94.57	+112.80
			- 94.54	- 112.75
			+0.03	+0.05

$$\text{Error of closure} = \sqrt{(0.03)^2 + (0.05)^2}$$

error of closure = ***0.0583***

② *Precesion of linear measurement:*

$$\text{Precision} = \frac{0.0583}{302.57}$$

$$\text{Precision} = \frac{1}{5190}$$

Precision = ***1:5190***

③ *Area in acres:*

LAT	DEP	DMD	DOUBLE AREA
+35.61	+48.05	+48.05	+1711.06
- 27.61	+64.73	+160.83	- 4440.46
- 66.94	- 37.31	+188.25	- 12601.46
+58.94	- 75.47	+75.47	+4448.20
0	0		2A = 10882.72

$A = 5441.36\ m^2$

Note: 1 acre = 4047 sq.m.

$$\text{Area} = \frac{5441.36}{4047}$$

Area = ***1.34 acres***

AREA OF CLOSED TRAVERSE

Problem 135:

① For a given closed traversed,
$\sum Lat = -0.44$
$\sum Deps = -0.37$
Perimeter = 2915.80 m.,
Total Deps = 1945.73
Total Lats = 1897.40.
For line AB = 483.52 m, its Latitude is 326.87 N. and dep. is 356.30 E. Determine the corrected latitude and departure of AB by compass rule.

② A given traversed has the following results:
Total Perimeter = 3615.40 m
Total Departures = 1842.64
Total Latitudes = 1868.94
$\sum Latitudes = +0.68$
$\sum Departures = +0.42$
For line BC distance equals 394.60 m, and its latitude is 249.40 N. and departure is 364.20 E. Compute the corrected departure and latitude of line BC, using transit rule.

Solution: TRANSIT RULE

① *Correction of latitude and departure AB:*
Corrected Latitude:

$$\frac{C}{(+)\,0.44} = \frac{483.52}{2915.80}$$

$C = +0.07$
Corrected lat = 326.87 + 0.07
Corrected lat = ***326.94***

Corrected Departure:

$$\frac{C}{0.37} = \frac{483.53}{2915.80}$$

$C = 0.06$
Corrected dep = 356.30 + 0.06
Corrected dep = ***356.36***

② *Correction of departure and latitude BC:*
Corrected Departure:

$$\frac{C}{-0.42} = \frac{364.20}{1842.64}$$

$C = -0.08$
Corrected dep = 364.20 - 0.08
Corrected dep = ***364.12***

Corrected Latitude:

$$\frac{C}{0.68} = \frac{249.40}{1868.94}$$

$C = 0.09$
Corrected lat = 249.40 - 0.09
Corrected lat = ***249.31***

Problem 136:

A closed traversed has the following data:
$\sum Lat = -0.56$
$\sum Dep = +0.34$
Total Latitude = 1726.8
Total Departure = 1876.3
Perimeter = 2628.5

For line DE:
Distance = 518.4 m.
Latitude = 259.20
Departure = 448.9

① Determine the corrected latitude of DE by compass rule.
② Compute the corrected latitude of DE by transit rule.
③ Compute the corrected departure of DE by compass rule.

Solution:

① *Corrected latitude of DE by compass rule:*

$$\frac{E}{+0.56} = \frac{518.40}{2628.5}$$

$E = +0.11$

Corrected latitude of DE = 259.2 + 0.11
Corrected latitude of DE = ***259.31***

② *Corrected latitude of DE by transit rule:*

$$\frac{E}{+0.56} = \frac{259.2}{1726.8}$$

$E = +0.08$

Corrected latitude of DE = 259.2 + 0.08
Corrected latitude of DE = ***259.28***

AREA OF CLOSED TRAVERSE

③ *Corrected departure of DE by compass rule:*

$$\frac{E}{-0.34} = \frac{518.4}{2628.5}$$

$$E = -0.07$$

Corrected departure = 448.9 - 0.07
Corrected departure = ***448.83***

Problem 137:

From the field notes of a closed traverse shown below, adjust the traverse using.

① Compute the relative error of closure.
② Compute the adjusted distance of line EA using Transit Rule.
③ Compute the adjusted bearing of line CD using Compass Rule.

STA. OCC.	STA. OBS.	Bearings	Distances
A	B	Due North	400.00 m.
B	C	N 45° E	800.00 m.
C	D	S 60° E	700.00 m.
D	E	S 20° W	600.00 m.
E	A	S 86°59' W	966.34 m.

Solution:

Lines	Bearings	Distance	LAT	DEP
A - B	Due North	400.00	+400.00	0
B - C	N 45° E	800.00	+565.69	+565.69
C - D	S 60° E	700.00	- 350.00	+606.22
D - E	S 20° W	600.00	- 563.82	- 205.21
E - A	S 86°59' W	966.34	- 50.86	- 965.00
	965.69	1171.91	+965.69	+1171.91
	964.68	1170.21	- 964.68	- 1170.21
	1930.37	2342.12	+ 1.01	+ 1.70
	ΣLAT	ΣDEP		

Total distance = 3466.34
Error in latitude = 1.01
Error in departure = 1.70

① *Relative error:*

$$\text{Error of closure} = \sqrt{(1.01)^2 + (1.7)^2}$$

Error of closure = 1.977

$$\text{Relative error} = \frac{1.977}{3466.34}$$

$$\text{Relative error} = \frac{\mathbf{1}}{\mathbf{1753.33}}$$

② *Adjusted distance of EA using Transit Rule:*

For A - B: (latitude)

$$\frac{C_1}{1.01} = \frac{400}{1930.37}$$

$$C_1 = 0.00052\,(400)$$

$$C_1 = 0.21 = C_{LAT}$$

Total distance = 3466.34

LINES	CORRECTION FOR LATITUDE			
A - B	C_1	=	0.000523 (400)	= +0.21
B - C	C_2	=	0.000523 (565.69)	= +0.30
C - D	C_3	=	0.000523 (350.00)	= +0.18
D - E	C_4	=	0.000523 (563.82)	= +0.29
E - A	C_5	=	0.000523 (50.86)	= +0.03
				1.01

Correction for Departure:
For line A - B:

$$\frac{C_1}{1.70} = \frac{0}{2342.12}$$

$$C_1 = 0.0007258\,(0)$$

$$C_1 = 0 \quad C_{DEP}$$

LINES	CORRECTION FOR DEPARTURE			
A - B	C_1	=	0.0007258 (0)	= 0
B - C	C_2	=	0.0007258 (565.69)	= 0.41
C - D	C_3	=	0.0007258 (606.22)	= 0.44
D - E	C_4	=	0.0007258 (205.21)	= 0.15
E - A	C_5	=	0.0007258 (965)	= 0.70
				1.70

AREA OF CLOSED TRAVERSE

ADJUSTED LATITUDES AND DEPARTURE
(uncorrected) (corrected)

Lines	LAT	DEP	LAT	DEP
	-0.21			
A - B	+400			
A - B	+400.00	0	+399.79	0
	- 0.30	- 0.41		
B - C	+565.69	+565.69	+565.39	+565.28
	- 0.18	- 0.44		
C - D	- 350.00	+606.22	- 350.18	+605.78
	- 0.29	+0.15		
D - E	- 563.82	- 205.21	- 564.11	- 205.36
	- 0.03	+0.70		
E - A	- 50.86	- 965.00	- 50.89	- 965.70
			0	0

Adjusted bearing of EA:

$$\tan \text{bearing} = \frac{Dep}{Lat}$$

$$\tan \text{bearing} = \frac{-965.70}{-50.89}$$

Bearing = S. 86°59' W.

Adjusted distance of EA:

$$\text{Distance} = \frac{965.70}{\sin 86°59'}$$

*Distance = **967.04 m.***

③ *Adjusted bearing of CD using Compass Rule:*

Correction for latitude:

Line A - B:

$$\frac{C_1}{1.01} = \frac{400}{3466.34}$$

$$C_1 = 0.000291374\,(400)$$

$$C_1 = 0.12$$

LINES	CORRECTION FOR LATITUDE			
A - B	C_1 =	0.000291374 (400)	=	0.12
B - C	C_2 =	0.000291374 (800)	=	0.24
C - D	C_3 =	0.000291374 (700)	=	0.20
D - E	C_4 =	0.000291374 (600)	=	0.17
E - A	C_5 =	0.000291374 (966.34)	=	0.28
				1.01

Correction for departure:

Line A - B:

$$\frac{C_1}{1.7} = \frac{40}{3466.34}$$

$$C_1 = 0.00049043\,(400)$$

$$C_1 = 0.20$$

LINES	CORRECTION FOR DEPARTURE			
A - B	C_1 =	0.00049043 (400)	=	0.20
B - C	C_2 =	0.00049043 (800)	=	0.40
C - D	C_3 =	0.00049043 (700)	=	0.34
D - E	C_4 =	0.00049043 (600)	=	0.29
E - A	C_5 =	0.00049043 (966.34)	=	0.47
				1.70

ADJUSTED LATITUDES AND DEPARTURE
(uncorrected) (corrected)

Lines	LAT	DEP	LAT	DEP
A - B	- 0.12	- 0.20		
	+400.00	0	+399.88	- 0.20
	- 0.24	- 0.40		
B - C	+565.69	+565.69	+565.45	+565.29
	- 0.20	- 0.34		
C - D	- 350.00	+606.22	- 350.20	+605.88
	- +0.17	+0.29		
D - E	- 563.82	- 205.21	- 563.99	- 205.50
	- +0.28	+0.47		
E - A	- 50.86	- 965.00	- 51.14	- 965.47
			0	0

$$\tan \text{bearing} = \frac{605.88}{350.20}$$

*Bearing = **S. 59°58' E***

Problem 138:

CORNER	COORDINATES	
	NORTHINGS	EASTINGS
1	1600	400
2	400	800
3	600	400
4	1400	1200

① Compute the bearing of line 2 - 3.
② Compute the distance of line 4 - 1.
③ Determine the area of the lot using DMD.

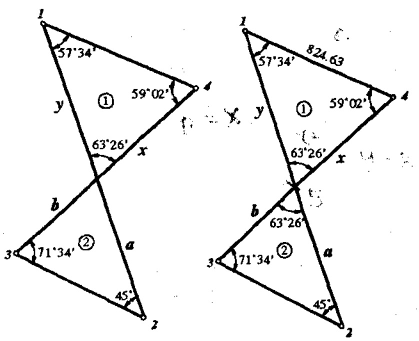
AREA OF CLOSED TRAVERSE

Solution:

Since the property line intersect each other, then we could only apply DMD, if we divide it into areas where no property lines will intersect.

Lines	LAT	DEP	Bearing	Distance
1 - 2	- 1200	+400	S 18°26' E	1265.02
2 - 3	+200	- 400	N 63°26' W	447.23
3 - 4	+800	+800	N 45° E	1131.37
4 - 1	+200	- 800	N 75°58' W	824.63

① *Bearing of line 2 - 3:*



For line 1 - 2:

$$\tan \text{Bearing} = \frac{400}{1200}$$

Bearing = S 18°26' E

$$\text{Distance} = \frac{400}{\sin 18°26'}$$

Distance = 1265.02

For line 2 - 3:

$$\tan \text{Bearing} = \frac{400}{200}$$

Bearing = N 63°26' W

② *Distance of line 4 - 1:*

$$\text{Distance} = \frac{400}{\sin 63°26'}$$

Distance = 447.23

For line 3 - 4:

$$\tan \text{Bearing} = \frac{800}{800}$$

Bearing = N 45° E

$$\text{Distance} = \frac{800}{\sin 45°}$$

Distance = 1131.37

For line 4 - 1:

$$\tan \text{Bearing} = \frac{800}{200}$$

Bearing = N 75°58' N

$$\text{Distance} = \frac{800}{\sin 75°58}$$

Distance = ***824.03***

③ *Area by DMD:*

Using Sine Law:

$$\frac{X}{\sin 57°32'} = \frac{824.63}{\sin 63°26'}$$

$X = 777.88$

$$\frac{Y}{\sin 59°02'} = \frac{824.63}{\sin 63°26'}$$

$Y = 790.56$

$a = 1265.02 - 790.56$

$a = 474.46$

$b = 1131.37 - 777.88$

$b = 353.49$

For Lot A:

Lines	Bearing	Distance	LAT	DEP
1 - 4	S 75°58' E	824.63	-.200	+800
4 - 5	S 45° W	777.88	- 550	- 550
5 - 1	N 18°66' W	790.56	+750	- 250

LINES	DMD	DOUBLE AREA
1 - 4	+800	- 160000
4 - 5	+1050	- 577500
5 - 1	+250	+187500

2A = 550000

$A = 275000 \text{ m}^2$

For Lot B:

Lines	Bearing	Distance	LAT	DEP
5 - 2	S 18°26' E	474.46	- 450	+150
2 - 3	N 63°26' W	447.23	+200	- 400
3 - 5	N 45° E	353.49	- 250	+250

LINES	DMD	DOUBLE AREA
5 - 2	+150	- 67500
2 - 3	- 100	- 20000
3 - 5	- 250	62500

$2A = 150000$
$A = 75000 \text{ m}^2$

Total area = 275000 + 75000
Total area = ***350000 m²***

Problem 139:

Given below is the technical description of lot 2081, Cebu Cadastre.

LINES	BEARINGS	DISTANCES
1 - 2	S 32°17' W	22.04 m.
2 - 3	S 36°25' W	10.00 m.
3 - 4	N 15°47' W	5.00 m.
4 - 5	N 73°07' E	19.95 m.

① Find the area of the lot by DMD method.
② Find the DPD of line 3 - 4.
③ Find the area of lot by DPD method.

Solution:

① *Area by DMD method:*
Departure = distance x *sin bearing*
Latitude = distance x *cos bearing*

Lines	Bearings	Dist	LAT	DEP	DMD
1 - 2	S32°17'W	22.04	-18.63	-11.77	-17.77
2 - 3	N36°25'W	10.00	+8.03	- 5.96	-29.50
3 - 4	N 15°47'W	5.00	+4.81	- 1.36	-36.82
4 - 1	N73°07'E	19.95	+5.79	-19.09	+19.09

To compute the DMD:

Lines		DMD	Double Area
1-2		-11.77	+219.275
2-3	- 11.77 - 11.77 - 5.96 =	-29.50	- 236.885
3-4	- 29.50 - 5.96 - 1.36 =	-36.82	-177.104
4-1	- 36.82 - 1.36 - 19.09 =	+19.09	-110.531

To compute for double area = DMD x *latitude*

Lines			Double Area
1 - 2	- 11.77 (-18.63)	=	219.275
2 - 3	- 29.50 (8.03)	=	- 236.885
3 - 4	- 36.82 (4.81)	=	- 177.104
4 - 1	- 19.09 (5.79)	=	- 110.531

Negative double areas
= 236.885 + 177.104 + 110.531
= 524.520 sq.m.

Positive double area = 219.275

$$Area = \frac{524.50 - 219.275}{2}$$

Area = ***152.622 sq.m.***

② *DPD of line 3 - 4:*

Lines	LAT	DEP	DPD	Double Area
1 - 2	- 18.63	- 11.77	- 18.63	+219.275
2 - 3	8.03	- 5.96	- 29.23	+174.211
3 - 4	4.81	- 1.36	- 16.39	+22.290
4 - 1	5.79	+19.09	- 5.79	+110.531

DPD of line 3 - 4 = ***- 16.39***

③ *Area by DPD method:*
Double Area = 219.275 + 174.211
+ 22.290 - 110.531
Double area = 305.245

$$Area = \frac{305.245}{2}$$

Area = ***152.622 sq.m.***

AREA OF CLOSED TRAVERSE

Problem 139-A:

From the given technical description of a lot.

LINES	BEARINGS	DISTANCES
AB	N.48°20'E.	529.60 m.
BC	N.87°00'E.	592.00 m.
CD	S.7°59'E.	563.60 m.
DE	S.80°00'W.	753.40 m.
EA	N.48°12'W.	428.20 m.

① Find the corrected bearing of line BC using transit rule.
② Find the corrected bearing of line DE using transit rule.
③ Find the corrected distance of line EA using transit rule.

Solution:

① *Corrected bearing of line BC using transit rule:*

Lines	Bearing	Distance	LAT	DEP
AB	N.48°20'E.	529.60	+352.08	+395.62
BC	N.87°00'E.	592.00	+30.98	+591.19
CD	S.7°59'E.	563.60	-558.14	+78.28
DE	S.80°00'W.	753.40	-130.83	-741.95
EA	N.48°12'W.	428.20	+285.41	-319.21
			+668.47	+1065.09
			-688.97	-1061.16
		Error =	-20.5	+3.93

Plus (+) 668.47 1065.09 minus

688.97 1061.16

1357.44 2126.25

Corrections using transit rule:

Line AB:

Latitude

$$\frac{C_1}{20.5} = \frac{352.08}{1357.44}$$

$C_1 = 5.32$

Departure

$$\frac{C_1}{3.93} = \frac{395.62}{2126.25}$$

$C_1 = 0.73$

Line BC:

Latitude

$$\frac{C_2}{20.5} = \frac{30.98}{1357.44}$$

$C_2 = 0.47$

Departure

$$\frac{C_2}{3.93} = \frac{591.19}{2126.25}$$

$C_2 = 1.09$

Line CD:

Latitude

$$\frac{C_3}{20.5} = \frac{558.14}{1357.44}$$

$C_3 = 8.42$

Departure

$$\frac{C_3}{3.93} = \frac{78.28}{2126.25}$$

$C_3 = 0.15$

Line DE:

Latitude

$$\frac{C_4}{20.5} = \frac{130.83}{1357.44}$$

$C_4 = 1.98$

Departure

$$\frac{C_4}{3.93} = \frac{74.95}{2126.25}$$

$C_4 = 1.37$

Line EA:

Latitude

$$\frac{C_5}{20.5} = \frac{285.41}{1357.44}$$

$C_5 = 4.31$

Departure

$$\frac{C_5}{3.93} = \frac{319.21}{2126.25}$$

$C_5 = 0.59$

LINES	CORRECTED LATITUDES
AB	+ 352.08 + 5.32 = + 357.40
BC	+ 30.98 + 0.47 = + 31.45
CD	- 558.14 + 8.42 = - 549.72
DE	- 130.83 + 1.98 = - 128.85
EA	+ 285.41 + 4.31 = + 289.72
	0

LINES	CORRECTED DEPARTURES
AB	+ 395.62 - 0.73 = + 394.89
BC	+ 591.19 - 1.09 = + 590.10
CD	+ 78.28 - 0.15 = + 78.13
DE	- 741.95 - 1.37 = - 743.32
EA	- 319.21 - 0.59 = - 319.80
	0

Corrected bearing of line BC:

$$\tan \text{bearing} = \frac{\text{Dep.}}{\text{Lat.}}$$

$$\tan \text{bearing BC} = \frac{590.10}{31.45}$$

Corrected Bearing BC = ***N. 86°57' E***

② *Corrected bearing of line DE:*

$$\tan \text{bearing} = \frac{\text{Dep.}}{\text{Lat.}}$$

$$\tan \text{bearing DE} = \frac{-743.32}{-128.85}$$

Corrected bearing DE = ***S. 80°10' W.***

③ *Corrected distance of line EA:*

$$\tan \text{bearing} = \frac{\text{Dep.}}{\text{Lat.}}$$

$$\tan \text{bearing EA} = \frac{-319.80}{+289.72}$$

Corrected bearing EA = N. 47°50' W.

$$\text{Distance} = \frac{\text{Dep.}}{\text{Sin bearing}}$$

$$\text{Distance} = \frac{319.80}{\text{Sin } 47°50'}$$

Distance = ***431.52 m.***

Check:

$$\text{Distance} = \sqrt{(\text{Lat})^2 + (\text{Dep})^2}$$

$$\text{Distance} = \sqrt{(289.72)^2 + (319.80)^2}$$

Distance = ***431.52 m.***

Problem 139-B:

Using the given data in the traverse shown:

POINTS	NORTHINGS	EASTINGS
A	75 m.	250 m.
B	425 m.	150 m.
C	675 m.	450 m.
D	675 m.	675 m.
E	425 m.	700 m.
F	175 m.	550 m.

① Compute the bearing of line BC.
② Compute the distance of line FA.
③ Compute the area enclosed by the straight line bounded by the points ABCDEFA.

Solution:

① *Bearing of line BC:*

LINES	LATITUDES	DEPARTURE
AB	425 - 75 = +350	150 - 250 = -100
BC	675 - 425 = +250	450 - 150 = +300
CD	675 - 675 = 0	675 - 450 = +225
DE	425 - 675 = -250	700 - 675 = +25
EF	175 - 425 = -250	550 - 700 = -150
FA	75 - 175 = -100	250 - 550 = -300

Bearing of line BC:

$$\tan \text{bearing} = \frac{\text{Dep.}}{\text{Lat.}}$$

$$\tan \text{bearing} = \frac{+300}{+250}$$

Bearing BC = ***N. 50°12' E.***

② *Distance of line FA:*

$$\text{FA} = \sqrt{(\text{Dep})^2 + (\text{Lat})^2}$$

$$\text{FA} = \sqrt{(-300)^2 + (-100)^2}$$

FA = ***316.23 m.***

③ *Area bounded the straight lines:*

Lines	LAT.	DEP.	DMD	Double Area
AB	+350	-100	-100	-100(350) = -35000
BC	+250	+300	+100	100(250) = + 25000
CD	0	+225	+625	625(0) = 0
DE	-250	+25	+875	875(-250) = -218750
EF	-250	-150	+750	750(-250) = -187500
FA	-100	-300	+300	300(-100) = -30000

2A = - 446250

A = ***223125 m²***

Problem 139-C:

In the traverse table below shows the Latitudes and Departures of the closed traverse.

LINES	LAT.	DEP.
AB	- 36.13	- 25.77
BC	+ 74.56	- 115.93
CD	+ 12.82	+ 0.39
DE	+ 19.90	+ 61.74
EA	- 68.40	+ 69.57

① Compute the corrected bearing of line BC using transit rule.
② Compute the corrected distance of line EA using transit rule.
③ Compute the area of the traverse by balancing the traverse by transit rule.

Solution:

① *Corrected bearing of line BC using transit rule:*

LINES	LAT.	DEP.
AB	- 36.13	- 25.77
BC	+ 74.56	- 115.93
CD	+ 12.82	+ 0.39
DE	+ 19.90	+ 61.74
EA	- 68.40	+ 69.57
	+ 107.28	+ 131.70
	- 104.53	- 141.70
	+ 2.75	- 10.00

107.28	131.70
104.53	141.70
211.81	273.40

Corrections using transit rule:

Line AB:

Latitude

$$\frac{C_1}{2.75} = \frac{36.13}{211.81}$$

$C_1 = 0.47$

Departure

$$\frac{C_1}{10} = \frac{25.77}{273.40}$$

$C_1 = 0.94$

Line BC:

Latitude

$$\frac{C_2}{2.75} = \frac{74.56}{211.81}$$

$C_2 = 0.97$

Departure

$$\frac{C_2}{10} = \frac{115.93}{273.40}$$

$C_2 = 4.24$

Line CD:

Latitude

$$\frac{C_3}{2.75} = \frac{12.82}{211.81}$$

$C_3 = 0.16$

Departure

$$\frac{C_3}{10} = \frac{0.39}{273.40}$$

$C_3 = 0.01$

Line DE:

Latitude

$$\frac{C_4}{2.75} = \frac{19.90}{211.81}$$

$C_4 = 0.26$

Departure

$$\frac{C_4}{10} = \frac{61.74}{273.40}$$

$C_4 = 2.26$

Line EA:

Latitude

$$\frac{C_5}{2.75} = \frac{68.40}{211.81}$$

$C_5 = 0.89$

Departure

$$\frac{C_5}{10} = \frac{69.57}{273.40}$$

$C_5 = 2.55$

AREA OF CLOSED TRAVERSE

LINES	CORRECTED LATITUDES	CORRECTED DEPARTURES
AB	- 36.13 + 0.47 = - 36.6	- 25.77 - 0.94 = - 24.83
BC	+ 74.56 - 0.97 = + 73.59	- 115.93 - 4.24 = - 111.69
CD	+ 12.82 - 0.16 = + 12.66	+ 0.39 + 0.01 = + 0.40
DE	+ 19.90 - 0.26 = + 19.64	+ 61.74 + 2.26 = + 64.00
EA	- 68.40 + 0.89 = - 69.29	+ 69.57 + 2.55 = + 72.12

Corrected bearing of BC:

$$\tan \text{bearing} = \frac{\text{Dep.}}{\text{Lat.}}$$

$$\tan \text{bearing BC} = \frac{-111.69}{+73.59}$$

Bearing BC = ***N 56°37' W***

② *Corrected distance of line EA:*

$$AE = \sqrt{(Dep)^2 + (Lat)^2}$$

$$AE = \sqrt{(72.12)^2 + (-69.29)^2}$$

AE = ***100.01 m.***

③ *Area of the traverse:*

LINES	LATITUDE	DEPARTURE	DMD	DOUBLE AREA
AB	- 36.6	- 24.83	- 24.83	- 24.83(- 36.6) = + 908.78
BC	+ 73.59	- 111.69	- 161.35	- 161.35(73.59) = - 11873.75
CD	+ 12.66	+ 0.40	- 272.64	- 272.64(12.66) = - 3451.62
DE	+ 19.64	+ 64.00	- 208.24	- 208.24(19.64) = - 4089.83
EA	- 69.29	+ 72.12	- 72.12	- 72.12(- 69.29) = + 4997.19

2A = - 13509.23

A = ***6754.62 m²***

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From the given data of a closed traverse

LINES	DISTANCE	BEARING
AB	368.76 m.	N.15°18'E.
BC	645.38 m.	S.85°46'E.
CD	467.86 m.	S.18°30'W
DA	593.00 m.	N.77°35'W

Using compass rule of balancing a traverse.

① Determine the corrected bearing of BC.
② Determine the corrected bearing of CD.
③ Determine the adjusted distance of BC.

Solution:

① Corrected bearing of BC:

LINE	BEARING	DISTANCE	LAT.	DEP.
AB	N.15°18'E.	368.76 m.	+355.69	+97.31
BC	S.85°46'E.	645.38 m.	-47.64	+643.62
CD	S.18°30'W	467.86 m.	-443.68	-148.45
DA	N.77°35'W	593.00 m.	+127.51	-579.13
		2075.00	+483.20	+740.93
			-491.32	-727.58
		Error =	-8.12	+13.35

Total distance = 2075.00 m.

Using compass rule of balancing:

Correction for Latitude:

$$\frac{C_1}{8.12} = \frac{368.76}{2075}$$

$$C_1 = \frac{8.12}{2075}(368.76) = 1.44$$

$$C_2 = \frac{8.12}{2075}(645.38) = 2.53$$

$$C_3 = \frac{8.12}{2075}(467.86) = 1.83$$

$$C_4 = \frac{8.12}{2075}(593) = \underline{2.32}$$

8.12

Correction for Departure:

$$\frac{C_1}{13.35} = \frac{368.76}{2075}$$

$$C_1 = \frac{13.35}{2075}(368.76) = 2.37$$

$$C_2 = \frac{13.35}{2075}(645.38) = 4.15$$

$$C_3 = \frac{13.35}{2075}(467.86) = 3.01$$

$$C_4 = \frac{13.35}{2075}(593) = \underline{3.82}$$

13.35

LINES	Corrected Lat.	Corrected Dep.
AB	$\boxed{+1.44}$ **+355.69** **= +357.13**	$\boxed{-2.37}$ **+97.31** **= +94.94**
BC	$\boxed{-2.53}$ **-47.64** **= -45.11**	$\boxed{-4.15}$ **+643.62** **= +639.47**
CD	$\boxed{-1.83}$ **-443.68** **= -441.85**	$\boxed{+3.01}$ **-148.45** **= -151.46**
DA	$\boxed{+2.32}$ **+127.51** **= +129.83**	$\boxed{+3.82}$ **-579.13** **= -582.95**
	= 0	**= 0**

Corrected bearing of line BC:

$$\tan \text{bearing} = \frac{\text{Dep.}}{\text{Lat.}} = \frac{+639.47}{-45.11}$$

Bearing = ***S.85°58'E.***

② Corrected bearing of CD:

$$\tan \text{bearing} = \frac{\text{Dep.}}{\text{Lat.}} = \frac{-151.46}{-441.85}$$

Bearing = ***S.18°55'W***

③ Adjusted distance of BC:

$$\text{Sin bearing} = \frac{\text{Dep.}}{\text{Dist.}}$$

$$\text{Dist} = \frac{\text{Dep.}}{\text{Sin bearing}} = \frac{639.47}{\text{Sin } 85°58'}$$

Dist = ***641.06 m.***

Problem 139-E:

A closed traverse has the following data:

LINES	DISTANCE	BEARING
AB	895	S. 70°29' E.
BC	315	S. 26°28' E.
CD	875	S. 65°33' W.
DE	410	N. 45°31' W.
EA	650	N. 10°00' E.

① Find the corrected bearing of line BC by using Transit Rule.
② Find the corrected bearing of line CD by using Transit Rule.
③ Find the corrected bearing of line EA by using Transit Rule.

Solution:

① Corrected bearing of line BC using Transit Rule:

LINES	BEARING	DISTANCE
AB	S. 70°29' E.	895
BC	S. 26°28' E.	315
CD	S. 65°33' W.	875
DE	N. 45°31' W.	410
EA	N. 10°00' E.	650

			CORRECTED	
Lines	LAT	DEP	LAT	DEP
AB	- 299	+843.58	-296.49	+840.57
BC	-281.99	+140.39	-279.62	+139.89
CD	-362.16	-796.53	-359.11	-799.37
DE	+287.29	-292.52	+289.71	-293.56
EA	+640.13	+112.87	+645.51	+112.47

-943.15	-1089.05	
+927.42	+1096.84	
1870.57	2184.89	Sum of lat & dep.
- 15.73	+7.79	Error

LAT

$$\frac{AB}{15.73} = \frac{299}{1870.57}$$

AB = 299(0.0084092)
AB = - 2.51 (to be subtracted
BC = 281.99(0.0084092)
BC = - 2.37 (subtracted)
CD = 362.16(0.0084092)
CD = - 3.05 (subtracted)
DE = 287.29(0.0084092)
DE = + 2.42 (added)
EA = 640.13(0.0084092)
EA = + 5.38 (added)
15.73

DEP

$$\frac{AB}{7.79} = \frac{843.58}{2184.89}$$

AB = 843.58(0.0035654)
AB = 3.01 (to be subtracted)
BC = 140.39(0.0035654)
BC = 0.50 (subtracted)
CD = 796.53(0.0035654)
CD = 2.84 (added)
DE = 292.52(0.0035654)
DE = 1.04 (added)
EA = 112.87(0.0035654)
EA = 0.40 (subtracted)
7.79

Corrected bearing for line BC:

$$\tan \text{bearing} = \frac{\text{dep}}{\text{lat}}$$

$$\tan \text{bearing} = \frac{+139.89}{-279.62}$$

Bearing = ***S. 26° 34' 42" E.***

② Corrected bearing for line CD:

$$\tan \text{bearing} = \frac{-799.37}{-359.11}$$

Bearing = ***S. 65° 48' 30" W.***

③ Corrected bearing for line EA:

$$\tan \text{bearing} = \frac{+112.47}{+645.51}$$

Bearing = ***N. 9° 53' 01" E***

AREA OF CLOSED TRAVERSE

Problem 140:

From the following traverse,

Line	Bearing	Azimuth	Distance
1 - 2	N 48°30' W	131°30'	81.00 m.
2 - 3	N 77°00' E	257°00'	66.00 m.
3 - 4	S 55°00' W	55°00'	94.00 m.
4 - 1	----	----	----

① Compute the bearing of line 4 - 1.
② Compute the distance of side 4 - 1.
③ Compute the area enclosed by the traverse.

Solution:

The sketch shows that the traverse lines 1 - 2 and 3 - 4 crossed each other, hence we could not adopt the DMD method of determining its area.

Lines	Bearing	Distance	LAT	DEP
1 - 2	N 48°30' W	81.00	+53.70	- 60.70
2 - 3	N 77°00' E	66.00	+14.85	+64.30
3 - 4	S 55°00' W	94.00	- 53.90	- 77.00
4 - 1			+14.65	- 73.40

① *Bearing of line 4 - 1:*

$$\tan \text{bearing} = \frac{dep}{lat}$$

$$\tan \text{bearing} = \frac{73.40}{14.65}$$

Bearing = ***S 78°42' E***

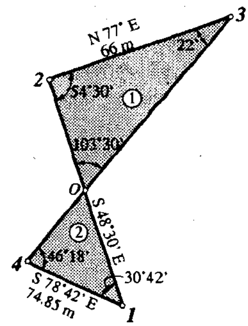
② *Distance of side 4 - 1:*

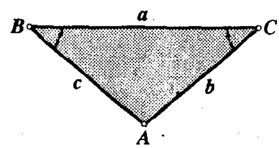
$$\text{Distance} = \frac{\text{Departure}}{\text{Sin bearing}}$$

$$\text{Distance} = \frac{73.40}{\text{Sin } 78°42'}$$

Distance (4 - 1) = ***74.85 m.***

③ *Area enclosed by the traverse:*





From Plane Trigonometry: 74.85

Area of triangle ABC $= \frac{1}{2} a\, c \sin B$

Using the Law of Sine:

$$\frac{c}{\text{Sin } C} = \frac{a}{\text{Sin } A}$$

$$c = \frac{a \text{ Sin } C}{\text{Sin } A}$$

$$\text{Area} = \frac{\frac{1}{2}(a)\,(a) \text{ Sin } C \text{ Sin } B}{\text{Sin } A}$$

$$\text{Area} = \frac{a^2 \text{ Sin } B \text{ Sin } C}{2 \text{ Sin } A}$$

Considering triangle 230:

$$A_1 = \frac{(66)^2 \text{ Sin } 22° \text{ Sin } 54°30'}{2 \text{ Sin } 103°20'}$$

A_1 = 683 sq.m.

Considering triangle 014:

$$A_2 = \frac{(74.85)^2 \text{ Sin } 46°18' \text{ Sin } 30°12'}{2 \text{ Sin } 103°30'}$$

A_2 = 1046 sq.m.

Total area = $A_1 + A_2$
A = 683 + 1046
A = ***1729 sq.m.***

AREA OF CLOSED TRAVERSE

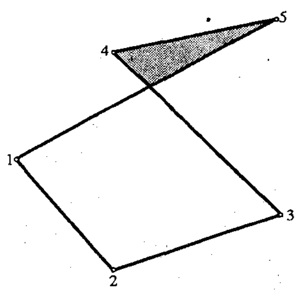
Problem 141:

From the closed traverse shown below, compute the following:

① The bearing of line 4 - 5.
② The distance of line 4 - 5.
③ The area enclosed by the line 3 - 4, 4 - 5 and 5 - 1.

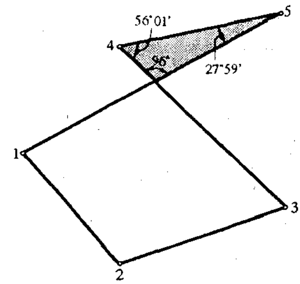
LINES	BEARING	DISTANCES
1 - 2	S 10°00' E	485.00
2 - 3	N 56°00' E	780.00
3 - 4	N 63°00 W	975.00
4 - 5	----	----
5 - 1	S 33°00' W	890.00

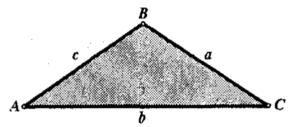
Solution:



① *Bearing of line 4 - 5:*

Lines	Bearing	Distances	LAT	DEP
5 - 1	S 33° W	890	- 746.42	- 484.73
1 - 2	S 10° E	485	- 477.63	+84.22
2 - 3	N 56° E	780	+436.17	+646.65
3 - 4	N 63° W	575	+442.64	- 868.73
			+345.84	+622.59
			+878.81	+730.87
			- 1224.05	- 1353.46
			- 345.24	- 622.59





$$\tan Bearing\,(4 - 5) = \frac{622.59}{345.24}$$

Bearing (4 - 5) = ***N 60°59' E***

② *Distance of line 4 - 5:*

$$Distance\,(4 - 5) = \frac{622.59}{\sin 60°59'}$$

Distance (4 - 5) = ***711.90 m.***

③ *Area enclosed by the line 3 - 4, 4 - 5 and 5 - 1:*

$$A = \frac{bc \sin A}{2}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$c = \frac{b \sin C}{\sin B}$$

$$A = \frac{b^2 \sin A \sin C}{2 \sin B}$$

Area of shaded section

$$A = \frac{(711.90)^2 \sin 56°01' \sin 27°\,59}{2 \sin 96°}$$

$$A = \mathbf{99169.28\ m^2}$$

AREA OF CLOSED TRAVERSE

Problem 142:

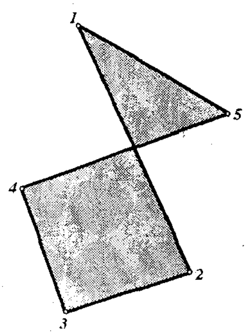
A Civil Engineer, in his haste, forgot to record the data of the closing line of his traverse, the field notes of which reflects the following record.

LINE	AZIMUTH	DISTANCE
1 - 4	321°40'	140.25 m.
2 - 3	51°27'	77.52 m.
3 - 4	130°50'	65.10 m.
4 - 5	225°0'	108.64 m.
5 - 1	----	----

① Compute the bearing of line 5 - 1.
② Compute the distance of line 5 - 1.
③ Compute the area enclosed by the traverse.

Solution:

Sketch the traverse and find out if the lines do not intersect each other, if so, then application of DMD in determining the area will not suffice.



① *Bearing of line 5 - 1:*

Lines	Bearing	Distance	LAT	DEP
1 - 2	S 30°20' E	140.25	- 110.02	+86.99
2 - 3	S 51°57' W	77.52	- 47.78	- 61.04
3 - 4	N 49°10' W	65.10	+42.57	- 49.26
4 - 5	N 45°00' E	108.64	+76.82	+76.82
5 - 1			+38.41	53.51

$$\text{tan bearing } (5-1) = \frac{53.51}{38.41}$$

Bearing (5 - 1) = ***N 54°20' W***

② *Distance of line 5 - 1:*

$$\text{Distance } (5-1) = \frac{53.51}{\sin 54°20'}$$

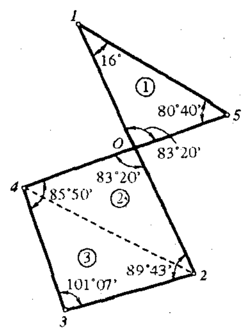
Distance (5 - 1) = ***65.86 m.***

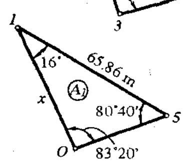
③ *Area enclosed by the traverse:*

$$A_1 = \frac{(65.86)^2 \sin 16° \sin 80°40'}{2 \sin 83°20'}$$

$A_1 = 593.91$ sq.m.

Using Sine Law:





$$\frac{X}{\sin 80°40'} = \frac{65.86}{\sin 83°20'}$$

$X = 65.43$

$$\frac{Y}{\sin 16°} = \frac{65.86}{\sin 83°20'}$$

$Y = 18.28$ m.

Distance 4 to 0 = 108.64 - 18.28
Distance 4 to 0 = 90.36
Distance 2 to 0 = 140.25 - 65.43
Distance 2 to 0 = 74.82

$$A_2 = \frac{90.36\,(74.82) \sin 83°20'}{2}$$

$A_2 = 3357.49$ sq.m.

$$A_3 = \frac{65.10\,(77.52) \sin 101°07'}{2}$$

$A_3 = 2475.90$ sq.m.

Total $A = A_1 + A_2 + A_3$
$A = 593.91 + 3357.49 + 2475.90$
$A =$ ***6427.30 sq.m.***

AREA OF CLOSED TRAVERSE

Problem 143:

Given the following technical description of an approved surveys and their corresponding coordinates.

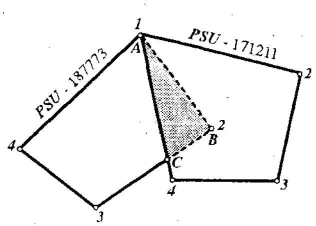
PSU - 187773

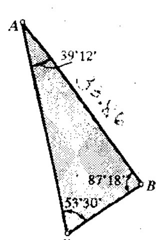
Corner	Bearings	Distance	Coordinates	
1 - 2	S 34°01' E	33.86	20079.84	19842.72
2 - 3	S 58°41' W	37.74	20051.77	19861.66
3 - 4	N 63°43' W	24.75	20032.16	19827.42
4 - 1	N 45°36' E	52.47	20043.12	19805.23

PSU - 171211

Corner	Bearings	Distance	Coordinates	
1 - 2	S 75°29' E	33.62	20079.84	19842.72
2 - 3	S 20°23' W	54.99	20071.41	19875.27
3 - 4	N 79°05' W	18.85	20019.86	19856.11
4 - 1	N 5°11' E	56.65	20023.43	19837.60

① Compute the location of the point of intersection of the overlapping areas from corner 4 of lot PSU-171211.
② Compute the location of the point of intersection of the overlapping areas from corner 5 of lot PSU-187773.
③ Compute the area of the overlapping portion.





Solution:

① *Location of the point of intersection of the overlapping areas from corner 4 of lot PSU-171211:*

$$\frac{AC}{\sin 87°18'} = \frac{33.86}{\sin 53°30'}$$

$AC = 42.08$

The point of intersection from corner 4

$= 56.65 - 42.08$

$= \mathbf{14.57}$

② *Location of the point of intersection of the overlapping areas from corner 5 of lot PSU-187773:*

$$\frac{BC}{\sin 39°12'} = \frac{33.86}{\sin 53°30'}$$

$BC = 26.62$

Point of intersection from corner 5

$= 37.74 - 26.62$

$= \mathbf{11.12}$

③ *Area of the overlapping portion:*

$$A = \frac{33.86\,(42.08)\sin 39°12'}{2}$$

A = ***450.20 sq.m.***

Problem 144:

Given the corrected latitudes and departure of a closed traverse.

LINES	LATITUDE	DEPARTURE
1 - 2	+ 80.16	- 40.12
2 - 3	- 40.13	- 36.82
3 - 4	+ 70.18	+ 50.42
4 - 5	- 30.14	+ 30.36
5 - 6	+ 60.20	- 52.34
6 - 1	- 140.27	+ 48.50

① Compute the DMD of line 3 - 4.
② Compute the DPD of line 4 - 5.
③ Compute the area of the closed traverse in acres.

AREA OF CLOSED TRAVERSE

Solution:

① *DMD of line 3 - 4:*

LINES	DEP	DMD
1 - 2	- 40.12	- 40.12
2 - 3	- 36.82	- 117.06
3 - 4	+ 50.42	**- 103.46**
4 - 5	+ 30.36	- 22.68
5 - 6	- 52.34	- 44.66
6 - 1	+ 48.50	- 48.50

DMD of line 3 - 4 = ***- 103.46***

② *DPD of line 4 - 5:*

LINES	LAT	DPD
1 - 2	+ 80.16	+ 80.16
2 - 3	- 40.13	+ 120.19
3 - 4	+ 70.18	+ 150.24
4 - 5	- 30.14	**+ 190.28**
5 - 6	+ 60.20	+ 220.34
6 - 1	- 140.27	+ 140.27

DPD of line 4 - 5 = ***+ 190.28***

③ *Area of lot:*

LINES	LAT	DMD	DOUBLE AREA (LAT x DMD)
1 - 2	+80.16	- 40.12	- 3216.02
2 - 3	- 40.13	- 117.06	+ 4697.62
3 - 4	+70.18	- 103.46	- 7260.82
4 - 5	- 30.14	- 22.68	+ 683.58
5 - 6	+60.20	- 44.66	- 2688.53
6 - 1	- 140.27	- 48.50	+ 6803.10

- 13165.37
+ 12184.30

$2A = -981.07$

$A = 490.54$

$$A = \frac{490.54}{4047}$$

$A = $ ***0.121 acres***

Problem 145:

From the given closed traverse:

LINES	BEARING	DISTANCE
AB	N. 20° E.	17.42
BC	N. 68° E.	18.46
CD	S. 22° E.	22.40
DE	S. 40° W.	12.60
EF	S. 62° W.	10.20
FA	—	—

① Compute the bearing of line FA.
② Compute the distance of line FA.
③ Compute the area of the closed traverse in acres.

Solution:

① *Bearing of line FA:*

Lines	Bearing	Distance	LAT.	DEP.
AB	N. 20° E.	17.42	+16.37	+5.96
BC	N. 68° E.	18.46	+6.92	+17.12
CD	S. 22° E.	22.40	- 20.77	+8.39
DE	S. 40° W.	12.60	- 9.65	- 8.10
EF	S. 62° W.	10.20	- 4.79	- 9.01
FA	—	—	+11.92	- 14.36

$$\tan \text{bearing} = \frac{Dep}{Lat}$$

$$\tan \text{bearing} = \frac{14.36}{11.92}$$

Bearing FA = ***N 50°18' W***

② *Distance FA:*

$$\text{Distance} = \frac{Dep}{\text{Sin bearing}}$$

$$\text{Distance} = \frac{14.36}{\text{Sin } 50°18'}$$

Distance = ***18.66 m.***

AREA OF CLOSED TRAVERSE

③ *Area of closed traverse:*

Lines	LAT.	DEP.	DMD	Double Area
AB	+16.37	+5.96	+5.96	+97.57
BC	+6.92	+17.12	+29.04	+200.96
CD	- 20.77	+8.39	+54.55	- 1133.00
DE	- 9.65	- 8.10	+54.84	- 529.21
EF	- 4.79	- 9.01	+37.73	- 180.73
FA	+11.92	- 14.36	+14.36	+171.17

$2A = -373.24$

$A = 186.62$

$A = \frac{186.62}{4047}$

$A = \mathbf{0.046\ acres}$

$\text{tangent bearing} = \frac{10.79}{53.47}$

tangent bearing = **S. 11°24' W**

② *Distance DA:*

$\text{Distance} = \frac{10.79}{\text{Sin } 11°24'}$

Distance = **54.55 m.**

③ *Area:*

$\text{Area} = \frac{3745.01 \text{ m}^2}{4047}$

Area = **0.925 acres**

Problem 146:

From the given data of a closed traversed, compute the following:

LINES	BEARING	DISTANCES
AB	S. 8°51' W.	126.90 m.
BC	N. 18°51' W.	90.20 m.
CD	N. 32°27' E.	110.80 m.
DA	—	—

① Bearing of line DA.
② Distance of line DA.
③ Area of lot ABCD in acres.

Solution:

① *Bearing DA:*

LINES	BEARING	DISTANCES
AB	S. 8°51' W.	126.90 m.
BC	N. 18°51' W.	90.20 m.
CD	N. 32°27' E.	110.80 m.
DA	—	—

Lines	LAT	DEP	DMD	2A
AB	- 125.39	- 19.52	- 19.52	+2447.61
BC	+ 85.36	- 29.14	- 68.18	- 5819.84
CD	+93.50	- 59.45	- 37.87	- 3540.85
DE	- 53.47	- 10.79	+10.79	- 576.94

$2A = 7490.02$

$A = 3745.01 \text{ m}^2$

Problem 147:

From the following closed traversed

LINES	BEARING	DISTANCES
1 - 2	N. 30° E.	120.20
2 - 3	N. 78° E.	90.20
3 - 4	S. 32° E.	88.40
4 - 1	---	---

① Compute the bearing of line 4 - 1.
② Compute the distance of line 4 - 1.
③ Compute the DMD of line 3 - 4.
④ Compute the area of the lot in hectares.

Solution:

① *Bearing of line 4 - 1:*

Lines	LAT	DEP	DMD	Double Area
1 - 2	+104.10	+ 60.10	+ 60.10	+3760102.41
2 - 3	+ 18.75	+ 88.23	+208.43	+3908.06
3 - 4	- 74.97	+ 46.84	+343.5	- 25752.20
4 - 1	- 47.88	- 195.17	+195.17	- 9344.74

$2A = 3728913.53$

$\text{tangent bearing} = \frac{195.17}{47.88}$

tangent bearing = **S. 76°13' W**

AREA OF CLOSED TRAVERSE

② *Distance 4 - 1:*

$$= \frac{195.17}{\text{Sin } 76°13'}$$

$= \mathbf{200.96}$

③ *DMD of line 3-4* = **+ 3434.5**

④ *Area* = 1,864,456.77
Area = **186.45 hectares**

Problem 148:

From the given tract of land having the following data, compute the following:

LINES	LATITUDE	DEPARTURE
A - B	+ 40	- 80
B - C	+ 80	- 40
C - D	- 30	+ 70
D - A	- 90	+ 50

① Double meridian distance of line CD.
② Double parallel distance of line CD.
③ Area of tract of land in acres.

Solution:

① *DMD of line CD:*

LAT	DEP	DMD	2A
+ 40	- 80	- 80	- 3200
+ 80	- 40	- 200	- 16000
- 30	+ 70	- 170	+ 5100
- 90	+ 50	- 50	+ 4500

2A = - 9600
A = **4800 m^2**

DMD of line CD = **- 170**

② *DPD of line CD:*

LAT	DEP	DPD
+ 40	- 80	+ 40
+ 80	- 40	+160
- 30	+ 70	+210
- 90	+ 50	+ 90

DPD of line CD = **+ 210**

③ *Area of lot in acres:*
Area = 4800 m^2

$$Area = \frac{4800}{4047}$$

Area = **1.186 acres**

Problem 149:

The given compound data of a five sided lot.

LINES	LAT	DEP	DMD	2A
AB	+57.81	+16.03	—	—
BC	x	+72.04	—	-1002.71
CD	y	+13.36	+189.50	-8108.71
DE	- 18.75	—	—	—
EA	+13.36	- 48.18	—	—

① Compute the bearing of line CD.
② Compute the DMD of line DE.
③ Compute the area of the 5sided lot in sq.meters.

Solution:

① *Bearing of CD:*
Double area = *Lat* x *DMD*
- 8108.71 = *y* (189.50)
y = - 42.79
Lat. CD = - 42.79
Dep. CD = +13.36

$$\tan \text{bearing} = \frac{Dep}{Lat}$$

$$\tan \text{bearing} = \frac{13.36}{42.79}$$

Bearing = **S 17°20' E**

② *DMD of line DE:*

+ 16.03
+ 72.04
+ 13.36

+101.43
- 48.18

- 53.25

Dep. of DE = **- 53.25**

AREA OF CLOSED TRAVERSE

③ *Area of the 5 sided lot in acres.*

Lines	LAT	DEP.	DMD	Double Area
AB	+ 57.81	+ 16.03	+ 16.03	+ 926.69
BC	- 9.63	+ 72.04	+104.10	- 1002.48
CD	- 42.79	+ 13.36	+189.50	- 8108.71
DE	- 18.75	- 53.25	+149.61	- 2805.19
EA	+ 13.36	- 48.18	+ 48.18	+ 643.68

2A = 10346.01
$A = 5173.005\ m^2$

Σ +LAT	Σ - LAT
+ 57.81	- 42.79
+ 13.36	- 18.75
+ 71.17	- 61.54

$x = 71.17 - 61.54$
$x = -9.63$
$A = 5173.005\ m^2$
$A = \frac{5173.005}{4047}$
$A =$ ***1.278 acres***

Problem 150:

Given below are the corresponding data of a computation for area of a given lot with missing data.

Course	LAT	DEP	DMD	2A
1 - 2	+60	+16	?	?
2 - 3	?	+70	?	- 1428
3 - 4	?	+14	+186	- 5580
4 - 5	- 28	?	?	?
5 - 1	+12	- 46	?	?

① Compute the bearing of line 3 - 4.
② Compute the DMD of line 4 - 5.
③ Compute the area of the whole lot in acres.

Solution:

① *Bearing of line 3 - 4:*
LAT (DMD) = Double area
Lat (186) = - 5580
Lat = - 30

$\tan bearing = \frac{dep}{lat}$

$\tan bearing = \frac{14}{-30}$

Bearing = ***S 25°01' E***

② *DMD of line 4 - 5:*

Course	Lat	Dep	DMD
1 - 2	+60	+16	+16
2 - 3	- 14	+70	+102
3 - 4	- 30	+14	+186
4 - 5	- 28	- 54	+146
5 - 1	+12	- 46	+46

Lat of 2 - 3:
60 + 12 - 30 - 28 = 14
Dep of 4 - 5:
16 + 70 + 14 - 46 = 54
DMD of line 4 -5 = ***+146***

③ *Area of lot:*

Lines	LAT	DEP	DMD	Double Area
1 - 2	+60	+16	+16	+960
2 - 3	- 14	+70	+102	- 1428
3 - 4	- 30	+14	+186	- 5580
4 - 5	- 28	- 54	+146	- 4088
5 - 1	+12	- 46	+46	+552

2A = 9584
$A = 4792\ m^2$

$Area = \frac{4792}{4047}$
Area = ***1.184 acres***

AREA OF CLOSED TRAVERSE

Problem 151

The following data of a relocation survey of Don Mariano Escudero is to be reconstructed.

Lines	LAT	DEP	DMD	Double Area
AB	- 310.95	+469.84	----	
BC	----	+112.87	----	- 673810.93
CD	----	- 1099.62	+65.80	+1895.04
DE	+576.94	----	----	----
EA	+345.42	+538.77	----	----

① Find the bearing of line CD.
② Find the DMD of line DE.
③ Compute the area of the whole lot.

Solution:

① *Bearing of line CD:*

Double area = Lat x DMD

$1895.04 = Lat\,(65.80)$

$Lat = +28.80$

$$\tan bearing = \frac{dep}{lat}$$

$$\tan bearing = \frac{-1099.62}{+28.80}$$

bearing = ***N. 88°30' W.***

② *DMD of line DE:*

LINES	DEP	DMD
AB	+469.84	+469.84
BC	+112.87	+1052.55
CD	- 1099.62	+65.8
DE	- 21.86	- 1055.68
EA	+538.77	- 538.77

DMD of line DE = ***- 1055.68***

③ *Area of whole lot:*

LINES	LAT	DMD	DOUBLE AREA
AB	- 310.95	+469.84	- 146096.75
BC	- 640.21	+1052.55	- 673853.04
CD	+28.80	+65.8	+1895.04
DE	+576.94	-1055.68	- 609064.02
EA	+345.42	- 538.77	- 186101.93

$2A = 1613220.70$

$A = \mathbf{806610.35\ m^2}$

Problem 152

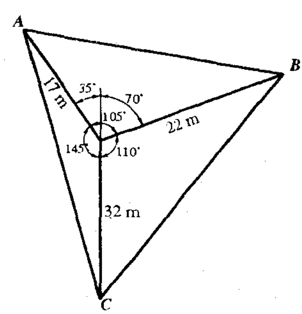
An engineer sets up a transit at a point inside a triangular lot and observes the bearings and distances of the corners A, B and C of the lot as follows:

CORNERS	BEARING	DISTANCES
A	N. 35° W.	17 m.
B	N. 70° E.	22 m.
C	Due South	32 m.

① Compute the area of the triangular lot.
② Compute the perimeter of the lot.
③ If the bearing from C to the point inside the triangular lot is due north, compute the bearing of CB.

Solution:

① *Area of the triangular lot:*



AREA OF CLOSED TRAVERSE

$$A_1 = \frac{17(22)\ \text{Sin } 105^\circ}{2}$$

$A_1 = 180.63$

$$A_2 = \frac{22(32)\ \text{Sin } 110^\circ}{2}$$

$A_2 = 330.77$

$$A_3 = \frac{32(17)\ \text{Sin } 145^\circ}{2}$$

$A_3 = 156.01$

$A = A_1 + A_2 + A_3$

$A = \mathbf{667.41\ m^2}$

② *Perimeter of the lot:*

$(AB)^2 = (17)^2 + (22)^2 - 2(17)(22)\ \text{Cos } 105^\circ$

$AB = 31.09$ m.

$(BC)^2 = (22)^2 + (32)^2 - 2(22)(32)\ \text{Cos } 110^\circ$

$BC = 44.60$ m.

$(AC)^2 = (17)^2 + (32)^2 - 2(17)(32)\ \text{Cos } 145^\circ$

$AC = 46.95$ m.

Perimeter = 31.09 + 44.60 + 46.95

Perimeter = ***122.64 m.***

③ *Bearing of CB:*

$$\frac{22}{\text{Sin } \theta} = \frac{44.60}{\text{Sin } 110^\circ}$$

$\theta = 27^\circ 37'$

Bearing of CB = ***N. 27°37' E.***

Missing Data

Problem 153:

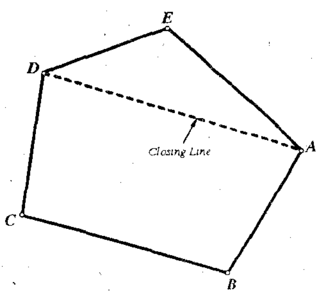
From the given closed traversed shown,

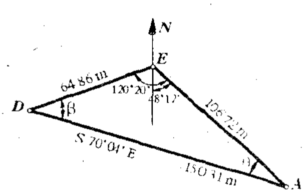
LINES	BEARING	DISTANCES
A - B	S. 35°30' W.	44.37 m.
B - C	N. 57°15' W.	137.84 m.
C - D	N. 1°45' E.	12.83 m.
D - E	?	64.86 m.
E - A	?	106.72 m.

① Compute the bearing of line DE.
② Compute the bearing of line EA.
③ Compute the area of the lot.

Solution:

Lines	Bearing	Distance	LAT	DEP
AB	S 35°30' W	44.37	- 36.12	- 25.77
BC	N 57°15' W	137.84	+74.57	-115.93
CD	N 1°45' E	12.83	+12.82	+0.39
DE	----	----	- 51.27	+141.31





MISSING DATA

① *Bearing of line DE:*
Bearing and distance of line DA:

$$\tan bearing = \frac{141.31}{51.27}$$

Bearing (DA) = S 70°04' E.

$$Distance\ (DA) = \frac{141.31}{\text{Sin } 70°04'} = 150.31 \text{ m.}$$

Considering triangle DCA:
Using Cosine Law:

$$(64.86)^2 = (106.72)^2 + (150.31)^2 - 2(150.31)(106.72) \cos ø$$

ø = 21°52"

Using Sine Law:

$$\frac{\text{Sin } B}{106.72} = \frac{\text{Sin } 21°52}{64.86}$$

B = 37°48'

$$\frac{\text{Sin } 21°52'}{64.86} = \frac{\text{Sin } \alpha}{150.31}$$

α = 120°20'

Bearing of line DE = ***N. 72°08' E.***

② *Bearing of line EA:*
Bearing of line EA = 70°04' - 21°52'
Bearing of line EA = ***S. 48°12' E.***

③ *Area of the lot:*

Lines	Bearing	Distance	LAT	DEP
AB	S 35°30' W	44.36	- 36.13	- 25.77
BC	N 57°15' W	137.84	+74.56	-115.93
CD	N 1°45' E	12.83	+12.82	+0.39
DE	N 72°08' E	64.86	+19.90	+61.74
EA	S. 48°12' E	106.72	- 71.15	+79.57

Lines	DMD	2A
AB	- 25.77	+ 931.07
BC	- 167.47	- 12486.56
CD	- 283.01	- 3628.19
DE	- 220.88	- 4395.51
EA	- 79.57	+ 5661.41

2A = 13917.78
A = 6958.89

Area of the lot = ***6958.89***

Problem 154:

From the given technical description shown.

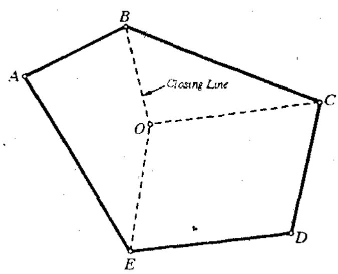
LINES	BEARING	DISTANCES
AB	N. 32°27' E.	110.8 m.
BC	?	83.6 m.
CD	S. 8°51' W.	126.9 m.
DE	S. 73°31' W.	?
EA	N. 18°44' W.	90.2 m.

① Compute the bearing of line BC.
② Compute the distance of line DE.
③ Compute the area of the lot.

Solution:

Lines	Bearing	Distance	LAT	DEP
OE	S 8°51' W	126.9	-125.39	- 19.52
EA	N 18°51' W	90.2	+85.42	- 28.96
AB	N 32°27' E	110.8	+93.50	+59.45
BO			- 53.53	- 10.97

Note: OE is equal and parallel to CD, likewise CO is equal and parallel to line DE.



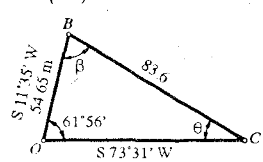
Bearing and distance of line (BO)

$$\tan bearing = \frac{10.97}{53.53}$$

Bearing of (BO) = S 11°35' W

$$Distance\ (BO) = \frac{10.97}{\text{Sin } 11°35'}$$

Distance (BO) = 54.65 m.

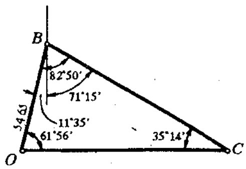


Consider the triangle BOC
Angle BOC = 73°31' - 11°35'
Angle BOC = 61°56'

$$\frac{\sin ø}{54.65} = \frac{\sin 61°56'}{83.6}$$

ø = 35°14'

① *Bearing of line BC:*



B = 180 - 61°56' - 35°14'
B = 180 - 90°10'
B = 82°50'

Bearing of line BC = ***S 71°15' E***

② *Distance of line DE:*

$$\frac{OC}{\sin 62°50'} = \frac{83.6}{\sin 61°56'}$$

OC = 94.00 m.
OC = DE
Distance of line DE = ***94.00 m.***

③ *Area of the lot:*

Lines	Bearing	Distance	LAT	DEP
AB	N 32°27' E	110.8	+93.50	+59.45
BC	S 71°15' E	83.60	- 26.87	+79.16
CD	S 8°51' W	126.90	- 125.39	- 19.52
DE	S 73°31' W	94.00	- 26.66	- 90.13
EA	N 18°44' W	90.20	+85.42	- 28.96

Lines	LAT	DMD	2A
AB	+93.50	+59.45	+5558.58
BC	- 26.87	+198.06	- 5321.87
CD	- 125.39	+257.70	- 32313.00
DE	- 26.66	+148.05	- 3947.01
EA	+85.42	+28.96	+2473.76

2A = 33549.54
A = 16774.77

Area of the lot = ***16774.77 sq.m.***

Problem 155:

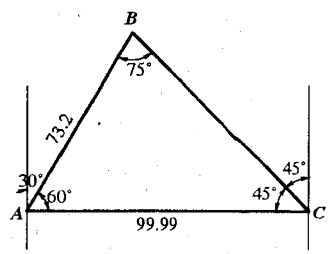
Given the technical description of a triangular lot ABC.

LINES	BEARING	DISTANCES
AB	N. 30° E.	73.2 m.
BC	S. 45° E.	?
CA	Due West	?

① Compute the missing side BC.
② Compute the missing side CA.
③ Compute the area of the lot in acres.

Solution:

① *Side BC:*



$$\frac{BC}{\sin 60°} = \frac{73.2}{\sin 45}$$

BC = ***89.65 m.***

② *Side CA:*

$$\frac{CA}{\sin 75°} = \frac{73.2}{\sin 45°}$$

CA = ***99.99 m.***

③ *Area of ABC:*

$$A = \frac{(73.2)(99.99) \sin 60°}{2}$$

$A = 3169.34 \text{ m}^2$

$$A = \frac{3169.34}{4047}$$

A = ***0.783 acres***

Note: $4047 \text{ m}^2 = 1$ acre

MISSING DATA

Problem 155-A:

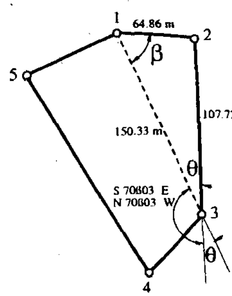
The technical description of a closed traverse is as follows.

LINE	DISTANCE (m)	BEARING
1 – 2	64.86	?
2 - 3	107.72	?
3 – 4	44.37	S. 35° 30' W.
4 – 5	137.84	N. 57° 15' W.
5 - 1	12.83	N. 1° 45' E.

① Find the bearing of line 1 – 2.
② Find the bearing of line 2 – 3.
③ Find the area of the closed traverse.

Solution:

① Bearing of line 1 – 2:



Lines	Bearing	Distance	LAT	DEP
3 – 4	S.35°30'W	44.37	-36.12	-25.77
4 - 5	N.57°15'W	137.84	+74.57	-115.93
5 - 1	N.1°45'E	12.83	+12.82	+0.39
1 – 5			-51.27	+141.31

Bearing of line 1-3:

$$\tan \text{bearing} = \frac{141.31}{51.27}$$

Bearing of line 1-3 = S. 70° 03' E.

$$\text{Distance} = \frac{141.31}{\text{Sin } 70° \, 03'}$$

Distance = 150.32 m.

Using Cosine Law:

$$(64.86)^2 = (150.32)^2 + (107.72)^2 - 2(150.32)(107.72) \text{ Cos } \theta$$

$\theta = 22° \, 09'$

$$\frac{107.72}{\text{Sin } \beta} = \frac{64.86}{\text{Sin } 22° \, 09'}$$

$\beta = 38° \, 47'$

Azimuth of line 1 to 3 = 360° - 70° 03'
Azimuth of line 1 to 3 = 289° 57'

Azimuth of line 1 – 2 = 289°57' – 38°47'
Azimuth of line 1 – 2 = 251° 10'
Bearing of line 1 – 2 = ***N. 71° 10' E.***

② Bearing of line 2 – 3:
Azimuth of line 2 – 3 = 289° 57' – 22° 09'
Azimuth of line 2 – 3 = 312° 06'
Bearing of line 2 – 3 = ***S 47° 54' E.***

③ Area of closed traverse:

Line	Bearing	Dist	LAT
1-2	N.71°10'E	64.86	+20.94
2-3	S.47°54'E	107.72	- 72.21
3-4	S.35°30'W	44.37	- 36.12
4-5	N.57°15'W	137.84	+74.57
5-1	N.1°45'E	12.83	+12.82
			+108.33
			- 108.33

Line	DEP	DMD	Double Area
1-2	+61.39	+61.39	+1285.51
2-3	+79.92	+202.70	- 14636.97
3-4	- 25.77	+256.85	- 9277.42
4-5	- 115.93	+115.15	+8586.74
5-1	+ 0.39	- 0.39	- 5.00
	+141.70		2A=-14047.14
	- 141.70		A=-7023.57

Area = ***7023.57 m²***

155-B CE Board Nov. 2007

A closed traversed shows tabulated values of latitudes and departures.

LINES	LATITUDE	DEPARTURE
1 - 2	+ 84.60	- - -
2 - 3	+ 95.32	- 56.11
3 - 4	+ 62.66	- 57.52
4 - 5	- 48.16	- 31.40
5 - 6	- 43.04	+ 59.70
6 -1	- - -	+47.63

① Compute the DMD of line 3 - 4.
② Compute the length of line 6 to 1.
③ Compute the bearing of line 6 to 1.

Solution:

① DMD of line 3 - 4.

Latitude of (6 - 1)

$\sum$(latitude) = 0

84.60 + 95.32 + 62.66 - 48.16 - 43.04 - y = 0

y = - 151.38

Departure of (1 - 2)

$\sum$(departure) = 0

x - 56.11 - 57.52 - 31.40+59.70+47.63 =0

x = 37.70

LINES	LATITUDE	DEPARTURE	DMD
1 - 2	+ 84.60	+ 37.70	37.70
2 - 3	+ 95.32	- 56.11	19.29
3 - 4	+ 62.66	- 57.52	- 94.34
4 - 5	- 48.16	- 31.40	-183.26
5 - 6	- 43.04	+ 59.70	-154.96
6 -1	- 151.38	+ 47.63	- 47.63

DMD of line 3 - 4 = ***- 94.34***

② Length of line 6 to 1.

$(\text{Distance})^2 = (\text{latitude})^2 + (\text{departure})^2$

$(D_{6-1})^2 = (-151.38)^2 + (47.63)^2$

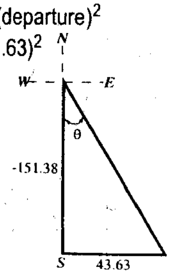
D_{6-1} = ***158.70 m.***

③ Bearing of line 6 to 1.

$\tan\theta = \dfrac{43.63}{151.38}$

$\theta = 17^\circ\ 29'$

$\therefore$ Bearing is ***S 17° 29' E***



155-C CE Board May 2007

Given the following descriptions of a four sided lot.

LINE	BEARING	DISTANCE
AB	N 30°30' E.	56.5 m.
BC	N 75°30' W	46.5 m.
CD	S 45°30' W	87.5 m.
DA	- - -	- - -

① What is the length of line DA?
② What is the bearing of line DA?
③ Compute the area of the enclosed traverse.

Solution:

① Length of line DA:

LINE	LAT	DEP
AB	+ 48.68	+ 28.68
BC	+ 11.64	- 45.09
CD	- 61.33	- 62.41
DA	+ 1.01	+ 78.82

Distance DA = $\sqrt{(+1.01)^2 + (78.82)^2}$

Distance DA = ***78.83 m.***

② Bearing of line DA:

$\tan \beta = \dfrac{78.82}{1.01}$

$\beta = 78^\circ 02'$

Bearing of DA = ***N 78°02' E.***

③ Area of the enclosed traverse:

LINE	LAT	DEP	DMD	PDA
AB	+ 48.68	+ 28.68	+ 28.68	+ 1396.14
BC	+ 11.64	- 45.09	+ 12.27	+ 142.82
CD	- 61.33	- 62.41	- 95.23	+ 5840.46
DA	+ 1.01	+ 78.82		2A = 7299.81

A = ***3,649.91 m²***

MISSING DATA

Problem 156:

A closed traverse has the following data:

LINES	BEARING	DISTANCES
AB	S. 15°36' W.	24.22 m.
BC	S. 69°11' E.	15.92 m.
CD	N. 57°58' E.	----
DA	S. 80°43' W.	----

① Find the distance DA in meters.
② Find the distance CD in meters.
③ Find the area in sq.m.

Solution:

① *Distance DA in meters:*

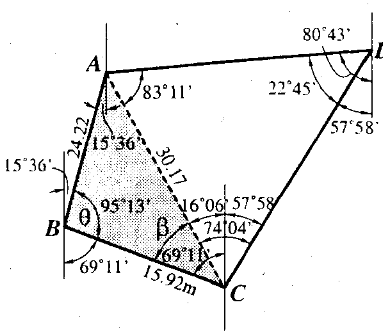
$ø = 180 - (15°36' + 69°11')$

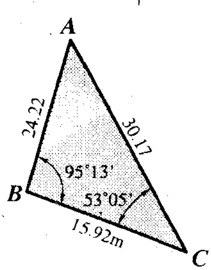
$ø = 95°13'$

Using Cosine Law:

$(AC)^2 = (24.22)^2 + (15.92)^2 - 2(24.22)(15.92) \cos 95°13'$

$AC = 30.17$





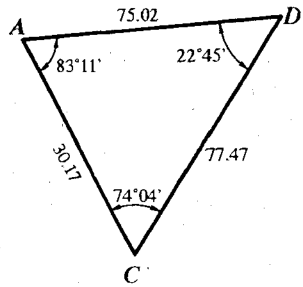
Using Sine Law:

$$\frac{30.17}{\sin 95°13'} = \frac{24.22}{\sin ß}$$

$ß = 53°05'$

Bearing CA = 69°11' - 53°05'
Bearing CA = N 16°06' W

Using Sine Law:



$$\frac{DA}{\sin 74°04'} = \frac{30.17}{\sin 22°45'}$$

DA = 75.02 m.

② *Distance CD in meters:*

$$\frac{30.17}{\sin 22°45'} = \frac{CD}{\sin 83°11'}$$

CD = 77.47

③ *Area in sq.m:*

LINES	BEARING	DISTANCES
AB	S. 15°36' W.	24.22 m.
BC	S. 69°11' E.	15.92 m.
CD	N. 57°58' E.	77.47 m.
DA	S. 80°43' W.	75.02 m.

Lines	LAT	DEP	DMD	2A
AB	- 23.33	- 6.51	- 6.51	+151.88
BC	- 5.66	+14.88	+1.86	- 10.53
CD	+41.09	+65.67	+82.41	+3386.23
DA	- 12.10	- 74.04	+74.04	- 895.88

2A = 2631.7

A = ***1315.85 m²***

MISSING DATA

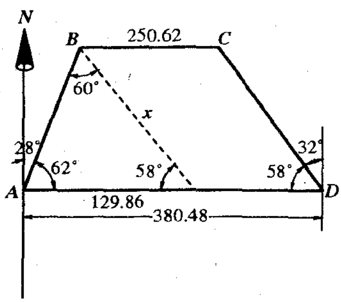
Problem 157:

From the given data on a four sided lot, compute the following:

LINES	BEARING	DISTANCES
A - B	N. 28° E.	—
B - C	Due East	250.62 m.
C - D	S. 32° E.	—
D - A	Due West	380.48 m.

① Missing distance AB.
② Missing distance CD.
③ Area of lot in acres.

Solution:



① *Distance AB:*

$$\frac{AB}{\sin 58^\circ} = \frac{129.86}{\sin 60^\circ}$$

AB = ***127.16 m.***

② *Distance CD:*

$$\frac{x}{\sin 62^\circ} = \frac{129.86}{\sin 60^\circ}$$

$x = 132.40$ m.

CD = ***132.40 m.***

③ *Area of lot:*

$$A = \frac{b^2 - b_1^2}{2\,(\text{Cot}\,\theta + \text{Cot}\,\beta)}$$

$$A = \frac{(380.48)^2 - (250.62)^2}{2\,(\text{Cot}\,62^\circ + \text{Cot}\,58^\circ)}$$

$A = 34084.72$

$$A = \frac{34084.72}{4047}$$

A = ***8.42 acres***

Problem 158:

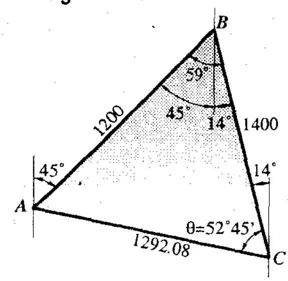
Given the following traverse notes for a closed traverse.

LINES	AZIMUTH	DISTANCE
AB	225°00'	1200 m.
BC	346°00'	1400 m.
CA	----	----

① Compute the total length of the traverse.
② Compute the azimuth of line CA.
③ Compute the area of the lot in acres.

Solution:

① *Total length of traverse:*



$$(AC)^2 = (1200)^2 + (1400)^2 - 2(1200)(1400)\,\text{Cos}\,59^\circ$$

AC = 1292.08 m.

Total length of traverse
= 1200 + 1400 + 1292.08
= ***3892.08 m.***

② *Azimuth of line CA:*

$$\frac{\sin\theta}{1200} = \frac{\sin 59^\circ}{1292.08}$$

$\theta = 52°45'$

Bearing = N. 66°45' W.

Azimuth of CA = ***113°15'***

③ *Area of lot:*

$$A = \frac{1200\,(1400)\,\sin 59^\circ}{2}$$

$A = 720020.53\ m^3$

$$A = \frac{720020.53}{4047}$$

A = ***177.91 acres***

MISSING DATA

Problem 159:

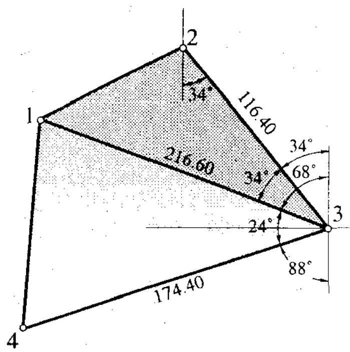
The data shown is the azimuth and distances of the corners of a lot.

LINES	AZIMUTH	DISTANCE
1 - 2	- - - -	- - - -
2 - 3	326°00'	116.40
3 - 4	88°00'	174.40
3 - 1	112°00'	216.60
4 - 1	- - - -	- - - -

① Compute the area of the lot in acres.
② Compute the missing distance of line 1 - 2.
③ Compute the missing distance of line 4 - 1.

Solution:

① *Area of the lot:*



$$Area = \frac{216.60(116.40)\ Sin\ 34^\circ}{2} + \frac{216.60(174.40)\ Sin\ 24^\circ}{2}$$

Area = 14731.50 m^2

$$Area = \frac{14731.50}{4047}$$

Area = ***3.64 acres***

② *Distance 1 - 2:*

$$(1-2)^2 = (216.60)^2 + (116.40)^2 - 2(216.60)(116.40)\ Cos\ 34^\circ$$

Distance 1 - 2 = ***136.60 m.***

③ *Distance 4 - 1:*

$$(4-1)^2 = (174.40)^2 + (216.60)^2 - 2(174.40)(216.60)\ Cos\ 24^\circ$$

Distance 4 - 1 = ***91.17 m.***

Problem 160:

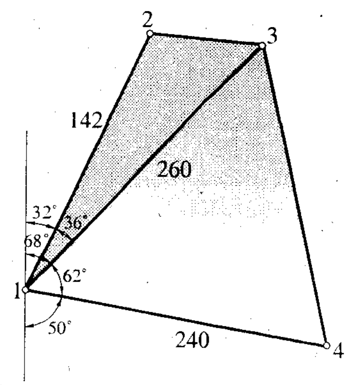
From the given data of a closed traversed with missing sides of a commercial lot in Dasmarinas Village.

LINES	BEARING	DISTANCES
1 - 2	N. 32° E.	142 m.
2 - 3	- - - -	- - - -
3 - 4	- - - -	- - - -
4 - 1	N. 50° W.	240 m.
1 - 3	N. 68° E.	260 m.

① Compute the area of the lot in square meters.
② Compute the distance of line 2 - 3.
③ Compute the distance of line 3 - 4.

Solution:

① *Area of lot in square meters:*



$$Area = \frac{142(260)\ Sin\ 36^\circ}{2} + \frac{260(240)\ Sin\ 62^\circ}{2}$$

Area = ***38,398.48 sq.m.***

② *Distance 2 - 3:*

$$(2-3)^2 = (142)^2 + (260)^2 - 2(142)(260)\ Cos\ 36^\circ$$

Line (2 - 3) = ***167.41 m.***

③ *Distance 3 - 4:*

$$(3-4)^2 = (260)^2 + (240)^2 - 2(260)(240)\ Cos\ 62^\circ$$

Line (3 - 4) = ***258.09 m.***

MISSING DATA

Problem 161:

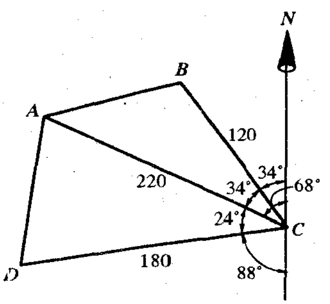
From the given technical description of lot ABCD as shown, compute the following:

LINES	BEARING	DISTANCES
AB	—	—
BC	S. 34° E.	120 m.
CD	S. 88° W.	180 m.
DA	—	—
AC	S. 68° E.	220 m.

① Area of lot ABCD in acres.
② Distance of AB.
③ Distance of DA.

Solution:

① *Area of lot:*



$$Area = \frac{(120)(220)\ \text{Sin}\ 34°}{2} + \frac{220\ (180)\ \text{Sin}\ 24°}{2}$$

$Area = 15434.73$

$$Area = \frac{15434.73}{4047}$$

Area = ***3.81 acres***

② *Distance AB:*

$(AB)^2 = (120)^2 + (220)^2 - 2(120)(220)\ \text{Cos}\ 34°$

AB = ***137.94 m.***

③ *Distance DA:*

$(DA)^2 = (180)^2 + (220)^2 - 2(180)(220)\ \text{Cos}\ 24°$

DA = ***91.91 m.***

Problem 162:

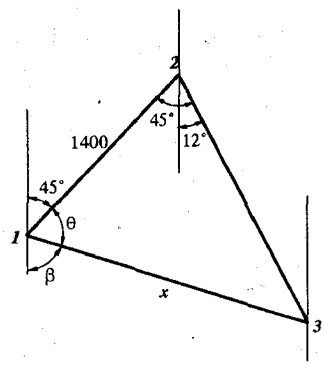
From the given closed traverse as shown, compute the following:

LINES	BEARING	DISTANCES
1 - 2	N. 45° E.	1400 m.
2 - 3	S. 12° E.	1800 m.
3 - 1	---	---

① Compute the area enclosed by the closed traverse in acres.
② Compute the total perimeter of the lot.
③ Compute the bearing of line 3 - 1.

Solution:

① *Area of closed traversed:*



$$Area = \frac{1400(1800)\ \text{Sin}\ 57°}{2}$$

$Area = 1056724.92\ m^2$

$$Area = \frac{1056724.92}{4047}$$

Area = ***261.11 acres***

② *Total perimeter of lot:*

$x^2 = (1400)^2 + (1800)^2 - 2(1400)(1800)\ \text{Cos}\ 57°$

$x = 1566.85$ m.

Total perimeter = 1400 + 1800 + 1566.85

Total perimeter = ***4766.85 m.***

MISSING DATA

③ *Bearing of 3-1:*

$$\frac{1800}{\text{Sin }\theta} = \frac{1566.85}{\text{Sin }57°}$$

$\theta = 74°\ 28'$

$ß = 180 - 45° - 74°28'$

$ß = 60°32'$

Bearing of 3-1 is ***N 60°32' W***

Problem 163:

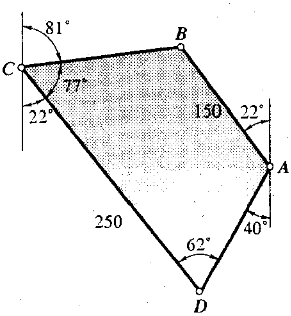
Given below is the technical description of a residential lot of Sta. Lucia Realty with some missing data:

LINES	BEARING	DISTANCES
AB	N. 22° W.	150 m.
BC	S. 81° W.	----
CD	S. 22° E.	250 m.
DA	N. 40° E.	----

① Compute the missing side BC.
② Compute the missing side DA.
③ Compute the area of the lot.

Solution:

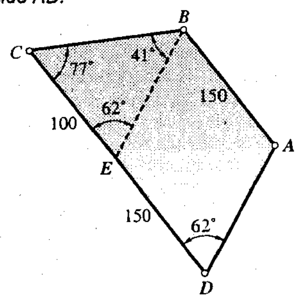
① *Side BC:*



$$\frac{BC}{\text{Sin }62°} = \frac{100}{\text{Sin }41°}$$

BC = ***134.58 m.***

② *Side AD:*



AD = BE

$$\frac{BE}{\text{Sin }77°} = \frac{100}{\text{Sin }41°}$$

BE = ***148.52 m.***

③ *Area of lot:*

$$A = \frac{(250)^2 - (150)^2}{2\,(\cot 77° + \cot 62°)}$$

A = ***26,226.84 sq.m.***

Problem 164:

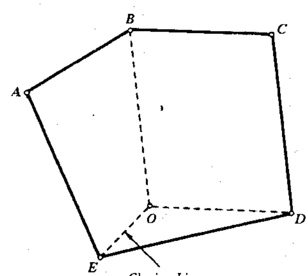
From the given technical description of a lot.

LINES	BEARINGS	DISTANCES
AB	N 48°20' E	529.60 m.
BC	----	592.00 m.
CD	S 7°59' E	563.60 m.
DE	----	753.40 m.
EA	N 48°12' W	428.20 m.

① Compute the bearing of line DE.
② Compute the bearing of line BC.
③ Compute the area of the lot.

Solution:

Draw a line BO parallel to CD at B:



MISSING DATA

① ***Bearing of line DE:***

Lines	Bearings	Distances	LAT	DEP
EA	N 48°12' W	428.20	+285.41	- 319.21
AB	N 48°20' E	529.60	+352.10	+395.60
BO	S 70°59' E	563.60	- 558.14	+78.24
			- 79.37	- 154.63

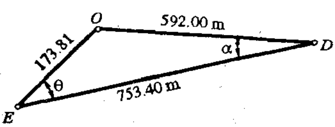
tan Bearing (OE) $= \dfrac{154.63}{79.37}$

Bearing (OE) = S 62°50' W

Distance (OE) $= \dfrac{154.63}{\text{Sin } 62°80'}$

OE = 173.81 m.

Consider triangle ODE:



Using Cosine Law:

$(592)^2 = (193.81)^2 + (753.40)^2$
$\quad - 2(173.81)(753.40) \text{ Cos } ø$

ø = 19°11'

Bearing (ED) = 62°50' + 19°11'

Bearing (ED) = N 82°01' E

Bearing of DE = ***S 82°01' W***

② *Bearing of line BC:*

$\dfrac{\text{Sin } \alpha}{173.81} = \dfrac{\text{Sin } 19°11'}{592.00}$

α = 5°32'

Bearing DO = 82°01' + 5°32'

Bearing DO = S 87°33' W

DO = BC

Bearing BC = ***N 87°33' E***

③ *Area of lot:*

LINES	BEARING	DISTANCES
AB	N. 48°20' E.	529.60
BC	N. 87°33' E.	592.00
CD	S. 7°59' E.	563.60
DE	S.82°01' W.	753.40
EA	N. 48°12' W.	428.20

Lines	LAT	DEP	DMD	2A
AB	+352.07	+395.60	+395.60	+139278.89
BC	+25.31	+591.44	+1382.64	+34994.62
CD	- 558.15	+78.28	+2052.36	- 1145524.73
DE	- 104.64	- 746.12	+1384.52	- 144876.17
AE	+285.41	- 319.20	+319.20	+91102.87

2A = - 1025024.52

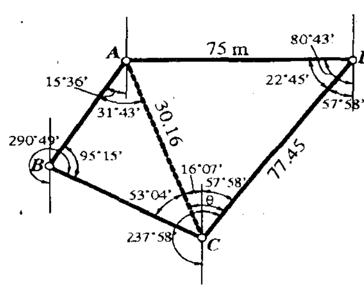
A = ***512,512.26 m²***

Problem 165:

LINES	AZIMUTH	DISTANCES
AB	15 - 36	----
BC	290° - 49	----
CD	237 - 58	77.45
DA	80 - 43	75.00

① Compute the distance BC.
② Compute the distance AB.
③ Compute the area by DMD method.

Solution:



MISSING DATA

① *Distance BC:*

Using Cosine Law:

$(AC)^2 = (75)^2 + (77.45)^2 - 2(75)(77.45)\cos 22°45'$

$AC = 30.16$ m.

$$\frac{\sin \phi}{75} = \frac{\sin 22°45'}{30.16}$$

$\phi = 74°05'$

$74°05' - 57°58' = 16°07'$

Bearing (AC) = S 16°07' E

Angle BAC = 16°07' + 15°36'

Angle BAC = 31°43'

Angle BCA = 69°11' - 16°07'

Angle BCA = 53°04'

Using Sine Law,

Considering triangle ABC:

$$\frac{30.16}{\sin 95°13'} = \frac{BC}{\sin 31°43'}$$

BC = ***15.92 m.***

② *Distance AB:*

$$\frac{30.16}{\sin 95°13'} = \frac{AB}{\sin 53°04'}$$

AB = ***24.21 m.***

③ *Area by DMD method:*

Lines	Bearings	Distances	LAT	DEP
AB	S 15°36' W	24.21 m	- 23.32	- 6.51
BC	S 69°11' E	15.92 m	- 5.66	+14.88
CD	N 57°58' E	75.45 m	- 41.07	+65.66
DA	S 80°43' W	75.00 m	- 12.00	- 74.03

Lines	LAT	DEP	DMD	Double Area
AB	- 23.32	- 6.51	- 6.51	+151.81
BC	- 5.66	+14.88	- 1.86	- 10.53
CD	+41.07	+65.66	+82.40	+3384.17
DA	- 12.09	- 74.03	+74.03	- 895.02

2A = 630.43

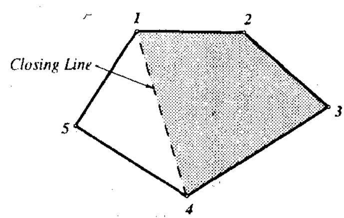
A = ***1315.22 m²***

Problem 166:

In the survey of a closed lot with five sides, the following data are given where in all the bearings and distances of all sides except the lengths of lines 4 - 5 and 5 - 1 were omitted.

LINES	BEARING	DISTANCES
1 - 2	S 73°21' E	247.20
2 - 3	S 40°10' E	154.30
3 - 4	S 26°42' W	611.90
4 - 5	N 14°20' W	----
5 - 1	N 12°20' E	----

① Compute the distance of line 4 - 1.
② Compute the distance of line 4 - 5.
③ Compute the distance of line 5 - 1.



Solution:

① *Distance of line 4 - 1:*

Lines	Bearing	Distance	LAT	DEP
1 - 2	S 73°21' E	247.20	- 70.83	+236.83
2 - 3	S 40°10' E	154.30	- 117.91	+99.53
3 - 4	S 26°42' W	611.90	- 546.65	- 274.94

$$\tan \text{bearing} = \frac{dep}{lat}$$

$$\tan \text{bearing} = \frac{61.42}{735.39}$$

Bearing (4 - 1) = N 4°47' W

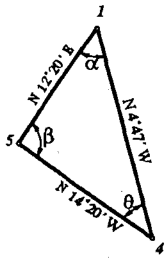
$$\text{Distance } (4 - 1) = \frac{\text{Dep}}{\sin 4°47'}$$

$$\text{Distance} = \frac{61.42}{\sin 4°47'}$$

Distance = ***746.53 m.***

MISSING DATA

② *Distance of line 4 - 5:*
Consider triangle 1 - 4 - 5:



$ø = 14°20' - 4°47'$
$ø = 9°33'$
$\alpha = 12°20' + 4°47'$
$\alpha = 17°07'$
$ß = 180 - 9°33' - 17°07'$
$ß = 153°20'$

Using Sine Law:

$$\frac{736.54}{\text{Sin } 153°20'} = \frac{(4-5)}{\text{Sin } 17°07'}$$

Distance (4 - 5) = ***483.02 m.***

③ *Distance of line 5 - 1:*

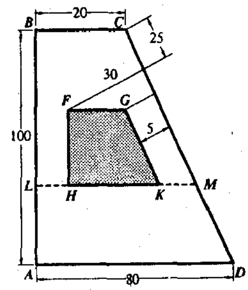
$$\frac{736.54}{\text{Sin } 153°20'} = \frac{(5-1)}{\text{Sin } 9°33'}$$

Distance (5 - 1) = ***272.88 m.***

Problem 167:

A trapezoidal lot abcd has the following technical description shown below. A 6 storey concrete building is to be constructed on the shaded portion as shown, wherein the cornerstone "F" can be located by measuring 25 m. from C along CD then 30 m. from CD. The building line HK falls along the subdivision line that divides the trapezoidal lot into two equal areas. GK is parallel to CD and is 5 m. from it.

LINE	BEARING	DISTANCE
AB	Due north	100 m.
BC	Due east	20 m.
CD	----	----
DA	Due west	80 m.



① Compute the distance of CD.
② Compute the bearing of CD.
③ Compute the floor area of the building.

Solution:

① *Distance of CD:*

$CD = \sqrt{(100)^2 + (60)^2}$
$CD =$ ***116.62 m.***

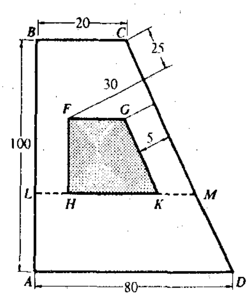
② *Bearing of CD:*

$$\tan \theta = \frac{60}{100}$$

$\theta = 30°58'$

Bearing CD = ***S. 30°58' E.***

③ *Floor area of the builidng:*



MISSING DATA

$$LM = \sqrt{\frac{mb_2^2 + nb_1^2}{m+n}}$$

$$LM = \sqrt{\frac{1(80)^2 + 1(20)^2}{1+1}}$$

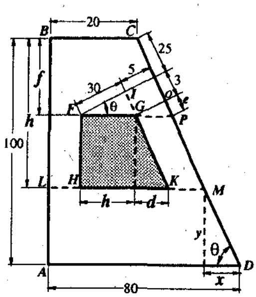
$LM = 58.30$ m.
$x = 80 - 58.30$
$x = 21.70$

$$\frac{x}{y} = \frac{60}{100}$$

$$y = \frac{100\,(21.70)}{60}$$

$y = 36.17$

Triangle FGI is similar to MCD



$$\frac{25}{a} = \frac{36.17}{21.70}$$

$a = 15$ m.
$b^2 = (25)^2 + (15)^2$
$b = 29.15$ m.

Triangle COP is similar to MED

$$\frac{5}{e} = \frac{36.17}{21.70}$$

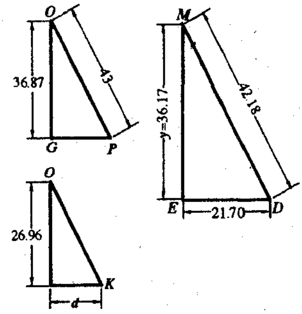
$e = 3$ m.

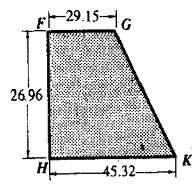
$$\frac{f}{42} = \frac{36.17}{42.18}$$

$f = 36.87$ m.
$b = 100 - 36.17$
$b = 63.83$ m.
$FH = 63.83 - 36.07$
$FH = 26.96$ m.

$$\frac{26.96}{d} = \frac{36.17}{21.70}$$

$d = 16.17$ m.
$HK = 45.32$ m.





Area of shaded section:

$$A = \frac{(45.32 + 29.15)\,(26.96)}{2}$$

$A = 1003.86$ sq.m.

Total floor area of building
$= 6\,(1003.86)$
$= \mathbf{6023.16\ sq.m.}$

Problem 168:

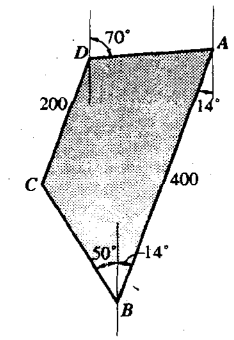
The following technical description of a lot shows some missing data in the office files.

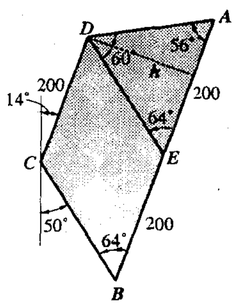
LINE	BEARING	DISTANCE
AB	S. 14° W.	400 m.
BC	N. 50° W.	- - -
CD	N. 14° E.	200 m.
DA	N. 70° E.	- - -

① Compute the distance of line BC.
② Compute the area of the lot in acres.
③ Compute the length of the dividing line which is parallel to line AB such that it divides the lot into two lots having a ratio of 1:2, the bigger lot adjacent to line CD.

Solution:

① *Distance BC:*





$DE = BC$

$$\frac{BC}{\text{Sin } 56°} = \frac{200}{\text{Sin } 60°}$$

$BC = \mathbf{191.46\ m.}$

② *Area:*

$$Area = \frac{(b_1 + b_2)\,h}{2}$$

$h = 191.46 \text{ Sin } 64°$

$h = 172.08$ m.

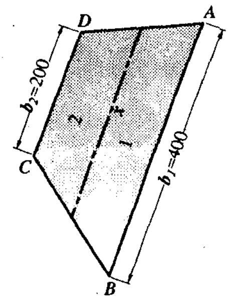
$$A = \frac{(200 + 400)(172.08)}{2}$$

$A = 51625$ sq.m.

$$A = \frac{51625}{4047}$$

$A = \mathbf{12.76\ acres}$

③ *Length of dividing line:*



$$x = \sqrt{\frac{mb_1^2 + nb_2^2}{m + n}}$$

$$x = \sqrt{\frac{2(400)^2 + (1)(200)^2}{2 + 1}}$$

$x = \mathbf{346.41\ m.}$

[illegible]

In the close traverse ABCDE, AB has a bearing of S 30' W and is 500 m. long, CB has a bearing of N 5'04' W and is 720 m. long, CD has a departure of 592.00 m. west and no latitude, DE is 800 m. long, and EA has a bearing of N 20' E.

LINES	BEARING	DISTANCE
AB	S 30' W	500
BC	S 5'04' E	720
CD	Due west	592
DE	- - - -	800
EA	N 20' E	- - - -

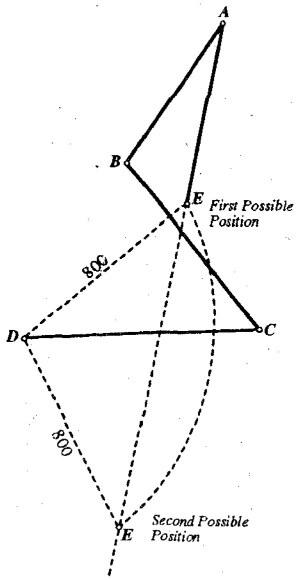
① Find one possible length of EA.
② Find one possible bearing of DE.
③ Find another possible bearing of DE.

Solution:

① *One possible length of EA:*

Lines	Bearing	Distance	LAT	DEP
AB	S 30' W	500	- 433.01	- 250
BC	S 5'04' E	720	- 717.19	+63.59
CD	Due west	592		- 592.00
DA	- - - -	- - - -	+1150.2	+778.41

MISSING DATA



$$\tan \text{bearing } DA = \frac{778.41}{1150.2}$$

$\tan$ *bearing DA* = N 34°05' E

$$\text{Distance } DA = \frac{778.41}{\sin 34°05'}$$

Distance DA = 1389.03

Considering triangle AED:

$$\frac{\sin \phi}{1389.03} = \frac{\sin 14°05'}{800}$$

$\phi = 25°$

$\alpha = 180 - 14°05 - 25°$

$\alpha = 140°55'$

$$\frac{AE}{\sin 140°55} = \frac{800}{\sin 14°05}$$

AE = ***2072.72 m.***

② *One possible bearing of DE:*

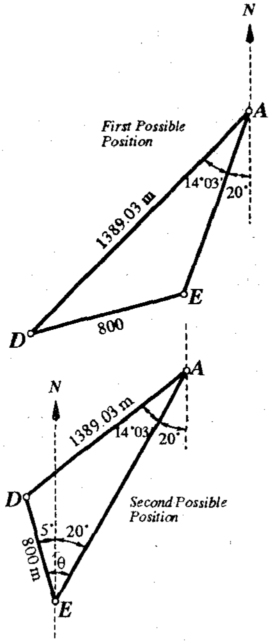
$$\frac{2072.72}{\sin \beta} = \frac{800}{\sin 14°05'}$$

$\beta = 39°05'$

Bearing DE = 34°05' + 39°05'

Bearing DE = ***N. 73°10' E.***

③ *Second possible bearing of DE*
= ***S. 5° E.***



Problem 170:

An engineer's notebook gives the following notes regarding a boundary line.

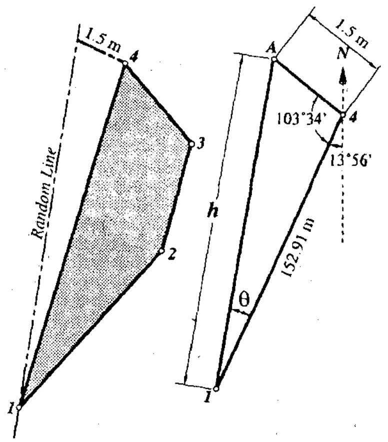
Course	Magnetic Bearing	Distance
1 - 2	N 16°30' E	105.30
2 - 3	N 15°15' E	33.50
3 - 4	N 7°10' W	15.20

Corners 1 and 4 can be divided on the ground. The engineer is to reset corners 2 and 3 where they were originally and determine the true bearings of all the courses. Date of survey unknown. Upon running a random line, the random line missed the true corner by 1.5 m. The bearing from the end of the random line to corner 4 was S 62°30' E.

① Compute the bearing of line 4 - 1.
② Compute the distance of line 4 - 1.
③ What was the magnetic declination at the time of the original survey?

MISSING DATA

Solution:



① *Bearing of line 4 - 1:*

Lines	Bearing	Distance	LAT	DEP
1 - 2	N 16°30' E	105.30	+100.96	+29.91
2 - 3	N 15°15' E	33.50	+32.32	+8.80
3 - 4	N 7°10' W	15.20	+15.08	- 1.90
4 - 1	----	----	- 148.36	- 36.82

$$\tan \textit{bearing}\,(4-1) = \frac{36.82}{148.36}$$

Bearing (4 - 1) = S 13°56' W

② *Distance of line 4 - 1:*

$$\textit{Distance}\,(4-1) = \frac{36.82}{\sin 13°56}$$

Distance (4 - 1) = 152.91 m.

③ *Magnetic declination:*

$$h^2 = (1.5)^2 + (152.91)^2 - 2(1.5)(152.91)\cos 103°34'$$

h = 153.27 m.

$$\frac{\sin 103°34'}{153.27} = \frac{\sin ø}{1.5}$$

ø = 0°33'

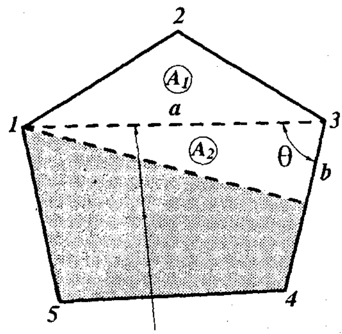
Since the random line is supposed to be the true position of 1 - 4 based on true bearing, then the magnetic declination during the survey is 0°33' E.

Subdivision

SUBDIVISION OF AREAS

a) ***To cut off an area from a given point.***

Let us say, that the entire area of the lot is 10,000 sq.m. It is required to divide the lot into two equal parts such that the division line shall pass thru corner one of the lot concern. Bearings and distances of all courses are known.



Since it is difficult to approximate the actual position of the subdivision line, it is therefore advisable to solve for the bearing and distance of line 3 to 1. Knowing the bearing of the line 3-1 and 3-4, ø could be computed. Let us say A_1 = 2000 sq.m. only, so we still have 3000 sq.m. more to be added in order to obtain the required area. A_2 therefore would be equal to 5000 - 2000 = 3000 sq.m. Knowing the distance "a" and the angle ø, we could compute the distance "b" from the relation.

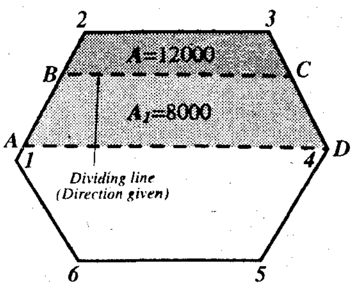
$$A_2 = \frac{ab \sin ø}{2}$$

$A_1 + A_2$ = 5000 sq.m.

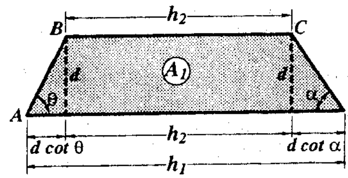
(the required area to be cut off)

b) ***To cut off an area by a line whose direction is given.***

This requires a longer computation in the sense that it would be difficult to assume the position of the subdivision line. Let us say an area of 12,000 sq.m. is to be segregated from the whole area, the direction of hte dividing line known (see sketch below). The total area of the lot is 40,000 sq.m. and the area to be segregated is adjacent to the line 2-3.



Thru the corner that seems likely to be the nearest line cutting the required area, a trial line AD is drawn whose direction is given. Distances AD and A_2 could be computed by using trigonometric principles. Considering lines A_2, 2-3, 3-4 and 4-A to be closed polygon, its area could be determined by using D.M.D. method. Let us say the area of lines A 234 A is 20,000 sq.m., so there is an excess area A_1 = 20,000 - 12,000 = 8,000 sq.m. The values of ø and α could be determined cause the bearings of lines A_2, 3-4 and 4-A is known, length of line 4-A is known.



$$h_2 = h_1 - d \cot ø - d \cot \alpha$$

$$h_2 = h_1 - d\,(\cot ø + \cot \alpha)$$

Given values:

A_1, h_1, ø and α

$$A_1 = \frac{(h_1 + h_2)\,d}{2}$$

$$A_1 = [h_1 + h_1 - d\,(\cot ø + \cot \alpha)]$$

We could solve for the value of "d"

$$d = AB \sin ø$$

$$4c = \frac{d}{\sin \alpha}$$

$$AB = \frac{d}{\sin ø}$$

$$2B = 2A - AB$$

$$3C = 3 - 4 - 4C$$

Points B and C is now the true position of the dividing line.

171: CE Board May 2005

Given the technical description of a triangular lot

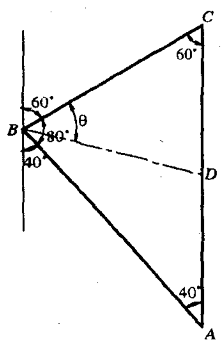
LINES	BEARING	DISTANCES
AB	N. 40° W.	?
BC	N. 60° E.	810 m.
CA	Due South	?

An area of 190000 m^2 is to be segregated along the side BC starting from B.

① Compute the location of the other end of the dividing line D along the side CA measured from C.
② Compute the bearing of the dividing line from B.
③ Compute the length of the dividing line.

Solution:

① *Distance CD:*



$$A = \frac{810\,x}{2}\,\text{Sin}\,60^\circ$$

$$190000 = \frac{180\ \text{Sin}\ 60^\circ}{2}\,x$$

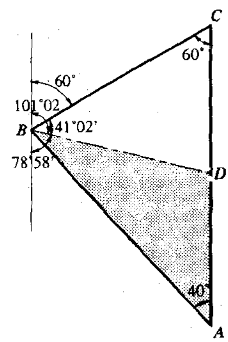
$x = \mathbf{541.71\ m.}$

② *Length of dividing line:*

$(BD)^2 = (541.71)^2 + (810)^2 - 2\,(541.71)(810)\ \text{Cos}\ 60^\circ$

$BD = \mathbf{741.68\ m.}$

③ *Bearing of dividing line:*



$$\frac{541.71}{\text{Sin}\ \theta} = \frac{714.68}{\text{Sin}\ 60^\circ}$$

$\theta = 41^\circ 02'$

Bearing of diving line BD = ***S. 78°58' E***

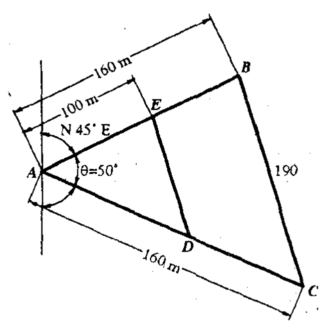
Problem 172:

A lot is bounded by 3 straight sides namely, AB, N. 45° E. 160 m. long, BC and CA = 190 m. long in clockwise direction. From point E, 100 m. from A and on side AB, a dividing line runs to D which is on side CA. The area of ADE is to be 2/5 of the total area of the lot. The total area of the lot is 11,643.88 m^2.

① Determine the distance from D to A.
② Compute the bearing of line AD.
③ Compute the distance DE.

Solution:

① *Distance DA:*



SUBDIVISION

$$\frac{100\,(DA)\,\text{Sin}\,\theta}{2} = \frac{2}{5}\,\frac{(160)(190)\,\text{Sin}\,\theta}{2}$$

$DA = \mathbf{121.60}$

② *Bearing of line AD:*

$$A = \frac{160\,(190)\,\text{Sin}\,\theta}{2}$$

$A = 11{,}643.88$

$\theta = 50^\circ$

Bearing of AD = **S. 85° E.**

③ *Distance DE:*

$(DE)^2 = (100)^2 + (121.6)^2 - 2(100)(121.6)\,\text{Cos}\,50^\circ$

$DE = \mathbf{95.68\ m.}$

Problem 173:

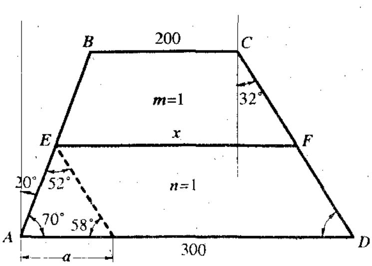
From the following data of a given 4 sided lot.

LINE	BEARING	DISTANCE
AB	N. 20° E.	?
BC	Due East	200 m.
CD	S. 32° W.	?
DA	Due West	300 m.

① Find the area of the lot in m^2.
② Find the length of the dividing line (EF) that is parallel to line DA which will divide the lot into two equal areas.
③ Determine the location of one end of the dividing line E from corner A along line AB.

Solution:

① *Area of lot:*



$$A = \frac{b_1^2 - b_2^2}{2\,(\cot\theta + \cot\beta)}$$

$$A = \frac{(300)^2 - (200)^2}{2\,(\cot 70^\circ + \cot 58^\circ)}$$

$A = \mathbf{25{,}282.16\ sq.m.}$

② *Length of dividing line:*

$$x = \sqrt{\frac{mb_1^2 + nb_2^2}{m + n}}$$

$$x = \sqrt{\frac{(1)(300)^2 + (1)(200)^2}{1 + 2}}$$

$x = \mathbf{254.95\ m.}$

③ *Distance AE:*

$a = 300 - 254.95$

$a = 45.05$

$$\frac{AE}{\text{Sin}\,58^\circ} = \frac{45.05}{\text{Sin}\,52^\circ}$$

$AE = \mathbf{48.48\ m.}$

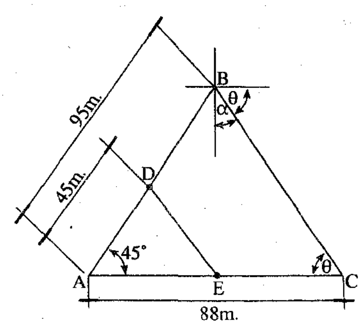
174. CE Board Nov. 2009

A lot is bounded by 3 straight sides A, B, C. AB is N. 45° E. 95 m. long and AC is due East, 88 m. long. From point D, 43 m. from A on side AB, a dividing line runs to E which is on side CA. The area ADE is to be 1/7 of the total area of the lot.

① Determine the distance DE.
② Determine the bearing of side BC.
③ Determine the distance AE.

Solution:

① *Distance DE:*



$$\frac{(AE)(43)\sin 45^\circ}{2} = \frac{1}{7}\,\frac{(95)(88)\sin 45^\circ}{2}$$

AE = 27.77 m.

Using Cosine Law:

$(DE)^2 = (43)^2 + (27.77)^2 - 2(43)(27.77)\cos 45^\circ$

DE = ***30.52 m.***

② *Bearing of line BC:*

$(BC)^2 = (95)^2 + (88)^2 - 2(95)(88)\cos 45^\circ$

BC = 70.33 m.

Using Sine Law:

$$\frac{95}{\sin\theta} = \frac{70.33}{\sin 45^\circ}$$

$\theta = 72^\circ 46'$

$\alpha = 90^\circ - 72^\circ 46'$

$\alpha = 17^\circ 14'$

Bearing of BC = ***S. 17°14' E.***

③ *Distance AE:*

AE = ***27.77 m.***

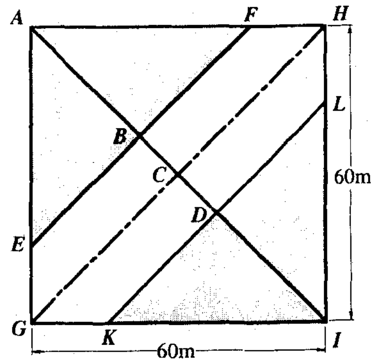
Problem 175:

The centerline of a proposed road having a bearing of N. 45° E. passes through the diagonal of a square lot AHIG having sides of 60 m x 60 m. If the area occupied by the proposed road is equal to 1200 sq.m.

① Compute the area of section AFE, where EF is parallel to the diagonal of the square lot.
② Compute the width of the road.
③ Compute the total perimeter of the proposed road inside the lot.

Solution:

① *Area of AFE:*



$AC = \frac{1}{2}\sqrt{2}\,(60)$

$AC = 42.43$ m.

Area of AGH $= \frac{1}{2}(60)(60) = A_1$

$A_1 = 1800\ m^2$

Area of AEF $= \frac{60(60) - 1200}{2}$

$A_2 = \mathbf{1200\ m^2}$

② *Width of the road:*

$$\frac{A_1}{A_2} = \frac{(AC)^2}{(AB)^2}$$

$$\frac{1800}{1200} = \frac{(42.43)^2}{(AB)^2}$$

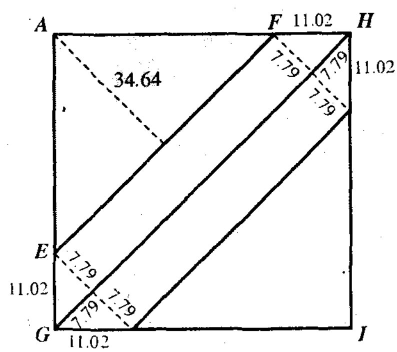
$AB = 34.64$

$BC = 42.43 - 34.64$

$BC = 7.79$ m.

$BD = 2(7.79)$

BD = ***15.58 m.*** *(width of road)*



③ *Total perimeter:*

$EF = GH - 7.79(2)$

$EF = 60\sqrt{2} - 15.58$

$EF = 69.27$ m.

$P = 69.27 + 11.02 + 11.02 + 69.27 + 11.02 + 11.02$

$P = \mathbf{182.62\ m.}$

Problem 176:

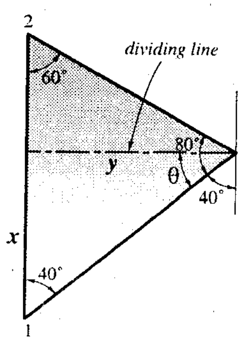
A triangular lot has the following Azimuths and distances.

LINES	AZIMUTH	DISTANCE
1 - 2	180°00'	----
2 - 3	300°00'	----
3 - 1	40°00'	960.22 m.

① The lot is to be divided such that the area of the southern portion would be 210,000 m^2. Compute the position of the other end of the dividing line if the line starts at corner 3 of the lot. Express the distance from corner 1.
② What is the length of the dividing line?
③ Compute the azimuth of the dividing line.

Solution:

① *Location of x from corner 1:*



$$\frac{x\,(960.22)\,\text{Sin}\ 40°}{2} = 210000$$

$x = \mathbf{680.47}$

② *Length of dividing line:*

$(y)^2 = (680.47)^2 + (960.22)^2 - 2(680.47)(960.22)\ \text{Cos}\ 40°$

$y = \mathbf{619.67\ m.}$

② *Bearing of dividing line:*

$$\frac{680.47}{\text{Sin}\ \theta} = \frac{619.67}{\text{Sin}\ 40°}$$

$\theta = 44°54'$

Bearing = S. 84°54' W.

Azimuth = **84°54'**

Problem 177:

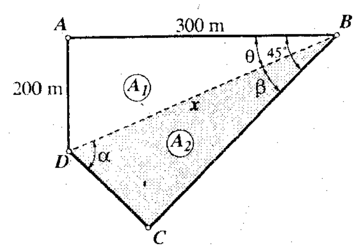
From the following technical descriptions shown.

LINES	BEARING	DISTANCES
AB	Due East	300 m.
BC	S. 45° W.	----
CD	----	----
DA	Due North	200 m.

① Compute the missing distance BC is the area of the lot is 43560 sq.m.
② Compute the distance of CD.
③ Compute the bearing CD.

Solution:

① *Distance BC:*



$\tan \phi = \frac{200}{300}$

$\phi = 33.69°$

$\beta = 45° - 33.69°$

$\beta = 11.31°$

$\sin 33.69° = \frac{200}{x}$

$x = 360.56$ m.

$A_1 = \frac{200(300)}{2}$

$A_1 = 30{,}000$

$A_2 = 43560 - 30000$

$A_2 = 13{,}560\ m^2$

$A_2 = \frac{x\,(BC)}{2} \sin \beta$

$13560 = \frac{360.56\,(BC) \sin 11.31°}{2}$

$BC = \boldsymbol{383.53\ m.}$

② *Distance of CD:*

$(CD)^2 = (360.56)^2 + (383.53)^2 - 2(360.56)(383.53) \cos 11.31°$

$CD = \boldsymbol{76.80\ m.}$

③ *Bearing of CD:*

$\frac{383.53}{\sin \alpha} = \frac{76.80}{\sin 11.31°}$

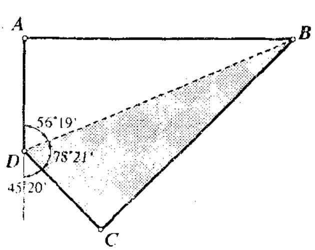
$\alpha = 78°21'$

Bearing BD = 45° + 11°19'

Bearing BD = 56°19'

Bearing BD = S.56°19' W.

Bearing DB = N. 56°19' E.



Bearing CD = ***N. 45°20' W.***

Problem 178:

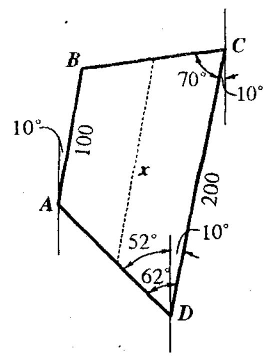
Subdivide the lot having the given technical description into two equal areas by a line parallel to the side AB.

LINES	BEARING	DISTANCES
AB	N. 10° E.	100 m.
BC	N. 80° E.	—
CD	S. 10° W.	200 m.
DA	N. 52° W.	—

① Compute the area of the whole lot in acres.
② Compute the length of the dividing line.
③ Compute the missing side BC.

Solution:

① *Area of whole lot:*



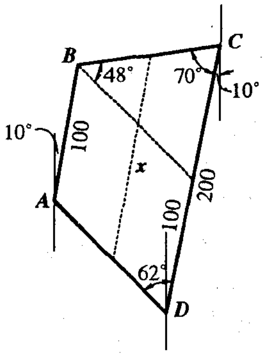
$A = \frac{b_2^2 - b_1^2}{2(\cot \theta + \cot \beta)}$

$A = \frac{(200)^2 - (100)^2}{2(\cot 62° + \cot 70°)}$

$A = \frac{16747.06\ m^2}{4047}$

$A = \boldsymbol{4.14\ acres}$

② *Length of dividing line:*



$$x = \sqrt{\frac{nb_1^2 + mb_2^2}{m+n}}$$

$$x = \sqrt{\frac{1(200)^2 + 1(100)^2}{1+1}}$$

x = ***158.11 m.***

③ *Side BC:*

$$\frac{BC}{\sin 62°} = \frac{100}{\sin 48°}$$

BC = ***118.81 m.***

Problem 179:

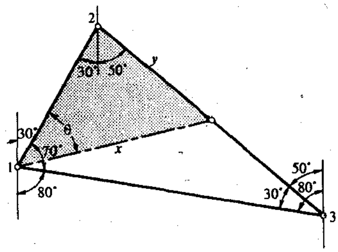
From the given technical description of lot 5025, Cebu Cadastre if the area of the northern side is only $\frac{2}{3}$ of the whole area of the lot.

LINES	BEARING	DISTANCES
1 - 2	N. 30° E.	1000 m.
2 - 3	S. 50° E.	—
3 - 4	N. 80° W.	—

① Compute the location of the dividing line from corner 2 if the dividing line starts from corner 1.
② Compute the length of the dividing line.
③ Compute the bearing of the dividing line from corner 1.

Solution:

① *Location of the dividing line from corner 2 if the dividing line starts from corner 1:*



$$A = \frac{(1000)^2 \sin 70° \sin 80°}{2 \sin 30°}$$

$A = 925416.58\ m^2$

$$A_1 = \frac{2}{3}(925416.58)$$

$A_1 = 616944.39\ m^2$

$$A_1 = \frac{1000\ (y) \sin 80°}{2}$$

$$616944.39 = \frac{\sin 80°\ (1000)\ y}{2}$$

y = ***1252.92 m.***

② *Length of the dividing line:*

$$x^2 = (1000)^2 + (1252.92)^2 - 2\ (1000)(1252.92) \cos 80°$$

x = ***1461.05 m.***

③ *Bearing of the dividing line from corner 1:*

$$\frac{y}{\sin \theta} = \frac{x}{\sin 80°}$$

$$\frac{1252.92}{\sin \theta} = \frac{1461.05}{\sin 80°}$$

$\theta = 57°37'$

Bearing of dividing line: (ø + 30°)
*= **N 87°37' E. from corner 1***

SUBDIVISION

Problem 180:

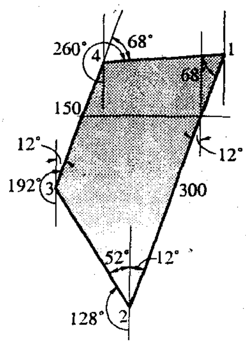
A four cornered lot has the following azimuth and distances with unknown non-adjacent sides.

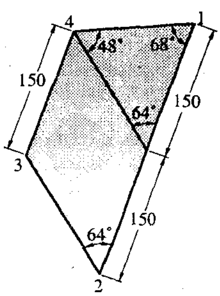
BOUNDARY	AZIMUTH	DISTANCE
1 - 2	12°00'	300 m.
2 - 3	128°00'	—
3 - 4	192°00'	150 m.
4 - 1	260°00'	—

① Compute the missing side 4 - 1.
② Compute the area of the lot in acres.
③ If the lot is to be subdivided into two lots having a ratio of 2:3, the smaller lot being on the South East portion, compute the length of the dividing line.

Solution:

① *Side 4 - 1:*





Using Sine Law:

$$\frac{4-1}{\text{Sin } 64°} = \frac{150}{\text{Sin } 48°}$$

$4 - 1 = \mathbf{181.42\ m.}$

② *Area of the lot:*

$$A = \frac{b_2^2 - b_1^2}{2(\cot\theta + \cot\beta)}$$

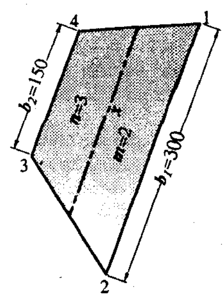
$$A = \frac{(300)^2 - (150)^2}{2(\text{Cot } 64° + \text{Cot } 68°)}$$

$A = 37846.56\ m^2$

$$A = \frac{37846.56}{4047}$$

$A = \mathbf{9.35\ acres}$

③ *Length of dividing line:*



$$x = \sqrt{\frac{nb_1^2 + mb_2^2}{m+n}}$$

$$x = \sqrt{\frac{3(300)^2 + 2(150)^2}{3+2}}$$

$x = \mathbf{251\ m.}$

Problem 181:

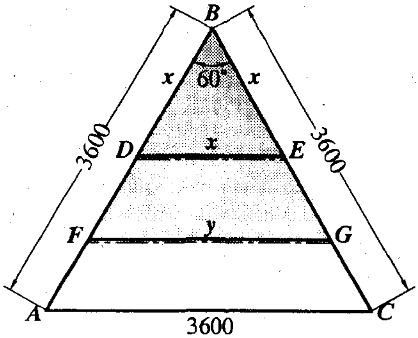
An equilateral triangular track of land has a length of one of its sides equal to 3600 m. long. It is required to divide the lot into 3 equal shares by a line parallel to one of the 3 sides.

① Compute the whole area of the lot ~~in acres~~.
② Compute the length of the dividing line on the portion near the vertex.
③ Compute the length of the dividing line on the second lot.

SUBDIVISION

Solution:

① *Area of whole lot:*



$$A = \frac{(3600)^2 \sin 60^\circ}{2}$$

$$A = \frac{5611844.62}{4047}$$

$$A = \mathbf{1386.67}$$

① *Length of DE:*

$$A = \frac{1}{3}(5611844.62)$$

$$A = 1870614.87$$

$$1840614.87 = \frac{x^2 \sin 60^\circ}{2}$$

$$x = \mathbf{2078.46\ m.}$$

③ *Length of dividing line y:*

$$A = \frac{2}{3}(5611844.62)$$

$$\frac{2}{3}(5611844.62) = \frac{y^2 \sin 60^\circ}{2}$$

$$y = \mathbf{2939.39\ m.}$$

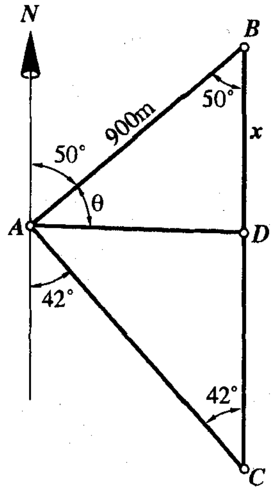
Problem 182:

An area of 200000 m^2 is to be segregated from the northern portion of triangular lot ABC. from corner A bearing and distance of AB is N. 50° E., 900 m., BC is due South and CA is N. 42° E.

① Compute the distance of the dividing line from corner B along line BC.
② Compute the length of the dividing line.
③ Compute the bearing of the dividing line from corner A.

Solution:

① *Distance of dividing line from corner B:*



$$\frac{900\,(x)\sin 50^\circ}{2} = 200000$$

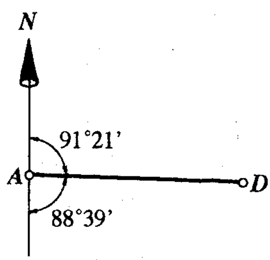
$$x = \mathbf{580.18\ m.}$$

② *Length of dividing line:*

$$(AD)^2 = (900)^2 + (580.18)^2 - 2(900)(580.18)\cos 50^\circ$$

$$AD = \mathbf{672.70\ m.}$$

③ *Bearing of dividing line:*



$$\frac{580.18}{\sin \theta} = \frac{672.70}{\sin 50^\circ}$$

$$\theta = 41^\circ 21'$$

$$50^\circ + 41^\circ 21' = 91^\circ 21'$$

Bearing = ***S. 88°39' E.***

Problem 183

Given technical description of a lot as follows.

LINES	AZIMUTH	DISTANCE
1 - 2	187°00'	27.90 m.
2 - 3	268°47'	34.12 m.
3 - 4	358°33'	27.72 m.
4 - 1	88°57'	38.21 m.

The area of the whole lot is 1000 sq.m. Subdivide the lot into two parts, the smallest part on the western side is to have an area of 400 m^2, whose dividing line is parallel to line 1 - 2 of the boundary.

① Compute the length of the dividing line.

② How far is the intersection point of the dividing line on the northern part of the lot from corner 2?

③ How far is the intersection point of the dividing line on the southern part of the lot from corner 1 of the boundary.

Solution:

① *Length of dividing line:*

$$A = \frac{bh}{2} + \frac{27.90\,h}{2}$$

$$400 = \frac{bh}{2} + \frac{27.90\,h}{2}$$

$$800 = bh + 27.90\,h$$

$$\tan 81°47' = \frac{h}{y}$$

$$y = 0.1444\,h$$

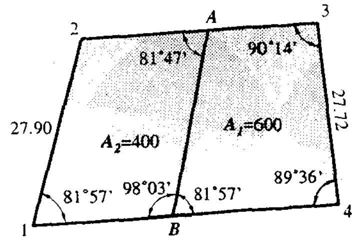
$$\tan 8°03' = \frac{x}{h}$$

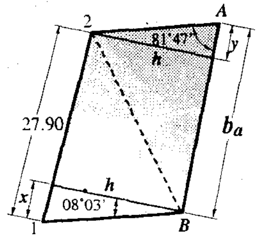
$$x = 0.1414\,\text{h}$$

$$b = 27.90 - x + y$$

$$b = 27.90 - 0.1414\,h + 0.1444\,\text{h}$$

$$b = 27.90 + 0.003\,h$$





$$800 = (27.90 + 0.003\,h)\,h + 27.90\,h$$

$$800 = 55.80\,h + 0.003\,h^2$$

$$h^2 + 18600\,h - 266666.67 = 0$$

$$h = \frac{-18600 \pm 18628.65}{2}$$

$$h = 14.33 \text{ m.}$$

$$b = 27.90 + 0.003\,(14.33)$$

$b = \mathbf{27.94\ m.}$ *(length of dividing line)*

② *Distance of A from corner 2:*

$$\text{Sin } 81°47' = \frac{h}{A - 2}$$

$$A - 2 = \frac{14.33}{\text{Sin } 81°47'}$$

$A - 2 = \mathbf{14.48\ m.}$

③ *Distance of B from corner 1:*

$$\text{Cos } 8°03' = \frac{h}{B - 1}$$

$$B - 1 = \frac{14.33}{\text{Cos } 8°03'}$$

$B - 1 = \mathbf{14.47\ m.}$

SUBDIVISION

Problem 184:

A parcel of land has a technical description as shown in the tabulated data.

LINES	BEARING	DISTANCE
1 - 2	S. 01°27' E.	27.72 m.
2 - 3	S. 88°57' W.	38.21 m.
3 - 4	N. 07°00' E.	27.90 m.
4 - 1	N. 88°47' E.	34.12 m.

The area of the lot is more or less 1000 sq.m. If the lot is to be subdivided into two parts such that the dividing line must start at the mid point of line 4 - 1 and must be parallel to line to 1 - 2 of the boundary.

① What is the distance of the subdividing line?
② What is the area of the lot subdivided on the eastern part?
③ What is the distance of the other end of the dividing line from corner 2 of the lot?

Solution:

① *Distance of dividing line:*
Using Cosine Law:

$y^2 = (17.06)^2 + (27.72)^2 - 2(17.06)(27.72)$ Cos 90°14'

$y = 32.61$ m.

Using Sine Law:

$$\frac{17.06}{\text{Sin } \theta} = \frac{32.61}{\text{Sin } 90°14'}$$

$\theta = 31°33'$

$\alpha = 89°36' - 31°33'$

$\alpha = 58°03'$

$\beta = 180° - 89°36'$

$\beta = 90°24'$

$\alpha = 180° - 90°24' - 58°03'$

$\alpha = 31°33'$

Using Sine Law:

$$\frac{32.61}{\text{Sin } 90°24'} = \frac{AB}{\text{Sin } 58°03'}$$

$AB = \mathbf{27.67\ m.}$

② *Area on the eastern part:*

$$A_2 = \frac{32.61\ (27.67)\ \text{Sin } 31°33'}{2}$$

$A_2 = 236.07\ m^2$

$$A_1 = \frac{17.06\ (27.72)\ \text{Sin } 90°14'}{2}$$

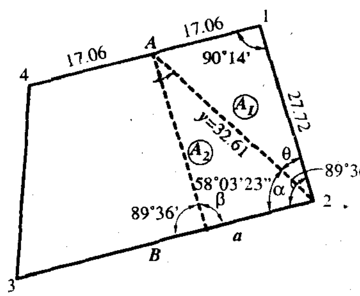
$A_1 = 236.45\ m^2$

Total area $= A_1 + A_2$
Total area $= 236.45 + 236.07$
Total area $= \mathbf{472.52\ m^2}$

③ *Distance of other end of dividing line from corner 2:*
Using Sine Law:

$$\frac{a}{\text{Sin } 31°33'} = \frac{32.61}{\text{Sin } 90°24'}$$

$a = \mathbf{17.06\ m.}$



Problem 185:

Given the field notes of the traverse for a four cornered lot.

LINES	AZIMUTH	DISTANCE
1 2	187°00'	27.89 m.
2 3	268°47'	34.12 m.
T - 1 3	246°27'	18.60 m.
3 4	358°33'	27.27 m.
4 1	88°57'	38.22 m.

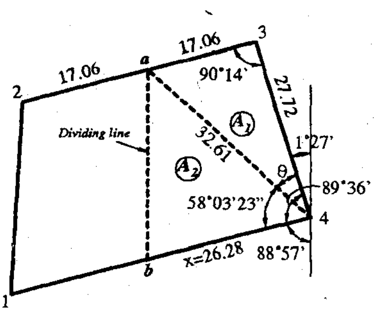
SUBDIVISION

Area of lot is 1000 sq.meters.
Coordinates of corner 1 (common Point)
Northings = 21412.63, Eastings = 19424.86

① Compute the length of the subdividing line.
② Compute the bearing of the dividing line from the mid - point of line 2 - 3 of the boundary.
③ Determine the bearing and distance from station T - 1 in order to layout the dividing point on the southern portion.

Solution:

① *Length of dividing line:*



$$A_1 = \frac{17.06\,(27.72)\,\text{Sin } 90°14'}{2}$$

$A_1 = 236.45\ m^2$
$A_2 = 600 - 236.45$
$A_2 = 363.55\ m^2$

Using Cosine Law:
$(4 - a)^2 = (17.06)^2 + (27.72)^2 - 2(17.06)(27.72)\text{ Cos } 90°14'$
$4 - a = 32.61$ m.

Using Sine Law:
$$\frac{17.06}{\text{Sin }\theta} = \frac{32.61}{\text{Sin } 90°14'}$$
$\theta = 31°32'37"$

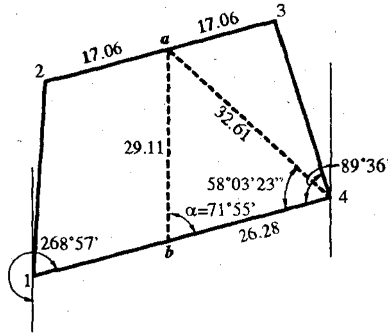
$$A_2 = \frac{32.61\,(x)\,\text{Sin } 58°03'23"}{2}$$

$$363.55 = \frac{x\,(32.61)\,\text{Sin } 58°03'23"}{2}$$

$x = 26.28$ m.

Length of dividing line
Using Cosine Law:
$(ab)^2 = (32.61)^2 + (26.28)^2 - 2(32.61)(26.28)\text{ Cos } 58°03'23"$
ab = ***29.11 m.***

② *Bearing of dividing line from mid - point of line 2 - 3:*



$$\frac{32.61}{\text{Sin }\alpha} = \frac{29.11}{\text{Sin } 58°03'23"}$$
$\alpha = 71°55'$

Azimuth of ba = 268°57'
- 71°55'
Azimuth of ba = 197°02'

Bearing ab = ***S 7°02' W***

③ *Bearing and distance from T - 1 to corner "b":*

LINES	BEARING	DISTANCE
1	N 7°00' E	27.89
2		
2 3	N 88°47' E	34.12
3 4	S 1°27' E	27.72
4 1	S 88°57' W	38.22

SUBDIVISION

LINES	Northings	Eastings
1	21412.63	17424.86
2	+ 27.69 21440.32	+ 3.40 19428.26
2 3	+ 0.72 21441.04	+ 34.11 19462.37
3 4	- 27.71 21413.33	+ 0.70 19463.07
4 1	- 0.70 21412.63	- 38.21 19424.86

LINES	BEARING	DISTANCE
T - 1	N 66°27' E	18.60
3		

LINES	Northings	Eastings
T - 1	21433.61	19445.32
3	+ 7.43 21441.04	- 17.05 19462.37

Coordinates of b:

4	S 88°57' W	26.28
b		

4	21413.33	19463.07
b	- 0.48 21412.85	- 26.28 19436.79

T - 1	21433.65	19445.32
b	21412.85 - 20.76	19436.79 - 8.53

$$tan\ bearing = \frac{8.53}{20.76}$$

Bearing (*T* - 1 to *b*) = S. 22°20' W.

$$Distance = \frac{8.53}{\sin 22°20'} = 22.45 \text{ m.}$$

Bearing and distance from T - 1 to dividing point on the southern portion is

S. 22°20' W., 22.45 m.

Problem 186

From the given technical description of lot 414, Cad 364 Cebu Cadastre the following data are recorded as follows.

Beginning at a point marked "1" being S. 37°33' W. 237.32 m. from BBM #1, Cad 364 Cebu Cadastre.

Thence N. 07°00' E. 27.89 m. to point 2,
Thence N 88°47' E. 34.12 m. to point 3,
Thence S. 01°27' E. 27.72 m. to point 4,
Thence S. 88°57' W. 38.22 m. to point of beginning.

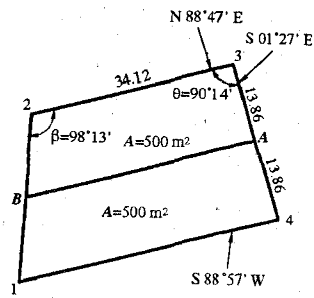
Containing an area of 1000 m^2 more or less.
BLLM#1 Cad 364 40000.00 N. 40000.00 E.
BBM#1 Cad 364 43095.02 N. 34691.42 E.

Subdivide the lot into two (2) equal parts, provided that the subdividing line must start at the center line of line 3-4 of boundary line.

① What is the distance of the subdividing line?
② What is the bearing of the subdividing line?
③ What is the distance be laid out from corner 2 of the boundary to the subdividing line?

Solution:

① *Distance of the subdividing line:*



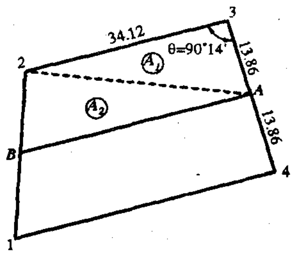
$\theta = 88°47' + 1°27'$

$\theta = 90°14'$

$\beta = 180° - 88°47' + 7°$

$\beta = 98°13'$

Distance of subdividing line AB:



$$A_1 = \frac{34.12\,(13.86)\,\text{Sin } 90°14'}{2}$$

$A_1 = 236.45\ m^2$

$A_2 = 500 - 236.45$

$A_2 = 263.55\ m^2$

	BEARING	DISTANCE
BBM #1	S. 37°33' W.	237.32
1		
1 / 2	N. 07°00' E.	27.89
2 / 3	N. 88°47' E.	34.12
3 / 4	S. 01°27' E.	27.72
4 / 1	S.88°57' W.	38.22

	Northings	Eastings
BBM #1	43095.02	34691.42
	- 188.15	- 144.64
1	42906.87	34546.78
1	+ 27.68	+ 3.40
2	42934.55	34550.18
2	+ 0.72	+ 34.11
3	42935.27	34584.29
3	- 27.70	+ 0.70
4	42907.57	34584.99
4	- 0.70	- 38.21
1	42906.87	34546.78

	BEARING	DISTANCE
Cor. 3	S. 01°27' E.	13.86
A		

	Northings	Eastings
Cor. 3	42935.27	34584.29
	- 13.86	+ 0.35
A	42921.41	34584.64

Bearing and distance from 2 to A:

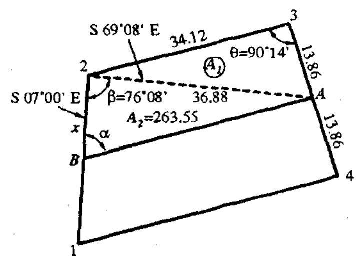
Cor. 2	42934.55	34550.18
A	42921.41	34584.64
	- 13.14	+ 34.46

$$\tan \text{bearing} = \frac{34.46}{13.14}$$

Bearing = S. 69°08' E.

$$\text{Distance} = \frac{34.46}{\text{Sin } 69°08'}$$

Distance = 36.88 m.



$\beta = 69°08' + 7°$

$\beta = 76°08'$

$$A_2 = \frac{(x)(36.88)\,\text{Sin } 76°08'}{2}$$

$$263.55 = \frac{x\,(36.88)\,\text{Sin } 76°08'}{2}$$

$x = 14.72$ m.

Using Cosine Law:

$$(AB)^2 = (14.72)^2 + (36.88)^2 - 2(14.72)(36.88)\,\text{Cos } 76°08'$$

AB = ***36.28 m.***

② *Bearing of subdividing line:*

Using Sine Law:

$$\frac{36.28}{\sin 76°08'} = \frac{36.88}{\sin \alpha}$$

$\alpha = 80°43'$

Bearing of subdividing line

$= 80°43' + 7°$

$=$ ***N. 87°43' E.***

③ *Distance to be laid out from corner 2 of the boundary to subdividing line:*

= ***14.72 m.***

Problem 187:

Given the following technical description of lot 245, Cad 230, Cebu Cadastre.

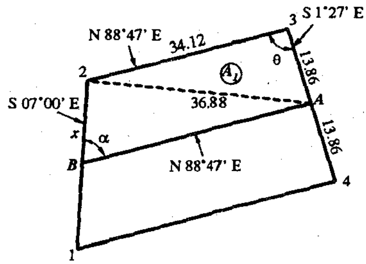
LINES	BEARING	DISTANCES
1 - 2	N. 07°00' E.	27.89 m.
2 - 3	N. 88°47' E.	34.12 m.
3 - 4	S. 01°27' E.	27.72 m.
4 - 1	S. 88°57' W.	38.22 m.

The line from BBM #20, Cad 240, Cebu Cadastre S. 37°33' W. 237.32 m. to cor. 1
Contains an area of 1000 sq.m. more or less.
Coordinates of BBM #20 43095.02 N. 34691.42 E.
Subdivide this lot into (two) parts, such that the dividing line must start at the center of line 3 - 4 of the boundary and it must be parallel to the line 3 - 4 of the boundary and it must be parallel to the line 2 -3 of the boundary.

① What is the distance of the subdividing line?
② What is the area of the lot to be subdivided on the northern part?
③ Compute the coordinates of the subdividing point on the Eastern part?

Solution:

① *Distance of subdividing line:*



$\theta = 88°47' + 1°27'$

$\theta = 90°14'$

	BEARING	DISTANCE
BBM #20	S. 37°33' W.	237.32
1		
1	N. 07°00' E.	27.89
2		
2	N. 88°47' E.	34.12
3		
3	S. 01°27' E.	27.72
4		
4	S.88°57' W.	38.22
1		

	Northings	Eastings
BBM #20	43095.02	34691.42
	- 188.15	- 144.64
1	42906.87	34546.78
1	42906.87	34546.78
2	+ 27.68	+ 3.40
2	42934.55	34550.18
3	+ 0.72	+ 34.11
3	42935.27	34584.29
4	- 27.70	+ 0.70
4	42907.57	34584.99
1	- 0.70	- 38.21
	42906.87	34546.78

SUBDIVISION

	BEARING	DISTANCE
Cor. 3	S. 01°27' E.	13.86
A		

	Northings	Eastings
Cor. 3	42935.27	34584.29
	- 13.86	+ 0.35
A	42921.41	34584.64

Bearing and distance from 2 to A

Cor. 2	42934.55	34550.18
A	42921.41	34584.64
	- 13.14	+ 34.46

$$\tan bearing = \frac{34.46}{13.14}$$

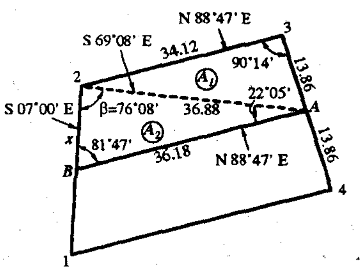
Bearing = S. 69°08' E.

$$Distance = \frac{34.46}{\text{Sin } 69°08}$$

Distance = 36.88 m.

$ß = 69°08' + 7'$

$ß = 76°08'$



$$\frac{AB}{\text{Sin } 76°08'} = \frac{36.88}{\text{Sin } 81°47'}$$

AB = ***36.18 m.***

② *Area of lot to be subdivided on the northern part:*

$$A_1 = \frac{(34.12)(13.86)\text{ Sin } 90°14'}{2}$$

$A_1 = 236.45\ m^2$

$$A_2 = \frac{(36.88)(36.18)\text{ Sin } 22°05'}{2}$$

$A_2 = 250.82\ m^2$

Total area = 236.45 + 250.82

Total area = ***487.27 m²***

③ *Coordinates of the subdividing point on the Eastern part:*

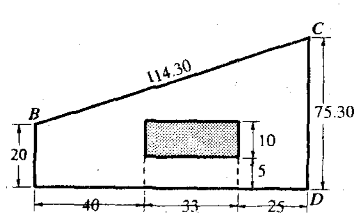
Coordinates of A = ***42921.41 N., 34584.64 E.***

Problem 188

Given below is a tabulation of the technical description of a residential lot located in Forbes Park. The owner wishes to sell the half portion of his lot in such a way that the dividing line should be parallel to the sides AB and CD.

① Determine the length of this dividing line.
② Determine also the area of the building nearer to the side AB that is being cut off by the dividing line.
③ Determine the area of the building nearer to the side CD that is being cut off by the dividing line.

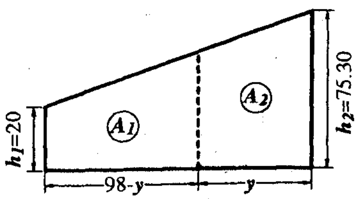
LINES	BEARINGS	DISTANCES
AB	Due north	20.00 m.
BC	N 61° E	114.30 m.
CD	Due south	75.30 m.
DA	Due west	98.00 m.



SUBDIVISION

Solution:

① *Length of this dividing line:*



$A_1 = A_2$

$n = m$

$n = 1$

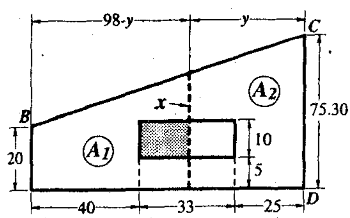
$m = 1$

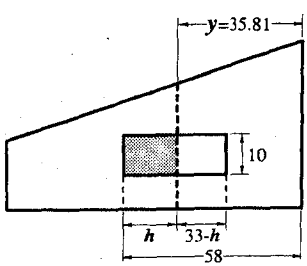
$$X = \sqrt{\frac{nb_2^2 + mb_1^2}{m+n}}$$

$$X = \sqrt{\frac{1(75.3)^2 + 1(20)^2}{1+1}}$$

$X = \mathbf{55.09\ m.}$

② *Area of building cut off near the line AB*





$$A_1 = \frac{(20 + x)(98 - y)}{2}$$

$$A_2 = \frac{(x + 75.3)\,y}{2}$$

$A_1 = A_2$

$$\frac{(20 + 55.09)(98 - y)}{2} = \frac{(55.09 + 75.3)\,y}{2}$$

$130.39y = 75.09\,(98) - 75.09y$

$205.48y = 7358.82$

$y = 35.81$ m.

$h = 58 - y$

$h = 58 - 35.81$

$h = 22.19$

Area of building cut off near the line AB

$= 10(22.19)$

$= \mathbf{221.90\ m^2}$

③ *Area of building cut off near CD:*

$A = 10\,(33 - 22.19)$

$A = \mathbf{108.10\ m^2}$

Problem 189:

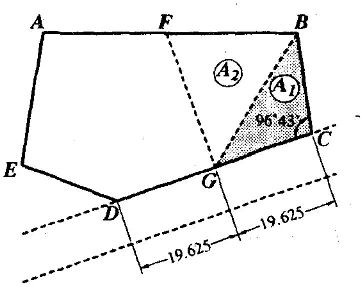
Given below is the technical description of a lot, having an area of 640.56 sq.m. It is required to subdivide this lot into two equal areas such that they will have equal frontage along the line C - D which adjoins a street.

LINES	BEARING	DISTANCES
AB	N 73°23' E	33.46 m.
BC	S 39°31' E	9.21 m.
CD	S 43°46' W	39.27 m.
DE	N 39°52' W	7.06 m.
EA	N 15°50' W	20.50 m.

① Compute the distance of the other end of the dividing line from corner B.
② Compute the distance of the dividing line.
③ Compute the bearing of the dividing line.

Solution:

① *Distance of the end of dividing line from B:*



$$A_1 = \frac{19.625\,(9.21)\,\text{Sin } 96°43'}{2}$$

$A_1 = 89.75\ m^2$

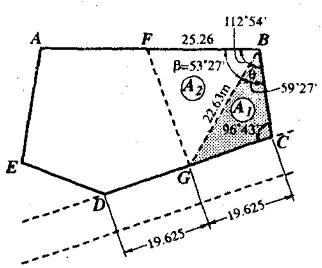
$$A_2 = \frac{640.56}{2} - 89.75$$

$A_2 = 230.53\ m^3$

$$(BG)^2 = (9.625)^2 + (9.21)^2 - 2(19.625)(9.21)\ \text{Cos } 96°43'$$

$BG = 22.63$ m.

Using Sine Law:



$$\frac{19.625}{\text{Sin } \theta} = \frac{22.63}{\text{Sin } 96°43'}$$

$\theta = 59°27'$

$ß = 112°54' - 59°27'$

$ß = 53°27'$

$$A_2 = \frac{(FB)\,(22.63)\,\text{Sin } 53°27'}{2}$$

$$230.53 = \frac{(FB)\,(22.63)\,\text{Sin } 53°27'}{2}$$

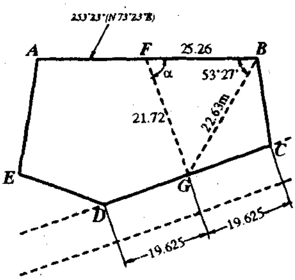
$FB =$ ***25.36 m.***

② *Distance of dividing line:*
Using Cosine Law:

$$(FG)^2 = (25.36)^2 + (22.63)^2 - 2(25.36)(22.63)\ \text{Cos } 53°27'$$

$FG =$ ***21.72 m.***

③ *Bearing of dividing line:*
Using Sine Law:



$$\frac{22.63}{\text{Sin } \alpha} = \frac{21.72}{\text{Sin } 53°27'}$$

$\alpha = 56°49'$

Azimuth of FG = 253°23' + 56°49'
Azimuth of FG = 310°12'

Bearing of FG = ***S. 49°48' E.***

Problem 190

Given the azimuth and the distance of a lot.

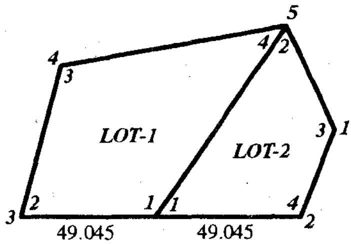
LINE	AZIMUTH	DISTANCE
BLLM 1 - 1	307°46'	198.66 m.
1 - 2	47°49'	30.48 m.
2 - 3	151°44'	111.37 m.
3 - 4	259°21'	29.09 m.
4 - 5	296°34'	59.68 m.
5 - 1	6°44'	56.70 m.
BLLM No. 1	Northings	Eastings
	20,000.00	20,000.00

① Compute the area of the lot.
② In the same lot, a dividing line is drawn from corner 5 to the midpoint of line 2 - 3. Find the azimuth of the dividing line.
③ Find the distance of the dividing line.

SUBDIVISION

Solution:

① *Area of lot:*



Line	Bearing	Distance	LAT	DEP
1 - 1	S 52°14 E	198.66	- 121.67	+157.04
1 - 2	S 47°49' W	30.48	- 20.47	- 22.59
2 - 3	N 28°16' W	111.37	+98.09	- 52.73
3 - 4	N 79°21' E	29.09	+5.38	+28.59
4 - 5	S 63°26' E	56.68	- 26.69	+53.38
5 - 1	S 6°44' W	56.70	- 56.31	- 6.65

Corners	LAT	DEP	COORDINATES	
BLLM 1	- 121.67	+157.04	20000.00	20000.00
			- 121.67	+157.04
1			19878.33	20157.04
	- 20.47	- 22.59	- 20.47	- 22.59
2			19857.86	20134.45
	+98.09	- 52.73	+98.09	- 52.73
3			19955.95	20081.72
	+5.38	+28.59	+5.38	+28.59
4			19961.33	20110.31
	- 26.69	+53.38	- 26.69	+53.38
5			19934.64	20163.69
	+56.31	- 6.65	- 56.31	- 6.65
1			19878.33	20157.04

LINE	LAT	DMD	DEP	DOUBLE AREA
1 - 2	- 20.47	- 22.59	- 22.59	+462.42
2 - 3	+98.09	- 97.91	- 52.73	- 9603.99
3 - 4	+5.38	- 122.05	+28.59	- 656.63
4 - 5	- 26.69	- 40.08	+53.38	+1069.74
5 - 1	- 56.31	+6.65	- 6.65	- 374.46

$2A = 9102.92$

$A = \mathbf{4551.46\ m^2}$

② *Azimuth of dividing line:*
For lot 1

Lines	Bearing	Distance	LAT	DEP
2 - 1	S 28°16' E	49.05	- 43.20	+23.23

Corner	LAT	DEP	Northings	Eastings
2 - 3			19955.95	20081.72
	- 43.20	+23.23	- 43.20	+23.23
1			19912.75	20104.95

BLLM 1	20000.00	20000.00
1	19912.75	20104.95
2	19955.95	20081.72
3	19961.33	20110.31
4	19934.64	20163.69
1	19912.75	20104.95

Corner	LAT	DMD	DEP	DOUBLE AREA
1 - 2	+43.20	- 23.23	- 23.23	- 1003.54
2 - 3	+5.38	- 17.87	- 28.59	- 96.14
3 - 4	- 26.69	+64.10	+53.38	- 1710.83
4 - 1	- 21.89	+58.74	- 58.74	- 1285.82

$2A = 4096.33$

$A = 2048.17\ m^2$

CORNERS	BEARING	DISTANCE
1 - 2	N 28°16' W	49.05 m.
2 - 3	N 79°21' E	29.09 m.
3 - 4	S 63°26' E	59.68 m.
4 - 1	S 69°34' W	62.69 m.

Line 4 - 1 (Dividing line)

$$\tan bearing = \frac{58.74}{21.89}$$

bearing = S 69°34' W

azimuth = **69°34'**

③ *Distance of dividing line:*

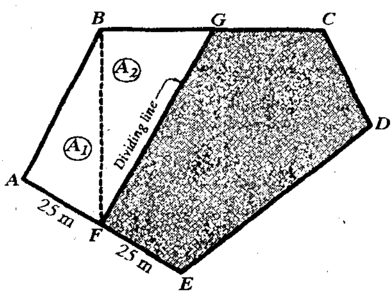
$$Distance = \frac{58.74}{\sin 69°34'}$$

Distance = **62.69 m.**

Problem 191:

A parcel of land, with boundaries as described below is to be subdivided into two lots of equal areas. The dividing line is to pass through a point midway between corner A and E, and through a point along the boundary BC.

LINE	LENGTH	BEARING
AB	60.00 m.	N 15°30' E
BC	72.69 m.	S 82°23' E
CD	44.83 m.	S 17°20' E
DE	56.45 m.	S 70°36' W
EA	50.00 m.	N 74°30' W



① Find the area of each lot.
② Find the distance of the dividing line.
③ Find the bearing of the dividing line.

Solution:

① *Area of each lot:*

LINES	BEARING	DISTANCES
AB	N 15°30' E	60.00
BC	S 82°23' E	72.69
CD	S 17°20' E	44.83
DE	S 70°36' W	56.45
EA	N 74°30' W	50.00

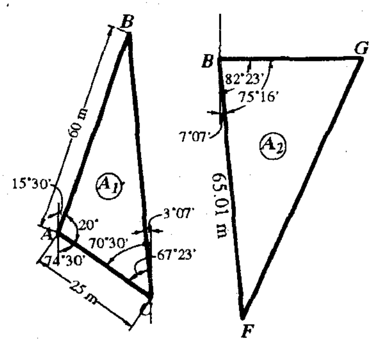
Lines	LAT	DEP	DMD	DOUBLE AREA
AB	+57.81	+16.03	+16.03	+926.09
BC	- 9.63	+72.04	- 104.10	- 1002.48
CD	- 42.79	+13.36	- 189.50	- 8108.71
DE	- 18.75	- 53.25	+149.61	- 2805.19
EA	+13.36	- 48.18	+48.18	+643.68

$2A = 10346.01$
$A = 5173.005\ m^2$

$$A = \frac{5173.005}{2}$$

$A =$ ***2586.50 sq.m.***

② *Distance of dividing line:*
Considering triangle ABF:



$$A_1 = \frac{60\,(25)}{2}$$

$A_1 = 750$ sq.m.
$A_2 = 2586.50 - 750$
$A_2 = 1836.50$ sq.m.

$$\tan ø = \frac{60}{25}$$

$ø = 67°23'$
$AF = FB \cos ø$

$$FB = \frac{25}{\cos 67°23'}$$

$FB = 65.01$ m.

Considering triangle BFG:
Bearing of FB: N 7°07' W

$$A_2 = \frac{65.01\,(BG) \sin 75°16'}{2}$$

$$BG = \frac{2\,(1836.50)}{65.01 \sin 75°16'}$$

$BG = 58.42$ m.

Using Cosine Law:

$(FG)^2 = (65.01)^2 + (58.42)^2 - 2(65.01)(58.42)\cos 75°16'$

FG = **75.55 m.** *(distance of dividing line)*

③ *Bearing of dividing line:*
Using Sine Law:

$$\frac{\sin\alpha}{BG} = \frac{\sin 75°16'}{FG}$$

$$\sin\alpha = \frac{58.42 \sin 75°16'}{75.55}$$

$\alpha = 48°24'$

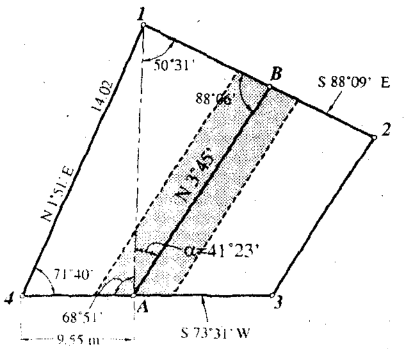
Bearing of dividing line FG = ***N 41°17' E***

Problem 192:

A proposed 10 m. service road crosses the property of JFN Holdings whose technical descriptions are as follows.

LINES	AZIMUTH	DISTANCE
1 - 2	271°51'	20.00 m.
2 - 3	1°51'	7.39 m.
3 - 4	73°31'	21.07 m.
4 - 1	181°51'	14.02 m.

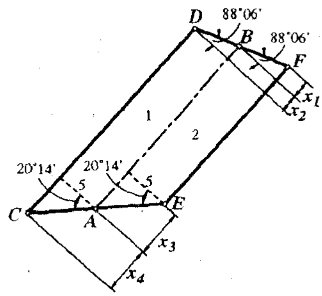
The centerline of the proposed service road crosses at 9.55 m. from corner 4 along the line 3 - 4 and runs in a direction of N 3°45' E.



① Compute the length of the center line of the road that traverses along the lot.
② How far is the other end of the center line intersecting the property boundary from corner 1.
③ If the property is located at Barangay Punta Princesa, where cost of land is P2000.00 per square meter, estimate the cost of the property to be expropriated for the service road.

Solution:

① *Length of the center line:*



Using Cosine Law:

$x^2 = (9.55)^2 + (14.02)^2 - 2(9.55)(14.02)\cos 71°40'$

$x = 14.27$ m.

Using Sine Law:

$$\frac{14.02}{\sin ø} = \frac{14.27}{\sin 71°40'}$$

$ø = 68°51'$

$ß = 180° - 71°40' - 68°51'$

$ß = 39°29'$

$$\frac{AB}{\sin 50°31'} = \frac{14.27}{\sin 88°06'}$$

AB = ***11.02 m.***

② *Distance of other end of center line of proposed road from corner 1:*

$$\frac{AB}{\sin 50°31'} = \frac{B-1}{\sin 41°23'}$$

$$B-1 = \frac{11.02 \sin 41°23'}{\sin 50°31'}$$

B - 1 = ***9.44 m.***

③ *Cost of property to be expropriated:*

$\tan 88°06' = \frac{5}{x_2}$

$x_2 = 0.17$

$x_1 = \frac{5}{\tan 88°06'}$

$x_1 = 0.17$

$\tan 20°14' = \frac{x_4}{5}$

$x_4 = 1.84$ m.

$x_5 = 1.84$ m.

$CD = AB - x_2 + x_4$

$CD = 11.02 - 0.17 + 1.84$

$CD = 12.69$

$EF = 11.02 - 1.84 + 0.17$

$EF = 9.35$

$A = A_1 + A_2$

$A_1 = \frac{(12.69 + 11.02)\,5}{2} = 59.275 \text{ m}^2$

$A_2 = \frac{(11.02 + 9.35)\,5}{2} = 50.925$

$A = 59.275 + 50.925$

$A = 110.20 \text{ m}^2$

Total Cost = 110.2 (2000)

Total Cost = ***P220400.00***

Problem 193:

In view of the desire of the government to relieve perennial vehicular and pedestrian traffic congestions of a city, a dead end road is to be extended and constructed with a road right of way of 20 m. which intersects several privately owned properties. One of these lot has the following field notes.

LINES	BEARING	DISTANCE
1 - 2	N 61°57' E	74.18 m.
2 - 3	----	----
3 - 4	S 9°03' W	54.13 m.
4 - 5	N 68°21' W	55.43 m.
5 - 1	N 13°56' W	58.85 m.

If the center line of the proposed road passes through point 3, and is parallel to the line 1 - 2.

① Compute the length of the boundary of the proposed road on the southern portion of the lot.

② Compute the length of the boundary of the proposed road on the northern portion of the lot.

③ Compute the total area of the proposed road to be expropriated from the property.

Solution:

Lines	Bearings	Distance	LAT	DEP
1 - 2	N 61°57' E	74.18	+34.88	+65.47
2 - 3	----	----	----	----
3 - 4	S 9°03' W	54.13	- 53.46	- 8.51
4 - 5	N 68°21' W	55.43	+20.45	- 51.52
5 - 1	N 13°56' W	58.85	+57.12	- 14.17
			+112.45	+65.47
			- 53.46	- 74.20
			+58.99	- 8.73

① *Length of the boundary of the proposed road on the southern portion of the lot:*

For line 2 - 3:

$\text{tangent bearing} = \frac{dep}{lat}$

$\text{tangent bearing} = \frac{8.73}{58.99}$

bearing = S 8°25' E

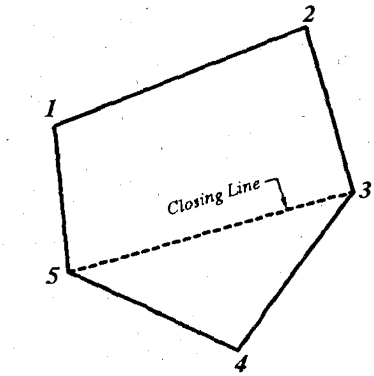
$\text{Distance} = \frac{dep}{\text{Sin bearing}}$

$\text{Distance} = \frac{8.73}{\sin 8°25'}$

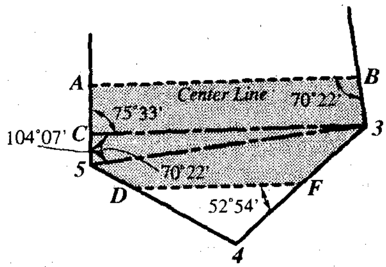
Distance = 59.64 m.

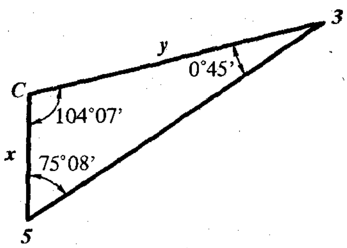
Complete Field Notes

LINES	BEARING	DISTANCE
1 - 2	N 61°57' E	74.18 m.
2 - 3	S 8°25' E	59.64 m.
3 - 4	S 9°03' W	54.13 m.
4 - 5	N 68°21' W	55.43 m.
5 - 1	N 13°56' W	58.85 m.



Lines	Bearings	Distance	LAT	DEP
3 - 4	S 9°03' W	54.13	- 53.46	- 8.51
4 - 5	N 68°21' W	55.43	+20.45	- 51.52
5 - 1	----	----	----	----
			- 33.01	- 60.03





$$tangent\ bearing = \frac{60.03}{33.01}$$

Bearing = N 61°12' E

$$Distance = \frac{60.03}{\text{Sin } 61°12'}$$

Distance = 68.50 m.

$$\frac{X}{\text{Sin } 0°45'} = \frac{68.50}{\text{Sin } 104°07'}$$

$X = 0.92$ m.

$$\frac{Y}{\text{Sin } 75°08'} = \frac{68.50}{\text{Sin } 104°07'}$$

$Y = 68.27$

$h_1 = 0.92 \text{ Sin } 75°53'$

$h_1 = 0.89$ m.

$h_2 = 10 - 0.89$

$h_2 = 9.11$ m.

$$\tan 52°54' = \frac{h_1}{a}$$

$$a = \frac{0.89}{\tan 52°54'}$$

$a = 0.67$ m.

$$\tan 52°54' = \frac{h_2}{d}$$

$$d = \frac{9.11}{\tan 52°54'}$$

$d = 6.89$ m.

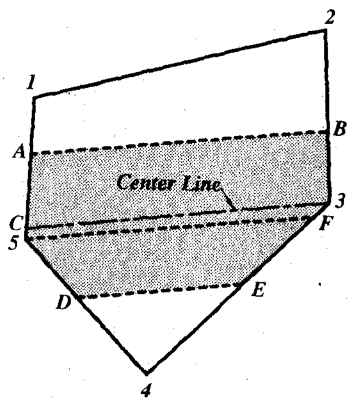
$$\tan 75°53' = \frac{10}{e}$$

$e = 2.51$ m.

$$\tan 70°22' = \frac{10}{f}$$

$f = 3.57$ m.

Distance of line F - 5:



$F - 5 = 68.27 - 0.67 + 0.22$

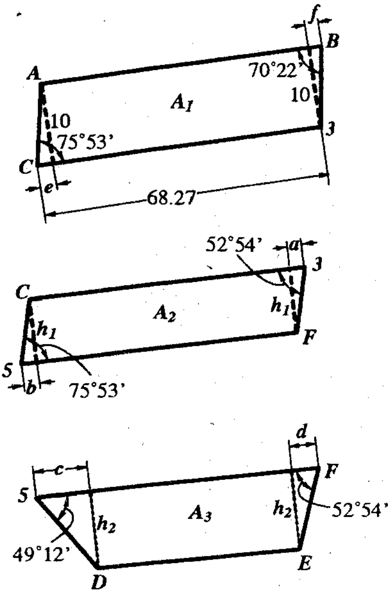
$F - 5 = 67.82$ m.

Distance DE:

$DE = 67.82 - 7.86 - 6.89$

$DE =$ ***53.07***

② *Lenght of the boundary of hte proposal road on the northern portion of the lot:*
Distance AB:



$AB = 68.27 - 2.51 + 3.57$

$AB = \mathbf{69.33\ m.}$

③ *Area of the proposed road to be expropriated from the property:*

$$A_1 = \frac{(69.33 + 68.27)(10)}{2}$$

$A_1 = 688$ sq.m.

$$A_2 = \frac{(68.27 + 67.82)(0.89)}{2}$$

$A_2 = 60.56$ sq.m.

$$A_3 = \frac{(67.82 + 53.07)(9.11)}{2}$$

$A_3 = 550.65$ sq.m.

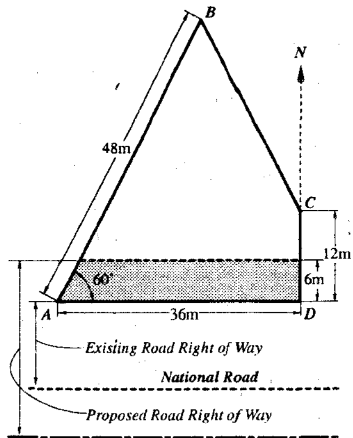
$A = A_1 + A_2 + A_3$

$A = 688 + 60.56 + 550.65$

$A = \mathbf{1299.21\ sq.m.}$

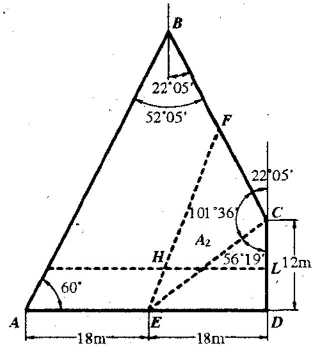
Problem 194:

Brothers A and B are to inherit a residential lot which shall be divided equally the area and the length of the frontage abutting the National Road as shown. As per Zoning Ordinance, a widening of road of right of way of 6 meters each on both sidewalks is required. Using DMD Method.



① Compute the bearing and distance of line BC.
② Compute the areas (after road widening) for each of the brother.
③ Compute the bearing and distance of the common sides of both lots.

Solution:



SUBDIVISION

① *Bearing and distance of EC:*

LINES	BEARING	DISTANCE
AB	N 30 E.	48 m.
BC	----	----
CD	Due South	12 m.
DA	Due West	36 m.

Lines	LAT	DEP	DMD	Double Area
AB	+41.57	+24	+24	+997.68
BC	- 29.57	+12	+60	- 1174.20
CD	- 12.00	0	+72	- 864.00
DA	0	- 36	+36	0

$2A = 1640.52$
$A = 820.26$

For line BC:

$$\tan bearing = \frac{Dep}{Latitude}$$

$$\tan bearing = \frac{12}{29.57}$$

$Bearing = S\ 22°05'\ E.$

$$Distance = \frac{12}{\sin 22°05'}$$

$Distance = 31.91$ m.

Bearing and distance of EC:
*Bearing = **S 22°05' E.***
*Distance = **31.91 m.***

② *Areas for each brother after widening:*

$$\tan ø = \frac{18}{12}$$

$ø = 56°19'$

$$Area\ required = \frac{820.26}{2} = 410.13 \text{ sq.m.}$$

$$A_1 = \frac{18\,(12)}{2}$$

$A_1 = 108$ sq.m.
$A_2 = 410.13 - 108.00$
$A_2 = 302.13$ sq.m.

$EC = \sqrt{(18)^2 + (12)^2}$
$EC = 21.63$ m.

$$A_2 = \frac{(FC)\,(EC)\,\sin 101°36'}{2}$$

$$302.13 = \frac{(FC)\,(21.63)\,\sin 101°36'}{2}$$

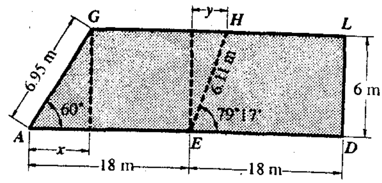
$FC = 28.52$ m.

$BF = 31.91 - 28.52$
$BF = 3.39$ m.
$(FE)^2 = (28.52)^2 + (21.63)^2 - 2(28.52)(21.63) \cos 101°36'$
$FE = 39.11$ m.

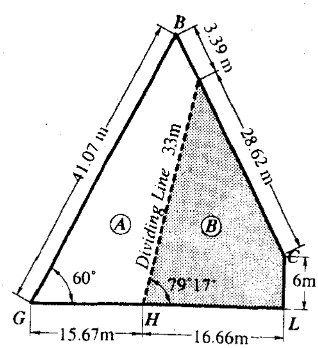
$$\frac{\sin\alpha}{21.63} = \frac{\sin 101°36'}{39.11}$$

$\alpha = 32°48'$

Bearing of FE $= 32°48' - 22°05'$
Bearing of FE $= S\ 10°43'\ W.$



$x = 6.93 \cos 60°$
$x = 3.47$ m.
$y = 6.11 \cos 79°17'$
$y = 1.14$ m.
$ME = 18 - 3.47$
$ME = 14.53$ m.
$GH = 14.53 + 1.14$
$GH = 15.67$ m.
$HL = 18 + 14.53 - 15.67$
$HL = 16.86$ m.
$BG = 48 - 6.93$
$BG = 41.07$ m.
$FH = 39.11 - 6.11$
$FH = 33$ m.
$CL = 6$ m.



LOT A:

LINES	BEARING	DISTANCE
HG	Due West	15.67
GB	N 30° E.	41.07
EF	S 22°05' E.	3.39
FH	S 10°43' W.	33.00

Lines	LAT	DEP	DMD	Double Area
HG	0	- 15.67	- 15.67	0
GB	+35.56	+20.54	- 10.80	- 384.05
EF	- 3.14	+1.27	+11.01	- 34.57
FH	- 32.42	- 6.14	+6.14	- 199.06

$2A = 617.68$
$A = 308.84\ m^2$

LOT B:

LINES	BEARING	DISTANCE
LH	Due West	16.86
HF	N 10°43' E.	33.00
FC	S 22°05' E.	28.52
CL	Due South	6.00

Lines	LAT	DEP	DMD	Double Area
LH	0	- 16.86	- 16.86	0
HF	32.42	+6.14	- 27.58	- 894.14
FC	- 26.42	+10.72	- 10.72	+283.22
CL	- 6.00	0	0	0

$2A = 610.92$
$A = 305.46\ m^2$

Areas for each brother after widening:

$A_1 = \mathbf{308.84\ m^2}$

$A_2 = \mathbf{305.46\ m^2}$

③ *Bearing and distance of common sides of both lots (line FE):*

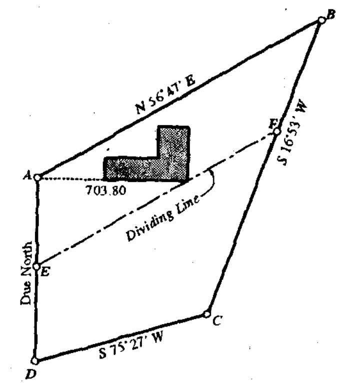
Bearing = S 10°43' W.

Distance = ***33.00 m.***

Problem 195

From the given technical description it is required to determine the location of the dividing line that will intersect the lines BC and DA such a way that it will pass through point O inside the lot which is due east of corner A and a distance of 703.80 m. This dividing line will divide the whole lot into two equal areas.

LINES	BEARING	DISTANCE
A - B	N 56°47' E	1276.90 m.
B - C	S 16°53' W.	1492.47 m.
C - D	S 75°27' W.	655.84 m.
D - A	Due North.	893.41 m.



① Compute the area of the whole lot.
② Compute the location of dividing line from corner A along the line AD.
③ Compute the location of the dividing line from corner B along line BC.

Solution:

① *Area of whole lot:*

Lines	LAT	DEP	DMD	Double Area
A - B	+699.49	+1068.26	+1068.26	+747237.19
B - C	- 1428.14	- 433.45	+1703.07	-2432222.39
C - D	- 164.76	- 634.81	+634.81	- 104591.30
D - A	+893.41	0	0	0

$2A = 1789596.50$
$A = \mathbf{894788.25}$

SUBDIVISION

② *Location of dividing line from corner A:*
Using Sine Law:

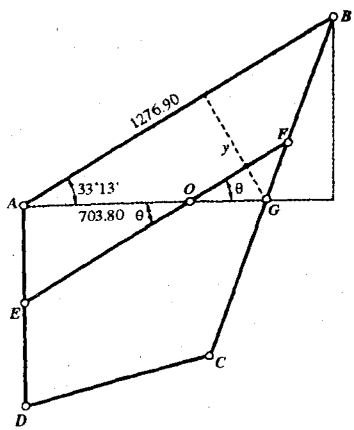
$$\frac{AG}{\text{Sin }39°54'} = \frac{1276.90}{\text{Sin }106°53'}$$

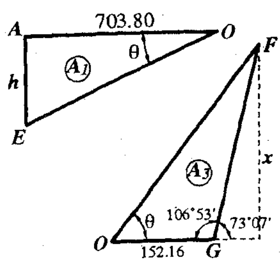
$AG = 855.96$
$OG = 855.96 - 703.90$
$OG = 152.16$

$$\text{Area of } ABFE = \frac{1}{2}(894788.25)$$

$\text{Area of } ABFE = 447394.125$

Area of triangle AOE:





$$A_1 = \frac{703.80\,(h)}{2}$$

$$\tan ø = \frac{h}{703.80}$$

$$A_1 = \frac{703.80\,(703.80)\tan ø}{2}$$

$A_1 = 247667.22 \tan ø$
$y = 855.96 \text{ Sin } 33°13'$
$y = 468.90$

Area of triangle ABG:

$$A_2 = \frac{1276.90\,y}{2}$$

$$A_2 = \frac{1276.90\,(468.9)}{2}$$

$A_2 = 299369.21$

Area of triangle OFG:

$$A_3 = \frac{152.16\,x}{2}$$

$x = FG \text{ Sin } 73°07'$

$$\frac{FG}{\text{Sin } ø} = \frac{152.16}{\text{Sin } ß}$$

$ß = 180 - (ø + 106°53')$
$ß = 73°07' - ø$

$$\frac{FG}{\text{Sin } ø} = \frac{152.16}{\text{Sin }(73°07' - ø)}$$

$$FG = \frac{152.16 \text{ Sin } ø}{\text{Sin } 73°07' \text{ Cos } ø - \text{Sin } ø \text{ Cos } 73°07'}$$

$$x = \frac{152.16 \text{ Sin } ø \text{ Sin } 73°07'}{\text{Sin } 73°07' \text{ Cos } ø - \text{Sin } ø \text{ Cos } 73°07'}$$

$$x = \frac{152.16 \text{ Sin } 73°07'}{\text{Sin } 73°07' \text{ Cot } ø - \text{Cos } 73°07'}$$

$$x = \frac{145.60}{0.96 \cot ø - 0.29}$$

$$A_3 = \frac{152.16\,(145.60)}{2\,(0.96 \cot ø - 0.29)}$$

$$A_3 = \frac{11077.25}{0.96 \cot ø - 0.29}$$

$A = A_1 + A_2 - A_3$

$$447394.125 = 247667.22 \tan ø + 299369.21 - \frac{11077.25}{0.96 \cot ø - 0.29}$$

$$148024.92 = 247676.22 \tan ø - \frac{11077.25}{0.96 \cot ø - 0.29}$$

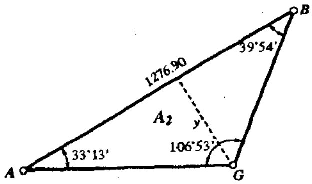
$142103.92 \cot ø - 42927.23$
$\quad = 237760.53 - 71823.49 \tan ø - 11077.25$
$142103.92 \cot ø + 71823.49 \tan ø = 269610.51$
$1.98 \cot ø + \tan ø - 3.75 = 0$
$\tan^2 ø - 3.75 \tan ø + 1.98 = 0$

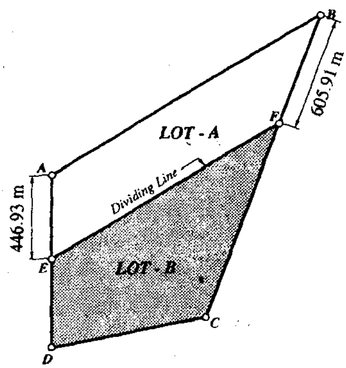
$$\tan ø = \frac{3.75 \pm \sqrt{(3.75)^2 - 4\,(1.98)}}{2}$$

$$\tan ø = \frac{3.75 \pm 2.48}{2}$$

$ø = 32°25'$
$h = 703.80 \tan 32°25'$
$h = \mathbf{446.93\ m.}$

③ *Location of dividing line from B:*





$$\frac{FG}{\sin 32°25'} = \frac{152.16}{\sin 40°42'}$$

$FG = 125.09$ m.

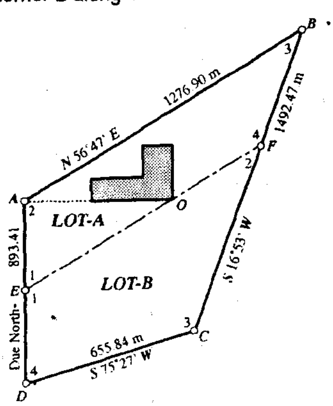
$$\frac{BG}{\sin 33°13'} = \frac{855.96}{\sin 39°54'}$$

$BG = 731.00$

$BF = 731 - 125.09$

$BF = 605.91$

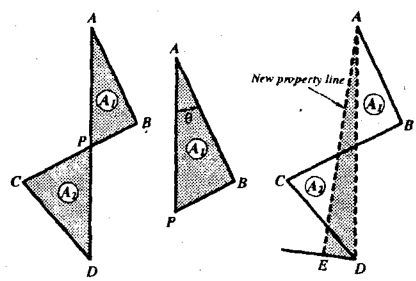
The dividing line is 446.93 m. from corner A along the line AD and ***605.91 m.*** from corner B along the line BC.



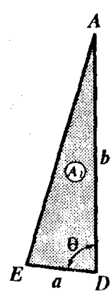
Straightening of Irregular Boundaries

STRAIGHTENING A BOUNDARY

The figure below shows an irregular boundary ABCD and is to be replaced by a single line AE. The general procedure is to solve for the choosing line AD and compute for the values of A_1 and A_2. When A_1 is greater than A_2 move the new property line towards the bigger area at A_1 but if A_2 is greater than A_1, move the property line towards A_2.



Since A_2 is greater than A_1 move the new property line to A_2.

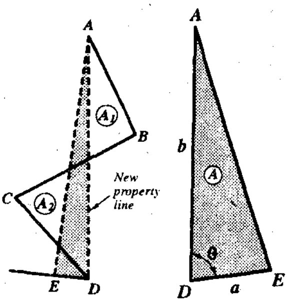


$$A = A_2 - A_1$$

$$A = \frac{ab \sin ø}{2}$$

STRAIGHTENING OF IRREGULAR BOUNDARIES

When A_1 is greater than A_2, move towards A_1.

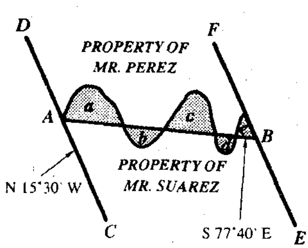


$A = A_1 - A_2$

$A = \dfrac{ab \text{ Sin } ø}{2}$

Problem 196:

Mr. Perez and Suarez own pieces of land which are adjacent to each other. The curve line in the figure represents a stream forming a boundary between the two pieces of property. It is proposed to place this stream in a storm drain and to straighten the boundary. Starting at point A, a random line AB was run and the area "a", "c", "d" and "e" were determined from offset measurements using Simpson's Rule to be 3040, 926, 2384, 1592 and 68 sq.m. respectively.
AB = 375 m. and has a bearing of S. 77°40' E.
CD = EF
Bearing of CD = N. 15°30' W.



① Find the distance BH (along the boundary EF) such that the straight line AH will provide the same areas in the properties of Mr. Perez and Mr. Suarez as was the case when the boundary was curve.
② Compute the length of line AH.
③ Compute the bearing of line AH.

Solution:

① *Distance BH:*

$A_1 = a + c + e$

$A_1 = 3040 + 2384 + 68$

$A_1 = 5492$ sq.m.

$A_2 = b + d$

$A_2 = 926 + 1592$

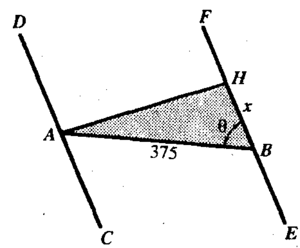
$A_2 = 2518$ sq.m.

$A = A_1 - A_2$

$A = 5492 - 2518$

$A = 2974$ sq.m.

Therefore, the dividing line should be moved towards the property of Mr. Perez.



$ø = 77°40' - 15°30'$

$ø = 62°10'$

$A = \dfrac{375\,(x) \text{ Sin } 62°10'}{2}$

$X = \dfrac{2\,(2974)}{375 \text{ Sin } 62°10'}$

$X =$ **17.93 m.**

② *Length of line AH:*

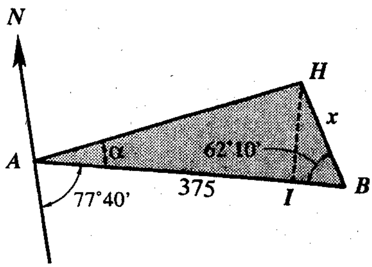
$\text{Sin } 62°10' = \dfrac{HI}{17.93}$

$HI = 15.86$ m.

$BI = 17.93 \text{ Cos } 62°10'$

$BI = 8.37$ m.

STRAIGHTENING OF IRREGULAR BOUNDARIES



$AI = 375 - 8.37$
$AI = 366.63$

$$\tan \alpha = \frac{15.86}{366.63}$$

$\alpha = 2°29'$

$$AH = \frac{366.63}{\text{Cos } 2°29'}$$

$AH =$ ***366.97 m.***

③ *Bearing of line AH:*
Bearing AH = 77°40' + 2°29'
Bearing AH = ***S 80°09' E***
$BI = 8.37$ m.

Problem 197:

The following is a set of notes of an irregular boundary of a piece of land. It is desired to straighten this crooked boundary line by substituting a straight line running from B to the line EF.

LINES	BEARINGS	DISTANCES
AB	S 89°14' E	373.62 m.
BC	N 13°10' E	100.27 m.
CD	N 0°17' E	91.26 m.
DE	N 27°39' E	112.48 m.
EF	N 72°12' W	346.07 m.
FG	S 5°07' W	272.42 m.

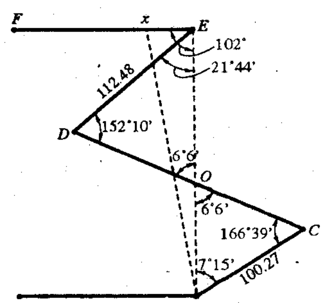
① Find the distance EB.
② Find the distance along EF from point E to the point where the new line cuts EF.
③ Find the bearing of the new boundary line BX.

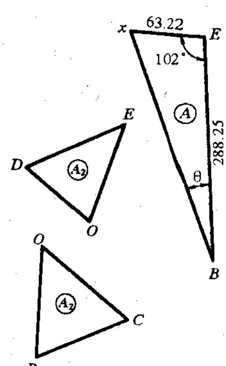
Solution:

① *Distance EB:*
Solve for the line EB:

LINES	LAT	
BC	100.27 Cos 13°10'	= +97.86
CD	91.26 Cos 0°11'	= +91.26
DE	112.48 Cos 27°39'	= +97.59
		+28.71

LINES	DEP	
BC	100.27 Sin 13°10'	= - 22.84
CD	91.26 Sin 0°11'	= - 0.25
DE	112.48 Sin 27°39'	= +52.20
		25.07





STRAIGHTENING OF IRREGULAR BOUNDARIES

$$\tan \text{bearing (EB)} = \frac{29.07}{286.71}$$

Bearing (EB) = S 5°55' W

$$\text{Distance (EB)} = \frac{286.71}{\text{Cos } 5°55'}$$

Distance (EB) = **288.25 m.**

② *Distance along EF from point E to the point where the new line cuts EF:*

From Plane Trigonometry:

$$A = \frac{c^2 \text{ Sin } A \text{ Sin } B}{2 \text{ Sin } C}$$

$$A_1 = \frac{(100.27)^2 \text{ Sin } 7°15' \text{ Sin } 166°39'}{2 \text{ Sin } 6°06'}$$

A_1 = 1378.42 sq.m.

$$A_2 = \frac{(112.48)^2 \text{ Sin } 21°44' \text{ Sin } 152°10'}{2 \text{ Sin } 6°06'}$$

A_2 = 10291.43 sq.m.

$A = A_2 - A_1$

A = 10291.43 - 1378.42

A = 8913.01 sq.m.

$$A = \frac{(EX)\ 288.25 \text{ Sin } 102°}{2}$$

$$EX = \frac{17826.02 \text{ Csc } 102°}{288.25}$$

EX = **63.22 m.**

③ *Bearing of BX:*

Using Cosine Law:

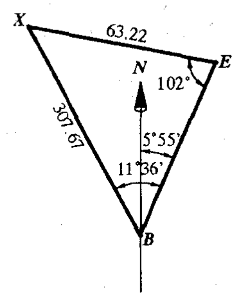
$(BX)^2 = (63.22)^2 + (288.25)^2 - 2(63.22)(288.25) \text{ Cos } 102°$

$(BX)^2 = 3996.77 + 83088.06 + 7577.56$

$(BX)^2 = 94662.39$

$BX = 307.67$

Using Sine Law:



$$\frac{307.67}{\text{Sin } 102°} = \frac{63.22}{\text{Sin } ø}$$

$$\text{Sin } ø = \frac{63.22 \text{ Sin } 78°}{307.67}$$

ø = 11°36'

Bearing of BX = 11°36' - 5°55' = 5°41'

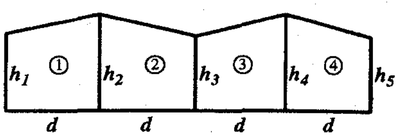
Bearing of BX = ***N 5°41' W***

Areas of Irregular Boundaries

Methods of computing Areas of Irregular Boundaries at Regular Intervals.

a) Trapezoidal Rule

d = common interval
h_1 = first offset
h_n = last offset



$$A_1 = \frac{(h_1 + h_2)\,d}{2}$$

$$A_2 = \frac{(h_2 + h_3)\,d}{2}$$

$$A_3 = \frac{(h_3 + h_4)\,d}{2}$$

$$A_4 = \frac{(h_4 + h_n)\,d}{2}$$

$$A = A_1 + A_2 + A_3 + A_4$$

$$A = \frac{d}{2}\left[(h_1 + h_2) + (h_2 + h_3) + (h_3 + h_4) + (h_4 + h_n)\right]$$

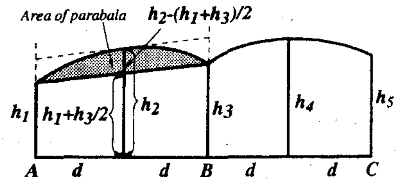
$$A = \frac{d}{2}\left[h_1 + 2h_2 + 2h_3 + 2h_4 + h_n\right]$$

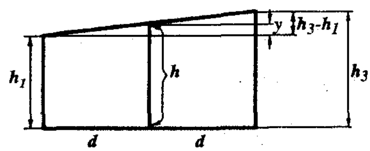
$$A = d\left[\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4\right]$$

$$A = d\left[\frac{h_1 + h_n}{2} + \Sigma h\right]$$

$\Sigma h = h_2 + h_3 + h_4$
Σh = sum of intermediate offsets.

b) Simpson's One Third Rule: *(Applicable only to even intervals or odd offsets)*





$$\frac{y}{d} = \frac{h_3 - h_1}{2d}$$

$$y = \frac{(h_3 - h_1)\,d}{2d}$$

$$y = \frac{h_3 - h_1}{2}$$

$$h = h_1 + y$$

$$h = h_1 + \frac{h_3 - h_1}{2}$$

$$h = \frac{h_3 + h_1}{2}$$

$$A_1 = \frac{(h_1 + h_3)\,2d}{2} + 2d\left[h_2 - \frac{(h_1 + h_3)}{2}\right]\frac{2}{3}$$

$$A_1 = 2d\left[\frac{(h_1 + h_3)}{2}\right] + \frac{(2h_2 - h_1 - h_3)}{2}\,\frac{4d}{3}$$

$$A_1 = d\,(h_1 + h_3) + \frac{d}{3}(4h_2 - 2h_1 - 2h_3)$$

$$A_1 = \frac{d}{3}(h_1 + h_3 + 4h_2)$$

For the next two interval

$$A_2 = \frac{d}{3}\left[h_3 + h_5 + 4h_4\right]$$

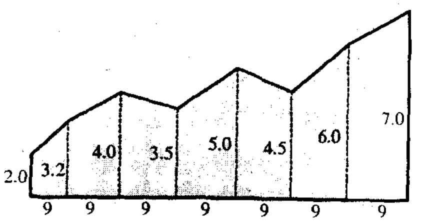
h_5 = last offset
h_1 = first offset
h_2 and h_4 = even offset
h_3 = odd offset

$$A = \frac{d}{3}\left[(h_1 + h_n) + 2\,\Sigma h \text{ odd} + 4\,\Sigma h \text{ even}\right]$$

AREAS OF IRREGULAR BOUNDARIES

Problem 198:

A series of perpendicular offsets were taken from a transit line to a curved boundary line. These offsets were taken 9 meters apart and were taken in the following order: 2 m., 3.2 m., 4 m., 3.5 m., 5 m. 4.5 m., 6 m., 7 m. Determine the area included between the transit line and the curved using:



① Simpson's One Third Rule.
② Trapezoidal Rule.
③ Compute the difference between Simpson's One Third Rule and Trapezoidal Rule.

Solution:

① ***Simpson's One Third Rule:***

$A_1 = \frac{d}{3}[h_1 + h_n + 2\sum h_{odd} + 4\sum h_{even}]$

$d = 9$ m.
$h_1 = 2$ m.
$h_n = 6$ m.
$\sum h_{odd} = 4 + 5$
$\sum h_{odd} = 9$ m.
$\sum h_{even} = 3.2 + 3.5 + 4.5$
$\sum h_{even} = 11.2$ m.

$A_1 = \frac{9}{3}[2 + 6 + 2(9) + 4(11.2)]$
$A_1 = 3(70.8)$
$A_1 = 212.40$ sq.m.
$A_2 = \frac{(6+7)}{2}9$
$A_2 = 58.8$ sq.m.

Total area = 212.40 + 58.5
Area = ***270.90 sq.m.***

② *Trapezoidal Rule:*

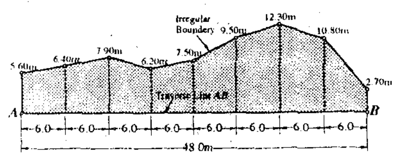
$A = d\left[\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + h_5 + h_6\right]$
$d = 9$ m.
$h_1 = 2$ m.
$h_n = 7$ m.

$A = 9\left[\frac{(2+7)}{2} + 3.2 + 4 + 3.5 + 5 + 4.5 + 6\right]$
$A = 9(30.7)$
$A =$ **276.3 sq.m.**

③ *Difference between Simpson's One Third Rule and Trapezoidal Rule:*
= 276.3 - 270.9
= ***5.4 m²***

Problem 199:

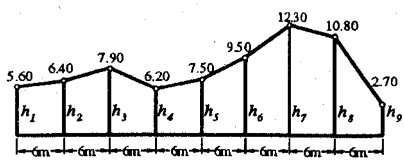
Shown in the accompanying sketch are the measured offsets from a traverse line AB to an irregular boundary and the spacing between the offsets. Determine the area bounded by the traverse line, the irregular boundary and the end offsets using:



① Trapezoidal Rule.
② Simpson's One-Third Rule.
③ Compute the difference between Trapezoidal Rule and Simpson's One-Third Rule.

Solution:

① *Trapezoidal Rule:*



$$A = \frac{d}{2}(h_1 + h_n + 2\sum h_{int})$$

$$A = \frac{6}{2}[5.60 + 2.70 + 2(6.40 + 7.90 + 6.20 + 7.50 + 9.50 + 12.30 + 10.80)]$$

$A = \mathbf{388.50\ m^2}$

② *Simpson's One-Third Rule:*

$$A = \frac{d}{3}(h_1 + h_n + 2\sum h_{odd} + 4\sum h_{even})$$

$$A = \frac{6}{3}[5.60 + 2.70 + 2(7.90 + 7.50 + 12.30) + 4(6.40 + 6.20 + 9.50 + 10.80)]$$

$A = \mathbf{390.60\ m^2}$

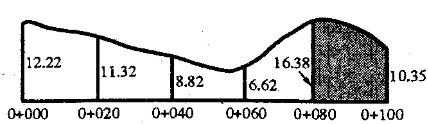
③ *Difference between Trapezoidal Rule and Simpson's One-Third Rule:*

Difference in area = 390.60 - 388.50

Difference in area = $\mathbf{2.1\ m^2}$

Problem 200:

Find the area of the figure shown using:



① Trapezoidal Rule.
② Simpson's One Third Rule.
③ Compute the difference of areas between the two methods.

Solution:

① *Trapezoidal Rule:*

$$A = d\left[\frac{h_2 + h_n + \sum h}{2}\right]$$

$d = 20$

$h_1 = 12.22$

$h_n = 10.35$

$\sum h = 11.32 + 8.82 + 6.52 + 16.38$

$\sum h = 43.04$

$$A = 20\left[\left(\frac{12.22 + 10.35}{2}\right) + 43.04\right]$$

$A = \mathbf{1086.50\ sq.m.}$

② *Simpson's One Third Rule:*
(Treat the last area as trapezoid)

$$A_1 = \frac{d}{3}[(h_1 + h_n) + 2\sum h_{odd} + 4\sum h_{even}]$$

$h_1 = 12.22$

$h_n = 16.38$

$d = 20$

$\sum h_{odd} = 8.82$

$\sum h_{even} = 11.32 + 6.52 = 17.84$

$$A_1 = \frac{20}{3}[12.22 + 16.38 + 2(8.82) + 4(17.84)]$$

$A_1 = 784$ sq.m.

$$A_2 = \frac{(16.38 + 10.35)(20)}{2}$$

$A_2 = 267.30$ sq.m.

$A = A_1 + A_2$

$A = 784 + 267.30$

$A = \mathbf{1051.30\ sq.m.}$

③ *Difference in area:*

Difference in area = 1086.50 - 1051.30

Difference in area = $\mathbf{35.20\ m^2}$

PLANE TABLE

PLANE TABLE

Five Methods of Orienting the Plane Table:

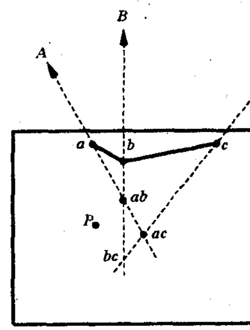
1. By the use of magnetic compass.
2. By backsighting.
3. By solving the three-point problem
   a) Trial Method (Lehman's Method)
   b) Bessel's Method
4. By solving the two-point problem
5. By using the Baldwin Solar Chart

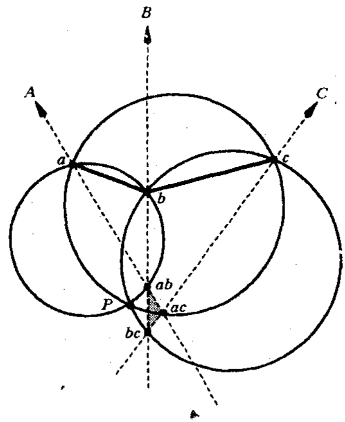
METHODS OF SOLVING THREE POINT PROBLEM WITH PLANE TABLE

A. *TRIAL METHOD* (LEHMAN'S METHOD)

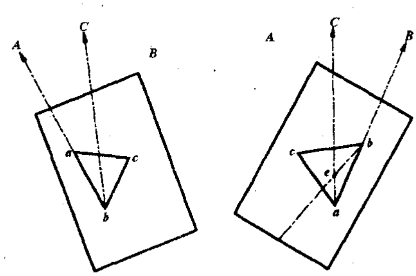
The 3 points A, B and C plotted on the tracing paper with any convenient scale. The tracing paper is then placed on top of the plane table and the plane table is set up over the station whose position is to be determined and is oriented either by compass or by estimation. Resection lines from the three stations A, B and C are drawn through the corresponding plotted points "a", "b" and "c". These lines will not intersect at a common point unless the trial orientation happens to be correct. Usually, a small triangle called the "triangle or error" is formed by the three lines. Let us say, the vertices of this triangle is ab, bc and ac as shown and point P is called the point sought, the position of which is to be determined as follows:

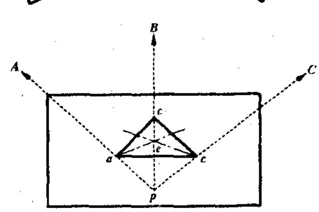
a) Draw a circle passing through points "a", "b" and "ab".
b) Draw a circle passing through points "b", "c" and "bc".
c) Draw another circle passing through points "a", "c" and "ac".
d) The three circles will intersect at point "P", the point sought.





B. *BESSEL'S METHOD*

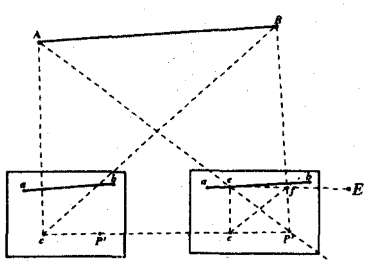


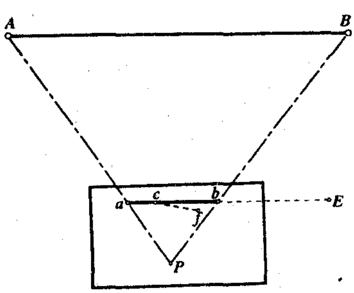


PLANE TABLE

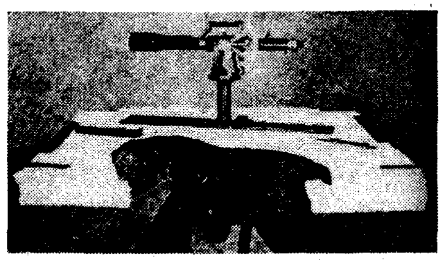
Points a, b and c are the plotted points of A, B and C on the ground. With the straight edge of the alidade placed along line "ab", turn the table until a backsight at A is taken as shown in the figure 1, with point "a" towards point "A" on the ground. Clamp the table, then take a foresight to point C and draw a line passing through "b". Reset the straight edge of the alidade at line "ab", turn the table and backsight at point "B" with "b" towards point "B" as shown on figure 2. Clamp the table and take a foresight towards point C and draw a line passing through point "a". The two lines drawn intersects at point "e". Set the straight edge along the line "ec" and take a backsight at C. Clamped the table. Then draw resection lines through "a" and "b", these two lines will intersect each other at point "P", then point sought.

ORIENTING THE PLANE TABLE BY SOLVING THE TWO POINT PROBLEM

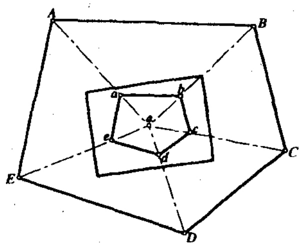




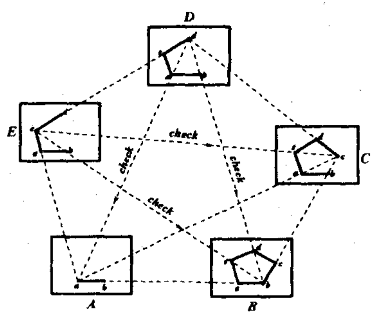
The location of points A and B are plotted on the plane table sheet at "a" and "b" as shown in the figure. These points must be plotted using convenient scale. The plane table is then set up over point C on the ground which points A and B are visible. The board is then oriented by either compass or by estimation. Point "c" corresponding to C on the ground is plotted by estimation on the plane table sheet. Point D is also established on the ground with the distance C to D estimated. With the table at "c", foresights are taken on A, B and D and lines are drawn on the sheet. The corresponding position of D is plotted on the sheet as "d". The table is then transformed to station D and is oriented tentatively by backsighting at C. Foresights are taken on points A and B and lines are drawn intersecting the previous lines drawn before, say at points e' and f'. The line joining e' and f' is parallel to the AB. With the straight edge of the alidade placed along line ef, a point E at some distance from the table is set on the line of sight. The alidade is then moved to the line ab and the board is turned until the same point E is sighted. The plane table is now properly oriented. By section through a and b, the correct position of the plane table is plotted at P.



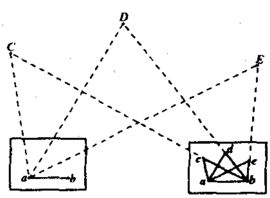
PLANE TABLE



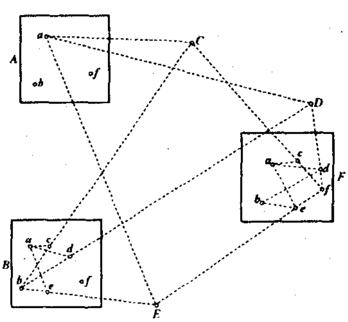
RADIATION WITH PLANE TABLE



TRAVERSE WITH PLANE TABLE



INTERSECTION WITH PLANE TABLE



GRAPHICAL TRIANGULATION

SOURCES OF ERRORS IN PLANE TABLE WORK:

1. Setting over a point.
2. Drawing rays.
3. Instability of the table.

Advantages of Plane Table

1. Relatively few points need be located because the map is drawn as the survey proceeds.
2. Contours and irregular objects can be presented accurately because the terrain is in view as the outlines are plotted.
3. As numerical values of angles are not observed, the consequent errors and mistakes in reading and recording are avoided.
4. As plotting is done in the field, omissions in the field data are avoided.
5. The useful principles of intersection and resection are made convenient.
6. Checks on the location of plotted points are obtained readily.
7. The amount of office work is relatively small.

Disadvantages of Plane Table

1. Plane table is very cumbersome and several accessories must be carried.
2. Considerable time is required for the topographer to gain proficiency.
3. The time required in the field is relatively large.
4. The usefulness of the method is limited to relatively open country.

ADJUSTMENTS OF THE PLANE TABLE ALIDADE:

1. **To make the axis of each Plate Level Parallel to the Plate:**

   Center the bubble of the plate level when manipulating the board. On the plane table sheet, mark a guide line alone one edge of the straight edge. Turn the alidade end for end, and again plane the straight edge along the guide line. If the bubble is off center, bring it back halfway by means of the adjusting screws. Again center the bubble by manipulation the board and repeat the test.

2. **To make the Vertical Cross-hair lie in a Plane Perpendicular to the Horizontal Axis.**

   Sight the vertical cross-hair on a well defined point about 100 m. away and swing the telescope through a small angle (vertical). If the point appears to depart from the vertical cross-hair loosen two adjacent screws of the cross-hair ring, and rotate the ring in the telescope tube until by further trial the point sighted traverses the entire length of the hair.

3. **(For alidade of Tube in Sleeve Type). To make the line of sight coincide with the axis of the Telescope Sleeve.**

   Sight the intersection of the cross-hairs on some well define point. Rotate the telescope in the sleeve through 180°. If the cross-hairs have apparently moved away from the point bring each hair halfway back to its origin position by means of the capstan screws, holding the cross-hair ring. The adjustment is made by manipulating opposite screws, bringing first one cross-hair and then the other to its estimated correct position. Again sight on the point and repeat the test.

4. **(For alidade on Tube-in Sleeve Type). To make the axis of the Striding Level parallel to the axis of the telescope Sleeve, and parallel to the Line of Sight.**

   Place the striding level on the telescope and center the bubble. Remove the level, turn it end for end, and replace it on the telescope tube. If the bubble if off center, bring it back halfway by means of the adjustment screw at one end of the level tube. Again center the bubble and repeat the test.

5. **(For alidade of Fixed tube Type). To make the axis of the telescope level parallel to the Line of Sight.**

   This adjustment is the same as the two-peg adjustment of the dumpy level.

6. **(For Alidade having a Fixed Vertical Vernier). To make the vertical vernier read zero when the Line of Sight is horizontal.**

   With the board level, center the bubble of the telescope level. If the vertical vernier does not read zero, loosen it and move it until it will read zero.

7. **(For Alidade having a movable Vertical Vernier with Control Level). To make the axis of the Vernier Control Level parallel to the axis of the Telescope when the Vernier reads zero.**

   Center the bubble of the telescope level, and move the vernier by means of its tangent screw until it reads zero, if it is off center move it to the center by means of the capstan screws at the end of the control level tube.

8. **(For Alidade having Tangent movement to Vertical Vernier Arm). This type of vernier needs no adjustment.**

TOPOGRAPHIC SURVEY

Topographic Survey

Topographic Survey - is a survey made in order to secure important data from which a topographic map could be made.

Scheme of work of a Topographic Survey:

1. Establishment of a horizontal control by measuring angular and linear measurements of a center point.
2. Establishment of the vertical control by determining the elevation of control points by leveling or using plane table.
3. Determining the elevations and location of some important features as many deem necessary for the preparation of the topographic map.
4. Computations of elevations, distances and angles as obtained from the previous field work undertaken.
5. Preparation of the topographic map, which is actually a representation of the terrestrial relief.

Relief - configuration of earth's surface.

Methods of representing relief:

1. Relief models
2. Shading
3. Hachure lines
4. Form lines
5. Contour lines

Contour - an imaginary line of constant elevation on the ground surface.

Contour line - a line on the map joining points of the same elevation.

Contour interval - on a given map, successive contour lines represents elevations which differs by a fixed vertical distance called contour interval.

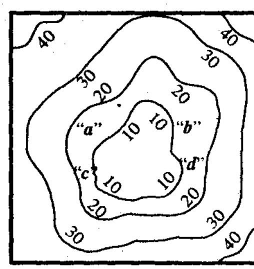
Hachures - artificial shade lines drawn in the direct of steepest slope for the purpose of representing a relief.

Saddle - a dip at the junction of two ridges.

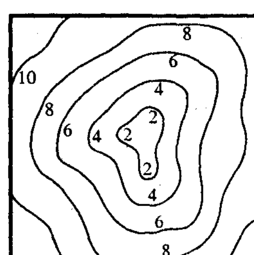
Thalwegs - are the lines where the two sides of a valley meet.

CHARACTERISTICS OR PROPERTIES OF CONTOURS

1. All points on the same contour have the same elevation.

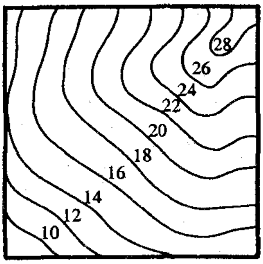


2. Every contour closes upon itself either within or outside the limits of the map.

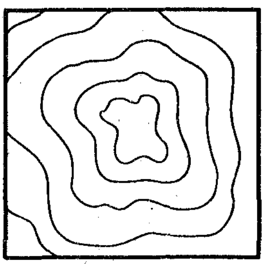


TOPOGRAPHIC SURVEY

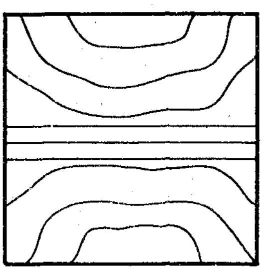
3. On uniform slopes, the contour lines are spaced uniformly.



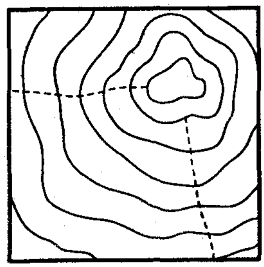
4. A single contour can not lie between two contour lines of higher or lower elevation.



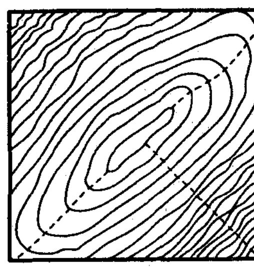
5. Along plane surfaces (such as those of railroad cuts and fills) the contour lines are straight and parallel to one another.



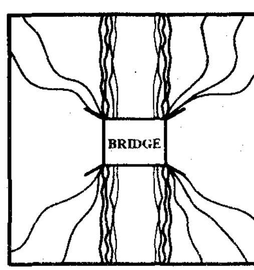
6. As contour lines represents level lines, they are perpendicular to the line of steepest slope. They are perpendicular to ridge and valley lines where they cross such lines.



7. On steep slopes contours are closely spaced and on gentle slopes, contours are spaced far apart.

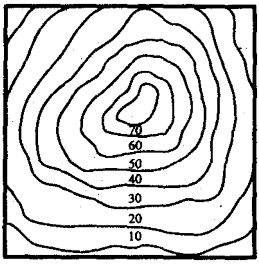


8. As contour lines represent contours of different elevation on the ground, they not merge or cross one another on the map, except in cases where there is an overhanging cliff or cave, or bridge abuttments.

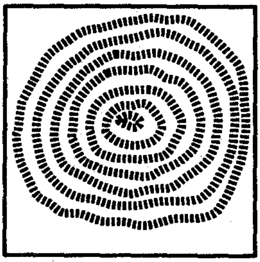


TOPOGRAPHIC SURVEY

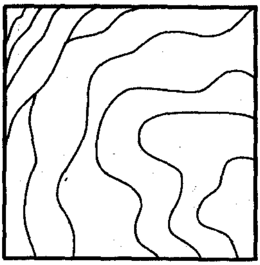
9. A closed contour indicate either a summit or depression. A hachured, closed contour line indicates a depression.



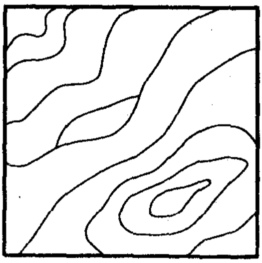
Hachure Lines



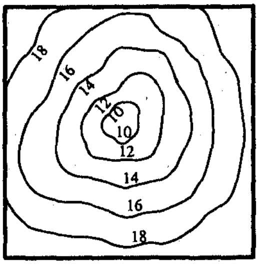
10. A contour never splits.



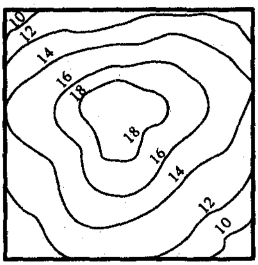
11. No two contours can run into one.



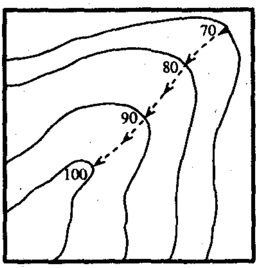
FIVE MOST COMMON TYPES OF GROUND FORMATION



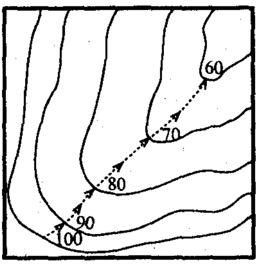
1. Depression



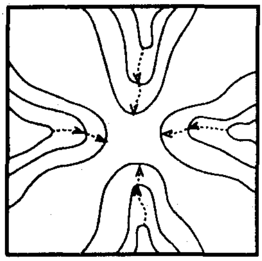
2. Summit of Hill



3. End of a Ridge



4. End of a Valley



5. Saddle

Four Systems of Ground Points for locating Contours:

1. *Control point system:*
   The ground points form an irregular system along ridge and valley lines and at other critical features of the terrain. The ground points are located in plan by radiation or intersection with transit or plane table and their elevations determined by trigonometric leveling or sometimes by direct leveling.

2. *Cross Profile System:*
   The ground points are on relatively short lines transverse to the main traverse. The distances from traverse to ground points are measured with the tape and the elevation of the ground points are determined by direct leveling.

3. *Trace Contour System:*
   in this system, the contours are traced out on the ground. The various contour points occupied by the rod are located by radiation using a transit or a plane table.

4. *Checker Board System:*
   This is used in areas whose topography is smooth. The tract is then divided into squared or rectangles with stakes set at the corners. The elevation of the ground is determined at these corners and at intermediate critical points where changes in slope occurs, usually by direct leveling.

Uses of Topographic Maps:

1. Cross-sections and profiles from contour maps.
2. Earthwork for grading areas.
3. Earthwork for roadway.
4. Reservoir areas and volume.
5. Route location.

Three General Methods Employed in Undertaking a Topographic Survey:

1. Transit and Level Method
2. Stadia Method
3. Plane Table Method

Problem 201:

A level rectangular field of 1800 sq.m. has for its sides in the ratio of 2:1. Three poles of equal heights are located at three consecutive corners. To measure the heights of the poles, a Civil Engineer set a transit (H.I. = 1.5 m.) at a point within the lot and took the angles of elevation of the top of the poles. The angles of elevation of the top of the three poles taken are 25°, 25° and 30°.

① Compute the height of each pole.

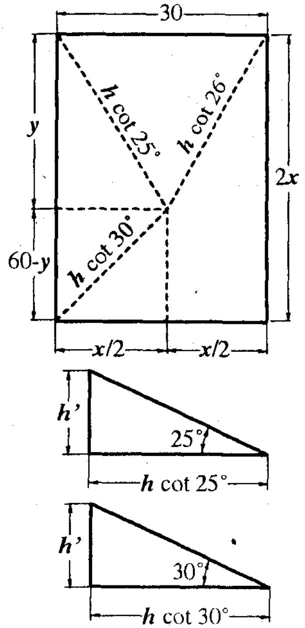
② Compute the distance of the transit from the nearest corner.

③ Compute the distance of the transit from the farthest corner.

TOPOGRAPHIC SURVEY

Solution:

① *Height of each pole:*



$3x^2 = 1800$

$x^2 = 900$

$x = 30$

① $(h \cot 25^\circ)^2 = y^2 + (15)^2$

$4.6h^2 = y^2 + 225$

② $(h \cot 30^\circ) = (60 - y)^2 + (15)^2$

$1.73h^2 = 3600 - 120y + y^2 + 225$

① $h^2 = 0.217y^2 + 48.91$

② $h^2 = 0.57y^2 - 69.36y + 2210.98$

$0.217y^2 + 48.91 = 0.57y^2 - 69.36y + 2210.98$

$0.353y^2 - 69.26y + 2162.07 = 0$

$y^2 - 196.5 + 6124.8 = 0$

$$y = \frac{196.5 \sqrt{(196.5)^2 - 4(6124.8)}}{2}$$

$$y = \frac{196.5 \pm 118.8}{2}$$

$y = 38.85$ m.

$h^2 = 0.217y^2 + 48.91$

$h^2 = 0.217(38.85)^2 + 48.91$

$h = 19.40$ m.

Height of each pole = 19.41 + 1.50

Height of each pole = ***20.90 m.***

② *Distance of the transit from the nearest corner:*

Distance of transit from nearest corner

= 19.40 cot 30°

= ***33.60 m.***

③ *Distance of the transit from the farthest corner:*

Distance of transit from farthest corner

= 19.40 cot 25°

= ***41.60 m.***

ROUTE SURVEYING

Route Surveying

Route Surveys - are surveys made for the purpose of locating any buildings, highways, canals, power transmission lines, pipe lines, and other utilities which are constructed for purposes of transportation or communications.

AN OUTLINE OF TYPICAL SEQUENCE OF OPERATIONS IN ROUTE SURVEYING:

1. ***Reconnaissance:***

   a. General routes are selected and horizontal and vertical controls are established.

   b. Reconnaisance report is made accompanying a reconnaissance map.

2. ***Preliminary survey:***

   a. There are survey parties that execute this phase of work.

      1. *Transit party* - runs the traverse.
      2. *Level party* - sets bench marks and determines the profile.
      3. *Topographic party* - runs the cross-sectioning work.

   b. A preliminary map is prepared for determination of possible cost of the project.

3. ***Location; survey***

   a. Five survey teams go out in the field.

      1. Transit party - stakes the location of circular curves with proper stationing.
      2. Level party - checks the selected bench mark and executes the profile work.
      3. Cross-section party - slope stakes are set on the ground.
      4. Land line party - property lines and other important details are indicated on the plan.
      5. Special team - takes care of the special surveys for structure.

   b. All survey works are consolidated with the following prepared:

      1. Location map
      2. Location profile
      3. Cross-sections
      4. Earthwork estimates
      5. Right of way maps
      6. Structure maps and plans

4. ***Construction survey:***

   a. Slope stakes for construction works are staked, spiral are laid and lines and grades for tract or pavement are defined.

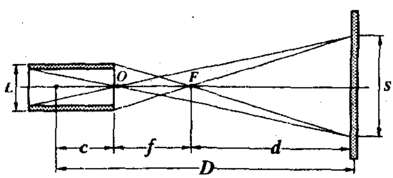
   b. Final plans are prepared, profile sections, as revised during construction.

STADIA SURVEYING

Stadia Surveying

Derivation of Stadia Formulas:

a) ***Horizontal Sights:***



F = principal focus
f = focal length
o = optical center
i = distance between stadia hairs
c = distance from optical center to center of instrument

By ratio and proportion:

$$\frac{f}{i} = \frac{d}{s}$$

$$d = \frac{f}{i} S$$

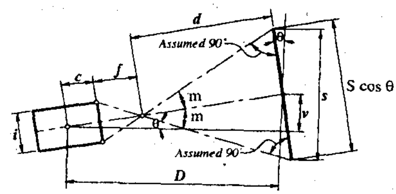
$$D = d + f + c$$

$$D = \frac{f}{i} S + (f + c)$$

$\frac{f}{i}$ = stadia interval factor
f + c = stadia constant
S = stadia interval or intercept

b) ***Inclined Sights:***

tan m = 0.006
m = 17' (too small and is negligible)



By ratio and proportion:

$$\frac{f}{i} = \frac{d}{S \cos ø}$$

$$d = \frac{f}{i} S \cos ø$$

$$H = (f + c + d) \cos ø$$

$$H = \frac{f}{i} S \cos^2 ø + (f + c) \cos ø$$

$$V = (d + f + c) \sin ø$$

$$V = \left[\frac{f}{i} S \cos ø + (f + c)\right] \sin ø$$

$$V = \frac{f}{i} S \frac{\sin 2ø}{2} + (f + c) \sin ø$$

Errors in Stadia Surveying

1. Stadia interval factor not that assumed.
2. Rod not of standard length
3. Incorrect stadia interval
4. Rod not held plumb
5. Unequal refraction

REQUISTIES FOR A GOOD TRANSIT USE FOR STADIA SURVEYING

1. The telescope be of excellent quality, with good illumination.
2. The magnifying power should be about 25 to 30.
3. The stadia hairs should be fixed and should be set accurately so that $\frac{f}{i} = 100$.
4. The transit should have a good compass needle.
5. The transit should have a complete vertical circle.

STADIA SURVEYING

Problem 202:

A survey party proceeded to do their stadia survey work as follows: the transit was set up at a point A and with the line of sight horizontal, took rod readings with the rod at points B and C, which were then measured to have taped distances from A to 200 m., and 60 m. respectively.

	Rod B	Rod C
Stadia Inteval	2.001 m.	0.600 m.

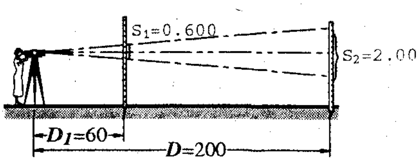
The distance from the center of the instrument to the principal focus was recorded as 0.30 m. Then they went on to survey other points, with some of the data recorded as follows: with the transit at point D, two points E and F were sighted.

	Rod E	Rod F
Stadia Inteval	2.120 m.	3.560 m.
Vertical Angle	+ 4°22'	- 3°17'

① Compute the stadia interval factor.
② Compute the horizontal distances DE and DF.
③ Compute the differences in elevation between points D and E and points D and F.

Solution:

① *Stadia interval factor:*



$$D=\frac{fS}{i}+(f+c)$$

$K=\frac{f}{i}=$ *stadia interval factor*

$R=(f+c)=$ *stadia constant*

$(f+c)=0.30$

$$D_1=KS_1+R$$
$$D_2=KS_2+R$$
$$D_2-D_1=(KS_2+R)-(KS_1+R)$$
$$K(S_2-S_1)=D_2-D_1$$
$$K=\frac{D_2-D_1}{S_2-S_1}$$
$$K=\frac{200-60}{2.001-0.600}$$

$K=$ ***99.93*** *(stadia interval factor)*

② *Horizontal distances DE and DF:*
Assume elevation of D = 100 m.

$$H=\frac{f}{i}S\cos^2\theta+(f+c)\cos\theta$$
$$H=99.93(2.12)\cos^2 4°22'+(0.30)\cos 4°22'$$

$H=210.92$ m.
Horizontal distance DE = ***210.92 m.***

$$H=\frac{f}{i}S\cos^2\theta+(f+c)\cos\theta$$
$$H=99.93(3.56)\cos^2 3°17'+(0.30)\cos 3°17'$$

$H=354.88$ m.
Horizontal distance DF = ***354.88 m.***

③ *Differences in elevation between points D and E and points D and F:*

$$V=\frac{f}{i}S\frac{\sin 2\theta}{2}+(f+c)\sin\theta$$
$$V=99.93(2.12)\frac{\sin 8°44'}{2}+(0.3)\sin 4°22'$$

$V=16.11$ m.
Elev. of E = 100 + *HI* + 16.11 - *HI*
Elev. of E = 116.11
Difference in elevation between D and E = ***16.11 m.***

$$V=\frac{f}{i}S\frac{\sin 2\theta}{2}+(f+c)\sin\theta$$
$$V=99.93(3.56)\frac{\sin 6°34'}{2}+(0.30)\sin 3°17'$$

$V=20.36$ m.
Elevation of F = 100 + *HI* - 20.36 - *HI*
Elevation of F = 79.64 m.
Difference in elevation between D and F = ***20.36 m.***

STADIA SURVEYING

Problem 203:

① A transit with a stadia constant equal to 0.30 is used to determine the horizontal distance between points B and C, with a stadia intercept reading of 1.85 m. the distance BC is equal to 182.87 m. Compute the stadia interval factor of the instrument.

② Using the same instrument, it was used to determine the difference in elevation between B and D having a stadia intercept reading of 2.42 m. at D at a vertical angle of + 6°30'. Compute the difference in elevation of B and D.

③ Compute also the horizontal distance between B and D.

Solution:

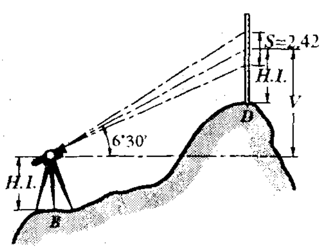
① *Stadia interval factor:*

$$D = \frac{f}{i} S + (f + c)$$

$$182.87 = \frac{f}{i}(1.85) + 0.30$$

$$\frac{f}{i} = \mathbf{98.69}$$

② *Diff. in elevation between B and D:*



$$V = \frac{f}{i} S \frac{\sin 2\theta}{2} + (f + c) \sin \theta$$

$$V = 98.69(2.42)\frac{\sin 13^\circ}{2} + 0.30 \sin 6.5^\circ$$

$$V = \mathbf{26.90\ m.}$$

③ *Horizontal distance between B and D:*

$$H = \frac{f}{i} S \cos^2 \theta + (f + c) \cos \theta$$

$$H = 98.69(2.42) \cos^2 6.5^\circ + 0.30 \cos 6.5^\circ$$

$$H = \mathbf{236.07\ m.}$$

Problem 204:

A Civil Engineer proceeded to do the stadia survey work to determine the topography of a certain area. The transit was set up at a point A, with the line of sight horizontal, took rod readings from the rods placed at B and C which is 200 m. and 60 m. from A respectively.

	Stadia Intercept
Rod at B	2.001 m.
Rod at C	0.600 m.

① Compute the stadia interval factor.

② Using the same instrument this was used for determining the elevation of point D with a stadia intercept of 2.12 m. and a vertical angle of +4°22'. If the elevation of the point where the instrument was set up is 100 m., compute the elevation of point D. Stadia constant is 0.30 m.

③ Compute the horizontal distance from the point where the instrument was set up to point D.

Solution:

① *Stadia interval factor:*

$$D = \frac{f}{i} S + (f + c)$$

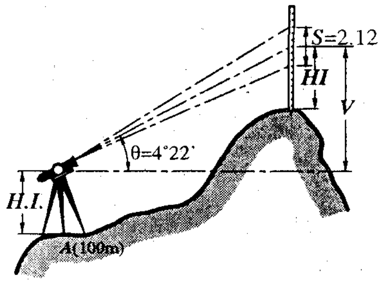
$$200 = \frac{f}{i}(2.001) + (f + c)$$

$$60 = \frac{f}{i}(0.600) + f + c$$

$$140 = (1.401)\frac{f}{i}$$

$\frac{f}{i} = \mathbf{99.93}$ *(stadia interval factor)*

② *Elev. of D:*



$$V = \frac{f}{i} S \frac{\sin 2ø}{2} + (f + c) \sin ø$$

$$V = 99.93(2.12) \frac{\sin 8°44'}{2} + (0.30) \sin 4°22'$$

$V = 16.11$

Elev. $D = 100 + HI + 16.11 - HI$

Elev. $D =$ ***116.11***

③ *Horizontal distance:*

$$H = \frac{f}{i} S \cos^2 ø + (f + c) \cos ø$$

$$H = 99.93(2.12) \cos^2 4°22' + (0.30) \cos 4°22'$$

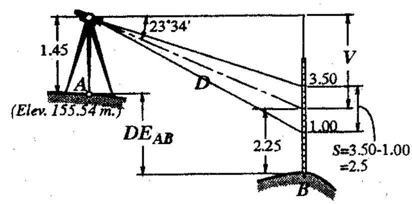
$H =$ ***210.92 m.***

Problem 205:

The upper and lower stadia hair readings on a stadia rod held at station B were observed as 3.50 and 1.00 m., respectively, with the use of a transit with an interval focusing telescope and having a stadia interval factor of 99.5. The height of the instrument above station A is 1.45 m. and the rod reading is taken at 2.25 m. If the vertical angle observed is - 23°34', determine the following:

① Inclined stadia distance.
② Difference in elevation between the two stations.
③ The elevation of station B, if the elevation of station A is 155.54 m. above mean sea level.

Solution:



$$\frac{f}{i} = 99.5$$

$f + c = 0$ *(interior focusing)*

① *Inclined stadia distance:*

$$D = \frac{f}{i} S \cos ø + (f + c)$$

$$D = 99.5(2.50) \cos 23°34' + 0$$

$D =$ ***228 m.***

② *Difference in elevation between the two stations:*

$$V = \frac{f}{i} S \frac{\sin 2ø}{2} + (f + c) \sin ø$$

$$V = 99.5(2.50) \frac{1}{2} \sin (2 \times 23°34') + 0$$

$V = 91.16$ m.

$DE_{AB} = 2.25 + 91.16 - 1.45$

$DE_{AB} =$ ***91.96 m.***

③ *Elevation of station B:*

Elev. at $B = 155.54 - 91.96$

Elev. at $B =$ ***63.58 m.***

Problem 206:

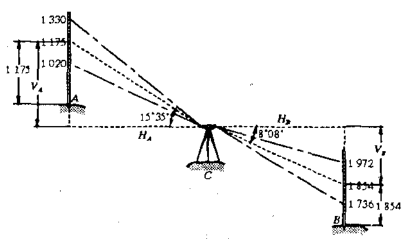
A transit with a stadia interval factor of 100.8 was set at C on the line between points A and B, and the following stadia readings were observed.

Position of Rod	Vertical Angle	HAIR READINGS		
		Upper	Middle	Lower
Rod at A	+15°35'	1.330	1.175	1.020 m.
Rod at B	- 8°08'	1.972	1.854	1.736 m.

If the stadia constant is 0.381 m., determine the following:

① Length of line AB.
② Difference in elevation between points A and B.
③ Find the horizontal distance from the transit to the rod held at B.

Solution:



$S_A = 1.330 - 1.020$
$S_A = 0.31$
$S_B = 1.972 - 1.736$
$S_B = 0.236$

① *Length of line AB:*

$$H = \frac{f}{i} S \cos^2 ø + (f + c) \cos ø$$

$H_A = 100.8\ (1.330 - 1.020) \cos^2 15°35' + 0.381 \cos 15°35'$
$H_A = 29.36$ m.
$H_B = 100.8\ (0.236) \cos^2 8°08' + 0.381 \cos 8°08'$
$H_B = 23.69$ m.
$H_{AB} = 29.36 + 23.69$
$H_{AB} = \mathbf{53.05}$

② *Difference in elevation between points A and B:*

$$V = \frac{f}{i} S \frac{1}{2} \sin 2ø + (f + c) \sin ø$$

$V_A = 100.8\ (0.31) \frac{1}{2} \sin (2 \times 15°35') + 0.381 \sin 15°35'$
$V_A = 8.19$ m.
$V_B = 100.8\ (0.236) \frac{1}{2} \sin (2 \times 8°08') + 0.381 \text{ Sin } 8°08'$
$V_B = 3.39$ m.

Diff. in elev. between A and B
$= 1.854 + 3.39 + 8.19 - 1.175$
$= \mathbf{12.259\ m.}$

③ *Horizontal distance from the transit to the rod held at B:*

$H_B = 100.8\ (0.236) \cos^2 8°08' + 0.381 \cos 8°08'$
$H_B = \mathbf{23.69\ m.}$

Problem 207:

A survey party proceeded to do their stadia survey work as follows. The transit was set up at A and with the line of sight horizontal, took rod readings at points B and C which is 300 m. and 80 m. respectively.

With rod at B the stadia interval was recorded to be 3.001 m. and with the rod at C the stadia interval was recorded to be 0.800 m. The distance from the instrument to the principal focus was recorded to be 0.30 m. Then they went to survey other points with some of the data recorded as follows with the transit at point D, the two points E and F were sighted.

Rod at E	Stadia interval	=	2.25 m.
	Vertical angle	=	+4°30'
Rod at F	Stadia interval	=	3.56 m.
	Vertical angle	=	- 3°30'

① Compute the stadia interval factor.
② Compute the horizontal distance DE.
③ Compute the difference in elevation between E and F assuming elevation of D = 350.42 m. above sea level.

Solution:

① *Stadia interval factor:*

$$S = \frac{f}{i} S + (f + c)$$

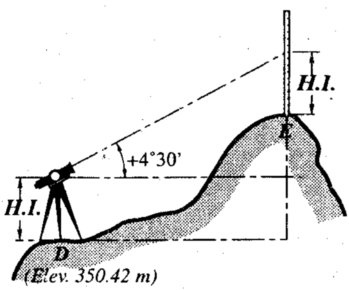
$$300 = \frac{f}{i}(3.001) + (f + c)$$

$$80 = \frac{f}{i}(0.80) + (f + c)$$

$$220 = \frac{f}{i}(2.201)$$

$\frac{f}{i}$ = **99.95** *(stadia interval factor)*

② *Horizontal Distance DE:*

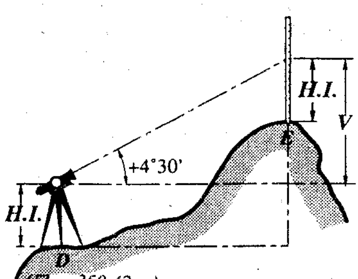


$$H = \frac{f}{i} S \cos^2 ø + (f + c) \cos ø$$

$$H = 99.95(2.25) \cos^2 4°30' + (0.30) \cos 4°30'$$

H = **223.80 m.**

③ *Diff. in elevation between E and F:*



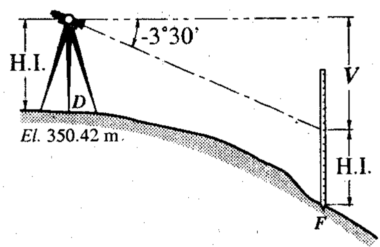
$$V = \frac{f}{i} S \frac{\sin 2ø}{2} + (f + c) \sin ø$$

$$V = 99.95(2.25) \frac{\sin 9°}{2} + 0.30 \sin 4°30'$$

V = 17.61 m.

Elev. E = 350.42 + *HI* + 17.61 - *HI*

Elev. E = 368.03 m.



$$V = \frac{f}{i} S \frac{\sin 2ø}{2} + (f + c) \sin ø$$

$$V = 99.95(3.56) \frac{\sin 7°}{2} + (0.30) \sin 3°30'$$

V = 21.70 m.

Elev. F = 350.42 + *HI* - *V* - *HI*

Elev. F = 350.42 - 21.70

Elev. F = 328.72 m.

Diff. in elev. between E and F

= 368.03 - 328.72

= ***39.31 m.***

HYDROGRAPHIC SURVEYING

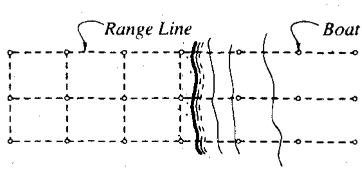
Hydrographic Surveying

Purpose of Hydrographic Surveying:

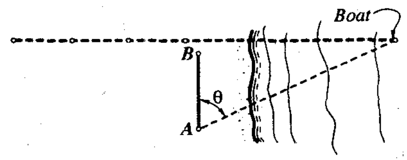
1. To determine shore lines of harbors, lakes and rivers from which to draw an outline map of the body of water.
2. To determine by means of soundings, the submerged relief of ocean bottoms.
3. To observe tidal conditions for the established of standard datum.
4. To obtain data, in case of rivers, related to the studies of flood control, power development, water supply and storage.
5. To locate channel depths and obstruction to navigators.
6. To determine quantities of underwater excavations.
7. To measure areas subject to scour or silting.
8. To indicate preferred locations of certain engineering works by stream discharge measurement.

Methods of locating soundings:

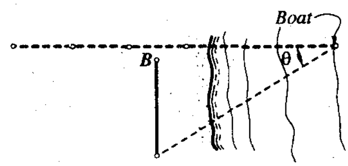
1. By means of a boat towed at uniform speed along a known range line at equal intervals of time.



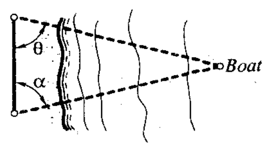
2. By means of range line and an angle from the shore.



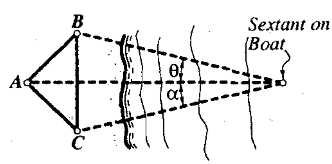
3. By means of range line and an angle from the boat.



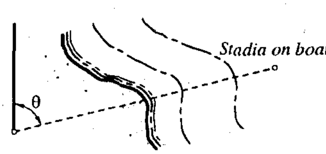
4. Two angles from the shore.



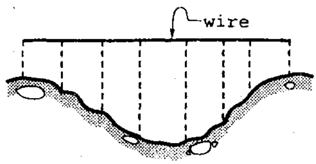
5. Two angles taken simultaneously at the boat by using a sextant, and three stations on the shore.



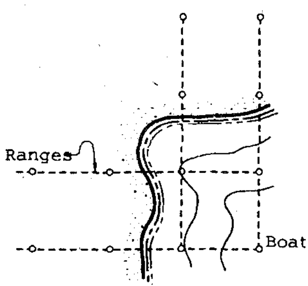
6. By transit and stadia.



7. By intersection of fixed ranges.



8. By a wire stretched along a river at known distances.



Hydrographic maps - is similar to the ordinary topographic map but it has its own particular symbols. The amount and kind of informations shown on the hydrographic map varies with the use of the map.

A hydrographic map contains the following informations:

1. Data used for elevation.
2. High and low water lines.
3. Soundings usually in feet and tenths, with a decimal point occupying the exact plotted location of the point.
4. Lines of equal depths, interpolated from soundings. On navigation charts the interval of line of equal depth is equal to one fanthom or six feet.
5. Conventional signs for land features as in topographic maps.
6. Light houses, navigation lights, bouys, etc., either shown by conventional signs or letters on the map.

Methods of Plotting Soundings:

1. By using the Two Polar Contractor
2. By using Two Tangent Protractors
3. By the Tracing Cloth Method
4. By using the Three Arm Protractor
5. By the use of Plotting Charts

Methods of Measuring Velocity in a Vertical Line:

1. ***Vertical-velocity-curve method:***
   Measurements of horizontal velocity are made at 0.5 beneath the surface and at each tenth of the depth from the surface to as near the bed of stream as the meter will operate. If the stream is relatively shallow, *measurements are taken at each one fifth* of the depth. These measured velocities are plotted as abscissas and the respective depths as ordinates. A smooth curve drawn through the plotted points defines the velocity at each point in the vertical. The are under this curve is equal to the product of the mean velocity and the total depth in that vertical line. This area may be computed by using a planimeter or by Simpson's One Third Rule. The vertical velocity curve method gives us the most precise method of determining mean velocity but requires only too much time.

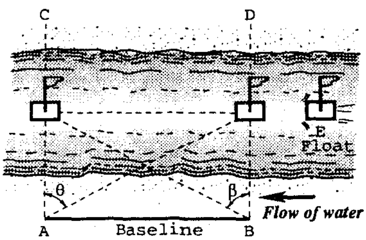
2. ***Two-tenths and Eight-tenths Method:***
   The current meter is lowered downward at 0.2 and 0.8 of the total depth where observations are made. The mean of this two velocities is taken as the mean horizontal velocity in that vertical.

3. ***Six-tenths Method:***
   Only one observation is made at a distance below the water surface equal to 0.6 the total depth of the stream. The velocity obtained at that particular depth is considered to be the mean velocity of vertical.

HYDROGRAPHIC SURVEYING

4. ***Integration Method:***
   The current meter is lowered at a uniform rate down to the bed of the stream and is raised also at the same rate up to the surface. The total time and the number of revolution during this interval constitute a measurement.
   This method is based upon the theory that all horizontal velocities in the vertical have acted equally upon the meter wheel thereby giving the average as the mean of all the velocity reading.

5. ***Subsurface Method:***
   In this particular method, the current meter is held at just sufficient depth below the surface usually 150 mm to 200 mm to avoid surface disturbance. The mean horizontal velocity is obtained by multiplying the sub-surface velocity by a coefficient. This coefficient varies with the depth and velocity of stream. This coefficient varies from 0.85 to 0.95.



Float Method of Measuring Stream Velocity

From the figure shown, a base line AB is well selected and is established near the bank of a river where no obstruction will interfere the line of sight during the observation period. Points C and D are established on the opposite side of the river such that the sections AC and BD are perpendicular to the line AB, hence they are parallel to each other. One transit is set up at A and the other at B. The transitman at B with vernier at zero, follows the float where it is being released at point E, at a distance of 15 m. above section BD. As the float approaches section BD, the transitman at A keeps the line of sight pointing at the float until the transitman at B shouts "shot" a the float passes section AB.

The transitman at A then clamps the lower plate, turns the line of sight to the signal station C and reads the angle ø. The transitman at B also follows the float, until the transitman at A gives the "get ready" signal and by means of the upper tangent screw angle B is measured the moment the float passes the section AC. The time that the float. passes the section BD and AC is also recorded.

The base line AB is then measured accurately and the position C and D is then plotted. The path of the float is either scaled or computed using trigonometric principles. The distance divided by the time gives the mean velocity of the float.

Three Distinct Methods of Determining the Flow Channels or in Open Channels or Stream:

1. ***Velocity-Area Method:***
   The velocities at any vertical line is observed by using a current meter based on the five different method of velocity measurement using current meters. The area of a certain section is obtained by sounding, or by stretching a wire across the stream and marking the points where observations were made referred from an initial zero point. The depths at this particular points are also measured. The area of the section could then be computed by dividing the section into triangles and trapezoids. The product of the area and the mean velocity gives us the discharge of flow of a certain section. The sum of all the discharges at all sections gives us the total discharge or flow.

2. ***Slope Method:***

   The slope method involves a determination of the following:

   a) Slope of water surface.
   b) Mean area of channel cross-section
   c) Mean hydraulic radius
   d) Character of stream bed and the proper selection of roughness coefficient

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Mean Velocity is computed by applying the Chezy Formula:

$$V = C\sqrt{RS}$$

where
- V = mean velocity
- C = coefficient of roughness of stream bed
- R = hydraulic radius
- $R = \frac{A}{P}$
- A = cross-sectional area of stream
- P = wetted perimeter of stream

Computing values of C by Kutter's Formula:

a) ***English:***

$$C = \frac{41.65 + \frac{0.00281}{s} + \frac{1.811}{n}}{1 + \frac{n}{\sqrt{R}}\left(41.65 + \frac{0.00281}{s}\right)}$$

- C = coefficient of roughness of stream bed
- n = retardation factor of the stream bed
- R = hydraulic radius
- s = slope of water surface

b) ***Metric:***

$$C = \frac{23 + \frac{0.00155}{s} + \frac{1}{n}}{1 + \frac{n}{\sqrt{R}}\left(23 + \frac{0.00155}{s}\right)}$$

Computing values of C by Manning's Formula:

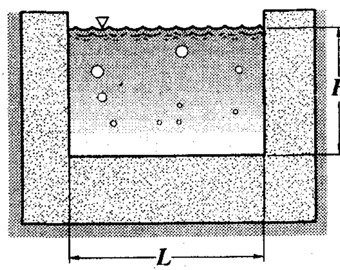
$$C = \frac{R^{1/6}}{n}$$

Discharge = Area x Velocity

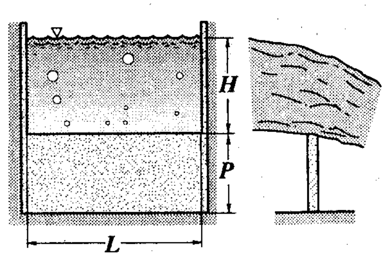
3. ***Weir Method:***

A weir method is an obstruction place in a channel, over which water must flow. Discharged of a stream using this method involves the necessary information.

a) Depth of water flowing over the crest of weir, H.
b) Length of crest, L for rectangular or trapezoidal weir.
c) Angle of side slopes if weir is triangular or trapezoidal.
d) Whether flat or sharp crested.
e) Height of crest above bottom of approach channel, P.
f) Width and depth of approach channel
g) Velocity of approach
h) Nature of end contractions

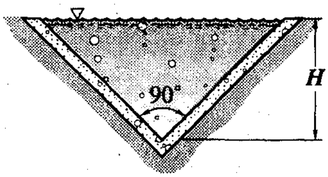


End Contracted Weir
$Q = 1.84\,(L - 0.2H)\,H^{3/2}$

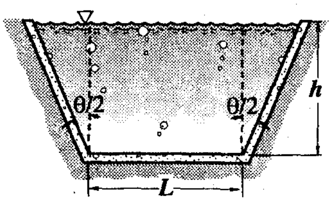


Suppressed Weir
$Q = CLH^{3/2}$

HYDROGRAPHIC SURVEYING



Triangular Weir
$Q = 1.4\,H^{2.5}$



Cipolletti Weir
$Q = 1.86\,LH^{3/2}$
when $\tan\frac{\varnothing}{2} = \frac{1}{4}$

$Q = 1.84\,LH^{3/2}$ (Francis Formula Neglecting Velocity of Approach)

$Q = 1.84\,L\left[(H + h_v)^{3/2} - h_v^{3/2}\right]$ (Considering Velocity of Approach)

Flow Measurements

Discharge measurements are made for the following purposes:

1. To determine a particular flow without regard to stage of stream.
2. To determine flows for several definite gage readings throughout the range of stage, in order to plot a rating curve for the station. From this curve the discharge for any subsequent period is computed from the curve of water stage developed in the recording gage.
3. To obtain a formula or coefficient of dams, or rating flumes.

Three common types of floats used in measuring stream velocity:

1. ***Surface floats*** - it is designed to measure surface velocities and should be made light in weight and of such a shape as to offer less resistance to floating debris, wind, eddy currents and other extraneous forces. The use of surface float is the quickest and the most economical method of measuring stream velocity.

2. ***Sub-surface floats*** - this is sometimes called a double floats. It consists of a small surface float from which is suspended a second float slightly heavier than water. The submerged float is a hollow cylinder, thus offering the same resistance in all directions and the minimum vertical resistance to rising currents.

3. ***Rod float*** - the rod float is usually a cylindrical tube of thin, copper or brass 25 mm to 50 mm in diameter. The tube is sealed at the bottom and in weighted with shot until it will float in an upright position with 50 mm to 150 mm, projecting above the surface of the water.

Instruments used for measuring difference in level of water:

1. Hook gauge
2. Staff gauge
3. Wire-Weight gauge
4. Float gauges
5. Automatic gauges
6. Piezometers
7. Plumb bob

Instruments used for measuring the velocity of flow:

1. ***Floats***
   a) surface float
   b) sub-surface float
   c) rod float

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2. ***Current meters***
   a) Those which the revolving element is cup-shaped, or of the anemometer type and acts under differential pressure.

   Types of current meter:

   1. Price meter
   2. Ellis meter
   3. Haskell meter
   4. Fteley meter
   5. Ott meter

Special Hydrographic Surveys

A. ***Measurement of dredged materials:***

1 ***Measurement in place:***

Soundings of fixed section are taken both before and after dredging and the change in the cross-sectional area is obtained by calculation or by using a planimeter. The volume of the material removed is computed by using the borrow pit method or by the end-area method.

2. ***Scow measurement:***

Each scow is numbered and the capacity of each is carefully determined. When the scow is filled to the capacity the inspector records the full measurements. Materials is scow is sometimes measured by the amount displaced in loading.

B. ***Measurements of Surface Current:***

Certain engineering problems require important information about the direction and velocity of currents at all tidal stages. This is done by locating the path and computing the velocity of floats from points whose locations are known and can be determined. Floats should be designed to give minimum wave resistance and to extend underwater to a sufficient depth to measure the current in question. The direction of the current may be determined by sextant angles from the boat between known signals and the floats.

C. ***Wire drag or Sweep:***

This method is used in harbor or a bay where corral reefs and pinnacle rocks are likely to occur. This consists of a wire of any length up to 120 m. which may be set at any desired depth. Depths are maintained by means of bouys placed at the wires and whose length can be adjusted. The drag is pulled through the water by means of a power launches, steering diverging forces to keep the drag taut. When an obstruction is met, the bouys are shown with the position of two straight lines intersecting at the obstruction. These intersection is located by sextant observations to reference points on the shore. Soundings are taken for the minimum depth.

D. ***Determination of stream slope:***

To determine surface slope, a gauge is installed on each side of the stream at the end of the section. The zero's of the gauge are connected to permanent bench marcks on the shore. The gauges are read simultenaously every ten to fifteen minutes for six to eight hours. The mean of these elevation at that point of the stream. The difference in elevation between the ends of the section divided by the distance is the slope.

Capacity of Existing Lakes or Reserviors:

1. ***Contour Method:***

A traverse is run from a shore line and the desired shore topography are located by stadia. Take sufficient number of soundings by any method suited for the particular job and plot the sub-ageous contour. The area inclosed between contours are determined by planimeter. The average area of two consecutive contours multiplied by the contour interval gives the partial volume. The summation of the partial volumes gives the total volume.

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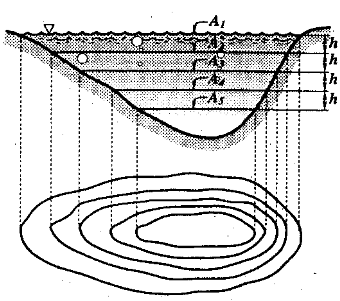
2. ***Cross-Section Method:***
The outline of the water line is obtained as in the contour method. The water line is then plotted and divided into approximate trapezoids and tri-angles. Soundings are taken along the boundary lines between each station and are plotted on cross section paper. A perpendicular distances between sections are then obtained by the end area method. The summation of these partial volumes gives the total volume.

Two General Methods of Determining the Capacity of a Lake or Reservoir:

1. ***Contour Method:***
   a) End-area method
   b) Prismoidal formula
2. ***Parallel Cross-Section Method:***
   a) End-area method
   b) Prismoidal formula

Contour Method:

a) ***End-area method:***



$$V_{1\_2} = \frac{(A_1 + A_2)\,h}{2}$$
$$V_{2\_3} = \frac{(A_2 + A_3)\,h}{2}$$
$$V_{3\_4} = \frac{(A_3 + A_4)\,h}{2}$$
$$V_{4\_5} = \frac{(A_4 + A_5)\,h}{2}$$

$$\text{Total } V = V_{1\_2} + V_{2\_3} + V_{3\_4} + V_{4\_5}$$
$$V = \frac{h}{2}(A_1 + A_2 + A_2 + A_3 + A_3 + A_4 + A_4 + A_5)$$
$$V = \frac{h}{2}(A + 2A_2 + 2A_3 + 2A_4 + A_5)$$

The areas A_1, A_2, etc. are determined by using a planimeter and h represents the contours interval. Area below A_5 is neglected.

b) ***Prismoidal Formula:***

$$V = \frac{L}{6}(A + 4A_m + A_2)$$

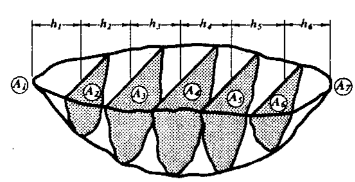
In this case the middle area A_m is the Area A_2 and A_4 while L is equivalent to 2h.

$$V_1 = \frac{2h}{6}[A_1 + 4A_2 + A_3]$$
$$V_2 = \frac{2h}{6}[A_3 + 4A_4 + A_5]$$

$$\text{Total } V = V_1 + V_2$$
$$V = \frac{h}{3}[A_1 + 4A_2 + A_3 + A_3 + 4A_4 + A_5]$$
$$V = \frac{h}{3}[A_1 + 2A_3 + 4A_2 + 4A_4 + A_5]$$

Parallel Cross-Section Method:

a) ***End-area method:***



Parallel ranges are laid out across the lake and soundings are then taken along the ranges. From the observed sounding the corresponding cross-sections could be plotted and its corresponding areas would then be computed.

$$V_1 = \frac{(A_1 + A_2)}{2} h_1$$

$$V_2 = \frac{(A_2 + A_3)}{2} h_2$$

$$V_3 = \frac{(A_3 + A_4)}{2} h_3$$

$$V_4 = \frac{(A_4 + A_5)}{2} h_4$$

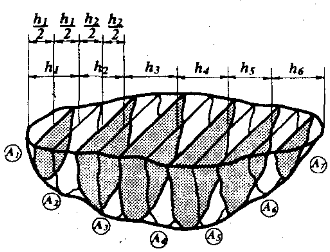
$$V_5 = \frac{(A_5 + A_6)}{2} h_5$$

$$V_6 = \frac{(A_6 + A_7)}{2} h_6$$

$$\textit{Total Volume} = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$$

b) ***Pismoidal Formula:***

The problem arises here in the determination of A_m, since the distances between parallel sections are not equal, it is therefore necessary to evaluate or interpolate the values of A_m.



$$V_1 = \frac{h_1}{6}(A_1 + 4A_m + A_2)$$

$$V_2 = \frac{h_2}{6}(A_2 + 4A_m + A_3)$$

$$V_3 = \frac{h_3}{6}(A_3 + 4A_m + A_4)$$

$$V_4 = \frac{h_4}{6}(A_4 + 4A_m + A_5)$$

$$V_5 = \frac{h_5}{6}(A_5 + 4A_m + A_6)$$

$$V_6 = \frac{h_6}{6}(A_6 + 4A_m + A_7)$$

$$\textit{Total Volume} = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$$

Problem 208:

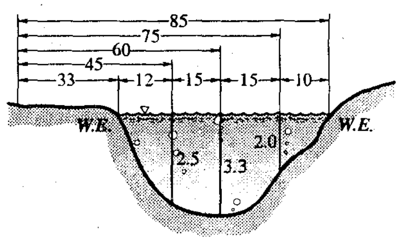
From the current meter notes taken on the Pasig River, the main outlet of the Laguna Lake. All measurements are in meters.

Dist.	Depth	Depth Obs.	Time (Sec.)	Rev.	Vel.
33 (WE)	0				
45	2.5	0.5 2.0	49.0 50.0	70 50	0.32 0.22
60	3.3	0.65 2.65	49.8 55.0	90 60	0.40 0.24
75	2.0	1.20	53.0	60	0.21
85 (WE)					

① Compute the total discharge in liters per second.
② Compute the total cross sectional area in sq.m.
③ Compute the mean velocity in m/sec.

Solution:

① *Total discharge:*



$$V_a = \frac{(0.32 + 0.22)}{2}$$

$$V_a = 0.27 \text{ m/sec}$$

$$V_b = \frac{(0.40 + 0.24)}{2}$$

$$V_b = 0.32 \text{ m/sec}$$

$$V_c = 0.21 \text{ m/sec}$$

Velocity:

$$V_1 = \frac{(0 + 0.27)}{2}$$

$$V_1 = 0.135$$

$V_2 = \frac{(0.27 + 0.32)}{2}$

$V_2 = 0.295$

$V_3 = \frac{(0.32 + 0.21)}{2}$

$V_3 = 0.265$

$V_4 = \frac{0 + 0.21}{2}$

$V_4 = 0.105$

Discharge: $Q = AV$

$Q_1 = 15\ (0.135) = 2.03$

$Q_2 = 43.5\ (0.295) = 12.83$

$Q_3 = 39.75\ (0.265) = 10.53$

$Q_4 = 10\ (0.105) = 1.05$

Total $Q = 2.03 + 12.83 + 10.53 + 1.05$

Total Q = 26.44 cu.m./sec.

1 cu.m. = 1000 liters

Total Q = ***26440 liters/sec***

② *Total area:*

$A_1 = \frac{12(2.5)}{}$

$A_1 = 15.00$

$A_2 = \frac{(2.5 + 3.3)(15)}{2}$

$A_2 = 43.50$

$A_3 = \frac{(3.3 + 2)(15)}{2}$

$A_3 = 39.75$

$A_4 = \frac{2(10)}{2}$

$A_4 = 10.00$

Total area:

$A = A_1 + A_2 + A_3 + A_4$

$A = 15 + 43.5 + 39.75 + 10$

A = ***108.25 sq.m.***

③ *Mean velocity:*

$V = \frac{Q}{A}$

$V = \frac{26.44}{108.25}$

V = ***0.244 m/sec.***

Problem 209

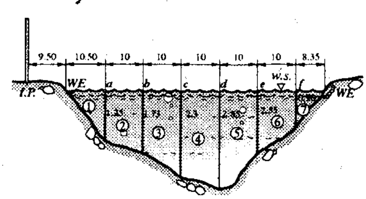
Below is the current meter notes for a river.

Dist. from I.P.	Depth of water	Depth of Obs.	No. of Rev.	Time in secs	Vel. at point
9.50	W.E.				
20.00	1.25	0.75	35	20.0	0.450
30.00	1.70	0.34	55	20.5	0.739
		1.36	50	17.2	0.720
40.00	2.30	0.46	95	21.0	1.251
		1.84	90	19.4	1.243
50.00	2.85	1.71	75	14.3	0.852
60.00	1.55	0.93	50	20.0	0.524
70.00	0.90	0.18	35	22.0	0.473
		0.72	30	18.2	0.469
78.35	W.E.				

① Compute the velocity at distance of 30 m. from I.P.
② Compute the discharge in liters/sec.
③ Compute the mean velocity in section.

Solution:

① *Velocity at distance of 30 m. from I.P.*



$V = \frac{0.739 + 0.720}{2}$

V = ***0.7295***

② *Discharge in liters/sec:*

$V_a = 0.45$

$V_b = \frac{0.739 + 0.720}{2} = 0.7295$

$V_c = \frac{1.251 + 1.243}{2} = 1.2465$

$V_d = 0.852$

$V_e = 0.524$

$V_f = \frac{0.473 + 0.469}{2} = 0.471$

$$V_1 = \frac{0 + 0.45}{2} = 0.225$$

$$V_2 = \frac{0.45 + 0.7295}{2} = 0.5898$$

$$V_3 = \frac{0.7295 + 1.2465}{2} = 0.988$$

$$V_4 = \frac{1.2465 + 0.852}{2} = 1.049$$

$$V_5 = \frac{0.852 + 0.524}{2} = 0.688$$

$$V_6 = \frac{0.524 + 0.471}{2} = 0.4975$$

$$V_7 = \frac{0.471 + 0}{2} = 0.2355$$

$$A_1 = \frac{10.5\,(1.25)}{2} = 6.5625$$

$$A_2 = \frac{(1.25) + (1.7)\,10}{2} = 14.75$$

$$A_3 = \frac{(1.7 + 2.3)\,10}{2} = 20$$

$$A_4 = \frac{(2.3 + 2.85)\,10}{2} = 25.75$$

$$A_5 = \frac{(2.85 + 1.55)\,10}{2} = 22$$

$$A_6 = \frac{(1.55 + 0.9)\,10}{2} = 12.25$$

$$A_7 = \frac{8.35\,(0.90)}{2} = 3.7575$$

$Q = AV$

Area	Velocity	Discharge
$A_1 = 6.5625$	$V_1 = 0.225$	$Q_1 = 1.477$
$A_2 = 14.75$	$V_2 = 0.5898$	$Q_2 = 8.700$
$A_3 = 20$	$V_3 = 0.988$	$Q_3 = 19.760$
$A_4 = 25.75$	$V_4 = 1.049$	$Q_4 = 27.012$
$A_5 = 22$	$V_5 = 0.688$	$Q_5 = 15.136$
$A_6 = 12.25$	$V_6 = 0.4975$	$Q_6 = 6.094$
$A_7 = 3.7575$	$V_7 = 0.2355$	$Q_7 = 0.885$
$A = 105.07\ m^2$		$Q = 79.064\ m^3/sec$
		$Q =$ ***79064 liters/sec***

③ *Mean velociy:*
$Q = AV$
$79.064 = (105.07)\,V$
$V =$ ***0.752 m/s***

Problem 210:

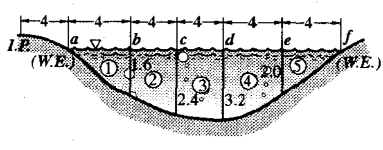
A stream flow measurement was conducted on a river using a current meter with the corresponding values of constants "a" and "b". The following are the observed data taken during the measurements. Only 0.6 method was used in observation.

Dist. from I.P.	Depth (m)	Meter Depth	Rev.	Time (sec)	Const a	Const b
4 (W.E.)	0					
8	1.6	0.96	10	50	0.232	0.022
12	2.4	1.44	22	55	0.232	0.022
16	3.2	1.92	35	52	0.232	0.022
20	2.0	1.20	28	53	0.232	0.022
24(W.E.)						

① Compute the velocity at a distance of 16 m. from W.E.
② Determine the discharge on the river.
③ Determine the mean velocity on the river.

Solution:

① *Velocity at distance of 16 m. from W.E.*



$V = aN + b$

$$V = 0.232\left(\frac{35}{52}\right) + 0.022$$

$V =$ ***0.1782 m/s***

② *Discharge of a certain river:*
$V_a = 0$
$V = aN + b$ *(Straight line equation for current meters)*

$$N = \frac{revolution}{sec}$$

$$V_b = 0.232\,\frac{(10)}{50} + 0.022$$

$V_b = 0.0684$ m/s

$V_c = 0.232\,\frac{(22)}{55} + 0.022$
$V_c = 0.1148$ m/s
$V_d = 0.232\,\frac{(35)}{52} + 0.022$
$V_d = 0.1782$ m/s
$V_e = 0.232\,\frac{(28)}{53} + 0.022$
$V_e = 0.1446$ m/s
$V_f = 0$

$V_1 = \frac{(V_a + V_b)}{2}$
$V_1 = \frac{(0 + 0.0684)}{2}$
$V_1 = 0.0342$ m/s
$V_2 = \frac{(V_b + V_c)}{2}$
$V_2 = \frac{(0.0684 + 0.1148)}{2}$
$V_2 = 0.0916$ m/s
$V_3 = \frac{(0.1148 + 0.1782)}{2}$
$V_3 = 0.1465$ m/s
$V_4 = \frac{(V_d + V_e)}{2}$
$V_4 = \frac{(0.1782 + 0.1446)}{2}$
$V_4 = 0.1614$ m/s
$V_5 = \frac{(V_e + V_f)}{2}$
$V_5 = 0.0723$ m/s

$A_1 = \frac{1.6\,(4)}{2}$
$A_1 = 3.2$
$A_2 = \frac{(1.6 + 2.4)\,(4)}{2}$
$A_2 = 8$
$A_3 = \frac{(2.4 + 3.2)\,4}{2}$
$A_3 = 11.2$
$A_4 = \frac{(3.2 + 2)\,4}{2}$
$A_4 = 10.4$
$A_5 = \frac{2\,(4)}{2}$
$A_5 = 4$

$Q_1 = A_1 V_1$
$Q_1 = 3.2\,(0.0342)$
$Q_1 = 0.10944$
$Q_2 = 8\,(0.0916)$
$Q_2 = 0.7328$
$Q_3 = A_3 V_3$
$Q_3 = 11.2\,(0.1465)$
$Q_3 = 1.6408$
$Q_4 = A_4 V_4$
$Q_4 = 10.4\,(0.1614)$
$Q_4 = 1.6786$
$Q_5 = 0.2892$

$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$
$Q = 0.10944 + 0.7328 + 1.6408 + 1.6786 + 0.2892$
$Q = 4.4508\ m^3/s$
$Q =$ ***4450.8 liters/sec***

③ *Mean velocity:*
$A = A_1 + A_2 + A_3 + A_4 + A_5$
$A = 3.2 + 8 + 11.2 + 10.4 + 4$
$A = 36.8$ sq.m.

$V = \frac{Q}{A}$
$V = \frac{4.4508}{36.8}$
$V =$ ***0.1209 m/s***

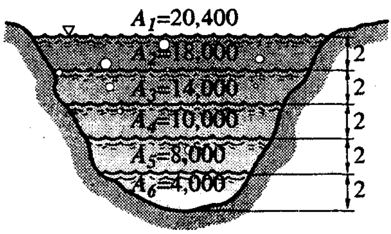
Problem 211:

The areas bounded by the water line of a reservoir is determined by using a planimeter. The contour interval is 2 m. A_1 = 20,400 sq.m., A_2 = 18,600 sq.m., A_3 = 14,300 sq.m., A_4 = 10,200 sq.m., A_5 = 8,000 sq.m. and A_6 = 4,000 sq.m. Determine the following:

① End area method.
② Prismoidal formula.
③ What is the difference of capacity of the reservoir using End area and by Prismoidal Formula.

Solution:

① *End area method:*



$$V_1 = \frac{2}{2}(20400 + 18600) = 39000$$

$$V_2 = \frac{2}{2}(18600 + 14300) = 32900$$

$$V_3 = \frac{2}{2}(14300 + 10200) = 24500$$

$$V_4 = \frac{2}{2}(10200 + 8000) = 18200$$

$$V_5 = \frac{2}{2}(8000 + 4000) = 12000$$

126600 m³

② *Prismoidal formula:*

$$V_1 = \frac{4}{6}[20400 + 4(18600) + 14300]$$

$$V_1 = 72733.33$$

$$V_2 = \frac{4}{6}[14300 + 4(10200) + 8000]$$

$$V_2 = 42066.67$$

$$V_3 = \frac{2}{2}(8000 + 4000)$$

$$V_3 = 12000$$

Prismoidal Formula

$= V_1 + V_2 + V_3$

$= 72733.33 + 42066.67 + 12000$

$=$ ***126800 m³***

③ *Difference of capacity of the reservoir using End area and by Prismoidal Formula:*

Diff. in volume = 126800 - 126600

Diff. in volume = ***200 m³***

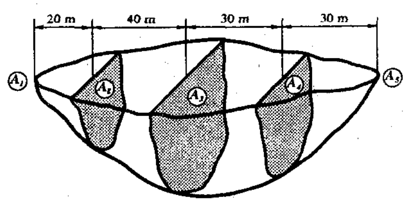
Problem 212:

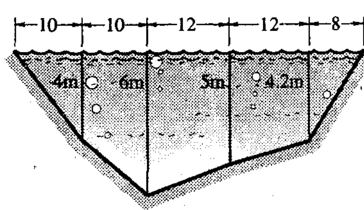
The NAWASA engineers conducted a hydrographic survey on one of the reservoirs in order to determine its capacity and to check out whether this reservoir is capable of serving the water needs of the adjacent municipalities. Parallel ranges were established along the reservoir and sounding was taken at certain distances as shown below. What would be the capacity of this reservoir by:

① End area method.

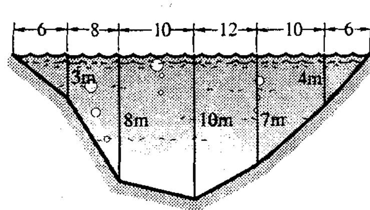
② Prismoidal formula.

③ Determine the difference in capacity between End area and Prismoidal formula.



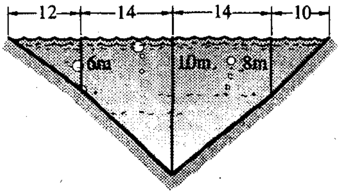


SECTION 2



SECTION 3

HYDROGRAPHIC SURVEYING



SECTION 4

Solution:

① *End area method:*

$$V_1 = \frac{(A_1 + A_2)\,h}{2}$$

$$A_1 = 0$$

$$A_2 = \frac{10\,(4)}{2} + \frac{(4+6)\,(10)}{2} + \frac{(6+5)\,(12)}{2} + \frac{(5+4.2)\,(12)}{2} + \frac{8\,(4.2)}{2}$$

$$A_2 = 20 + 50 + 66 + 55.2 + 16.8$$

$$A_2 = 208 \text{ sq.m.}$$

$$V_1 = \frac{(0 + 208)\,(20)}{2}$$

$$V_1 = 2080 \text{ cu.m.}$$

$$V_2 = \frac{(A_2 + A_3)\,h}{2}$$

$$A_3 = \frac{8\,(3)}{2} + \frac{(3+8)\,(8)}{2} + \frac{(8+10)\,(10)}{2} + \frac{(10+7)\,(12)}{2} + \frac{(7+4)\,(10)}{2} + \frac{4\,(6)}{2}$$

$$A_3 = 12 + 44 + 90 + 102 + 55 + 12$$

$$A_3 = 315 \text{ sq.m.}$$

$$V_2 = \frac{(208 + 315)\,(40)}{2}$$

$$V_2 = 10{,}460 \text{ cu.m.}$$

$$V_3 = \frac{(A_3 + A_4)\,h}{2}$$

$$A_4 = \frac{6\,(12)}{2} + \frac{(6+10)\,(14)}{2} + \frac{(10+8)\,(14)}{2} + \frac{8\,(10)}{2}$$

$$A_4 = 36 + 112 + 126 + 40$$

$$A_4 = 314 \text{ sq.m.}$$

$$V_3 = \frac{(315 + 314)\,(30)}{2}$$

$$V_3 = 9435 \text{ cu.m.}$$

$$V_4 = \frac{(A_4 + A_5)\,h}{2}$$

$$V_4 = \frac{(314 + 0)\,(30)}{2}$$

$$V_4 = 4710 \text{ cu.m.}$$

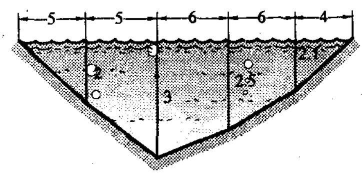
Total volume $= V_1 + V_2 + V_3 + V_4$

$$V = 2078 + 10456 + 9435 + 4710$$

$$\boldsymbol{V = 26{,}679 \text{ cu.m.}}$$

② *Prismoidal formula:*

Note: To solve for A_m: compute the dimensions of A_m using the average values of the sections (1) and (2).



FROM 1 & 2 Am

$$A_m = \frac{2\,(5)}{2} + \frac{(2+3)\,(5)}{2}\,\frac{(2.5+3)\,(6)}{2} + \frac{(2.5+2.1)\,(6)}{2} + \frac{4\,(2.1)}{2}$$

$$A_m = 5 + 12.5 + 16.5 + 13.8 + 4.2$$

$$A_m = 52 \text{ sq.m.}$$

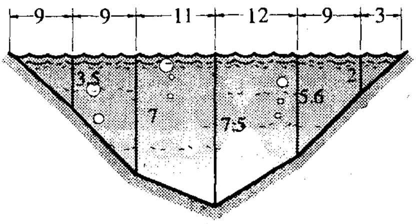
$$V_1 = \frac{L}{6}\,(A_1 + A_4 + A_2)$$

$$V_1 = \frac{20}{6}\,[0 + 4(52) + 208]$$

$$V_1 = \frac{20\,(208 + 208)}{6}$$

$$V_1 = 1386.67 \text{ cu.m.}$$

From (2) to (3). Use average dimensions of sections (2) and (3).



Am

$$A_m = \frac{9\,(3.5)}{2} + \frac{(3.5+7)\,(9)}{2} + \frac{(7+7.5)\,(11)}{2}$$
$$+ \frac{(7.5+5.6)\,(12)}{2} + \frac{(5.6+2)\,(9)}{2} + \frac{2\,(3)}{2}$$

$A_m = 15.75 + 47.25 + 79.75 + 78.6 + 34.2 + 3$

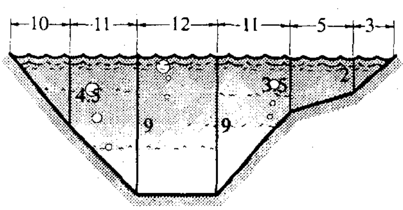
$A_m = 258.55$ sq.m.

$$V_2 = \frac{(A_2 + 4\,A_m + A_3)\,h}{6}$$

$$V_2 = \left[\frac{208 + 4\,(258.55) + 315}{6}\right](40)$$

$V_2 = 10{,}381.33$ cu.m.

From (3) and to (4). Use average dimensions of sections (3) and (4).



$$A_m = \frac{4.5\,(10)}{2} + \frac{(4.5+9)\,(11)}{2} + \frac{(9+9)\,(12)}{2}$$
$$+ \frac{(9+3.5)\,(11)}{2} + \frac{(3.5+2)\,(5)}{2} + \frac{2\,(3)}{2}$$

$A_m = 22.5 + 74.25 + 54 + 68.75 + 13.75 + 3$

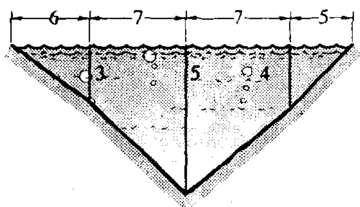
$A_m = 236.25$ sq.m.

$$V_3 = \frac{4\,(A_3 + 4\,A_m + A_3)\,h}{6}$$

$$V_3 = \frac{30}{6}\,[315 + 4\,(236.25) + 314]$$

$V_3 = 7870$ cu.m.

From (4) to (5). Use average dimensions of sections (4) and (5).



$$A_m = \frac{3\,(6)}{2} + \frac{(3+5)\,7}{2}\,\frac{(5+4)\,(7)}{2} + \frac{4\,(5)}{2}$$

$A_m = 9 + 28 + 31.5 + 10$

$A_m = 78.5$ sq.m.

$$V_4 = \frac{(A_4 + 4\,A_m + A_5)\,h}{6}$$

$$V_4 = \frac{30}{6}\,[314 + 4\,(78.5) + 0]$$

$V_4 = 5\,(314 + 314)$

$V_4 = 3140$ cu.m.

Total volume $= V_1 + V_2 + V_3 + V_4$

$V = 1386.67 + 10381.33 + 7870 + 3140$

$V =$ ***22,778 cu.m.***

③ *Difference in capacity:*

Difference in capacity = 26685 - 22778

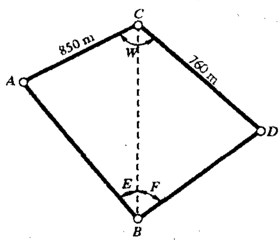
Difference in capacity = ***3,907 cu.m.***

THREE POINT PROBLEM

Three Point Problem

Problem 213:

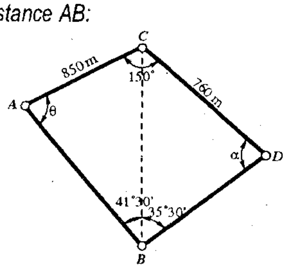
A, C and D are three triangulation shore signals whose positions were determined by the angles W = 150° and the sides AC = 850 m. and CD = 760 m. A sounding at B was taken from a boat and the angles E = 41°30' and the angles E = 41°30' and F = 35°30' were measured simultaneously by two sextants from the boat to the three shore signals from the shore.

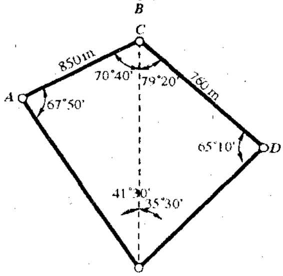


① Find the distance AB.
② Find the distance BD.
③ Find the distance CB.

Solution:

① *Distance AB:*





Considering triangle ACB:

❶ $CB = \dfrac{850 \text{ Sin } ø}{\text{Sin } 41°30'}$

Considering triangle BC:

❷ $\dfrac{CB}{\text{Sin } \alpha} = \dfrac{760}{\text{Sin } 35°30'}$

$CB = \dfrac{760 \text{ Sin } \alpha}{\text{Sin } 35°30'}$

❶ *and* ❷

$\dfrac{850 \text{ Sin } ø}{\text{Sin } 41°30'} = \dfrac{760 \text{ Sin } \alpha}{\text{Sin } 35°30'}$

$\text{Sin } ø = 1.02 \text{ Sin } \alpha$

$ø + \alpha + 150° + 41°30' + 35°30' = 360$

$ø + \alpha = 133°$

$\alpha = (133° - ø)$

$\text{Sin } ø = 1.02 \text{ Sin } (133° - ø)$

$\text{Sin } ø = 1.02 (\text{Sin } 133° \text{ Cos } ø - \text{Cos } 133° \text{ Sin } ø)$

$\text{Sin } ø = 0.746 \text{ Cos } ø + 0.696 \text{ Sin } ø$

$0.304 \text{ Sin } ø = 0.746 \text{ Cos } ø$

$\tan ø = 2.454$

$ø = 67°50'$

$\alpha = 133° - 67°50'$

$\alpha = 65°10'$

$\dfrac{850}{\text{Sin } 41°30'} = \dfrac{AB}{\text{Sin } 70°40'}$

AB = ***1210.45 m.***

② *Distance BD:*

$\dfrac{BD}{\text{Sin } 79°20'} = \dfrac{760}{\text{Sin } 35°30'}$

BD = ***1286.14 m.***

③ *Distance CB:*

$\dfrac{CB}{\text{Sin } 67°50'} = \dfrac{850}{\text{Sin } 41°30'}$

CB = ***1187.98 m.***

THREE POINT PROBLEM

Problem 214:

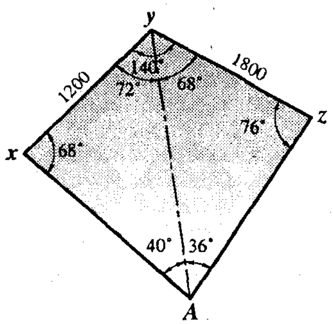
A hydrographic survey was conducted to locate the position of soundings. Three stations XYZ were established on the seashore and the soundings were observed at point A using a small boat. The foll. data were recorded in order to plot the position of the soundings.

Distance XY = 1200 m.
Distance YZ = 1800 m.
Angle XYZ = 140°
Angle XAY = 40°
Angle ZAY = 36°
Angle YXA = 68°

① Compute the angle YZA.
② Compute the distance AX.
③ Compute the distance AY.

Solution:

① *Angle YZA:*



Angle YZA = 180 - 68 - 36
Angle YZA = ***76°***

② *Distance AX:*

$$\frac{AX}{\sin 72°} = \frac{1200}{\sin 40°}$$

AX = ***1775.50 m.***

③ *Distance AY:*

$$\frac{1200}{\sin 40°} = \frac{AY}{\sin 68°}$$

AY = ***1730.93 m.***

Problem 215:

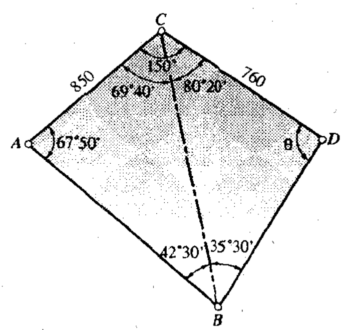
Three shore stations A, C and D are triangulation points whose position as observed from B where soundings are observed and the angles were measured using two sextants from the boat at B to the three shore signals. The following data were recorded during the sounding observation.

Angle ACD = 150°
Angle ABC = 42°30'
Angle CBD = 35°30'
Angle CAB = 67°50'
Distance AC = 850 m.
Distance CD = 760 m.

① Compute the angle CDB.
② Compute the distance AB.
③ Compute the distance BD.

Solution:

① *Angle CDA:*



$\theta + 150° + 67°50' + 42°30' + 35°30 = 360°$

$\theta =$ ***64°10'***

② *Distance AB:*

ACB = 180° - 67°50' - 42°30'
ACB = 69°40'

$$\frac{850}{\sin 42°30'} = \frac{AB}{\sin 69°40'}$$

AB = ***1179.76 m.***

③ *Distance BD:*

$$\frac{760}{\sin 35°30'} = \frac{BD}{\sin 80°20'}$$

BD = ***1290.18 m.***

THREE POINT PROBLEM

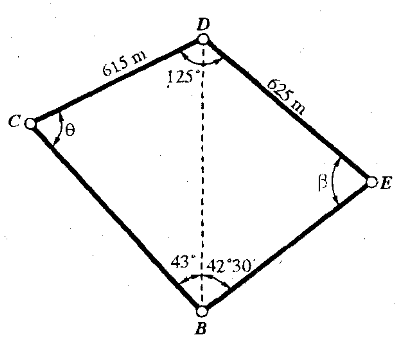
Problem 216:

Three shore stations C, D and E are triangulation observation points with CD = 615 m. and DE = 625 m. Angle EDC is 125. A hydrographer at point D wanted to know his position with respect to the triangulation points, he measured angle CBD = 43° and DBE = 42°30'.

① Compute the angle DCB.
② Compute the angle CDB.
③ Compute the distance BC.

Solution:

① *Angle DCB:*



$$\frac{BD}{\sin \phi} = \frac{615}{\sin 43°}$$

$$BD = \frac{615 \sin \phi}{\sin 43°}$$

$$BD = 901.76 \sin \phi$$

$$\frac{BD}{\sin \beta} = \frac{625}{\sin 42°30'}$$

$$BD = 925.12 \sin \beta$$

$$901.76 \sin \phi = 925.12 \sin \beta$$

$$\phi + \beta + 125° + 43° + 42°30' = 360°$$

$$\phi + \beta = 149°30'$$

$$\beta = 149°30' - \phi$$

$$901.76 \sin \phi = 925.12 \sin (149°30' - \phi)$$

$$901.76 \sin \phi = 925.12 (\sin 149°20' \cos \phi - \sin \phi \cos 149°30')$$

$$901.76 \sin \phi = 169.53 \cos \phi + 797.11 \sin \phi$$

$$104.65 \sin \phi = 469.53 \cos \phi$$

$$\tan \phi = 4.487$$

ϕ = ***77°27'*** *(angle DCB)*

② *Angle CDB:*

$$\beta = 149°30' - 77°27'$$

Angle CDB = 180° - 77°27' - 43°

Angle CDB = ***59°33'***

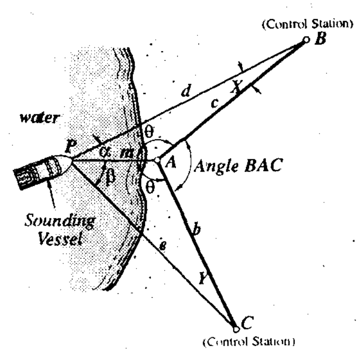
③ *Distance BC:*

$$\frac{BC}{\sin 59°33'} = \frac{615}{\sin 43°}$$

BC = ***777.38 m.***

Problem 217:

In the accompanying figure, A, B and C are three known control stations and P is the position of a sounding vessel which is to be located. If *b* = 6,925.50 m., *c* = 6,708.40 m, angle BAC = 112°45'25", angle alpha = 25°32'40", and angle beta = 45°35'50".

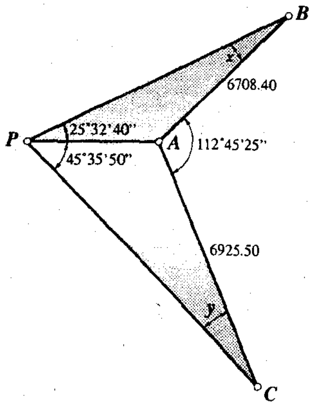


① Compute the value of angle x.
② Compute the value of angle y.
③ Compute the length of line AP.

THREE POINT PROBLEM

Solution:

① *Angle x:*



$\frac{AP}{\text{Sin } x} = \frac{6708.40}{\text{Sin } 25°32'40"}$

$AP = 15557.11 \text{ Sin } x$

$\frac{AP}{\text{Sin } y} = \frac{6925.50}{\text{Sin } 45°35'50"}$

$AP = 9693.62 \text{ Sin } y$

$9693.62 \text{ Sin } y = 15557.11 \text{ Sin } y$

$\text{Sin } y = 1.605 \text{ Sin } x$ ①

$x + (360 - 112°45'25") + y$
$+ 45°35'50" + 25°32'40" = 360$

$y = 41°36'55" - x$ ②

$\text{Sin } (41°36'55" - x) = 1.605 \text{ Sin } x$

$\text{Sin } 41°36'55" \text{ Cos } x - \text{Cos } 41°36'55" \text{ Sin } x$
$= 1.605 \text{ Sin } x$

$\text{Sin } 41°36'55" \text{ Cos } x$
$= (1.605 + \text{Cos } 41°36'55") \text{ Sin } x$

$\frac{\text{Sin } 41°36'55"}{1.605 + \text{Cos } 41°36'55"} = \tan x$

$x = \mathbf{15°45'50.17"}$

② *Angle y:*
Subt. to equation ②

$y = 41°36'55" - x$

$y = 41°36'55" - 15°45'50.17"$

$y = \mathbf{25°51'04.83"}$

③ *Length of line AP:*

$AP = 15557.11 \text{ Sin } x$

$AP = 15557.11 \text{ Sin } 15°45'50.17"$

$AP = \mathbf{4226.47 \text{ m.}}$

Problem 218:

Given the following data for a three point resection problem.

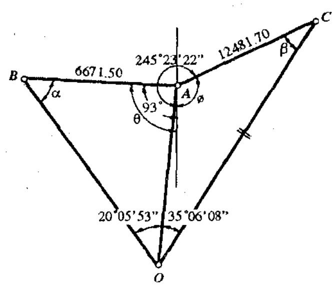
LINE	AZIMUTH	DISTANCE
AB	93°00'	6671.50 m.
AC	245°23'22"	12481.70 m.

The instrument is at 0, south of A, B and C with angle BOA = 20°05'53" and angle AOC = 35°06'08".

① Find the length of line AO.
② Find the angle ACO.
③ Find the length of line OC.

Solution:

① *Length of AO:*



$\omega = 360 - (245°23'22" - 93)$

$\omega = 207°36'38"$

$\frac{AO}{\text{Sin } \alpha} = \frac{6671.50}{\text{Sin } 20°05'53"}$

$AO = 19414.9 \text{ Sin } \alpha$

$\frac{AO}{\text{Sin } \beta} = \frac{12481.70}{\text{Sin } 35°06'08"}$

$AO = 21705.91 \text{ Sin } \beta$

$21705.91 \text{ Sin } \beta = 19414.9 \text{ Sin } \alpha$

$\text{Sin } \beta = 0.894 \text{ Sin } \alpha$ ①

$\alpha + \omega + \beta + 35°06'08" + 20°05'53" = 360$

$\beta = 97°11'21" - \alpha$

THREE POINT PROBLEM

$\sin(97°11'21" - \alpha) = 0.894 \sin\alpha$

$\sin 97°11'21" \cos\alpha - \cos 97°11'21" \sin\alpha$
$= 0.894 \sin\alpha$

$\sin 97°11'21" \cos\alpha$
$= (0.894 + \cos 97°11'21") \sin\alpha$

$$\frac{\sin 97°11'21"}{0.894 + \cos 97°11'21"} = \tan\alpha$$

$\alpha = 52°13'34.41"$

$AO = 19414.9 \sin 52°13'34.41"$
$AO = \mathbf{15346.22\ m.}$

② *Angle ACO:*
$\beta = 97°11'21" - 52°13'34.41"$
$\beta = \mathbf{44°57'46.59"}$

③ *Length of line OC:*
$AO = 21705.91 \sin 44°57'46.59"$
$AO = 15338.47$ m

$$Ave.\ AO = \frac{15346.22 + 15338.47}{2}$$

$Ave.\ AO = 15342.32$ m.

$\theta = 180° - 20°05'53" - 52°13'34.41"$

$\theta = 107°40'32.5"$

$$\frac{OB}{\sin 107°40'32.5"} = \frac{6671.50}{\sin 20°05'53"}$$

$OB = 18498.33$

$\varnothing = 180 - 35°06'08" - 44°57'46.59"$
$\varnothing = 99°56'5.41"$

$$\frac{OC}{\sin 99°56'5.41"} = \frac{12481.70}{\sin 35°06'08"}$$

$OC = \mathbf{21380.42\ m.}$

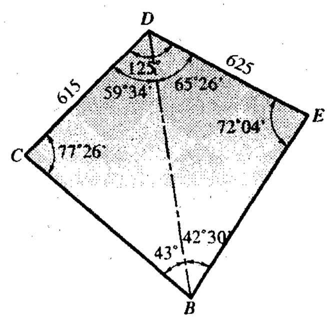
Problem 219:

Triangulation stations C, D and E are established by the Phils Coast and Geodetic Survey with observation points recorded as CD = 615 m. and DE = 625 m. A hydrographer at point B wanted to know his position with respect to the triangulation stations, he then measured angles CBD = 43° and DBE = 42°30'. Angle EDC is 125°. Angle DCB = 77°26'.

① Compute the distance BC.
② Compute the distance BD.
③ Compute the distance BE.

Solution:

① *Distance BC:*



$$\frac{BC}{\sin 59°34'} = \frac{615}{\sin 43°}$$

$BC = \mathbf{777.52\ m.}$

② *Distance BD:*

$$\frac{BD}{\sin 77°26'} = \frac{615}{\sin 43°}$$

$BD = \mathbf{880.16\ m.}$

③ *Distance BE:*

$$\frac{BE}{\sin 65°26'} = \frac{625}{\sin 42°30'}$$

$BE = \mathbf{841.37\ m.}$

Mine Surveying

Vein - a relatively thin deposit of mineral between definite boundaries.

Strike - the line of intersection of the vein with a horizontal plane.

Dip - the vertical angle between the plane of the vein and horizontal plane measured perpendicular to the strike.

Outcrop - the portion of the vein exposed at the ground surface.

Drift - an inclined passage driven in a particular direction.

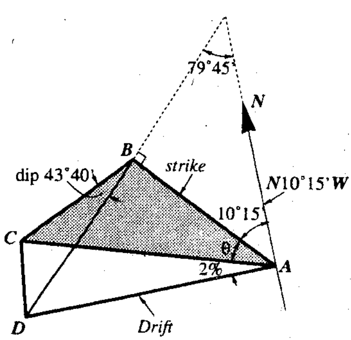
Problem 220:

A vein has a strike of N. 10°15' W. and a dip of 43°40' W. A drift in the vein has a grade of 2%.

① What is the bearing of the vertical plane containing the dip.
② Determine the horizontal angle between the strike and the vertical projection of the drift.
③ Determine the bearing of the drift.

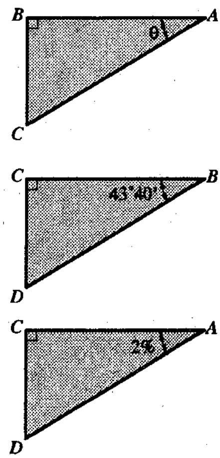
Solution:

① *Bearing of dip:*



Bearing = 90° - 10°15'
Bearing = ***S. 79°45' W.***

② *Angle between strike and drift:*



$$\text{Sin } ø = \frac{BC}{AC}$$

$$\tan 43°40' = \frac{CD}{BC}$$

$$BC = CD \cot 43°40'$$

$$\frac{2}{100} = \frac{CD}{AC}$$

$$AC = 50\ CD$$

$$\text{Sin } ø = \frac{BC}{AC}$$

$$\text{Sin } ø = \frac{CD \cot 43°40'}{CD\ 50}$$

$$\boldsymbol{ø = 1°12'}$$

③ *Bearing of drift:*
Bearing = 10°15' + 1°12'
Bearing = ***N. 11°27' W.***

Problem 221:

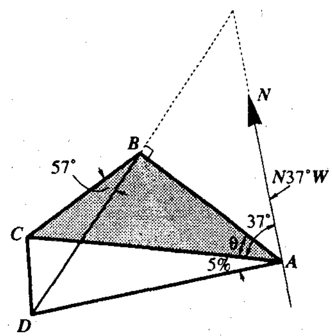
A vein has a dip of 57° W. The bearing of a drift is N. 37° W. having a grade of 5% with the plane of the vein.

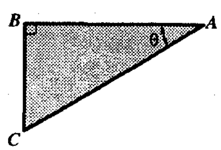
① Compute the horizontal angle between the strike and the vertical projection of the drift.
② Compute the bearing of the strike.
③ Compute the bearing of the vertical plane containing the dip.

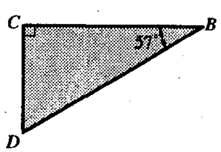
MINE SURVEYING

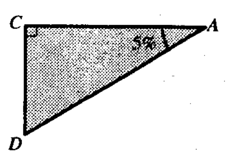
Solution:

① *Horizontal angle between the strike and the vertical projection of drift:*









$$\sin ø = \frac{BC}{AC}$$

$$\tan 57° = \frac{CD}{BC}$$

$$BC = CD \cot 57°$$

$$\frac{5}{100} = \frac{CD}{AC}$$

$$AC = 20\,CD$$

$$\sin ø = \frac{BC}{AC}$$

$$\sin ø = \frac{CD \cot 57°}{20\,CD}$$

$$ø = \mathbf{1°52'}$$

② *Bearing of strike:*
Bearing = 37° - 1°52'
Bearing = ***N. 35°08' W.***

③ *Bearing of the vertical plane containing the dip:*
Bearing = 90° - 35°08'
Bearing = ***S. 54°52' W.***

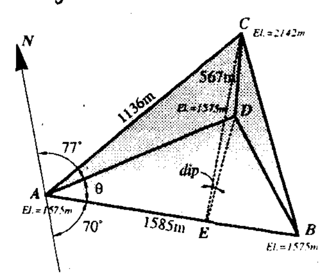
Problem 222:

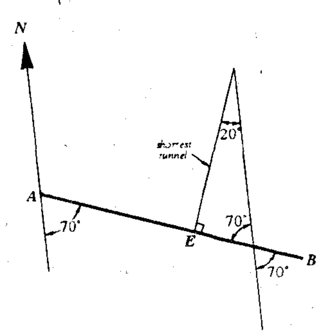
The centerline of a mine tunnel runs through A and B, each 1575 m. in altitude, with a distance between AB equal to 1585 m. The tunnel bears S. 70° E. from A. On the other side of the hill, 1136 m., N. 77° E. of A is a point C at 2142 m. altitude.

① Determine the bearing of the shortest possible tunnel from C to AB.
② Determine the dip of the shortest possible tunnel from C to AB.
③ Determine the length of the shortest possible tunnel from C to AB.

Solution:

① *Bearing of shortest tunnel from C to AB:*



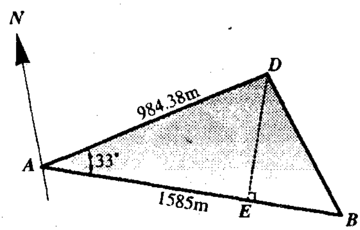


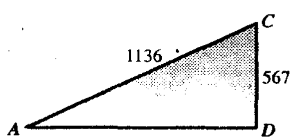
$\theta = 180° - 77° - 70°$

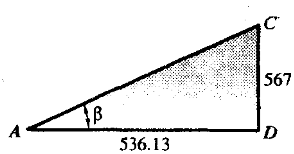
$\theta = 33°$

Bearing = S. 20° W.

② *Dip of shortest tunnel from C to AB:*







$CD = 2142 - 1575$

$CD = 567$ m.

$(AD)^2 = (1136)^2 - (567)^2$

$AD = 984.38$

$DE = 984.38 \text{ Sin } 33°$

$DE = 536.13$ m.

$\tan ß = \dfrac{567}{536.13}$

$ß = $ **46°36'** *(dip)*

Shortest tunnel from C to AB:

$(EC)^2 = (536.13)^2 + (567)^2$

*EC = **780.37 m.***

Problem 223:

Drill holes are bored through points A, B and C until it strikes the mineral ores. Point A is 400 m. due south of B and point C is 300 m. N. 60° E. of B.

POINTS	GROUND ELEVATION	DEPTH OF HOLE
A	450 m.	165 m.
B	470 m.	187 m.
C	485 m.	203 m.

① Compute the difference in elevation of the surface of ore at A and C.
② Compute the bearing of strike.
③ Compute the angle of the dip.

Solution:

① *Difference in elevation of the surface of ore at A and C:*

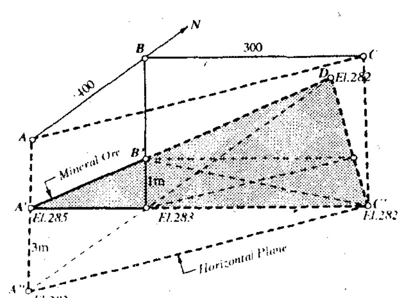
Elevation of mineral ores:

POINTS	ELEVATION OF ORES
A'	450 - 165 = 285 m.
B'	470 - 187 = 283 m.
C'	485 - 203 = 282 m.

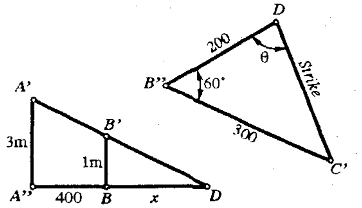
Diff. of elevation of the surface of ore at A and C = 285 - 282

Diff. of elevation of the surface of ore at A and C= **3 m.**

② *Bearing of strike:*



MINE SURVEYING



$$\frac{400 - x}{3} = \frac{x}{1}$$

$3x = 400 + x$

$2x = 400$

$x = 200$

$(C'D) = (200)^2 + (300)^2 - 2(200)(300) \cos 60°$

$C'D = 264.58$ m.

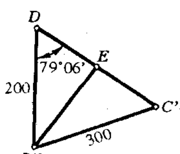
Using Sine Law:

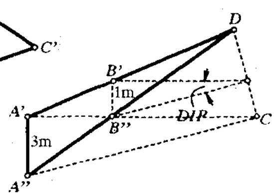
$$\frac{\sin ø}{300} = \frac{\sin 60°}{264.58}$$

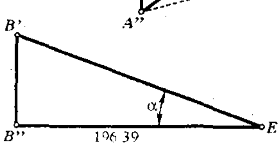
$o = 79°06'$

Bearing of strike = ***N. 79°06' W.***

③ *Angle of the dip:*







$B''E = 200 \sin 79°06'$

$B''E = 196.39$ m.

$$\tan \text{dip} = \frac{1}{196.39}$$

dip = *0°17'*

Problem 224:

From point A at the opening of a tunnel, a surface traverse is run on the side hill and an underground traverse is run through the tunnel. Both traverse are oriented from the same meridian. Below are the computed latitudes and departures. Consider the line AF is a straight line on the surface and the slope AF is uniform upward. Also assume that all points of the under ground traverse are on the same elevation while point F is 33 meters above A.

SURFACE TRAVERSE		
Line	Lat.	Dep.
A - B	+ 31.2	+ 40.5
B - C	- 17.4	+ 34.6
C-D	- 3.7	+ 66.0
D - E	+ 22.8	+ 39.9

UNDERGROUND TRAVERSE		
Line	Lat.	Dep.
A - 1	- 82.5	+ 25.0
1 - 2	- 18.8	+ 50.4
2 - 3	0	+ 40.0
3 - 4	- 15.7	+ 40.0

① Find the bearing of line AF.
② Find the distance of line AF.
③ Compute the shortest distance of a shaft from point 4 in the tunnel of the surface along line AF.

Solution:

① *Bearing of line AF:*

Line AF:

Lat. = 31.2 - 17.4 - 3.7 + 22.8 + 12.1

Lat. = + 45.0

Dep. = 40.5 + 34.6 + 66.0 + 39.9 + 55

Dep. = + 236.5

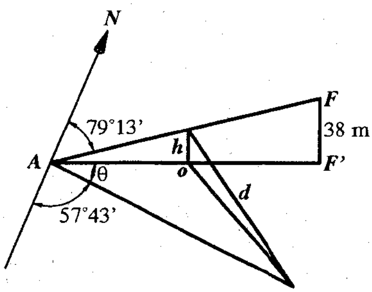
$$\tan ø = \frac{Dep.}{Lat.}$$

$$ø = \frac{236.3}{45.0}$$

$ø = 79°13'$

Bearing = ***N. 79°13' E.***

② *Distance AF:*



$$\text{Distance } AF = \frac{\text{Departure}}{\text{Sin } 79°13'}$$

Distance AF = ***240.55 meters***

③ *Shortest distance of a shaft from point 4 in the tunnel of the surface along line AF.*
Line A - 4:
Lat. = - 82.5 - 8.8 - 5.7
Lat. = - 117.0
Dep. = 25.0 + 50.4 + 40.0 + 70.4
Dep. = 185.8

$$\tan \text{bearing } A - 4 = \frac{185.8}{117}$$

tan bearing A - 4 = S 57°48' E.

$$\text{Distance } A - 4 = \frac{\text{Departure}}{\text{Sin } 57°48'}$$

Distance A - 4 = 219.58 m.
ø = 180° - (79°13' + 57°48')
ø = 42°59'
AO = 219.58 Cos 42°59'
AO = 160.63 meters

$$\frac{h}{160.33} = \frac{33}{240.55}$$

h = 22.037 meters

Distance 04 = (*A* - 4) Sin 42°59'
Distance 04 = 219.58 Sin 42°59'
Distance 04 = 149.7 meters.

$$\tan \alpha = \frac{h}{0 - 4}$$

$$\alpha = \frac{22.037}{149.7}$$

α = 8°22' 26"

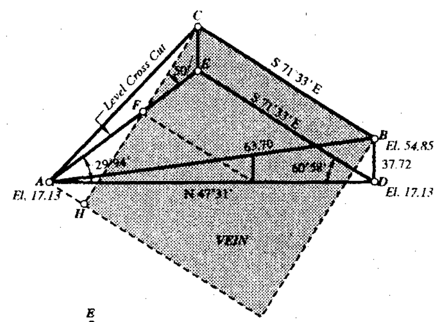
$$d = \frac{0 - 4}{\text{Cos } 8°22' 26''}$$

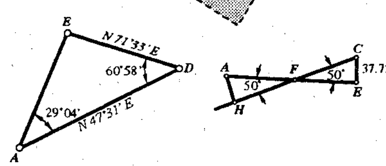
d = ***151.32 meters.*** *(shortest distance of the shaft)*

Problem 225:

A line from A elevation 17.13 m. intersects the vein at B, elevation 54 - 85 m. The bearing and slope distance of AB are N. 47°31' E. and 63.70 m. respectively. Strike is S. 71°33' E. Dip is 50° SW.

① Compute the direction of the shortest level cross cut from A to the vein.
② Compute the length of the shortest level cross cut from A to the vein.
③ Determine the shortest distance from A to the vein.





Solution:

① *Direction of the shortest level cross cut from A to the vein:*

Direction of the shortest level = 47°31' - 29°04'
Direction of the shortest level = ***N. 18°27' E.***

② *Length of the shortest level cross cut from A to the vein:*

$AD = \sqrt{(63.70)^2 - (37.72)^2}$
AD = 51.33 m.
AE = 51.33 Cos 29°04'
AE = 44.87 m.
EF = 37.72 Cot 50°
EF = 31.65 m.
AF = 44.87 - 31.65
AF = ***13.22 m.*** *(shortest level cross out from A to the vein)*

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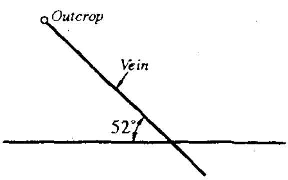
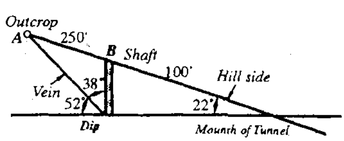
③ *Shortest distance from A to the vein:*
$AH = AF \sin 50°$
$AH = 13.22 \sin 50°$
$AH = \mathbf{10.13\ m.}$

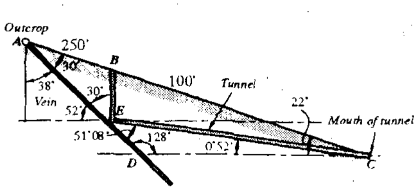
Problem 226:

A vein dips to the west at an angle of 52°. A hill side assumed to be sloping uniformly has an angle of depression of 22°. From the outcrop of the vein, the slope distance along the hillside to the top of the shaft and mouth of the tunnel are respectively 250 ft. and 350 ft. If the tunnel is driven at right angles to the strike and the shaft is sunk vertically.

① Determine the height of the shaft.
② Determine the length of a + 1.5% tunnel to meet the vein.
③ Determine the distance from the outcrop to the bottom of the shaft.







Solution:

① *Height of the shaft:*

$$\frac{BE}{\sin 30°} = \frac{250}{\sin 38°}$$

$BE = \mathbf{203.03\ m.}$

② *Length of a + 1.5% tunnel to meet the vein:*

$$\frac{CD}{\sin 30°} = \frac{350}{\sin 128°}$$

$CD = 222.08$

$$\frac{CE}{\sin 128°} = \frac{222.08}{\sin 51°08"}$$

$CE = \mathbf{224.76\ ft.}$

③ *Distance from the outcrop to the bottom of the shaft.*

$$\frac{AE}{\sin 112°} = \frac{203.03}{\sin 30°}$$

$AE = 376.49\ m.$

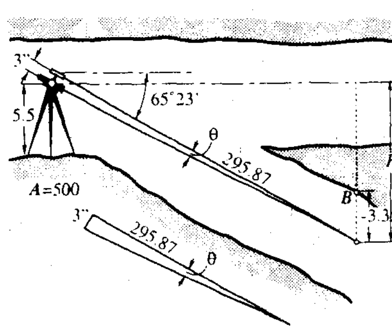
Problem 227:

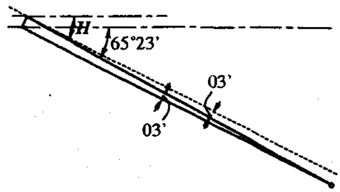
A point B at the bottom of a winze has a vertical angle of - 65°23' sighted with the top telescope of a mining transit. The slope distance to a point B from the instrument at A is 295.87 ft. The eccentricity of the telescope is 3 inches.

① Compute the corrected vertical angle.
② Compute the elevation of B if A is at elevation 500 ft. and the H.I. is + 5.5 ft. and $H.P_t$. is - 3.3 ft.
③ Determine the horizontal distance between A to B.

Solution:

① *Corrected vertical angle:*





$$\tan \phi = \frac{3/12}{295.87}$$

$\phi = 0°03'$

Corrected vertical angle = 65°23' - 03'

Corrected vertical angle = ***65°20'***

② *Elevation of B if A is at elevation 500 ft. and the H.I. is + 5.5 ft. and H.P$_t$. is - 3.3 ft.*

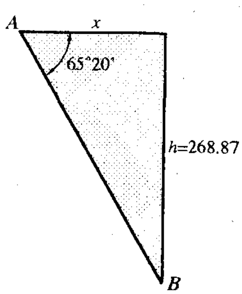
$h = 295.87 \sin 65°20'$

$h = 268.87$

El. of B = 500 + 5.5 - 268.87 + 3.3

El. of B = ***239.93 ft.***

③ *Horizontal distance between A to B:*



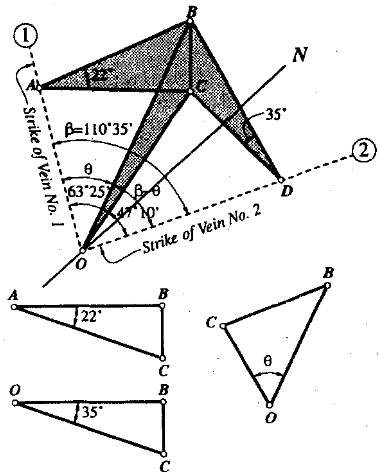
$$\tan 65°20' = \frac{268.87}{x}$$

$x =$ ***123.48 ft.***

Problem 228:

Vein number one strikes N 63°25' W. and dip 22° NE. Vein number two strikes N 47°10' E. and dips 35° NW.

① Determine the direction of the line of intersection of the veins.
② Determine its slope.



Solution:

① *Direction of the line of intersection of the veins:*

$\beta = 63°25' + 77°10'$

$\beta = 110°35'$

In Δ ABC:

$BC = AB \tan 22°$

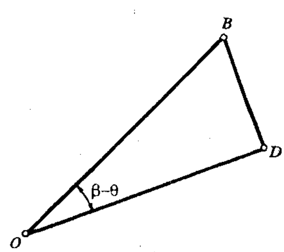
In Δ BOD:

$BC = BD \tan 35°$

In Δ ABO:

$AB = OB \sin \phi$

In Δ OBD:



$BD = OB \text{ Sin } (ß - ø)$

$OB = \dfrac{AB}{\text{Sin } ø}$

$OB = \dfrac{BD}{\text{Sin } (ß - ø)}$

$\dfrac{AB}{\text{Sin } ø} = \dfrac{BD}{\text{Sin } (ß - ø)}$

$\dfrac{BC \text{ Cot } 22°}{\text{Sin } ø} = \dfrac{BC \text{ Cot } 35°}{\text{Sin } (ß - ø)}$

$\dfrac{\text{Sin } (ß - ø)}{\text{Sin } ø} = \dfrac{\text{Cot } 35°}{\text{Cot } 22°}$

$\dfrac{\text{Sin } (ß - ø)}{\text{Sin } ø} = 0.578$

$\dfrac{\text{Sin } ß \text{ Cos } ø - \text{Sin } ø \text{ Cos } ß}{\text{Sin } ø} = 0.578$

$\dfrac{\text{Sin } 110°35' \text{ Cos } ø - \text{Sin } ø \text{ Cos } 110°35'}{\text{Sin } ø} = 0.578$

$\text{Cot } ø \text{ Sin } 69°25' + \text{Cos } 69°25' = 0.578$

$\text{Cot } ø = \dfrac{0.578 - 0.352}{0.93616}$

$\text{Cot } ø = 0.241$

$ø = 76°27'$

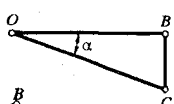
$ß - ø = 34°08'$

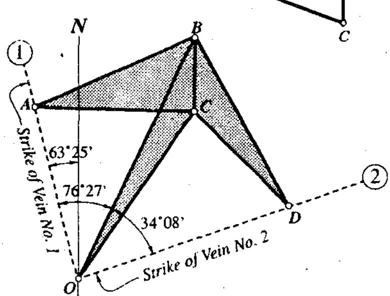
Bearing of the line of intersection

$= 76°27' - 63°25'$

$= N. 13°02' E.$

② *Slope of the line intersection:*





$\tan \alpha = \dfrac{BC}{OB}$

$\tan \alpha = \dfrac{AB}{AB} \tan 22° \text{ Sin } ø$

$\tan \alpha = \tan 22° \text{ Sin } 76°27'$

$\alpha = \mathbf{21°27'}$ *(slope)*

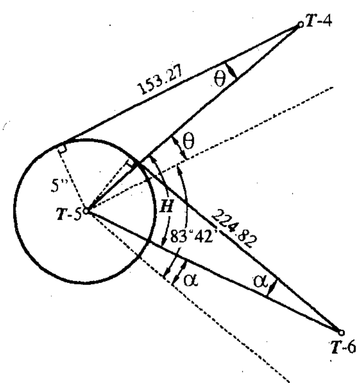
Problem 229:

A horizontal angle is measured with a side telescope and the following measurements are recorded.

Sta. Occ.	Sta. Obs.	Hor. Angles	Inclined Dist	Vert Angles	H.I.	H.P.T.
	T - 6	83-42	224.82	-4°27'		- 1.0
	T - 4	0	153.27	+5°23'		
T - 5					+5.0	

If the eccentricity of the side telescope is 5 inches, compute the corrected horizontal angle.

Solution:



$\tan ø = \dfrac{\frac{5}{12}}{153.27}$

$ø = 0°9'21"$

$\tan \alpha = \dfrac{\frac{5}{12}}{224.82}$

$\alpha = 0°06'22"$

$H = 83°42' - \alpha + ø$

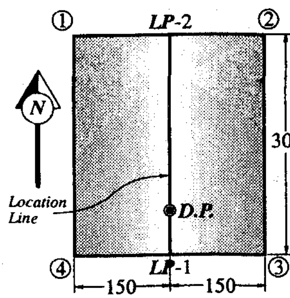
$H = 88°42' - 0°06'22" + 0°09'21"$

$H = \mathbf{88°44'59"}$

Problem 230:

A mineral lode claim 300 m. by 300 m. has three of its corners 1, 3 and 4 situated on hill-tops of equal elevations of 400 m. The discovery post (D.P.) is located somewhere on the location line which is parallel to the west or east boundary and divides the square lode into equal areas. Vertical angle readings at corners 1, 3 and 4 taken and were recorded as 45°, 60° and 60° respectively. If the height of the instrument is 1.5 m.

① Determine the location of the discovery post from the location post number 2.
② Det. the elevation of the discovery post.
③ Determine the distance of the discovery post from corner L.



Solution:

① *Location of discovery post from the location post number 2:*

❶ $(150)^2 + x^2 = (h \cot 45°)^2$

$(150)^2 + x^2 = h^2$

❷ $(150)^2 + (300 - x)^2 = (h \cot 60°)^2$

$(150)^2 + (300 - x)^2 = \dfrac{h^2}{3}$

❶ & ❷

$h^2 - x^2 = \dfrac{h^2}{3} - (300 - x)^2$

$3h^2 - 3x^2 = h^2 - 3\,(90000 - 600x + x^2)$

$3h^2 - 3x^2 = h^2 - 270000 + 1800x - 3x^2$

❸ $2h^2 - 1800x + 270000 = 0$

❶ & ❸

$h^2 = (150)^2 + x^2$

$2\,[(150)^2 + x^2] - 1800x + 270000 = 0$

$2\,(22500 + x^2) - 1800x + 270000 = 0$

$45000 + 2x^2 - 1800x + 270000 = 0$

$2x^2 - 1800x + 315000 = 0$

$x^2 - 900x + 157500 = 0$

$x = \dfrac{900 \pm \sqrt{810000 - 630000}}{2}$

$x = \dfrac{900 \pm \sqrt{180000}}{2}$

$x = \dfrac{900 \pm 424}{2}$

$x = \dfrac{476}{2}$

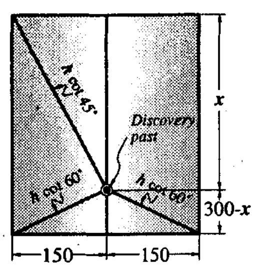
$x = 238$

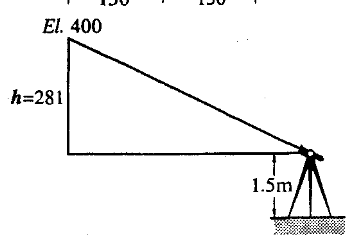
$x = \dfrac{1323}{2}$

$x = 612 > 300$ *(absurd)*

Use $x =$ ***238***

② *Elevation of discovery post:*





$h^2 = (150)^2 + x^2$

$h^2 = 22500 + (238)^2$

$h^2 = 22500 + 56700$

$h = 281$

Elev. of Discovery Post = 400 - 281 - 1.5

Elev. of Discovery Post = ***117.5 meters***

③ *Distance of discovery post from cor. 1:*

Distance = $h \cot 45°$

Distance = $281 \cot 45°$

Distance = ***281 m.***

PRACTICAL ASTRONOMY

Practical Astronomy

Methods of determining time:

1. Time by transit of a star across the meridian.
2. Time by transit of the sun.
3. Time by altitude of the sun.
4. Time by measured altitude of the star.
5. Time by transit of a star across the vertical circle through the Polaris.
6. Time by two stars at equal altitude.

Methods of determining longitude:

1. By time signals.
2. By transportation of time piece.

Methods of determining latitudes:

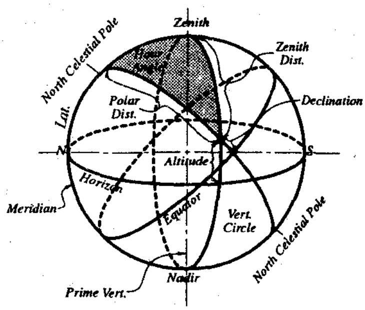
1. By altitude of the sun at noon.
2. By a circumpolar star at time of transit.
3. By altitude of polaris at any hour angle.
4. By circum-meridian altitudes.

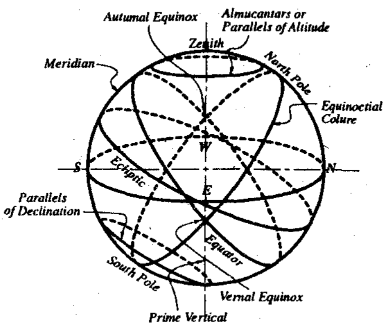
Methods of determining azimuths:

1. By an altitude of the sun.
2. By an altitude of the star.
3. By Polaris at greatest elongation.
4. By a circumpolar star at any hour angle.

Definitions:

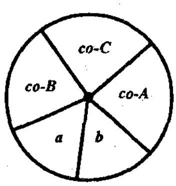
1. ***Vertical circles*** - are great circles passing from the zenith through the star or sun.
2. ***Hour circle*** - are great circles through the poles.
3. ***Zenith*** - the point where the vertical produce upward, pierces the celestial sphere.
4. ***Horizon*** - the great circle on the celestial sphere cut by a plane through the earth's center at right angles to the vertical.
5. ***Equator*** - the great circle of the celestial sphere cut by a plane through the earth's center perpendicular to the axis.
6. ***Meridian of an observer*** - the great circle of the celestial sphere which passes through the poles and the observer's zenith.
7. ***Ecliptic*** - the great circle of the celestial sphere which the sun appear to describe in its annual eastward motion among the stars.
8. ***Equinoxes*** - the point of intersection of the equator and the ecliptic.
9. ***Autmunal equinox*** - the point where the sun crosses the equator in September.
10. ***Declination*** - the angular distance from the equator measured on an hour circle through the point.
11. ***Polar distance*** - is the complement of the declination.
12. ***Altitude*** - the angular distance below or above the horizon measured on a vertical circle through the point.
13. ***Zenith distance*** - complement of the altitude.
14. ***Hour angle*** - the arc of the equator measured from the meridian westward to the hour circle through the point.
15. ***Nadir*** - the point where the plumb line of the transit when prolonged downward.
16. ***Celestial sphere*** - is an imaginary sphere whose center is the center of the earth and whose radius is infinite.





Derivation of the Formula for Determining the Azimuth of Polaris at Elongation:

From the PZS Triangle, using the Napiers rule:

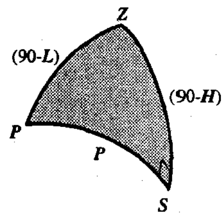


sin b = cos (co - b) cos (co - c)
sin z = sin B sin c

$$\sin Z = \frac{\sin p}{\cos L}$$

sin Z = sin p sec L

Let b = p
B = Z
(90 - L) = c
A = P
(90 - H) = a
C = S
sin p = sin Z sin (90 - LO)
= sin Z cos L
Z' = p" sec L



Note:

At western elongation, subtract Z from 180° to obtain the azimuth of the star, and at eastern elongation, add Z to 180° to obtain the true azimuth of the star.

Determination of Azimuth by Polaris at Greatest Elongation:

Set the transit at one end of the line to be observed and level it carefully. Find the star and sight the vertical hair on it. As the star moves almost vertically (upward for eastern elongation and downward for western elongation) it requires slow motion of the tangent screw to keep the vertical hair on the star. Follow it until it seems to move vertically, which should be about the time given the table of Ephemires. Lower the telescope and set a mark in line with the cross hair on the ground. Reverse the telescope and sight the star again and then set another point along the first side. The point halfway between these two should be the point in the vertical plane of the star at elongation. With the values of declination and latitude given, silve for the value of Z, using the relation that Z = P sec L. When the observation is at western elongation, just subtract the value of Z from ° to obtain the true azimuth of the star, and if it is observed to be on the eastern elongation, just add the value of Z to ° thus obtaining the true azimuth of the star that particular time of observation.

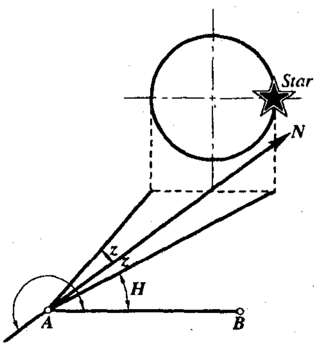
Determination of Azimuth by Polaris at either Upper of Lower Culmination

The direction of the meridian may be determined by observing with a transit at the instant when Polaris and some other stars are in the same vertical plane and then waiting for a certain time until Polaris will be on the meridian. At this instant Polaris is sighted and its direction is then marked on the ground by means of a stake. The observation to determine when the two stars are in the same vertical plane is done by the approximate method by first pointing the vertical hair on Polaris and then lower the telescope by pointing the star to be observed. At upper culmination the Ursa Minor is exactly below the Polaris and at lower culmination, the Cassiopeia is also located directly below the Polaris. This would be repeated until the Polaris and the star other than Polaris, are located on the same vertical hair. The telescope now is pointing the true meridian, and this is marked on the ground.

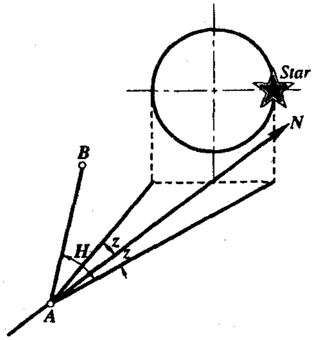
POLARIS AT EASTERN ELONGATION

$Z" = P" \sec L$

H = measured horizontal angle between star and object.

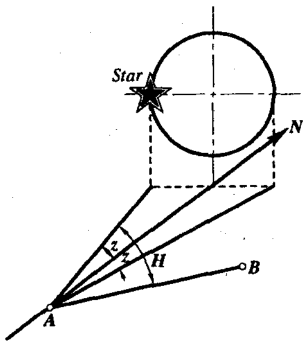


Azimuth of AB = 180 + Z + H

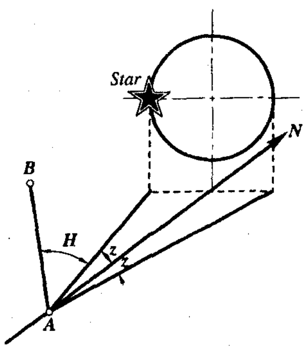


Azimuth of AB = 180 + Z - H

POLARIS AT WESTERN ELONGATION

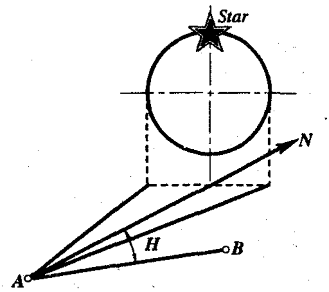


Azimuth of AB = 180 - Z + H

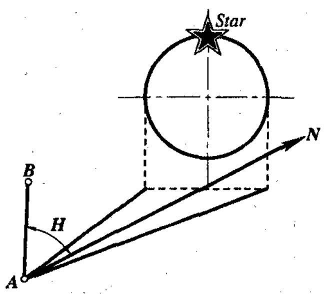


Azimuth of AB = 180 - Z - H

POLARIS AT UPPER CULMINATION

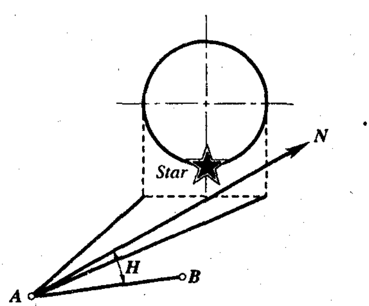


Azimuth AB = 180 + H

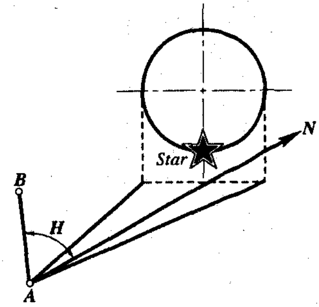


Azimuth AB = 180 - H

POLARIS AT LOWER CULMINATION



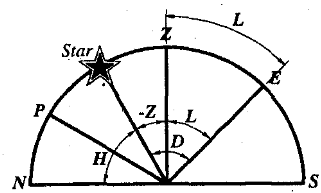
Azimuth AB = 180 + H



Azimuth AB = 180 - H

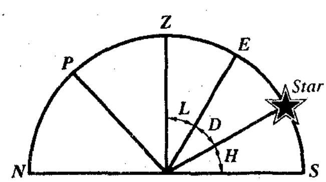
Determination of Latitude

1) *Star between Zenith and Pole*



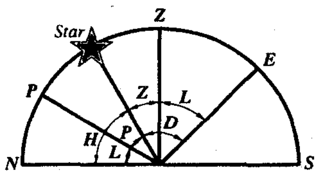
L = D - Z
L = latitude
D = declination
Z = 90 - H
H = vertical angle

2) *Star between the south and equator*



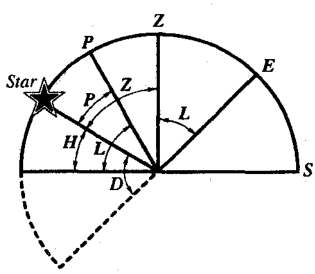
L = 90 - (H + D)
L = Z - D
Z = 90 - H

3) *Polaris at upper culmination*



L = H - P
P = 90 - D (polar distance)
L = D - Z
Z = 90 - H

4) *Polaris at lower culmination*



L = H + P
P = 90 - D

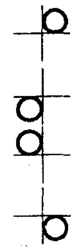
Determination of Azimuth by Solar Observation

Set up the transit at one end of the line whose azimuth is to be determined. With the telescope in the normal position, orient the telescope due south. Sight the other end of the line and record the magnetic azimuth of such line. Then rotate the instrument and point approximately to the position of the sun. Taking precautions that observing the sun directly through the telescopic eyepiece may result injury to the eye. Good observations can be made by bringing the sun's image to a focus on a white card held several inches in the rear of the telescope. Sight the sun in the following order and recording each observation the values of vertical angles, horizontal angle and time. Using the tangent method, the cross wires shall be made tangent to the left and lower right as shown in the following sets of observations.

(TANGENCY METHOD)

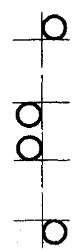
SET I

Position of Telescope	Position of Sun's Image
D	
D	
R	
R	



SET II

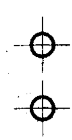
Position of Telescope	Position of Sun's Image
R	
R	
D	
D	



CENTER METHOD

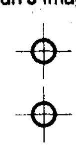
SET I

Position of Telescope	Position of Sun's Image
D	
R	



SET II

Position of Telescope	Position of Sun's Image
R	
D	



Sight the other end of the line again and check whether the reading is still the same as that of the previous one. It must give the same reading otherwise the instrument is disturbed. Take note that the interval of time between any two consecutive sighting shall not exceed 2 minutes, if it does, discard that observation. The date of observation must also be recorded since values of the North Polar Distance from the table is obtained from this date.

Solar Azimuth Formula:

$$\cot\frac{A}{2} = \sqrt{\text{Sec S Sec (S - P) Sin (S - H) Sin (S - L)}}$$

A = azimuth of sun if observed in the afternoon.
360 - A = azimuth of sun if observed in the morning.

$$S = \frac{P + H + L}{2}$$

P = *corrected north polar distance*

Correction applied = Diff. in hours from 8:00 A.M. or 2:00 P.M. multiplied by the variation per hour, which is to be added when observed between June 21 to Dec. 21, and to be subtracted when observed between Dec. 21 to June 21.

H = corrected altitude of sun, corrected for parallax and refraction which is always subtracted from the observed altitude.

L = Latitude of place of observation.

Determination of Time from Solar Observation

$$\text{Tan}\frac{t}{2} = \frac{\sqrt{\text{Sec (S - P) Sin (S - H)}}}{\text{Cot}\frac{A}{2}}$$

$$\text{Tan}\frac{t}{2} = \sqrt{\text{Cos S Sec (S - P) Sin (S - H) Csc (S - L)}}$$

12 - t = local apparent time *(when observed in the morning)*
t = local apparent time *(when observed in the afternoon)*

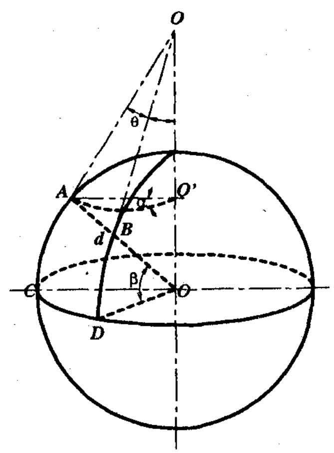
$$S = \frac{P + H + L}{2}$$

P = corrected north Polar distance
H = corrected altitude of sun
L = latitude of place of observation

CONVERGENCY OF MERIDIAN

Convergency of Meridian = is the meeting of two tangents at each point of different longitude as they recede to the north pole.

Angular convergence in seconds = diff. in long in sec. x Sine middle latitude.



ø = angle of convergency
ß = latitude angle of AB
α = difference in longitude between A and B
OB = R = radius of earth approximately 20,890,000 ft.

AB = O'B α

$$\alpha = \frac{AB}{O'B}$$

Angle BOD = Angle BCO' = ß

$$\text{Sin } \beta = \frac{BO'}{BC}$$

$$BC = \frac{BO'}{\text{Sin } \beta}$$

PRACTICAL ASTRONOMY

With negligible error

$AB = BC\ ø$

$ø = \frac{AB}{BC}$

But $AB = BO'\ \alpha$

$BC = \frac{BO'}{\sin ß}$

$ø = \frac{AB}{BC}$

$ø = \frac{BO'\ \alpha \sin ß}{BO'}$

$ø = \alpha \sin ß$

$BO' = R \cos ß$

$\alpha = \frac{AB}{BO'}$

Let AB = d *(distance between A and B)*

$\alpha = \frac{d}{R \cos ß}$

$ø = \frac{d \sin ß}{R \cos ß}$

$ø = \frac{d \tan ß}{R}$ *(radians)*

$ø = 32.38\ d \tan ß$ *(seconds)*

$ø = 32.38 \tan ß$ *(convergency correction)*

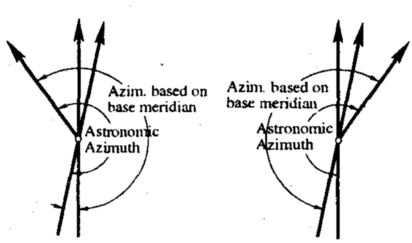
For d = 1 kilometers
If d = kilometers,
R = in feet

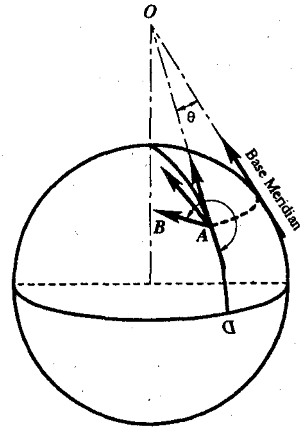
$ø = \frac{3280\ d \tan ß}{20890000}\ \frac{180}{\pi}\ (3600)$

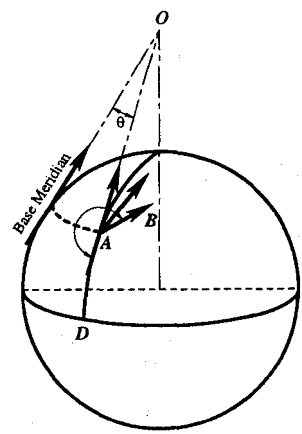
$ø = 32.38 \tan ß$

To obtain azimuth based on base meridian, subtract the covergency correction if the line is on the east of base meridian.

To obtain azimuth based on the base meridian, add the convergency correction if the line is on the west of base meridian.



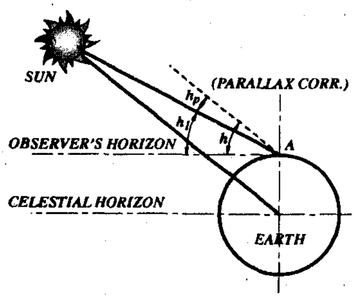




Parallax and Refraction

Parallax Correction:

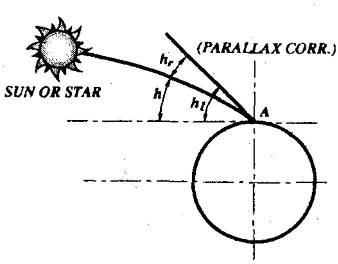
It is assumed that the celestial sphere is of infinite radius and that vertical angle measured from a station on the earth's surface is the sam eas that if it would be measured from the center of the earth. But for stellar or solar observations these angles are not equal. There is an error in this observed vertical angle due to the fact it is observed on the surface and not on the cetner of the earth. This error is called parallax.



h = corrected altitude
h_1 = observed altitude
h_p = parallax correction
$h = h_1 + h_p$ *(parallax correction is added)*

Refraction Correction:

When a ray of light emanating from a celestial body passes through the atmosphere of the earth, the ray is bent downward as shown on the figure. Hence the sun or star appears to be higher above the observer's horizon than they actually are. The angle of deviation of the ray from its direction on entering the earth's atmosphere to its direction at the surface of the earth is called the refraction of the ray.

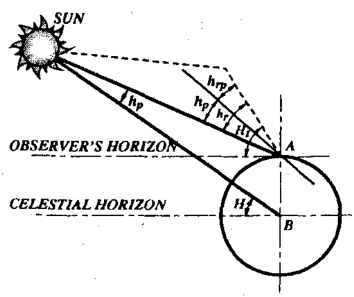


h = corrected altitude
h_1 = observed altitude
h_r = refraction correction
$h = h_1 - h_r$ *(refraction correction is subtracted to the observed altitude)*

Combined Correction due to Parallax and Refraction

H = corrected altitude
H_1 = observed altitude
h_r = refraction correction
h_p = parallax correction
h_{rp} = combined parallax and refraction correction.

$H = H_1 - h_r + h_p$
$H = H_1 - (h_r - h_p)$



$h_{rp} = h_r - h_p$
$H = H_1 - h_{rp}$

Combined correction due to parallax and refraction is always subtracted to the observed altitude.

Problem 231:

The following notes were taken during solar observation.

Position of Telescope	Time	Horizontal Angle	Vertical Angle
D	8:32:00	158°22'	35°21'
D	8:32:30	159°04'	34°55'
R	8:33:05	159°10'	35°36'
R	8:33:40	158°37'	35°13'

PRACTICAL ASTRONOMY

Sta. Occ.: T - 1
Sta. Obs: T - 2
Date of observation: Dec. 12,2004
Initial reading: 00° - 00'
Latitude of T - 1: 12°50'27"
North Polar Distance: 107°02'32"
Hourly variation: - 43.9"
Parallax and refraction: 01'10"

① Compute the corrected north polar distance.
② Compute the azimuth of sun.
③ Compute the azimuth of T - 1 to T - 2.

Solution:

① *Corrected north polar distance:*

	Time
D	8:32:00
D	8:32:30
R	8:33:05
R	8:33:40
Mean =	8:32:48

Diff. in time = 8:32:48
- 8:00:00
0-32:48

Diff. in time = 0.5467 hrs.

Correction for NPD
= *diff. in time* x *variation per hour*
Correction for NPD = 0.5467 (- 43.9) = - 24"
Corrected NPD = 107°02'32"
- 24"
P = **107°02'08"**

② *Azimuth of sun:*

	Horizontal Angle	Vertical Angle
D	158°22'	35°21'
D	159°04'	34°55'
R	159°10'	35°36'
R	158°37'	35°13'
Mean =	158°48'15"	35°16'15"

Corrected H = 35°16'15"
- 1-10"
H = 35°15'05"
P = 107°02'08"
L = 12°50'27"
$2S$ = 155-07-40
S = 77-33-50
P = 107-02-08
$S - P$ = 29-28-18
S = 77-33-50
H = 35-15-05
$S - H$ = 42-18-45
S = 77-33-50
L = 12-50-27
$S - L$ = 64-43-23

$$\text{Cot}\frac{1}{2}A = \sqrt{\text{Sec } S \text{ Sec } (S-P) \text{ Sin } (S-H) \text{ Sin } (S-L)}$$

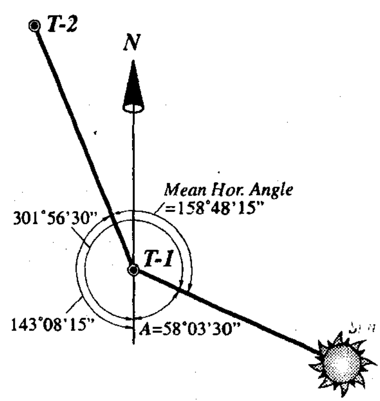
$$\text{Cot}\frac{1}{2}A = \sqrt{\text{Sec } 77°33'50'' \text{ Sec } 29°28'18'' \text{ Sin } 42°18'45'' \text{ Sin } 64°43'23''}$$

$$\frac{1}{2}A = 29\text{-}01\text{-}45$$

$$A = 58\text{-}03\text{-}30$$

Azimuth of sun = 360 - A
Azimuth of sun = **301°56'30"**

③ *Azimuth of* $T_1 - T_2$:



Azimuth $T_1 - T_2$ = 301°56'30"
- 158°48'15"
Azimuth $T_1 - T_2$ = **143°08'15"**

Problem 232:

The following notes were taken during solar observation.

Position of Telescope	Time	Horizontal Angle	Vertical Angle
D	3:35:00	102°59'	36°09'
D	3:45:30	103°39'	35°28'
R	3:46:00	103°57'	36°19'
R	3:46:30	103°19'	35°40'

Date of observation: May 19, 2005
Sta. Occupied: T - 1
Sta. Observed: T - 2
Initial reading: 178°36'
Latitude of T - 1 = 14°53'25"
North Polar Distance = 70°36'24"
Hourly Variation = - 38.57"
Parallax and refraction correction = - 1'09"

① Compute the azimuth of sun.
② Compute the azimuth of T_1 to T_2.
③ Compute the magnetic declination of the place of observation.

Solution:

① *Azimuth of sun:*

	Horizontal Angle
D	102-59
D	103-39
R	103-57
R	103-19
Mean =	103°28'30"

	Time
D	3:45:00
D	3:45:30
R	3:46:00
R	3:46:30
Mean =	3:45:45

	Altitude (H)
D	36-09
D	35-28
R	36-19
R	35-40
Mean =	35°54'00"

Corrected H = 35°54'00"
- 1'09"
Corrected H = 35°52'51"

Diff. in time = 3:45:45 - 2:00:00
Diff. in time = 1:45:45 = 1.7625 hrs.

Correction for North Polar Distance
= *Diff. in hrs* x *Variation per hour*
Correction for NPD = 1.7625 (- 38.57") = 1'08"

Corrected NPD:
P = 70°36'24"
- 1'08"
P = 70°35'16"

P = 70°35'16"
H = 35°52'51"
L = 14°53'25"
$2S$ = 121°21'32"
S = 60°40'46"
P = 70°35'16"
$S - P$ = 9°54'30"
S = 60°40'46"
H = 35°52'51"
$S - H$ = 24°47'55"
S = 60°40'46"
L = 14°53'23"
$S - L$ = 45°17'21"

$$\text{Cot}\,\frac{1}{2}A = \sqrt{\text{Sec}\,S\;\text{Sec}\,(S-P)\;\text{Sin}\,(S-H)\;\text{Sin}\,(S-L)}$$

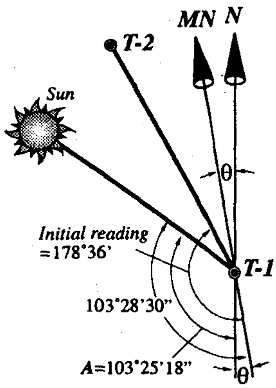
$$\text{Cot}\,\frac{1}{2}A = \sqrt{\text{Sec}\,60°40'46"\;\text{Sec}\,9°54'30"\;\text{Sin}\,24°47'55"\;\text{Sin}\,45°17'21")}$$

$$\frac{1}{2}A = 51°42'39"$$

A = ***103°25'18"***

PRACTICAL ASTRONOMY

② *Azimuth of $T_1 - T_2$:*



Note: Azimuth of sun = A if observation is in the afternoon
Azimuth of sun = 360 - A if observation is in the morning

$\theta = 103°28'30" - 103°25'18"$

$\theta = 03'12"$

Azimuth of $T_1 - T_2$ = 178°36' - 03'12"
*Azimuth of $T_1 - T_2$ = **178°32'48"***

③ *Magnetic declination:*
*Magnetic declination = **03'12" W.***

Problem 233:

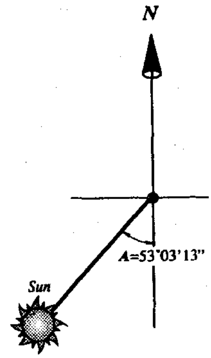
The following data were recorded for an observation of the sun.

Station Occupied: A
Station Observed: B
Time of observation: 2:04:12 P.M.
Altitude of Sun: 51°04'00"
Latitude of place of Observation: 10°22'00"
Declination of Sun: - 13°23'23"
Initial reading: 00°00'
Horizontal angle clockwise from station B to the sun = 142°20'00"

① Compute the true azimuth of sun.
② Compute the true azimuth of line AB.
③ Is the watch too slow or too fast and by how much?

Solution:

① *Azimuth of sun:*



$H = 51°04'00"$
$L = 10°22'00"$
$P = 103°23'23"$
$2S = 164°49'23"$
$S = 82°24'42"$
$S - P = 20°58'41"$
$S - H = 31°20'42"$
$S - L = 72°02'42"$

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } S \text{ Sec }(S-P) \text{ Sin }(S-L) \text{ Sin }(S-H)}$$

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } 82°24'42" \text{ Sec } 20°58'41" \text{ Sin } 72°02'42" \text{ Sin } 31°20'42"}$$

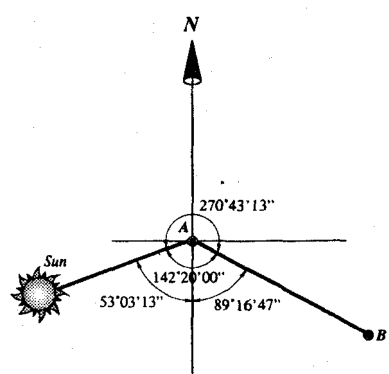
$$\text{Cot}\frac{A}{2} = 2.00$$

$$\frac{A}{2} = 26°31'37"$$

$A = 53°03'13"$

*Azimuth of sun = **53°03'13"***

② *True azimuth of AB:*



True azimuth of AB = 360° - 89°16'47"
True azimuth of AB = **270°43'13"**

③ *Time of observation:*

$$\tan\frac{t}{2} = \sqrt{\text{Cos } S \text{ Sec } (S-P) \text{ Sin } (S-H) \text{ Csc } (S-L)}$$

$$\tan\frac{t}{2} = \sqrt{\text{Cos } 82°24'42" \text{ Sec } 20°58'41" \text{ Sin } 72°02'42" \text{ Sin } 31°20'42"}$$

$$\frac{t}{2} = 15°32'27"$$

$t = 31°04'55"$
$t = 2^h04^m20^s$ *(time of observation)*
2:04:20 - 2:04:12 = 8 sec.

Watch is too slow by 8 seconds.

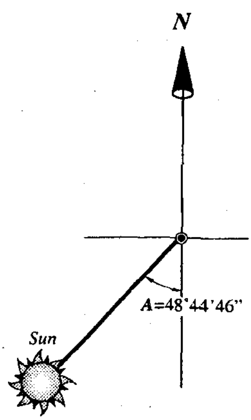
Problem 234:

An observation of the sun was made at latitude 41°20' N. The corrected altitude of sun during observation is 46°48' and its corrected North Polar distance is 82°02'.

① Compute the azimuth of sun if observed in the afternoon.
② Compute the azimuth of sun if observed in the morning.
③ Compute the time of observation if it was observed in the morning.

Solution:

① *Azimuth of sun if observed in the afternoon:*



L = 41°20'
H = 46°48'
P = 82°02'
2S = 170°10'
S = 85°05'
S - P = 3°03'
S - H = 37°05'
S - L = 43°45'

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } S \text{ Sec } (S-P) \text{ Sin } (S-L) \text{ Sin } (S-H)}$$

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } 85°05' \text{ Sec } 3°03' \text{ Sin } 43°45' \text{ Sin } 37°05'}$$

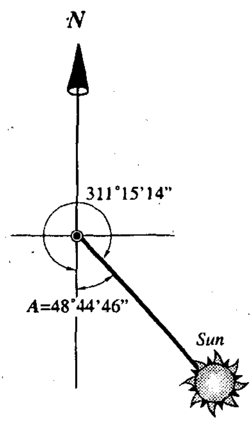
$$\text{Cot}\frac{A}{2} = 2.2072$$

$$\frac{A}{2} = 24°22'23"$$

$A = 48°44'46"$

Azimuth = **48°44'46"** *if observed in P.M.*

② *Azimuth if observed in the morning:*



Azimuth = 360 - A
Azimuth = 360 - 48°44'46"
Azimuth = **311°15'14"** *if observed in A.M.*

③ *Time of observation:*

$$\tan\frac{t}{2} = \sqrt{\text{Cos } S \text{ Sec } (S-P) \text{ Sin } (S-H) \text{ Csc } (S-L)}$$

$$\tan\frac{t}{2} = \sqrt{\text{Cos } 85°05' \text{ Sec } 3°03' \text{ Sin } 37°05' \text{ Csc } 43°45'}$$

$\tan\frac{t}{2} = 0.27357$

$\frac{t}{2} = 15°18'$

$t = 30°36'$

$t = 2^h02^m24^s$

Time = 12:00:00
 - 2:02:24
 9:57:36 A.M.

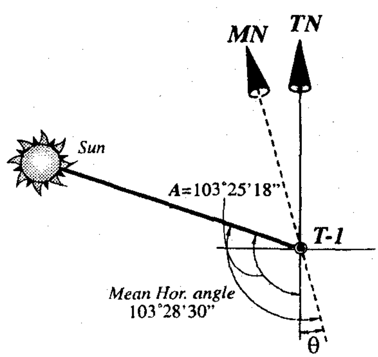
Problem 235:

A solar observation was observed in the afternoon. Station occupied is T-1 and station observed is T-2. Initial reading of mark is 178°36' 00". Angle A as computed from the formula is equal to 103°25' 18". The computed mean horizontal angle is 103°28' 30". North polar distance from table is 70°36' 24". Variation per hour is - 38.57". Time of observation is 3:45:45 P.M.

① Compute the corrected north polar distance.
② Compute the declination at the instant of observation.
③ Compute the true azimuth of line T-1 to T-2.

Solution:

① *Corrected North polar Distance:*

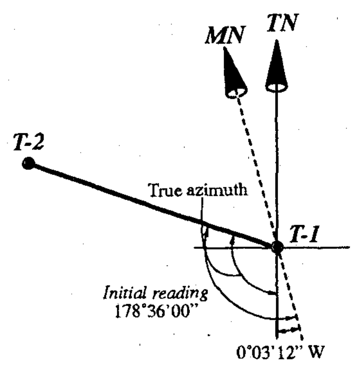


Diff. in time = 3:45:45 - 2:00:00
Diff. in time = 1.7625 hrs.

Correction for NPD = variation per hour x Difference in hours
Correction = - 38.57 (1.7625)
Correction = - 67.98"
Correction = - 1' 08"

Corrected NPD = 70°36' 24"
 - 00°01' 08"
Corrected NPD = ***70°35' 16"***

② *Declination at the instant of observation:*



Note: Angle A is on the west if observed on the afternoon

θ = 103°28' 30" - 103°25' 18"

θ = ***00°03' 12" W.*** *(declination)*

③ *True azimuth:*
True azimuth of T-1 to T-2
= 178°36' 00" - 0°03' 12"
= ***178°32' 48"***

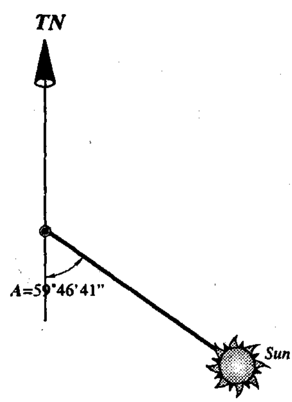
Problem 236:

The computed value of angle A by using the formula is equal to 59°46' 41". Initial reading on the horizontal verneir is 00°00'. The altitude of sun at the instant of observation was recorded to be 36°49' 45". Parallax and refraction correction is 01'07" variation per hour is + 49.01". Mean horizontal angle is 313°48' 45". Time of observation is 8:41:52 A.M.

① Compute the corrected altitude of the sun.
② Compute the azimuth of sun.
③ Compute the azimuth of mark.

Solution:

① *Corrected altitude:*



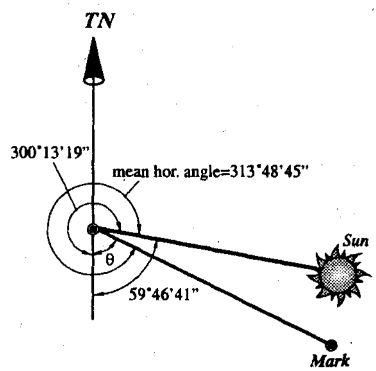
Recorded altitude H	=	36°49' 45"
Parallax & refraction		01' 07"
Corrected H		36°48' 38"

② *Azimuth of sun:*

Azimuth of sun = 360° - 59°46' 41"

Azimuth of sun = ***300°13' 19"***

③ *Azimuth of mark:*



θ = 313°48' 45" - 300°13' 19"

θ = 13°35' 26"

Azimuth of mark = 360° - 13°35' 26"
Azimuth of mark = ***346°24' 34"***

Problem 237:

A solar observation was observed using a theodolite and tabulated the following results and computations.

Initial horizontal circle reading = 00°00'
Sta. occupied = BLLM #1
Sta. observed = BLLM #2
Latitude of sta. occupied = 14°33' 20"

Horizontal Circle Reading	Time	Zenith Angle
359-02-00	8:32:07	48-33-48
358-19-47	8:32:32	48-49-50
178-19-44	8:33:06	311-50-20
179-02-25	8:33:30	311-25-18

Computed angle A = 104°23' 34"

① If the parallax and refraction is 0' 58", what is the corrected value of H.
② Compute the true bearing of the sun.
③ Compute the azimuth of BLLM #1 to BLLM #2.

Solution:

① *Corrected vertical angle H:*

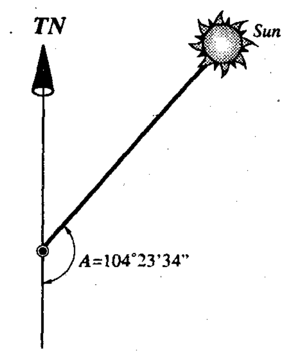
Note: $H = 90° - \text{zenith angle}$

$H_1 = 90° - 48°33'48" = 41°26'12"$
$H_2 = 90° - 48°49'50" = 41°10'10"$
$H_3 = 90° - 48°09'40" = 41°50'20"$
$H_4 = 90° - 48°34'42" = 41°25'18"$
$\qquad\qquad\qquad\qquad 165°52'00"$

$$\text{Average } H = \frac{165°52'00"}{4}$$

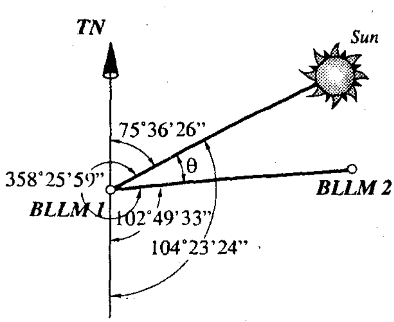
Average H = 41°28' 00"
Corrected H = 41°28' 00" - 0°58'
Corrected H = ***41°27' 02"***

② *True bearing of sun:*



Azimuth of sun = 360° - 104°23' 34"
Azimuth of sun = 255°36' 26"
True bearing = ***N. 75°36' 26" E.***

③ *Azimuth of BLLM #1 to BLLM #2:*



Horizontal angle:
359-02-00
358-19-47
358-19-44
359-02-25
1433°43' 56"

$$\text{Ave. Horizontal angle} = \frac{1433°43'56"}{4}$$

Ave. Horizontal angle = 358°25' 59"
θ = 104°23' 24" - 102°49' 33"
θ = 1°33' 51"

Azimuth of BLLM #1 to BLLM # 2
= 255°36' 26" + 1°33' 51"
= ***257°10' 17"***

Problem 238:

A solar observation was performed in determining the direction of the line from BLLM #1 to BLLM #2.

Latitude of BLLM # 1 : 17°16'4.80" N
Longitude of BLLM # 2 : 121°13'50.8" E.
Initial horizontal circle reading : 00°00'00"
NPD = 51°13'46"
Variation/hour = 00°00'09"
Parallax & Refraction correction = 00°00'59.39"

Field Data:

SET 1

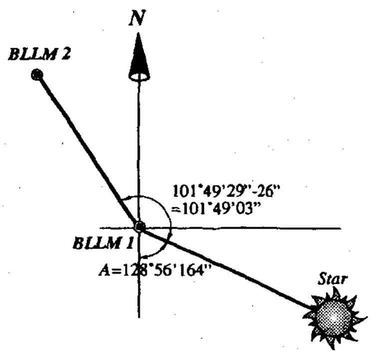
Time	Hor. Circle Reading	Vertical Angle
8:14:36	101 - 50 - 25	47 - 06 - 12
8:15:09	101- 48 - 33	47- 01 - 11

SET 2

Time	Hor. Circle Reading	Vertical Angle
8:15:12	101 - 47 - 33	43°58' 01"
8:15:51	101 - 45 - 57	46°57' 17"

① Compute the azimuth of sun.
② Compute the true azimuth of BLLM No. 1 to BLLM No. 2.
③ Compute the probable error of azimuth.

Solution:



Set 1		
Time	Hor. Circle Reading	Vertical Angle
8:14:36	101 - 50 - 25	47 - 06 - 12
8:15:09	101 - 48 - 33(sub.180°)	47 - 01 - 11 (sub.360°)
8:14:52.5	101 - 49 - 29	47 - 03 - 41.5

Corrected H = 47°- 03'- 41.5" - 59.39"
H = 47°02'42'11"

Diff. in time = 8:14:52.5
8:00:00
14:52.5 = 0.2479 hrs.

Correction for NPD = 0.2479 (09")
Correction for NPD = 2.23"

$P = 51°13'46" - 2.23"$
$P = 51°13'43.77"$
$H = 47°02'42.11"$
$L = 17°16'4.80"$
$2S = 115°32'30.6"$
$S = 57°46'15.34"$
$P = 51°13'43.77"$
$S - P = 6°32'31.57"$
$S = 57°46'15.34"$
$H = 47°02'42.11"$
$S - H = 10°43'33.23"$
$S = 57°46'15.34"$
$L = 17°16'4.80"$
$S - L = 40°30'10.54"$

$$\text{Cot}\frac{A}{2} = \sqrt{\sec S \ \text{Sec}(S\text{-}P)\ \text{Sin}(S\text{-}H)\ \text{Sin}(S\text{-}L)}$$

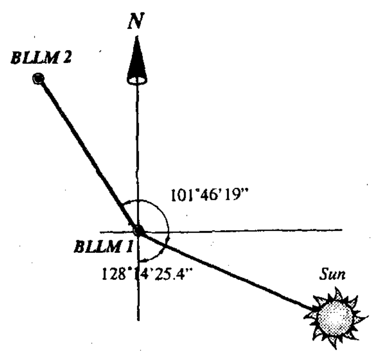
$$\text{Cot}\frac{A}{2} = \sqrt{0.228140761}$$

$$\frac{A}{2} = 64°28'8.24"$$

$$A = 128°56'16.4"$$

Azimuth of sun = 360° - 128°56'16.4"
Azimuth of sun = 231°03'43.6"

Azimuth of $BLLM_1$ *to* $BLLM_2$
= 231°03'43.6" - 101°49'29"
= 129°14'14.6"



Set 2		
Time	Hor. Circle Reading	Vertical Angle
8:15:12	101 - 47 - 33	43°58'01"
8:15:51	101 - 45 - 57	46°57'17"
8:15:31.5	101 - 46 - 45	45°27'39"

Diff. in hrs. = 8:15:31.5 - 8:00:00
Diff. in hrs. = 0.25875
Corr. = 0.25875 (09")
Corr. = 2.33"
P = 51°13'46"
- 2.33"
51°13'43.67"

Corrected H = 45°27'39"
- 59.39"
H = 45°26'39.61"
P = 51°13'43.67"

$L = 17°16'4.80''$
$2S = 113°56'28.10''$
$S = 56°58'14.09''$
$P = 51°13'43.67''$
$S - P = 5°44'30.33''$
$S = 56°58'14.09''$
$H = 45°26'39.61''$
$S - H = 11°31'34.4''$
$S = 56°58'14''$
$L = 17°16'4.80''$
$S - L = 39°42'9.2''$

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } S \text{ Sec } (S-P) \text{ Sin } (S-H) \text{ Sin } (S-L)}$$

$$\text{Cot}\frac{A}{2} = \sqrt{0.235359069}$$

$$\frac{A}{2} = 64°07'12.74''$$

$$A = 128°14'25.4''$$

Azimuth of sun = 360° - 128°14'25.4"
Azimuth of sun = 231°45'34.6"

① *True azimuth of sun:*

$$= \frac{231°\text{-}03'\text{-}43.6'' + 231°\text{-}45'\text{-}34.6''}{2}$$

True azimuth of sun = ***231°24' 39"***

② *Azimuth of BLLM #1 to BLLM #2:*
True azimuth of BLLM No. 1 to BLLM No. 2
= 231°45'34.6" - 101°46'45"
= 129°58'49.6"

True azimuth of BLLM No. 1 to BLLM No. 2

$$= \frac{129°58'49.6'' + 129°14'14.6''}{2}$$

= ***129°36' 32.1"***

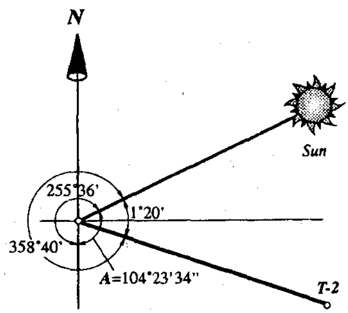
③ *Probable error of azimuth:*
Probable error of azimuth
= 0.33725 x *difference in azimuth*
Difference in azimuth
= 129°58'49.6" - 129°14' 14.6"
Difference in azimuth = 44' 35" = 2675"

Probable error of azimuth = 0.33725(2675)
Probable error of azimuth = 902"
Probable error of azimuth = ***15' 02"***

Problem 239:

From the field notes of a solar observation using wild T-2 theodolite, the following data are observed.

Sta. Occupied : T-1
Sta. Observed : T-2
Latitude of T-1 : 14°33'40.73"
Initial Horizontal Circle Reading = 00°00'00"
Date of Observation : May 22, 1993
North Polar Distance : 69°39'26.40"
Hourly Variation : 0°00'29.64"
Correction for Parallax and Refraction
= 0°00'56" and 0°00'55"



SET I

Position of Telescope	Time	Hor. Circle Reading	Zenith Angle
Direct Upper Right	8:32:07	359°02'00"	48°33'48"
Direct Lower Left	8:32:31	358°19'47"	48°49'59"
Reverse Upper Left	8:33:09	178°19'44"	311°50'43"
Reverse Lower Right	8:33:36	179°02'25"	311°25'17"

SET II

Position of Telescope	Time	Hor. Circle Reading	Zenith Angle
Reverse Upper Right	8:33:54	179°02'12"	311°34'03"
Reverse Lower Left	8:34:14	178°20'09"	311°34'03"
Direct Upper Left	8:34:44	358°20'54"	47°46'17"
Direct Lower Right	8:35:08	359°03'57"	48°12'50"

① Compute the corrected altitude for set I.
② Compute the true azimuth of T_1 to T_2.
③ Compute the probable error.

Solution:

① *Corrected altitude for set I.*

Note: Vertical angle = 90° - zenith angle

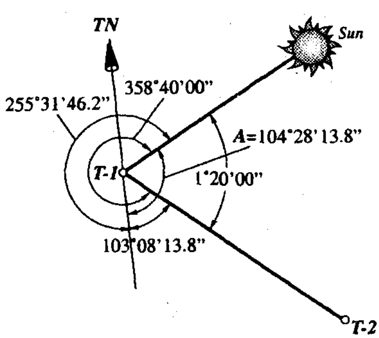
Position of Telescope	Time	Horizontal Angle
D	8:32:07	359-02-00
D	8:32:31	358-17-47
R	8:33:09	358-19-44
R	8:33:36	359-02-25
	8:32:50.7	358-40-00

Position of Telescope	Zenith Angle	Vertical Angle
D	48-33-48	41-26-12
D	48-49-59	41-10-01
R	48-09-17	41-50-43
R	48-34-43	41-25-17
		41-28-03.25

Corrected altitude for set I = 41°28'03.25" - 0°00'56"

Corrected altitude H = ***41°27'07.25"***

② *True azimuth of T_1 to T_2.*



Diff. in time = 8:32:50.7 - 8:00:00

Diff. in time = 32:50.7

Diff. in time = 0.5474166 hrs.

Correction for NPD = Variation per hour x Diff. in hrs.

Correction for NPD = 29.64 (0.5474166)

Correction for NPD = 16.23"

Uncorrected NPD	=	69°-39'-26.40"
Correction	=	16.23"
Corrected NPD	=	69°-39'-10.17"

$$S = \frac{P + H + L}{2}$$

$P = 69°39'10.17"$

$H = 41°27'7.25"$

$L = 14°33'40.73"$

$2S = 125°39'58.1"$

$S = 62°49'59"$

$S - P = 6°49'11.17"$

$S - H = 21°22'51.75"$

$S - L = 48°16'18.27"$

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } S \text{ Sec } (S - P) \text{ Sin } (S - H) \text{ Sin } (S - L)}$$

$$\text{Cot}\frac{A}{2} = 0.77469$$

$$\frac{A}{2} = 52°11'6.91"$$

$A = 104°28'13.8"$

Azimuth of sun = 360° - 104°28'13.8"

Azimuth of sun = 255°31'46.2"

True azimuth of T_1 to T_2 = 255°31'46.2"
+ 1°20'00"

True azimuth of T_1 to T_2 = 256°51'46.2"

SET II

Position of Telescope	Time	Horizontal Angle
R	8:33:36	359-02-12
R	8:34:14	358-20-09
D	8:34:44	358-20-54
D	8:35:08	359-03-57
	8:34:25.2	358-41-00

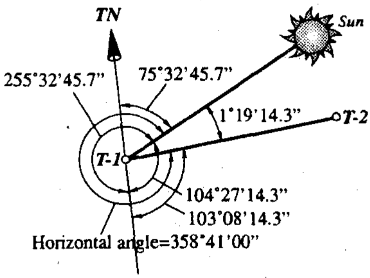
Position of Telescope	Zenith Angle	Vertical Angle
R	47-58-58	42-01-02
R	48-25-57	41-34-03
D	47-46-17	42-13-43
D	48-12-50	41-47-10
		41-53-59.5

Corrected H = 41°53'59.5"
- 0°00'55"

Corrected H = 41°53'04.5"

Diff. in time = 8:34:25.5 - 8:00:00

Diff. in time = 34'25.5" = 0.57375 hrs.



Correction for NPD = 29.64" (0.57375)
Correction for NPD = 17"
Corrected NPD (P) = 69°39'-26.40"
- 17"

P = 69°39'9.40"
H = 41°53'9.40"
L = 14°33'40.73"
$2S$ = 126°05'54.63"

S = 63°02'57.32"
$S - P$ = 6°36'12.08"
$S - H$ = 21°09'52.82"
$S - L$ = 48°29'16.59"

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } S \text{ Sec }(S - P) \text{ Sin }(S - H) \text{ Sin }(S - L)}$$

$$\text{Cot}\frac{A}{2} = 0.774925257$$

$$\frac{A}{2} = 52°13'37.17"$$

$$A = 104°27'14.3"$$

Azimuth of sun = 360° - 104°27'14.3"
Azimuth of sun = 255°32'45.7"

Azimuth of T_1 to T_2 = 255°32'45.7"
+ 1°19'00"
Azimuth of T_1 to T_2 = 256°51'45.7"

Average azimuth

$$= \frac{256°51'46.2" + 256"51'45.7"}{2}$$

Average azimuth = ***256°51'46"***

③ *Probable error:*
Diff. in azimuth = 256°51'46.2" - 256°51'45.7"
Diff. in azimuth = 0.5"
Probable error = 0.33725 x *Diff. in azimuth*
Probable error = 0.33725 (0.5")
Probable error = ***0.169"***

Problem 240:

A solar observation was made at BBM # 50 with a latitude of 10°23' 05.29" N. and longitude of 124°58' 42" E. The mean altitude is 51°05' 00", the North polar distance of the place of observation is 103°24' 30.24" and variation per hour is - 38.24" for a difference of time of 1:45:20. If the parallax and refraction is - 38.92".

① Compute the azimuth of sun which is equal to A.
② What will be the value of "t".
③ If the equation of time is - 0°14' 15.8", what is the local mean time.
③ What is the standard time at 120° E. meridian.

Solution:

① *Azimuth of sun:*
Diff. in hours = 1:45.20
Diff. in hours = 1.756 hrs.
Correction for NPD = - 38.24(1.756)
Correction for NPD = - 67.15" = - 01'7.15"

P = 103°24' 30.24"
- 01' 23.09"
P = 103°23' 23.09"
H = 51°05' 00"
- 38.92"
H = 51°04' 21.08"
L = 10°23' 5.29"
P = 103°23' 23.09"
$2S$ = 164°50' 49.4"
S = 82°25' 24.73"
P = 103°23' 23.09"
$S - P$ = 20°57' 58.36"
S = 82°25' 24.73"
H = 51°04' 21.08"
$S - H$ = 31°21' 3.65"
S = 82°25' 24.73"
L = 10°23' 5.29"
$S - L$ = 72°02' 19.44"

$$\text{Cot}\frac{A}{2} = \sqrt{\text{Sec } S \text{ Sec } (S\text{-}P) \text{ Sin } (S\text{-}L) \text{ Sin } (S\text{-}H)}$$

$$\text{Cot}\frac{A}{2} = 2.0$$

$$\frac{A}{2} = 26.51^\circ$$

$$A = \mathbf{53^\circ 01'}$$

② *Value of t:*

$$\tan\frac{t}{2} = \sqrt{\text{Cos } S \text{ Sec } (S\text{-}P) \text{ Sin } (S\text{-}H) \text{ Csc } (S\text{-}L)}$$

$$\frac{t}{2} = 15.53^\circ$$

$$t = \mathbf{31^\circ 03' 36''}$$

③ *Local mean time:*

$t = 31^\circ 03' 36''$

$t = 2^h\, 04^m\, 14.4^s$

Local mean time $= 2^h\, 04^m\, 14.4^s - (-14'15.8'')$

Local mean time $= 2^h\, 18^m\, 30.2^s$

Local mean time = **2:18:30.2**

③ *Standard time at 120th meridian:*

Diff. in longitude = 124°58' 42" - 120°

Diff. in longitude = 4°58' 42"

Diff. in longitude $= \frac{4.978^\circ}{15}$

Diff. in longitude $= 0^h\, 19^m\, 54.8^s$

Standard time = 2:18:30.2
- 19:54.8

Standard time = **1:58:35.4**

Problem 241:

From the given data of a Solar observation using a Wild T-2 theodolite
Sta. Occ. BLLM # 1
Sta. Obs. BLLM # 2
Latitude of BLLM # 1 = 14°20' 13.6"
NPD = 68°22' 42.4"
Hourly variation : 26.64"
Initial reading = 0°00' 00"

Horizontal Angle Reading	Time	Vertical Angle
359°02' 00"	8:32:07	48°42' 30"
358°19' 42"	8:33:05	48°49' 50"
358°20' 21"	8:33:09	48°50' 32"
179°01' 36"	8:33:34	48°45' 40"

① Compute the corrected North Polar Distance.
② What will be the azimuth of the Sun.
③ What will be the azimuth of the mark.

Solution:

① *Corrected North Polar Distance:*

Horizontal Angle	Time	Vertical Angle
358°40' 54.7"	Ave: 8:32:58.75	48°47' 08" (Ave)

Diff. in time = 8:32:58.75 - 8:00

Diff. in time = 00 - 32:58.75

Diff. in time = 0.5496 hrs.

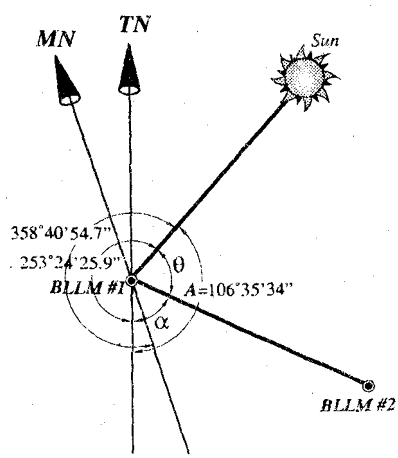
Corrected for NPD = 0.5496(26.64)

Corrected for NPD = - 14.64"

Corrected NPD = 68°22' 42.4" - 14.64"

P = **68°22' 27.76"**

② *Azimuth of sun:*



H = 48°47' 08"
P = 68°22' 27.76"
L = 14°20' 13.6"
2S = 131°29' 49.3"
S = 65°44' 54.65"

$P = 68°22'27.76"$
$S\text{-}P = 2°37'33.11"$
$S = 65°44'54.65"$
$H = 48°47'08"$
$S\text{-}H = 16°57'46.65"$
$S = 65°44'54.65"$
$L = 51°24'41.05"$
$S\text{-}L = 51°24'41.05"$

$$\cos\frac{A}{2} = \sqrt{\text{Sec } S \text{ Sec } (S\text{-}P) \text{ Sin } (S\text{-}H) \text{ Sin } (S\text{-}L)}$$

$$\frac{A}{2} = 53.30'$$

$A = 106°35'34.1"$

Azimuth of Sun = 360° - 106°35'34.1"

Azimuth of Sun = **253°24'25.9"**

③ *Azimuth of Mark:*

358°40'54.7"
\- 253°24'25.9"

$\alpha = 105°16'28.8"$

$\theta = 106°35'34.1" - 105°16'28.8"$

$\theta = 1°19'5.2"$

$\theta = 1°19'5.2"$

Azimuth of Mark = 253°24'25.9"
+ 1°19'5.2"

Azimuth of Mark = **254°43'31.2"**

Problem 242:

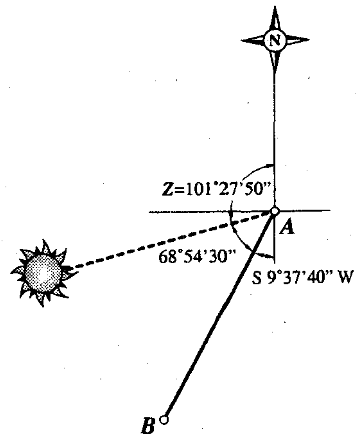
An observation was made to determine the azimuth of the line AB by observing the altitude of sun in the afternoon. The following data were observed.

Latitude of place of observation = 42°29'30" N.
Longitude of place of observation = 124°20'30" E.
Mean Horizontal Angle from station B to the sun = 68°54'30" (clockwise)
Mean altitude of Sun (corrected) = 43°16'48"
Declination of Sun = 20°52'44"

① Compute the true bearing of sun from the North.
② Compute the true azimuth of sun.
③ Compute the true azimuth of AB.

Solution:

① *True bearing of sun from the North:*



$$\cos Z = \frac{\sin D}{\cos L \cos h} - \tan L \tan h$$

Note:
Z = true bearing of sun from the north
NW if observed in the afternoon
NE if observed in the morning
$D = 20°52'44"$
$L = 42°29'30"$
$h = 43°16'48"$

$$\cos Z = \frac{\sin D}{\cos L \cos h} - \tan L \tan h$$

$$\cos Z = \frac{\sin 20°52'44"}{\cos 42°29'30" \quad \cos 43°16'50"}$$

$\cos Z = -0.19875$

$Z =$ **101°27'50"**

② *True azimuth of the sun:*

True azimuth of the sun
= 180°00'00" - 101°27'50"

True azimuth of the sun = **78°32'10"**

③ *True azimuth of AB:*

True azimuth of AB = 78°32'10" - 68°54'30"

True azimuth of AB = **9°37'40"**

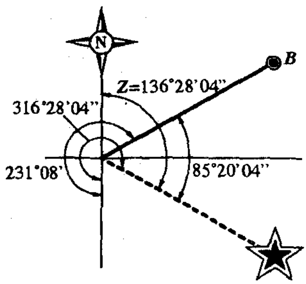
Problem 243:

A stellar observation was made to determine the azimuth of line AB by observing an altitude of star which was recorded to be 28°36' 48" (star bearing to East). The latitude of place of observation was found to be 39°14' 12" N and the declination of star at the instant of observation was - 10°52' 18" S. The horizontal angle between station B to the star measured clockwise is 85°20' 04".

① Compute the bearing of star from the north.
② Compute the azimuth of star.
③ Compute the azimuth of AB.

Solution:

① *Bearing of star from the north:*



$$\text{Cos } Z = \frac{\text{Sin } D}{\text{Cos L Cos H}} - \tan L \tan H$$

$$\text{Cos } Z = \frac{\text{Sin } (-10°57' 18")}{\text{Cos } 39°14'12" \text{ Cos } 28°36'48"} - \tan 39°14'12" \tan 28°36'48"$$

$Z = \mathbf{136°28' 04"}$ *East of North*

② *Azimuth of star:*

Azimuth of star = 136°28' 04" + 180

Azimuth of star = ***316°28' 04"***

③ *Azimuth of AB:*

Azimuth of AB = 316°28' 04 - 85°20' 04"

Azimuth of AB = ***231°08' 00"***

Problem 244:

In a certain locality, a Geodetic Engineer desiring to verify the true azimuth of BLLM No. 1 to BLLM No. 2 observed Polaris at Eastern Elongation and obtained the following data:

Vertical angle of Polaris:
Direct = 10°21' 30"
Reverse = 10° 22' 20"
Station occupied: BLLM No. 1
Mark: BLLM No. 2 (right of star)
Latitude of BLLM No. 1: 14°45' 00"
Co-declination of Polaris for the date of observation was 1°15' 40"

① Find the azimuth of Polaris measured from the north.
② Find the true azimuth of BLLM No. 1 to BLLM No. 2.
③ Compute the latitude of the place of observation when Polaris attains its upper culmination with an altitude of 43°37'. Index correction is - 30" and refraction is 1' 01".

Solution:

① *Azimuth of Polaris measured from the north.*

$$H = \frac{10°21' 30" + 10°22' 20"}{2} = 10°21' 55"$$

$Z" = P" \sec L$

$P = 1°15' 40"$

$P" = 3600 + 15(60) + 40$

$P" = 4540$

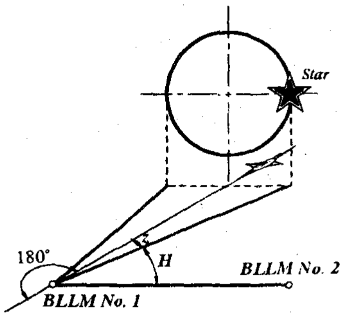
$Z" = P" \sec L$

$Z" = 4540 \sec 14°45'$

$Z = 4695"$

$Z = \mathbf{1°18' 15"}$

② *True azimuth of BLLM No. 1 to BLLM No. 2.*

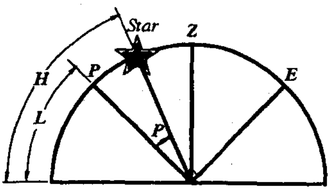


True azimuth of BLLM No. 1 to BLLM No. 2
$= 180 + Z + H$

True azimuth $= 180° + 1°18'15" + 15°21'55"$

True azimuth = **$191°40'10"$**

③ *Latitude of Observation:*



Altitude	$= 43°37'00"$
Index correction	$= +\ 00'30"$
Refraction correction	$= -\ 01'01"$
Corrected H	$= 43°36'29"$

$L = H - P$

$P = 1°15'40"$

$L = 43°36'29" - 1°15'40"$

L = **$42°20'49"$**

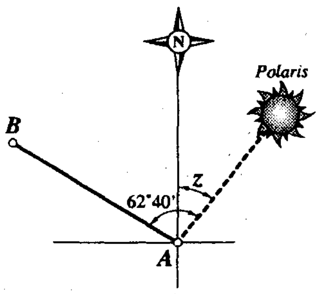
Problem 245:

Polaris is observed at a certain hour angle equal to 45°30' at a certain place whose latitude is 42°20' and a declination of 86°40'. A horizontal angle was measured from the line AB clockwise towards Polaris (East of North) and was recorded to be 62°40'.

① Compute the bearing of Polaris measured from north.
② Compute the true azimuth of Polaris.
③ Compute the true azimuth of AB.

Solution:

① *Bearing of Polaris measured from north:*



$$\tan Z = \frac{\sin t}{\cos L \tan D - \sin L \cos t}$$

$$\tan Z = \frac{\sin 45°30'}{\cos 42°20' \tan 86°40' - \sin 42°20' \cos 45°30'}$$

$Z = 3°20'25"$

Bearing of Polaris = **N. $3°20'25"$ E.**

② *True azimuth of Polaris:*

True azimuth of Polaris = **$183°20'25"$**

③ *True azimuth of AB:*

True azimuth of AB $= 183°20'25" - 62°40'00"$

True azimuth of AB = **$120°40'25"$**

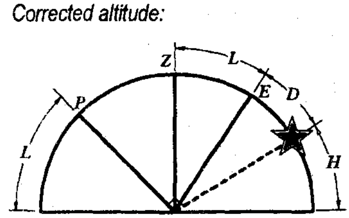
Problem 246:

The observed meridian altitude of a star on April 10, 1990 was 39°24', star bearing south. Refraction correction is 1'11". The declination of the star at that instant was - 8°29'21".

① Determine the corrected altitude.
② Determine the latitude of the place of observation.
③ Determine the hour angle of the star.

Solution:

① *Corrected altitude:*



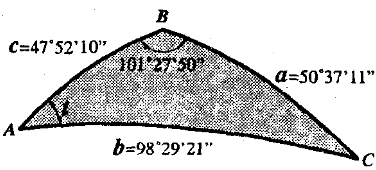
Observed altitude	= 39°24' 00"
Refraction Correction	= - 1' 11"
Altitude H	= ***39°22' 49"***

② *Latitude of the place of observation.*

H	= 39°22' 49"
D	= 8°29' 21"
90 - *L*	= 47°52' 10"

H + *D* = 47°52' 10"
90 - *L* = 47°52' 10"
L = 90 - 47°52' 10"
L = ***42°09' 50"***

③ *Hour angle of the star.*



$$\frac{\text{Sin } b}{\text{Sin } B} = \frac{\text{Sin } a}{\text{Sin } A}$$

$$\frac{\text{Sin } 98°29'21''}{\text{Sin } 101°27'50''} = \frac{\text{Sin } 50°37'11''}{\text{Sin } A}$$

A = 49°59' 21"

Hour angle t = 49.989°

$$t = \frac{49.989°}{15°}$$

$t = 3^h\ 19^m\ 58^s$

Problem 247:

An observed altitude of Polaris at an hour angle of 51°20' 46.5" was recorded to be 43°28' 30". Index error is + 01'00". Declination of Polaris at this instant is + 89°04' 55.6". Refraction correction is 01'00".

① Determine the corrected altitude of Polaris.
② Compute the hour angle in hours, minutes and seconds.
③ Determine the latitude of place of observation.

Solution:

① *Corrected altitude of Polaris.*

Obs. altitude = 43°28' 30"

Index error = - 01'00"

Refraction Corr. = - 01'00"

Corrected altitude:

h = 43°28' 30" - 01' - 01'

h = ***43°26' 30"***

② *Hour angle in hours, minutes and seconds.*

Hour angle:

t = 51°20' 46.5"

$$t = \frac{51.34625°}{15°}$$

$t = 3^h 25' 23.1''$

③ *Latitude of place of observation.*

p = 90° - *D*

p = 90° - 89°04' 55.6"

p = 55' 04.6"

p = 55.08'

$$L = h - p \text{ Cos } t + \frac{1}{2} \text{ Sin } 1' \ p^2 \text{ Sin}^2 t \text{ tan } h$$

L = latitude of place of observation

h = corrected altitude of polaris

t = hour angle in degrees, minutes and seconds

p = polar distance in minutes

$$L = h - p \text{ Cos } t + \frac{1}{2} \text{ Sin } 1' \ p^2 \text{ Sin}^2 t \text{ tan } h$$

$$L = 43°26'30'' - 55.08 \text{ Cos } 51°20'46.5''$$

$$+ \frac{1}{2} \text{ Sin } 1' \ (55.08)^2 \text{ Sin}^2 51°20'46.5'' \text{ tan } 43°26'30''$$

L = 43°26' 30" - 34.40' + 0.25'

L = 43°26' 30" - 34' 39"

L = ***42°51' 51" N***

Problem 248:

The measured altitude of the suns upper limb when on the meridian at a point in longitude 73° W was 47°10' 00" on March 10, 2005.

Declination of sun = + 3°35' 02"
Index correction = + 02' 00"
Refraction and parallax = - 0'18"
Sun's semi-diameter = + 12'06"

① Determine the latitude of place of observation.
② Determine the local sidereal time if the hour angle of the sun during observation was - 32°15' 45" and the apparent right ascension of the sun is $5^h\ 47^m\ 30^s$.
③ Determine the Greenwich standard time if the equation of time is $5^m\ 30^s$.

Solution:

① *Latitude of place of observation:*

$L = 90 - H + D$

Observed H	=	47°10' 00"
Index Correction	=	- 02' 00"
Refraction and Parallax	=	- 0'18"
Suns semi-diameter	=	+ 12'06"
Corrected H	=	47° 19' 48"
D	=	+ 3° 35' 02"

$L = 90 - H + D$
$L = 90° - 47°19'48" + 3°35'02"$
$L = $ **46°15' 14"**

② *Sidereal time:*

Sidereal time = hour angle + right ascension
Hour angle = - 32° 15' 45"
Hour angle = $-2^h\ 09^m\ 03^s$
Sidereal time = $-2^h\ 09^m\ 03^s + 5^h\ 47^m\ 30^s$
Sidereal time = $\mathbf{3^h\ 38^m\ 27^s}$

③ *Greenwich standard time:*

$$\text{Greenwich apparent time} = \frac{73°00'00"}{15°}$$

Greenwich apparent time = $4^h\ 52^m\ 00^s$

Greenwich apparent time
= *Greenwich civil time + equation of time*
$4^h\ 52^m\ 00^s = \text{G.C.T.} + 5^m\ 30^s$
$\text{G.C.T.} = 4^h\ 46^m\ 30^s$

Greenwich civil time = Greenwich standard time
Greenwich standard time = $\mathbf{4^h\ 46^m\ 30^s}$

Problem 249:

In preparation for an observation on the star gives a low altitude is calculated for the star at the moment of observation. The calculated altitude of the star is 17°36.8'. The calculated refraction correction for that altitude is 00°03'.

① What altitude will be measured when the star is observed if these calculated values are correct.
② If the declination of the star at the moment of observation is 12°25' and the latitude of point of observation is 42°21' N, compute the bearing of the star.
③ Compute the hour angle of the star at the instant of observation.

Solution:

① *Altitude of star:*

Calculated altitude	=	17° 36.8'
Refraction Corr.	=	- 03'
Corrected H	=	**17°33.8'**

② *Bearing of star:*

$$\text{Cos } Z = \frac{\text{Sin } D}{\text{Cos L Cos H}} - \tan L \tan H$$

$$\text{Cos } Z = \frac{\text{Sin } 12°25'}{\text{Cos } 42°21' \text{ Cos } 17°33.8'} - \tan 42°21' \tan 17°33.8'$$

$Z = 89°02' 44"$

Bearing of star = **N. 89°02' 44" E.**

③ *Hour angle of the star:*

$$\text{Cos } t = \frac{\text{Sin } H}{\text{Cos D Cos L}} - \tan D \tan L$$

$$\text{Cos } t = \frac{\text{Sin } 17°33.8'}{\text{Cos } 12°25' \text{ Cos } 42°21'} - \tan 12°25' \tan 42° 21'$$

t = *77°26' 37"*

Problem 250:

The right ascension of Polaris is $1^h\,47^m\,10.3^s$ and a declination of 89°01' 56". If the hour angle is $2^h\,42^m$.

① What is the azimuth of Polaris east of north at a place in latitude 48°16' N and longitude 120°30' W.
② What is the sidereal time at the instant of observation.
③ If the local apparent time at the place of observation was $4^h\,50^m\,20^s$, what is the Greenwich apparent time?

Solution:

① *Azimuth of Polaris:*

$$\tan Z = \frac{\sin t}{\cos L \tan D - \sin L \cos t}$$

$t = 2^h\,42^m$

$t = 40°\,30'$

$$\tan Z = \frac{\sin 40°30'}{\cos 48°16' \tan 89°01'56" - \sin 48°16' \cos 40°30'}$$

$Z = \mathbf{0°57'\,29"}$

② *Sidereal time:*

Sidereal time = Right ascension + hour angle

Sidereal time = $1^h\,47^m\,10.3^s + 2^h\,42^m$

Sidereal time = $\mathbf{4^h\,29^m\,10.3^s}$

③ *Greenwich apparent time:*

$$\text{Diff. in longitude} = \frac{120°30'}{15}$$

Diff. in longitude = $8^h\,02^m$

Greenwich apparent time
= local apparent time + diff. in longitude

Greenwich apparent time
= $4^h\,50^m\,20^s + 8^h\,02'\,00"$

Greenwich apparent time = $\mathbf{12^h\,52^m\,20^s}$

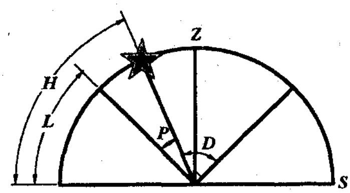
Problem 251:

On July 12, 1905, BLLM No. 1 was established and its latitude was determine from a meridian observation of a star on that date. The following notes of observation are:

	Vertical Angle
Telescope direct =	41.35'
Telescope reversed =	41°36'
Refraction correction =	01'
Declination of star =	62°14'29"

Compute the latitude of BLLM No. 1

Solution:



41°35'30" *(average value)*
- 01'00" *(refraction correction)*
H = 41°34'30" *correction altitude*

$P = 90 - D$
$P = 90° - 62°14'29"$
$P = 27°45'31"$
$L = H - P$
$L = 41°34'30" - 27°45'31"$
$L = \mathbf{13°48'59"}$ *(latitude of BLLM No. 1)*

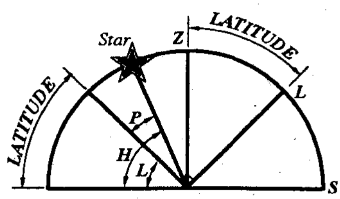
Problem 252:

Compute the latitude of the place of observation P-1 from the following date. Altitude of Polaris on Nov. 3, 1967 was observed to be 15°50'08" at Upper Culmination.

Index error of instrument = 0
Correction for parallax = 01'00"
Polar distance of Polaris = 0°05'18"

PRACTICAL ASTRONOMY

Solution:



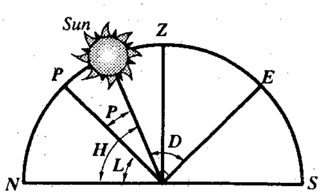
L = latitude of place of observation
L = H - P
H = 15°50'08" + parallax correction
H = 15°50' 08" + 01'
H = 15°51'18"
L = 15°51'08" - 05'18"
L = **15°45'50"**

Problem 253:

From the following given data, it is required to determine the latitude of place of observation. The star observed is Polaris as it attains its upper culmination.

Altitude of Polaris at Upper Culmination = 43°37'
Index Error = - 30"
Declination = + 88°44'35"

Solution:



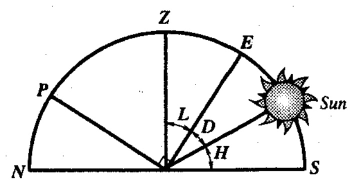
H = 43°37'
Index Error = - 30"

Corrected H = 43°37'30"
D = 88°43'35"
P = 90 - D = 1°15'25"
L = H - P
H = 43°37'30"
P = 1°15'25"
L = **42°22'05"** (latitude of place of observation)

Problem 254:

A Civil Engineer wants to know the latitude of a station point. He then observed a star which crosses south of its zenith and found the angle of elevation of the star to be 50°20'00". If the declination of the star of the star at the instant of meridian passage was - 15°30'15" what is the latitude of the point?

Solution:



H = 50°20'00"
D = - 15°30'15"
L = 90° - (H + D)
Corrected H = 50°20'00" - C_f
C_f = correction for refraction

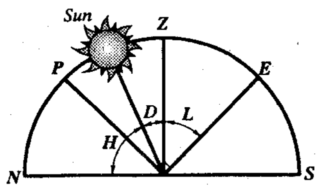
Corrected H = 50°20'00" - 00°00'46.5"
Corrected H = 50°19'13.5"
L = 90° - (50°19'13.5" + 15°30'15")
L = **24°10'31.5"** Latitude of the place

Problem 255:

It is necessary to determine the latitude of a traverse survey and for this purpose the star Ursa Majores was observed on the meridian on a certain date. The following data were taken:

H = 41°36' (direct)
H = 41°37' (reversed)
Correction for refraction = 1'36"
Declination of star = 62°14'29"

Solution:



Mean value of H = 41°36'30"
Corrected H = 41°36'30" - 1'36"
Corrected H = 41°34'54"

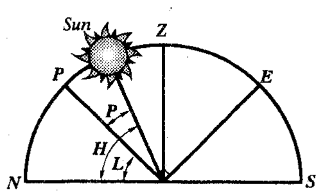
$Z = 90 - H$
Z = 90 - 41°34'54"
Z = 48°25'06"
$L = D - Z$
L = 62°14'29" - 48°25'06"
L = **13°49'23"** *latitude of the place*

Problem 256:

Compute the latitude of the place of observation P-1 from the following data. Altitude of Polaris on November 3, 1967 is 15°50'08" at upper culmination.

Index error = 0
Parallax correction = 1'00"
Polar distance of Polaris = 0°05'18"

Solution:

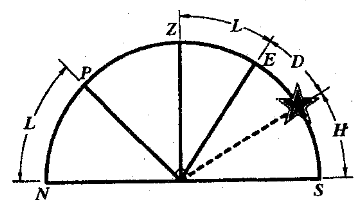


H = 15°50'08" + *Parallax correction*
H = 15°51'08"
P = 05'18"
$L = H - P$
L = 15°51'08" - 05'18"
L = **15°45'50"**

Problem 257:

The observed meridian altitude of a star on April 10, 1990 was 39°24', star bearing south. Refraction correction is 1'11". The declination of the star at that instant was - 8°29'21". Determine the latitude of the place of observation.

Solution:



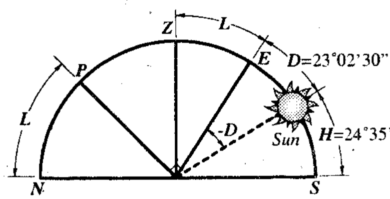
Observed altitude = 39°24'00"
Refraction Corr. = 1'11"
H = 39°24'00" - 0°01'11"

H = 39°22'49"
D = 8°29'21"
$H + D$ = 47°52'10"
$90 - (H + D) = L$
Lat = **42°07'50" N**

Problem 258:

An observation was made on the sun at noon and the recorded altitude is 24°35'18" the sun being south of the equator. Determine the latitude of the place of observation, parallax and refraction = 02', semi-diameter = +16'18", declination of sun is - 23°02'30".

Solution:



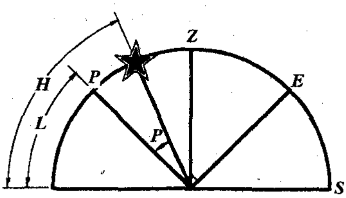
Corrected altitude = H

Obs. altitude	= 24°21'00"
Parallax and refraction	= - 2'00"
Semi-diameter	= +16'18"
H	= 24°35'18"
D	= 23°02'30"
H + D	= 47°37'48"
L	= 90 - (*H* + *D*)
L	= 90 - 47°37'48"
L	= ***42°22'12"***

Problem 259:

An observation of Polaris was made at its upper culmination and recorded the altitude to be 43°37'. Index correction is - 30" and refraction is 1'01". If the polar distance of the star at the instant of observation was 0°55'20", determine the latitude of place of observation.

Solution:



Observed altitude	= 43°37'00"
Index Correction	= + 30"
	43°37'30"
Refraction Corr.	= - 1'01"
H	= 43°36'29"
L	= *H* - *P*

L = 43°36'29" - 0°55'20"

L = ***42°41'09"***

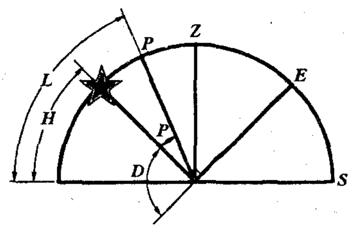
Problem 260:

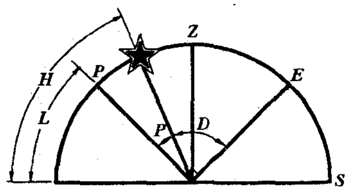
The zenith distance to Polaris when at lower culmination was found on June 22, 1958 with a theodolite to be 76°03'32".

① Compute the latitude of the observer. Refraction corr. = 3'40.4". Polar distance of Polaris is 0°56'5.3".

② Compute the latitude if it was observed on Upper Culmination.

Solution:





① *Latitude of the observer.:*

Altitude = 90 - zenith distance

Altitude = 90° - 76°03'37"

Altitude = 13°56'23"

Corrected H = 13°56'23"

- 3'40.4"

H = 13°52'42.6"

L = H + P

L = 13°52'42.6" + 0°56'05.3"

L = 14°48'47.9"

② *Latitude if it was observed on Upper Culmination.*

L = H - P

L = 13°52'42.6"

- 0°56'05.3"

L = ***12°56'37.3"***

SIMPLE CURVES

SIMPLE CURVES

RAILROAD AND HIGHWAY CURVES

In highway or railroad construction, the curves most generally used presently are circular curves although parabolic and other curves are sometimes used. These types of curves are classified as Simple, Compound, Reversed or Spiral curves.

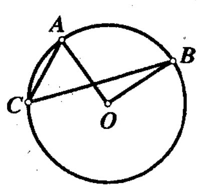
A. ***Simple Curve:***

A simple curve is a circular arc, extending from one tangent to the next. The point where the curve leaves the first tangent is called the "point of curvature" (P.C.) and the point where the curve joins the second tangent is called the "point of tangency" (P.T.). The P.C. and P.T. are often called the tangent points. If the tangent be produced, they will meet in a point of intersection called the "vertex". The distance from the vertex to the P.C. or P.T. is called the "tangent distance". The distance from the vertex to the curve is called the "external distance" (measured towards the center of curvature). While the line joining the middle of the curve and the middle of the chord line joining the P.C. and P.T. is called the "middle ordinate".

Geometry of the Circular Curves:

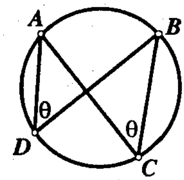
In the study of curves, the following geometric principles should be emphasized:

1. An inscribed angle is measured by one half its intercepted arc.



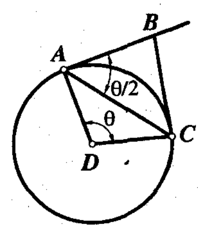
$\angle ACB = \frac{1}{2}\ \angle AOB$

2. Inscribed angles having the same or equal intercepted arcs are equal.



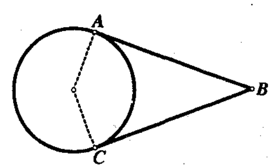
$\angle ADB = \angle ACB$

3. An angle formed by a tangent and a chord is measured by one half its intercepted arc.



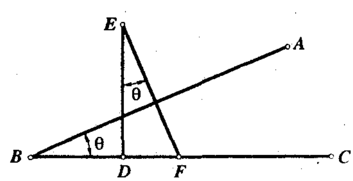
$\angle BAC = \frac{1}{2}\ \angle ADC$

4. Tangents from an external point a circle are equal.



AB = BC

5. Angles whose sides are perpendicular each to each are either equal or supplementary.



$\angle ABC = \angle FED$

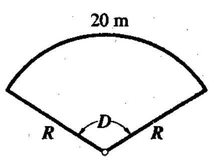
SIMPLE CURVES

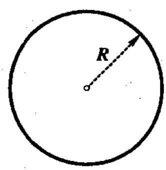
Sharpness of the curve is expressed in any of the three ways:

1. ***Degree of Curve:*** *(Arc Basis)*

   Degree of curve is the angle at the center subtended by an arc of 20 m. is the Metric system or 100 ft. in the English system. This is the method generally used in Highway practice.

   a. ***Metric System:***





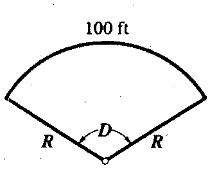
By ratio and proportion:

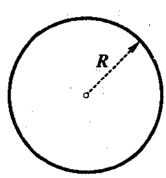
$$\frac{20}{D}=\frac{2\pi R}{360}$$

$$D=\frac{360(20)}{2\pi R}$$

$$D=\frac{1145.916}{R}$$

   b. ***English System:***





$$\frac{100}{D}=\frac{2\pi R}{360}$$

$$D=\frac{360(100)}{2\pi R}$$

$$D=\frac{1145.916(5)}{R}$$

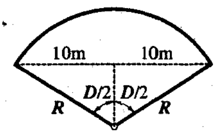
(5 times the metric system)

$$D=\frac{5729.58}{R}$$

2. ***Degree of Curve:*** *(Chord Basis)*

   Degree of curve is the angle subtended by a chord of 20 meters in Metric System or 100 ft. in English System.

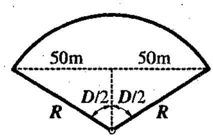
   a. ***Metric System:***



$$\sin\frac{D}{2}=\frac{10}{R}$$

$$R=\frac{10}{\sin\frac{D}{2}}$$

   b. ***English System:***



$$\sin\frac{D}{2}=\frac{50}{R}$$

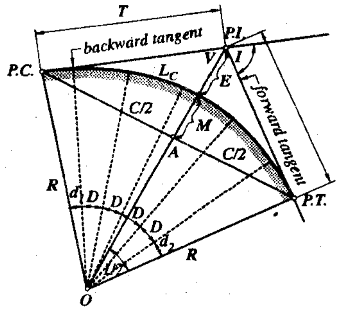
$$R=\frac{50}{\sin\frac{D}{2}}$$

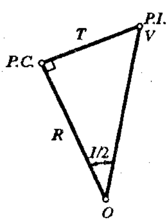
3. ***Radius = Length of radius is stated***

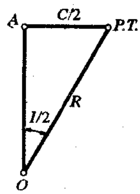
Elements of a simple curve:

P.C. = point of curvature
P.T. = point of tangency
P.I. = point of intersection
R = radius of the curve
D = degree of the curve
T = tangent distance
I = angle of intersection
E = external distance
M = middle ordinate
L_c = length of curve
C = long chord
c_1 and c_2 = sub-chord
d_1 and d_2 = sub-angle

SIMPLE CURVES







1. ***Tangent distance:***

$$\tan\frac{I}{2}=\frac{T}{R}$$

$$T=R\tan\frac{I}{2}$$

2. ***External distance:***

$$\text{Cos}\frac{I}{2}=\frac{R}{OV}$$

$$OV=R\,\text{Sec}\frac{I}{2}$$

$$E=OV-R$$

$$E=R\,\text{Sec}\frac{I}{2}-R$$

$$E=R\left(\text{Sec}\frac{I}{2}-1\right)$$

3. ***Middle Ordinate:***

$$\text{Cos}\frac{I}{2}=\frac{AO}{B}$$

$$AO=R\,\text{Cos}\frac{I}{2}$$

$$M=R-AO$$

$$M=R-R\,\text{Cos}\frac{I}{2}$$

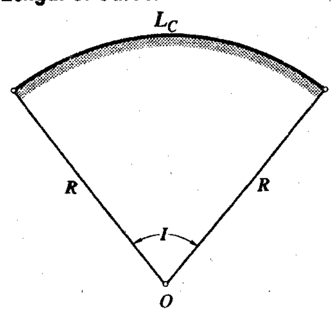
$$M=R\left(1-\text{Cos}\frac{I}{2}\right)$$

4. ***Length of Chord:***

$$\text{Sin}\frac{I}{2}=\frac{\frac{C}{2}}{R}$$

$$C=2R\,\text{Sin}\frac{I}{2}$$

5. ***Length of Curve:***



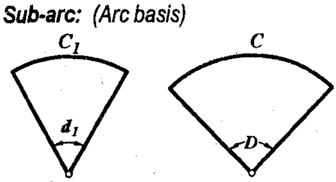
$$\frac{L_c}{I}=\frac{20}{D}$$

$$L_c=\frac{20\,I}{D}\quad \textit{(metric)}$$

$$\frac{L_c}{I}=\frac{100}{D}$$

$$L_c=\frac{100\,I}{D}\quad \textit{(English)}$$

6. ***Sub-arc:*** *(Arc basis)*



$$\frac{C_1}{d_1}=\frac{C}{D}$$

$$d_1=\frac{C_1 D}{C}\quad \textit{(degrees)}$$

$$\frac{d_1}{2}=\frac{C_1 D}{2C}\,(60)\quad \textit{(minutes)}$$

$$\frac{d_1}{2}=\frac{C_1 D\,(60)}{2(20)}\quad \textit{(metric system)}$$

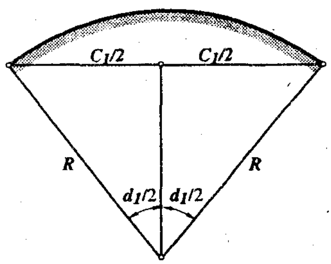
$$\frac{d_1}{2}=1.5\,C_1 D$$

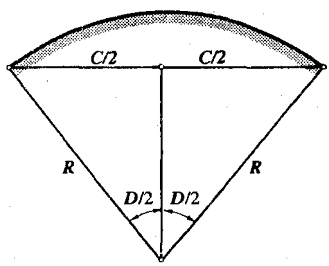
$$\frac{d_1}{2}=\frac{C_1 D\,(60)}{2(100)}$$

$$\frac{d_1}{2}=0.3\,C_1 D\quad \textit{(English system)}$$

SIMPLE CURVES

7. ***Sub-chords:*** *(Chord basis)*





$$\sin\frac{d_1}{2}=\frac{C_1}{2R}$$

$$\sin\frac{D}{2}=\frac{C}{2R}$$

$$2R=\frac{C}{\sin\frac{D}{2}}$$

$$\sin\frac{d_1}{2}=\frac{C_1\sin\frac{D}{2}}{C}$$

$$\sin\frac{d_1}{2}=\frac{C_1\sin\frac{D}{2}}{20}\ \textit{(Metric)}$$

$$\sin\frac{d_1}{2}=\frac{C_1\sin\frac{D}{2}}{100}\ \textit{(English)}$$

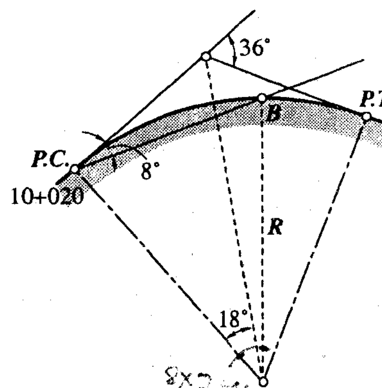
$$C_1=\frac{20\sin\frac{d_1}{2}}{\sin\frac{D}{2}}\ \textit{(Metric)}$$

$$C_1=\frac{100\sin\frac{d_1}{2}}{\sin\frac{D}{2}}$$

261. CE Board Nov. 2004

A simple curve has a central angle of 36° and a degree of curve of 6°.

① Find the nearest distance from the mid point of the curve to the point of intersection of the tangents.
② Compute the distance from the mid point of the curve to the mid point of the long chord joining the point of curvature and point of tangency.
③ If the stationing of the point of curvature is at 10 + 020, compute the stationing of a point on the curve which intersects with the line making a deflection angle of 8° with the tangent through the P.C.



Solution:

① *Distance from mid point of curve to P.I.:*

$$R=\frac{1145.916}{6}$$

$$R=190.99$$

$$E=R\left(\sec\frac{I}{2}-1\right)$$

$$E=190.99\,(\sec 18^\circ-1)$$

$$E=\mathbf{9.83\ m.}$$

② *Distance from mid point of curve to the mid point of long chord:*

$$M=R\left(1-\cos\frac{I}{2}\right)$$

$$M=190.99\,(1-\cos 18^\circ)$$

$$M=\mathbf{9.35\ m.}$$

③ *Stationing of B:*

$S = R\theta$

$S = \dfrac{190.99\,(16)\,\pi}{180}$

$S = 53.33$ m.

$C = \pi r \frac{\theta}{180°}$

$= \pi(190.99)\frac{16}{180}$

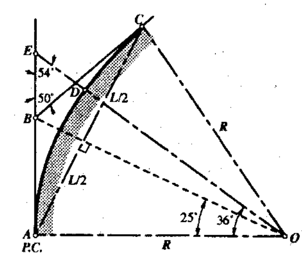
Sta. of B = (10 + 020) + (53.33)

Sta. of B = ***10 + 073.33***

262. CE Board Nov. 2004

A simple curve of the proposed extension of Mantabahadra Highway have a direction of tangent AB which is due north and tangent BC bearing N. 50° E. Point A is at the P.C. whose stationing is 20 + 130.46. The degree of curve is 4°.

① Compute the long chord of the curve.
② Compute the stationing of point D on the curve along a line joining the center of the curve which makes an angle of 54° with the tangent line passing thru the P.C.
③ What is the length of the line from D to the intersection of the tangent AB.



Solution:

① *Long chord:*

$R = \dfrac{1145.916}{4}$

$R = 286.48$ m.

$\dfrac{L}{2} = R\,\text{Sin}\,25°$

$L = 2\,(286.48)\,\text{Sin}\,25°$

$L = \boldsymbol{242.14}$ ***m.***

② *Stationing of D:*

$S = R\theta$

$S = \dfrac{286.48\,(36)\,\pi}{180}$

$S = 180$ m.

Sta. of D = (20 + 130.46) + 180

Sta. of D = ***20 + 310.46***

③ *Distance DE:*

$\text{Cos}\,36° = \dfrac{286.48}{OE}$

$OE = 354.11$ m.

$DE = 354.11 - 286.48$

$DE = \boldsymbol{67.63}$ ***m.***

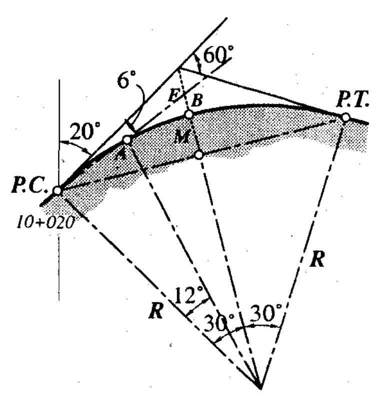
263. CE Board May 2005

The tangents of a simple curve have bearings of N. 20° E. and N. 80° E. respectively. The radius of the curve is 200 m.

① Compute the external distance of the curve.
② Compute the middle ordinate of the curve.
③ Compute the stationing of point A on the curve having a deflection angle of 6° from the P.C. which is at 1 + 200.00.

Solution:

① *External distance:*



SIMPLE CURVES

$$E = R\left(\sec\frac{I}{2} - 1\right)$$
$$E = 200\,(\sec 30° - 1)$$
$$E = \mathbf{30.94\ m.}$$

② *Middle ordinate:*

$$M = R\left(1 - \cos\frac{I}{2}\right)$$
$$M = 200\,(1 - \cos 30°)$$
$$M = \mathbf{26.79\ m.}$$

③ *Stationing of point A:*

$$S = R\theta$$
$$S = \frac{200\,(12)(\pi)}{180}$$
$$S = 41.89$$

Sta. A = (1 + 200.00) + 41.89
Sta. A = **1 + 241.89**

Problem 264:

The tangent distance of a 3° simple curve is only 1/2 of its radius.

① Compute the angle of intersection of the curve.
② Compute the length of curve.
③ Compute the area of the fillet of a curve.

Solution:

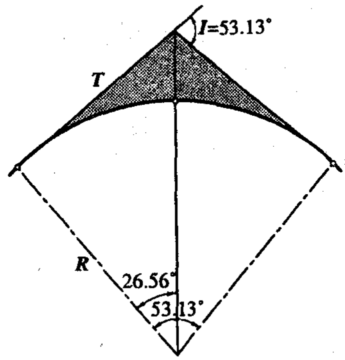
① *Angle of intersection:*

$$T = R\tan\frac{I}{2}$$
$$\frac{1}{2}R = R\tan\frac{I}{2}$$
$$\tan\frac{I}{2} = \frac{1}{2}$$
$$\frac{I}{2} = 26.56°$$
$$I = \mathbf{53.13°}$$

② *Length of curve:*

$$\frac{L_c}{I} = \frac{20}{D}$$
$$L_c = \frac{20\,(53.13)}{3}$$
$$L_c = \mathbf{354.20\ m.}$$

③ *Area of fillet of a curve:*



$$A = \frac{TR\,(2)}{2} - \frac{\pi R^2\,(I)}{360°}$$
$$R = \frac{1145.916}{3}$$
$$R = 381.972$$
$$T = \frac{1}{2}(381.972)$$
$$T = 190.986$$
$$A = \frac{190.986\,(381.972)(2)}{2} - \frac{\pi(381.972)^2\,(53.13°)}{360°}$$
$$A = \mathbf{5304.04\ sq.m.}$$

Problem 265:

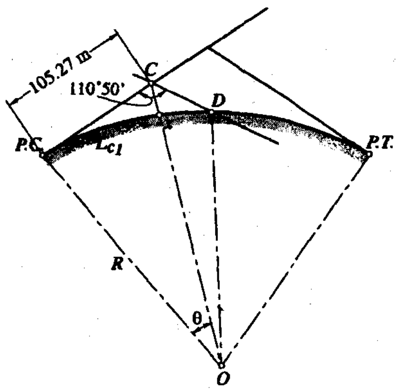
A 5° curve intersects a property line CD at point D. The back tangent intersects the property line at point C which is 105.27 m. from the P.C. which is at station 2 + 040. The angle that the property line CD makes with the back tangent is 110°50'.

① Compute the length of curve from the P.C. to the point of intersection of the line from the center of the curve to point C and the curve.
② Compute the distance CD.
③ Compute the stationing of point D on the curve.

SIMPLE CURVES

Solution:

① *Length of curve from P.C:*



$$R = \frac{1145.916}{D}$$

$$R = \frac{1145.916}{5°}$$

$$R = 229.18 \text{ m.}$$

$$\tan\theta = \frac{105.27}{229.18}$$

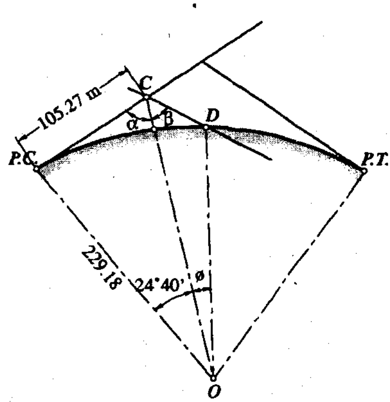
$$\theta = 24°40'$$

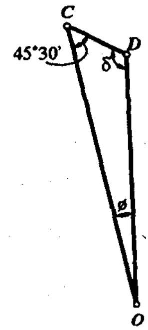
$$Lc_1 = R\theta$$

$$Lc_1 = \frac{229.18\,(24°40')\,\pi}{180}$$

$$Lc_1 = \mathbf{98.68\ m.}$$

② *Distance CD:*





$$\alpha = 90 - 24°40'$$

$$\alpha = 65°20'$$

$$\beta = 110°50' - 65°20'$$

$$\beta = 45°30'$$

$$229.18 = OC \cos 24°40'$$

$$OC = 252.20 \text{ m.}$$

$$\frac{229.18}{\sin 45°30'} = \frac{252.20}{\sin\delta}$$

$$\delta = 128°17'$$

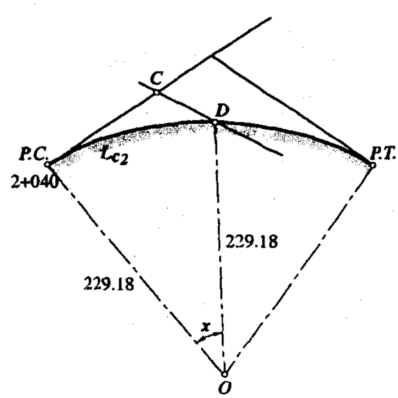
$$ø = 180° - 45°30' - 128°17'$$

$$ø = 6°13'$$

$$\frac{CD}{\sin 6°13'} = \frac{229.18}{\sin 45°30'}$$

$$CD = \mathbf{34.80\ m.}$$

③ *Stationing of D:*



Angle $x = 24°40' + 6°13'$

Angle $x = 30°53'$

SIMPLE CURVES

$$Lc_2 = \frac{R \times \pi}{180}$$

$$Lc_2 = \frac{229.18(30°53')\pi}{180}$$

$$Lc_2 = 148.24 \text{ m.}$$

Station of D = (2 + 040) + (148.24)
Station of D = ***(2 + 188.24)***

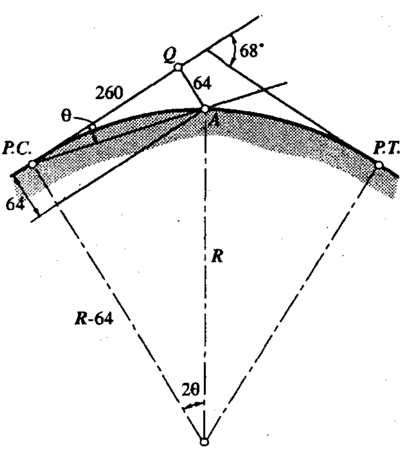
Problem 266:

The perpendicular offset distance from point A on a simple curve to Q on the tangent line is 64 m. If the distance from the P.C. to Q on the tangent is 260 m.

① Compute the radius of the simple curve.
② Compute the length of curve from P.C. to A.
③ If the angle of intersection of the curve is 64°, compute the length of long chord from PC. To PT.

Solution:

① *Radius of curve:*



$$\tan\theta = \frac{64}{260}$$

$$\theta = 13.83°$$

$$2\theta = 27°39'$$

$$\text{Cos } 27°39' = \frac{R - 64}{R}$$

$$R = \mathbf{560.13 \text{ m.}}$$

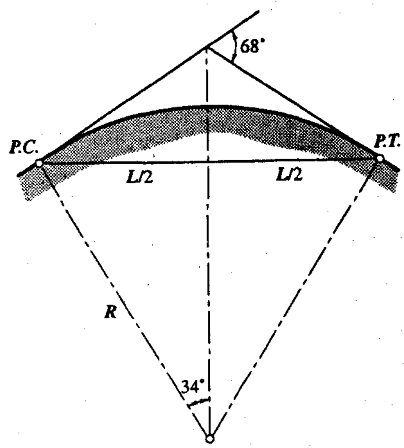
② *Length of curve from PC. to A:*

$$S = R\,2\theta$$

$$S = \frac{560.13\,(27°39')\,\pi}{180}$$

$$S = \mathbf{270.31 \text{ m.}}$$

③ *Length of long chord:*



$$\text{Sin } 34° = \frac{L}{2\,(560.13)}$$

$$L = \mathbf{928.74 \text{ m.}}$$

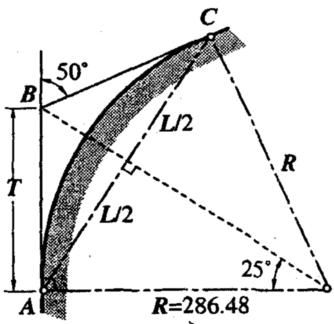
Problem 267:

A simple curve have tangents AB and BC intersecting at a common point B. AB has an azimuth of 180° and BC has an azimuth of 230°. The stationing of the point of curvature at A is 10 + 140.26. If the degree of curve of the simple curve is 4°.

① Compute the length of the long chord from A.
② Compute the tangent distance AB of the curve.
③ Compute the stationing of a point "x" on the curve on which a line passing through the center of the curve makes an angle of 58° with the line AB, intersects the curve at point "x".

Solution:

① *Length of long chord:*



$$R = \frac{1145.916}{4^\circ}$$

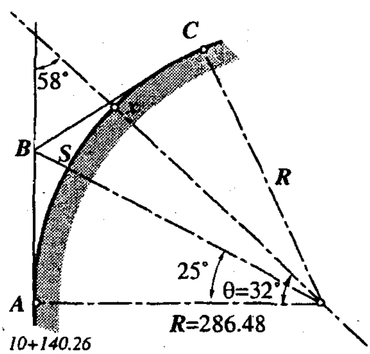
$R = 286.48$ m.

$$\frac{L}{2} = R \operatorname{Sin} 25^\circ$$

$L = 2(288.48) \operatorname{Sin} 25^\circ$

$L = \mathbf{242.14\ m.}$

② *Distance AB:*



$$\tan 25^\circ = \frac{AB}{286.48}$$

$AB = \mathbf{133.59\ m.}$

③ *Stationing of x:*

$S = R\theta$

$$S = \frac{286.48\,(32)\,\pi}{180}$$

$S = 160$ m.

Sta. of x = (10 + 140.26) + (160)

Sta. of x = **10 + 300.26**

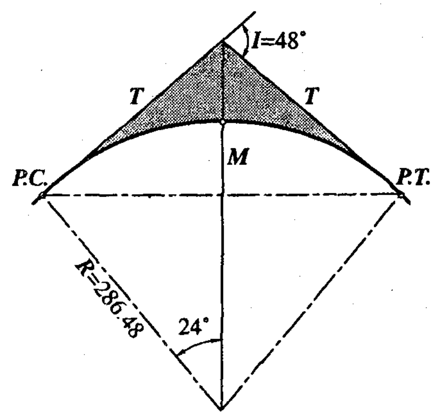
Problem 268:

A simple curve has a radius of 286.48 m. Its distance from P.C. to P.T. along the curve is equal to 240 m.

① Compute the central angle of the curve. Use arc basis.
② Compute the distance from the mid-point of the long chord to the mid-point of the curve.
③ Compute the area bounded by the tangents and the portion outside the central curve in acres.

Solution:

① *Central angle:*



$$\frac{Lc}{I} = \frac{20}{D}$$

$$D = \frac{1145.916}{R}$$

$$D = \frac{1145.916}{286.48}$$

$D = 4^\circ$

$$\frac{240}{I} = \frac{20}{4}$$

$I = \mathbf{48^\circ}$

② *Distance from mid point of curve to mid point of long chord:*

$$M = R\left(1 - \cos\frac{I}{2}\right)$$

$M = 286.48\,(1 - \cos 24^\circ)$

$M = \mathbf{24.76\ m.}$

SIMPLE CURVES

③ *Area bounded by the tangents and outside the central curve:*

$T = R \tan 24°$

$T = 286.48 \tan 24°$

$T = 127.55$

$$Area = \frac{TR\,(2)}{2} - \frac{\pi R^2 I}{360}$$

$$Area = \frac{127.55\,(286.48)(2)}{2} - \frac{\pi\,(286.48)^2(48)}{360}$$

$Area = \mathbf{2162.8}$

② *External distance:*

$$E = R\left(\sec\frac{I}{2} - 1\right)$$

$E = 336.49\,(\sec 25° - 1)$

$E = \mathbf{34.79\ m.}$

③ *Length of long chord:*

$$\frac{L}{2} = R \sin 25°$$

$L = 2\,(336.49) \sin 25°$

$L = \mathbf{284.41\ m.}$

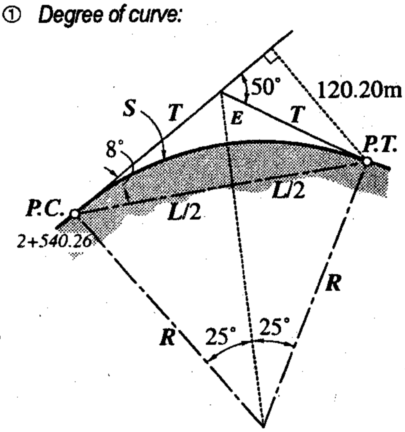
Problem 269:

The offset distance of the simple curve from the P.T. to the tangent line passing through the P.C. is equal to 120.20 m. The stationing of P.C. is at 2 + 540.26. The simple curve has an angle of intersection of 50°.

① Compute the degree of curve.
② Compute the external distance.
③ Compute the length of long chord.

Solution:

① *Degree of curve:*



$$\sin 50° = \frac{120.20}{T}$$

$T = 156.91$ m.

$T = R \tan 25°$

$156.91 = R \tan 25°$

$R = 336.49$ m.

$$D = \frac{1145.916}{336.49}$$

$D = \mathbf{3°24'}$

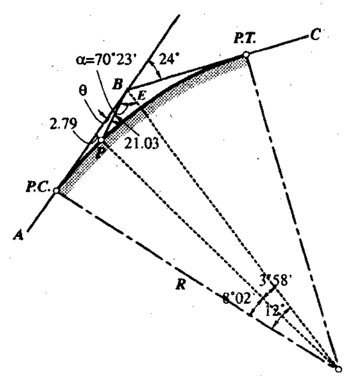
Problem 270:

Two tangents AB and BC intersect at an angle of 24°. A point P is located 21.03 m from point B and has a perpendicular distance of 2.79 m. from line AB.

① Calculate the radius of the simple curve connecting the two tangents and passing point P.
② Find the length of chord connecting PC and point P.
③ Compute the area bounded by the curve and the tangent lines.

Solution:

① *Radius of curve:*



$$\sin\theta = \frac{2.79}{21.03}$$

$\theta = 7°37'$

$\alpha = 90° - 12° - \theta$

$\alpha = 70°23'$

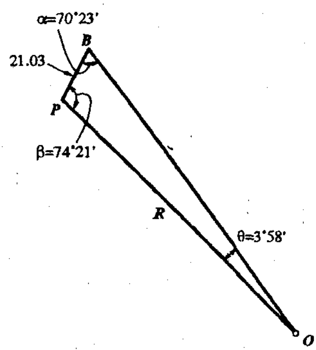
$OB = R + E$

$OB = R + R\left(\sec\frac{I}{2} - 1\right)$

$OB = R + R\left(\sec\frac{24}{2} - 1\right)$

$OB = 1.0223\,R$

In Δ *OPB*



$$\frac{OB}{\sin \beta} = \frac{R}{\sin \alpha}$$

$$\frac{1.0223\,R}{\sin \beta} = \frac{R}{\sin 70°23'}$$

$\beta = 105°39'$

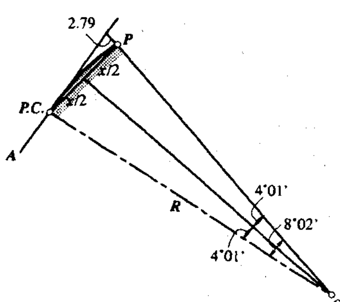
$\phi = 180 - \alpha - \beta$

$\phi = 3°58'$

$$\frac{21.03}{\sin 3°58'} = \frac{R}{\sin 70°23'}$$

$R = \mathbf{286.36\ m.}$

② *Length of chord:*



$$\sin 4°01' = \frac{x}{2\,(286.36)}$$

$x = \mathbf{40.12\ m.}$

③ *Area bounded by the curve and the tangent lines:*

$$A = \frac{RT(2)}{2} - \frac{\pi R^2 (24°)}{360°}$$

$T = R \tan 12°$

$T = 286.36 \tan 12°$

$T = 60.87$

$$A = 286.36\,(60.87) - \frac{\pi\,(286.36)^2\,(24)}{360°}$$

$A = \mathbf{256.26\ m^2}$

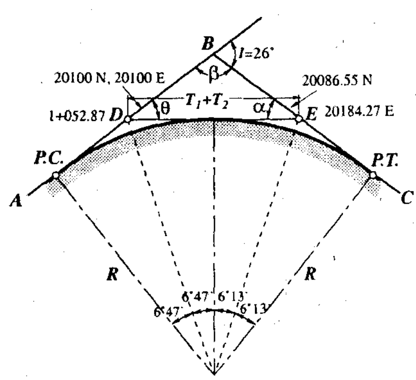
Problem 271:

A simple curve connects two tangents AB and BC with bearings N 85°30' E and S 68°30' E respectively. Point D along line AB has a coordinate of 20100 N and 20100 E while point E along line BC has coordinates of 20086.55 N and 20184.27 E.

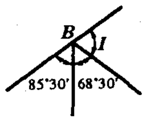
① Find the distance of line BD.
② Solve for the degree of simple curve that shall be tangent to the three lines AB, DE and BC.
③ If point D is at station 1 + 052.87 determine the stationing of PT.

Solution:

① *Distance of line BD:*



SIMPLE CURVES



LINE	LAT	DEP
DE	13.45	+84.27

$$\tan \textit{bearing} = \frac{84.27}{13.45}$$

Bearing of DE = S 80°56' E

$$\textit{Distance DE} = \frac{84.27}{\text{Sin } 80°56'}$$

Distance DE = 85.34 m.

$I = 180° - (85°30' + 68°30')$
$I = 26°$
$\beta = 180° - 26°$
$\beta = 154°$
$\alpha = 80°56' - 68°30'$
$\alpha = 12°26'$
$\theta = 180° - 154° - 12°26'$
$\theta = 13°34'$

$$\frac{BD}{\sin \alpha} = \frac{DE}{\sin \beta}$$

$$\frac{BD}{\sin 12°26'} = \frac{85.34}{\sin 154°}$$

BD = 41.91 m.

② *Degree of curve:*
$T_1 + T_2 = DE$
$R \tan 6°47' + R \tan 6°13' = 85.34$
$R = 374.50$

$$D = \frac{1145.916}{374.50}$$

D = 3°04'

③ *Stationing of PT:*
$T_1 = 374.5 \tan 6°47'$
$T_1 = 44.55$ m.

$$Lc = \frac{20\, I}{D}$$

$$Lc = \frac{20\,(26)}{3.06}$$

$Lc = 169.93$

Sta. at PT = sta. at point D - T_1 + *Lc*
Sta. at PT = (1 + 052.87) - 44.55 + 169.93
Sta. at PT = ***(1 + 178.25)***

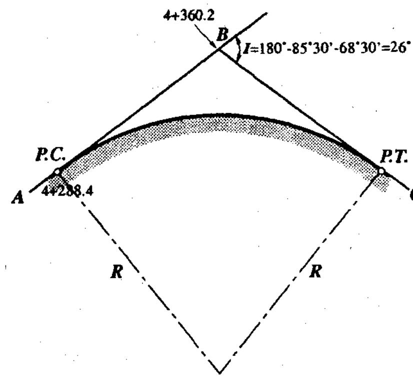
Problem 272:

A simple curve connects two tangents AB and BC with bearings N 85°30' E and S68°30' E respectively. If the stationing of the vertex is 4 + 360.2 and the stationing of PC is 4 + 288.4.

① Determine the radius.
② Determine the external distance.
③ Determine the middle ordinate.
④ Determine the chord distance.
⑤ Determine the length of curve.

Solution:

① *Radius:*



$T = (4 + 360.2) - (4 + 288.4)$
$T = 71.8$ m.

$$T = R \tan \frac{I}{2}$$

$$71.8 = R \tan \frac{26}{2}$$

$R = 311$ m.

② *External distance:*

$$E = R\left(\sec \frac{I}{2} - 1\right)$$

$$E = 311\left(\sec \frac{26}{2} - 1\right)$$

E = 8.18 m.

③ *Middle ordinate:*

$$M = R\left(1 - \cos \frac{I}{2}\right)$$

$$M = 311\left(1 - \cos \frac{26}{2}\right)$$

M = 7.97 m.

④ *Chord distance:*

$$C = 2R \sin \frac{I}{2}$$

$$C = 2(311) \text{ Sin} \frac{26}{2}$$

$$C = \mathbf{139.92\ m.}$$

⑤ *Length of curve:*

$$L_c = RI \frac{\pi}{180}$$

$$L_c = (311)\ 26 \frac{\pi}{180}$$

$$L_c = \mathbf{141.13\ m.}$$

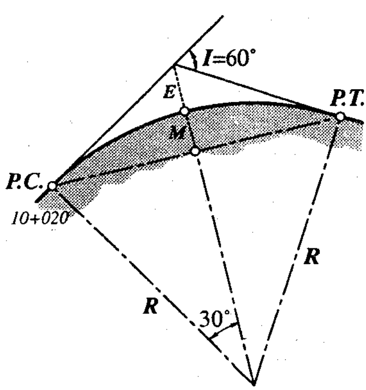
Problem 273:

A simple curve has a central angle of 36° and a degree of curve of 6°.

① Find the nearest distance from the mid-point of the curve to the point of intersection of the tangents.
② Compute the distance from the mid-point of the curve to the mid-point of the long chord joining the point of tangency and point of curvature.
③ If the stationing of the point of curvature is at 10 + 020, compute the stationing at a point on the curve which intersects with the line making a deflection angle of 8° with the tangent through the P.C.

Solution:

① *Distance from mid point of curve to P.I.*



$$R = \frac{1145.916}{D}$$

$$R = \frac{1145.916}{6}$$

$$R = 190.99$$

$$E = R\,(\sec I/2 - 1)$$

$$E = 190.99\,(\sec 30° - 1)$$

$$E = \mathbf{29.55\ m.}$$

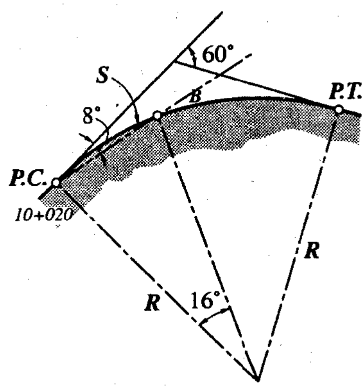
② *Distance of mid point of curve to mid point of long chord:*

$$M = R\,(1 - \cos\ I/2)$$

$$M = 190.99\,(1 - \cos 30°)$$

$$M = \mathbf{25.59\ m.}$$

③ *Stationing of B:*



$$S = R\,\theta$$

$$S = \frac{190.99\,(16)\,\pi}{180}$$

$$S = 53.33$$

Sta. of B = 10 + (020) + (53.33)
Sta. of B = **10 + 073.33**

Problem 274:

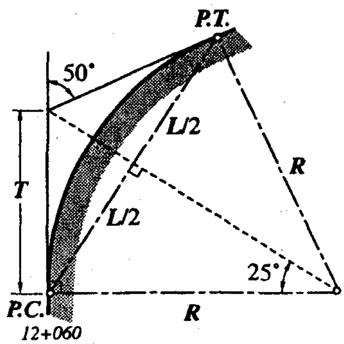
The tangent thru the P.C. has a direction due north and the tangent through the P.T. has a bearing of N. 50° E. It has a radius of 200 m. Using arc basis. Stationing of P.C. is 12 + 060.

SIMPLE CURVES

① Compute the tangent distance of the curve.
② Compute the long chord of the curve.
③ If a line making an angle of 62° with the tangent thru the P.C. intersects the curve at point B, what is the stationing of B if this line passes through the center of the curve.

Solution:

① *Tangent distance:*

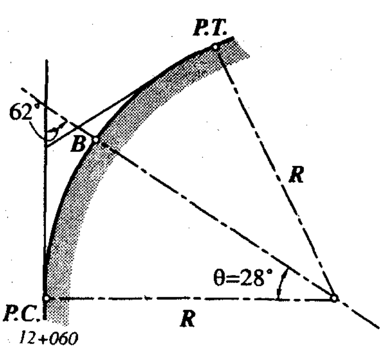


$T = R \tan 25°$
$T = 200 \tan 25°$
$T = \mathbf{93.26\ m.}$

② *Long chord:*

$\text{Sin } 25° = \dfrac{L}{2R}$
$L = 2(200) \text{ Sin } 25°$
$L = \mathbf{169.05\ m.}$

③ *Stationing of B:*



$S = R\,\theta$
$S = \dfrac{200\,(28)\,\pi}{180}$
$S = 97.74$ m.

Sta. of B = (12 + 060) + (97.74)
Sta. of B = **12 + 157.74**

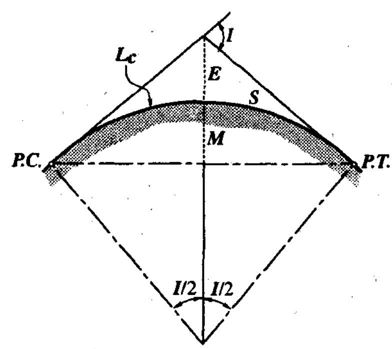
Problem 275:

The length of curve of a simple curve having a degree of curve 4° is equal to 210 m.

① Compute the middle ordinate of the curve.
② Compute the external distance of the curve.
③ Compute the area of the fillet of the curve.

Solution:

① *Middle ordinate:*



$\dfrac{L_c}{I} = \dfrac{20}{D}$
$\dfrac{210}{I} = \dfrac{20}{4}$
$I = 42°$
$M = R\,(1 - \text{Cos } I/2)$
$R = \dfrac{1145.916}{4} = 286.48$
$M = 286.48\,(1 - \text{Cos } 21°)$
$M = \mathbf{19.03\ m.}$

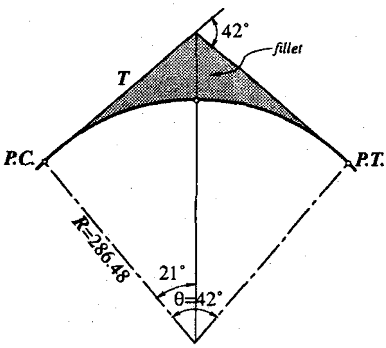
② *External distance:*

$E = R\,(\text{Sec}\ I/2 - 1)$

$E = 286.48\ (\text{Sec}\ 21° - 1)$

$E = 20.38\ m.$

③ *Area of fillet of curve:*



$T = R \tan 21°$

$T = 286.48 \tan 21°$

$T = 109.97$ m.

$$A = \frac{TR\,(2)}{2} - \frac{\pi R^2 \theta}{360}$$

$$A = \frac{(109.97)(286.48)(2)}{2} - \frac{\pi\,(286.48)^2\,(42)}{360}$$

$A = 1423.69\ m^2$ say $\mathbf{1424\ m^2}$

Problem 276:

The deflection angles of two intermediate points R and S on the curve measured from the tangent passing through the P.C. are 6°15' and 12°15' respectively. The chord distance between R and S is 20 m. (Standard in metric system) while the long chord is 100 m. meters long.

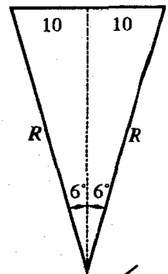
① Compute the radius of the curve.

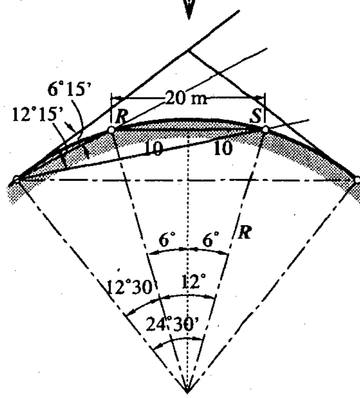
② Compute the angle of intersection of the simple curve.

③ Compute the tangent distance.

Solution:

① *Radius of the curve:*

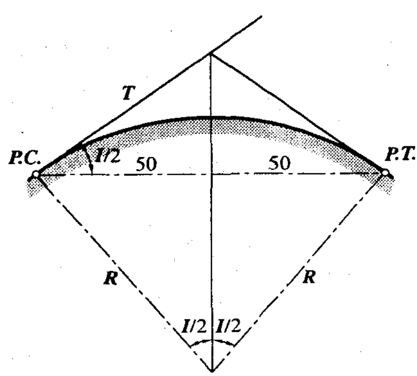




$$\text{Sin}\ 6° = \frac{10}{R}$$

$R = 95.67\ m.$

② *Angle of intersection:*



SIMPLE CURVES

$$\text{Sin}\frac{I}{2}=\frac{70}{R}$$

$$\text{Sin}\frac{I}{2}=\frac{50}{95.67}$$

$$\frac{I}{2}=31.5^\circ$$

$$I=\mathbf{63^\circ}$$

③ *Tangent distance:*

$$T=R\tan\frac{I}{2}$$

$$T=95.67\tan 31.5^\circ$$

$$T=\mathbf{58.63\ m.}$$

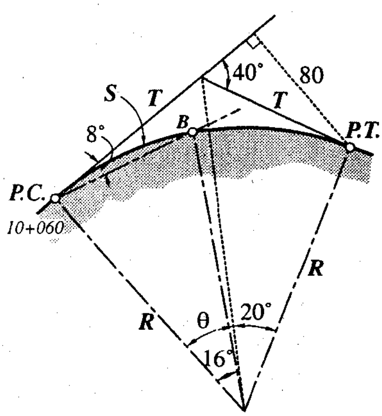
Problem 277:

A simple curve has a central angle of 40°. The stationing at the point of curvature is equal to 10 + 060. The offset distance from the P.T. to the tangent line passing thru the P.C. is 80 m. long.

① Compute the tangent distance of the curve.
② Compute the degree of curve.
③ The deflection angle from the tangent at the P.C. to point B on the curve is equal to 8°, what would be the stationing of point B.

Solution:

① *Tangent dsistance:*



$$80=T\,\text{Sin}\,40^\circ$$

$$T=\mathbf{124.46}$$

② *Degree of curve:*

$$T=R\tan 20^\circ$$

$$124.46=R\tan 20^\circ$$

$$R=341.95$$

$$D=\frac{1145.916}{341.95}$$

$$D=\mathbf{3.35^\circ}$$

③ *Stationing of B:*

$$S=R\theta$$

$$S=\frac{341.95\,(16)\,\pi}{180}$$

$$S=95.49$$

Sta. of B = (10 + 060) + (95.49)
Sta. of B = ***10 + 155.49***

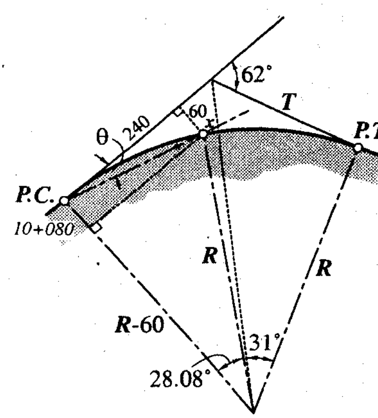
Problem 278:

Two tangents making an angle of 62° from each other is connected by a simple curve. A point "x" on the curve is located by a distance along the tangent from the P.C. equal to 240 m. and an offset from the tangent equal to 60 m. The P.C. is at station 10 + 080.

① Compute the radius of the curve.
② Compute the tangent distance of the curve.
③ Compute the stationing of point "x" on the curve.

Solution:

① *Radius of curve:*



$\tan\theta = \dfrac{60}{240}$

$\theta = 14.04°$

$2\theta = 28.08°$

$R - 60 = R\cos 28.08°$

$0.1177R = 60$

$R = \mathbf{509.70\ m.}$

② *Tangent distance:*

$T = R\tan 31°$

$T = 509.70\tan 31°$

$T = \mathbf{306.26\ m.}$

③ *Stationing of point x:*

$S = R\theta$

$S = \dfrac{509.70\,(28.08)\,\pi}{180}$

$S = 249.80$

Sta. of x = (10 + 080) + (249.80)

Sta. of x = ***10 + 329.8***

Problem 279:

A simple curve having a radius of 229.18 m. has a back tangent of N. 28° E. and a forward tangent of N. 66° E. A property line running parallel to the back tangent crosses the centerline of the curve at a distance of 10 m. from it. If the P.C. of the curve is at 10 + 120.60.

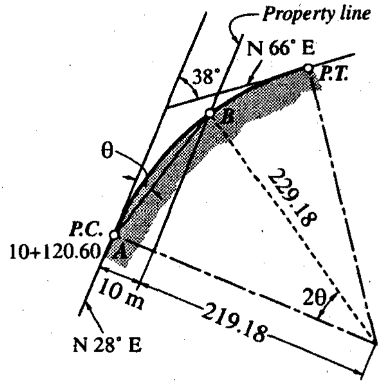
① What is the deflection angle at the point of intersection of the property line and the curve measured from the tangent at sta. 10 + 120.60.

② What is the stationing at the point of intersection of the property line and the curve?

③ Compute the chord distance from P.C. to the point of intersection of the property line and the curve?

Solution:

① *Deflection angle at the P.C.:*



$\cos 2\theta = \dfrac{219.18}{229.18}$

$2\theta = 16.988°$

$\theta = \mathbf{8.49°}$ *(deflection angle)*

② *Stationing at B:*

$S = R(2\theta)$

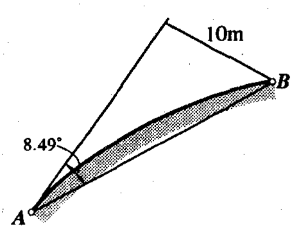
$S = \dfrac{229.18\,(16.988)°\,\pi}{180}$

$S = 67.95$ m.

Sta. of B = (10 + 120.60) + 67.95

Sta. of B = ***10 + 188.55***

③ *Chord distance from P.C. to B:*



$\sin 8.49° = \dfrac{10}{AB}$

$AB = \mathbf{67.73\ m.}$

SIMPLE CURVES

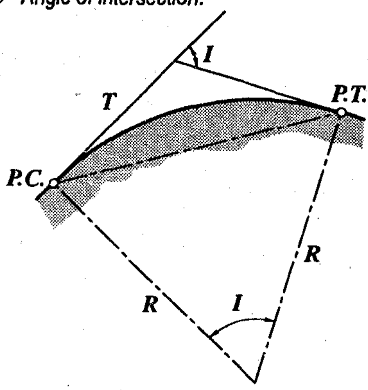
Problem 280:

The radius of a simple curve is twice its tangent distance. If the degree of curve is 4°.

① What is the angle of intersection of the curve.
② Compute the length of curve.
③ Determine the area enclosed by the curve.

Solution:

① *Angle of intersection:*



$T = R \tan \frac{I}{2}$

$T = 2T \tan \frac{I}{2}$

$\tan \frac{I}{2} = 0.5$

$\frac{I}{2} = 26.56°$

$I = 53.13°$

$I = \mathbf{53°08'}$

② *Length of curve:*

$R = \frac{1145.916}{4°} = 286.48$ m.

$\frac{L_c}{I} = \frac{20}{D}$

$L_c = \frac{20(53.13)}{4}$

$L_c = \mathbf{265.65\ m.}$

③ *Area enclosed by the curve:*

$A = \frac{53.13° \pi (286.48)^2}{360°}$

$A = \mathbf{380.54\ m^2}$

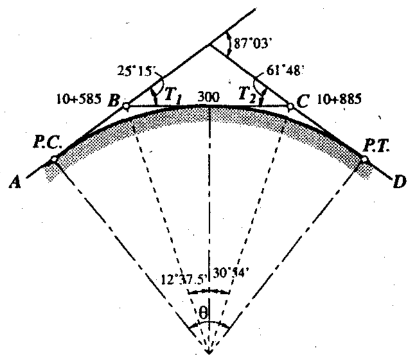
Problem 281:

Three tangent lines AB, BC and CD of a traverse have azimuths of 228°15', 253°30' and 315°18' respectively. The stationing of B is (10 + 585) and that of C is (10 + 885). A proposed highway curve is to connect these three tangents.

① Compute the radius of the simple curve that connects these tangents.
② Compute the stationing of the P.C.
③ Compute the length of curve from P.C. to P.T.

Solution:

① *Radius of curve:*



$T_1 + T_2 = 300$

$R \tan 12°37.5' + R \tan 30°54' = 300$

$R = \mathbf{364.75}$

② *Stationing of P.C.* = (10 + 585) - T_1

$T_1 = 364.75 \tan 12°37.5'$

$T_1 = 81.70$ m.

Sta. of P.C. = (10 + 585) - (81.70)

Sta. of P.C. = ***(10 + 503.3)***

③ *Length of curve:*

$S = R\theta$

$S = \frac{364.75\ (87°03')\ \pi}{180}$

$S = \mathbf{554.17\ m.}$

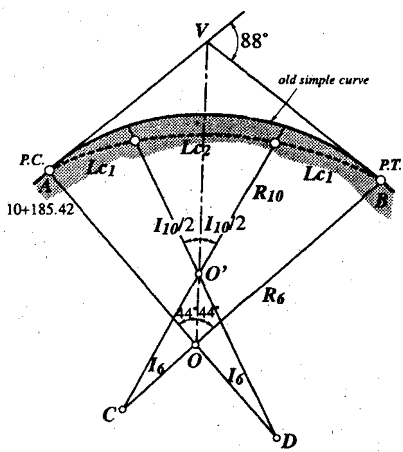
Problem 282:

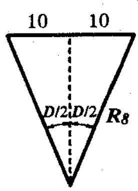
An 8° simple curve connecting two tangents that intersect at an angle of 88° is to be replaced by a symmetrical three centered compound curve having 6° end curves and a 10° curve at the center maintaining the same P.C. Use chord basis.

① Find the central angle of the 10° center curve.
② Find the central angle of the 6° end curves.
③ Find the stationing of the P.T. if P.C. is at 10 + 185.42.

Solution:

① *Central angle of 10° center curve:*





$OA = R_8$

$$\text{Sin}\,\frac{D}{2} = \frac{20}{2\,R_8}$$

$$\text{Sin}\,\frac{8^\circ}{2} = \frac{10}{R_8}$$

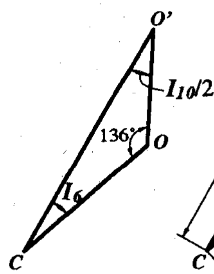
$R_8 = 143.47$ m.

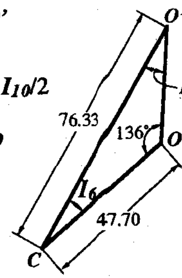
$$\text{Sin}\,\frac{6^\circ}{2} = \frac{10}{R_6}$$

$R_6 = 191.07$ m.

$$\text{Sin}\,\frac{10^\circ}{2} = \frac{10}{R_{10}}$$

$R_{10} = 114.74$ m.





$O'C = R_6 - R_{10}$
$O'C = 191.07 - 114.74$
$O'C = 76.33$
$OC = R_6 - R_8$
$OC = 191.07 - 143.37$
$OC = 47.70$

Using Sine Law:

$$\frac{47.70}{\text{Sin}\,\frac{I_{10}}{2}} = \frac{76.33}{\text{Sin}\,136^\circ}$$

$$\frac{I_{10}}{2} = 25^\circ 44'$$

$I_{10} = \mathbf{51^\circ 28'}$

② *Central angle of 6° end curves:*

$$I_6 + \frac{I_{10}}{2} + 136^\circ = 180^\circ$$

$I_6 + 25^\circ 44' + 136^\circ = 180^\circ$

$I_6 = \mathbf{18^\circ 16'}$

③ *Stationing of P.T.:*

$$Lc_1 = \frac{20\,I_1}{D_1}$$

$$Lc_1 = \frac{20\,(18^\circ 16')}{6^\circ}$$

$Lc_1 = 60.89$ m.

$$Lc_2 = \frac{20\,I_2}{D_2}$$

$$Lc_2 = \frac{20\,(51^\circ 28')}{10^\circ}$$

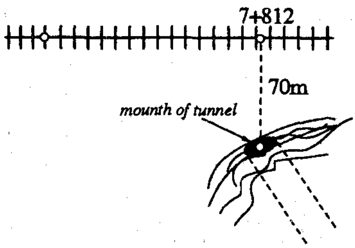
$Lc_2 = 102.93$ m.

$P.T. = (10 + 185.42) + 60.89 + 102.93 + 60.89$
$P.T. = \mathbf{(10 + 410.13)}$

SIMPLE CURVES

Problem 283:

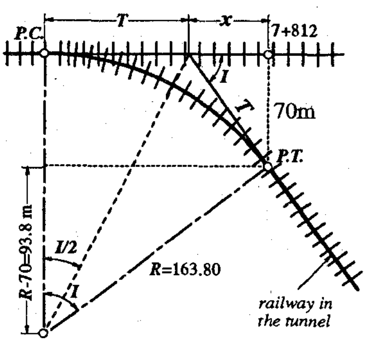
A 7° circular turnout is to connect a railway track, leading due east, to the mouth of the tunnel which is 70 m. from station 7 + 812 as shown on the figure. Use chord basis.



① Determine the stationing of the point of deviation.
② Determine the stationing of the mouth of the tunnel.
③ What is the direction of the railway in the tunnel if it is used for hauling.

Solution:

① *Stationing of the point of deviation:*



$$\text{Sin}\frac{D}{2}=\frac{10}{R}$$

$$\text{Sin } 3.5° = \frac{10}{R}$$

$R = 163.80$ m.

$$\text{Cos } I = \frac{93.8}{163.80}$$

$I = 55°04'$

$$\tan 55°04' = \frac{70}{x}$$

$x = 48.89$ m.

$$\tan\frac{I}{2}=\frac{T}{R}$$

$T = 163.80 \tan 27°32'$

$T = 85.39$ m.

Sta. of point of deviation (P.C.)
$= (7 + 812) - (85.39 + 48.89)$
$= \mathbf{7 + 677.72}$

② *Stationing of mouth of tunnel:*

$$\frac{L_c}{I}=\frac{20}{D}$$

$$L_c=\frac{55°04'(20)}{7°}$$

$L_c = 157.33$ m.

Sta. of mouth of tunnel = (7 + 677.72) + (157.33)
Sta. of mouth of tunnel = ***7 + 835.05***

③ *Direction of railway in the tunnel:*
$90° - 55°04' = 34°56'$
Direction is ***S. 34°56' E.***

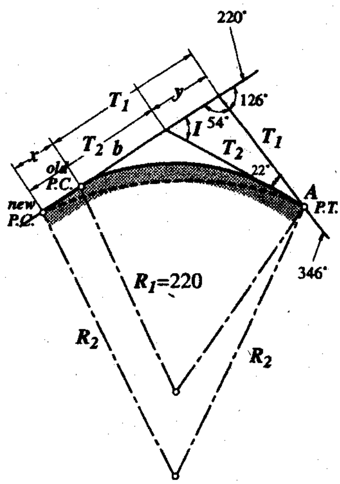
Problem 284:

A horizontal curve with radius equal to 220 m. and intersection angle of 126° is to be realigned by rotating the forward tangent through an angle of 22° counter clockwise along the P.T. If the azimuths of the back and forward tangents are 220° and 346° respectively, stationing of old P.C. is 10 + 721.20.

① Compute the central angle of the new curve.
② Compute the radius of the new curve.
③ What is the stationing of the new P.C.

Solution:

① *New cental angle of new curve:*



Old central angle = 346 - 220
Old central angle = 126°

New central angle:
$I = 180° - 54° - 22°$
$I = \mathbf{104°}$

② *Radius of new curve:*

$$T_1 = R_1 \tan\frac{I_1}{2}$$
$$T_1 = 220 \tan 63°$$
$$T_1 = 431.77$$

Using Sine Law:

$$\frac{431.77}{\sin 104°} = \frac{T_2}{\sin 54°}$$
$$T_2 = 360 \text{ m.}$$
$$\frac{y}{\sin 22°} = \frac{431.77}{\sin 104°}$$
$$y = 166.70 \text{ m.}$$
$$T_2 = R_2 \tan\frac{I_2}{2}$$
$$360 = R_2 \tan 52°$$
$$R_2 = \mathbf{281.26 \text{ m.}}$$

③ *Sta. of new P.C.:*

$$T_1 - y = b$$
$$b = 431.77 - 166.70$$
$$b = 265.07 \text{ m.}$$

$$x = T_2 - b$$
$$x = 360 - 265.07$$
$$x = 94.93 \text{ m.}$$

Sta. of new P.C. = (10 + 721.20) - 94.93
Sta. of new P.C. = ***10 + 626.27***

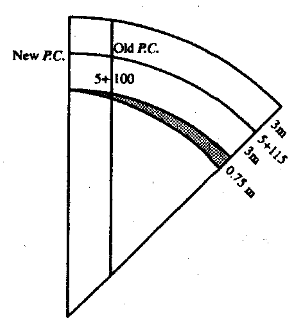
Problem 285:

A circular road having a curve of 8° curvature is to be 6 m. wide on the tangents and 6.75 m. wide along the main part of the curve. The P.C. is at station 5 + 100 and widening is to be completed at station 5 + 115.

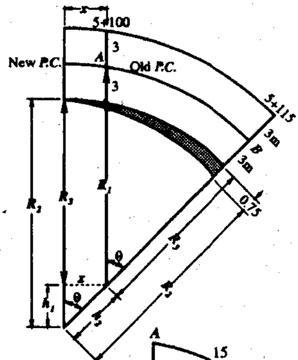
① Compute the stationing of the P.C. of the approach curve at the inner edge.
② Compute the radius of the approach curve at the inner edge.
③ Compute the degree of curve of the approach curve at the inner edge.

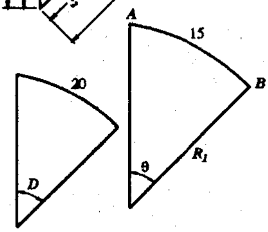
Solution:

① *Stationing of the P.C:*



SIMPLE CURVES





$D = 8°$

$\frac{15}{ø} = \frac{20}{D}$

$ø = \frac{15\,(8)}{20}$

$ø = 6°$

$\tan 6° = \frac{x}{h_1}$

$h_1 = x \text{ Cot } 6°$

$h_1 = 9.51\,x$

$\text{Sin } 6° = \frac{x}{h_2}$

$h_2 = x \text{ Csc } 6°$

$h_2 = 9.57\,x$

① $R_2 = R_3 + h_2 - 0.75$
$R_2 = R_3 + h_1$

② $R_2 = R_3 + 9.51\,x$

① $R_2 = R_3 + 9.57\,x - 0.75$
$R_3 + 9.51\,x + R_3 + 9.57\,x - 0.75$
$0.06\,x = 0.75$
$x = 12.5$ m.

Sta. of new P.C. = 5 + 100
− 12.5
New P.C. = ***5 + 087.5***

② *Radius of the approach curve:*

$R_1 = \frac{1145.916}{D}$

$R_1 = \frac{1145.916}{8}$

$R_1 = 143.24$ m.

$R_3 = R_1 - 3$

$R_3 = 143.24 - 3$

$R_3 = 140.24$ m.

$R_2 = R_3 + 9.51\,x$

$R_2 = 140.24 + 9.51\,(12.5)$

$R_2 = \mathbf{259.12\ m.}$

③ *Degree of curve:*

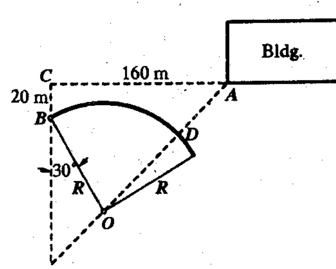
$D = \frac{1145.916}{R}$

$D = \frac{1145.916}{259.12}$

$D = \mathbf{4°25'}$

Problem 286:

From the figure shown:



① Compute the radius R of the simple curve so that the curve will pass through a point which is 30 m. from the edge of the building. The center line of the curve passes through BO.

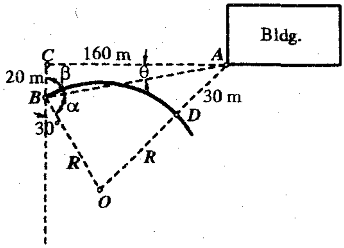
② Compute the central angle of the curve.

③ Compute the area of ACBD.

SIMPLE CURVES

Solution:

① *Radius of the simple curve:*



$$\tan \phi = \frac{20}{160}$$

$\phi = 7°08'$

$\beta = 90° - 7°08'$

$\beta = 82°52'$

$\alpha = 180° - 82°52' - 30°$

$\alpha = 67°08'$

$AB = \sqrt{(20)^2 + (160)^2}$

$AB = 161.25$ m.

Using Cosine Law:

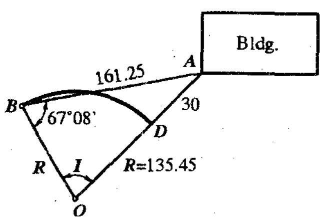
$(30 + R)^2 = (161.25)^2 + (R)^2$
$\quad - 2R\,(161.25) \cos 67°08'$

$900 + 60R + R^2 = 26001.56 + R^2 - 125.32\,R$

$185.32R = 25101.56$

$R =$ ***135.45 m.***

② *Central angle of curve:*



$$\frac{161.25}{\sin I} = \frac{135.45}{\sin 67°08'}$$

$I =$ ***63°54'***

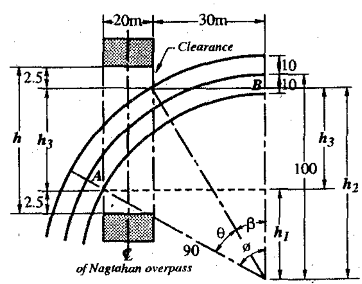
③ *Area of ACBD:*

$$A = \frac{20\,(160)}{2} + \frac{161.25\,(135.45) \sin 67°08'}{2} - \frac{\pi\,(135.45)^2 (63°54')}{360°}$$

$A =$ ***1431.70 m²***

Problem 287:

Piers of the proposed overpass in Nagtahan, are to be placed with a clearance of 2.5 m. of the existing Ramon Magsaysay Avenue as shown.



① Determine the min. distance between the piers when the radius of the curve is 100 m. Width of roadway is 20 m.

② Find the angle θ.

③ Find the area of the road between A and B.

Solution:

① *Min. distance:*

$h_1^2 = (90)^2 - (50)^2$

$h_1 = 74.83$ m.

$h_2^2 = (110)^2 - (30)^2$

$h_2 = 105.83$ m.

$h_3 = h_2 - h_1$

$h_3 = 105.83 - 74.83$

$h_3 = 31$ m.

- *Min. distance between piers = h*

$h = h_3 + 2.5 + 2.5$

$h = 31 + 5$

$h =$ ***36 m.*** *(clear distance between piers)*

② *Angle θ:*

$$\text{Cos } \phi = \frac{74.83}{90}$$

$$\phi = 33°45'$$

$$\text{Cos } \beta = \frac{h_1 + h_3}{90 + 20}$$

$$\text{Cos } \beta = \frac{74.83 + 31}{110}$$

$$\beta = 15°50'$$

$$\theta = 33°45' - 15°50'$$

$$\theta = \mathbf{17°55'}$$

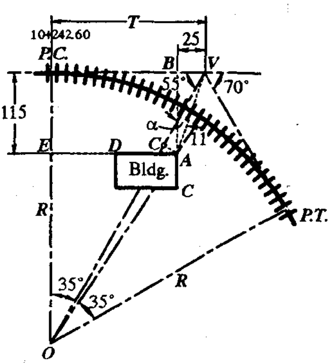
③ *Area of the road between A and B:*

$$A = \frac{\pi (110)^2 (33°45')}{360°} - \frac{\pi (90)^2 (33°45')}{360°}$$

$$A = \mathbf{1178.10\ m^2}$$

Problem 288:

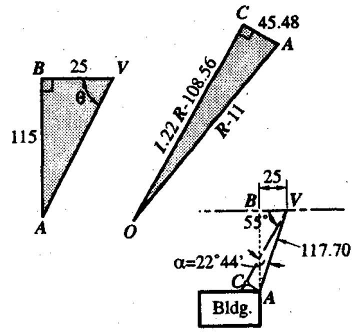
Given a building and railroad track curve located as shown.



① Find the smallest radius of rail track curve that will clear the building by 11 m.
② If AC = 50 m. and DE = 105 m., find the area of the building.
③ If the stationing of the PC is at 10 + 242.60, what is the stationing of the P.T.

Solution:

① *Smallest radius of rail track curve:*



$$AV = \sqrt{(115)^2 + (25)^2}$$

$$AV = 117.70 \text{ m.}$$

$$\tan \phi = \frac{115}{25}$$

$$\phi = 77°44'$$

$$\alpha = 77°44' - 55°$$

$$\alpha = 22°44'$$

$$AC = 117.70 \sin 22°44'$$

$$AC = 45.48 \text{ m.}$$

$$VC = 117.70 \cos 22°44'$$

$$VC = 108.56 \text{ m.}$$

$$OV \cos 35° = R$$

$$OV = 1.22\,R$$

$$OC = OV - VC$$

$$OC = 1.22\,R - 108.56$$

$$OA = R - 11$$

$$(R - 11)^2 = (1.22R - 108.56)^2 + (45.48)^2$$

$$R = \mathbf{431\ m.}$$

② *Area of the building:*

$$\tan 35° = \frac{T}{R}$$

$$T = 431 \tan 35°$$

$$T = 301.79 \text{ m.}$$

$$AD + ED + VB = T$$

$$AD + 105 + 25 = 301.79$$

$$AD = 171.79 \text{ m.}$$

Area of bldg. $= 50(171.79)$

Area of bldg. $= \mathbf{8589.5\ m^2}$

③ *Stationing of P.T.:*

$L_c = RI$

$L_c = \dfrac{431(70)\pi}{180}$

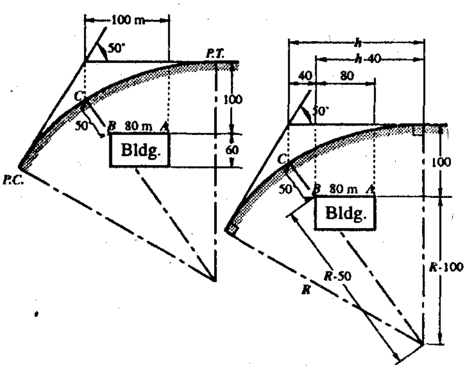
$L = 526.57$ m.

$PC = (10 + 242.60) + (526.57)$

$PC = \mathbf{10 + 769.17}$

Problem 289:

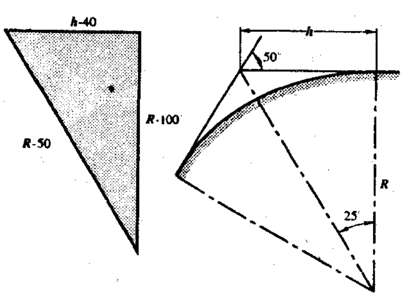
The two intersecting streets are to be connected by a simple curve. The centerline of the curve is to be located such that point B of the building is to be 50 m. from the curve as shown in the figure.



① Compute the radius of the curve.
② Compute the tangent distance.
③ Find the stationing of C, if PC is at 10 + 240.26.

Solution:

① *Radius of the curve:*



$(R - 50)^2 = (h - 40)^2 + (R - 100)^2$

$R^2 - 100R + 2500 = h^2 - 80h + 1600$
$\quad + R^2 - 200R + 10000$

$100R = h^2 - 80h + 9100$

$\tan 25° = \dfrac{h}{R}$

$h = 0.466\,R$

$100R = (0.466R)^2 - 80(0.466R) + 9100$

$100R = 0.217R^2 - 37.28R + 9100$

$R^2 - 632.63R + 41935.48 = 0$

$R = \dfrac{632.63 \pm 482.16}{2}$

$R = \mathbf{557.56\ m.}$

② *Tangent distance:*

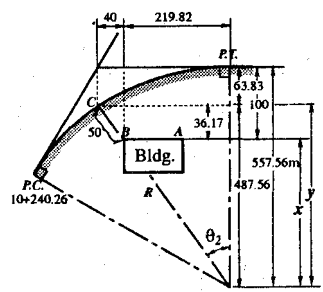
$h = 0.466\,R$

$h = 0.466\,(557.56)$

$h = 259.82$ m.

$T = \mathbf{259.82\ m.}$

③ *Stationing of C:*



$x^2 + (219.82) = (507.56)^2$

$x = 457.49$

$y^2 + (259.82)^2 = (557.86)^2$

$y = 493.66$

$\cos\theta = \dfrac{493.66}{557.56}$

$\theta = 27°42'$

$\beta = 50° - 27°42'$

$\beta = 22°18'$

$L_c = \dfrac{R\,\beta\,\pi}{180}$

$L_c = \dfrac{557.56\,(22°18')\,\pi}{180°}$

$L_c = 217$ m.

Sta. of C $= (10 + 240.26) + 217$

Sta. of C $= \mathbf{10 + 457.26}$

SIMPLE CURVES

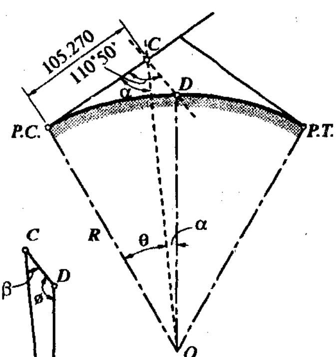
Problem 290:

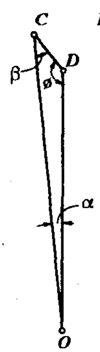
A 5° curve intersects a property line CD at point D. The back tangent intersects the property line at point C which is 105.270 m. from the P.C. which is at station 2 + 040. The angle that the property line CD makes with the back tangent is 110°50'.

① Determine the distance CD.
② Determine the stationing of point D.
③ Determine the deflection angle of point D from the P.C.

Solution:

① *Distance CD:*





$$R = \frac{114.916}{D}$$

$$R = \frac{1145.916}{5}$$

$R = 229.18$ m.

$$\tan\theta = \frac{105.27}{229.18}$$

$\theta = 24°40'$

$\alpha = 90° - 24°40'$

$\alpha = 65°20'$

$\beta = 110°50' - 65°20'$

$\beta = 45°30'$

$$\cos 24°40' = \frac{229.18}{OC}$$

$OC = 252.19$ m.

Using Sine Law:

$$\frac{229.18}{\sin 45°30'} = \frac{259.19}{\sin ø}$$

$ø = 128°18'$

$\alpha = 180° - 128°18' - 45°30'$

$\alpha = 6°12'$

$$\frac{CD}{\sin 6°12'} = \frac{229.18}{\sin 45°30'}$$

CD = 34.70 m.

② *Stationing of point D:*

$$L_c = \frac{R(\theta + \alpha)\pi}{180}$$

$$L_c = \frac{229.18\,(24°40' + 6°12')\,\pi}{180°}$$

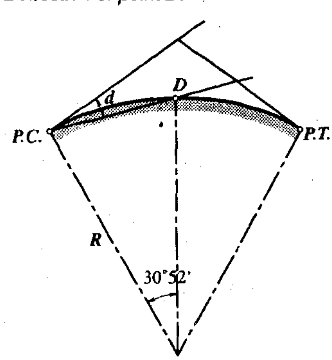
$$L_c = \frac{229.18\,(30°52')\,\pi}{180}$$

$L_c = 123.46$ m.

Sta. of point D = (2 + 040) + (123.46)

Sta. of point D = ***2 + 163.46***

③ *Deflection of point D:*



$$d = \frac{1}{2}(30°52')$$

$d =$ ***15°26'***

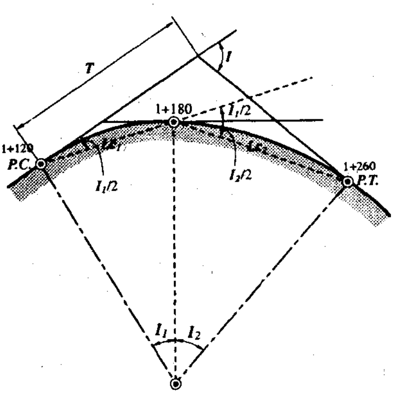
Problem 291:

A simple curve is to be laid out by deflection angle method. The P.C. of the 3° curve is located at station 1 + 120. Stations 1 + 140, 1 + 160, and 1 + 180 can be located with the transit set up at the P.C. Due to obstruction beyond 1 + 180, the transit must be transferred to station 1 + 180 in order to lay out the other stations of the curve. With the telescope in inverted position a backsight is taken on the P.C. with reading 00°00' and then the telescope is plunged back to normal position.

① Compute the angle of intersection of the simple curve.
② Compute the deflection angle that should be turned to locate the position of the P.T. which is at station 1 + 260.
③ Compute the tangent distance of the simple curve.

Solution:

① *Angle of intersection of the simple curve:*



Deflection angle at 1 + 180 to locate P.T.

$$= \frac{I_1}{2} + \frac{I_2}{2}$$

$LC_1 = (1 + 180) - (1 + 120)$

$LC_1 = 60$ m.

$$\frac{20}{D} = \frac{LC_1}{I_1}$$

$$I_1 = \frac{60(3°)}{20}$$

$I_1 = 9°$

$LC_2 = (1 + 260) - (1 + 180)$

$LC_2 = 80$ m.

$$\frac{20}{D} = \frac{LC_2}{I_2}$$

$$I_2 = \frac{(80)(3)}{20}$$

$I_2 = 12°$

$I = I_1 + I_2$

$I = 9° + 12°$

$\boldsymbol{I = 21°}$

② *Deflection angle:*

$$\text{Deflection angle} = \frac{21}{2}$$

Deflection angle = **10°30' R**

③ *Tangent distance of the simple curve:*

$$T = R \tan\frac{I}{2}$$

$$R = \frac{1145.916}{D}$$

$$R = \frac{1145.916}{3}$$

$R = 381.97$ m.

$$T = R \tan\frac{I}{2}$$

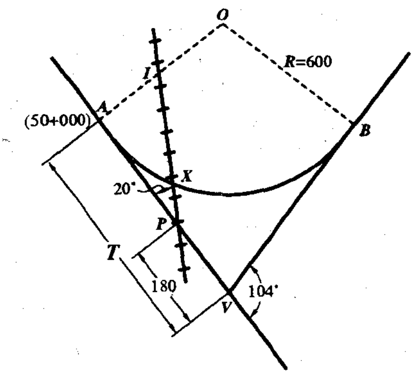
$T = 381.97 \tan 10°30'$

$\boldsymbol{T = 70.8}$ **m.**

Problem 292:

A straight railroad IP intersects the curve highway route AB. Distance on the route are measured along the arc. Using the data in the figure.

SIMPLE CURVES



① Compute the distance OI.
② Compute the distance PX.
③ Compute the stationing of point x.

Solution:

① *Distance OI:*

$\tan 52' = \dfrac{T}{R}$

$T = 600 \tan 52'$

$T = 767.96$ m.

$AP = 767.96 - 180$

$AP = 587.96$ m.

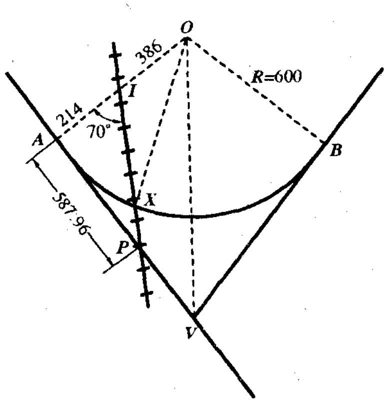
$\tan 20' = \dfrac{AI}{587.96}$

$AI = 214$ m.

$OI = 600 - 214$

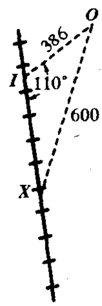
$OI =$ ***386 m.***

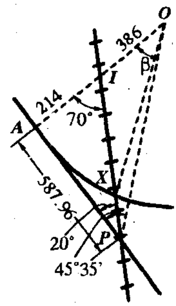
② *Distance PX:*

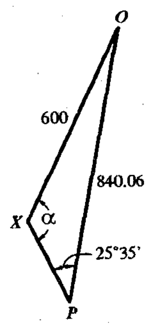


$\text{Sin } 70' = \dfrac{587.96}{PI}$

$PI = 625.69$ m.







$\tan ß = \dfrac{587.96}{600}$

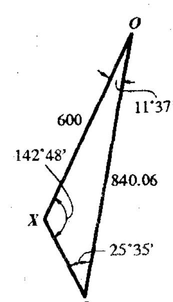
$ß = 44'25'$

$\text{Cos } 44'25' = \dfrac{600}{OP}$

$OP = 840.06$ m.

$\dfrac{840.06}{\text{Sin } \alpha} = \dfrac{600}{\text{Sin } 25'35'}$

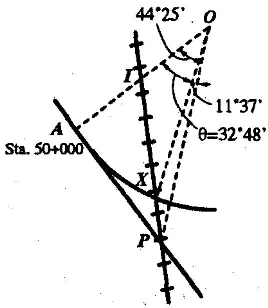
$\alpha = 142'48'$



$$\frac{PX}{\sin 11°37'} = \frac{600}{\sin 25°35"}$$

$PX = \mathbf{279.79\ m.}$

③ *Stationing of point x:*



$\theta = 44°25' - 11°37'$

$\theta = 32°48'$

$$AX = \frac{R\,\theta\,\pi}{180}$$

$$AX = \frac{600\,(32°48')\,\pi}{180}$$

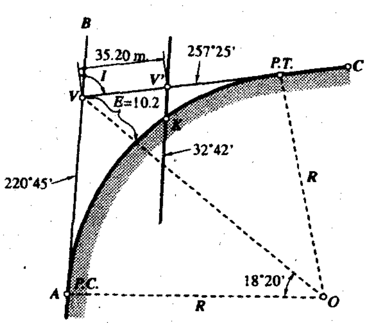
$AX = 343.48$ m.

Stationing of point x = (50 + 000) + (343.48)

Stationing of point x = ***50 + 343.48***

Problem 293:

In the figure shown the azimuth of AB = 220°45', azimuth of VC is 257°25', VV' = 35.20 m. VK with the simple curve. Stationing V is 10 + 283.68, E = 10.20 m.



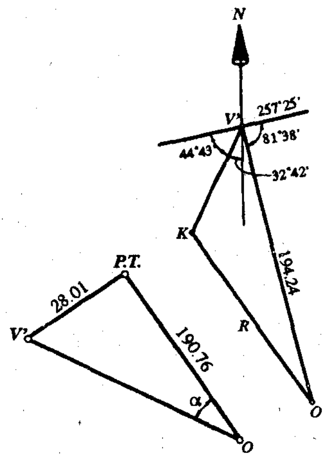
① Compute the radius R.

② Compute the stationing at PT if stationing of PC is 10 + 220.47.

③ Compute the stationing of point K.

Solution:

① *Radius:*



$I = 257°25' - 220°45'$

$I = 36°40'$

$E = 10.20$

$OV = R + E$

$$\cos 18°20' = \frac{R}{OV}$$

$R = (R + E)\cos 18°20'$

$R = R\cos 18°20' + 10.20\cos 18°20'$

$$R = \frac{10.20\,(0.949)}{0.051}$$

$R = \mathbf{190.76\ m.}$

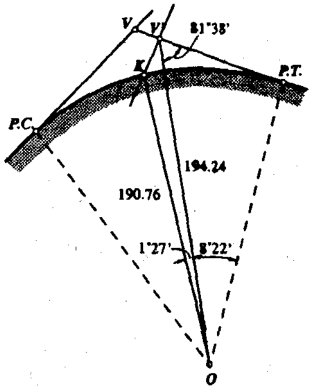
② *Stationing at PT:*

$$T = R\tan\frac{1}{2}$$

$T = 190.76\tan 18°20'$

$T = 63.21$ m.

SIMPLE CURVES



Stationing of $PC = (10 + 283.68) - 63.21$

Stationing of $PC = 10 + 220.47$

$VV' = 35.20$

$V'P.T. = 63.21 - 35.20$

$V'P.T. = 28.01$

$$\tan \alpha = \frac{28.01}{190.76}$$

$\alpha = 8°22'$

$$\cos 8°22' = \frac{190.76}{OV}$$

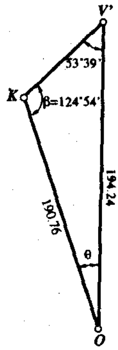
$OV = 194.24$ m.

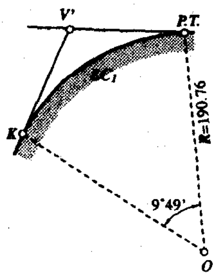
$$\frac{190.76}{\sin 53°39'} = \frac{194.24}{\sin B}$$

$B = 124°54'$

$ø = 180° - 124°54' - 53°39'$

$ø = 1°27'$





Arc P.T. to K:

$Lc_1 = R\,ø$

$$Lc_1 = \frac{190.76\,(9.817)\,\pi}{180}$$

$Lc_1 = 32.60$ m.

$$D = \frac{1145.916}{R}$$

$$D = \frac{1145.916}{190.76}$$

$D = 6°$

$$L_c = \frac{20\,I}{D}$$

$$L_c = \frac{20\,(36.67)}{6}$$

$L_c = 122.22$ m.

Sta. of $P.T. = (10 + 220.47) + 122.22$

Sta. of $P.T. = \mathbf{10 + 342.69}$

③ *Stationing of point K:*

Sta. of $K = (10 + 342.69) - 32.60$

Sta. of $K = \mathbf{10 + 310.09}$

Problem 294:

Two tangents intersecting at V with bearings N 75°12' E and S 78°36' E are connected with a 4° simple curve. Without changing the direction of the two tangents and with the same angle of intersection, it is required to shorten the curve to 100 m. starting from the P.C.

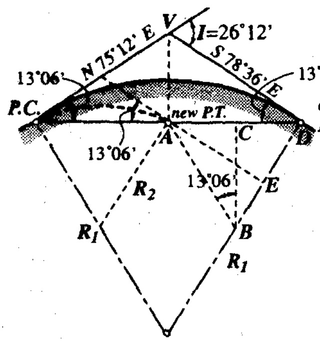
① Compute the change of length of the radius.

② By how much shall the PT be move and in what direction?

③ What is the distance between two parallel tangents?

Solution:

① *Change of length of radius:*



$$R_1 = \frac{1145.916}{D}$$
$$R_1 = \frac{1145.916}{4}$$
$R_1 = 286.48$ m.
$$L_c = \frac{20\,I}{D}$$
$$D = \frac{20\,(26^\circ 12')}{100}$$
$D = 5.24^\circ$
$$R_2 = \frac{1145.916}{5.24^\circ}$$
$R_2 = 218.69$ m.
$AB = R_1 - R_2$
$AB = 286.48 - 218.69$
$AB = \mathbf{67.79}$ *(change of length of radius)*

② *Distance and direction where P.T. must moved:*

$$\text{Sin } 13^\circ 06' = \frac{AC}{AB}$$
$AC = 67.79 \text{ Sin } 13^\circ 06'$
$AC = 15.36$ m.
$AD = 2\,(15.36)$
$AD = 30.72$

Therefore P.T. must be moved at a distance of ***30.72 m.*** *at an angle of* ***13°06'*** *from the second tangent.*

③ *Distance between two parallel tangents:*
$DE = AD \text{ Sin } 13^\circ 06'$
$DE = 30.72 \text{ Sin } 13^\circ 06'$
$DE = \mathbf{6.96\ m.}$

Problem 295:

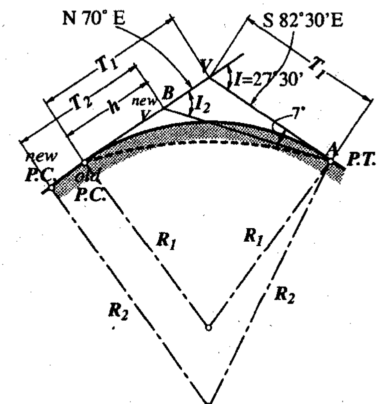
The bearing of the back tangent of a simple curve is N 70° E. while the forward tangent has a bearing of S 82°30' E. The degree of curve is 4.5°. Stationing of P.C. is at 10 + 345.43. It is proposed to decrease the central angle by changing the direction of the forward tangent by an angle of 7°, in such a way that the position of the P.T. of the forward tangent and the direction of the back tangent shall remain unchanged.

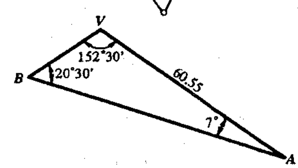
① Compute the new angle of intersection.
② Compute the new radius of curve.
③ Compute the stationing of new P.C.

Solution:

① *New angle of intersection:*
$I = 180^\circ - 70^\circ - 82^\circ 30'$
$I = 27^\circ 30'$
New $I_2 = 27^\circ 30' - 7^\circ 00'$
New $I_2 = \mathbf{20^\circ 30'}$

② *New radius of curve:*





$$R_1 = \frac{1145.916}{4.5}$$
$R_1 = 254.65$ m.
$$T_1 = R_1 \tan \frac{I_1}{2}$$
$T_1 = 254.65 \tan 13^\circ 45'$
$T_1 = 62.31$ m.

Considering triangle AVB:
$$\frac{AB}{\text{Sin } 152^\circ 30'} = \frac{62.31}{\text{Sin } 20^\circ 30'}$$
AB = 82.16
$T_2 = 82.16$
$$R_2 = \frac{82.16}{\tan 10^\circ 15'}$$
$R_2 = \mathbf{454.35}$ *(new radius)*

SIMPLE CURVES

③ *Stationing of new P.C:*

$$\frac{VB}{\sin 7^\circ} = \frac{62.31}{\sin 20^\circ 30'}$$

$VB = 21.68$

$h = T_1 - VB$

$h = 62.31 - 21.68$

$h = 40.63$

$T_2 - h = 82.16 - 40.63$

$T_2 - h = 41.53$

Sta. of new P.C. = (10 + 345.43) - (41.53)

Sta. of new P.C. = **10 + 303.90**

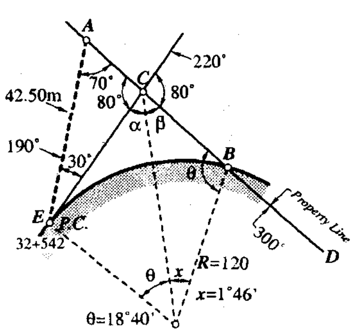
Problem 296:

A property line AD intersects a simple curve at point B whose radius is 120 m. long. The azimuth of the property line is 300° while that of the back tangent is 220°. A random line was run from the P.C. of the curve towards the property line which intersects at point A, with an azimuth and distance of 190° and 42.50 m. respectively.

① Compute the distance AC.
② Compute the central angle of arc EB.
③ Compute the stationing of B if P.C. is at (3 + 025.42).

Solution:

① *Distance AC:*



Considering triangle ACE:

$$\frac{42.50}{\sin 80^\circ} = \frac{EC}{\sin 70^\circ}$$

$EC = 40.55$ m.

$$\frac{42.50}{\sin 80^\circ} = \frac{AC}{\sin 30^\circ}$$

$AC = \mathbf{21.58\ m.}$

② *Angle of arc EB:*

Considering triangle OEC:

$$\tan \phi = \frac{40.55}{120}$$

$\phi = 18^\circ 40'$

$\alpha = 90^\circ - 18^\circ 40'$

$\alpha = 71^\circ 21'$

$\beta = 180^\circ - 80^\circ - 71^\circ 20'$

$\beta = 180^\circ - 151^\circ 20'$

$\beta = 28^\circ 40'$

Considering OEC:

$$\cos \phi = \frac{120}{OC}$$

$$OC = \frac{120}{\cos 18^\circ 40'}$$

$OC = 126.67$

Considering triangle OBC:

$$\frac{120}{\sin 28^\circ 40'} = \frac{126.67}{\sin \theta}$$

$\theta = 149^\circ 34'$

Angle X = 180° - 149°34' - 28°40'

Angle X = 180° - 178°14'

Angle X = 1°46'

Arc EB = 18°40' + 1°46'

Arc EB = **20°26'**

③ *Stationing of B:*

$$EB = \frac{120\,(20^\circ 26')\,\pi}{180}$$

$EB = 42.79$ m.

Stationing of B = (3 + 025.42) + (42.79)

Stationing of B = **3 + 068.21**

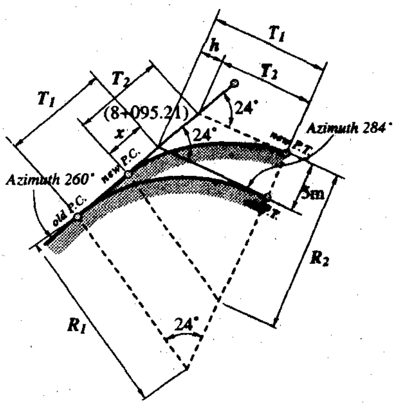
Problem 297:

Two tangents intersecting at station 8 + 095.21 have azimuth of 260° and 284° respectively, and are to be connected with a 5° simple curve. If the new P.T. is to be shifted 5 m. directly opposite of the old P.T. without changing the directions of the tangents.

① Compute the new tangent distance.
② Compute the radius of the new curve.
③ Compute the stationing of new P.C.

Solution:

① *New tangent distance:*



$$I = 284° - 260$$
$$I = 24°$$
$$R_1 = \frac{1145.916}{5°}$$
$$R_1 = 229.18 \text{ m.}$$

$$\tan 12° = \frac{T_1}{R_1}$$
$$T_1 = 229.18 \tan 12°$$
$$T_1 = 48.71 \text{ m.}$$
$$\tan 24° = \frac{5}{h}$$
$$h = 11.23 \text{ m.}$$
$$T_2 = 48.71 - 11.23$$
$$T_2 = \mathbf{37.48\ m.}$$

② *Radius of new curve:*

$$\tan 12° = \frac{T_2}{R_2}$$
$$R_2 = \frac{37.48}{\tan 12°}$$
$$R_2 = \mathbf{176.33\ m.}$$

③ *Stationing of new P.C.:*

$$VV' = \frac{5}{\sin 24°}$$
$$VV' = 12.29 \text{ m.}$$
$$x = T_2 - 12.29$$
$$x = 37.48 - 12.29$$
$$x = 25.19 \text{ m.}$$

Stationing of new P.C. = (8 + 095.21) - 25.19
Stationing of new P.C. = **8 + 070.02**

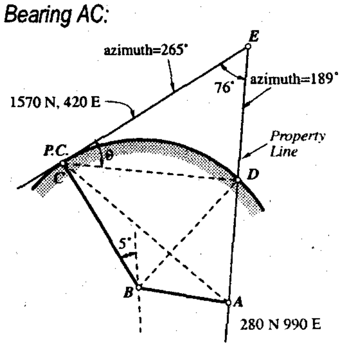
Problem 298:

The tangent through the P.C. has an azimuth 265° which intersects a property line at E, having an azimuth of 189°. The property line intersects the curve point D and passes also through point A having a coordinate of 280 N. and 990 E. The coordinate of P.C. is 1570 N. and 420 E. The radius of the simple curve is 1100 meters with a central angle of 59°. Bearing CB is S. 5° E.

① Compute the bearing AC.
② Compute the bearing of DB.
③ Compute the angle that the tangent through the P.C. makes with the chord connecting the P.C. and point D of the curve.

Solution:

① *Bearing AC:*



SIMPLE CURVES

Departure of line AC = 990 - 420 = 570
Latitude of line AC = 1570 - 280 = 1290

$$\text{tan bearing } AC = \frac{\text{Departure}}{\text{Latitude}}$$

$$\text{tan bearing} = \frac{570}{1290}$$

Bearing (AC) = **N. 23°50' W.**

② *Bearing DB:*
Departure of line BC = 1100 Sin 5°
Departure of line BC = 95.9 m.
Latitude of line BC = 1100 Cos 5°
Latitude of line BC = 1095.8 m.

Departure of B = 420 + 95.9
Departure of B = 515.9 E. *(Dep.)*
Latitude of B = 1570 - 1095.8
Latitude of B = 474.2 N. *(Lat.)*
Departure of line AB = 990 - 515.9 = 474.10
Latitude of line AB = 474.2 - 280 = 194.20

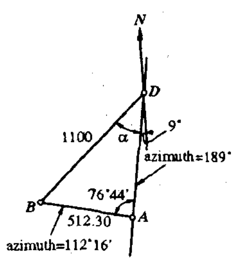
$$\text{tan bearing } (AB) = \frac{474.10}{194.20}$$

bearing (AB) = N. 67°44' W.

$$\text{Distance } (AB) = \frac{474.10}{\text{Sin } 67°44'}$$

AB = 512.30 m.

Considering triangle ABD:



$$\frac{512.30}{\text{Sin } \alpha} = \frac{1100}{\text{Sin } 70°44'}$$

$$\text{Sin } \alpha = \frac{512.30 \text{ Sin } 76°44'}{1100}$$

$\alpha = 26°58'$

Bearing of DB = 26°58' + 9°
Bearing of DB = **S. 35°58' W.**

③ *Angle:*
Angle CBD = 40°58'

$$\varnothing = \frac{1}{2} \text{ of angle } CBD$$

$$\varnothing = \frac{1}{2}(40°58')$$

$\varnothing$ = **20°29'**

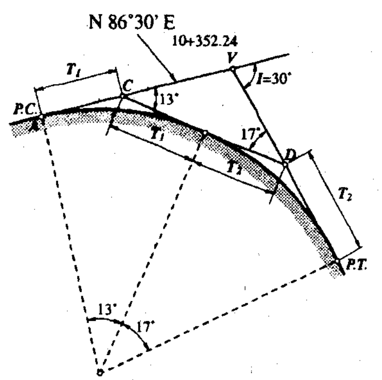
Problem 299:

Two tangents intersecting at V which is inaccessible has an angle of intersection of 30°. Two points C and D are laid out on the tangent through the P.C. and P.T respectively. The bearing and distance of the line joining C and D is S 80°30' E. 86.42 m. The highway engineer would like to construct a highway curve which shall be tangent to the two tangent line as well as the line CD. The bearing of the tangent through the P.C. is N. 86°30'E.

① Find the radius of the curve.
② Find the distance from P.C. to the P.I.
③ Find the stationing of the P.T. if P.I. is at station 10 + 352.24.

Solution:

① *Radius of the curve:*



$T_1 + T_2 = 86.42$ m.
$T_1 = R \tan 6°30'$
$T_2 = R \tan 8°30'$

$R(\tan 6°30' + \tan 8°30') = 86.42$

$$R = \frac{86.42}{0.263}$$

$R = \mathbf{328.59}$

② *Distance from P.C. to the P.I.:*

$T_1 = 328.59 \tan 6°30'$

$T_1 = 37.44$ m.

Considering triangle CVD:

$$\frac{86.42}{\text{Sin } 150°} = \frac{CV}{\text{Sin } 17°}$$

$CV = 50.53$ m.

$AV = T_1 + CV$

$AV = 37.44 + 50.53$

$AV = \mathbf{87.97\ m.}$

③ *Stationing of the P.I.:*

Stationing P.C. = (10 + 352.24) - (87.97)

Stationing P.C. = 10 + 264.27

$$R = \frac{1145.916}{D}$$

$$D = \frac{1145.916}{328.59}$$

$D = 3.49°$

$$L_c = \frac{20\, I}{D}$$

$$L_c = \frac{20\,(30)}{3.49}$$

$L_c = 171.92$ m.

Stationing of P.T. = (10 + 264.27) + (171.92)

Stationing of P.T. = **10 + 436.19**

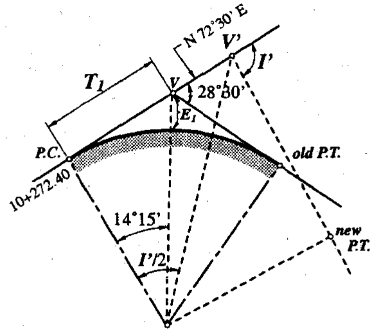
Problem 300:

Two tangents intersects at an angle of 28°30' at V whose stationing is at 10 + 362.40. If the external distance were increased by 6 m.

① Compute the new external distance.
② Compute the new bearing of second tangent line with the first tangent line remaining in the same direction in order that the degree of curve does not change.
③ Compute the stationing of the new P.T.

Solution:

① *New external distance:*



$T_1 = (10 + 362.40) - (10 + 272.40)$

$T_1 = 90$ m.

$T_1 = R \tan 14°15'$

$$R = \frac{90}{\tan 14°15'}$$

$R = 354.38$ m.

$$\text{Cos } 14°15' = \frac{R}{OV}$$

$$OV = \frac{354.38}{\text{Cos } 14°15'}$$

$OV = 365.63$

$E_1 = OV - R$

$E_1 = 365.63 - 354.38$

$E_1 = 11.25$ m.

New external distance:

$E_2 = 11.25 + 6$

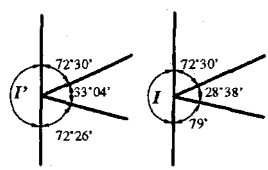
$E_2 = \mathbf{17.25}$

② *New bearing:*

$OV' = R + E_2$

$OV' = 354.38 + 17.25$

$OV' = 371.63$ m.



$$\text{Cos } \frac{I'}{2} = \frac{R}{OV'}$$

$$\text{Cos } \frac{I'}{2} = \frac{354.38}{371.63}$$

$\frac{I'}{2} = 17°32'$

$I' = 35°04'$

Old bearing of 2nd tangent line is = S. 79° E.
New bearing of 2nd tangent line is = ***S. 72°26' E.***

③ *Stationing of the new P.T.:*

$L_c = \frac{20\,I}{D}$

$D = \frac{1145.916}{354.38}$

$D = 3.23°$

$L_c = \frac{20\,(35°04')}{3.23°}$

$L_c = 217.15$ m.

New P.T. = (10 + 272.40) + 217.15

New P.T. = ***10 + 489.55***

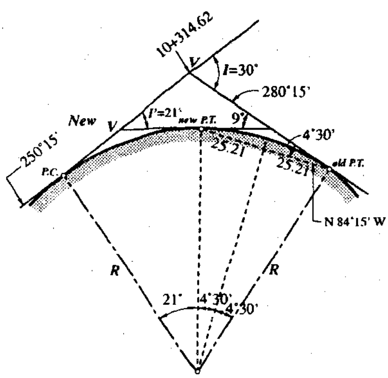
Problem 301:

Two tangents of a simple curve have azimuth of 250°15' and 280°15' respectively and intersects at V at station 10 + 314.62. It is required to shorten the curve to point C on the curve having a direction of N 84°15' W, 50.42 m. from the old P.T. without changing the degree of curve and the P.C.

① Compute the new angle of intersection of the tangents.
② Compute the stationing of new P.T.
③ Compute the stationing of new vertex.

Solution:

① *New angle intersection of the tangents:*



$\sin 4°30' = \frac{25.21}{R}$

$R_1 = 321.31$ m.

$T_1 = R_1 \tan\frac{I'}{2}$

$T_1 = 321.31 \tan 15°$

$T_1 = 86.09$ m.

$I' = 30 - 9 = 21°$

New angle of intersection = I' ***= 21°***

② *Stationing of new P.T.:*

Sta. of P.C. = (10 + 314.62) - 86.09

Sta. of P.C. = 10 + 228.53

$D = \frac{1145.916}{R}$

$D = \frac{1145.916}{321.31}$

$D = 3.57°$

$L_c = \frac{20\,I}{D}$

$L_c = \frac{20\,(21)}{3.57}$

$L_c = 117.65$

Sta. of new P.T. = (10 + 228.53) + 117.65

Sta. of new P.T. = ***10 + 346.18***

③ *Stationing of new vertex:*

$T_2 = R\tan\frac{I'}{2}$

$T_2 = 321.31 \tan\frac{21°}{2}$

$T_2 = 59.55$

Sta. of new vertex = (10 + 228.53) + 59.55

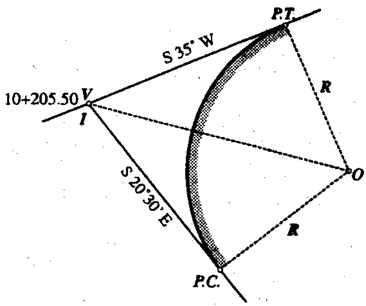
Sta. of new vertex = ***10 + 288.08***

Problem 302:

The bearings of two tangents are S. 20°30' E. and S. 35°00' W. If the degree of curve is 10° for a chord of 10 meters and the stationing at the vertex is 10 + 205.50.

① Find the stationing at the P.T.
② Find the length of the last subchord, and its corresponding sub-angle.
③ Find the deflection distance to the fourth station.

Solution:



$I = 20°50' + 35°00'$
$I = 55°30'$
$\frac{I}{2} = 27°45'$
$D = 10°$
$C = 10$ meters

① *Stationing at the P.T.*

$R = \frac{10}{2\ \text{Sin}\ \frac{D}{2}}$

$R = \frac{5}{\text{Sin}\ 5°} = 57.37$ m.

$T = R \tan 27°45'$
$T = 57.37 \tan 27°45'$
$T = 30.18$ m.

Station of P.C. = (10 + 205.50) - (30.18)
P.C. = 10 + 175.32

$L = \frac{10(I)}{D}$

$L = \frac{10(55.5)}{10} = 55.50$ m.

Station of P.T. = (10 + 175.32) + (55.50)
Station of P.T. = **10 + 230.82**

② *Length of the last subchord, to the fourth station:*

$c_2 = (10 + 230.82) - (10 + 230.00)$
$c_2 = 0.82$ m.

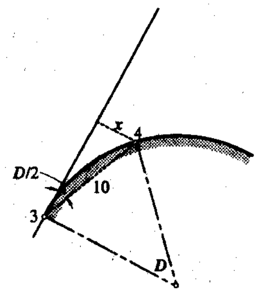
$\frac{d_2}{c_2} = \frac{D}{C}$

$d_2 = \frac{c_2 D}{C}$

$d_2 = \frac{0.82(10)}{10}$

$d_2 = 0.82°$
$d_2 = \mathbf{0°49'}$

③ *Deflection distance to the fourth station:*



$\text{Sin}\ \frac{D}{2} = \frac{x}{10}$

$x = 10\ \text{Sin}\ \frac{D}{2}$

But: $\text{Sin}\ \frac{D}{2} = \frac{C}{2R}$

$x = \frac{10\ (10)}{2R}$

$x = \frac{100}{2(57.37)}$

$x = 0.87$ m.

Deflection distance = 2x
Deflection distance = 2(0.87)
Deflection distance = ***1.74 m.***

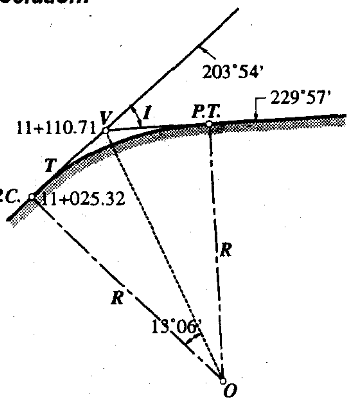
Problem 303:

Two intersecting tangents have azimuths 203°45' and 229°57', respectively. The stationing of the vertex and the point of curvature P.C. are respectively 11 + 110.71 and 11 + 025.32.

① If the external distance of this curved were decreased by 5 meters, what would be the change in the direction of the second tangent, with the first tangent line remaining in the same direction in order that the degree of curve does not change?
② Determine the azimuth of the new forward tangent.
③ Compute the stationing of new P.T.

SIMPLE CURVES

Solution:



① *Change in direction of the second tangent:*

$I = 229°57' - 203°45'$
$I = 26°12'$
$T = 85.39$
$T = R \tan 13°06'$
$85.39 = R \tan 13°06'$
$R = 366.94$
$OV = R + E_1$

$$\text{Cos } 13°06' = \frac{R}{R + E_1}$$

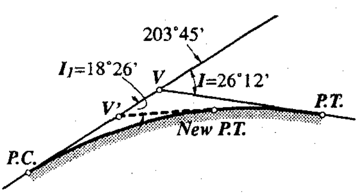
$R = R \text{ Cos } 13°06' + E_1 \text{ Cos } 13°06'$
$366.94 = 396.94 \text{ Cos } 13°06' + E_1 \text{ Cos } 13°06'$
$E_1 = 9.80$ m.

New $E = 9.80 - 5$
New $E = 4.80$ *(new external distance)*

$$E = R\left(\text{Sec}\frac{I_1}{2} - 1\right)$$

$$4.80 = 3.66.94\left(\text{Sec}\frac{I_1}{2} - 1\right)$$

$I_1 = 9°13'$
$I_1 = 18°26'$ *(new angle of intersection)*



The change in direction of the second tangent is 26°12' - 18°26' = ***7°46'***

② *Azimuth of new forward tangent:*

$= 203°45' + 18°26'$
$= \mathbf{222°11'}$

③ *Stationing of new P.T.*

New $I = 18°26'$

$$D = \frac{1145.916}{R}$$

$$D = \frac{1145.916}{366.94}$$

$D = 3.12°$

$$L_c = \frac{20\,I}{D}$$

$$L_c = \frac{20\,(18°26')}{3.12°}$$

$L_c = 118.16$

Sta. of new P.T. = (11 + 025.32) + 118.16
Sta. of new P.T. = ***11 + 143.48***

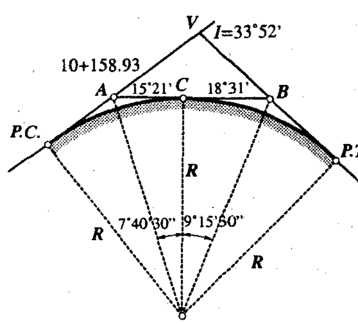
Problem 304:

In the figure shown AB = 103.20 m. with A at station 10 + 158.93. The angle VAC is 15°21' and angle VBC is 18°31', where C is the point to which a simple curve is to be constructed tangent to the line AB.

① Determine the radius of the curve.
② Determine the stationing of point C.
③ Determine the stationing of P.T.

Solution:

① *Radius of curve:*



$$\tan 7°40'30" = \frac{AC}{R}$$
$$AC = R \tan 7°40'30"$$
$$\tan 9°15'30" = \frac{BC}{R}$$
$$BC = R \tan 9°15'30"$$
$$AC + BC = 103.20$$
$$R \tan 7°40'30" + R \tan 9°15'30" = 103.20$$
$$R(0.13476 + 0.16301) = 103.20$$
$$R = \mathbf{346.58\ m.}$$

② *Stationing of Point C:*

$$T = R \tan \frac{I}{2}$$
$$T = 346.58 \tan 16°56'$$
$$T = 105.52 \text{ m.}$$
$$AC = 346 \tan 7°40'30"$$
$$AC = 46.70 \text{ m.}$$

Stationing of P.C.:

P.C. = (10 + 158.93) - AC
P.C. = (10 + 158.93) - 46.70
P.C. = 10 + 112.23

$$D = \frac{1145.916}{R} = \frac{1145.916}{346.58}$$
$$D = 3.31°$$
$$L_{c_1} = \frac{I(20)}{D}$$
$$L_{c_1} = \frac{(15°21')}{3.31°}(20)$$
$$L_{c_1} = 92.75 \text{ m.}$$

Stationing of C:

C = (10 + 112.23) + 92.75
C = **10 + 204.98**

③ *Stationing of P.T.:*

$$L = \frac{I(20)}{D} = \frac{(33°52')(20)}{3.32}$$
$$L = 205.07$$

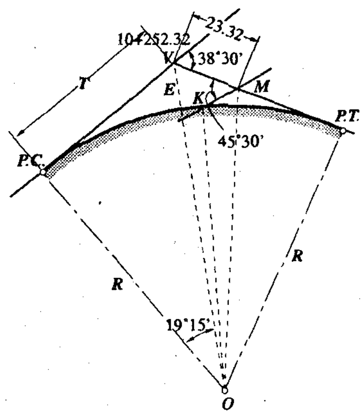
P.T. = (10 + 112.23) + 204.63
P.T. = **10 + 316.86**

Problem 305:

A circular curve has the following properties. External distance is 18 m. angle of intersection of tangents is 38°30'. Stationing of vertex V (intersection of the tangents) is 10 + 252.32. A line MK intersects the forward tangent at M and the circular curve at K. Point M is 12.32 m. from V and the angle VMK is 45°30'.

① Find the radius of the curve.
② Find the distance MK.
③ Find the stationing of K.

Solution:



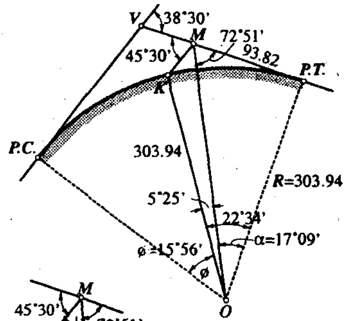
① *Radius of curve:*

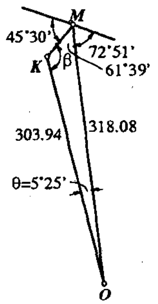
$$\text{Cos } 19°15' = \frac{R}{OV}$$
$$OV = R + E$$
$$OV = R + 18$$
$$\text{Cos } 19°15' = \frac{R}{R + 18}$$
$$R = \mathbf{303.94}$$

② *Distance MK:*

$$T = R \tan 19°15'$$
$$T = 303.94 \tan 19°15'$$
$$T = 106.14 \text{ m.}$$

M to P.T. = 106.14 - 12.32
M to P.T. = 93.82 m.





$$\tan \alpha = \frac{93.82}{303.94}$$

$\alpha = 17°09'$

$$\cos 17°09' = \frac{303.94}{OM}$$

$OM = 318.08$

Using Sine Law:

$$\frac{318.08}{\sin \beta} = \frac{303.94}{\sin 61°39'}$$

$\beta = 112°56'$

$\theta = 180° - 112°56' - 61°39'$

$\theta = 5°25'$

$$\frac{MK}{\sin 5°25'} = \frac{303.94}{\sin 61°39'}$$

$MK = \mathbf{32.60\ m.}$

③ *Stationing of K:*

Sta. of P.C. = (10 + 252.32) - (106.14)

Sta. of P.C. = 10 + 146.18

$$L_c = \frac{R\,ø\,\pi}{180}$$

$$L_c = \frac{303.94\,(15°56')\,\pi}{180}$$

$L_c = 84.52$

Sta. of K = (10 + 146.18) + 84.52

Sta. of K = ***10 + 230.70***

Problem 306:

Two tangents intersecting at V whose stationing is at 1 + 027.32 has the angle of intersection of 26°12'. It is to be connected with a 4° curve based on the arc basis. Without changing the directions of the two tangents, it is required to shorten the curve to 100 m. starting from the same P.C.

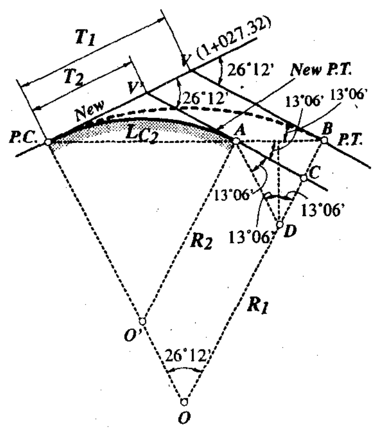
① By how much shall the P.T. be moved and in what direction with respect to the second tangent.

② What is the distance between the two parallel tangents.

③ What is the stationing of the new point of tangency.

Solution:

① *Distance the P.T. is moved:*



$$R_1 = \frac{1145.916}{D}$$

$$R_1 = \frac{1145.916}{4}$$

$R_1 = 286.48$ m.

$L_{c_2} = 100$ m.

$$L_{c_2} = \frac{20\,I_2}{D_2}$$

$$D_2 = \frac{20(26.2')}{100}$$

$$D_2 = 5.24'$$

Construct AD parallel to 00'

$$R_2 = \frac{1145.916}{5.24}$$

$$R_2 = 218.69 \text{ m.}$$

$$AD = R_1 - R_2$$

$$AD = 286.48 - 218.69$$

$$AD = 67.79$$

$$\text{Sin } 13°06' = \frac{AB}{2\,AD}$$

$$AB = 2(67.79) \text{ Sin } 13°06'$$

*AB = **30.73 m.** distance the P.T. is moved at an angle of 13°06' from the 2nd tangent.*

② *Distance between the two parallel tangents:*

$$BC = AB \text{ Sin } 13°06'$$

$$BC = 30.73 \text{ Sin } 13°06'$$

$$BC = \mathbf{6.965\ m.}$$

③ *Stationing of the new point of tangency.*

$$T_1 = R_1 \tan 13°06'$$

$$T_1 = 286.48 \tan 13°06'$$

$$T_1 = 66.67 \text{ m.}$$

Stationing of P.C.

$$P.C. = (1 + 027.32) - 66.67$$

$$P.C. = 0 + 960.65$$

$$L_{c_2} = 100$$

Stationing of New P.T.

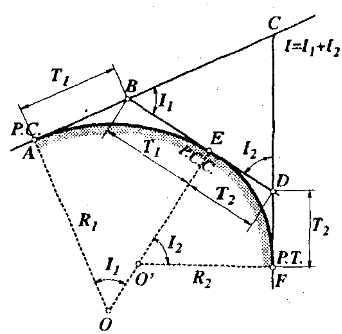
$$\text{New } P.T. = (0 + 960.65) + 100$$

$$P.T. = \mathbf{1 + 060.65.1}$$

COMPOUND CURVES

Compound Curve consists of two or more consecutive simple curves having different radius, but whose centers lie on the same side of the curve, likewise any two consecutive curves must have a common tangent at their meeting point. When two such curves lie upon opposite sides of the common tangent, the two curves then turns a reversed curve. In a compound curve, the point of the common tangent where the two curves join is called the point of compound curvature (P.C.C.)

Elements of a compound curve:



R_1 = radius of the curve AE

R_2 = radius of the curve EF

T_1 = tangent distance of the curve AE

T_2 = tangent distance of the curve EF

$BD = T_1 + T_2$ = common tangent

I_1 = central angle of curve AE

I_2 = central angle of curve EF

I = angle of intersection of tangents AC and CF.

$$T_1 = R_1 \tan \frac{I_1}{2}$$

$$T_2 = R_2 \tan \frac{I_2}{2}$$

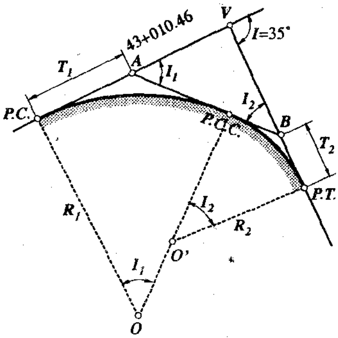
COMPOUND CURVES

Problem 307:

The common tangent AB of a compound curve is 76.42 m. with an azimuth of 268°30'. The vertex V being inaccessible. The azimuth of the tangents AV and VB was measured to be 247°50' and 282°50', respectively. If the stationing of A is 43 + 010.46 and the degree of the first curve was fixed at 4° based on the 20 m. chord. Using chord basis.

① Determine the stationing of the P.C.
② Determine the stationing of the P.C.C.
③ Determine the stationing of the P.T.

Solution:



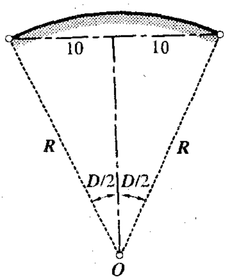
① *Stationing of the P.C.*

$I_1 = 268°30' - 247°50'$

$I_1 = 20°40'$

$I_2 = 282°50' - 268°30'$

$I_2 = 14°20'$



$D_1 = 4°$

$\text{Sin } \dfrac{D_1}{2} = \dfrac{10}{R_1}$

$\text{Sin } 2° = \dfrac{10}{R_1}$

$R_1 = 286.56$

$T_1 = R_1 \tan \dfrac{I_1}{2}$

$T_1 = 286.56 \tan 10°20'$

$T_1 = 52.25 \text{ m.}$

$P.C. = (43 + 010.46) - 52.25$

$P.C. = \mathbf{42 + 958.21}$

② *Stationing of the P.C.C.*

$T_1 + T_2 = 76.42$

$T_2 = 76.42 - 52.25$

$T_2 = 24.17$

$T_2 = R_2 \tan \dfrac{I_2}{2}$

$24.17 = R_2 \tan 7°10'$

$R_2 = 192.233 \text{ m.}$

$\text{Sin } \dfrac{D_2}{2} = \dfrac{10}{R_2}$

$\text{Sin } \dfrac{D_2}{2} = \dfrac{10}{192.23}$

$\dfrac{D}{2} = 2°59'$

$D_2 = 5°58'$

$L_{c_1} = \dfrac{I_1 (20)}{D_1}$

$L_{c_1} = \dfrac{20°40' (20)}{4}$

$L_{c_1} = \dfrac{20.667(20)}{4}$

$L_{c_1} = 103.34$

$P.C.C. = (42 + 958.21) + 103.34$

$P.C.C. = \mathbf{103.34}$

③ *Stationing of the P.T.*

$L_{c_2} = \dfrac{I_2 (20)}{D_2}$

$L_{c_2} = \dfrac{14°20 (20)}{5°58'}$

$L_{c_2} = \dfrac{14.33(20)}{5.966}$

$L_{c_2} = 48.10$

$P.T. = (43 + 06.55) + 48.10$

$P.T. = \mathbf{43 + 109.65}$

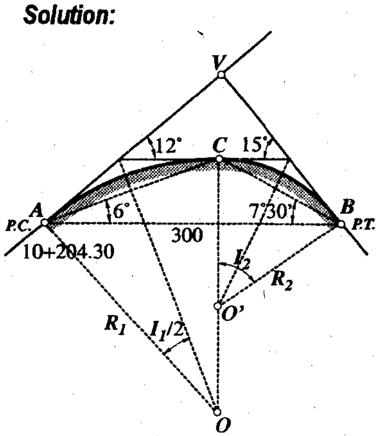
COMPOUND CURVES

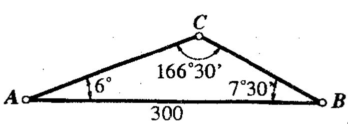
Problem 308:

The long chord from the P.C. to the P.T. of a compound curve is 300 meters long and the angles it makes with the longer and shorter tangents are 12° and 15° respectively. If the common tangent is parallel to the long chord.

① Find the radius of the first curve.
② Find the radius of the 2nd curve.
③ If stationing of P.C. is 10 + 204.30, find the stationing of P.T.

Solution:





① *Radius of the first curve:*

$I_1 = 12°$

$I_2 = 15°$

Considering triangle ABC:

$$\frac{300}{\text{Sin } 166°30'} = \frac{BC}{\text{Sin } 6°}$$

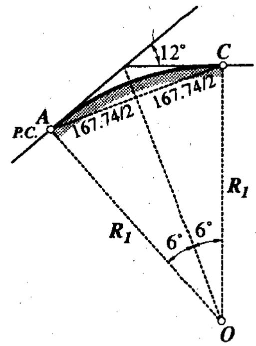
$$BC = \frac{300 \text{ Sin } 6°}{\text{Sin } 166°30'}$$

$BC = 134.33$ m.

$$\frac{300}{\text{Sin } 166°30'} = \frac{AC}{\text{Sin } 7°30'}$$

$$AC = \frac{300 \text{ Sin } 7°30'}{\text{Sin } 166°30'}$$

$AC = 167.74$ m.

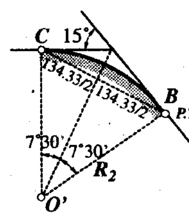


$$\text{Sin } \frac{I_1}{2} = \frac{AC}{2R_1}$$

$$R_1 = \frac{167.74}{2 \text{ Sin } 6°}$$

$R_1 = $ ***802.36 m.***

② *Radius of the 2nd curve:*



$$R_2 = \frac{BC}{2 \text{ Sin } 7°30'}$$

$$R_2 = \frac{134.33}{2 \text{ Sin } 7°30'}$$

$R_2 = $ ***514.55 m.***

③ *Stationing of P.T.*

$$L_{c_1} = \frac{R_1 I_1 \pi}{180}$$

$$L_{c_1} = \frac{802.36(12°)\pi}{180°}$$

$L_{c_1} = 168.05$ m.

$$L_{c_2} = \frac{R_2 I_2 \pi}{180}$$

$$L_{c_2} = \frac{514.55(15°)\pi}{180}$$

$L_{c_2} = 134.71$ m.

Sta. of P.T. = (10 + 204.30) + 168.05 + 134.71

Sta. of P.T. = ***10 + 507.06***

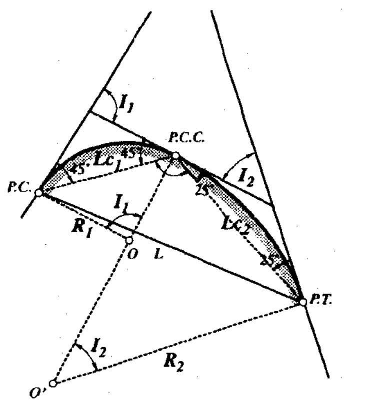
COMPOUND CURVES

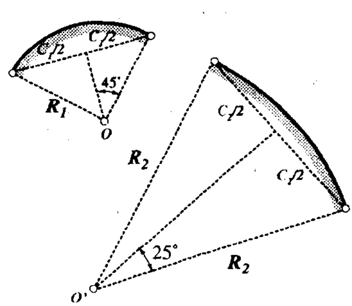
Problem 309:

The locating engineer a railroad curve runs a 6° curve to the P.C.C., 300 m. long from the P.C. of the compound curve, thence from the P.C.C., a 1°40' curve was run towards to the P.T. 600 m. long. Use arc basis.

① It is required to determine the length of the long chord connecting the P.C. and P.T.
② Find the angle that the long chord makes with the first tangent.
③ Find the angle that the long chord makes with the 2nd tangent.

Solution:





① *Length of the long chord connecting the P.C. and P.T.*

$L_{c_1} = 300$

$D_1 = 6°$

$L_{c_1} = \frac{20\, I_1}{D_1}$

$I_1 = \frac{300(6)}{20}$

$I_1 = 90°$

$L_{c_2} = 600$

$D_2 = 1°40'$

$L_{c_2} = \frac{20\, I_2}{D_2}$

$I_2 = \frac{600(1.667)}{20}$

$I_2 = 50°$

$R_1 = \frac{1145.916}{D_1}$

$R_1 = \frac{1145.916}{6}$

$R_1 = 190.99$ m.

$\text{Sin } 45° = \frac{c_1}{2\, R_1}$

$c_1 = 2\, R_1 \text{ Sin } 45°$

$c_1 = 2(190.99) \text{ Sin } 45°$

$c_1 = 270.10$ m.

$R_2 = \frac{1145.916}{D_2}$

$R_2 = \frac{1145.916}{1.66°}$

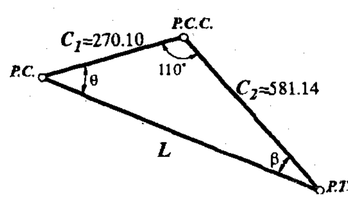
$R_2 = 687.55$ m.

$\text{Sin } 25° = \frac{c_2}{2\, R_2}$

$c_2 = 2\, R_c \text{ Sin } 25°$

$c_2 = 2(687.55) \text{ Sin } 25°$

$c_2 = 581.14$

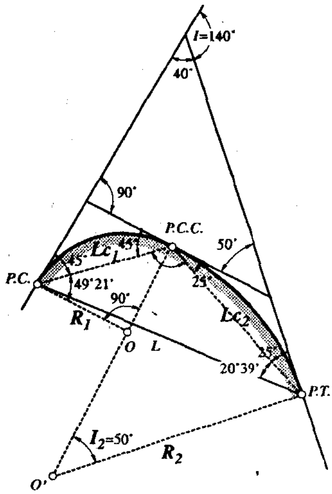


$L^2 = (270.10)^2 + (581.14)^2 - 2(270.10)(581.14) \text{ Cos } 110°$

$L = \mathbf{719.76\ m.}$

COMPOUND CURVES

② *Angle that the long chord makes with the first tangent:*



$$\frac{581.14}{\sin\theta}=\frac{719.76}{\sin 110°}$$

$\theta = 49°21'$

$$\frac{\sin B}{270.10}=\frac{\sin 110°}{719.76}$$

$B = 20°39'$

The angle of the long chord makes with the first tangent line is

$45° + 49°21' = \mathbf{94°21'}$

③ *Angle that the long chord makes with the 2nd tangent line is* $25° + 20°39' = \mathbf{45°39'}$

Problem 310:

Given the following compound curve with the vertex V, inaccessible. Angles VAD and VDA are equal to 16°20' and 13°30' respectively. Stationing of A is 1 + 125.92. Degree of curve are 3°30' for the first curve and 4°00' for the second curve.

① It is desired to substitute the compound curve with a simple curve that shall end with the same P.T., determine the total length of curve of the simple curve.
② It is desired to substitute the compound curve with a simple curve that shall be tangent to the two tangent lines as well as the common tangent AD. What is the radius of the simple curve.
③ What is the stationing of the new P.C.

Solution:

① *Total length of curve of the simple curve:*

$D_1 = 3°30'$

$I_1 = 16°20'$

$$R_1 = \frac{1145.916}{D_1}$$

$$R_1 = \frac{1145.916}{3°30'}$$

$R_1 = 327.40$

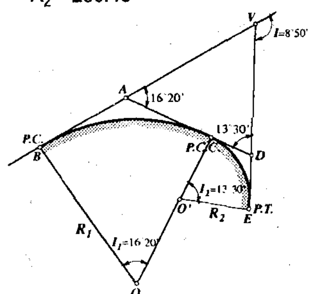
$D_2 = 4°00'$

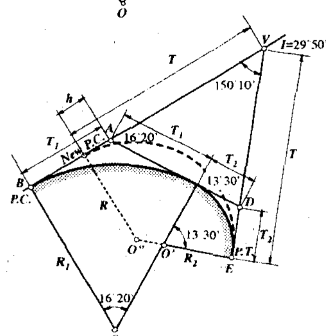
$I_2 = 13°30'$

$$R_2 = \frac{1145.916}{D_2}$$

$$R_2 = \frac{1145.916}{4}$$

$R_2 = 286.48$





COMPOUND CURVES

$$T_1 = R_1 \tan\frac{I_1}{2}$$
$$T_1 = 327.40 \tan 8°10'$$
$$T_1 = 46.98$$
$$T_2 = R_2 \tan\frac{I_2}{2}$$
$$T_2 = 286.48 \tan 6°45'$$
$$T_2 = 33.91$$

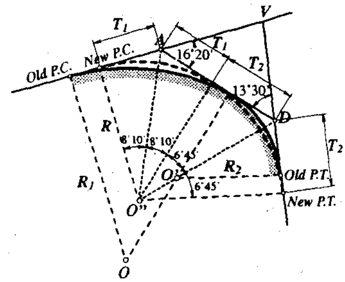
$$AD = T_1 + T_2$$
$$AD = 46.98 + 33.91$$
$$AD = 80.89$$
$$\frac{VA}{\text{Sin } 13°30'} = \frac{80.89}{\text{Sin } 150°10'}$$
$$VA = 37.96$$
$$\frac{VD}{\text{Sin } 16°20'} = \frac{80.89}{\text{Sin } 150°10'}$$
$$VD = 45.73$$

$$T = VD + T_2$$
$$T = 45.73 + 33.91$$
$$T = 79.64$$
$$T = R \tan\frac{I}{2}$$
$$R = \frac{T}{\tan 14°55'}$$
$$R = \frac{79.64}{\tan 14°55'}$$
$$R = 298.96$$

$$D = \frac{1145.916}{298.96}$$
$$D = 3°50'$$
$$h = T - VA$$
$$h = 79.64 - 37.96$$
$$h = 41.68$$

Length of curve:
$$L = \frac{I(20)}{D}$$
$$L = \frac{29°50'(20)}{3°50'}$$
$$L = \frac{29.833(20)}{3.833}$$
$$L = \mathbf{155.66}$$

② *Radius of the simple curve:*



$$T_1 + T_2 = 80.89$$
$$T_1 = R \tan 8°10'$$
$$T_2 = R \tan 6°45'$$
$$R(\tan 8°10' + \tan 6°45') = 80.90$$
$$R = \mathbf{308.89}$$

③ *Stationing of the new P.C.*
$$T_1 = R \tan 8°10'$$
$$T_1 = 308.89 \tan 8°10'$$
$$T_1 = 44.33$$

Stationing of new P.C.
P.C. = (1 + 125.98) - (44.33)
P.C. = **1 + 081.65**

Problem 311:

A compound curve connects three tangents having an azimuths of 254°, 270° and 280° respectively. The length of the chord is 320 m. long measured from the P.C. to the P.T. of the curve and is parallel to the common tangent having an azimuth of 270°. If the stationing of the P.T. is 6 + 520.

① Determine the total length of the curve.
② Determine the stationing of the P.C.C.
③ Determine the stationing of the P.C.

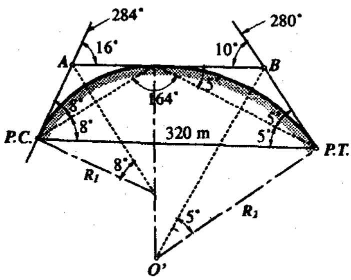
Solution:

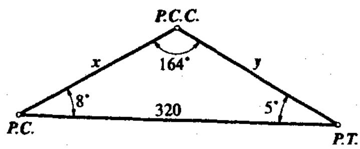
① *Total length of the curve:*
$$\frac{320}{\text{Sin } 164°} = \frac{x}{\text{Sin } 5°}$$
$$x = \frac{320 \text{ Sin } 5°}{\text{Sin } 164°}$$
$$x = 101.18 \text{ m.}$$

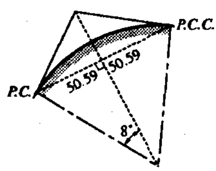
$$\frac{320}{\text{Sin } 164^\circ} = \frac{y}{\text{Sin } 8^\circ}$$

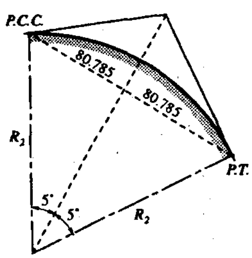
$$y = \frac{320 \text{ Sin } 8^\circ}{\text{Sin } 164^\circ}$$

$$y = 161.57 \text{ m.}$$









$$\text{Sin } 8^\circ = \frac{50.59}{R_1}$$

$$R_1 = \frac{50.59}{\text{Sin } 8^\circ}$$

$$R_1 = 363.50 \text{ m.}$$

$$D_1 = \frac{1145.916}{R_1}$$

$$D_1 = \frac{1145.916}{363.50}$$

$$D_1 = 3.15^\circ$$

$$L_{c_1} = \frac{20\, I_1}{D_1}$$

$$L_{c_1} = \frac{20(16)}{3.15^\circ}$$

$$L_{c_1} = 101.59 \text{ m.}$$

$$\text{Sin } 5^\circ = \frac{80.785}{R_2}$$

$$R_2 = \frac{80.785}{\text{Sin } 5^\circ}$$

$$R_2 = 926.90 \text{ m.}$$

$$D_2 = \frac{1145.916}{R_2}$$

$$D_2 = \frac{1145.916}{926.90}$$

$$D_2 = 1.24^\circ$$

$$L_{c_2} = \frac{20\, I_2}{D_2}$$

$$L_{c_2} = \frac{20(10)}{1.24}$$

$$L_{c_2} = 161.29 \text{ m.}$$

Total length of curve = 161.29 + 101.59
Total length of curve = ***262.88 m.***

② *Stationing of the P.C.C.*
P.C.C. = (6 + 520) - 161.29
P.C.C. = ***6 + 358.71***

③ *Stationing of the P.C.*
P.C. = (6 + 358.71) - 101.59
P.C. = ***6 + 257.12***

Problem 312:

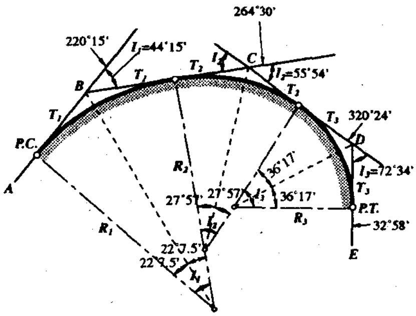
A turn around pattern which fits with the topography is provided in a highway by connecting four tangents with a compound curve consisting of three simple curves. The azimuths of AB = 220°15', BC = 264°30', CD = 320°24' and DE = 32°58'. The radius of the last curve is four times sharper than the first curve. The distance BC = 303 m. and CD = 200 m.

① Compute the radius of the 3rd curve.
② Compute the radius of the second curve.
③ If PC is at 12 + 152.60, what is the stationing of the P.T.

COMPOUND CURVES

Solution:

① *Radius of third curve:*



$I_1 = 264°30' - 220°15'$
$I_1 = 44°15'$
$I_2 = 320°24' - 264°30'$
$I_2 = 55°54'$
$I_3 = 360° - 320°24' + 32°58'$
$I_3 = 72°34'$

$T_1 + T_2 = 303$
$R_1 \tan 22°7.5' + R_2 \tan 27°57' = 303$
$0.407\ R_1 + 0.530\ R_2 = 303$
$T_2 + T_3 = 200$
$R_2 \tan 27°57' + R_3 \tan 36°17' = 200$
$0.530\ R_2 = 0.734\ R_3 = 300$

$R_1 = 4\ R_3$
$0.407\ (4\ R_3) + 0.530\ R_2 = 303$
$1.628\ R_3 + 0.530\ R_2 = 303$
$\underline{0.734\ R_3 + 0.530 R_2 = 200}$

$0.894\ R_3 = 103$
$R_3 = \mathbf{115.21}$

② *Radius of 2nd curve:*
$R_1 = 4\ (115.21)$
$R_1 = 460.84$ m.

$0.407\ R_1 + 0.530\ R_2 = 303$
$0.407(460.84) + 0.530\ R_2 = 303$
$R_2 = \mathbf{217.81\ m.}$

③ *Stationing of the P.T.*

$$L_{c_1} = \frac{R_1\ I_1\ \pi}{180}$$
$$L_{c_1} = \frac{460.84\ (44°15')\ \pi}{180°}$$
$$L_{c_1} = 355.91 \text{ m.}$$

$$L_{c_2} = \frac{R_2\ I_2\ \pi}{180}$$
$$L_{c_2} = \frac{217.81\ (55°54')\ \pi}{180°}$$
$$L_{c_2} = 212.50 \text{ m.}$$
$$L_{c_3} = \frac{R_3\ I_3\ \pi}{180}$$
$$L_{c_3} = \frac{115.21\ (72°34')\ \pi}{180°}$$
$$L_{c_3} = 145.92 \text{ m.}$$

Sta. of P.T. = (12 + 152.60) + 355.91 + 212.50 + 145.92
Sta. of P.T. = ***12 + 654.43***

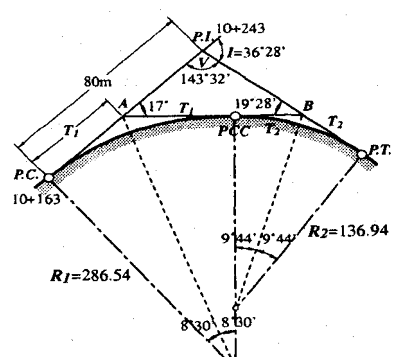
Problem 313:

On a rail road line, two tangents that intersect at station 10 + 243 so as to form an angle of 36°28' are to be connected by a compound curve consisting of two simple curves. The simple curve beginning at the P.C. which is at station 10 + 163 is to be a 4° curve whose degree is based on a 20 m. chord and is to have a central angle of 17°. Using chord basis.

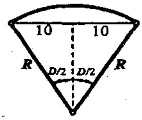
① What should be the radius of the other simple curve that ends at the P.T.?
② Compute the stationing of the P.C.C.
③ What is the length of the tangent from the P.I. to the P.T. of the compound curve?

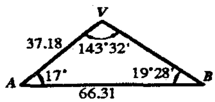
Solution:

① *Radius of second curve:*



COMPOUND CURVES





$D_1 = 4°$

$\sin \frac{D_1}{2} = \frac{10}{R_1}$

$\text{Sin } 2° = \frac{10}{R_1}$

$R_1 = 286.54$ m.

$\tan 8°30' = \frac{T_1}{286.54}$

$T_1 = 42.82$ m.

$AV = 80 - 42.82$

$AV = 37.18$ m.

$\frac{AB}{\text{Sin } 143°32'} = \frac{37.18}{\text{Sin } 19°28'}$

$AB = 66.31$ m.

$T_1 + T_2 = 66.31$

$T_2 = 66.31 - 42.82$

$T_2 = 23.49$ m.

$\tan 9°44' = \frac{T_2}{R_2}$

$R_2 = \frac{23.49}{\tan 9°44'}$

$R_2 =$ ***136.94 m.***

② *Stationing of P.C.C.:*

$S = R_1\theta$

$S = \frac{286.54\,(17°)(\pi)}{180°}$

$S = 85.02$ m.

Stationing of P.C.C. = (10 + 163) + 85.02

Stationing of P.C.C. = ***10 + 248.02***

③ *Distance from P.I. to P.T. of compound curve:*

$\frac{VB}{\text{Sin } 17°} = \frac{37.18}{\text{Sin } 19°28'}$

$VB = 32.62$ m.

Distance from P.I. to P.T. = $VB + T_2$

Distance = 32.62 + 23.49

Distance = ***56.11 m.***

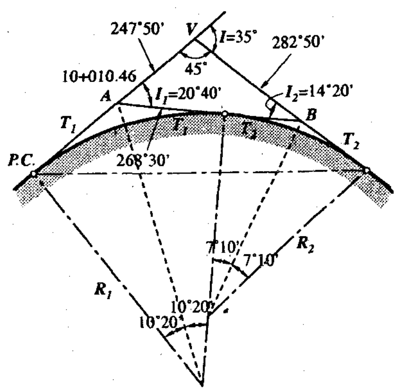
Problem 314:

The common tangent AB of a compound curve is 76.42 m. with an azimuth of 268°30'. The vertex V being inaccessible. The azimuth of the tangents AV and VB was measured to be 247°50' and 282°50' respectively. The stationing at A is 10 + 010.46 and the degree of the first curve is 4° based on the 20 m. chord. Use chord basis.

① Compute the stationing of P.C.C.
② Compute the radius of the second curve.
③ Compute the stationing of the P.T.

Solution:

① *Stationing of P.C.C.:*



$I = 282°50' - 247°50'$

$I = 35°$

$I_1 = 268°30' - 247°50'$

$I_1 = 20°40'$

$I_2 = 180° - 145° - 20°40'$

$I_2 = 14°20'$

$\text{Sin } \frac{D_1}{2} = \frac{10}{R_1}$

$\text{Sin } 2° = \frac{10}{R_1}$

$R_1 = 286.56$ m.

$T_1 = R_1 \tan 10°20'$

$T_1 = 286.56 \tan 10°20'$

$T_1 = 52.25$ m.

COMPOUND CURVES

$$S_1 = \frac{R_1 I_1 \pi}{180}$$
$$S_1 = \frac{286.56(20^\circ 40')\pi}{180}$$
$$S_1 = 103.36 \text{ m.}$$

Sta. of P.C. = (10 + 010.46) - 52.25
Sta. of P.C. = 9 + 958.21
Stationing of P.C.C. = (9 + 958.21) + 103.36
Stationing of P.C.C. = ***10 + 061.57***

② *Radius of second curve:*
$T_1 + T_2 = 76.42$
$T_2 = 76.42 - 52.25$
$T_2 = 24.17$ m.
$T_2 = R_2 \tan 7^\circ 10'$
$24.17 = R_2 \tan 7^\circ 10'$
$R_2 =$ ***192.22 m.***

③ *Stationing of P.T.:*
$$S_2 = \frac{R_2 I_2 \pi}{180}$$
$$S_2 = \frac{192.22(14^\circ 20')\pi}{180}$$
$$S_2 = 48.09 \text{ m.}$$

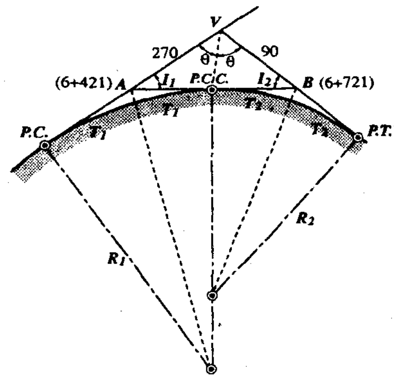
Stationing of P.T. = (10 + 061.57) + 48.09
Stationing of P.T. = ***10 + 109.66***

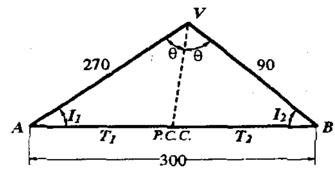
Problem 315:

In a compound curve, the line connecting the P.I. at point V and the P.C.C. is an angle bisector. AV is 270 meters and BV = 90 m. The stationing of A is 6 + 421 and that of B is 6 + 721. Point A is along the tangent passing thru the P.C. while point B is along the tangent passing thru the P.T. The P.C.C. is along line AB.

① Compute the radius of the first curve passing thru the P.C.
② Compute the radius of the second curve passing thru the P.T.
③ Determine the length of the long chord from P.C. to P.T.

Solution:
① *Radius of first curve:*





$AB = (6 + 721) - (6 + 421)$
$AB = 300$ m.
$T_1 + T_2 = 300$

In any triangle the angle bisector divides the opposite sides into segments whose ratio is equal to that of the other sides.

$$\frac{T_1}{270} = \frac{T_2}{90}$$
$T_1 = 3T_2$
$T_1 + T_2 = 300$
$3T_2 + T_2 = 300$
$T_2 = 75$ m.
$T_1 = 225$

Using Cosine Law:
$(90)^2 = (270)^2 + (300)^2 - 2(270)(300) \cos A$
$A = 17^\circ 09'$

Using Sine Law:
$$\frac{270}{\sin B} = \frac{90}{\sin 17^\circ 09'}$$
$B = 62^\circ 11'$
$I_1 = 17^\circ 09'$
$I_2 = 62^\circ 11'$

$T_1 = R_1 \tan \frac{I_1}{2}$

$225 = R_1 \tan \frac{17°09'}{2}$

$R_1 = \mathbf{1492.15\ m.}$

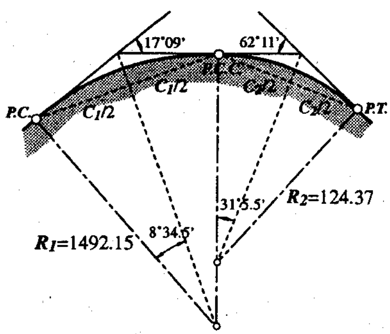
② *Radius of second curve:*

$T_2 = R_2 \tan \frac{I_2}{2}$

$75 = R_2 \tan \frac{62°11'}{2}$

$R_2 = \mathbf{124.37\ m.}$

③ *Length of long chord from P.C. to P.T.:*

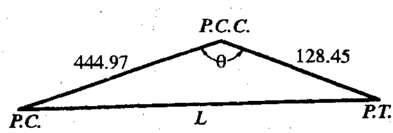


$\sin 8°34.5' = \frac{C_1}{2(1492.15)}$

$C_1 = 444.97$ m.

$\sin 31°5.5' = \frac{C_2}{2(124.37)}$

$C_2 = 128.45$ m.



$\theta = 180° - 31°5.5' - 8°34.5'$

$\theta = 140°20'$

Using Cosine Law:

$L^2 = (444.97)^2 + (128.45)^2 - 2(444.97)(128.45) \cos 140°20'$

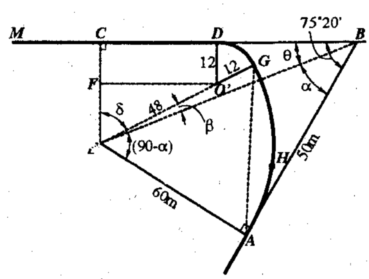
$L = \mathbf{550\ m.}$

Problem 316:

Two intersecting streets, one straight and the other simple curve, is to be connected by a small curve to round off the street corner, as shown in the figure. An existing station of the curve "A" was occupied by a transit and a backsight taken to a second point "H" on the curve. An angle equal to the deflection angle to H, was deflected setting line AB tangent to the curve at A. Point B is the intersection with the straight street MB. The instrument was then transferred to B, and the angle ABD was measured and found to be 75°20'. The tangent line AB was also measured and found to be 50 m. If the radius of the simple curve street is 60 m. and the radius of the small curve connecting the simple curve and the straight street is 12 m.

① Find the length of BD.
② Find the angle GEA.
③ Find the deflection angle BAG.

Solution:



① *Length of BD:*

$\tan \alpha = \frac{60}{50}$

$\alpha = 50°12'$

$\theta = 75°20' - 50°12'$

$\theta = 25°08'$

$EB = \frac{60}{\sin 50°12'}$

$EB = 78.10$

$BC = 78.10 \cos 25°08'$

$BC = 70.70$

$CE = 78.10 \sin 25°08'$

$CE = 33.17$

COMPOUND CURVES

$FE = 33.17 - 12.00$
$FE = 21.17$

$$\cos\delta = \frac{FE}{EO'}$$

$$\cos\delta = \frac{21.17}{48.00}$$

$\delta = 63°50'$

Angle CEB $= 90° - 25°08'$
Angle CEB $= 64°52'$
$\beta = 64°52' - 63°50'$
$\beta = 1°02'$

$CD = FO'$
$CD = 48 \sin 63°50'$
$CD = 43.08'$
$BD = BC - CD$
$BD = 70.70 - 43.08$
$BD = \mathbf{27.62\ m.}$

② *Angle GEA:*

Angle GEA $= (90° - \alpha) + 1°02'$
Angle GEA $= (90° - 50°12') + 1°02'$
Angle GEA $= \mathbf{40°50'}$

③ *Deflection angle BAG:*

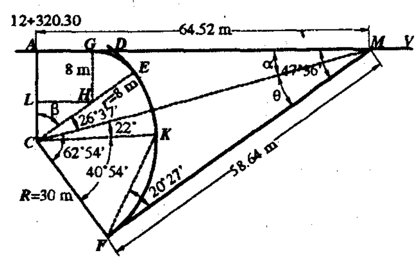
Deflection angle BAG $= \frac{1}{2}(40°50')$
Deflection angle BAG $= \mathbf{20°25'}$

Problem 317:

In the figure shown, AV is straight road and DF is a curved street. The radius of the curved street is 30 m. A circular curve of 8 m. radius is to be introduced at H to round off the intersection. AM is 64.52 m. and FM is 58.64 m. The angle AMF is equal to 47°36'. The stationing of A is 12 + 320.30. Deflection angle of point K from F is 20°27'.

① Find the stationing of point G.
② Find the stationing of point E.
③ Find the stationing of point K.

Solution:



① *Stationing of point G:*

$$\tan\theta = \frac{30}{58.64}$$

$\theta = 27°06'$

$\alpha + \theta = 47°36'$

$\alpha = 47°36' - 27°06'$

$\alpha = 20°30'$

$$\tan 20°30' = \frac{AC}{64.52}$$

$AC = 64.52 \tan 20°30'$
$AC = 24.12$ m.
$LC = 24.12 - 8$
$LC = 16.12$ m.
$CH = 30 - 8$
$CH = 22$ m.

$(LH)^2 = (CH)^2 - (LC)^2$
$(LH)^2 = (22)^2 - (16.12)^2$
$LH = 27.3$ m.
$LH = AG = 27.3$ m.

Stationing at G:
$G = (12 + 320.30) + 27.30$
$G = \mathbf{12 + 347.60}$

② *Stationing of point E:*

$$\cos\beta = \frac{16.12}{22}$$

$\beta = 42°53'$

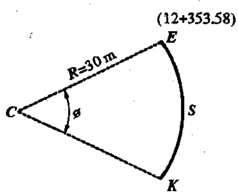
$$GE = \frac{8(42.883)\pi}{180}$$

$GE = 5.98$ m.

Stationing at E $= (12 + 347.60) + 5.98$
Stationing at E $= \mathbf{12 + 353.58}$

COMPOUND CURVES

③ *Stationing of point K:*



$ø = 62°54' + 26°37'$

$ø = 89°31'$

$$S = \frac{R\,ø\,\pi}{180}$$

$$S = \frac{30\,(89°31')\,\pi}{180°}$$

$S = 46.87$ m.

Stationing of K = (12 + 353.58) + 46.87

Stationing of K = ***(12 + 400.45)***

Problem 318:

Two tangents that intersect at an angle of 44°36' are to be connected by a compound curve. The tangent at the beginning of the curve at the P.C. is 125.70 m. long and that at the P.T. is 155.6 m. long. The degree of curve of the first curve on the P.C. is 4°. Using arc basis.

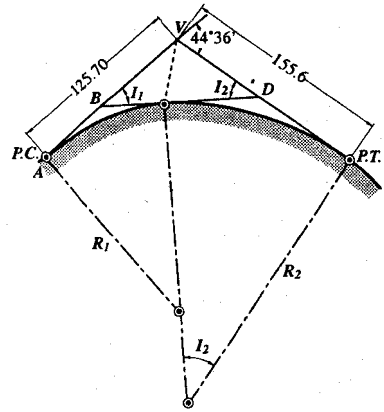
① Compute for the radius of the 2nd curve.

② Compute the central angle of the 2nd curve.

③ Compute the central angle of the first curve.

Solution:

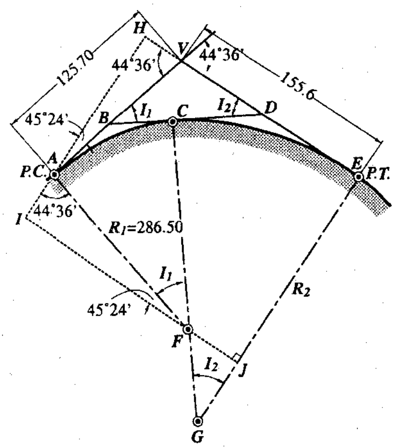
① *Radius of 2nd curve:*



$$R_1 = \frac{1145.916}{D_1}$$

$$R_1 = \frac{1145.916}{4}$$

$R_1 = 286.50$ m.



$AH = 125.70 \text{ Sin } 44°36'$

$AH = 88.26$ m.

$VH = 125.70 \text{ Cos } 44°36'$

$VH = 89.50$ m.

$AI = 286.50 \text{ Cos } 44°36'$

$AI = 204$ m.

$IF = 286.50 \text{ Sin } 44°36'$

$IF = 201.17$ m.

COMPOUND CURVES

$FJ + IF = VH + 155.60$
$FJ + 201.17 = 89.50 + 155.60$
$FJ = 43.93$ m.
$EJ = AI + AH$
$EJ = 204 + 88.26$
$EJ = 292.26$ m.
$JG = R_2 - EJ$
$FG = R_2 - R_1$
$FG = R_2 - 286.50$
$FJ = 43.93$
$JG = R_2 - 292.26$

Considering triangle FJG:
$(JG)^2 + (FJ)^2 = (FG)^2$
$(R_2 - 292.26)^2 + (43.93)^2 = (R_2 - 286.50)^2$
$R_2^2 - 584.52R_2 + 85415.91 + 1929.84$
$= R_2^2 - 573R_2 + 82082.25$
$11.52R_2 = 5263.5$
$R_2 = \mathbf{456.90\ m.}$

② *Central angle of 2nd curve:*
$JG = R_2 - 292.26$
$JG = 456.90 - 292.26$
$JG = 164.64$ m.

$$\tan I_2 = \frac{FJ}{JG}$$

$$\tan I_2 = \frac{43.93}{164.64}$$

$I_2 = \mathbf{14°56'}$

③ *Central angle of first curve:*
$I_1 + I_2 = 44°36'$
$I_1 = 44°36' - 14°56'$
$I_1 = \mathbf{29°40'}$

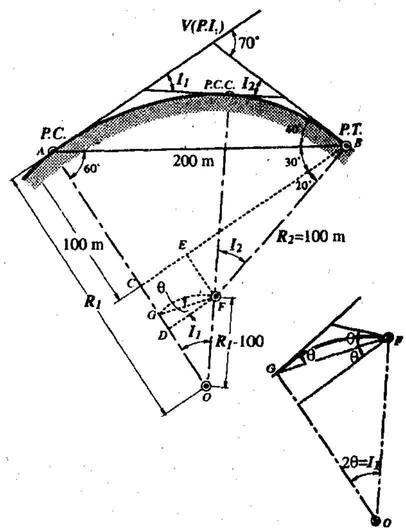
Problem 319:

Two tangents AV and VB are connected by a compound curve at points A (P.C.) and B (P.T.). Point V is the point of intersection of the tangents (P.I.). Angle VAB = 30° and angle VBA = 40°. Distance AB is 200 m. and the radius of the second curve R_2 = 100 m.

① Determine the central angle of the first curve.
② Determine the central angle of the 2nd curve.
③ Determine the radius of the first curve.

Solution:

① *Central angle of first curve:*



$AC = 200 \cos 60°$
$AC = 100$
$BC = 200 \sin 60°$
$BC = 173.20$
$CD = EF$
$EF = 100 \sin 20°$
$EF = 34.20$
$BE = 100 \cos 20°$
$BE = 93.97$
$GD = AC + CD - 100$
$GD = 100 + 34.20 - 100$
$GD = 34.20$
$DF = BC - BE$
$DF = 173.20 - 93.97$
$DF = 79.23$ m.

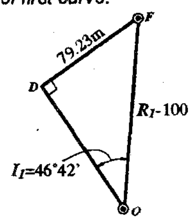
$$\tan\theta = \frac{GD}{DF}$$

$$\tan\theta = \frac{34.20}{79.23}$$

$\theta = 23°21'$
$I_1 = 2\theta$
$I_1 = 2\,(23°21')$
$I_1 = \mathbf{46°42'}$

② *Central angle of 2nd curve:*
$I_1 + I_2 = 70°$
$I_2 = 70 - 46°42'$
$I_2 = \mathbf{23°18'}$

③ *Radius of first curve:*



$$\sin 46^\circ 42' = \frac{79.23}{OF}$$

$OF = 108.87$ m.

$OF = R_1 - 100$

$108.87 = R_1 - 100$

$R_1 = \boldsymbol{208.87}$ ***m.***

Problem 320:

Two tangents that intersect at an angle of $42^\circ 36'$ are to be connected by a compound curve consisting of a 4° curve and a $5^\circ 30'$ curve. If the tangent at the beginning of the curve to the point of intersection of the tangent is 110 m. long.

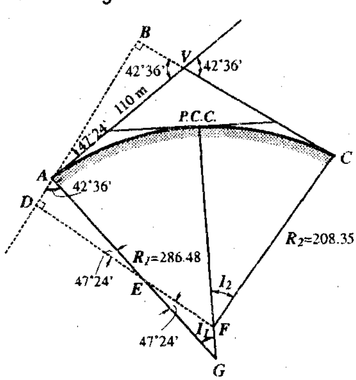
① Compute the central angle of the first curve.

② Compute the central angle of the second curve.

③ Compute the length of the tangent from the end of the $5^\circ 30'$ curve to the point of intersection of the tangents.

Solution:

① *Central angle of first curve:*



$$R_1 = \frac{1145.916}{D_1}$$

$$R_1 = \frac{1145.916}{4}$$

$R_1 = 286.48$

$$R_2 = \frac{1145.916}{5.5}$$

$R_2 = 208.35$

$VB = 110 \cos 42^\circ 36'$

$VB = 80.97$ m.

$AB = 110 \sin 42^\circ 36'$

$AB = 74.46$ m.

$AD + AB = R_2$

$AD + 74.46 = 208.35$

$AD = 133.89$ m.

$$\cos 42^\circ 36' = \frac{AD}{AE}$$

$$AE = \frac{133.89}{\cos 42^\circ 36'}$$

$AE = 181.89$ m.

$EG = R_1 - AE$

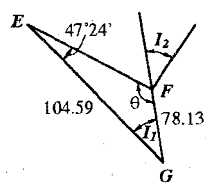
$EG = 286.48 - 181.89$

$EG = 104.59$ m.

$FG = R_1 - R_2$

$FG = 286.48 - 208.35$

$FG = 78.13$



Using Sine Law:

$$\frac{78.13}{\sin 47^\circ 24'} = \frac{104.59}{\sin \theta}$$

$\theta = 99^\circ 48'$

$I_1 = 180^\circ - 47^\circ 24' - 99^\circ 48'$

$I_1 = \boldsymbol{32^\circ 48'}$

② *Central angle of 2nd curve:*

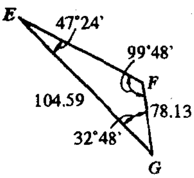
$I_1 + I_2 = 42^\circ 36'$

$I_2 = 42^\circ 36' - 32^\circ 48'$

$I_2 = \boldsymbol{9^\circ 48'}$

COMPOUND CURVES

③ *Distance of tangent from end of 5°30' curve to the P.I.:*



$$\frac{EF}{\text{Sin } 32°48'} = \frac{78.13}{\text{Sin } 47°24'}$$
$EF = 57.50$ m.

$VC + VB = DE + EF$
$DE = 181.89 \text{ Cos } 47°24'$
$DE = 123.12$ m.
$VC + 80.97 = 123.12 + 57.50$
$VC =$ ***99.65 m.***

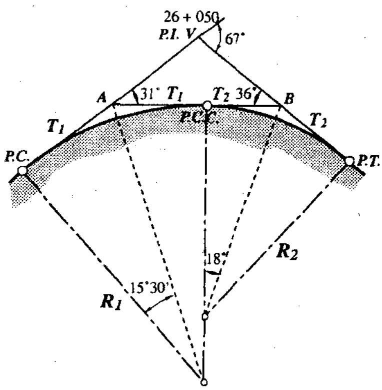
Problem 321:

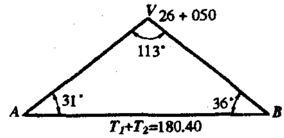
Two tangents intersect at station 26 + 050. A compound curve laid on their tangents has the foll. data:
$I_1 = 31°$, $I_2 = 36°$, $D_1 = 3°$, $D_2 = 5°$

① Compute the stationing of the P.C.
② Compute the stationing of the P.T.
③ If the P.T. is moved 15 m. out from the center, compute the stationing of the new P.T. with the P.C.C. remaining unchanged.

Solution:

① *Stationing of the P.C.:*





$$R_1 = \frac{1145.916}{3°}$$
$R_1 = 381.97$ m.
$$R_2 = \frac{1145.916}{5°}$$
$R_2 = 229.18$ m.

$$\tan 15°30' = \frac{T_1}{R_1}$$
$T_1 = 381.97 \tan 15°30'$
$T_1 = 105.93$ m.
$T_2 = R_2 \tan 18°$
$T_2 = 229.18 \tan 18°$
$T_2 = 74.47$ m.

$AB = T_1 + T_2$
$AB = 105.93 + 74.47$
$AB = 180.40$
$$\frac{AV}{\text{Sin } 36°} = \frac{180.40}{\text{Sin } 113°}$$
$AV = 115.19$ m.

Sta. of P.C. = (26 + 050) - (115.19 + 105.93)
Sta. of P.C. = ***25 + 828.88***

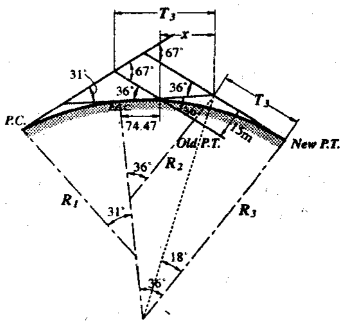
② *Stationing of P.T.:*
$$L_1 = \frac{20\, I_1}{D_1}$$
$$L_1 = \frac{20\,(31)}{3}$$
$L_1 = 206.67$

$$L_2 = \frac{20\, I_2}{D_2}$$
$$L_2 = \frac{20\,(36)}{5}$$
$L_2 = 144$

Sta. of P.T. = (25 + 828.88) + (206.67 + 144)
Sta. of P.T. = ***(26 + 179.55)***

COMPOUND CURVES

③ *Stationing of new P.T.:*



$$\text{Sin } 36° = \frac{15}{x}$$
$x = 25.52$ m.
$T_3 = 74.47 + 25.52$
$T_3 = 99.99$
$$\tan 18° = \frac{99.99}{R_3}$$
$R_3 = 307.74$ m.
$$L_3 = \frac{307.74\,(36)\,\pi}{180}$$
$L_3 = 193.36$ m.

Sta. of new PT
$= (25 + 828.88) + (206.67 + 193.36)$
Sta. of new PT = ***26 + 228.91***

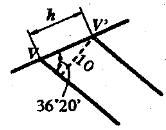
Problem 322:

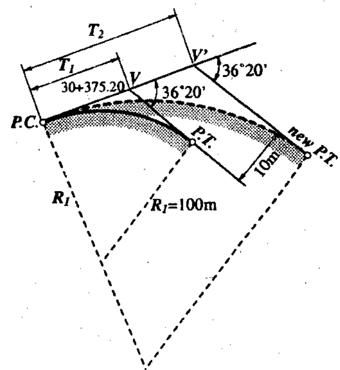
The highway engineer of a certain road construction decided to use a radius of 100 m. in laying out a simple curve having an angle of intersection of 36°20'. The stationing of the vertex is 30 + 375.20 after verifying the actual conditions of the proposed route, it was found out that the P.T. should be moved out in a parallel tangent having a perpendicular distance of 10 meters with the angle of intersection remaining the same while the curve shall have the same P.C.

① Compute the new tangent distance.
② Compute the new radius of curve.
③ Compute the stationing of new P.T.

Solution:

① *New tangent distance:*





$T_1 = R_1 \tan 18°10'$
$T_1 = 100 \tan 18°10'$
$T_1 = 32.81$ m.
$$h = \frac{10}{\text{Sin } 36°20'}$$
$h = 16.9$ m.

New tangent distance = 16.9 + 32.81
New tangent distance = ***49.71 m.***

② *New radius of curve:*
$T_2 = T_1 + h$
$T_2 = 32.8 + 16.90$
$T_2 = 49.71$ m.
$T_2 = R_2 \tan 18°10'$
$$R_2 = \frac{49.71}{\tan 13°10'}$$
$R_2 =$ ***151.40 m.***

③ *Stationing of new P.T.:*
Stationing of P.C. (30 + 375.20) - 32.81
Stationing of P.C. = 30 + 342.39

$$D = \frac{1145.916}{R}$$
$$D = \frac{1145.916}{151.40}$$
$D = 7.57°$

COMPOUND CURVES

$$L_c = \frac{20\,I}{D}$$
$$L_c = \frac{20\,(36.33)}{7.57^\circ}$$
$$L_c = 95.99 \text{ m.}$$

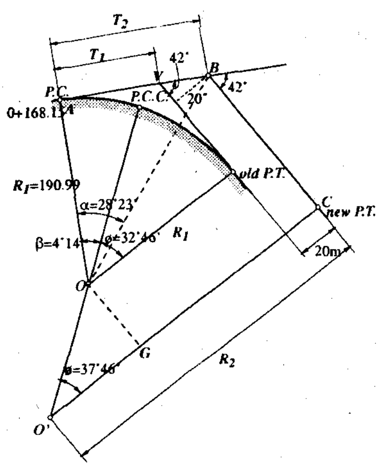
Stationing of new P.T.:
New P.T. = (30 + 342.39) + 95.99
*New P.T. = **30 + 438.38***

Problem 323:

A 6° simple curve connects two tangents that intersect at an angle of 42°. If the P.C. which is at station 0 + 168.15 is to be retained and a 4° curve is to connect the first curve with the second tangent that is shifted 20 meters to a parallel position outward. Use arc basis.

① Find the stationing of old P.T.
② Find the stationing of the P.C.C.
③ Find the stationing of the new P.T.

Solution:
① *Stationing of old P.T.*



$$R_1 = \frac{1145.916}{D_1}$$
$$R_1 = \frac{1145.916}{6}$$
$$R_1 = 190.99 \text{ m.}$$

$$L_c = \frac{R_1 I_1 \pi}{180}$$
$$L_c = \frac{190.99\,(42^\circ)\,\pi}{180^\circ}$$
$$L_c = 140 \text{ m.}$$

Stationing of old P.T. = (0 + 168.15) + 140
*Stationing of old P.T. = **(0 + 308.15)***

② *Stationing of the P.C.C.*
$T_1 = 190.99 \tan 21^\circ$
$T_1 = 73.31$ m.
$VB \sin 42^\circ = 20$
$VB = 29.89$ m.

$T_2 = T_1 + VB$
$T_2 = 73.31 + 29.89$
$T_2 = 103.20$ m.
$$\tan\alpha = \frac{T_2}{R_1}$$
$$\tan\alpha = \frac{103.20}{190.99}$$
$\alpha = 28^\circ 23'$
$$\cos 28^\circ 23' = \frac{R_1}{OB}$$
$$OB = \frac{190.99}{\cos 28^\circ 23'}$$
$OB = 217.09$ m.
$$R_2 = \frac{1145.916}{D_2}$$
$$R_2 = \frac{1145.916}{4}$$
$R_2 = 286.48$ m.

$GC = R_1 + 20$
$GC = 190.99 + 20$
$GC = 210.99$ m.
$O'G = R_2 - GC$
$O'G = 286.48 - 210.99$
$O'G = 75.49$ m.
$OO' = R_2 - R_1$
$OO' = 286.48 - 190.99$
$OO' = 95.49$
$$\cos ø = \frac{O'G}{OO'}$$
$$\cos ø = \frac{75.49}{95.49}$$
$ø = 37^\circ 46'$
$\beta = 42^\circ - 37^\circ 46'$
$\beta = 4^\circ 14'$

$$L_{c_1} = \frac{R_1\, \beta\, \pi}{180}$$

$$L_{c_1} = \frac{190.91\,(4°14')\,\pi}{180°}$$

$$L_{c_1} = 14.11 \text{ m.}$$

Stationing of P.C.C. = (0 + 168.15) + 14.11
Stationing of P.C.C. = ***0 + 182.26***

③ *Stationing of new P.T.*

$$L_{c_2} = \frac{R_2\, \varnothing\, \pi}{180}$$

$$L_{c_2} = \frac{286.48\,(37°46')\,\pi}{180°}$$

$$L_{c_2} = 188.83 \text{ m.}$$

Stationing of new P.T. = (0 + 182.26) + 188.83
Stationing of new P.T. = ***0 + 371.09***

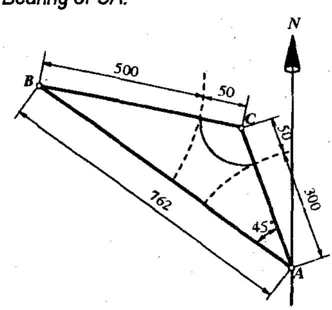
Problem 324:

Two simple curve are both convex to the Northeast. The first has a radius of 300 m. and its center designated as A. The second curve has a radius of 500 m. and its center designated as B. They have been so located that the line AB connecting their centers is 762 m. long and has a bearing of N 45° W. A third simple curve, concave to the Northeast with a radius of 50 m. and its center designated as C, is to be located so that it will be tangent to the other two curves.

① Compute the bearing of CA.
② Compute the angle ACB.
③ Compute the bearing of CB.

Solution:

① *Bearing of CA:*

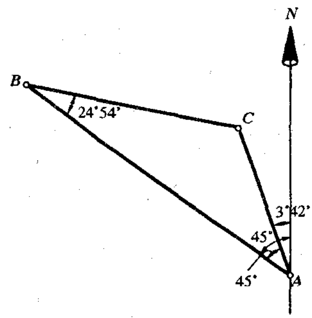


$b = 350$ m.
$a = 550$ m.
$c = 762$ m.

$$s = \frac{a + b + c}{2}$$

$$s = \frac{550 + 350 + 762}{2}$$

$s = 831$
$(s - a) = 281$
$(s - b) = 481$
$(s - c) = 69$



$$\text{Sin}\,\frac{A}{2} = \sqrt{\frac{(b - b)\,(c - c)}{bc}}$$

$$\text{Sin}\,\frac{A}{2} = \frac{(48)\,(69)}{(370)\,(762)}$$

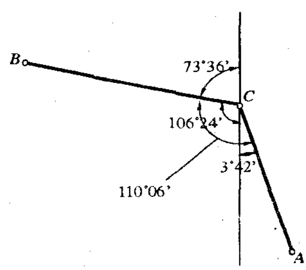
$$\text{Sin}\,\frac{A}{2} = 0.1234$$

$$\frac{A}{2} = 20.65°$$

$A = 41.3°$
$\angle A = 41°18'$

Bearing of CA = ***S 3°42' E.***

② *Angle ACB:*



COMPOUND CURVES

$$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$$

$$\sin\frac{B}{2} = \sqrt{\frac{(281)(69)}{(550)(762)}}$$

$$\sin\frac{B}{2} = \sqrt{0.0463}$$

$$\sin\frac{B}{2} = 0.215$$

$$\frac{B}{2} = 12.45^\circ$$

$$\angle B = 24^\circ 54'$$

$$\angle C = 180^\circ - (45^\circ + 24^\circ 54')$$

$$\angle C = 110^\circ 06'$$

*Angle ACB = **110°06'***

③ *Bearing CB:*
*Bearing of CB = **N 73°36' W.***

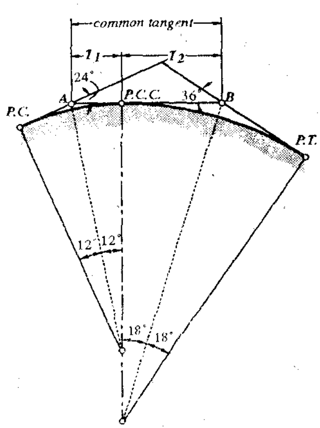
Problem 325:

Given a compound curve $I_1 = 24^\circ$, $I_2 = 36^\circ$, $D_1 = 6^\circ$, $D_2 = 4^\circ$.

① Compute the length of the common tangent of the curve.
② Compute the stationing of P.C.C. if P.C. is at 10 + 420.
③ Compute the stationing of P.T.

Solution:

① *Length of the common tangent of the curve:*



$$R_2 = \frac{1145.916}{4^\circ}$$

$$R_2 = 286.48 \text{ m.}$$

$$R_1 = \frac{1145.916}{6^\circ}$$

$$R_1 = 190.99$$

$$T = 190.99 \tan 12^\circ + 286.48 \tan 18^\circ$$

$$T = \mathbf{133.68\ m.}$$

② *Stationing of P.C.C.:*

$$\frac{L_1}{I_1} = \frac{20}{D_1}$$

$$L_1 = \frac{20(24)}{6}$$

$$L_1 = 80$$

Sta. of P.C.C. = (10 + 420) + 80
*Sta. of P.C.C. = **10 + 500***

③ *Stationing of P.T.:*

$$\frac{L_2}{I_2} = \frac{20}{D_2}$$

$$L_2 = \frac{36^\circ(20)}{4^\circ}$$

$$L_2 = 180$$

Stationing of P.T. = (10 + 500) + 180
*Stationing of P.T. = **10 + 680***

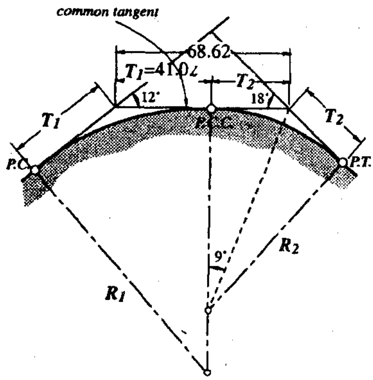
Problem 326:

The length of the common tangent of a compound curve is equal to 68.62 m. The common tangent makes an angle of 12° and 18° respectively to the tangents of the compound curve. If the length of the tangent of the first curve (on the side of P.C.) is equal to 41.02 m.

① Compute the radius of the second curve.
② Compute the radius of the first curve.
③ Compute the stationing of the P.T. if PC is at 20 + 042.20.

Solution:

① *Radius of the second curve:*



$T_2 = 68.62 - 41.02$
$T_2 = 27.60$
$T_2 = R_2 \tan 9°$
$27.60 = R_2 \tan 9°$
$R_2 = \mathbf{174.26\ m.}$

② *Radius of the first curve:*

$T_1 = 41.02$
$T_1 = R_1 \tan 6°$
$R_1 = \dfrac{41.02}{\tan 6°}$
$R_1 = \mathbf{390.28\ m.}$

③ *Stations of the P.T.:*

$L_1 = \dfrac{R_1 I_1 \pi}{180}$
$L_1 = \dfrac{390.28\,(12)\,\pi}{180}$
$L_1 = 81.74$ m.
$L_2 = \dfrac{R_2 I_2 \pi}{180}$
$L_2 = \dfrac{174.26\,(18)\,\pi}{180}$
$L_2 = 54.75$ m.

Sta. of P.T. = (20 + 042.20) + (81.74) + (54.75)
Sta. of P.T. = ***(20 + 178.69)***

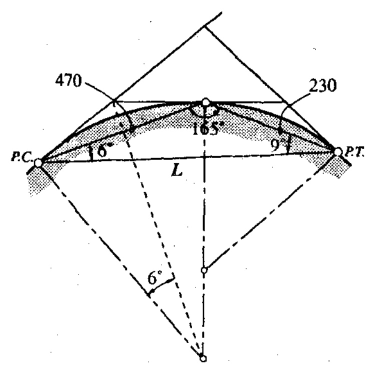
Problem 327:

A compound curve has a length of chord of the first curve (passing thru the P.C. to P.T.) of 470 m. If the angle that the long chord makes with the chord from the P.C. to P.C.C. and from P.T. to the P.C.C. are 6° and 9° respectively. Assume the long chord is parallel to the common tangent.

① Find the length of the long chord.
② Find the radius of first curve.
③ Find the radius of second curve.

Solution:

① *Length of the long chord:*



$\dfrac{470}{\text{Sin } 9°} = \dfrac{L}{\text{Sin } 165°}$
$L = \mathbf{777.61\ m.}$

② *Radius of first curve:*

$\text{Sin } 6° = \dfrac{470}{2R_1}$
$R_1 = \mathbf{2248.19\ m.}$

③ *Radius of second curve:*

$\dfrac{C_2}{\text{Sin } 6°} = \dfrac{470}{\text{Sin } 9°}$
$C_2 = 314.05$
$\text{Sin } 9 = \dfrac{314.05}{2R_2}$
$R_2 = \mathbf{1003.77\ m.}$

COMPOUND CURVES

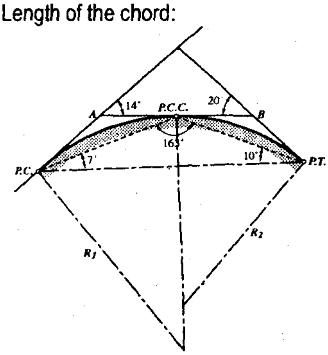
Problem 327-A:

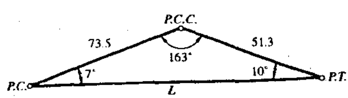
The common tangent of a compound curve makes an angle of 14° and 20° with the tangent of the first curve and the second curve respectively. The length of chord from the P.C. to P.C.C. is 73.5 m. and that from P.C.C. to P.T. is 51.3 m.

① Find the length of the chord from the P.C. to the P.T. if it is parallel to the common tangent.
② Find the radius of the first curve.
③ Find the radius of the second curve.

Solution:

① Length of the chord:





$L^2 = (73.5)^2+(51.3)^2- 2(73.5)(51.3) \cos 163°$

$L =$ ***123.5 m.***

② Radius of the first curve.

$$\sin 7° = \frac{73.5}{2R_1}$$

$R_1 =$ ***301.55 m.***

③ Radius of the second curve.

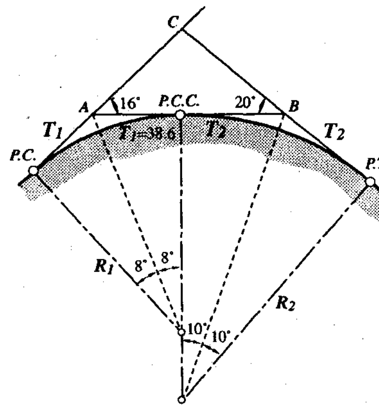
$$\sin 10° = \frac{51.3}{2R_2}$$

$R_2 =$ ***147.71 m.***

Problem 327-B:

A compound curve has a common tangent 84.5 m. long which makes angles of 16° and 20° with the tangents of the first curve and the second curves respectively. The length of the tangent of the first curve is 38.6 m. What is the radius of the second curve.

Solution:



$38.6 + T_2 = 84.5$

$T_2 = 45.9$ m.

$T_2 = T \tan 10°$

$45.9 = R_2 \tan 10°$

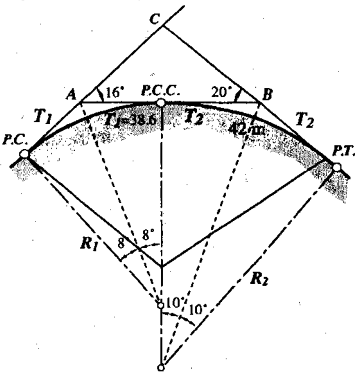
$R_2 =$ ***260.3 m.***

Problem 327-C:

A compound curve has a common tangent of 84.5 m. long which makes an angle of 16° and 20° with the tangents of the first curve and the second curve respectively. The length of the tangent of the second curve is 42 m.

① What is the radius of the first curve.
② Find the radius of the second curve.
③ Find the length of curve from P.C. to P.T.

Solution:



① Radius of the first curve.

$84.5 = T_1 + 42$

$T_1 = 42.5$

$T_1 = R_1 \tan 8°$

$42.5 = R_1 \tan 8°$

$R_1 =$ ***302.4 m.***

② Radius of the second curve.

$\tan 10° = \dfrac{42}{R_2}$

$R_2 =$ ***238.19 m.***

③ Length of curve from P.C. to P.T.

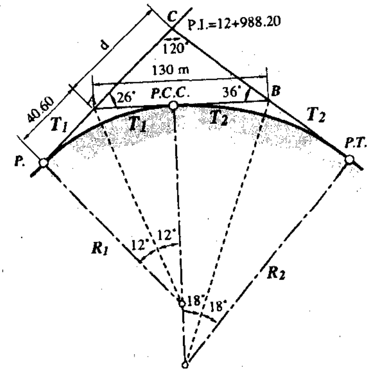
$L = \dfrac{302.4(16)\pi}{180} + \dfrac{238.19(20)\pi}{180}$

$L =$ ***167.59 m.***

Problem 327-D:

Given a compound curve $I_1 = 24°$, $I_2 = 36°$, $D_1 = 6°$, $D_2 = 4°$.

① Compute the length of the common tangent of the compound curve. Use arc basis.
② Compute the sta. of P.C. if P.I. is at sta. 12 + 988.20.
③ Compute the sta. of P.T.



Solution:

① Length of the common tangent of the compound curve. Use arc basis.

$R_1 = \dfrac{1145.916}{6°}$

$R_1 = 190.99$

$R_2 = \dfrac{1145.916}{4°}$

$R_2 = 286.48$

$AB = R_1 \tan \dfrac{I_1}{2} + R_2 \tan \dfrac{I_2}{2}$

$AB = 190.99 \tan 12° + 286.48 \tan 18°$

$AB = 40.60 + 93.08$

$AB =$ ***133.68 m.***

② Sta. of P.C. if P.I. is at sta. 12 + 988.20.

$\dfrac{d}{\text{Sin } 36°} = \dfrac{133.68}{\text{Sin } 120°}$

$d = 90.73$

Sta. P.C. = 12 + 988.20 - (90.73 + 60)

P.C. = ***12 + 856.87***

③ Compute the sta. of P.T.

$P.T. = (12 + 856.87) + \dfrac{190.99(24)\pi}{180}$

$+ \dfrac{286.48(36)\pi}{180}$

P.T. = ***13 + 116.87***

COMPOUND CURVES

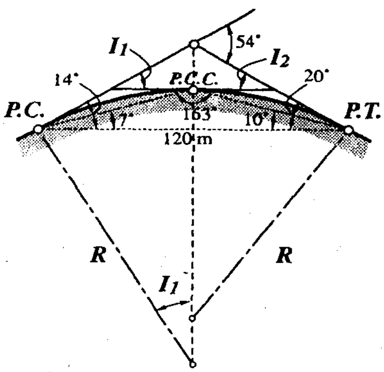
Problem 328:

The long chord of a compound curve is 120 m. long which makes and angle of 14° from the tangent of the first curve passing through the P.C. and 20° from the tangent of the second curve passing through the P.T. If the common tangent is parallel to the long chord.

① Compute the length of chord from P.C. to P.C.C.
② Compute the length of chord from PCC to P.T.
③ Compute the difference in radius of the first and second curve.

Solution:

① *Length of chord from PC to PCC:*



$$\frac{C_1}{\text{Sin } 10°} = \frac{120}{\text{Sin } 163°}$$

$C_1 = \textbf{71.27 m.}$

② *Length of chord from PCC to PT:*

$$\frac{C_2}{\text{Sin } 17°} = \frac{120}{\text{Sin } 163°}$$

$C_2 = \textbf{120}$

③ *Diff. in radius of 1st and second curve:*

$$\text{Sin } 7° = \frac{71.27}{2R_1}$$

$R_1 = 292.40$ m.

$$\text{Sin } 10° = \frac{120}{2R_2}$$

$R_2 = 345.53$

Diff. in radius = 345.53 - 292.40
Diff. in radius = **53.13 m.**

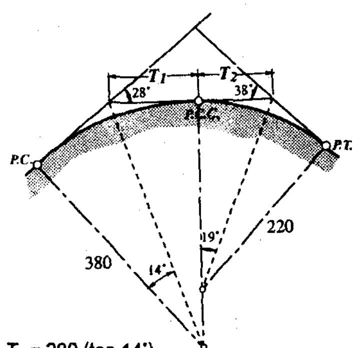
Problem 329:

A compound curve has the following data:
$I_1 = 28°$, $I_2 = 38°$, $R_1 = 380$ m., $R_2 = 220$ m.
If P.C. is at sta. 20 + 100

① Compute the length of the common tangent.
② Compute the sta. of P.C.C.
③ Compute the sta. of P.T.

Solution:

① *Length of the common tangent:*



$T_1 = 380$ (tan 14°)
$T_1 = 94.74$ m.
$T_2 = 220$ tan 19°
$T_2 = 75.75$
$T_1 + T_2 = \textbf{170.49 m.}$

② *Sta. of P.C.C.*

$$L_1 = \frac{R_1 I_1 \pi}{180}$$

$$L_1 = \frac{380(28)\pi}{180} = 185.70$$

P.C.C. = (20 + 000) + (185.70)
P.C.C. = **20 + 185.70**

③ *Sta. of P.T.*

$$L_2 = \frac{R_2 I_2 \pi}{180}$$

$$L_2 = \frac{220(38)\pi}{180} = 145.91$$

P.T = (20 + 185.70) + 145.91
P.T. = **20 + 331.61**

COMPOUND CURVES

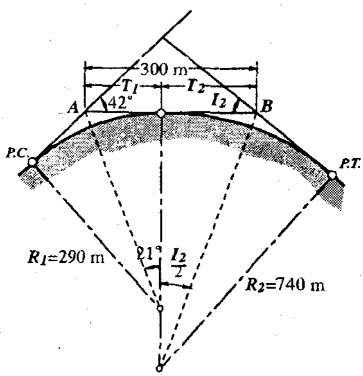
Problem 330:

A compound curve passes thru a common tangent AB having a length of 300 m. The radius of the first curve is equal to 290 m. and a central angle of 42°. If the radius of the second curve is 740 m.

① Compute the length of the tangent of the second curve.
② Compute the central angle of second curve.
③ Compute the stationing of the PT if PC is at (20 + 542.20).

Solution:

① *Tangent of second curve:*



$T_1 = R_1 \tan 21°$
$T_1 = 290 \tan 21°$
$T_1 = 111.32$ m.
$T_2 = 300 - 111.32$
$T_2 =$ ***188.68 m.***

② *Central angle of second curve:*

$$\tan\frac{I_2}{2} = \frac{T_2}{R_2}$$

$$\tan\frac{I_2}{2} = \frac{188.68}{740}$$

$$\frac{I_2}{2} = 14.30°$$

$I_2 = 28.60°$
$I_2 =$ ***28°36'***

③ *Stationing of the PT:*

$$L_1 = \frac{R_1 I_1 \pi}{180}$$

$$L_1 = \frac{290\,(42)\,\pi}{180}$$

$L_1 = 212.58$ m.

$$L_2 = \frac{R_2 I_2 \pi}{180}$$

$$L_2 = \frac{740\,(28°36')\,\pi}{180}$$

$L_2 = 369.38$ m.

Sta. of PT = (20 + 542.20) + (212.58) + (369.38)
Sta. of PT = ***(21 + 124.16)***

Problem 331:

The common tangent AB of a compound curve makes an angle with the tangents of the compound curve of 25°30' and 30°00' respectively. The stationing of A of 10 + 362.42. The degree of curve of the first curve is 4°30' while that of the second curve is 5°. It is required to change this compound curve with a simple curve that shall end at the same P.T. while the direction of the tangents remains the same.

① Find the radius of the simple curve.
② Find the stationing of the new P.C.
② Find the stationing of P.T.

Solution:

① *Radius of simple curve:*

$$R_1 = \frac{1145.916}{D_1}$$

$$R_1 = \frac{1145.916}{4.5°}$$

$R_1 = 254.65$ m.

$$R_2 = \frac{1145.916}{5°}$$

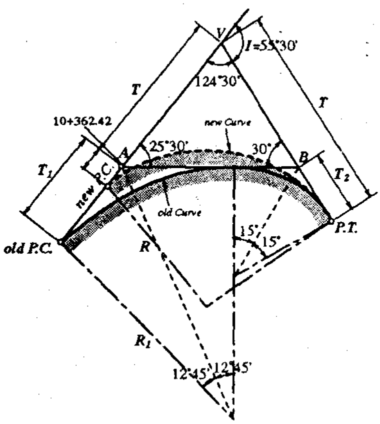
$R_2 = 229.18$ m.

$$\tan 15° = \frac{T_2}{R_2}$$

$T_2 = 229.18 \tan 15°$
$T_2 = 61.41$ m.

COMPOUND CURVES

$T_1 = R_1 \tan 12°45'$
$T_1 = 57.62$ m.



$AB = T_1 + T_2$
$AB = 61.41 + 57.62$
$AB = 119.03$ m.

$$\frac{VB}{\text{Sin } 25°30'} = \frac{119.03}{\text{Sin } 124°30'}$$

$VB = 62.18$

New tangent of the simple curve:
$T = VB + T_2$
$T = 62.18 + 61.41$
$T = 123.59$

$$R = \frac{T}{\tan \frac{I}{2}}$$

$$R = \frac{123.59}{\tan 27°45'}$$

$R =$ ***234.91 m.***

② *Stationing of new P.C.*
Old P.C. = (10 + 362.42) - (57.62)
Old P.C. = 10 + 304.80

$$\frac{AV}{\text{Sin } 30°} = \frac{119.03}{\text{Sin } 124°30'}$$

$AV = 72.22$ m.

Sta. of V = (10 + 362.42) + (72.22)
Sta. of V = 10 + 434.64
Sta. of new P.C. = (10 + 434.64) - (123.59)
Sta. of new P.C. = ***10 + 311.05***

③ *Stationing of P.T.*

$$L_c = \frac{R I \pi}{180}$$

$$L_c = \frac{234.91\,(55°30')\,\pi}{180°}$$

$L_c = 227.55$

Sta. of P.T. = (10 + 311.05) + 227.55
Sta. of P.T. = ***(10 + 538.60)***

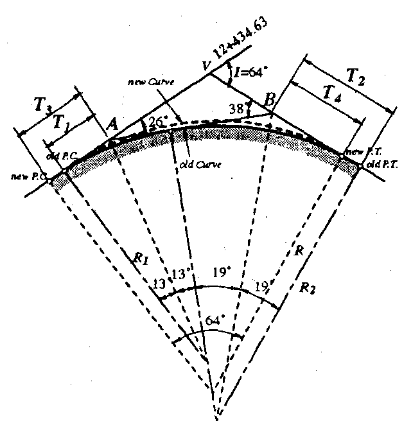
Problem 332:

A common tangent AB of a compound curve makes an angle with the tangents of 26° and 38° respectively. The degree of curve of the first curve is 5°30' while that of the second curve it is 3°30'. The stationing of the point of intersection of the tangents is at 12 + 434.63. If it is desired to substitute a simple curve that shall be tangent of the two tangent lines as well as the common tangent.

① Find the radius of the simple curve.
② Find the stationing of the new P.C.
③ Find the stationing of the new P.T.

Solution:

① *Radius of simple curve:*

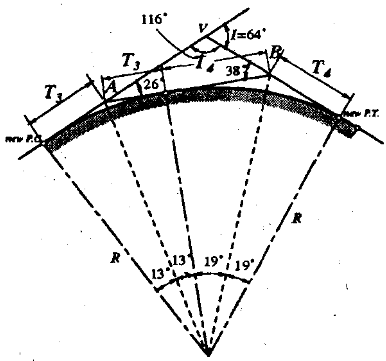


$$R_1 = \frac{1145.916}{D_1}$$

$$R_1 = \frac{1145.916}{5°30'}$$

$R_1 = 208.35$ m.

$$R_2 = \frac{1145.916}{3°30'}$$
$$R_2 = 327.40 \text{ m.}$$



$$T_1 = R_1 \tan\frac{I_1}{2}$$
$$T_1 = 208.35 \tan 13°$$
$$T_1 = 48.10 \text{ m.}$$
$$T_2 = R_2 \tan\frac{I_2}{2}$$
$$T_2 = 327.40 \tan 19°$$
$$T_2 = 112.73 \text{ m.}$$

$$AB = T_1 + T_2$$
$$AB = 48.10 + 112.73$$
$$AB = 160.83$$
$$T_3 = R \tan 13°$$
$$T_4 = R \tan 19°$$
$$T_3 + T_4 = AB$$
$$R \tan 13° + R \tan 19° = 160.83$$
$$\boldsymbol{R = 279.61 \text{ m.}}$$

② *Stationing of new P.C.*

$$T_3 = 279.61 \tan 13°$$
$$T_3 = 64.55$$
$$\frac{AV}{\sin 38°} = \frac{160.83}{\sin 116°}$$
$$AV = 110.17 \text{ m.}$$

Sta. of new P.C. = (12 + 434.63) - (110.17) - (64.55)

Sta. of new P.C. = ***12 + 259.91***

③ *Stationing of new P.T.*

$$D = \frac{1145.916}{279.61}$$
$$D = 4.10°$$
$$L_c = \frac{20\,I}{D}$$
$$L_c = \frac{20\,(64)}{4.10}$$
$$L_c = 312.20 \text{ m.}$$

Sta. of new P.T. = (12 + 259.91) + 312.20

Sta. of new P.T. = ***12 + 572.11***

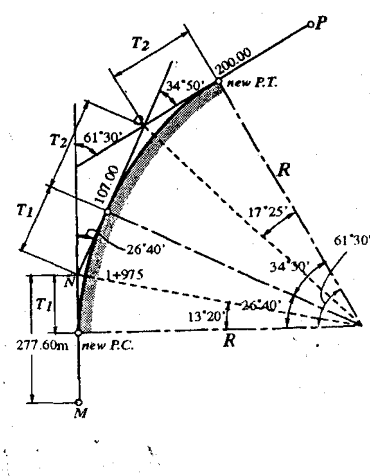
Problem 353

Traverse lines MN, NO, OP, are centerlines of a portion of a proposed highway. Respectively the bearings and distances are; MN due North, 277.60 meters; NO, N 26°40' E, 107.00 meters, OP, N 61°30' E, 200.00 meters. A previously designed compound curve connected these three tangent lines with the P.C.C. at station 2 + 012.00. It is desired to revise the system into a single circular curve that will still be tangent to the three lines.

① Determine the radius of the simple curve.
② Determine the stationing of the new P.C. if station N is at 1 + 975.00.
③ Determine the stationing of the new P.T.

Solution:

① *Radius of simple curve:*



COMPOUND CURVES

$T_1 + T_2 = 107$
$T_1 = R \tan 13°20'$
$T_2 = R \tan 17°25'$

$R \tan 13°20' + R \tan 17°25' = 107$
$R = \mathbf{194.30\ m.}$

② *Stationing of new P.C.*
$T_1 = R \tan 13°20'$
$T_1 = 194.30 \tan 13°20'$
$T_1 = 46.05$ m.

Sta. of new P.C. = (1 + 975) - 46.05
Sta. of new P.c. = ***1 + 928.95***

③ *Stationing of new P.T.*

$$L_c = \frac{R\,ø\,\pi}{180}$$

$$L_c = \frac{194.30\,(61.5°)\,\pi}{180°}$$

$$L_c = 208.56$$

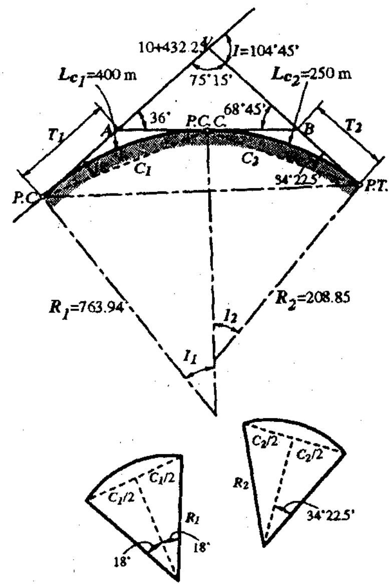
Sta. of new P.T. = (1 + 928.95) + 208.56
Sta. of new P.T. = ***2 + 137.51***

Problem 334:

A compound curve is laid out 480 m. from the P.C. to the P.C.C. having a radius of 763.94 m. then from the P.C.C. another curve was laid out to the P.T. 250 m. long with a radius of 208.85 m. If the stationing of the point of intersection of the tangents is 10 + 432.25.

① Determine the stationing of the P.C.
② Determine the length of the long chord from the P.C. to the P.T.
③ Determine the angle that the long chord makes with the tangent.

Solution:
① *Stationing of P.C.*



$$D_1 = \frac{1145.916}{763.94}$$

$$D_1 = 1.5°$$

$$D_2 = \frac{1145.916}{208.35}$$

$$D_2 = 5.5°$$

$$L_{c_1} = \frac{20\,I_1}{D_1}$$

$$I_1 = \frac{480\,(1.5)}{20}$$

$$I_1 = 36°$$

$$L_{c_2} = \frac{20\,I_2}{D_2}$$

$$I_2 = \frac{250\,(5.5)}{20}$$

$$I_2 = 68°45'$$

$$T_1 = R_1 \tan\frac{I_1}{2}$$

$$T_1 = 763.94 \tan 18°$$

$$T_1 = 248.22 \text{ m.}$$

$T_2 = 208.35 \tan 34°22.5$
$T_2 = 142.53$

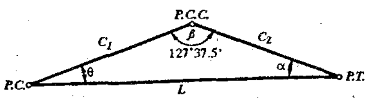
$AB = T_1 + T_2$
$AB = 248.22 + 142.53$
$AB = 390.75$

$$\frac{VA}{\text{Sin } 68°45'} = \frac{390.75}{\text{Sin } 75°15'}$$

$VA = 376.59$

Sta. of P.C. = (10 + 432.25) - (376.59) - (248.22)
Sta. of P.C. = ***9 + 807.44***

② *Length of long chord from P.C. to P.T.*



$$\text{Sin } 18° = \frac{C_1}{2\,R_1}$$

$C_1 = 2\,(763.94)\text{ Sin } 18°$
$C_1 = 472.14$ m.

$$\text{Sin } 34°22.5' = \frac{C_2}{2\,R_2}$$

$C_2 = 2\,(200.35)\text{ Sin } 34°22.5'$
$C_2 = 235.21$ m.

$ß = 180° - 18° - 34°22.5'$
$ß = 127°37.5'$
$L^2 = C_1^2 + C_2^2 - 2\,(C_1C_2)\text{ Cos } 127°37.5$
$L^2 = (472.14)^2 + (235.21)^2$
$\quad - 2(472.14)(235.21)\text{ Cos } 127°37.5$
$L =$ ***643.30 m.***

③ *Angle that the long chord makes with tangent at the P.T.*

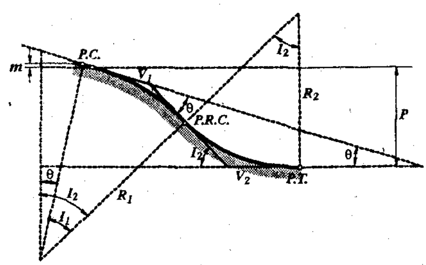
$$\frac{643.30}{\text{Sin } 127°37.5} = \frac{235.21}{\text{Sin } ø}$$

$ø = 16°50'$
$\alpha = 180° - 127°37.5' - 16°50'$
$\alpha =$ ***35°32.5'***

REVERSED CURVES

A ***reversed curve*** is formed by two circular simple curves having a common tangent but lies on opposite sides. The method of laying out a reversed curve is just the same as the deflection angle method of laying out simple curves. At the point where the curve reversed in its direction is called the Point of Reversed Curvature. After this point has been laid out from the P.C., the instrument is then transferred to this point (P.R.C.). With the transit at P.R.C. and a reading equal to the total deflection angle from the P.C. to the P.R.C., the P.C. is backsighted. If the line of sight is rotated about the vertical axis until horizontal circle reading becomes zero, this line of sight falls on the common tangent. The next simple curve could be laid out on the opposite side of this tangent by deflection angle method.

Elements of a Reversed Curve:



R_1 and R_2 = radii of curvature
D_1 and D_2 = degree of curve
V_1 and V_2 = points of intersection of tangents
θ = angle between converging tangents
$I_2 - I_1 = \theta$
P.C. = point of curvature
P.T. = point of tangency
P.R.C. = point of reversed curvature
$L_c = L_{c_1} + L_{c_2}$ = length of reversed curve
m = offset
P = distance between parallel tangents

REVERSED CURVES

Four types of reversed curve problems:

1. Reversed curve with equal radii and parallel tangents.
2. Reversed curve with unequal radii and parallel tangents.
3. Reversed curve with radii and converging tangents.
4. Reversed curve with unequal radii and converging tangents.

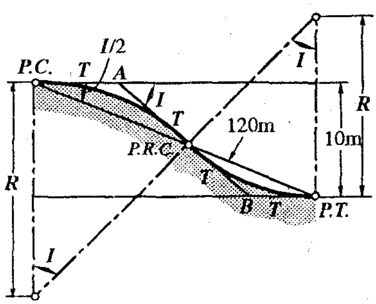
Problem 335:

Two parallel tangents 10 m. apart are connected by a reversed curve. The chord length from the P.C. to the P.T. equals 120 m.

① Compute the length of tangent with common direction.
② Determine the equal radius of the reversed curve.
③ Compute the stationing of the P.R.C. if the stationing of A at the beginning of the tangent with common direction is 3 + 420.

Solution:

① *Length of tangent with common direction:*



$$\text{Sin}\,\frac{I}{2} = \frac{10}{120}$$

$$\frac{I}{2} = 4.78^\circ$$

$$I = 9^\circ 34'$$

$$\text{Sin}\,9^\circ 34' = \frac{10}{AB}$$

$$AB = \mathbf{60.17\ m.}$$

② *Radius of reversed curve:*

$$2T = AB$$

$$2T = 60.17$$

$$T = 30.085$$

$$T = R\tan\frac{I}{2}$$

$$30.085 = R\tan 4.78^\circ$$

$$R = \mathbf{359.78\ m.}$$

③ *Stationing of P.R.C.*

$$L_c = \frac{R\,I\,\pi}{180}$$

$$L_c = \frac{359.78(9^\circ 34')\,\pi}{180^\circ}$$

$$L_c = 60.07\ \text{m.}$$

Stationing PC = (3 + 420) - 30.085

Sta. PC = (3 + 389.92)

Stationing of PRC = (3 + 389.92) + 60.07

Stationing of PRC = **3 + 449.99**

Problem 336:

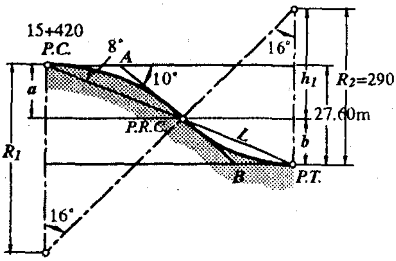
In a rail road layout, the centerline of two parallel tracks are connected with a reversed curve of unequal radii. The central angle of the first curve is 16° and the distance between parallel tracks is 27.60 m. stationing of the P.C. is 15 + 420 and the radius of the second curve is 290 m.

① Compute the length of the long chord from the P.C. to P.T.
② Compute the radius of the first curve.
③ Compute the stationing of the P.T.

REVERSED CURVES

Solution:

① *Length of long chord:*



$$\sin 8° = \frac{27.60}{L}$$

$L = \mathbf{198.31}$

② *Radius of first curve:*

$a = R_1 - R_1 \cos 16°$

$a = R_1 (1 - \cos 16°)$

$b = R_2 - R_2 \cos 16°$

$b = R_2 (1 - \cos 16°)$

$a + b = 27.60$

$R_1 (1 - \cos 16°) + R_2 (1 - \cos 16°) = 27.60$

$(R_1 + R_2)(1 - \cos 16°) = 27.60$

$R_1 + R_2 = 712.47$

$R_1 = 712.47 - 290$

$R_1 = \mathbf{422.47\ m.}$

③ *Stationing of PT:*

$$Lc_1 = \frac{422.47(16)\,\pi}{180}$$

$Lc_1 = 117.98$

$$Lc_2 = \frac{290(16)\,\pi}{180}$$

$Lc_2 = 80.98$

Sta. of P.R.C. = (15 + 420) + 117.98

Sta. of P.R.C. = 15 + 537.98

Sta. of PT = (15 + 537.98) + 80.98

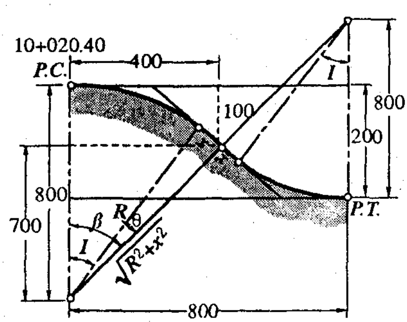
Sta. of PT = **15 + 618.96**

Problem 397:

Two parallel tangents have directions of due east and are 200 m. apart, are connected by a reversed curve having equal radius of 800 m. The P.C. of the curve is on the upper tangent while the P.T. is at the lower tangent. If the horizontal distance parallel to the tangent from the P.C. to the P.T. of the reversed curve is 800 m.

① Compute the distance of the intermediate tangent between the curves.
② Compute the distance between the centers of the reversed curve.
③ Compute the stationing of P.T. if sta. of the P.C. is 10 + 020.40.

Solution:



① *Length of intermediate tangent:*

$$\left(\sqrt{R^2 + x^2}\right)^2 = (400)^2 + (700)^2$$

$(800)^2 + x^2 = (400)^2 + (700)^2$

$x = 100$ m.

$2x = \mathbf{200\ m.}$

② *Distance between the centers of the reversed curve:*

$D = 2\sqrt{(800)^2 + x^2}$

$D = 2\sqrt{(800)^2 + (100)^2}$

$D = 2(806.23)$

$D = \mathbf{1612.45\ m.}$

③ *Stationing of P.T.*

$$\cos \beta = \frac{700}{806.23}$$

$\beta = 29°45'$

$$\tan \theta = \frac{100}{800}$$

$\theta = 7°08'$

$I = 29°45' - 7°08'$

$I = 22°37'$

$$L_c = \frac{R\,I\,\pi}{180}$$

$$L_c = \frac{800\,(22°37')\,\pi}{180°}$$

$L_c = 315.79$ m.

Sta. of P.T. = (10 + 020.40) + 315.79 + 200 + 315.79

Sta. of P.T. = ***10 + 851.98***

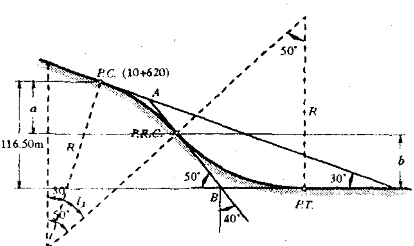
Problem 338:

Two tangents converge at an angle of 30°. The direction of the second tangent is due east. The distance of the P.C. from the second tangent is 116.50 m. The bearing of the common tangent is S. 40° E.

① Compute the central angle of the first curve.
② If a reversed curve is to connect these two tangents, determine the common radius of the curve.
③ Compute the stationing of the P.T. if P.C. is at station 10 + 620.

Solution:

① *Central angle of the first curve:*



$I_1 = 50° - 30°$

$I_1 = 20°$

② *Radius of curve:*

$a = R\cos 30° - R\cos 50°$

$a = R(\cos 30° - \cos 50°)$

$a = 0.223\,R$

$b = R - R\cos 50°$

$b = R(1 - \cos 50°)$

$b = 0.357\,R$

$a + b = 116.50$

$0.223\,R + 0.357\,R = 116.50$

$R =$ ***200.86 m.***

③ *Stationing of PT:*

$$L_1 = \frac{200.86\,(20)\,\pi}{180}$$

$L_1 = 70.11$ m.

$$L_2 = \frac{200.86\,(50)\,\pi}{180}$$

$L_2 = 175.28$ m.

Sta. of PT = (10 + 620) + 70.11 + 175.28

Sta. of PT = ***10 + 865.39***

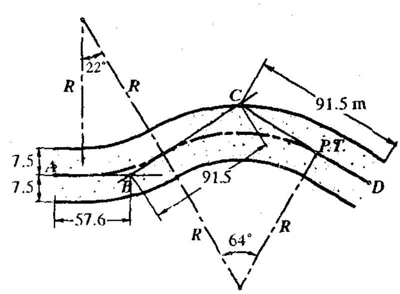
Problem 339:

Given broken lines AB = 57.6 m., BC = 91.5 m. and CD = 91.5 m. arranged as shown. A reverse curve is to connect these three lines thus forming the center line of a new road.

① Find the length of the common radius of the reverse curve.
② If the P.C. is at Sta. 10 + 000, what is the stationing of P.T.
③ What is the total area included in the right of way in this section of the road (A to D) if the road width is 15 m.

Solution:

① *Length of the common radius of the reverse curve:*



REVERSED CURVES

$T_1 = R \tan 11°$
$T_2 = R \tan 32°$
$T_1 + T_2 = 91.5$
$R(\tan 11° + \tan 32°) = 91.5$
$R = \mathbf{111.688\ m.}$

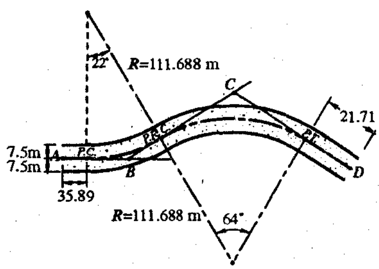
② *Sta. of P.T.*
P.C. to P.R.C. $= L_1$

$$L_1 = 111.688\,(22°)\frac{\pi}{180}$$
$L_1 = 42.885$ m.
$$L_2 = \frac{111.688\,(64°)\,\pi}{180°}$$
$L_2 = 124.757$ m.

Total length of reverse curve:
$L_1 + L_2 = 167.642$ m.

Stationing of P.R.C. = (10 + 000) + 42.885
Stationing of P.R.C. = 10 + 042.885
Stationing of P.T. = (10 + 042.885) + 124.757
Stationing of P.T. = ***10 + 167.642***

③ *Area included in the right of way:*



$T_1 = 111.688 \tan 11°$
$T_1 = 21.710$ m.
$T_2 = 111.688 \tan 32°$
$T_2 = 69.79°$

A to P.C. = 57.6 - 21.710
A to P.C. = 35.89 m.
P.T. to D = 91.5 - 69.790
P.T. to D = 21.710 m.

$$A = \frac{\pi}{360}(R_1^2 - R_2^2)\,ø$$
$ø = 22° + 64°$
$ø = 86°$

$R_1 = 111.688 + 7.5$
$R_1 = 119.188$ m.
$R_2 = 111.688 - 7.5$
$R_2 = 104.188$ m.

$$A = \frac{\pi}{360}(86)\left[(119.188)^2 - (104.188)^2\right]$$
$A = 2514.63\ m^2$

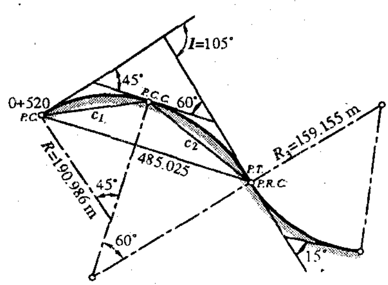
Total area = 2514.63 + 35.89(15) + 21.710(15)
Total area = ***3378.63*** m^2

Problem 340:

Three simple curve are connected to each other such that the first and the second form a compound curve while the second and the third formed a reversed curve. The distance between the point of curvature and the point of tangency of the compound curve which is also the point of reversed curvature of the reversed curve is 485.025 meters. If the angle of convergence between the second and the third tangents is 15°, $I_1 = 45°$, $R_1 = 190.986$ m., $I_2 = 60°$, $R_3 = 159.155$ m. and stationing at P.C. is 0 + 520.

① Determine the stationing at P.T. at the end of the long chord.
② Determine the angle between the long chord of the compound curve and the first tangent.
③ Determine the angle between the long chord of the compound curve and the second tangent.

Solution:



REVERSED CURVES

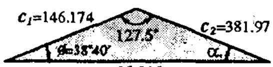
① *Stationing at P.T.*

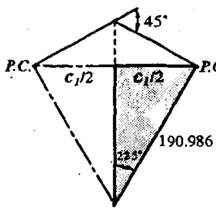
$\text{Sin } 22.5° = \dfrac{C_1}{2R_1}$

$C_1 = 2\,(190.986)\text{ Sin } 22.5°$

$C_1 = 146.174\text{ m.}$

Using Cosine Law:





$(485.025)^2 = (146.174)^2 + C_2^2$
$\quad - 2(146.174)\, C_2 \text{ Cos } 127.5°$

$213882.41 = C_2^2 + 177.97\, C_2$

$C_2 = 381.97\text{ m.}$

Using Sine Law:

$\dfrac{381.97}{\text{Sin } \theta} = \dfrac{485.025}{\text{Sin } 127.5°}$

$\theta = 38°40'$

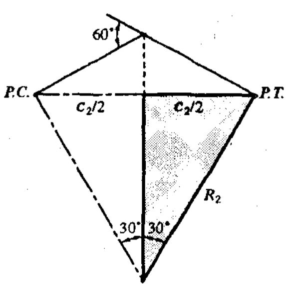
$\alpha = 180° - 127.5° - 38°40'$

$\alpha = 13°50'$

$Lc_1 = \dfrac{R_1\, I_1\, \pi}{180}$

$Lc_1 = \dfrac{190.986\,(45)\,\pi}{180}$

$Lc_1 = 150\text{ m}$



$Lc_2 = \dfrac{R_2\, I_2\, \pi}{180}$

$\text{Sin } 30° = \dfrac{C_2}{2\,R_2}$

$R_2 = \dfrac{C_2}{2\text{ Sin } 30°}$

$R_2 = \dfrac{381.97}{2\text{ Sin } 30°}$

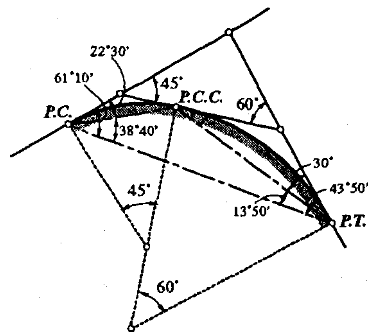
$R_2 = 381.97\text{ m.}$

$Lc_2 = \dfrac{381.97\,(60)\,\pi}{180}$

$Lc_2 = 400\text{ m.}$

Sta. at the P.T. = (0 + 520) + (150) + (400)
Sta. at the P.T. = ***1 + 070***

② *The angle that the long chord makes with the first tangent =* ***61°10'***



③ *The angle that the long chord makes with the second tangent =* ***43°50'***

Problem 341:

A reversed curve with diverging tangents is to be designed to connect three traverse lines for a portion of the proposed highway. The lines $T_{10} - T_{11}$ is 185 m, $T_{11} - T_{12}$ is 122.40 m, and $T_{12} - T_{13}$ is 285 m. The azimuth are Due East, 242°00' and 302°00' respectively.

Type of Pavement = Item 311 (Portand Concrete)
Number of lanes = Two (2) lanes
Width of Pavement = 3.05 m. per lane
Thickness of Pavement = 20 cms.
Unit Cost = P460.00 per square meter

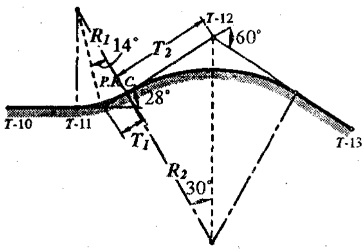
It is necessary that the P.R.C. (point of Reversed Curvature) must be one fourth (1/4) the distance of $T_{11} - T_{12}$ from T_{11}.

REVERSED CURVES

① Compute the radius of the first curve.
② Compute the radius of the second curve.
③ Compute the cost of the concrete pavement along the curves (reversed) from the P.C. to the P.T., based on the given highway cost index and specifications.

Solution:

① *Radius of first curve:*

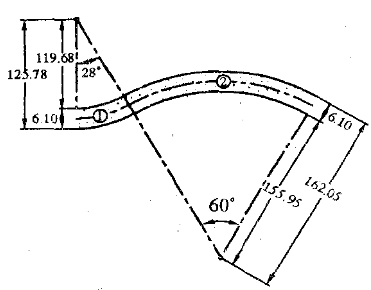


$$T_1 = \frac{1}{4}(122.40)$$
$$T_1 = 30.6 \text{ m.}$$
$$T_2 = 122.40 - 30.6$$
$$T_2 = 91.8 \text{ m.}$$
$$\tan 14° = \frac{T_1}{R_1}$$
$$R_1 = \frac{30.6}{\tan 14°}$$
$$R_1 = \mathbf{122.73\ m.}$$

② *Radius of second curve:*

$$\tan 30° = \frac{T_2}{R_2}$$
$$R_2 = \frac{91.8}{\tan 30°}$$
$$R_2 = \mathbf{159\ m.}$$

③ *Cost of concrete pavement:*



Area of Sector:

$$A = \frac{\pi r^2 \varnothing}{360}$$
$$A_1 = \frac{\pi \varnothing}{360}\left[(125.78)^2 - (119.68)^2\right]$$
$$A_1 = \frac{\pi (28)}{360}\left[(125.78)^2 - (119.68)^2\right]$$
$$A_1 = 365.86 \text{ sq.m.}$$
$$A_2 = \frac{\pi (60)}{360}\left[(162.05)^2 - (155.95)^2\right]$$
$$A_2 = 1015.68 \text{ sq.m.}$$
$$A = 365.86 + 1015.68$$
$$A = 1381.54 \text{ sq.m.}$$

Total Cost = 460 (1381.54)
Total Cost = ***P635,508.40***

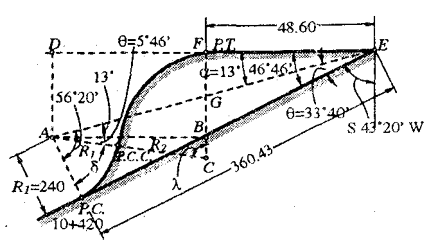
Problem 342:

Two tangents intersects at an angle of 46'40' are to be connected by a reversed curved. The tangent distance from the point of intersection of the tangents to the P.T. of the reversed curve is 48.60 m. The bearing of the back tangent is S 43'20' W. The radius of the curve through the P.C. is 240 m. and the distance from the point of intersection of tangents to the P.C. of the reversed curve is 360.43 m.

① Determine the radius of the other branch of the curve.
② If the stationing of P.C. is 10 + 420, what is the sta. of the P.R.C.
③ What is the sta. of the P.T.

Solution:

① *Radius of the other branch of the curve:*



REVERSED CURVES

$\tan\theta = \dfrac{240}{360.43}$

$\theta = 33^\circ 40'$

$\alpha = 46^\circ 40' - 33^\circ 40'$

$\alpha = 13^\circ$

$AE = \dfrac{240}{\text{Sin } 33^\circ 40'}$

$AE = 432.89$ m.

$AD = 432.89 \text{ Sin } 13^\circ$

$AD = 97.38$ m.

$DE = 432.89 \text{ Cos } 13^\circ$

$DE = 421.80$ m.

$DF = DE - 48.60$

$DF = 421.80 - 48.60$

$DF = 373.20$; $AB = 373.20$

$BC = R_2 - BF$

$BC = R_2 - 97.38$

$(R_1 + R_2)^2 = (AB)^2 + (BC)^2$

$(240 + R_2)^2 = (373.20)^2 + (R_2 - 97.38)^2$

$57600 + 480 R_2 + R_2^2$

$= 139278.24 + R_2^2 - 194.76 R_2 + 9482.86$

$R_2 = \mathbf{135.10}$

② *Stationing of P.R.C.*

$BC = R_2 - 97.38$

$BC = 135.10 - 97.38$

$BC = 37.72$

$AC = 135.10 + 240$

$AC = 375.10$

$\tan ø = \dfrac{BC}{AB}$

$\tan ø = \dfrac{37.72}{373.20}$

$ø = 5^\circ 46'$

$\tan 13^\circ = \dfrac{FG}{48.60}$

$FG = 11.22$

$\text{Cos } 13^\circ = \dfrac{48.60}{EG}$

$EG = 49.88$ m.

$AG = EA - EG$

$AG = 432.89 - 49.88$

$AG = 383.01$ m.

$\delta = 56^\circ 20' - ø - 13^\circ$

$\delta = 56^\circ 20' - 5^\circ 46' - 13^\circ$

$\delta = 37^\circ 34'$

$Lc_1 = \dfrac{R_1\,\delta\,\pi}{180}$

$Lc_1 = \dfrac{240\,(37^\circ 34')\,\pi}{180^\circ}$

$Lc_1 = 157.36$ m.

Sta. of P.R.C. = (10 + 420) + 157.36

Sta. of P.R.C. = ***10 + 577.36***

③ *Stationing of P.C.*

$Lc_2 = \dfrac{R_2\,\lambda\,\pi}{180}$

$\lambda = 90^\circ - 5^\circ 46'$

$\lambda = 84^\circ 14'$

$Lc_2 = \dfrac{135.10\,(84^\circ 14')\,\pi}{180^\circ}$

$Lc_2 = 198.62$ m.

Sta. of P.T. = (10 + 577.36) + 198.62

Sta. of P.T. = ***10 + 775.98***

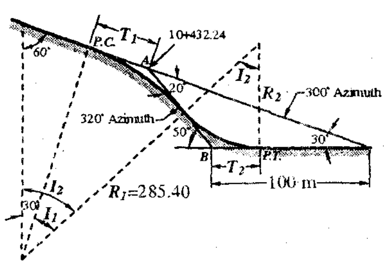
Problem 343:

Two converging tangents have azimuth of 300° and 90° respectively, while that of the common tangent is 320°. The distance from the point of intersection of the tangents to the P.I. of the 2nd curve is 100 m. while the stationing of the P.I. of the first curve is 10 + 432.24. If the radius of the first curve is 285.40 m.

① Determine the radius of the 2nd curve.
② Determine the stationing of P.R.C.
③ Determine the stationing of P.T.

Solution:

① *Radius of 2nd curve:*



REVERSED CURVES

$R_1 = 285.40$
$I_1 + 30 = I_2$
$I_2 = 50$
$I_1 = 50 - 30$
$I_1 = 20$
$$\frac{AB}{\sin 30^\circ} = \frac{100}{\sin 20^\circ}$$
$AB = 146.19$
$$T_1 = R_1 \tan \frac{I_1}{2}$$
$T_1 = 285.40 \tan 10$
$T_1 = 50.32$ m.

$T_2 = R_2 \tan 25^\circ$
$T_1 + T_2 = 146.19$
$T_2 = 146.19 - 50.32$
$T_2 = 95.87$ m.
$$R_2 = \frac{95.87}{\tan 25^\circ}$$
$R_2 = \mathbf{205.59\ m.}$

② *Sta. of P.R.C.*
$$D_1 = \frac{1145.916}{285.40}$$
$D_1 = 4.02^\circ$
$$D_2 = \frac{1145.916}{205.59}$$
$D_2 = 5.57^\circ$

$$Lc_1 = \frac{20\ I_1}{D_1}$$
$$Lc_1 = \frac{20\ (20)}{4.02}$$
$Lc_1 = 99.50$
$$Lc_2 = \frac{20\ (50)}{5.57}$$
$Lc_2 = 179.53$

P.C. = (10 + 432.24) - 50.32
P.C. = 10 + 381.92
P.R.C. = (10 + 381.92) + 99.50
P.R.C. = **10 + 481.42**

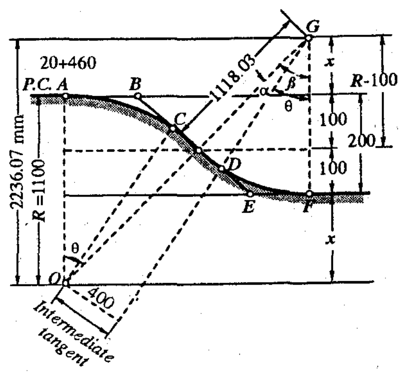
③ *Sta. of P.T.*
P.T. = (10 + 481.42) + 179.53
P.T. = **10 + 660.95**

Problem 344:

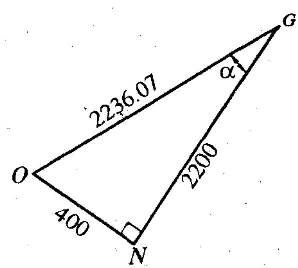
Two parallel railway 200 m. apart were to be connected by equal turnouts. If the intermediate tangent is 400 m. and the radius of curve is 1100 m.

① Determine the central angle of the reverse curve.
② If the P.C. is at sta. 20 + 460, find the sta. of the middle of the intermediate tangent.
③ Find the sta. of P.T.

Solution:



① *Central angle of the reverse curve:*



$$\tan \alpha = \frac{400}{2\ (1100)}$$
$\alpha = 10^\circ 18'$
$OG = \sqrt{(400)^2 + (2200)^2}$
$OG = 2236.07$
$2236.07 \cos \beta = 2x + 200$
$x + 200 = R$
$x = R - 200$

REVERSED CURVES

$2236.07 \cos \beta = 2(R - 200) + 200$
$2236.07 \cos \beta = 2R - 400 + 200$
$2236.07 \cos \beta = 2R - 200$
$1118.03 \cos \beta = R - 100$

$$\cos \beta = \frac{R - 100}{1118.035}$$

$$\cos \beta = \frac{1100 - 100}{1118.035}$$

$$\cos \beta = \frac{1000}{1118.035}$$

$\beta = 26°34'$
$\phi = 26°34' - 10°18'$
$\boldsymbol{\phi = 16°16'}$

② *Sta. of the middle of intermediate tangent.*

$$L = \frac{R \phi \pi}{180}$$

$$L = \frac{1100\,(16°16')\,\pi}{180°}$$

$L = 312.30$ m.

Sta. = (20 + 460) + 312.30 + 200
Sta. = ***20 + 972.30***

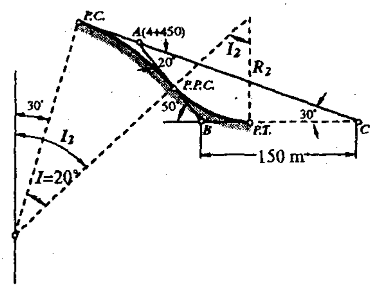
③ *Sta. of the P.T.*
Sta. of P.T. = (20 + 972.30) + 200 + 312.30
Sta. of P.T. = ***21 + 484.60***

Problem 345:

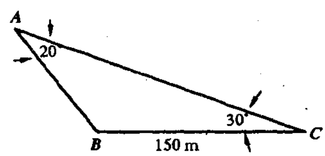
A reverse curve connects two converging tangents intersecting at an angle of 30°. The distance of this intersection from the P.I. of the curve is 150 m. The deflection angle of the common tangent from the back tangent is 20° R, and the azimuth of the common tangent is 320°. The degree of curve of the second simple curve is 6° and the stationing of the point of intersection of the first curve is 4 + 450.

① Determine the radius of the first curve.
② Determine the stationing of P.R.C.
③ Determine the stationing of the P.T.

Solution:



① *Radius of first curve:*



$$\frac{AB}{\sin 30°} = \frac{150}{\sin 20°}$$

$$AB = \frac{150 \sin 30°}{\sin 20°}$$

$AB = 219.29$ m.

$T_1 + T_2 = 219.29$

$$T_1 = R_1 \tan \frac{I_1}{2}$$

$$T_2 = R_2 \tan \frac{I_2}{2}$$

$I_2 = I_1 + 30°$
$50° = I_1 + 30°$
$I_1 = 20°$

$$R_1 \tan \frac{I_1}{2} + R_2 \tan \frac{I_2}{2} = 219.29$$

$R_1 \tan 10° + R_2 \tan 25° = 219.29$

$$R_2 = \frac{1145.916}{D_2}$$

$$R_2 = \frac{1145.916}{6}$$

$R_2 = 190.99$ m.

$R_1 \tan 10° + 190.99 \tan 25° = 219.29$
$0.1763 R_1 + 89.06 = 219.29$
$\boldsymbol{R_1 = 738.68}$ ***m.***

REVERSED CURVES

② *Stationing of P.R.C.*

$$D_1 = \frac{1145.916}{R_1} = \frac{1145.916}{738.68}$$
$$D_1 = 1.55°$$
$$T_1 = R_1 \tan\frac{I_1}{2}$$
$$T_1 = 738.68 \tan 10°$$
$$T_1 = 130.25 \text{ m.}$$

Sta. of P.C. = (4 + 450) - 130.25
Sta. of P.C. = 4 + 319.75

$$L_1 = \frac{(20)(20)}{1.55} = 258.06 \text{ m.}$$

Sta. of P.R.C. = (4 + 319.75) + 258.06
Sta. of P.R.C. = **4 + 577.81**

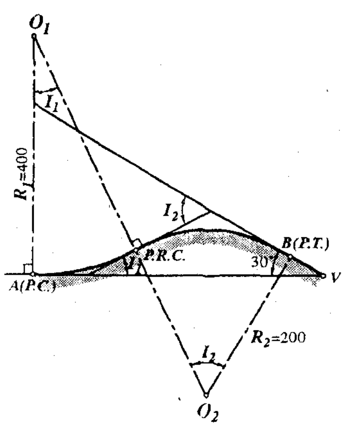
③ *Stationing of P.T.*

$$L_2 = \frac{20\, I_2}{D_2}$$
$$L_2 = \frac{20\,(50)}{6}$$
$$L_2 = 166.67 \text{ m.}$$

Sta. of P.T. = (4 + 577.81) + 166.67
Sta. of P.T. = **4 + 744.48**

Problem 346:

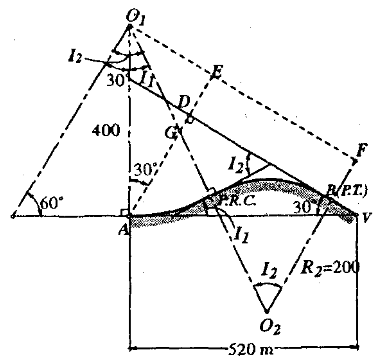
A reverse curve shown having radius R_1 = 400 m. and R_2 = 200 m. long is to connect the two tangents AV and VB with angle of intersection of the tangents equal to 30°. AV = 520 m.



① Determine the central angle of the second curve.
② Determine the central angle of the first curve.
③ Determine the distance VB.

Solution:

① *Central angle of 2nd curve:*



AD = 520 Sin 30°
AD = 260 m.
EA = 400 Cos 30°
EA = 346.41
DE = 346.41 - 260
DE = 86.41 = FB

$$\text{Cos } I_2 = \frac{O_2 F}{O_1 O_2}$$
$$\text{Cos } I_2 = \frac{(200 + 86.41)}{200 + 400}$$
$$I_2 = \mathbf{61°29'}$$

② *Central angle of 1st curve:*

I_1 + 30° = I_2
I_1 = 61°29' - 30°
I_1 = **31°29'**

③ *Distance VB:*

$$\text{Sin } I_2 = \frac{FO_1}{400 + 200}$$
FO_1 = 600 Sin 61°29'
FO_1 = 527.21 m.
EO_1 = 400 Sin 30°
EO_1 = 200

REVERSED CURVES

$FE = FO_1 - EO_1$
$FE = 527.21 - 200$
$FE = 327.21 = DB$

In triangle VDA:

$$\text{Cos } 30° = \frac{VB + DB}{AV}$$

$$\text{Cos } 30° = \frac{VB + 327.21}{520}$$

$VB = \mathbf{123.12\ m.}$

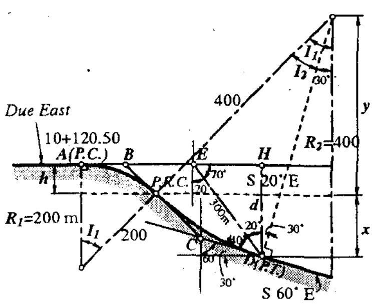
Problem 347:

A reversed curve is connecting the two tangent lines AB and CD having directions of due East and S. 60° E. respectively. The radius of the first curve at A (P.C.) is 200 m. and that of the second curve at D (P.T.) is 400 m. Stationing of A (P.C.) is at 10 + 120.50. If ED is 300 m. long and has a bearing of S. 20° E.

① Find the central angle of first curve.
② Find the central angle of second curve.
③ Find the stationing of P.T.

Solution:

① *Central angle of first curve:*



$$\text{Sin } 70° = \frac{d}{300}$$

$d = 281.91$ m.
$h = 200 - 200 \text{ Cos } I_1$
$y = 400 \text{ Cos } I_1$
$x = 400 \text{ Cos } 30° - y$
$x = 346.41 - 400 \text{ Cos } I_1$

$x + h = d$
$346.41 - 400 \text{ Cos } I_1 + 200 - 200 \text{ Cos } I_1 = 281.91$
$600 \text{ Cos } I_1 = 264.50$
$I_1 = \mathbf{63°51'}$

② *Central angle of 2nd curve:*

$I_1 = I_2 + 30°$
$I_2 = 63°51' - 30°$
$I_2 = \mathbf{33°51'}$

③ *Stationing of P.T.*

$$Lc_1 = \frac{R_1 I_1 \pi}{180}$$

$$Lc_1 = \frac{200\,(63°51')\,\pi}{180°}$$

$Lc_1 = 222.88$ m.

$$Lc_2 = \frac{R_2 I_2 \pi}{180}$$

$$Lc_2 = \frac{400\,(33°51')\,\pi}{180°}$$

$Lc_2 = 236.32$ m.

$P.R.C. = (10 + 120.50) + 222.88$
$P.R.C. = 10 + 343.38$

$P.T. = (10 + 343.38) + 236.32$
$P.T. = \mathbf{10 + 579.70}$

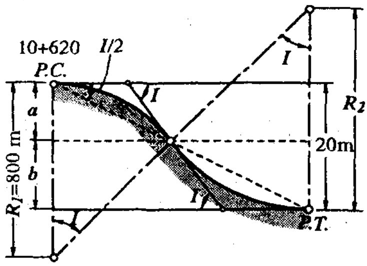
Problem 348:

Two parallel tangents 20 m. apart are to be connected by a reversed curve. The radius of the first curve at the P.C. has a radius of 800 m. and the total length of the chord from the P.C. to the P.T. is 300 m. Stationing of the P.C. is 10 + 620.

① Find the central angle of each curve.
② Find the radius of the curve passing thru the P.T.
③ What is the stationing of the P.T.

Solution:

① *Central angle of each curve:*



$$\sin\frac{I}{2}=\frac{20}{300}$$

$$\frac{I}{2}=3°49'$$

$$I = 7°38'$$

② *Radius of the curve passing thru the P.T.*

$a = 800 - 800\cos 7°38'$

$a = 7.09$ m.

$b = 20 - 7.09$

$b = 12.91$

$b = R_2 - R_2\cos 7°38'$

$12.91 = R_2(1 - \cos 7°38')$

$R_2 =$ ***1456.85 m.***

③ *Sta. of P.T.*

$$Lc_1=\frac{R_1 I \pi}{180}$$

$$Lc_1=\frac{800\,(7°38')\,\pi}{180°}$$

$Lc_1 = 106.58$

$$Lc_2=\frac{1456.85\,(7°38')\,\pi}{180°}$$

$Lc_2 = 194.09$

Sta. of P.T. = (10 + 620) + 106.58 + 194.09

Sta. of P.T. = ***10 + 920.67***

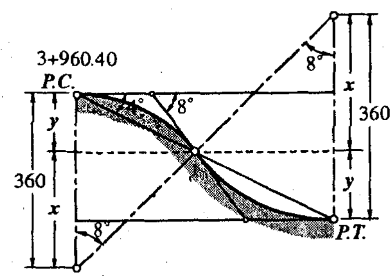
Problem 349:

Two parallel tangents are connected by a reversed curve having equal radii of 360 m.

① If the central angle of the curve is 8° compute the distance between parallel tangents.
② Compute the length of chord from the P.C. to the P.T.
③ If P.C. is at sta. 3 + 960.40, what is the stationing of the P.T.

Solution:

① *Distance between parallel tangents:*



$x = 360\cos 8°$

$x = 356.50$

$y = 360 - 356.50$

$y = 3.50$

$2y =$ ***7.0 m.***

② *Length of chord from P.C. to P.T.*

$$\sin 4° = \frac{2\,(3.50)}{L}$$

$L =$ ***100.35 m.***

③ *Sta. of P.T.*

$$L_c=\frac{R\,\theta\,\pi}{180}$$

$$L_c=\frac{360\,(8°)\,\pi}{180°}$$

$L_c = 50.27$

Sta. of P.T. = (3 + 960.40) + 50.27 + 50.27

Sta. of P.T. = ***4 + 060.94***

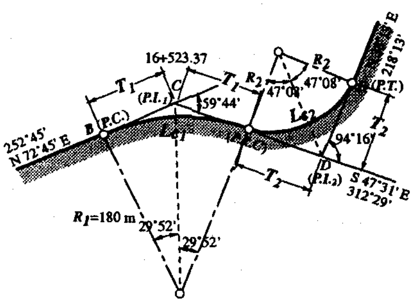
REVERSED CURVES

Problem 349-A:

The common tangent CD of a reversed curve is 280.5 m. and has an azimuth of 312° 29'. BC is a tangent of the first curve whose azimuth is 252° 45'. DE is a tangent of the second curve whose azimuth is 218° 13'. The radius of the first curve is 180 m. The $P.I._1$ is at sta. 16 + 523.37. B is at PC and E is at P.T.

a) What is the stationing of the P.C.
b) What is the stationing of the P.R.C.
c) What is the stationing of the P.T.

Solution:



a) Stationing of P.C.

CD = 280.5 m.

$280.5 = 180 \tan 29\ 52' + R_2 \tan 47\ 08'$

$R_2 = 164.41$

$T_1 = 180 \tan 29\ 52'$

$T_1 = 103.37$

$T_2 = 280.5 - 103.37$

$T_2 = 177.13$

P.C. = (16 + 523.37) - (103.37)

P.C. = ***16 + 420***

b) Stationing of P.R.C.

$$L_{c_1} = \frac{380(59°\ 44')\ \pi}{180}$$

$L_{c_1} = 187.66$

PRC = (16 + 420) + (187.66)

PRC = ***16 + 607.66***

c) Stationing of P.T.

$$L_{c_2} = \frac{164.41(94°\ 16')\ \pi}{180}$$

$L_{c_2} = 270.50$

P.T. = (16 + 607.66) + 270.50

P.T. = ***16 + 878.16***

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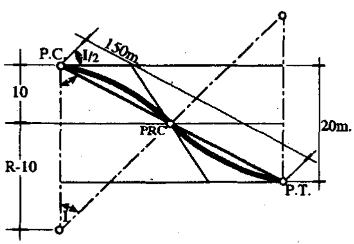
Two parallel tangents 20 m. apart are to be connected by a reversed curve with equal radius at the P.C. and P.I. The total length of chord from the P.C. to the P.T. is 150 m. Stationing of the P.C. is 10 + 200.

① Find the radius of the reversed curve.
② Find the length of cord from P.C. to P.R.C.
③ Find the stationing of the P.T.

Solution:

① *Central angle of each curve:*



$$\sin\frac{I}{2} = \frac{20}{150}$$

$$\frac{I}{2} = 7°40'$$

$$I = 15°\ 20'$$

$$R - R\cos 15°20' = 10$$

$$R\,(1 - \cos 15°20') = 10$$

$$R = \boldsymbol{280.93\ m.}$$

② *Length of chord from P.C. to P.R.C.*

$$L = \frac{150}{2}$$

$$L = \boldsymbol{75\ m.}$$

③ *Sta. of P.T.*

$$Lc = \frac{R\,I\,(\pi)}{180}$$

$$Lc = \frac{280.93\,(15°20')\,\pi}{180°}$$

$$Lc = 75.18\ m.$$

P.T. = (10 + 200) + (75.18) + (75.18)

P.T. = 10 + 350.36

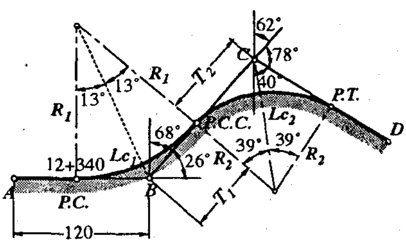
REVERSED CURVES

Problem 350:

A reversed curve with diverging tangents is to pass through three lines to form a center line of a proposed road. The first line AB has a bearing of N. 88° E. and a distance of 120 m., BC has a bearing of N. 62° E., and a distance of 340 m., while that of CD has a bearing of S. 40° E. and a distance of 300 m. If the first tangent has a distance of only 1/4 that of the common tangent measured from the point of intersection of the first curve.

① Compute the radius of the first curve.
② Compute the radius of the second curve.
③ Compute the stationing of the P.T. if P.C. is at station 12 + 340.

Solution:
① *Radius of 1st curve:*



$$T_1 = \frac{1}{4}(340)$$
$$T_1 = 85 \text{ m.}$$
$$T_1 = R_1 \tan 13°$$
$$R_1 = \mathbf{368.18 \text{ m.}}$$

② *Radius of 2nd curve:*
$$T_2 = 340 - 85$$
$$T_2 = 255 \text{ m.}$$
$$T_2 = R_2 \tan 39°$$
$$R_2 = \mathbf{314.90}$$

③ *Sta. of P.T.*
$$Lc_1 = \frac{R_1 I_1 \pi}{180}$$
$$Lc_1 = \frac{368.18\,(26°)\,\pi}{180°}$$
$$Lc_1 = 167.07 \text{ m.}$$

$$Lc_2 = \frac{R_2 I_2 \pi}{180}$$
$$Lc_2 = \frac{314.90\,(78°)\,\pi}{180°}$$
$$Lc_2 = 428.69$$

Total length of curve:
$$Lc = Lc_1 + Lc_2$$
$$Lc = 167 + 428.69$$
$$Lc = 595.76$$

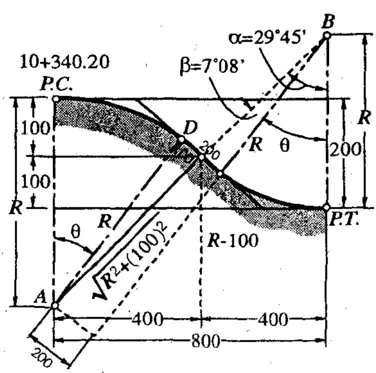
$$\text{Sta. of P.T.} = (12 + 340) + 595.76$$
$$\text{Sta. of P.T.} = \mathbf{12 + 935.76}$$

Problem 351:

A reversed curve is to connect two tangents which are parallel to each other and are 200 m. apart with directions of due east. There is an intermediate tangent of 200 m. in between the reversed curve and the horizontal distance of the P.C. and P.T. measured parallel to the tangents is 800 m. long. The P.C. of the reversed curve is on the upper tangent while the P.T. of the reversed curve is at the lower tangent.

① Compute the common radius of the reversed curve.
② Compute the central angle of the reversed curve.
③ Compute the stationing of the P.T. if the P.C. is at station 10 + 340.20.

Solution:
① *Common radius of curve:*



$$R^2 + (100)^2 = (R - 100)^2 + (400)^2$$
$$R^2 + (100)^2 = R^2 - 200R + (100)^2 + (400)^2$$
$$200R = 400\ (400)$$
$$R = \mathbf{800\ m.}$$

② *Central angle of the curve:*

$$\tan \beta = \frac{200}{1600}$$
$$\beta = 7°08'$$
$$(AB)^2 = (200)^2 + (1600)^2$$
$$AB = 1612.45$$
$$BD = \frac{1}{2}(1612.45)$$
$$BD = 806.23$$
$$\cos \alpha = \frac{700}{806.23}$$
$$\alpha = 29°45'$$
$$\theta = 29°45' - 7°08'$$
$$\theta = \mathbf{22°37'}$$

③ *Sta. of P.T.*

$$Lc_1 = \frac{R\,\theta\,\pi}{180}$$
$$Lc_1 = \frac{800\ (22°37')\,\pi}{180°}$$
$$Lc_1 = 315.79$$

Sta. of P.T. = (10 + 340.20) + 315.79
+ 200 + 315.79
Sta. of P.T. = ***11 + 171.78***

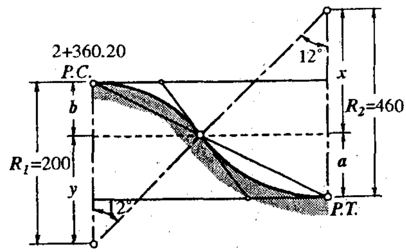
Problem 352:

A reverse curve has a radius of the curve passing through the P.C. equal to 200 m. and that of the second curve passing through the P.T. is 460 m. long. If the central angle of curve is 12.

① Find the perpendicular distance between the two parallel tangents.
② If the stationing of the P.C. is 2 + 360.20, find the stationing of the P.R.C.
③ Find the stationing of the P.T.

Solution:

① *Distance between parallel tangents:*



$$x = 460 \cos 12°$$
$$x = 449.95$$
$$y = 200 \cos 12°$$
$$y = 195.63$$

$$a = 460 - 449.95$$
$$a = 10.05$$
$$b = 200 - 195.63$$
$$b = 4.37$$

Distance between the parallel tangents
= 10.05 + 4.37
= ***14.42 m.***

② *Stationing of P.R.C.*

$$Lc_1 = \frac{R_1\,\theta\,\pi}{180}$$
$$Lc_1 = \frac{200\ (12°)\,\pi}{180°}$$
$$Lc_1 = 41.89$$

Sta. of P.R.C. = (2 + 360.20) + 41.89
Sta. of P.R.C. = **2 + 402.09**

③ *Sta. of P.T.*

$$Lc_2 = \frac{R_2\,\theta\,\pi}{180}$$
$$Lc_2 = \frac{460\ (12°)\,\pi}{180°}$$
$$Lc_2 = 96.34$$

Sta. of P.T. = (2 + 402.09) + 96.34
Sta. of P.T. = **2 + 498.43**

REVERSED CURVES

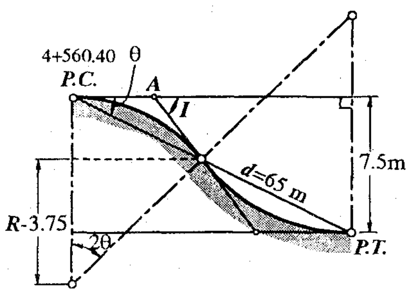
Problem 353:

The perpendicular distance between two parallel tangents of a reversed curved is 7.5 m. and the chord distance from the P.C. to the P.T. is equal to 65 m.

① Compute the central angle of the reversed curve.
② Compute the common radius of the reversed curve.
③ If the sta. of the P.C. is at 4 + 560.40, find the stationing of the P.T.

Solution:

① *Central angle of the reversed curve:*



$$\text{Sin } \theta = \frac{7.5}{6.5}$$

$$\theta = 6.63^\circ$$

$$2\theta = \mathbf{13^\circ 15'} \text{ (central angle)}$$

② *Radius of the curve:*

$$\text{Cos } 13^\circ 15' = \frac{R - 3.75}{R}$$

$$R - 3.75 = 0.913\,R$$

$$R = \mathbf{140.87\ m.}$$

③ *Staioning of P.T.*

$$L_c = \frac{R\,(2\theta)\,\pi}{180}$$

$$L_c = \frac{140.87\,(13^\circ 15')\,\pi}{180^\circ}$$

$$L_c = 32.58$$

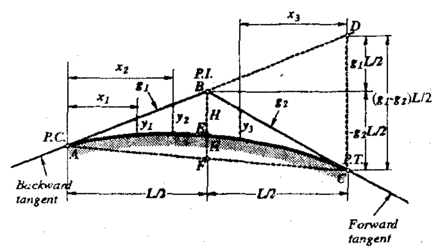
Sta. of P.T. = (4 + 560.40) + 32.58 + 32.58
Sta. of P.T. = **4 + 625.56**

PARABOLIC CURVES

SYMMETRICAL PARABOLIC CURVES

In highway practice, abrupt change in the vertical direction of moving vehicles should be avoided. In order to provide gradual change in its vertical direction, a parabolic vertical curve is adopted on account of its slope which varies at constant rate with respect to horizontal distances.

Properties of Vertical Parabolic Curves:



1. The vertical offsets from the tangent to the curve are proportional to the squares of the distances from the point of tangency.

$$\frac{y_1}{(x_1)^2} = \frac{H}{\left(\frac{L}{2}\right)^2} \qquad \frac{y_1}{(x_1)^2} = \frac{y_2}{(x_2)^2}$$

$$\frac{y_2}{(x_2)^2} = \frac{H}{\left(\frac{L}{2}\right)^2} \qquad \frac{y_3}{(x_3)^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$

2. The curve bisects the distance between the vertex and the midpoint of the long chord.

From similar triangles:

$$\frac{BF}{\frac{L}{2}} = \frac{CD}{L} \qquad BF = \frac{CD}{2}$$

PARABOLIC CURVES

From the first property of the curve:

$$\frac{BE}{\left(\frac{L}{2}\right)^2}=\frac{CD}{L^2}$$

$$BE=\frac{CD\,L^2}{4\,L^2}$$

$$BE=\frac{CD}{4}$$

$$H=\frac{1}{4}CD$$

But $\frac{CD}{2}=BF$

$$BE=\frac{1}{2}\left(\frac{CD}{2}\right)$$

$$BE=\frac{1}{2}BF$$

3. If the algebraic difference in the rate of grade of the two slopes is positive, that is $(g_1 - g_2)$, we have a "summit" curve, but if it is negative, we have a "sag curve".

4. The length of curve of a parabolic vertical curve, refers to the horizontal distance from the P.C. to the P.T.

5. The stationing of vertical parabolic curves is measured not along the curve but along a horizontal line.

6. For a symmetrical parabolic curve, the number of stations to the left must be equal to the number of stations to the right, of the intersection of the slopes or forward and backward tangent.

7. The slope of the parabola varies uniformly along the curve, as shown by differentiating the equation of the parabolic curve.

$$y=k\,x^2$$

$$\frac{dy}{dx}=2\,kx$$

The second derivative is $\frac{d^2y}{2}=2k$

where $\frac{d^2y}{dx^2}$ = rate of change of grade or slope.

Therefore the rate of change of slope is constant and equal to:

$$r=\frac{g_2-g_1}{L}$$

$$r=2\,k$$

8. The maximum offset H = 1/8 the product of the algebraic difference between the two rates of grade and the length of curve:

From the figure: $H=BE=\frac{1}{4}CD.$

$$CD=(g_1-g_2)\frac{L}{2}$$

$$H=\frac{1}{4}CD$$

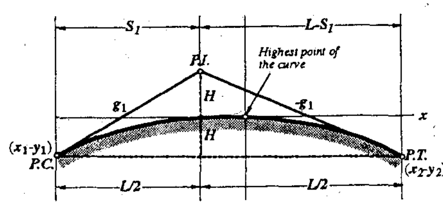
$$H=\frac{1}{4}\left(\frac{L}{2}\right)(g_1-g_2)$$

$$H=\frac{1}{8}L\,(g_1-g_2)$$

Location of highest or lowest point of the Curve

a) *From the P.C.*

$$S_1=\frac{g_1\,L}{g_1-g_2}$$



The slope of the tangent at P.C. is g_1.
From the equation of the parabola,

$$y=k\,x^2$$

$$\frac{dy}{dx}=2\,kx$$

where $\frac{dy}{dx}=g_1$

$$x=S_1$$

PARABOLIC CURVES

$$\frac{dy}{dx} = 2\,kx$$
$$g_1 = 2\,k\,(S_1)$$

① $g_1 = 2\,k\,S_1$

The slope of the tangent at P.T. is g_2.

$$\frac{dy}{dx} = g_2$$
$$x = L - S_1$$
$$\frac{dy}{dx} = -\,2\,kx$$
$$g_2 = -\,2\,k(L - S_1)$$

② $g_2 = -\,2\,k(L - S_1)$

Divide equation ① by ②

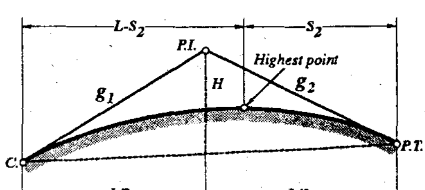
$$\frac{g_1}{g_2} = \frac{2\,k\,S_1}{-\,2k\,(L - S_1)}$$
$$\frac{g_1}{g_2} = \frac{S_1}{-\,(L - S_1)}$$
$$-\,g_1 L + g_1 S_1 = S_1 g_2$$
$$S_1\,(g_1 - g_2) = g_1 L$$

$$S_1 = \frac{g_1\,L}{g_1 - g_2}$$

location of the highest point of the curve from the P.C.

b) *From the P.T.*

$$S_2 = \frac{g_2\,L}{g_2 - g_1}$$



The slope at the P.C. is g_1:

$$\frac{dy}{dx} = 2\,kx$$

At the P.C.

$$x = (L - S_2)$$
$$\frac{dy}{dx} = g_1$$
$$\frac{dy}{dx} = 2\,kx$$

① $g_1 = +\,2k(L - S_2)$

At the P.T.

$$x = S_2$$
$$\frac{dy}{dx} = g_2$$
$$\frac{dy}{dx} = -\,2\,kx$$

② $g_2 = -\,2k(S_2)$

Divide equation ① by ②

$$\frac{g_1}{g_2} = \frac{2k\,(L - S_2)}{-\,2k\,(S_2)}$$
$$-\,g_1 S_2 = g_2 L - g_2 S_2$$
$$S_2\,(g_2 - g_1) = g_2 L$$

$$S_2 = \frac{g_2\,L}{g_2 - g_1}$$

location of the highest or lowest point of the curve from the P.T.

Problem 354:

A parabolic curve has a descending grade of - 0.8% which meets an ascending grade of 0.4% at sta. 10 + 020. The max. allowable change of grade per 20 m. station is 0.15. Elevation at station 10 + 020 is 240.60 m.

① What is the length of the curve?
② Compute the elevation of the lowest point of the curve.
③ Compute the elevation at station 10 + 000.

PARABOLIC CURVES

Solution:

① *Length of curve:*

$$n = \frac{g_2 - g_1}{r}$$

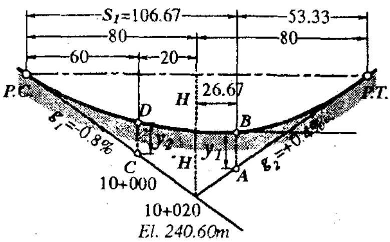
$$n = \frac{0.4 - (-0.8)}{0.15}$$

$n = 8$

$L = 20\,(80)$

$L = \mathbf{160\ m.}$

② *Elevation of lowest point of curve:*



$$S = \frac{g_1 L}{g_1 - g_2}$$

$$S = \frac{-0.008\,(160)}{-0.008 - 0.004}$$

$S = 106.67$

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{160}{8}(-0.008 - 0.004)$$

$H = 0.24$

$$\frac{H}{(80)^2} = \frac{y_1}{(53.33)^2}$$

$$\frac{0.24}{(80)^2} = \frac{y}{(53.33)^2}$$

$y = 0.11$

Elev. $A = 240.60 + 26.67(0.004)$

Elev. $A = 240.71$ m.

Elev. of lowest point of curve = 240.71 + 0.11

Elev. of lowest point of curve = **240.82 m.**

③ *Elevation of station 10 + 000:*

$$\frac{y_2}{(60)^2} = \frac{0.24}{(80)^2}$$

$y_2 = 0.135$

Elev. $D = 240.60 + 20(0.008) + 0.135$

Elev. D = **240.895 m.**

Problem 355:

A symmetrical vertical summit curve has tangents of + 4% and - 2%. The allowable rate of change of grade is 0.3% per meter station. Stationing and elevation of P.T. is at 10 + 020 and 142.63 m. respectively.

① Compute the length of curve.
② Compute the distance of the highest point of curve from the P.C.
③ Compute the elevation of the highest point of curve.

Solution:

① *Length of curve:*

$$\text{Rate of change} = \frac{g_1 - g_2}{n}$$

$$0.3 = \frac{4 - (-2)}{n}$$

$n = 20$ *stations*

Length of curve = 20 (20)

Length of curve = **400 m.**

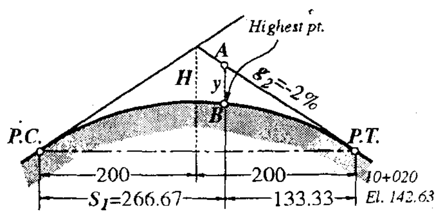
② *Sta. of highest point of curve:*

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{0.04\,(400)}{0.04 - (-0.02)}$$

S_1 = **266.67 m.** *from P.C.*

③ *Elevation of highest point of curve:*



$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{400}{8}(0.04 + 0.02)$$

$H = 3$

$$\frac{H}{(200)^2} = \frac{y}{(133.33)^2}$$

$$y = \frac{3\,(133.33)^2}{(200)^2}$$

$$y = 1.33$$

Elev. at highest point
$= 142.63 + 133.33\,(0.02) - 1.33$

Elev. at highest point = ***143.97***

Problem 356:

A vertical parabolic sag curve of Lapulapu underpass has a grade of - 4% followed by a grade of +2% intersecting at station 12 + 150.60 at elevation 124.80 m. above sea level. The change of grade of the sag curve is restricted to 0.6%.

① Compute the length of curve.
② Compute the elevation of the lowest point of the curve.
③ Compute the elevation at station 12 + 125.60.

Solution:

① *Length of curve:*

$$r = \frac{g_2 - g_1}{n}$$

$$0.6 = \frac{2 - (-4)}{n}$$

$$n = 10$$

$$L = 20\,(10)$$

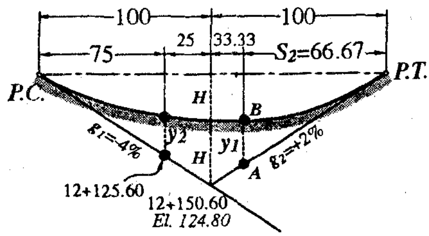
$$L = \mathbf{200\ m.}$$

② *Elevation of lowest point of curve:*

$$S_2 = \frac{g_2 L}{g_2 - g_1}$$

$$S_2 = \frac{0.02\,(200)}{0.02 = 0.04}$$

$S_2 = 66.67$ m. *from P.T.*



$$H = \frac{L}{8}(g_2 - g_1)$$

$$H = \frac{200}{8}(0.02 + 0.04)$$

$$H = 1.5 \text{ m.}$$

$$\frac{y_1}{(66.67)^2} = \frac{1.5}{(100)^2}$$

$$y_1 = 0.67 \text{ m.}$$

Elev. B $= 124.80 + 0.02(33.33) + 0.67$

Elev. B = ***126.14 m.***

③ *Elevation of sta. 12 + 125.60:*

$$\frac{y_2}{(75)^2} = \frac{1.5}{(100)^2}$$

$$y_2 = 0.84$$

Elev. D $= 124.80 + 0.04(25) + 0.84$

Elev. D = ***126.64 m.*** *(elev. at 12 + 125.60)*

Problem 357:

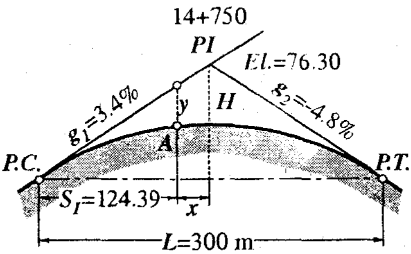
A vertical summit parabolic curve has its P.I. at station 14 + 750 with elevation of 76.30 m. The grade of the back tangent is 3.4% and forward tangent of - 4.8%. If the length of curve is 300 m.

① Compute the location of the vertical curve turning point from the P.I.
② Compute the elevation of the vertical curve turning point in meters.
③ Compute the stationing of the vertical curve turning point.

PARABOLIC CURVES

Solution:

① *Location of vertical curve turning point:*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{0.034(300)}{0.034 + 0.048}$$

$$S_1 = 124.39 \text{ m.}$$

$x = 150 - 124.39$

$x =$ ***25.61 m.*** *from the P.I.*

② *Elev. of vertical curve turning point:*

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{300}{8}(0.039 + 0.048)$$

$$H = 3.075$$

$$\frac{y}{(124.39)^2} = \frac{3.075}{(150)^2}$$

$y = 2.11$ m.

Elev. of vertical curve turning point
$= 76.30 - 25.61(0.034) - 2.11$

Elev. of vertical curve turning point
$=$ ***73.32 m.***

③ *Stationing of the vertical curve turning point:*

$(14 + 750) - (25.61) =$ ***14 + 724.39***

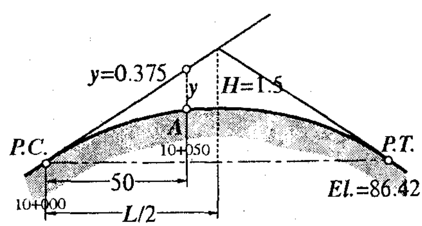
Problem 358:

A vertical summit parabolic curve has a vertical offset of 0.375 m. from the curve to the grade tangent at sta 10 + 050. The curve has a slope of + 4% and - 2% grades intersecting at the P.I. The offset distance of the curve at P.I. is equal to 1.5 m. If the stationing of the P.C. is at 10 + 000.

① Compute the required length of curve.
② Compute the horizontal distance of the vertical curve turning point from the point of intersection of the grades.
③ Compute the elevation of the vertical curve turning point if the elevation of P.T. is 86.42 m.

Solution:

① *Required length of curve:*

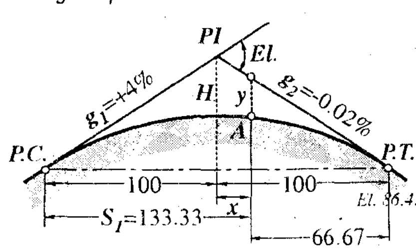


$$\frac{y}{(50)^2} = \frac{H}{(L/2)^2}$$

$$\frac{0.375}{(50)^2} = \frac{1.5}{(L/2)^2}$$

$L =$ ***200 m.***

② *Highest point of curve:*



PARABOLIC CURVES

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$
$$S_1 = \frac{0.04(200)}{0.04 + 0.02}$$
$S_1 = 133.33$ m.

$x = 133.33 - 100$
$x = \mathbf{33.33\ m.}$

③ *Elev. of highest point of curve:*
$$\frac{y}{(66.67)^2} = \frac{1.5}{(100)^2}$$
$y = 0.67$

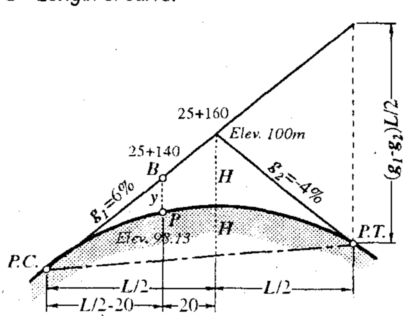
Elev. $A = 86.42 + 66.67(0.02) - 0.67$
Elev. $A = \mathbf{87.08\ m.}$

Problem 359:

A symmetrical parabolic summit curve connects two grades of +6% and - 4%. It is to pass through a point "P" on the curve at station 25 + 140 having an elevation of 98.134 m. If the elevation of the grade intersection is 100 m. with a stationing of 25 + 160.

① Compute the length of the curve.
② Compute the stationing of the highest point of the curve.
③ Compute the elevation of station 25 + 120 on the curve.

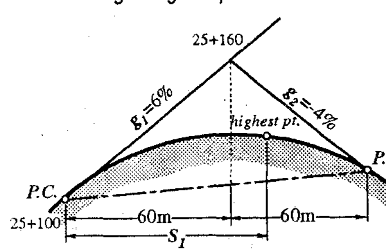
Solution:
① *Length of curve:*



By ratio and proportion:
$$\frac{2H}{\frac{L}{2}} = \frac{(g_1 - g_2)\frac{L}{2}}{L}$$
$$H = \frac{(g_1 - g_2)L}{8}$$
$$H = \frac{[0.06 - (-0.04)]L}{8}$$
$H = 0.0125L$

Elev. of $B = 100 - 0.06(20)$
Elev. of $B = 98.80$
$y = 98.80 - 98.134$
$y = 0.666$
$$\frac{y}{\left(\frac{L}{2} - 20\right)^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$
$$\frac{0.666}{\left(\frac{L}{2} - 20\right)^2} = \frac{0.0125L}{\left(\frac{L}{2}\right)^2}$$
$$\frac{0.666}{\left(\frac{L}{2} - 20\right)^2} = \frac{4(0.0125)L}{L^2}$$
$$\left(\frac{L}{2} - 20\right)^2 = 13.32L$$
$$\frac{L^2}{4} - 20L + 400 - 13.32L = 0$$
$L^2 - 133.28L + 1600 = 0$
$L = \mathbf{120\ m.}$

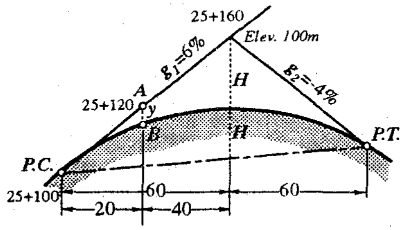
② *Stationing of highest point of curve:*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$
$$S_1 = \frac{0.06(120)}{0.06 - (-0.04)}$$
$S_1 = 72$ m. *from P.C.*

Sta. of highest point = (25 + 100) + 72
Sta. of highest point = **25 + 172**

③ *Elevation of station 25 + 120:*



$H = 0.0125\,L$
$H = 0.0125\,(120)$
$H = 1.5$
$\frac{y}{(20)^2} = \frac{1.5}{(60)^2}$
$y = \frac{1.5\,(20)^2}{(60)^2}$
$y = 0.167$ m.

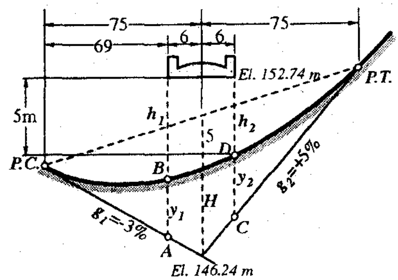
Elev. of A = 100 - 40 (0.06)
Elev. of A = 97.6 m.
Elev. of B = 97.6 - 0.167
Elev. of B = ***97.433 m.***

Problem 360:

A - 3% grade meets a +5% grade at a vertex (El. 146.24) directly under an over pass bridge whose underside is at elev. 152.74, and carries another road across the grades at right angles.

① What is the longest parabolic curve that can be used to connect the two grades and at the same time provide at least 5 m. of clearance under the bridge at its center line?
② If the underside of the bridge is level and is 12 m. wide, find the actual clearance at the left edge of the bridge.
③ If the underside of the bridge is level and is 12 m. wide, find the actual clearance at the right edge of the bridge.

Solution:
① *Length of curve:*



$H + 5 = 152.74 - 146.24$
$H + 5 = 6.5$
$H = 1.5$ m.
$H = \frac{L}{8}(g_2 - g_1)$
$1.5 = \frac{L}{8}(0.035 + 0.03)$
$L = \frac{1.5\,(8)}{0.08}$
$L = 150$ m.

② *Clearance at left edge:*
Elev. A = 146.24 + 0.03 (6)
Elev. A = 146.42

$\frac{y_1}{(69)^2} = \frac{1.5}{(75)^2}$
$y_1 = 1.27$

Elev. B = 146.42 + 1.27
Elev. B = 147.69
$h_1 = 152.74 - 147.69$
h_1 = ***5.05 m.***

③ *Clearance at right edge:*
Elev. C = 146.24 + 0.05 (6)
Elev. C = 146.54
$\frac{y_2}{(69)^2} = \frac{1.5}{(75)^2}$
$y_2 = 1.27$

Elev. D = 146.24 + 0.05 (6) + 1.27
Elev. D = 147.81
$h_2 = 152.74 - 147.81$
h_2 = ***4.93 m.***

PARABOLIC CURVES

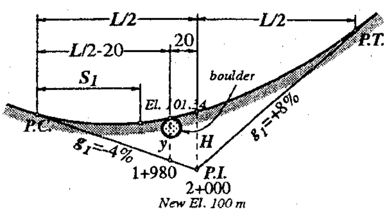
Problem 361:

A grade descending at the rate of - 4% intersects another grade ascending at the rate of +8% at station 2 + 000, elevation 100 m. A vertical curve is to connect the two such that the curve will clear a boulder located at station 1 + 980, elevation 101.34 m.

① Determine the necessary length of the curve.
② Determine the station of the location of a sewer to be laid out.
③ Compute the elevation of station where the sewer is to be place.

Solution:

① *Length of the curve:*



$Elev.\ 1 + 980 = 100 + 20\ (0.04)$
$Elev.\ 1 + 980 = 100.80$
$y = 101.34 - 100.80$
$y = 0.54$

$$\frac{y}{\left(\frac{L}{2} - 20\right)^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$

$$\frac{0.54}{H} = \frac{\left(\frac{L}{2} - 20\right)^2}{\left(\frac{L}{2}\right)^2}$$

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{L}{8}(0.04 + 0.08)$$

$$H = \frac{L}{8}(0.12)$$

$$\frac{8\ (0.54)}{L\ (0.12)} = \frac{\left(\frac{L}{2} - 20\right)^2 (4)}{L^2}$$

$$\frac{36}{L} = \frac{4\left(\frac{L^2}{4} - 40\frac{(L)}{2} + 400\right)}{L^2}$$

$36L = L^2 - 80L + 1600$
$L^2 - 116L + 1600 = 0$

$$L = \frac{116 + \sqrt{(116)^2 - 4\ (1)(1600)}}{2}$$

$$L = \frac{116 + \sqrt{7056}}{2}$$

$$L = \frac{116 + 84}{2}$$

$L = \mathbf{100\ m.}$

② *Station of the location of a sewer:*

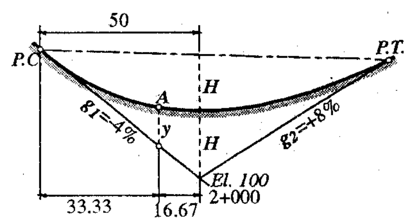
$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{-0.04\ (100)}{-0.04 - 0.08}$$

$S_1 = 33.33$

Sta. $= 2 + 000 - 16.67$
Sta. $= \mathbf{1 + 983.33}$

③ *Elevation of station where sewer is located:*



$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{100}{8}(-0.04 - 0.08)$$

$H = -1.5$ *(sag curve)*

$$\frac{y}{(33.33)^2} = \frac{1.5}{(50)^2}$$

$y = 0.667$

Elev. of A $= 100 + 16.67\ (0.04) + 0.667$
Elev. of A $= \mathbf{101.334\ m.}$

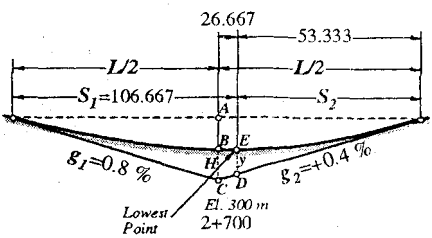
PARABOLIC CURVES

Problem 362:

On a railroad a - 0.8% grade meters a +0.4% grade station 2 + 700 whose elevation of 300 m. The maximum allowable change in grade per station having a length of 20 m. is 0.15.

① Compute the length of curve.
② Compute the stationing where a culvert be located.
③ At what elevation must the invert of the culvert be set if the pipe has a diameter of 0.9 m. and the backfill is 0.3 m. depth. Neglect thickness of pipe.

Solution:



① *Length of curve:*
one station = 20 m. long
r = rate of change per station

$$r = \frac{g_2 - g_1}{n} = 0.15$$

$$r = \frac{0.4 - (-0.8)}{n} = 0.15$$

$$n = \frac{1.2}{0.15}$$

$$n = 8 \text{ stations}$$

$$L = 8\,(20)$$

$$L = \mathbf{160\ m.}$$

② *Stationing of the lowest point on the curve:*

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

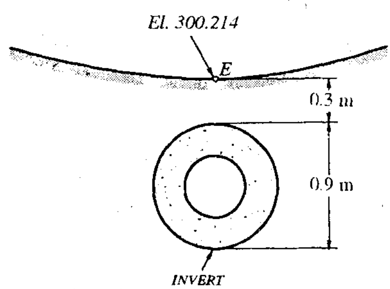
$$S_1 = \frac{-0.008\,(160)}{-0.008 - 0.004}$$

$$S_1 = 106.667$$

Sta. of P.C. = = (2 + 700) - 80
Sta. of P.C. = 2 + 620

Sta. of lowest point = (2 + 620) + 106.667
Sta. of lowest point = **2 + 726.667**

③ *Elevation of the invert of the culvert:*



$$H = \frac{L}{8}(g_2 - g_1)$$

$$H = \frac{160}{8}(0.004 + 0.008)$$

$$H = 0.24\text{ m.}$$

$$\frac{H}{(80)^2} = \frac{y}{(53.333)^2}$$

$$\frac{0.24}{(80)^2} = \frac{y}{(53.333)^2}$$

$$y = 0.107\text{ m.}$$

Elev. E = 300 + 0.004 (26.667) + 0.107
Elev. E = 300.214
Elev. of invert = 300.214 - 0.3 - 0.9
Elev. of invert = ***299.014 m.***

Problem 363:

The grade of a symmetrical parabolic curve from station 9 + 000 to the vertex V at sta. 9 + 100 minus 6% and from station 9 + 100 to 9 + 200 is minus 2%. The elevation at the vertex is 100.00 m. H is required to connect these grade lines with a vertical parabolic curve that shall pass 0.80 m. above the vertex.

PARABOLIC CURVES

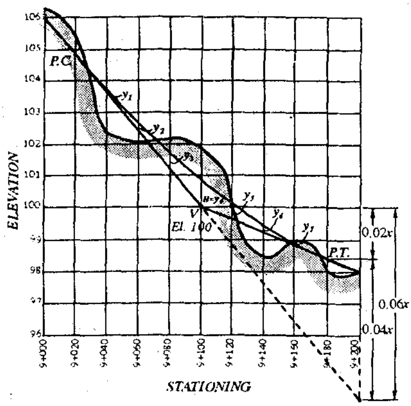
Station	Ground Elevation	Station	Ground Elevation
9 + 000	106.20	9 + 120	99.65
9 + 020	105.37	9 + 140	98.36
9 + 040	102.49	9 + 160	99.00
9 + 060	102.00	9 + 180	98.00
9 + 080	102.18	9 + 200	98.00
9 + 100	101.80		

① Compute the length of the vertical parabolic curve.

② Determine the amount of cut at station 9 + 080.

③ Determine the amount of fill at station 9 + 140.

Solution:



① *Length of the vertical parabolic curve:*

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{2x}{8}[-0.06 - (-0.02)]$$

$$0.80 = \frac{2x}{8}(-0.04)$$

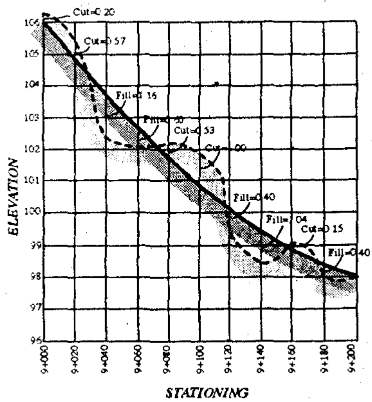
$$0.08x = 0.080\,(8)$$

$$x = 80 \text{ m.}$$

$$2x = 160 \text{ m.}$$

$$L = 160 \text{ m.}$$

② *Depth of cut at station 9 + 080:*



$$H = \frac{L}{8}(g_2 - g_1)$$

$$H = \frac{160}{8}(-0.02 + 0.06)$$

$$H = 0.80$$

$$\frac{y_3}{(60)^2} = \frac{0.80}{(80)^2}$$

$$y_3 = 0.45$$

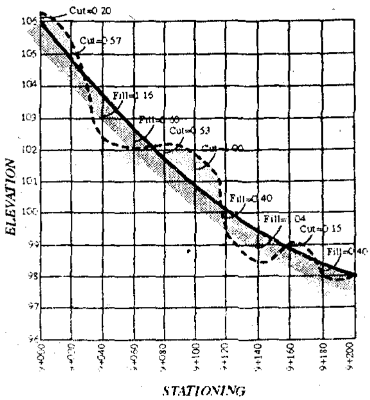
Elev. of 9 + 080 = 100 + 20 (0.06) + 0.45

Elev. of 9 + 080 = 101.65 m.

Depth of cut = 102.18 - 101.65

Depth of cut = ***0.53 m.***

③ *Depth of fill at sta. 9 + 140:*



$$\frac{y_4}{(40)^2}=\frac{H}{(80)^2}$$

$$\frac{y_4}{(40)^2}=\frac{0.80}{(80)^2}$$

$$y_4=0.20$$

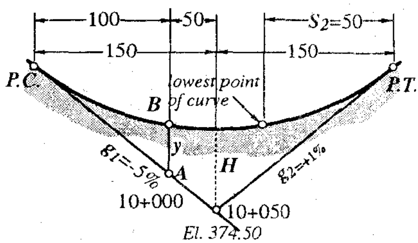
Elev. of sta. 9 + 140 = 100 - 40 (0.02) + 0.20
Elev. of sta. 9 + 140 = 99.40 m.
Depth of fill = 99.40 - 98.63
Depth of fill = ***1.04 m.***

Problem 364:

A grade of - 5% is followed by a grade of + 1%, the grades intersecting at station 10 + 050 of elevation 374.50 m. The change of grade is restricted to 0.4% in 20 m.

① Compute the length of vertical parabolic sag curve.
② How far is the lowest point of the curve from the P.T.
③ What is the elevation at station 10 + 000.

Solution:
① *Length of curve:*



$$r=\frac{g_2-g_1}{n}$$

$$0.4=\frac{1+5}{n}$$

$$n=15$$

$$L=20(15)$$

$$L=\mathbf{300\ m.}$$

② *Lowest point of curve:*

$$S_2=\frac{g_2\,L}{g_2-g_1}$$

$$S_2=\frac{0.01\,(300)}{0.01+0.05}$$

S_2 = ***50 m.*** *from P.T.*

③ *Elevation of station 10 + 000:*

$$H=\frac{L}{8}(g_2-g_1)$$

$$H=\frac{300}{8}(0.01+0.05)$$

$$H=2.25$$

$$\frac{y}{(100)^2}=\frac{2.25}{(150)^2}$$

$$y=1.0$$

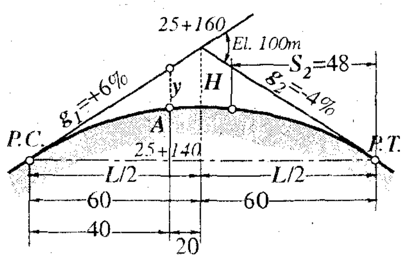
El. B = 374.50 + 0.05(50) + 1.0
El. B = ***378 m.***

Problem 365:

A vertical summit curve has its highest point of the curve at a distance 48 m. from the P.T. The back tangent has a grade of + 6% and a forward grade of - 4%. The curve passes thru point A on the curve at station 25 + 140. The elevation of the grade intersection is 100 m. at station 25 + 160.

① Compute the length of curve.
② Compute the stationing of P.T.
③ Compute the elevation of point A on the curve.

Solution:
① *Length of curve:*



PARABOLIC CURVES

$$S_2 = \frac{g_2 L}{g_2 - g_1}$$

$$48 = \frac{-0.04L}{-0.04 - 0.06}$$

$L =$ ***120 m.***

② *Stationing of P.T.*

Sta. of P.T. = (25 + 160) + (60)

Sta. of P.T. = ***25 + 220***

③ *Elev. of A on the curve:*

$$H = \frac{L}{8}(g_2 - g_1)$$

$$H = \frac{120(0.06 + 0.04)}{8}$$

$H = 1.5$

$$\frac{y}{(40)^2} = \frac{1.5}{(60)^2}$$

$y = 0.67$

Elev. of A = 100 - 20(0.06) - 0.67

Elev. of A = ***98.13 m.***

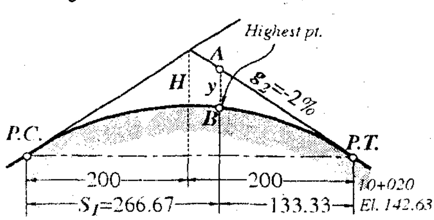
Problem 366:

A symmetrical vertical summit curve has tangents of +4% and - 2%. The allowable rate of change of grade is 0.3% per meter station. Stationing and elevation of P.T. is at 10 + 020 and 142.63 m. respectively.

① Compute the length of curve.
② Compute the distance of the highest point of curve from the P.C.
③ Compute the elevation of the highest point of curve.

Solution:

① *Length of curve:*



$$Rate\ of\ change = \frac{g_1 - g_2}{n}$$

$$0.3 = \frac{4 - (-2)}{n}$$

n = 20 stations

L = **20 (20)**

L = *400 m.*

② *Sta. of highest point of curve:*

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{0.04(400)}{0.04 - (-0.02)}$$

S_1 = ***266.67 m.*** *from P.C.*

③ *Elevation of highest point of curve:*

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{400}{8}(0.04 + 0.02)$$

$H = 3$

$$\frac{H}{(200)^2} = \frac{y}{(133.33)^2}$$

$$y = \frac{3(133.33)^2}{(200)^2} = 1.33$$

Elev. of highest point
= 142.63 + 133..33(0.02) - 1.33

Elev. of highest point = ***143.97***

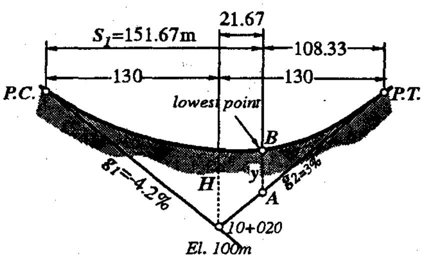
Problem 367:

A vertical symmetrical sag curve has a descending grade of - 4.2% and an ascending grade of +3% intersecting at station 10 + 020, whose elevation is 100 m. The two grade lines are connected by a 260 m. vertical parabolic sag curve.

① At what distance from the P.C. is the lowest point of the curve located?
② What is the vertical offset of the parabolic curve to the point of intersection of the tangent grades.
③ If a 1 m. diam. culvert is placed at the lowest point of the curve with the top of the culvert buried 0.60 m. below the subgrade, what will be the elevation of the invert of the culvert?

Solution:

① *Lowest point of the curve from P.C.*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{-0.042(260)}{-0.042 - 0.03}$$

S_1 = ***151.67*** *from P.C.*

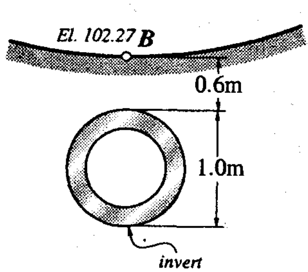
② *Nearest distance of curve from pt. of intersection of grades*

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{260}{8}(-0.042 - 0.03)$$

H = ***- 2.34 m.*** *(sag curve)*

③ *Elevation of invert.*



$$\frac{y}{(108.33)^2} = \frac{2.34}{(130)^2}$$

$y = 1.62$ m.

Elev. of B = 100 + 21.67(0.03) + 1.62

Elev. of B = 102.27 m.

Elev. of invert of culvert.

Elev. = 102.27 - 1.6

Elev. = ***100.67 m.***

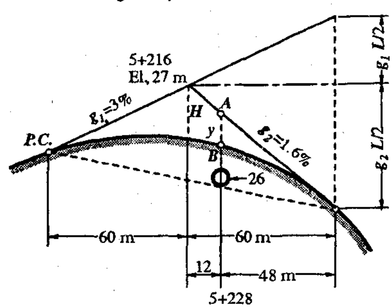
Problem 368:

A horizontally laid circular pipe culvert having an elevation of its top to be 26.0 m. crosses at right angles under a proposed 120 m. high way parabolic curve. The point of intersection of the grade lines is at station 5 + 216 and its elevation is 27.0 m. while the culvert is located at station 5 + 228. The backward tangent has a grade of 3% and the grade of the forward tangent is - 1.6%.

① Compute the stationing of the highest point of curve.
② Compute the elevation of the highest point of curve.
③ Under this conditions, what will be the depth of cover over the pipe?

Solution:

① *Sta. of highest point of curve:*



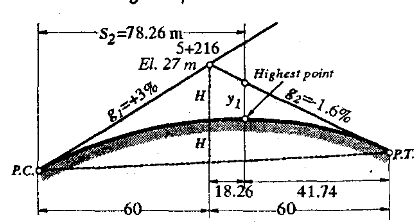
$$S = \frac{g_1 L}{g_1 - g_2}$$

$$S = \frac{0.03\,(120)}{0.03 + 0.016} = 78.26$$

Sta. of highest point = (5 + 216) + 18.26

Sta. of highest point = ***5 + 234.26***

② *Elev. of highest point of curve:*



PARABOLIC CURVES

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{120}{8}(0.03 + 0.016)$$

$$H = 0.69$$

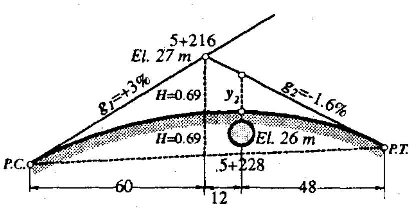
$$\frac{y_1}{(41.74)^2} = \frac{0.69}{(60)^2}$$

$$y_1 = 0.33 \text{ m.}$$

Elev. of highest point = 27 - 18.26 (0.016) - 0.33

Elev. of highest point = ***26.378 m.***

③ *Depth of cover over the pipe:*

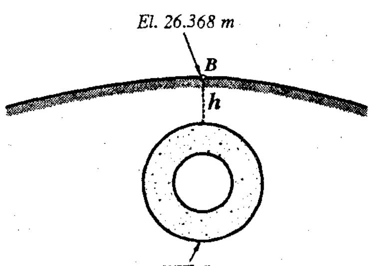


$$\frac{y_2}{(48)^2} = \frac{0.69}{(60)^2}$$

$$y_2 = 0.44$$

Elev. of B = 27 - 12 (0.016) - 0.44

Elev. of B = 26.368 m.



Depth of cover:

$h = 26.368 - 26$

$\boldsymbol{h = 0.368}$ ***m.***

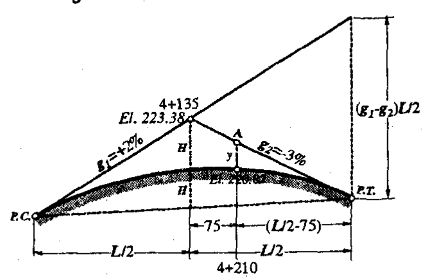
Problem 369:

A vertical highway curve is to pass through a railway at grade. The crossing must be at station 4 + 210 and at elevation 220.82 m. The initial grade of the highway is +2% and meets a - 3% grade at station 4 + 135 at an elevation of 223.38 m. The rate of change must not exceed 2%.

① What length of curve will meet these conditions?
② What is stationing of the highest point of the curve?
③ What is the elevation of the highest point of the curve?

Solution:

① *Length of curve:*



Elev. of A = 223.38 - 0.03 (75)

Elev. of A = 221.13 m.

$y = 221.13 - 220.82$

$y = 0.31$

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{L}{8}(0.02 + 0.03)$$

$$H = 0.00625\,L$$

$$\frac{H}{\left(\frac{L}{8}\right)^2} = \frac{y}{\left(\frac{L}{2} - 75\right)^2}$$

$$\frac{(4)\,0.00625\,L}{L^2} = \frac{0.31}{\left(\frac{L}{2} - 75\right)^2}$$

$$\frac{0.025}{L} = \frac{0.31}{\left(\frac{L}{2} - 75\right)^2}$$

$$\left(\frac{L}{2}-75\right)^2 = 12.4L$$
$$\frac{L^2}{4} - 75L + 5625 = 12.4L$$
$$L^2 - 349.6L + 22500 = 0$$
$$L = 264.55$$

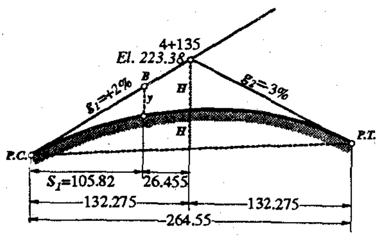
$$r = \frac{g_1 - g_2}{L}$$
$$r = \frac{2-(-3)}{264.55}$$
$$r = 0.0189$$
$$r = \mathbf{1.89\%} < 2\% \text{ ok}$$

② *Stationing of highest point of curve:*

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$
$$S_1 = \frac{0.02\,(264.55)}{0.02-(-0.03)}$$
$$S_1 = 105.82 \text{ from P.C.}$$

$$\text{Sta. of highest point} = (4+135) - \frac{264.55}{2} + 105.82$$
$$\text{Sta. of highest point} = \mathbf{4+108.545}$$

③ *Elevation of highest point of curve:*



$$H = 0.00625\,L$$
$$H = 0.00625\,(264.55)$$
$$H = 1.65$$
$$\frac{H}{(132.275)^2} = \frac{y}{(105.82)^2}$$
$$\frac{1.65}{(132.275)^2} = \frac{y}{(105.82)^2}$$
$$y = 1.056 \text{ m.}$$

Elev. of B = 223.38 - 76.455 (0.02)
Elev. of B = 222.85 m.

Elev. of highest point of curve at C
= 222.85 - 1.056
= ***221.794 m.***

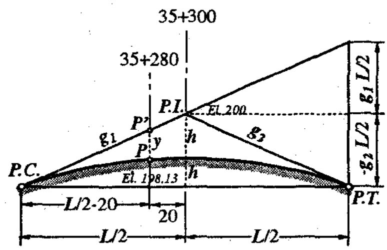
Problem 370:

A symmetrical parabolic summit curve connects two grades of +6% and - 4%. It is to pass through a point "P" the stationing of which is 35 + 280 and the elevation is 193.13 m. If the elevation of the grade intersection is 200 m. with stationing 35 + 300.

① Determine the length of the curve.
② Determine the stationing and elevation of P.C.
③ Determine the stationing and elevation of P.T.

Solution:

① *Length of curve:*



$$p' = 200 - 20\,(0.06)$$
$$p' = 198.80$$
$$y = 198.80 - 198.13$$
$$y = 0.67$$
$$h = \frac{1}{4}(g_1 - g_2)\frac{L}{2}$$
$$h = \frac{L}{8}(g_1 - g_2)$$
$$h = \frac{L}{8}(0.06 + 0.04)$$
$$h = \frac{0.10\,L}{8}$$
$$h = \frac{0.05\,L}{4}$$
$$h = 0.0125\,L$$

$$\frac{\left(\frac{L}{2}-20\right)^2}{y} = \frac{\left(\frac{L}{2}\right)^2}{h}$$

PARABOLIC CURVES

$$\frac{\left(\frac{L}{2}-20\right)^2}{0.67}=\frac{\left(\frac{L}{2}\right)^2}{0.0125\,L}$$

$$\left(\frac{L}{2}-20\right)^2=13.4\,L$$

$$\frac{L^2}{4}-20L+400=13.4L$$

$$L^2-113.6L+1600=0$$

$$L=\frac{240}{2}$$

$$L=\mathbf{120\ m.}$$

② *Stationing and elevation of P.C.*
Sta. of P.C. = (35 + 280) - 60
Sta. of P.C. = 35 + 220

Elev. of P.C. = 200 - 60 (0.06)
Elev. of P.C. = ***196.40***

③ *Stationing and elevation of P.T.*
Sta. of P.T. = (35 + 300) + 60
Sta. of P.T. = ***35 + 360***

Elev. of P.T. = 200 - 60 (0.04)
Elev. of P.T. = ***197.60***

Problem 371:

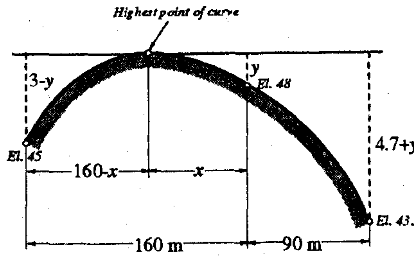
A vertical parabolic curve is designed to pass thru three points on the profile of an existing unimproved road with stationing and corresponding elevations as follows.

Points	Stations	Elevations
P_1	5 + 432	45.00 m.
P_2	5 + 592	48.00 m.
P_3	5 + 682	43.30 m.

① Determine the stationing of the highest point of the curve.
② Determine the elevation of the highest point of the curve.
③ Determine the elevation of the P.T. if the grade of the back tangent is +5% with P.C. at station 5 + 432 and elevation 45.00 m. The length of the parabolic curve connected by the tangents is 200 m.

Solution:

① *Stationing of highest point of curve:*



Using squared property of parabola.

❶ $\frac{y}{x^2}=\frac{(4.7+y)}{(90+x)^2}$

❷ $\frac{y}{x^2}=\frac{3+y}{(160-x)^2}$

❶ $4.7x^2+x^2y=y\,(8100+180x+x^2)$
$4.7x^2-180xy-8100y=0$ *(multiply by 3)*
$14.1x^2-540xy-24300y=0$

❷ $3x^2+yx^2=y\,(25600-320x+x^2)$
$3x^2+320xy-25600y=0$ *(multiply by 4.7)*
$14.1x^2+1504xy-120320y=0$

❶ & ❷

$$\begin{array}{r} 14.1x^2-\ \ 540xy-\ \ 24300y=0 \\ 14.1x^2+1504xy-120320y=0 \\ \hline -2044xy+\ 96020y=0 \end{array}$$

$$x=\frac{96020}{2044}$$

$$x=46.98\text{ m.}$$

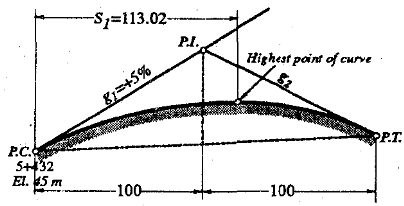
Sta. of highest point = (5 + 592) - (46.98)
Sta. of highest point = ***5 + 545.02***

② *Elevation of highest point of curve:*

❶ $4.7x^2-180xy-8100y=0$
$4.7\,(46.98)^2-180\,(46.98)\,y-8100y=0$
$y=0.63$ m.

Elev. of highest point of curve = 48 + 0.63
Elev. of highest point of curve = ***48.63 m.***

③ *Elevation of P.T.*



$S_1 = 160 - 46.98$
$S_1 = 113.02$

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$113.02 = \frac{0.05\,(200)}{0.05 - g_2}$$

$0.05 - g_2 = 0.088$
$g_2 = -\,0.038$
$g_2 = -\,3.8\%$

Elev. P.I. $= 45 + 100\,(0.05)$
Elev. P.I. $= 50$ m.
Elev. P.T. $= 50 - 100\,(0.028)$
Elev. P.T. = ***46.2 m.***

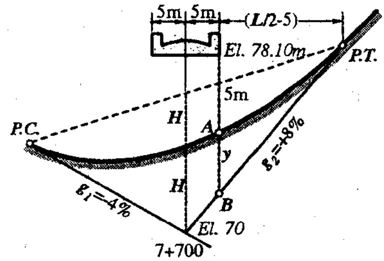
Problem 372:

An uderpass crossing a reinforced concrete bridge along the Shaw Blvd. has a downward grade of - 4% meeting an upward grade of +8% at the vertex V at elevation 70 m. and stationing of 7 + 700, exactly underneath the center line of the bridge having a width of 10 m. If the required minimum clearance under the bridge is 5 m. and the elevation of the bottom of the bridge is 78.10 m.

① Determine the length of the vertical parabolic curve that shall connect the two tangents.
② Determine the stationing of the point where a catch basin will be placed.
③ Determine the elevation of the point where a catch basin will be placed.

Solution:

① *Length of vertical parabolic curve:*



Elev. of A $= 78.10 - 5$
Elev. of A $= 73.10$ m.
Elev. of B $= 70 + 0.08\,(5)$
Elev. of B $= 70.40$ m.
$y = 73.10 - 70.40$
$y = 2.7$

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{L}{8}(-\,0.04 - 0.08)$$

$H = -\,0.015\,L$ *(sag curve)*

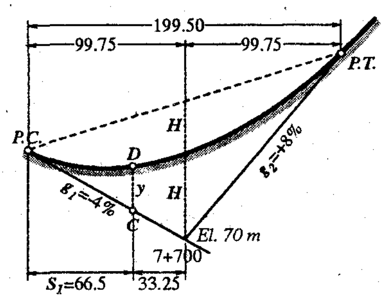
$$\frac{y}{\left(\frac{L}{2} - 5\right)^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$

$$\left(\frac{L}{2} - 5\right)^2 = 45L$$

$$\frac{L^2}{4} - 5L + 25 = 45L$$

$L^2 - 200L + 100 = 0$
$L =$ ***199.50 m.***

② *Stationing of catch basin:*



PARABOLIC CURVES

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$
$$S_1 = \frac{-0.04\,(199.5)}{-0.04 - 0.08}$$
$S_1 = 66.5$ m. from P.C.

Sta. of the point where catch basin is placed:
$= (7 + 700) - 33.25$
$= \mathbf{7 + 666.75}$.

③ *Elevation of the point where catch basin is placed:*
$$\frac{y}{(66.5)^2} = \frac{H}{(99.75)^2}$$
$H = 0.015\,(199.5)$
$H = 2.99$
$$\frac{y}{(66.5)^2} = \frac{2.99}{(99.75)^2}$$
$y = 1.33$ m.

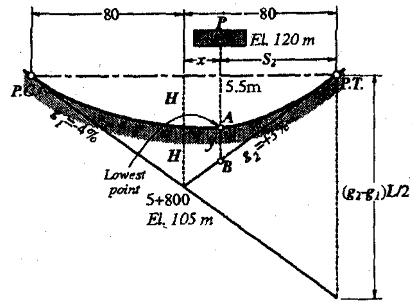
Elev. of the point where catch basin is placed:
Elev. D = Elev. $C + y$
Elev. $C = 70 + 33.25\,(0.04)$
Elev. $C = 71.33$ m.
Elev. of $D = 71.33 + 1.33$
Elev. of $D = \mathbf{72.66\ m.}$

Problem 373:

Point "P" is the location of the center line of an existing highway. An underpass is to be designed perpendicular to the existing highway with a vertical parabolic curve such that its lowest point is directly below "P" with a vertical clearance of 5.5 m. Stationing of the P.I. is 5 + 800 and has an elevation of 105 m. The slope of the tangent passing thru the P.C. is - 4% and that of the P.T. is +3%.

① Determine the length of the vertical parabolic curve.
② Determine the stationing of point "P" being on the right side of the curve if it has an elevation of 120 m.
③ Determine the elevation of the P.T. of the curve.

Solution:
① *Length of vertical parabolic curve:*



$$\frac{2H}{\frac{L}{2}} = \frac{\frac{L}{2}(g_2 - g_1)}{L}$$
$$H = \frac{L}{8}(g_2 - g_1)$$
$$H = \frac{L}{8}[0.03 - (-0.04)]$$
$H = 0.00875\,L$

$$S_2 = \frac{g_2 L}{g_2 - g_1}$$
$$S_2 = \frac{0.03\,(L)}{0.03 - (-0.04)}$$
$S_2 = 0.429\,L$
$$\frac{y}{(S_2)^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$
$$\frac{y}{(0.429\,L)^2} = \frac{0.00875\,L}{\left(\frac{L}{2}\right)^2}$$
$y = 0.00644\,L$

Elev. of $A = 120 - 5.5$
Elev. of $A = 114.5$ m.
Elev. of $B = 105 + x\,(0.03)$
$$x = \frac{L}{2} - S_2$$
$x = 0.5\,L - 0.429\,L$
$x = 0.071\,L$

Elev. of $B = 105 + 0.071\,L\,(0.03)$
Elev. of $B = 105 + 0.00213\,L$
Elev. of A = Elev. of $B + y$
$114.5 = 105 + 0.00213\,L + 0.00644\,L$
$L = \mathbf{1108.52\ m.}$

② *Stationing of point "P":*
$x = 0.071\,L$
$x = 0.071\,(1108.52)$
$x = 78.70$ m.

Sta. of point "P" = (5 + 800) + 78.70
Sta. of point "P" = ***(5 + 878.70)***

③ *Elevation of P.T.*

$$\text{Elev. of P.T.} = 105 + \frac{1108.52}{2}(0.03)$$

Elev. of P.T. = ***121.63 m.***

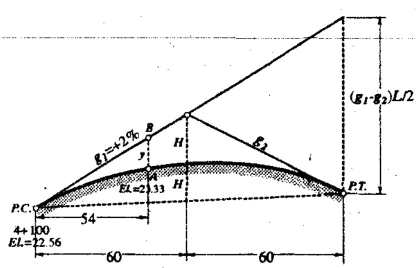
Problem 374:

A symmetrical parabolic curve passes through point A whose elevation is 23.23 m. at a distance of 54 m. from the P.C. The elevation of the P.C. at station 4 + 100 is 22.56 m. The grade of the back tangent is +2% and the length of curve is 120 m.

① Compute the grade of the forward tangent.
② Compute the stationing of the highest point of the curve.
③ Compute the elevation of the highest point of the curve.

Solution:

① *Grade of forward tangent:*



Elev. of B = 22.56 + 54 (0.02)
Elev. of B = 23.64 m.
$y = 23.64 - 23.23$
$y = 0.41$ m.

$$\frac{0.41}{(54)^2} = \frac{H}{(60)^2}$$

$H = 0.506$ m.

$$\frac{2H}{\frac{L}{2}} = \frac{(g_1 - g_2)\frac{L}{2}}{L}$$

$$H = \frac{L}{8}(g_1 - g_2)$$

$$0.506 = \frac{120}{8}(0.02 - g_2)$$

$0.02 - g_2 = 0.0337$
$g_2 = -0.014$
$g_2 = \boldsymbol{-1.4\%}$

② *Stationing of highest point of curve:*

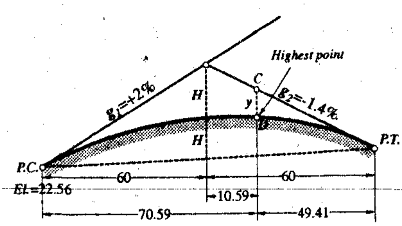
$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{0.02\,(120)}{0.02 - (-0.014)}$$

$S_1 = 70.59$ m. from P.C.

Sta. of highest point = (4 + 100) + 70.59
Sta. of highest point = ***4 + 170.59***

③ *Elevation of highest point of curve:*



Elev. of C = 22.56 + 60 (0.02) - 10.59 (0.014)
Elev. of C = 23.61 m.

$$\frac{H}{(60)^2} = \frac{y}{(49.41)^2}$$

$$\frac{0.506}{(60)^2} = \frac{y}{(49.41)^2}$$

$y = 0.343$ m.

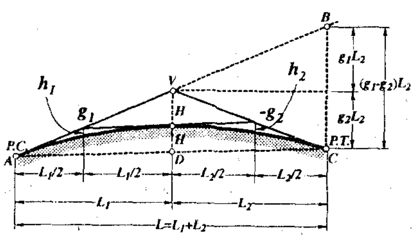
Elev. of D = 23.61 - 0.343
Elev. of D = ***23.267 m.***

PARABOLIC CURVES

Unsymmetrical Parabolic Curves

A vertical highway curve is at times designed to include a particular elevation at a certain station where the grades of the forward and backward tangents have already been established. It is therefore necessary to use a curve with unequal tangents or a compound curve which is usually called "unsymmetrical" or asymmetrical parabolic curve where one parabola extends from the P.C. to a point directly below the vertex and a second parabola which extends from this point to the P.I. In order to make the entire curve smooth and continuous, the two parabolas are so constructed so that they will have a common tangent at the point where they joined, that is at a point directly below the vertex.

Let us consider the figure shown below:



L_1 = length of the parabolic curve on the left side of the vertex.

L_2 = length of the parabolic curve on the right side of the vertex.

g_1 = slope of backward tangent

g_2 = slope of forward tangent

Considering triangles AVD and ABC.

$$\frac{2H}{L_1} = \frac{(g_1 - g_2)\,L_2}{L_1 + L_2}$$

$$H = \frac{(g_1 - g_2)\,L_2}{2(L_1 + L_2)}$$

$$h_1 = \frac{1}{4}H$$

$$h_2 = \frac{1}{4}H$$

Solving for L_1:

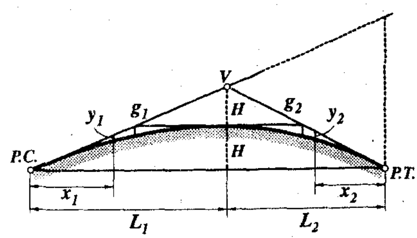
$$\frac{2H}{L_1} = \frac{(g_1 - g_2)\,L_2}{L_1 + L_2}$$

$$2HL_1 + 2HL_2 = L_1 L_2 (g_1 - g_2)$$

$$L_1\left[L_2 (g_1 - g_2) - 2H\right] = 2HL_2$$

$$L_1 = \frac{2HL_2}{L_2 (g_1 - g_2) - 2H}$$

Applying the squared property of parabola, in solving for the vertical offsets of the parabola.



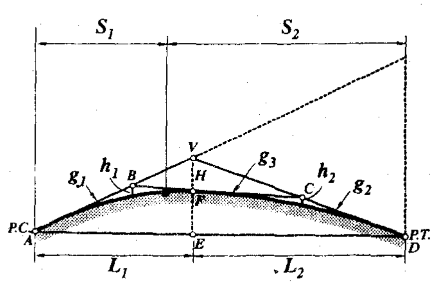
$$\frac{y_1}{(x_1)^2} = \frac{H}{(L_1)^2}$$

$$\frac{y_2}{(x_2)^2} = \frac{H}{(L_2)^2}$$

Location of the highest or lowest point of the curve.

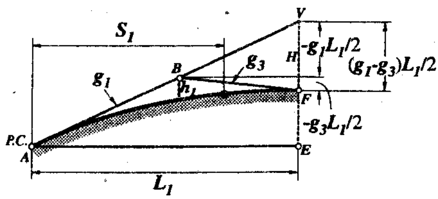
a) *From the P.C. when* $\frac{L_1 g_1}{2} < H$

$$S_1 = \frac{g_1 L_1^2}{2H}$$



PARABOLIC CURVES

Let g_3, be the slope of the common tangent of the parabolic curve.



Considering the symmetrical parabola AVF, the location of the highest point of the sag is obtained from the relation.

① $S_1 = \dfrac{g_1 L_1}{g_1 - g_3}$

Substitutintg these values and solving for g_3, we have:

② $H = \dfrac{L_1}{2}(g_1 - g_3)$

$2H = L_1 g_1 - L_1 g_3$

$g_3 L_1 = L_1 g_1 - 2H$

$g_3 = \dfrac{L_1 g_1 - 2H}{L_1}$

③ $g_3 = g_1 - \dfrac{2H}{L_1}$

From equation ① substitute equation ③.

$$S_1 = \frac{g_1 L_1}{g_1 - g_3}$$

$$S_1 = \frac{g_1 L_1}{g_1 - \left(g_1 - \frac{2H}{L_1}\right)}$$

$$S_1 = \frac{g_1 L_1}{g_1 - \frac{(g_1 L_1 - 2H)}{L_1}}$$

$$S_1 = \frac{g_1 L_1^2}{\frac{g_1 L_1 - g_1 L_1 + 2H}{L_1}}$$

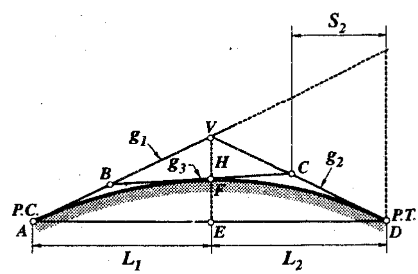
$$\boxed{S_1 = \frac{g_1 L_1^2}{2H}}$$

S_1 = location of the highest or lowest point of the curve from the P.C.

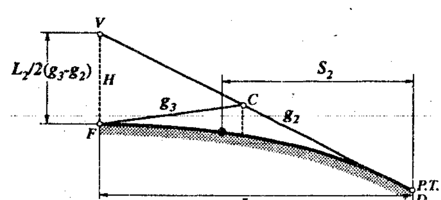
Likewise, the location of the lowest or highest point of the curve could be computed from the P.T. of the curve, this holds true when $\dfrac{L_1 g_1}{2}$ is greater than H. Considering the figure shown, let us assume that the highest or lowest point of the curve is found on the right side of the parabola.

b) *From the P.T. when* $\dfrac{L_1 g_1}{2} > H$

$$\boxed{S_2 = \frac{g_2 L_2^2}{2H}}$$



Considering the right side of the parabola, VFCD.



① $S_2 = \dfrac{g_2 L_2}{g_3 - g_2}$

② $H = \dfrac{L_2}{2}(g_3 - g_2)$

Solving for g_3 in equation ②.

$2H = L_2 g_3 - L_2 g_2$

③ $g_3 = \dfrac{2H + L_2 g_2}{L_2}$

PARABOLIC CURVES

Substitute equation ③ in ①.

$$S_2 = \frac{g_2 L_2}{g_3 - g_2}$$

$$S_2 = \frac{g_2 L_2}{\frac{2H + L_2 g_2}{L_2} - g_2}$$

$$S_2 = \frac{g_2 (L_2)^2}{2H + L_2 g_2 - (L_2)^2 g_2}$$

$$S_2 = \frac{g_2 L_2}{2H} \text{ from the P.T.}$$

When $\frac{L_1 g_1}{2} > H$, the highest or lowest point of the curve is located on the right side of the curve.

① When $\frac{L_1 g_1}{2} > H$

Use: $S_2 = \frac{g_2 L_2^2}{2H}$ *(from the P.T.)*

② When $\frac{L_1 g_1}{2} < H$

Use: $S_1 = \frac{g_1 L_1^2}{2H}$ *(from the P.C.)*

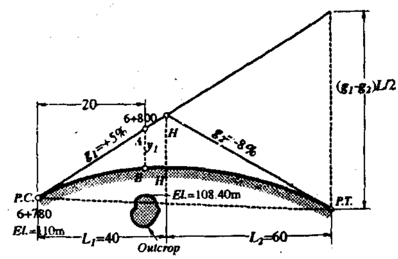
Problem 375:

An unsymmetrical parabolic curve has a forward tangent of - 8% and a back tangent of +5%. The length of curve on the left side of the curve is 40 m. long while that of the right side is 60 m. long. The P.C. is at station 6 + 780 and has an elevation of 110 m. An outcrop is found at station 6 + 800 has an elevation of 108.40 m.

① Compute the height of fill needed to cover the outcrop.
② Compute the elevation of curve at station 6 + 820.
③ Compute the elevation of the highest point of the curve.

Solution:

① *Height of fill needed to cover the outcrop:*



$$\frac{2H}{L_1} = \frac{L_2 (g_1 - g_2)}{L_1 + L_2}$$

$$H = \frac{L_1 L_2 (g_1 - g_2)}{2 (L_1 + L_2)}$$

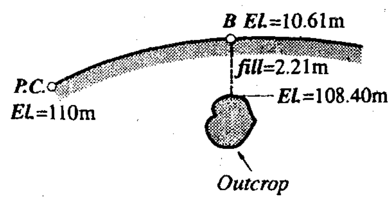
$$H = \frac{40 (60) [0.05 - (-0.08)]}{2 (40 + 60)}$$

$$H = 1.56$$

$$\frac{H}{(40)^2} = \frac{y_1}{(20)^2}$$

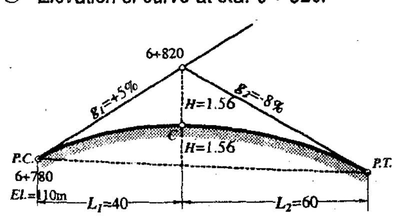
$$y_1 = \frac{1.56 (20)^2}{(40)^2}$$

$$y_1 = 0.39$$



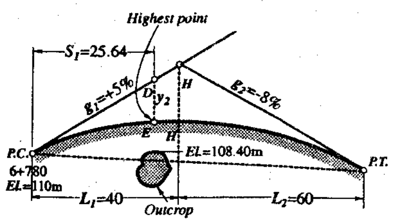
Elev. of B = 110 + 0.05 (20) - 0.39
Elev. of B = 110.61 m.
Depth of fill at the outcrop = 110.61 - 108.40
Depth of fill = **2.21 m.**

② *Elevation of curve at sta. 6 + 820:*



Elev. of C = 110 + 40 (0.05) - 1.56
Elev. of C = **110.44 m.**

③ *Elevation of highest point of curve:*



$$\frac{L_1 g_1}{2} = \frac{40\,(0.05)}{2} = 1.0 < H$$

$$S_1 = \frac{g_1 L_1^2}{2H} \text{ from P.C.}$$

$$S_1 = \frac{0.05\,(40)^2}{2\,(1.56)}$$

$$S_1 = 25.64 \text{ m.}$$

$$\frac{H}{(40)^2} = \frac{y_2}{(25.64)^2}$$

$$y_2 = \frac{1.56\,(25.64)^2}{(40)^2}$$

$$y_2 = 0.64$$

Elev. of E = 110 + 0.05 (25.64) - 0.64
Elev. of E = ***110.642 m.***

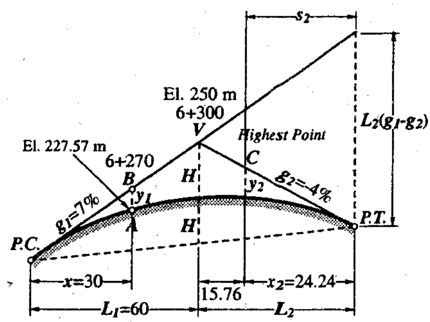
Problem 376:

A forward tangent having a slope of - 4% intersects the back tangent having a slope +7% at point V at stations 6 + 300 having an elevation of 230 m. It is required to connect the two tangents with an unsymmetrical parabolic curve that shall pass through point A on the curve having an elevation of 227.57 m. at station 6 + 270. The length of curve is 60 m. on the side of the back tangent.

① Determine the length of the curve on the side of the forward tangent.
② Determine the stationing of the highest point of the curve.
③ Determine the elevation of the highest point of the curve.

Solution:

① *Length of curve:*



Elev. B = 230 - 30 (0.07)
Elev. B = 227.90 m.

$$y_1 = 227.90 - 227.57$$

$$y_1 = 0.33$$

$$\frac{y_1}{x^2} = \frac{H}{(L_1)^2}$$

$$\frac{0.33}{(30)^2} = \frac{H}{(60)^2}$$

$$H = \frac{0.33\,(60)^2}{(30)^2}$$

$$H = 1.32 \text{ m.}$$

$$\frac{2H}{L_1} = \frac{L_2\,(g_1 - g_2)}{L_1 + L_2}$$

$$\frac{2\,(1.32)}{60} = \frac{L_2\,(0.07 + 0.04)}{60 + L_2}$$

$$2.64\,(60 + L_2) = 60\,(0.11)\,L_2$$

$$158.4 + 2.64\,L_2 = 6.6\,L_2$$

$$3.96\,L_2 = 158.4$$

$$L_2 = \mathbf{40\ m.}$$

② *Stationing of the highest point of the curve:*

$$\frac{L_1 g_1}{2} = \frac{60\,(0.07)}{2} = 2.1 > H$$

Therefore the highest point of hte curve is on the right side.

$$S_2 = \frac{g_2 L_2^2}{2H}$$

$$S_2 = \frac{0.04\,(40)^2}{2\,(1.32)}$$

S_2 = 24.24 m. *from P.T.*

PARABOLIC CURVES

Sta. of P.T. = (6 + 300) + 40
Sta. of P.T. = 6 + 340
Sta. of highest point = (6 + 340) - 24.24
Sta. of highest point = **6 + 315.76**

③ *Elevation of the highest point of the curve:*

$$\frac{y_2}{(x_2)^2} = \frac{H}{(L_2)^2}$$

$$\frac{y_2}{(24.24)^2} = \frac{1.32}{(40)^2}$$

$$y_2 = 0.48$$

Elev. of C = 230 - 15.76 (0.04)
Elev. of C = 229.37 m.
Elev. of highest point = 229.37 - 0.48
Elev. of highest point = **228.89 m.**

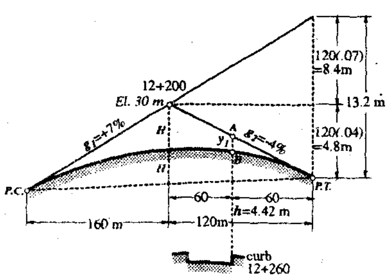
Problem 377:

V_1 station 12 + 200 and elevation 30 m. It is required to connect the two tangents with an unsymmetrical vertical parabolic curve 160 m. on the side of the back tangent and 120 m. on the side of the forward tangent. The curve must provide a vertical clearance of at least 4.42 m. above the right curb of an underpass, the stationing of which is 12 + 260.

① Determine the elevation of the curb.
② If the curb elevation is 22.6385 m., compute the length of curve on the side of the forward tangent in order to fulfill the requirement.
③ Compute the stationing of the highest point of the curve for the second condition.

Solution:

① *Elevation of curb:*



$$\frac{2H}{160} = \frac{13.20}{280}$$

$$H = 3.77 \text{ m.}$$

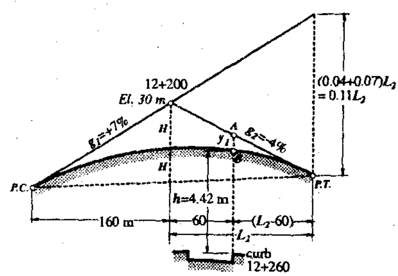
$$\frac{y_1}{(60)^2} = \frac{H}{(120)^2}$$

$$y_1 = \frac{3.77\,(60)^2}{(120)^2}$$

$$y_1 = 0.94$$

Elev. of B = 30 - 60 (0.04) - 0.94
Elev. of B = 26.66 m.
Elev. of curb = 26.66 - 4.42
Elev. of curb = **22.24 m.**

② *Length of curve on the forward tangent:*



Elev. of A = 30 - 0.04 (60)
Elev. of A = 37.6 m.
Elev. of B = 22.6385 + 4.42
Elev. of B = 27.0585

$$y_2 = 27.60 - 27.0585$$

$$y_2 = 0.5415$$

$$\frac{y_2}{(L_2 - 60)^2} = \frac{H}{(L_2)^2}$$

$$\frac{2H}{160} = \frac{0.11\,L_2}{(160 + L_2)}$$

$$H = \frac{8.8\,L_2}{160 + L_2}$$

$$\frac{0.5415}{(L_2 - 60)^2} = \frac{8.8\,L_2}{(160 + L_2)\,(L_2)^2}$$

$$0.5415\,(160 + L_2)\,L_2 = 8.8\,(L_2 - 60)^2$$

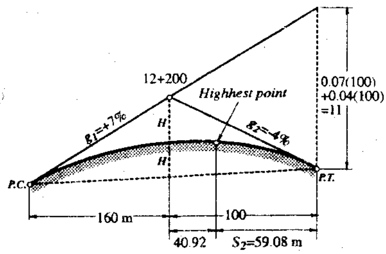
$$86.64L_2 + 0.5415L_2^2 = 8.8\,(L_2^2 - 120\,L_2 + 3600)$$

$$8.2585L_2^2 - 1142.64L_2 + 31680 = 0$$

$$L_2^2 - 138.359L_2 + 3836.048 = 0$$

$$L_2 = \mathbf{100\ m.}$$

③ *Stationing of highest point of curve based on second condition:*



$$\frac{2H}{160}=\frac{11}{260}$$

$$H=3.385$$

$$\frac{L_1\,g_1}{2}=\frac{160\,(0.07)}{2}$$

$$\frac{L_1\,g_1}{2}=5.6>H$$

The highest point of curve is on the right side.

$$S_2=\frac{g_2\,L_2^{\,2}}{2H}$$

$$S_2=\frac{0.04\,(100)^2}{2\,(3.385)}$$

$$S_2=59.08\text{ m.}$$

Sta. of highest point of curve

= (12 + 200) + (40.92)

= ***12 + 240.92***

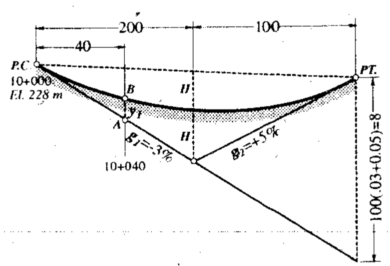
Problem 378:

A - 3% grade meets a +5% grade near an underpass. In order to maintain the minimum clearance allowed under the bridge and at the same time introduce a vertical transition curve in the grade line, it is necessary to use a curve that lies 200 m. on one side of the vertex of the straight grade and 100 m. on the other. The station of the beginning of the curve (200 m. side) is 10 + 000 and its elevation is 228 m.

① Determine the elevation at station 10 + 040.
② If the uphill edge of the under side of the bridge is at station 10 + 220 and at elevation 229.206 m., what is the vertical clearance under the bridge at this point?
③ Determine the stationing of the lowest point of the curve.

Solution:

① *Elevation at station 10 + 040:*



$$\frac{8}{300}=\frac{2H}{200}$$

$$H=2.67$$

$$\frac{y_1}{(40)^2}=\frac{2.67}{(200)^2}$$

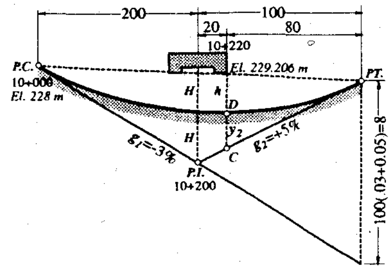
$$y_1=0.107$$

Elev. of sta. 10 + 040 = 228 - 0.03 (40) + 0.107

Elev. of sta. 10 + 040 = ***226.907 m.***

PARABOLIC CURVES

② *Clearance under the bridge:*



$$\frac{y_2}{(80)^2}=\frac{H}{(100)^2}$$

$$\frac{y_2}{(80)^2}=\frac{2.67}{(100)^2}$$

$$y_2 = 1.71$$

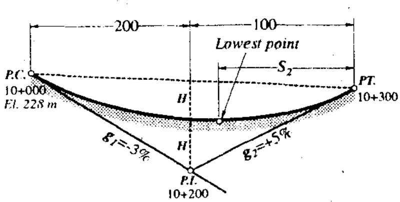
Elev. of P.I. = 228 - 200 (0.03)
Elev. of P.I. = 222 m.
Elev. of C = 222 + 0.05 (20)
Elev. of C = 223 m.
Elev. of D = 223 + 1.71
Elev. of D = 224.71 m.

Vertical clearance h = 229.206 - 224.71
Vertical clearance h = ***4.496 m.***

③ *Stationing of lowest point of curve:*

$$\frac{L_1\,g_1}{2}=\frac{200\,(0.03)}{2}=3\text{ m.} > 2.67$$

The lowest point of curve is on the right side.



$$S_2=\frac{g_2\,L_2^{\,2}}{2H}$$

$$S_2=\frac{0.05\,(100)^2}{2\,(2.67)}$$

$$S_2 = 93.63\text{ m.}$$

Sta. of lowest point of curve = (10 + 300) - 93.63
Sta. of lowest point curve = ***10 + 206.37***

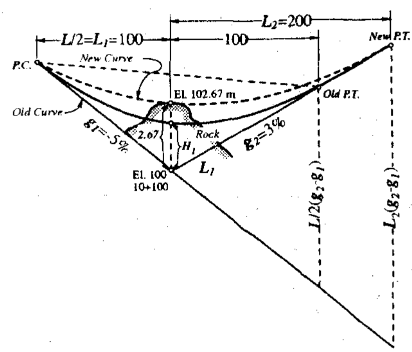
Problem 379:

In a certain road construction undertaken by the Bureau of Public Highway it was decided to connect a forward tangent of 3% and a back tangent of - 5% by a 200 m. symmetrical parabolic curve. It was discovered that the grade intersection at station 10 + 100 whose elevation is 100 m. falls on a rocky section with an outcrop of 2.67 m. directly above the grade intersection. To avoid rock excavation, the project engineer decided to adjust the vertical parabolic curve in such a way that the curve will just clear the rock without altering the position of P.C. and the grade of the tangents.

① Determine the total length of the new parabolic curve.
② Determine the stationing and elevation of the new P.T.
③ Determine the elevation of the lowest point of the curve.

Solution:

① *Total length of new parabolic curve:*



$$\frac{2H_1}{\frac{L}{2}}=\frac{\frac{L}{2}(g_2-g_1)}{L}$$

$$H_1=\frac{L\,(g_2-g_1)}{8}$$

$H_1 = 2$ m. < 2.67 *(it will hit the boulder)*

Construct an unsymmetrical parabolic curve

$$\frac{2H_2}{L_1}=\frac{L_2(g_2-g_1)}{L_1+L_2}$$

$$\frac{2(2.67)}{100}=\frac{L_2(0.03+0.05)}{5.34}$$

$$1.50L_2=L_2+100$$

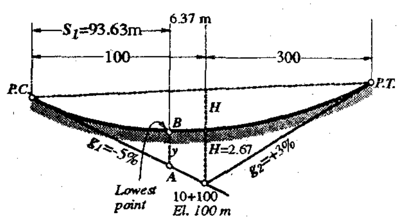
$$L_2=200\text{ m.}$$

Total length of curve = 100 + 200
Total length of curve = ***300 m.***

② *Stationing and elevation of the new P.T.*
Sta. of new P.T. = ***10 + 300***

Elev. of new P.T. = 100 + 200 (0.03)
Elev. of new P.T. = ***106 m.***

③ *Elevation of lowest point of curve:*



$$\frac{L_1 g_1}{2}=\frac{100(0.05)}{2}=2.5<H$$

The lowest point of curve is on the left side.

$$S_1=\frac{g_1 L_1^2}{2H}$$

$$S_1=\frac{0.05(100)^2}{2(2.67)}$$

$$S_1=93.63\text{ m.}$$

$$\frac{y}{(93.63)^2}=\frac{H}{(100)^2}$$

$$\frac{y}{(93.63)^2}=\frac{2.67}{(100)^2}$$

$$y=2.34$$

Elev. of lowest point of curve
= 100 + 0.05 (6.37) + 2.34
= ***102.66 m.***

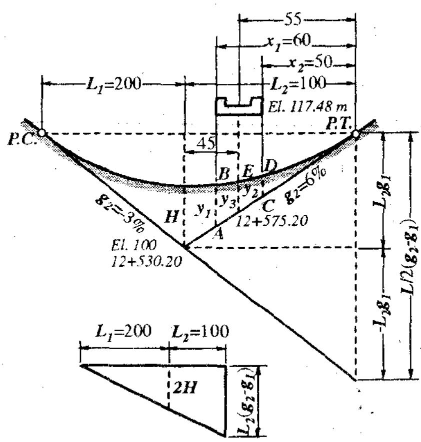
Problem 380:

A forward tangent of +6% was designed to intersect a back tangent of - 3% at a proposed underpass along Epifanio de lost Santos Avenue so as to maintain a minimum clearance allowed under a bridge which crosses perpendicular to the underpass. A 200 m. curve lies on the side of the back tangent while a 100 m. curve lies on the side of the forward tangent. The stationing and elevation of the grade intersection is 12 + 530.20 and 100 m. respectively. The center line of the bridge falls at station 12 + 575.20. The elevation of the underside of the bridge is 117.48.

① Determine the clearance of the bridge at the left side if it has a width of 10 m.
② Determine the clearance of the bridge at the right side.
③ Determine the clearance of the bridge at the center of the bridge.

Solution:

① *Clearance on the left side of the bridge:*



PARABOLIC CURVES

$$\frac{2H}{L_1}=\frac{L_2(g_2-g_1)}{L_1+L_2}$$
$$H=\frac{L_1L_2(g_2-g_1)}{2(L_1+L_2)}$$
$$H=\frac{200(100)(0.06-0.03)}{2(200+100)}$$
$$H=3\text{ m.}$$
$$\frac{y_1}{(x_1)^2}=\frac{H}{(L_2)^2}$$
$$\frac{y_1}{(60)^2}=\frac{3}{(100)^2}$$
$$y_1=1.08$$
$$\frac{y_2}{(50)^2}=\frac{3}{(100)^2}$$
$$y_2=0.75$$

Elev. of A = 100 + 0.06 (40)
Elev. of A = 102.40
Elev. of B = 102.4 + 1.08
Elev. of B = 103.48

Clearance on the left side = 117.48 - 103.48
Clearance on the left side = ***14 m.***

② *Clearance on the right side of the bridge:*
Elev. of C = 100 + 0.06 (50)
Elev. of C = 103
Elev. of D = 103 + 0.75
Elev. of D = 103.75

Clearance at right side = 117.48 - 103.75
Clearance at right side = ***13.73 m.***

③ *Clearance at the center:*
$$\frac{y_3}{(55)^2}=\frac{3}{(100)^2}$$
$$y_3=0.91$$

Elev. of E = 100 + 45 (0.06) + 0.91
Elev. of E = 103.61 m.

Clearance at the center = 117.48 - 103.61
Clearance at the center = ***13.87 m.***

Sight Distance

Sight distance = is the length of roadway ahead visible to the driver. For purpose of design and operation it is termed stopping sight distance and passing sight distance.

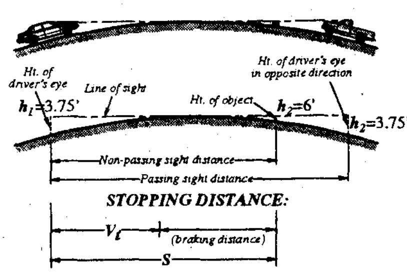
1.)

Stopping Sight Distance:

Stopping Sight Distance is the total distance traveled during three time intervals.

a. The time for the driver to perceive the hazard.
b. The time to react
c. The time to stop the vehicle after the brakes are applied.

Based on the National Safety Council, average driver reaction time is 3/4 seconds.

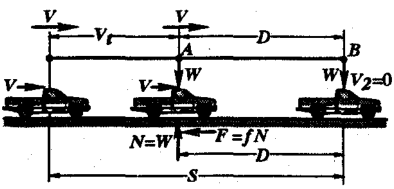


S = Vt + D

A car moving at a certain velocity V after seeing an object ahead of him, will still travel a distance Vt before he starts applying the brakes. The braking distance depends upon the speed and type of pavements, in this case we have to consider the coefficient of friction (f) between the tires and the pavement. The time t (sec) is called the perception-reaction time as is approximately 3/4 of a seconds.

SIGHT DISTANCE

Using work - energy equation in solving for the braking distance d.



Safe Stopping Distance

① Distance traversed during perception plus brake reaction time.

$d = Vt$

V = running speed in kph

t = reaction time

t = perception time + action time

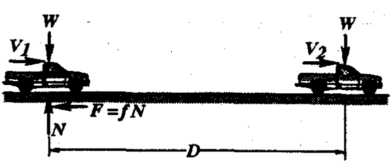
$t = 1.5 + 1.0$

$t = 2.5$ sec.

d = distance traversed during perception time plus brake reaction time in meters

$d = Vt$

② Distance required for stopping after brakes are applied (braking distance)



Positive work - Neg. work $= \frac{1}{2}\frac{W}{g}(V_2^2 - V_1^2)$

$$O - DF = \frac{1}{2}\frac{W}{g}(0 - V^2)$$

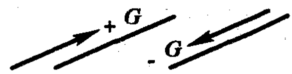
$$D f N = \frac{W V^2}{2g}$$

$N = W$

$$D = \frac{W V^2}{2g f W}$$

$D = \frac{V^2}{2g f}$ if it is moving on a horizontal plane.

$D = \frac{V^2}{2g(f \pm G)}$ if moving at a certain grade



Safe stopping distance: $S = d + D$

$$S = Vt + \frac{V^2}{2g(f \pm G)}$$

S = stopping distance in meters

t = perception-reaction time in seconds

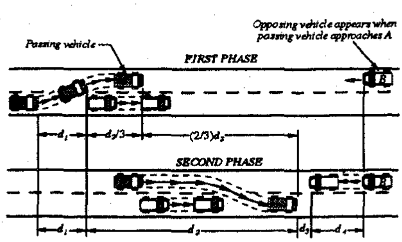
V = velocity of vehicle in meters per second

f = coefficient of friction between tires and pavement

$g = 9.81$ meters/sec^2.

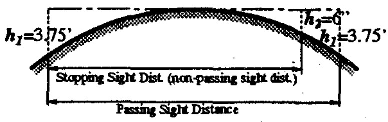
2.) ***Passing Sight Distance:***

Passing Sight Distance is a shortest distance sufficient for a vehicle to turn out of a traffic lane, pass another vehicle, and then turn back to the same lane safely and comfortably without interfering with the overtaken vehicle or an on incoming vehicle traveling at the design speed should it come into view after the passing maneuver is started.



SIGHT DISTANCE

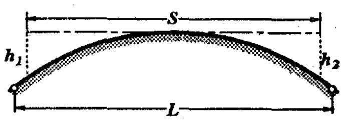
Stopping Sight Distance



① ***Stopping Sight Distance:***
(Summit Parabolic Curve)

h_1 = 3.75 ft. (height of drivers eye above the pavement)
h_1 = 1.14 m.
h_2 = 6 inches (height of object above pavement)
h_2 = 0.15 m.

A. When $S < L$



❶

$$L = \frac{AS^2}{100\left(\sqrt{2\,h_1} + \sqrt{2\,h_2}\right)^2}$$

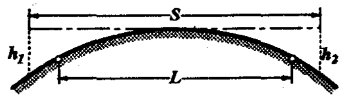
when h_1 = 1.14 m.
h_2 = 0.15 m.
L = in meters
S = in meters

❷

$$L = \frac{AS^2}{1400}$$

when h_1 = 3.75 ft.
h_2 = 6 inches
$A = g_1 - g_2$
L = in feet
S = in feet

B. When $S > L$



❶

$$L = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A}$$

when h_1 = 1.14 m.
h_2 = 0.15 m.
L = in meters
S = in meters

❷

$$L = 2S - \frac{1400}{A}$$

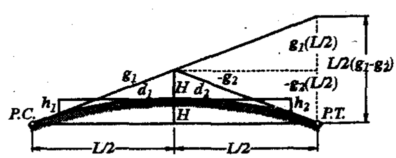
when h_1 = 3.75 ft.
h_2 = 6 inches
$A = g_1 - g_2$
L = in feet
S = in feet

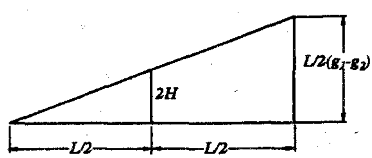
SIGHT DISTANCES (Vertical Curves)

① When

$S < L$

Sight distance is less than the length of curve.





SIGHT DISTANCE

$$\frac{2H}{\frac{L}{2}} = \frac{\frac{L}{2}(g_1 - g_2)}{L}$$

$$4H = \frac{L}{2}(g_1 - g_2)$$

$$H = \frac{L}{8}(g_1 - g_2)$$

Express the slope in percent not decimals

$$\frac{A}{100} = g_1 - g_2$$

$$H = \frac{L}{8}\frac{A}{100}$$

$$H = \frac{AL}{800}$$

Ex:

$g_1 = +2\%$
$g_2 = -3\%$
$A = g_1 - g_2$
$A = 2 - (-3)$
$A = 5$

Using the squared property of the parabola:

$$\frac{h_1}{d_1^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$

$$\frac{h_2}{d_2^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$

$$d_1^2 = \frac{h_1 L^2}{4H}$$

$$d_1^2 = \frac{h_1 L^2 (800)}{4AL}$$

$$d_1^2 = \frac{200 h_1 L}{A}$$

$$d_1 = \sqrt{\frac{200 h_1 L}{A}}$$

$$\frac{h_2}{d_2^2} = \frac{H}{\left(\frac{L}{2}\right)^2}$$

$$d_2^2 = \frac{h_2 L^2}{4H}$$

$$d_2^2 = \frac{h_2 L^2}{4\frac{AL}{800}}$$

$$d_2^2 = \frac{200 h_2 L}{A}$$

$$d_2 = \sqrt{\frac{200 h_2 L}{A}}$$

$$S = d_1 + d_2$$

$$S = \sqrt{\frac{200 h_1 L}{A}} + \sqrt{\frac{200 h_2 L}{A}}$$

$$S = \sqrt{\frac{100 L}{A}}\sqrt{2 h_1} + \sqrt{\frac{100 L}{A}}\sqrt{2 h_2}$$

$$S = \sqrt{\frac{100 L}{A}}\left(\sqrt{2 h_1} + \sqrt{2 h_2}\right)$$

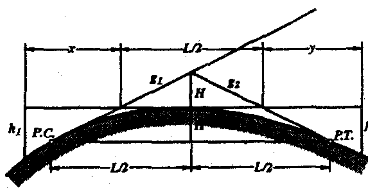
$$S^2 = \frac{100 L}{A}\left(\sqrt{2 h_1} + \sqrt{2 h_2}\right)^2$$

$$\boxed{L = \frac{S^2 A}{100\left(\sqrt{2 h_1} + \sqrt{2 h_2}\right)^2}}$$

② When

$\boxed{S > L}$

Sight distance is greater than the length of curve.



$$h_1 = x\, g_1$$

$$x = \frac{h_1}{g_1}$$

If g_1 = is in percent

$$x = \frac{h_1}{\frac{g_1}{100}}$$

$$x = \frac{100 h_1}{g_1}$$

$$y = \frac{h_2 (100)}{g_2}$$

$$S = \frac{L}{2} + x + y$$

$$S = \frac{L}{2} + \frac{h_1 (100)}{g_1} + \frac{h_2 (100)}{g_2}$$

$$S = \frac{L}{2} + 100\left(\frac{h_1}{g_1} + \frac{h_2}{g_2}\right)$$

SIGHT DISTANCE

Use scalar value of g_2. Actual value is negative for g_2.

$$S = \frac{L}{2} + 100\left(\frac{h_1}{g_1} + \frac{h_2}{g_2}\right)$$

$$\frac{dS}{dg} = 0 + 100\left[\frac{h_1(1)\,g_1(0)}{g_1^2} + \frac{h_2(-1) - g_2(0)}{g_2^2}\right] = 0$$

$$\frac{h_1}{g_1^2} - \frac{h_2}{g_2^2} = 0$$

$$g_1^2 = \frac{h_1 g_2^2}{h_2}$$

$$g_2 = \frac{h_2 g_1^2}{h_1}$$

$$g_2 = \sqrt{\frac{h_2}{h_1}}\, g_1$$

$$A = g_1 - (-g_2) = g_1 + g_2$$

$$A = \sqrt{\frac{h_2}{h_1}}\, g_1 + g_2$$

$$A = \left(\sqrt{\frac{h_2}{h_1}} + 1\right) g_1$$

$$A = \left(\frac{\sqrt{h_2} + \sqrt{h_1}}{\sqrt{h_1}}\right) g_1$$

$$g_2 = \sqrt{\frac{h_2}{h_1}}\, \frac{A\sqrt{h_1}}{(\sqrt{h_2} + \sqrt{h_1})}$$

$$g_2 = \frac{A\sqrt{h_2}}{(\sqrt{h_2} + \sqrt{h_1})}$$

$$S = \frac{L}{2} + 100\left(\frac{h_1}{g_1} + \frac{h_2}{g_2}\right)$$

$$S = \frac{L}{2} + 100\,\frac{\left[\dfrac{h_1}{A\sqrt{h_1}} + \dfrac{h_2}{A\sqrt{h_2}}\right]}{(\sqrt{h_2} + \sqrt{h_1})\,(\sqrt{h_2} + \sqrt{h_1})}$$

$$S = \frac{L}{2} + 100\,\frac{(\sqrt{h_2} + \sqrt{h_1})}{A}\left[\frac{h_1}{\sqrt{h_1}} + \frac{h_2}{\sqrt{h_2}}\right]$$

$$S = \frac{L}{2} + \frac{100}{A}\left(\frac{h_1\sqrt{h_2}}{\sqrt{h_1}} + h_1 + h_2 + \frac{\sqrt{h_1}\,h_2}{\sqrt{h_2}}\right)$$

$$S = \frac{L}{2} + \frac{100}{A}\left(h_1 + \frac{h_1\sqrt{h_2}}{\sqrt{h_1}} + \frac{\sqrt{h_1}\,h_2}{\sqrt{h_2}} + h_2\right)$$

$$S = \frac{L}{2} + \frac{100}{A}\left[h_1 + \frac{h_1 h_2 + h_1 h_2}{\sqrt{h_1}\sqrt{h_2}} + h_2\right]$$

$$S = \frac{L}{2} + \frac{100}{A}\left[h_1 + \frac{2 h_1 h_2}{\sqrt{h_1 h_2}} + h_2\right]$$

$$S = \frac{L}{2} + \frac{100}{A}\left[h_1 + \frac{2 h_1 h_2 \sqrt{h_1 h_2}}{\sqrt{h_1 h_2}\sqrt{h_1 h_2}} + h_2\right]$$

$$S = \frac{L}{2} + \frac{100}{A}\left[h_1 + 2\sqrt{h_1 h_2} + h_2\right]$$

$$S = \frac{L}{2} + \frac{100}{A}\left[\sqrt{h_1} + \sqrt{h_2}\right]^2$$

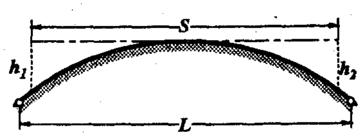
$$\frac{L}{2} = S - \frac{100}{A}(\sqrt{h_1} + \sqrt{h_2})^2$$

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$

SIGHT DISTANCES

①

$S < L$



a.

$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$$

b.

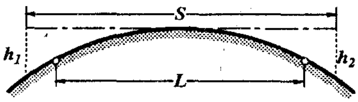
$$L = \frac{S^2(g_1 - g_2)}{8h}$$

when $h_1 = h_2 = h$

SIGHT DISTANCE

②

S > L



a.

$$L = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A}$$

b.

$$L = 2S - \frac{4\,(h_1 + h_2)}{g_1 - g_2}$$

c.

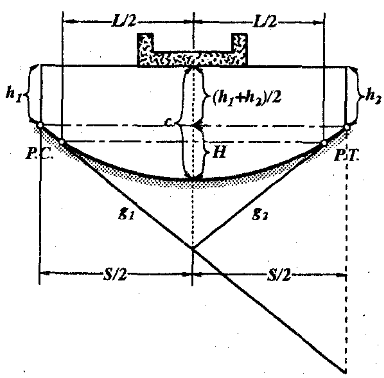
$$L = \frac{2S\,(g_1 - g_2) - 8h}{g_1 - g_2}$$

when $h_1 = h_2 = h$

Passing Sight Distance for Vertical Sag Curve at Underpass

① When passing sight distance is greater than the length of curve.

S < L



Using the previous relation of parabolic curves.

Where
S = length of passing sight distance
L = Length of curve
h_1 = height of drivers eye
h_2 = height of object
C = vertical clearance from the lowest point of underpass to the curve.

$$H = C - \frac{(h_1 + h_2)}{2}$$

$$y = \frac{1}{4}\left(\frac{L}{2}\right)(g_2 - g_1)$$

$$y = \frac{L}{8}(g_2 - g_1)$$

By ratio and proportion:

$$\frac{\frac{S}{2}}{H + y} = \frac{S}{\frac{S}{2}(g_2 - g_1)}$$

$$\frac{S}{2H + \frac{L}{8}(g_1 - g_2)} = \frac{2S}{S\,(g_2 - g_1)}$$

$$S\,(g_2 - g_1) = 4H + \frac{L}{2}(g_2 - g_1)$$

$$\frac{L}{2}(g_2 - g_1) = S\,(g_2 - g_1) - 4H$$

$$L = \frac{2S\,(g_1 - g_2) - 8H}{(g_2 - g_1)}$$

AASHO Specs.

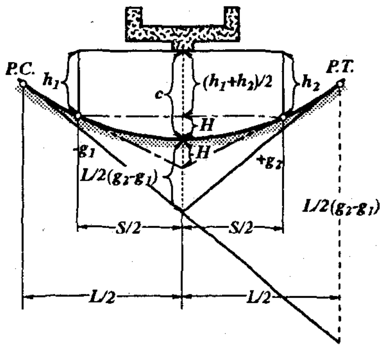
when C = 14 ft.
h_1 = 6 ft.
h_2 = 1.5 ft.
$A = g_2 - g_1$

$$L = 2S - \frac{82}{A}$$

SIGHT DISTANCE

② When the passing sight distance is less than the length of curve.

$S > L$



Using the squared property of parabola.

$$\frac{\frac{L}{8}(g_2-g_1)}{\left(\frac{L}{2}\right)^2} = \frac{H}{\left(\frac{S}{2}\right)^2}$$

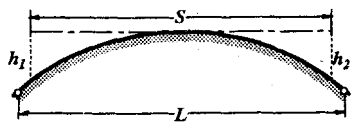
$$\frac{4L\,(g_2-g_1)}{8\,L^2} = \frac{4H}{S^2}$$

$$L = \frac{S^2\,(g_2-g_1)}{8H}$$

Passing Sight Distance (Summit Parabolic Curve)

A. when

$S < L$



①

$$L = \frac{AS^2}{100\left(\sqrt{2\,h_1} + \sqrt{2\,h_2}\right)^2}$$

when $h_1 = 1.14$ m.
$h_2 = 1.14$ m.
L = in meters
S = in meters

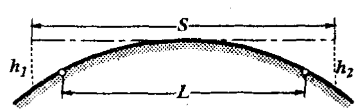
②

$$L = \frac{AS^2}{3000}$$

when $h_1 = 3.75$ ft.
$h_2 = 3.75$ ft.
$A = g_1 - g_2$
L = in feet
S = in feet

B. when

$S > L$



SIGHT DISTANCE

①

$$L = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A}$$

when $h_1 = 1.14$ m.
$h_2 = 1.14$ m.
L = in meters
S = in meters

②

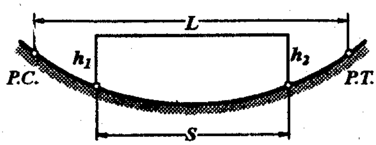
$$L = 2S - \frac{3000}{A}$$

when $h_1 = 3.75$ ft.
$h_2 = 3.75$ ft.
$A = g_1 - g_2$
L = in feet
S = in feet

Sight Distance for sag parabolic curve:

A. when

$$S < L$$



$$L = \frac{AS^2}{400 + 3.5\,S}$$

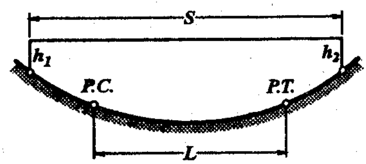
L = in feet
S = in feet
$A = g_2 - g_1$

$$L = \frac{AS^2}{122 + 3.5\,S}$$

L = in meters
S = in meters

B. when

$$S > L$$



$$L = 2S - \frac{(400 + 3.5\,S)}{A}$$

L = in feet
S = in feet

$$L = 2S - \frac{(122 + 3.5S)}{A}$$

L = in meters
S = in meters

C. ***Max. velocity of car moving in a vertical sag curve.***

$$L = \frac{V^2 A}{46.50}$$

V = velocity in mph
$A = g_2 - g_1$
L = length of curve in feet

$$L = \frac{AV^2}{395}$$

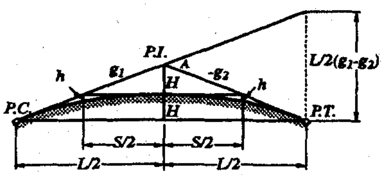
L = in meters
V = in kph

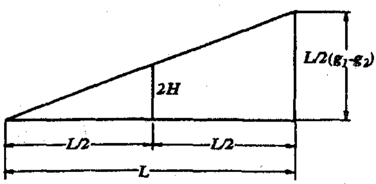
SIGHT DISTANCE

SIGHT DISTANCE (Another Formula but with some results)

① When

$S < L$





S = sight distance
L = length of curve
h = height of drivers eye above the pavement

By ratio and proportion:

$$\frac{2H}{\frac{L}{2}} = \frac{\frac{L}{2}(g_1 - g_2)}{L}$$

$$H = \frac{L}{8}(g_1 - g_2)$$

Using the squared property of parabola:

$$\frac{H}{(\frac{L}{2})^2} = \frac{h}{(\frac{S}{2})^2}$$

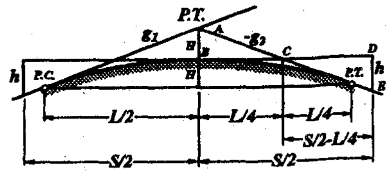
$$\frac{\frac{L}{8}(g_1 - g_2)}{\frac{L^2}{4}} = \frac{h}{\frac{S^2}{4}}$$

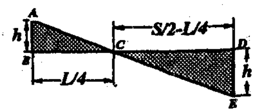
$$Lh = \frac{1}{8}(g_1 - g_2)\,S^2$$

$$L = \frac{S^2(g_1 - g_2)}{8h}$$

② When

$S > L$





Considering triangles ABC and CDE:

By ratio and proportion:

$$\frac{H}{\frac{L}{4}} = \frac{h}{\frac{S}{2} - \frac{L}{4}}$$

$$H = \frac{L}{8}(g_1 - g_2)$$

$$\frac{\frac{L}{8}(g_1 - g_2)}{\frac{L}{4}} = \frac{h}{\frac{2S - L}{4}}$$

$$\frac{(g_1 - g_2)}{8} = \frac{h}{2S - L}$$

$$2S(g_1 - g_2) - L(g_1 - g_2) = 8h$$

$$L(g_1 - g_2) = 2S(g_1 - g_2) - 8h$$

$$L = \frac{2S(g_1 - g_2) - 8h}{(g_1 - g_2)}$$

SIGHT DISTANCE

Problem 381:

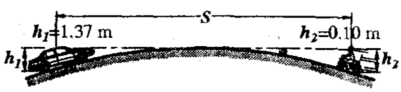
A 5% grade intersects a - 3.4% grade at station 1 + 990 of elevation 42.30 m. Design a vertical summit parabolic curve connecting the two tangent grades to conform with the following safe stopping sight distance specifications.

Design velocity = 60 kph
Height of driver's eye from the road pavement = 1.37 m.
Height of an object over the pavement ahead = 100 mm.
Perception-reaction time = $\frac{3}{4}$ sec.
Coefficient of friction between the road pavement and the tires = 0.15.

① Determine the stopping sight distance.
② Determine the length of curve.
③ Determine the elevation of highest point on curve.

Solution:

① *Stopping sight distance:*



$$S = Vt + \frac{V^2}{2g\,(f + G)}$$

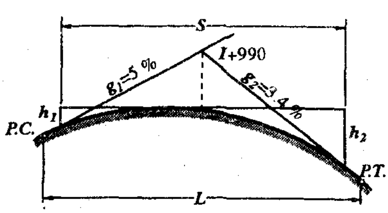
$$V = \frac{60000}{3600}$$

V = 16.67 m/s

$$S = 16.67\left(\frac{3}{4}\right) + \frac{(16.67)^2}{2\,(9.81)\,(0.15 + 0.05)}$$

S = **83.32** ***m.***

② *Length of curve:*



Assume $S < L$

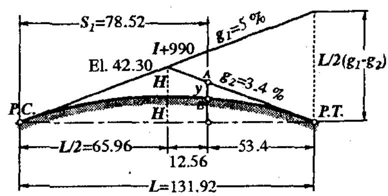
$$L = \frac{AS^2}{100(\sqrt{2\,h_1} + \sqrt{2\,h_2})^2}$$

$A = g_1 - g_2$
$A = 5 - (-3.4)$
$A = 8.4$

$$L = \frac{(8.4)\,(83.32)^2}{100\,(\sqrt{2\,(1.37)} + \sqrt{2\,(0.10)})^2}$$

L = ***131.92 m.*** > 83.32 *ok as assumed.*

③ *Elevation of highest point of curve:*



$$S_1 = \frac{g_1\,L}{g_1 - g_2}$$

$$S_1 = \frac{0.05\,(131.92)}{0.05 + 0.034}$$

S_1 = 78.52 m.

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{131.92\,(0.05 + 0.034)}{8}$$

H = 1.39

$$\frac{y}{(53.4)^2} = \frac{1.39}{(65.96)^2}$$

y = 0.91

Elev. of highest point
= 42.30 - 12.56 (0.034) - 0.91
Elev. of highest point = ***40.963 m.***

SIGHT DISTANCE

Problem 382:

The length of sag parabolic curve is 130 m. with a design speed of 100 kph. The back tangent has a slope of - 2.5%.

① Compute the slope of the forward tangent.
② Compute the distance of the lowest point of the curve from the P.C.
③ Compute the length of the sight distance.

Solution:

① *Slope of forward tangent:*

$$L = \frac{AV^2}{395}$$

$$130 = \frac{A\,(100)^2}{395}$$

$$A = 5.135$$

$$A = g_1 + g_2$$

$$5.135 = 2.5 + g_2$$

$$g_2 = \mathbf{2.635\%}$$

② *Distance of lowest point of curve from P.C:*

$$S = \frac{g_1 L}{g_1 - g_2}$$

$$S = \frac{-0.025\,(130)}{-0.025 - 0.02635}$$

$$S = \mathbf{63.29\ m.}$$

③ *Length of sight distance:*

Ass: $S < L$

$$L = \frac{AS^2}{122 + 3.5S}$$

$$130 = \frac{5.135\,S^2}{122 + 3.5S}$$

$$5.135\,S^2 = 15860 + 455S$$

$$S^2 - 88.61S - 3088.61 = 0$$

$$S = \mathbf{115.38\ m.}\quad ok$$

Problem 383:

The design speed of a sag parabolic curve is 100 kph. The downward tangent grade is - 2%. The length of curve is 126 m.

① Compute the upward tangent grade of the parabolic sag curve.
② Compute the length of sight distance.
③ At what distance from the P.C. is the lowest point of the curve.

Solution:

① *Upward tangent grade:*

$$L = \frac{AV^2}{395}$$

$$126 = \frac{A\,(100)^2}{395}$$

$$A = 4.98$$

$$A = g_2 + 2$$

$$4.98 = g_2 + 2$$

$$g_2 = \mathbf{2.98\%}$$

② *Sight distance:*

$$L = \frac{AS^2}{122 + 3.5\,S}$$

$$126 = \frac{4.98\,S^2}{122 + 3.5\,S}$$

$$15372 + 441\,S = 4.98\,S^2$$

$$S^2 - 88.55\,S - 3086.75 = 0$$

$$S = \mathbf{115.32\ m.}$$

③ *Distance of lowest point from P.C.*

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{-0.02\,(126)}{-0.02 - 0.0298}$$

$$S_1 = \mathbf{50.60\ m.}\ \text{from P.C.}$$

Problem 384:

A vertical curve has a descending grade of - 1.2% starting from the P.C. and an ascending grade of +3.8% passing thru the P.T. The curve has a sight distance of 180 m.

① Compute the length of the vertical curve.
② Compute the max. velocity of the car that could pass thru the curve.
③ Compute the distance of the lowest point of the curve from the P.C.

Solution:

① *Length of curve:*

$S < L$

$$L = \frac{AS^2}{122 + 3.5\,S}$$

$A = g_2 - g_1$

$A = 3.5 - (-1.2)$

$A = 5$

$$L = \frac{(5)(180)^2}{122 + 3.5\,(180)}$$

$L = \mathbf{215.43\ m.}$

② *Max. velocity:*

$$L = \frac{AV^2}{395}$$

$$215.43 = \frac{5\,V^2}{395}$$

$V = \mathbf{130.46\ kph}$

③ *Distance of lowest point of curve from P.C.*

$$S = \frac{g_1 L}{g_1 - g_2}$$

$$S = \frac{-0.012\,(215.43)}{-0.012 - 0.038}$$

$S = \mathbf{51.70\ m.}$

Problem 385:

The design speed of a vertical sag curve is equal to 100 kph. The tangent grades of the curve are - 2% and +3% respectively.

① Compute the length of curve in meters.
② Compute the sight distance in meters.
③ Compute the length of minimum visibility in meters.

Solution:

① *Length of curve:*

$$L = \frac{AV^2}{395}$$

$$L = \frac{5\,(100)^2}{395}$$

$L = \mathbf{126.6\ m.}$

② *Sight distance:*

$$L = \frac{AS^2}{122 + 3.5\,S}$$

$$126.6 = \frac{5\,S^2}{122 + 3.5\,(S)}$$

$15445.2 + 443.1\,S = 5\,S^2$

$S^2 - 88.62\,S - 3089.04 = 0$

$S = \mathbf{115.39}$

③ *Minimum visibility:*

$$Min.\ visibility = \frac{115.39}{2}$$

Min. visibility = **57.70 m.**

Problem 386:

A vertical summit curve has a back tangent of +2% and a forward tangent of - 3% intersecting at station 10 + 220.60 m. and elevation of 200 m. The design speed of the curve is 80 kph. Assuming coefficient of friction is 0.30 and a perception reaction time of 2.5 sec.

① Compute the safe stopping sight.
② Compute the length of curve.
③ Compute the elevation of highest point of curve.

Solution:

① *Safe stopping sight distance:*

$V = 80$ kph

$$V = \frac{80000}{3600}$$

$V = 22.22$ m/s

$$S = Vt + \frac{V^2}{2g\,(f+G)}$$

$$S = 22.22\,(2.5) + \frac{(22.22)^2}{2\,(9.81)\,(0.30+0.02)}$$

$S = \mathbf{134.19\ m.}$

② *Length of curve:*
For safe stopping sight distance:
$h_1 = 1.14$ m.
$h_2 = 0.15$ m.

Assume S < L

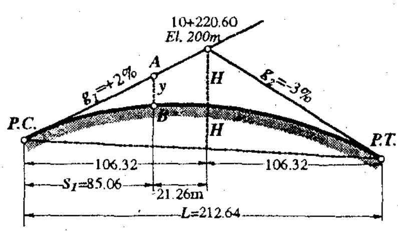
$$L = \frac{AS^2}{100\,(\sqrt{2\,h_1} + \sqrt{2\,h_2})^2}$$

$A = g_1 - g_2$
$A = 2 - (-3)$
$A = 5$

$$L = \frac{5\,(134.19)^2}{100\,(\sqrt{2\,(1.4)} + \sqrt{2\,(0.15)})^2}$$

$L = \mathbf{212.64\ m.}$
$S < L$ ok

③ *Elevation of highest point:*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{0.02\,(212.64)}{0.02 + 0.03}$$

$S_1 = 85.06$ m.

$$H = \frac{L}{8}\,(g_1 - g_2)$$

$$H = \frac{212.64\,(0.02+0.03)}{8}$$

$H = 1.33$ m.

$$\frac{y}{(85.06)^2} = \frac{1.33}{(106.33)^2}$$

$y = 0.85$ m.

Elev. of highest point of curve:
Elev. B = 200 - 0.02 (21.26) - 0.85
Elev. B = ***198.72 m.***

Problem 387:

A vertical summit curve have grades of +3% and - 2% which intersects a station 20 + 040. If the stopping sight distance is equal to 163.78 m.

① Compute the maximum speed that a vehicle could move along this curve to avoid collision if the perception reaction time of the driver is 2.0 sec. and the coefficient of friction is equal to 0.25.
② Compute the length of curve.
③ Compute the stationing of the highest point of curve.

Solution:

① *Max. speed that a vehicle could move along the curve:*

$$S = Vt + \frac{V^2}{2g\,(f+G)}$$

$$163.78 = V\,(2) + \frac{V^2}{2\,(9.81)\,(0.25+0.03)}$$

$0.182\,V^2 + 2\,V - 163.78 = 0$
$V^2 = 10.989\,V - 899.89 = 0$
$V = 25$ m/s

$$V = \frac{25\,(3600)}{1000}$$

$V = \mathbf{90\ kph}$

② *Length of curve:*
Assume S < L

$$L = \frac{AS^2}{100\,(\sqrt{2\,h_1} + \sqrt{2\,h_2})^2}$$

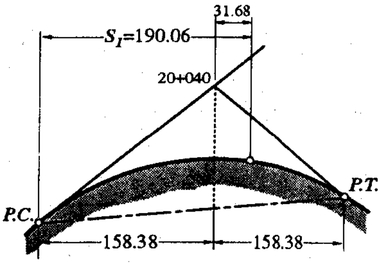
$h_1 = 1.14$ m.
$h_2 = 0.15$ m.

$A = 3 - (-2)$
$A = 5$

$$L = \frac{5\,(163.78)^2}{100\,(\sqrt{2\,(1.4)} + \sqrt{2\,(0.15)})^2}$$

$L = \mathbf{316.76\ m.}$

③ *Stationing of highest point of curve:*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{0.03\,(316.76)}{0.03 + 0.02}$$

$$S_1 = 190.06$$

Sta. of highest point of curve = (20 + 040) - 31.68
Sta. of highest point of curve = ***20 + 008.32***

Problem 388:

A vertical summit curve has tangent grades of +2.5% and - 1.5% intersecting at station 12 + 460.12 at an elevation of 150 m. above sea level. If the length of curve is 182 m.

① Compute the length of the passing sight distance.
② Compute the stationing of the highest point of the curve.
③ Compute the elevation of the highest point of the curve.

Solution:

① *Passing sight distance:*

$h_1 = 1.14$ m.
$h_2 = 1.14$ m.

Assume $S > L$

$$L = 2S - \frac{200\,(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$

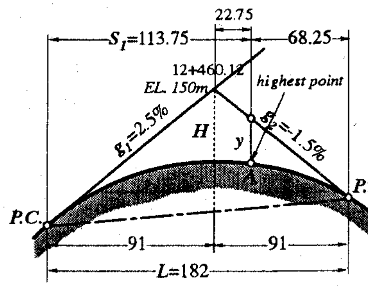
$$A = g_1 - g_2$$
$$A = 2.5 - (-1.5)$$
$$A = 4$$

$$182 = 2S - \frac{200\,(\sqrt{1.14} + \sqrt{1.14})^2}{4}$$

$$2S = 182 + 228$$

S = ***205 m.*** > 182 *ok*

② *Stationing of highest point of curve:*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{0.025\,(182)}{0.025 + 0.015}$$

$$S_1 = 113.75 \text{ m.}$$

Sta. of highest point = (12 + 460.12) + 22.75
Sta. of highest point = ***12 + 482.87***

③ *Elevation of highest point of curve:*

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{182\,(0.025 + 0.015)}{8}$$

$$H = 0.91 \text{ m.}$$

$$\frac{y}{(68.25)^2} = \frac{0.91}{(91)^2}$$

$$y = 0.51 \text{ m.}$$

Elev. A = 150 - 22.75 (0.015) - 0.51
Elev. A = ***149.15 m.***

Problem 389:

A vertical sag parabolic curve has a length of 141 m. with tangent grades of - 1.5% and +2.5% intersecting at station 12 + 640.22 and elevation of 240 m. above sea level.

① Compute the length of the sight distance.
② Compute the maximum speed that a car would travel to avoid collision.
③ Compute the stationing of the lowest point of the curve.

SIGHT DISTANCE

Solution:

① *Length of sight distance:*

Assume S > L

$$L = 2S - \frac{(122 + 3.5S)}{A}$$

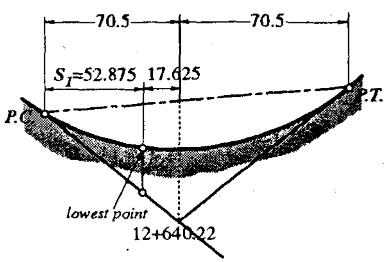
$$A = g_2 - g_1$$
$$A = 2.5 - (-1.5)$$
$$A = 4$$

$$141 = 2S - \frac{(122 + 3.5S)}{4}$$
$$564 = 8S - 122 - 3.5S$$
$$4.5S = 686$$
$$S = 152.44 \text{ m.} > 141 \text{ ok}$$

② *Max. speed:*

$$L = \frac{AV^2}{395}$$
$$141 = \frac{4V^2}{395}$$
$$V = \mathbf{118\ kph}$$

③ *Stationing of lowest point of curve:*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$
$$S_1 = \frac{-0.015(141)}{-0.015 - 0.025}$$
$$S_1 = 52.875 \text{ m. from P.C.}$$

Stationing of lowest point of curve

$= (12 + 640.22) - 17.625$

$= \mathbf{12 + 622.595}$

Problem 390:

The sight distance of a sag parabolic curve is 155.32 m. long with grade tangents of - 2% and +2.98% respectively.

① Compute the length of the parabolic curve.
② Compute the max. velocity of car that can maneuver on the curve to avoid possible collision of any approaching cars.
③ Compute the perception-reaction time of the driver to avoid collision. Assume coefficient of friction between tires and pavement to be 0.38.

Solution:

① *Length of sag curve:*

Assume: L > S

$$L = \frac{AS^2}{122 + 3.5S}$$
$$A = 2.98 - (-2)$$
$$A = 4.98$$
$$L = \frac{4.98(115.32)^2}{122 + 3.5(115.32)}$$
$$L = \mathbf{126\ m.} > 115.32 \text{ ok}$$

② *Max. speed of the car:*

$$L = \frac{AV^2}{395}$$
$$126 = \frac{4.98V^2}{395}$$
$$V = 100 \text{ kph}$$

③ *Perception reaction time:*

$$S = Vt + \frac{V^2}{2gf}$$
$$V = \frac{100000}{3600}$$
$$V = 27.78 \text{ m/s}$$
$$115.32 = 27.78t + \frac{(27.78)^2}{2(9.81)(0.38)}$$
$$t = \mathbf{0.42\ sec.}$$

Problem 391:

A parabolic sag curve has a sight distance of 115 m. The tangent grades of the curve are - 2% and +3%.

① Compute the length of the curve.
② Compute the max. speed that a car could move along this curve to prevent skidding.
③ If the P.C. is at station 10 + 540 and elevation of 120 m., find the elevation of the lowest point of the curve.

Solution:

① *Length of curve:*

$$L = \frac{AS^2}{122 + 3.5\,S}$$

$$A = g_2 - g_1$$
$$A = 3 - (-2)$$
$$A = 5$$

$$L = \frac{5\,(115)^2}{122 + 3.5\,(115)}$$

$L = \mathbf{126.07\ m.} > S$ *ok*

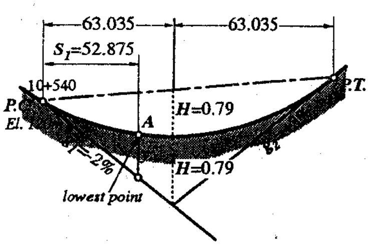
② Max. speed:

$$L = \frac{AV^2}{395}$$

$$126.07 = \frac{5\,V^2}{395}$$

$V = \mathbf{99.80\ kph}$

③ *Elev. of lowest point of curve:*



$$S_1 = \frac{g_1\,L}{g_1 - g_2}$$

$$S_1 = \frac{-0.02\,(126.07)}{-0.02 - 0.03}$$

$S_1 = 50.43$ m.

$$H = \frac{L}{8}(g_2 - g_1)$$

$$H = \frac{126.07}{8}(0.03 + 0.02)$$

$$H = 0.79$$

$$\frac{y}{(50.43)^2} = \frac{0.79}{(63.05)^2}$$

$y = 0.51$ m.

Elev. of lowest point = 120 - 0.02 (50.43) + 0.51

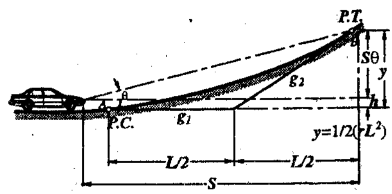
Elev. of lowest point = ***119.50 m.***

HEAD LAMP SIGHT DISTANCE

Sight Distance Related to Height of the Beam of a Vehicles Headlamp

Case 1:

$S > L$



L = Length of curve
S = head lamp sight distance
h = ht. of headlamps above road surface
ø = angle.the beam tilts upward above the longitudinal axis of the car.

$r = \frac{g_2 - g_1}{L}$ rate of change of grade

Change of grade = $g_2 - g_1$

To get the offset y, the change of grade from A to B is multiplied by the average horizontal distance $\frac{L}{2}$.

$$y = (g_2 - g_1)\frac{L}{2}$$

If we multiply by $\frac{L}{L}$

$$y = \frac{g_2 - g_1}{2L}L^2$$

$$y = \frac{1}{2}rL^2$$

$$y = h + S\,ø$$

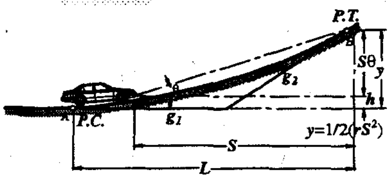
$$\frac{(g_2 - g_1)\,L}{2} = h + S\,ø$$

$$\boxed{L = \frac{2\,(h + S\,ø)}{g_2 - g_1}}$$

HEAD LAMP SIGHT DISTANCE

Case 2: When

$S < L$



$$r = \frac{g_2 - g_1}{L}$$

$$y = \frac{1}{2} r S^2$$

$$y = \frac{\frac{1}{2}(g_2 - g_1)\, S^2}{L}$$

$$y = S\,ø + h$$

$$\frac{(g_2 - g_1)\, S^2}{2L} = S\,ø + h$$

$$L = \frac{S^2\,(g_2 - g_1)}{2\,(S\,ø + h)}$$

Problem 392:

The sag vertical curve has a back tangent grade of - 3% and a forward tangent grade of +2% is to be designed on the basis of that the head lamp sight distance of a car traveling along the curve equals the minimum safe stopping distance with following specifications.

Design velocity = 120 kph
Perception reaction time = 3/4 sec.
Coefficient of friction between road pavement and tires is 0.15

① Compute the head lamp sight distance.
② If the head lamps are 0.75 m. above the road surface and their beams tilt upward at an angle of 1° above the longitudinal axis of the car, compute the length of curve.
③ If the P.C. is at station 10 + 540 and elevation 100 m., compute the elevation of the lowest point of the curve.

Solution:

① *Head lamp sight distance:*

$$\text{Safe stopping distance} = Vt + \frac{V^2}{2g\,f}$$

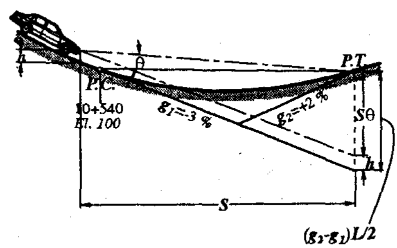
$$V = \frac{120000}{3600}$$

$$V = 33.33 \text{ m/s}$$

$$S = 33.33\left(\frac{3}{4}\right) + \frac{(33.33)^2}{2\,(9.81)\,(0.15)}$$

$$S = \mathbf{402.47\ m.}$$

② *Length of curve:*



$$Sø + h = (g_2 - g_1)\,\frac{L}{2}$$

$$L = \frac{2\,(Sø + h)}{g_2 - g_1}$$

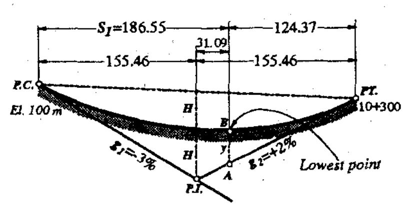
$$ø = \frac{1^\circ\,\pi}{180}$$

$$ø = 0.01745$$

$$L = \frac{2\,[(402.47)(0.01745) + 0.75]}{0.02 + 0.03}$$

L = ***310.92 m.*** < 402.47 m. *ok*

③ *Elevation of lowest point of curve:*



HEAD LAMP SIGHT DISTANCE

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{-0.03\,(310.92)}{-0.03 - 0.02}$$

$$S_1 = 186.55$$

$$H = \frac{L}{8}(g_2 - g_1)$$

$$H = \frac{310.92}{8}(0.02 + 0.03)$$

$$H = 1.94$$

$$\frac{y}{(124.37)^2} = \frac{1.94}{(155.46)^2}$$

$$y = 1.24 \text{ m.}$$

Elev. of lowest point of curve:
Elev. of P.I. = 100 - 0.03 (155.46)
Elev. of P.I. = 95.43 m.
Elev. of A = 95.34 + 0.02 (31.09)
Elev. of A = 95.96 m.
Elev. of B = 95.96 + 1.24
Elev. of B = ***97.2 m.***

Problem 393:

The Land Transportation Office (LTO) requires that cars must switch on their head lights when traveling at night time to avoid accidents. Car specifications requires head light to have an angle of tilt of 1° above the longitudinal axis of the car and at a height of 0.90 m. above the pavement. If a car passes through a vertical sag parabolic curve 153 m. long having grade tangents of - 4.4% and + 3.2% respectively.

① What max. speed could a car maneuver on this curve to prevent sliding if the coefficient of friction between tires and pavement is 0.15?
② Compute the max. head lamp sight distance to avoid collision along the curve.
③ If a driver approaching this curve sees an object ahead of him, find the time taken from the instant the object is visible to the driver to the instant the brakes are applied effectively.

Solution:

① *Max. speed:*

$$A = g_2 - g_1$$

$$A = 3.2 - (4.4)$$

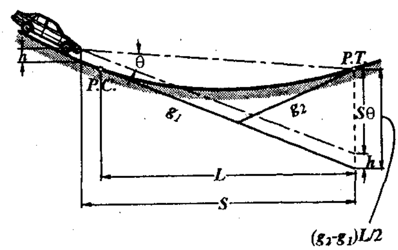
$$A = 7.6$$

$$L = \frac{AV^2}{395}$$

$$153 = \frac{7.6\,V^2}{395}$$

$$V = \mathbf{89.17\ kph}$$

② *Max. head lamp sight distance:*



$$S\phi + h = \frac{(g_2 - g_1)\,L}{2}$$

$$\phi = \frac{1°\,(\pi)}{180°}$$

$$\phi = 0.01745$$

$$S\,(0.01745) + 0.90 = \frac{(0.022 + 0.028)\,(153)}{2}$$

$$S = \mathbf{167.62}$$

③ *Perception time:*

$$V = \frac{89170}{3600}$$

$$V = 24.77 \text{ m/s}$$

$$S = Vt + \frac{V^2}{2g\,f}$$

$$167.62 = 24.77\,t + \frac{(24.77)^2}{2\,(9.81)(0.15)}$$

$$t = \mathbf{1.65\ sec.}$$

HEAD LAMP SIGHT DISTANCE

Problem 394:

A cargo truck approaches a sag parabolic curve at a speed of 100 kph. The length of the curve is 180 m. long with grade tangents of - 3% and +2% respectively. The intersection of the grade tangents is a 10 + 430 with an elevation of 240.60 m. The driver has to switch on the beam lights at night time travel with the beam light making an angle of tilt of 0.85° above the longitudinal axis of the car. The drivers perception reaction time is 0.78 sec.

① Assuming a coeff. of friction of 0.18, compute the length of the head lamp sight distance.
② How high was the head lamp above the pavement at this instant?
③ What is the max. design speed that a car could maneuver on this curve?

Solution:

① *Head lamp sight distance:*

$$S = Vt + \frac{V^2}{2g\,f}$$

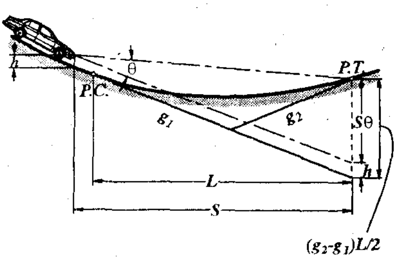
$$V = \frac{100000}{3600}$$

$$V = 27.78 \text{ m/s}$$

$$S = 27.78\,(0.78) + \frac{(27.78)^2}{2\,(9.81)\,(0.18)}$$

$$S = \mathbf{240.19\ m.}$$

② *Height of head lamp:*



$$S\text{ø} + h = \frac{(g_2 - g_1)\,L}{2}$$

$$\text{ø} = \frac{0.85\,(\pi)}{180^\circ}$$

$$\text{ø} = 0.0148$$

$$240.19\,(0.0148) + h = \frac{(0.02 + 0.03)\,(180)}{2}$$

$$h = \mathbf{0.495\ m.}$$

③ *Max. design speed:*

$$L = \frac{AV^2}{395}$$

$$A = 2 - (-3)$$

$$A = 5$$

$$180 = \frac{5\,V^2}{395}$$

$$V = \mathbf{119.2\ kph}$$

Problem 395:

Given the following data for a head lamp sight distance:
Grade of back tangent = - 2.9%
Grade of forward tangent = +2.1%
Design velocity = 110 kph
Coefficient of friction between pavement and tires = 0.14
Perception time is 0.80 seconds.
Angle of tilt of the beam light = 1° above the longitudinal axis of the car.
Length of curve = 280 m.
Stationing of P.C. = 10 + 620 at elev. 200 m.

① Compute the head lamp sight distance.
② Compute the height of the head lamp above the road surface.
③ Compute the elevation of the lowest point of the curve.

Solution:

① *Head lamp sight distnace:*

$$S = Vt + \frac{V^2}{2g\,f}$$

$$V = \frac{110000}{3600}$$

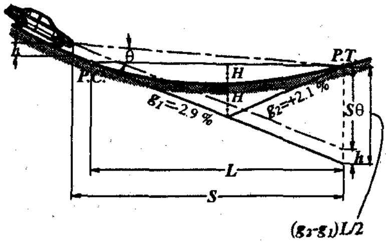
$$V = 30.56$$

$$S = 30.56\,(0.80) + \frac{(30.56)^2}{2\,(9.81)\,(0.14)}$$

$$S = \mathbf{364.45\ m.}$$

HEAD LAMP SIGHT DISTANCE

② *Height of head lamp:*



$$S\phi + h = \frac{(g_2 - g_1)\,L}{2}$$

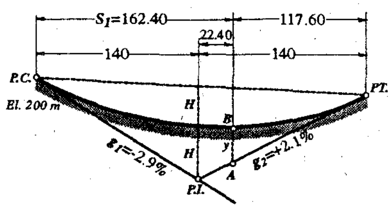
$$\phi = \frac{1^\circ(\pi)}{180^\circ}$$

$$\phi = 0.017$$

$$h + 364.45\,(0.017) = \frac{(0.021 + 0.029)\,(280)}{2}$$

h = ***0.80 m.***

③ *Elevation of lowest point of curve:*



$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$S_1 = \frac{-0.029\,(280)}{-0.029 - 0.021}$$

S_1 = 162.40 m.

$$H = \frac{L}{8}(g_2 - g_1)$$

$$H = \frac{280}{8}(0.021 + 0.029)$$

H = 1..75 m.

$$\frac{y}{(117.6)^2} = \frac{1.75}{(140)^2}$$

y = 1.23

Elev. of lowest point of curve
= 200 - 0.029 (140) + 0.021 (22.40) + 1.23
= ***197.64 m.***

Problem 396:

The design speed of a vertical sag parabolic curve is 100 kph. The tangent grades of the curve are - 2.2% and + 2.8% respectively.

① Compute the length of curve.
② If the perception time of the driver is 0.75 seconds and the coefficient of friction between the tires and the pavement is 0.16, compute for the head lamp sight distance assuming that the head lamp is 0.70 m. above the road surface.
③ Compute the angle that the beam light tilts above the longitudinal axis of the car.

Solution:

① *Length of curve:*

$$L = \frac{AV^2}{395}$$

$A = g_2 - g_1$

$A = 2.8 - (-2.2)$

$A = 5$

$$L = \frac{(5)\,(100)^2}{395}$$

L = ***126.58 m.***

② *Head lamp sight distance:*

$$S = Vt + \frac{V^2}{2g\,f}$$

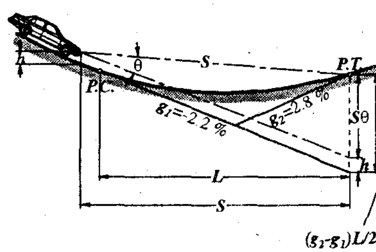
$$V = \frac{100000}{3600}$$

V = 27.78 m/s

$$S = 27.78\,(0.75) + \frac{(27.78)^2}{2\,(9.81)\,(0.16)}$$

S = ***266.67 m.***

③ *Angle the head lamp tilts:*



HEAD LAMP SIGHT DISTANCE

$$S\phi + h = \frac{(g_2 - g_1)L}{2}$$

$$266.67\phi + 0.70 = \frac{(0.028 + 0.022)(126.58)}{2}$$

$$\phi = 0.00924$$

$$\phi = \frac{0.00924\,(180)}{\pi}$$

$$\phi = \mathbf{0.53^\circ}$$

Problem 397:

The sag vertical curve has a back tangent grade of - 3% and a forward tangent grade of +2% is to be designed on the basis that the head lamp sight distance of a car traveling along the curve equals the minimum safe stopping distance with the following specifications.

Velocity = 120 kph

Perception reaction time = $\frac{3}{4}$ sec.

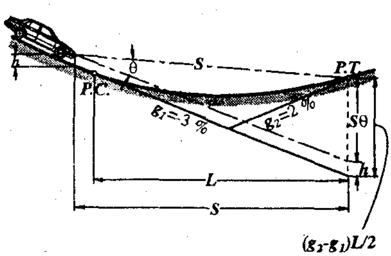
Coefficient of friction between road pavement and tires is 0.15.

The head lamps are 0.75 m. above the road surface and their beams tilts upward at an angle of one degree above the longitudinal axis of the car.

① Compute the head lamp sight distance.
② Compute the length of the sag curve.
③ Compute the max. velocity of the car that could pass thru the sag curve in kph.

Solution:

① *Head lamp sight distance:*



$$S = Vt + \frac{V^2}{2gf}$$

$$V = \frac{120000}{3600}$$

$$V = 33.33 \text{ m/s}$$

$$S = 33.33\left(\frac{3}{4}\right) + \frac{(33.33)^2}{2\,(9.81)(0.15)}$$

$$S = \mathbf{402.47\ m.}$$

② *Length of curve:*

$$L = \frac{2\,(h + S\phi)}{g_2 - g_1}$$

$$\phi = \frac{1^\circ(\pi)}{180}$$

$$\phi = 0.017 \text{ rad}$$

$$L = \frac{2\,[0.75 + (402.47)(0.017)]}{(0.02) - (-0.03)}$$

$$L = \mathbf{310.98\ m.}$$

③ *Max. velocity:*

$$L = \frac{AV^2}{395}$$

$$A = 2 - (-3)$$

$$A = 5$$

$$310.98 = \frac{(5)\,V^2}{395}$$

$$V = \mathbf{156.74\ kph}$$

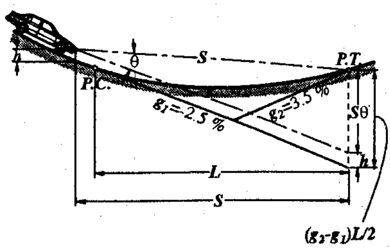
Problem 398:

The length of the sag curve having grades of - 2.5% and +3.5% is equal to 310 m.

① Compute the length of the head lamp sight distance if the head lamps are 0.80 m. above the road surface and their beams tilts upward at an angle of 1.2 degrees above the longitudinal axis of the car.
② Compute the max. speed of the car passing thru sag curve.
③ Determine the perception reaction time in seconds if the coefficient of friction between the road pavement and tires is 0.25.

Solution:

① *Head lamp sight distance:*



$$L = \frac{2\,(h + S\phi)}{g_2 - g_1}$$

$$\phi = \frac{1.2\,\pi}{180}$$

$$\phi = 0.021 \text{ rad}$$

$$310 = \frac{2\,[0.80 + S\,(0.021)]}{(0.035) - (-0.025)}$$

$$0.80 + 0.021\,S = 9.3$$

$$S = \mathbf{404.76\ m.}$$

② *Max. speed of car:*

$$L = \frac{AV^2}{395}$$

$$A = 3.5 - (-2.5)$$

$$A = 6$$

$$310 = \frac{6\,V^2}{395}$$

$$V = \mathbf{142.86\ kph}$$

③ *Perception reaction time:*

$$S = Vt + \frac{V^2}{2g\,f}$$

$$V = \frac{142.86\,(1000)}{3600}$$

$$V = 39.68 \text{ m/s}$$

$$S = Vt + \frac{V^2}{2g\,f}$$

$$404.76 = 39.68\,t + \frac{(39.68)^2}{2\,(9.81)(0.25)}$$

$$t = \mathbf{2.11\ sec.}$$

Problem 399:

The design velocity of a sag curve is 150 kph. The curve has tangent grades of - 3.2% and +1.8%. The perception reaction time is 2.1 seconds.

① Compute the length of the sag curve.
② Compute the minimum safe stopping distance if the coeff. of friction between tires and road is 0.40.
③ Compute the height of the head lamp above the road surface if the beams of the head lamp tilts at an angle of 1.1 degrees above the longitudinal axis of the car.

Solution:

① *Length of sag curve:*

$$L = \frac{AV^2}{395}$$

$$A = 1.8 - (-3.2)$$

$$A = 5$$

$$L = \frac{(5)\,(150)^2}{395}$$

$$L = \mathbf{284.81\ m.}$$

② *Min. safe stopping distance:*

$$S = Vt + \frac{V^2}{2g\,f}$$

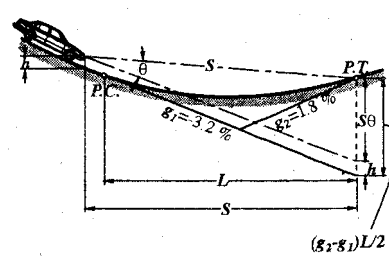
$$V = \frac{150\,(1000)}{3600}$$

$$V = 41.67 \text{ m/s}$$

$$S = 41.67\,(2.1) + \frac{(41.67)^2}{2\,(9.81)\,(0.40)}$$

$$S = \mathbf{308.76}$$

③ *Height of head lamp:*



HEAD LAMP SIGHT DISTANCE

$$L = \frac{2\,(h + S\phi)}{g_2 - g_1}$$

$$\phi = \frac{1.1\,(\pi)}{180}$$

$$\phi = 0.0192 \text{ rad}$$

$$284.81 = \frac{2\,[h + 308.76\,(0.0192)]}{0.018 - (-0.032)}$$

$$h + 5.928 = 7.120$$

$$h = \mathbf{1.19\ m.}$$

Problem 400:

A parabolic sag curve has grade tangents of - 3% and +2% intersecting at station (10 + 120.60) having an elevation of 326.42 m. A car approaches the curve at a speed of 80 kph. If the driver is driving on the night time he has to switch on the head light to avoid collision of cars moving towards his direction. The coeff. of friction between the tires and the pavement is 0.20. The perception-reaction time of the driver is 3/4 of a seconds.

① Compute the head lamp sight distance considering the downward slope.
② If the length of the curve is 200 m. how high is the head lamp above the pavement if the beam light has an angle of tilt of 0.90° above the longitudinal axis of the car.
③ Determine the max. speed that a car could safely travel along this curve.

Solution:

① *Head lamp sight distance:*

$$S = Vt + \frac{V^2}{2g\,(f + G)}$$

$$V = \frac{80000}{3600}$$

$$V = 22.22 \text{ m/s}$$

$$S = \frac{22.22\,(3)}{4} + \frac{(22.22)^2}{2\,(9.81)\,(0.20 - 0.03)}$$

$$S = \mathbf{164.69\ m.}$$

② *Height of head lamp above the pavement:*

$$L > S$$

$$L = \frac{S^2\,(g_2 - g_1)}{2\,(S\phi + h)}$$

$$200 = \frac{(164.69)^2\,(0.02 + 0.03)}{2\left[(164.69)\,\frac{(0.9)\,\pi}{180} + h\right]}$$

$$\frac{164.29\,(0.9)\,\pi}{180} + h = 3.39$$

$$h = \mathbf{0.80\ m.}$$

③ *Max. speed that a car could safely travel:*

$$L = \frac{AV^2}{395}$$

$$A = g_2 - g_1$$

$$A = 2 - (-3)$$

$$A = 5$$

$$200 = \frac{5\,V^2}{395}$$

$$V = \mathbf{125.70\ kph}$$

Problem 401:

A vertical summit curve has tangent grades of +2.8% and - 1.6%. A motorist whose eyesight is 1.5 m. above the roadway sighted the top of a visible object 100 mm at the right side of the summit.

① Determine the length of curve of a sight distance of 130 m.
② Determine the stationing of highest point of curve if the P.C. is at sta. 10 + 040 and has an elevation of 100 m.
③ Determine the elevation of the highest point of the curve.

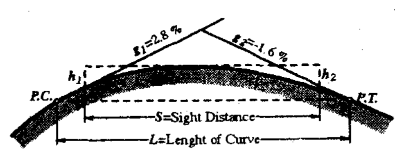
Solution:

① *Length of curve of a sight distance of 130 m.*

$h_1 = 1.5$ m.

$h_2 = 100$ mm

$h_2 = 0.10$ m.



HEAD LAMP SIGHT DISTANCE

Assume sight distance is lesser than the length of curve:

$A = 2.8 - (-1.6)$

$A = 4.4$

$$L = \frac{AS^2}{100\,(\sqrt{2\,h_1} + \sqrt{2\,h_2})^2}$$

$$L = \frac{4.4\,(130)^2}{100\,(\sqrt{3} + \sqrt{0.2})^2}$$

$L =$ ***156.574 m.*** > 130 m. *(ok)*

② *Stationing of highest point of curve:*

$$S_1 = \frac{g_1\,L}{g_1 - g_2}$$

$$S_1 = \frac{0.028\,(156.574)}{0.028 + 0.016}$$

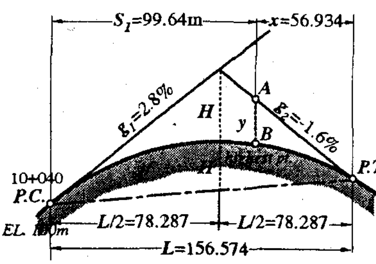
$S_1 = 99.64$ m.

Stationing of the highest point of curve:

$= (10 + 040) + 99.64$

$=$ ***10 + 139.64***

③ *Elevation of highest point of curve:*



$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{156.574}{8}$$

$H = 0.86$ m.

$$\frac{y}{(59.934)^2} = \frac{0.86}{(78.287)^2}$$

$y = 0.45$ m.

Elev. of P.T. $= 100 + 78.287\,(0.028) - 78.287\,(0.016)$

Elev. of P.T. $= 100.94$ m.

Elevation of highest point of curve:

Elev. B = Elev. A - y

Elev. A $= 100.94 + 0.016\,(56.934)$

Elev. A $= 101.85$ m.

Elev. B $= 101.85 - 0.45$

Elev. B $=$ ***101.40 m.***

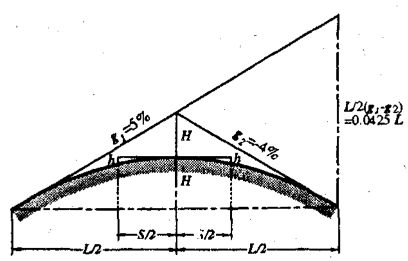
Problem 402:

A vertical curve connects a +5% grade to a - 4% grade as shown. The station of the point of vertical intersection of the tangents is at sta. 72 + 000 and the elevation of the point of intersection is 100 m. Determine the length of curve for a sight distance of 150 m., the height of object and observer being 1.5 m. above the pavement. If on the right side of the summit of the curve is an object having a height of 0.60 m.

① Compute the stationing of the highest point of the curve.

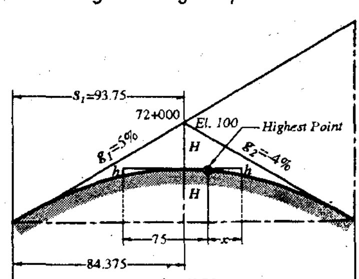
② Compute the length of the non-passing sight distance.

③ Compute the stationing of the non-passing distance.

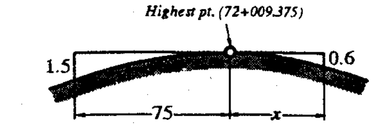


Solution:

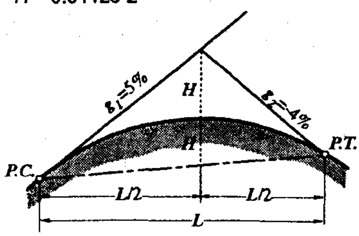
① *Stationing of the highest point of the curve:*

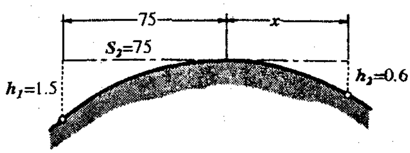


HEAD LAMP SIGHT DISTANCE



$$H=\frac{L}{8}(g_1-g_2)$$
$$H=\frac{L}{8}(0.05+0.04)$$
$$H=0.01125\,L$$





Using the squared property of parabola:

$$\frac{H}{\left(\frac{L}{2}\right)^2}=\frac{h}{\left(\frac{S}{2}\right)^2}$$

$$\frac{\frac{L}{8}(g_1-g_2)}{\left(\frac{L}{2}\right)^2}=\frac{h}{\left(\frac{S}{2}\right)^2}$$

$$Lh=\frac{S^2(g_1-g_2)}{8}$$

$$L=\frac{S^2(g_1-g_2)}{8h}$$

$$L=\frac{(150)^2(0.05+0.04)}{8\,(1.5)}$$

$L = 168.75 > S = 150$ ok.

$$\text{Sta. of P.C.}=72+000-\frac{168.75}{2}$$

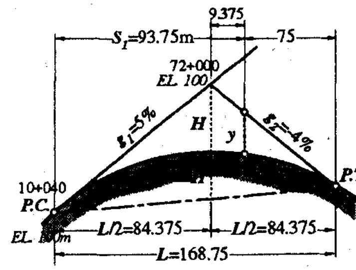
Sta. of P.C. = 71 + 915.625

$$S_1=\frac{g_1L}{g_1-g_2}$$

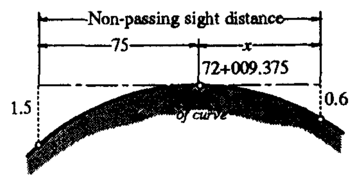
$$S_1=\frac{0.05\,(168.75)}{0.05+0.04}$$

$S_1 = 93.75$

Sta. of highest point = (71 + 915.625) + 93.75
Sta. of highest point = ***72 + 009.375***



② *Length of non-passing sight distance:*



$$\frac{1.5}{(75)^2}=\frac{0.6}{(x)^2}$$

$x = 47.43$

Non-passing sight distance = 75 + 47.43
Non-passing sight distance = ***122.43 m.***

③ *Stationing of non-passing sight distance:*
(72 + 009.375) + 47.43 = 72 + 056.805
(72 + 009.375) - 75 = ***71 + 934.375***

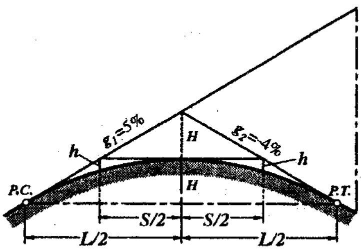
Problem 403:

A grade ascending at the rate of 5% meets another grade descending at the rate of 4% at the vertex of elevation 20.00 m. and stationing 5 + 000. The height of the eyes of the drivers above the pavement at each end of the sight distance which is 150 m. long is 1.5 m.

① Compute the length of the vertical parabolic curve.
② Compute the stationing of the highest point of the curve.
③ Compute the elevation of the highest point of the curve.

HEAD LAMP SIGHT DISTANCE

Solution:



① *Length of curve:*

Assume S < L

$$\frac{H}{\left(\frac{L}{2}\right)^2} = \frac{h}{\left(\frac{S}{2}\right)^2}$$

$$\frac{\frac{L}{8}(g_1 - g_2)}{\left(\frac{L}{2}\right)^2} = \frac{h}{\left(\frac{S}{2}\right)^2}$$

$$L = \frac{S^2 (g_1 - g_2)}{8h}$$

$A = g_1 - g_2$

$A = 5 - (-4)$

$A = 9$

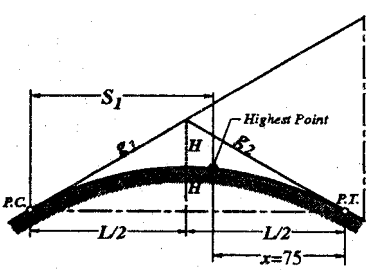
$$L = \frac{9\,(150)^2}{100\left(\sqrt{2\,(1.5)} + \sqrt{2\,(1.5)}\right)^2}$$

$L = 168.75 > 150$ *(ok)*

$$L = \frac{(150)^2\,(0.05 + 0.04)}{8\,(1.5)}$$

$L = \mathbf{168.75} > 150$ *(ok)*

② *Location of highest point of curve:*



$$S_1 = \frac{g_1 L}{(g_1 - g_2)}$$

$$S_1 = \frac{0.05\,(168.75)}{(0.05 + 0.04)}$$

$S_1 = 93.75$ m.

$$\text{Sta. of P.C.} = 5 + 00 - \frac{168.75}{2}$$

Sta. of P.C. = 4 + 915.625

Sta. of highest point of curve

= (4 + 915.625) + 93.75

= ***5 + 009.375***

③ *Elev. of highest point of curve:*

$$H = \frac{L}{8}(g_1 - g_2)$$

$$H = \frac{168.75}{8}(0.05 + 0.04)$$

$H = 1.90$ m.

$$\frac{H}{\left(\frac{L}{2}\right)^2} = \frac{y}{x^2}$$

$$\frac{1.90}{(84.375)^2} = \frac{y}{(75)^2}$$

$y = 1.50$ m.

Elev. of P.T. = 20 - 0.04 (84.375)

Elev. of P.T. = 16.625 m.

Elev. of B = 16.625 + (0.04)(75)

Elev. of B = 19.625 m.

Elev. of highest point of curve = 19.625 - 1.5

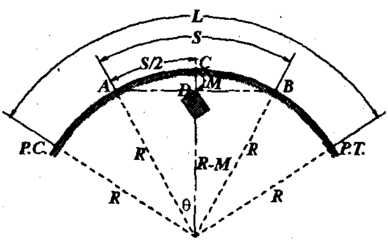
Elev. of highest point of curve = ***18.125***

SIGHT DISTANCE

SIGHT DISTANCES (Horizontal Curves)

① When

$S < L$



S = sight distance
L = length of curve

$(AC)^2 = M^2 + (AD)^2$

$(AD)^2 = R^2 - (R - M)^2$

$(AD)^2 = R^2 - (R^2 - 2\,RM + M^2)$

$(AD)^2 = 2\,RM - M^2$

$AC = \frac{1}{2}S$ (approximately)

$\left(\frac{S}{2}\right)^2 = M^2 + 2\,RM - M^2$

$\frac{S^2}{4} = 2\,RM$

$$M = \frac{S^2}{8\,R}$$

M = clear distance from center of roadway to the obstruction
S = sight distance along the center of roadway
R = radius of center - line curve
L = length of curve
D = degree of curve

$\text{Cos } \varnothing = \frac{R - M}{R}$

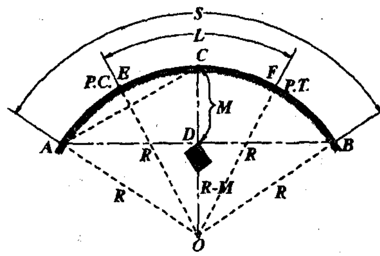
$R - M = R \text{ Cos } \varnothing$

$M = R - R \text{ Cos } \varnothing$

$M = R\,(1 - \text{Cos } \varnothing)$

② When

$S > L$



$L + 2d = S$

$d = \frac{S - L}{2}$

$(AC)^2 = (AD)^2 + M^2$

$(AD)^2 = (AO)^2 - (R - M)^2$

$(AO)^2 = (AE)^2 + R^2$

$(AD)^2 = (AE)^2 + R^2 - (R - M)^2$

$(AD)^2 = d^2 + R^2 - R^2 + 2\,RM - M^2$

$(AD)^2 = d^2 + 2\,RM - M^2$

$(AC)^2 = (AD)^2 + M^2$

$(AC)^2 = d^2 + 2\,RM - M^2 + M^2$

$(AC)^2 = d^2 + 2\,RM$

$(AC)^2 = \left(\frac{S - L}{2}\right)^2 + 2\,RM$

Let $AC = \frac{S}{2}$

$\frac{S^2}{4} = \frac{(S - L)^2}{4} + 2\,RM$

$S^2 = S^2 - 2\,SL + L^2 + 8\,RM$

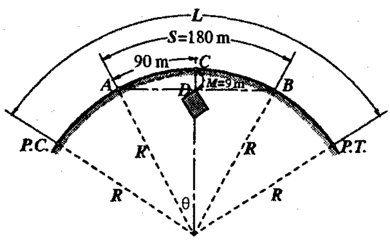
$8\,RM = 2\,SL - L^2$

$$M = \frac{L\,(2S - L)}{8\,R}$$

Problem 404:

The clearance to an obstruction is 9 m. and the desirable sight distance when rounding a horizontal curve is 180 m. Determine the minimum radius of the horizontal curve.

Solution:



$$M = \frac{S^2}{8R}$$

$$R = \frac{S^2}{8M}$$

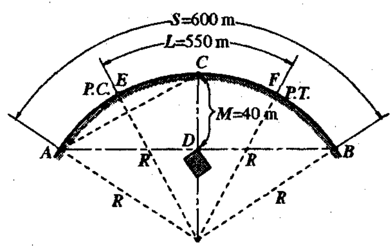
$$R = \frac{(190)^2}{8(9)}$$

$R = $ ***450 m.*** *(min. radius of horizontal curve)*

Problem 405:

The clearance to an obstruction is 40 m. and the desirable sight distance when rounding a horizontal curve is 600 m. Determine the minimum radius of horizontal curve if the length of curve is 550 m. long.

Solution:



$$M = \frac{L(2S - L)}{8R}$$

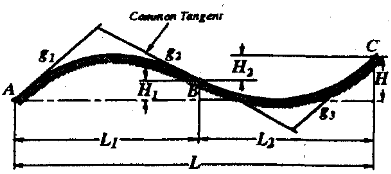
$$R = \frac{L(2S - L)}{8M}$$

$$R = \frac{550\,[2(600) - 550]}{8(40)}$$

$R = $ ***1117.19 m.***

REVERSED VERTICAL PARABOLIC CURVE

Reversed Vertical Parabolic Curve



g_1 = grade of approaching tangent
g_3 = grade of receding tangent
g_2 = grade of common tangent
L_1 = length of first curve
L_2 = length of second curve
L = total length of curve
$L = L_1 + L_2$
H_1 = difference in elevation between A and B
H_2 = difference in elevation between B and C
r_1 = rate of change of grade of first curve
r_2 = rate of change of grade of second curve

$$r_1 = \frac{g_2 - g_1}{L_1}$$

$$r_2 = \frac{g_3 - g_2}{L_2}$$

$$L = L_1 + L_2$$

$$L = \frac{g_2 - g_1}{r_1} + \frac{g_3 - g_2}{r_2}$$

$$L_1 = \frac{g_2 - g_1}{r_1}$$

$$L_2 = \frac{g_3 - g_2}{r_2}$$

$$\text{Elev. B} = \text{Elev. A} + \left[\frac{g_1 L_1}{2} - \left(-\frac{g_2 L_1}{2}\right)\right]$$

$$\text{Elev. B} - \text{Elev. A} = \frac{(g_1 + g_2) L_1}{2}$$

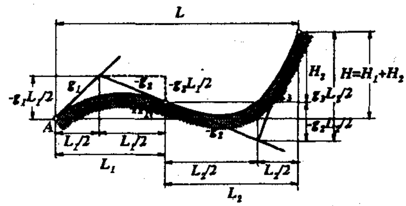
$$H_1 = \frac{(g_1 + g_2) L_1}{2}$$

$$\text{Elev. C} = \text{Elev. B} + \left[\frac{g_3 L_2}{2} - \left(-\frac{g_2 L_2}{2}\right)\right]$$

$$\text{Elev. C} - \text{Elev. B} = \frac{(g_3 + g_2) L_2}{2}$$

$$H_2 = \frac{(g_3 + g_2) L_2}{2}$$

$$H = H_1 + H_2$$



$$H_1 = \frac{g_1 L_1}{2} - \left(\frac{-g_2 L_1}{2}\right)$$

$$H_1 = \frac{(g_1 + g_2) L_1}{2}$$

$$H_2 = \frac{g_3 L_2}{2} - \left(\frac{-g_2 L_2}{2}\right)$$

$$H_2 = \frac{(g_3 + g_2) L_2}{2}$$

$$r_1 = \frac{g_2 - g_1}{L_1}$$

$$L_1 = \frac{g_2 - g_1}{r_1}$$

$$r_2 = \frac{g_3 - g_2}{L_2}$$

$$L_2 = \frac{g_3 - g_2}{r_2}$$

$$H_1 = \frac{(g_1 + g_2) L_1}{2}$$

$$H_1 = \frac{(g_1 + g_2)(g_2 - g_1)}{2 r_1}$$

$$H_1 = \frac{g_2^2 - g_1^2}{2 r_1}$$

$$H_2 = \frac{(g_3 + g_2) L_2}{2}$$

$$H_2 = \frac{(g_3 + g_2)(g_3 - g_2)}{2 r_2}$$

$$H_2 = \frac{g_3^2 - g_2^2}{2 r_2}$$

$$H = H_1 + H_2$$

REVERSED VERTICAL PARABOLIC CURVE

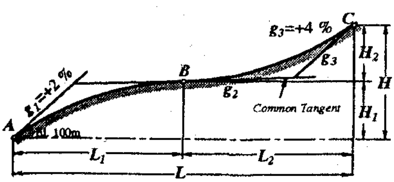
Problem 406:

From the given reversed parabolic curve shown, the rate of change of grade of the first curve is - 0.5 and the rate of change of grade of the second curve is 0.5. If the difference in elevation between A and C is 18 m., and the grade of approaching tangent is +2% and the grade of the receding tangent is +4%.

① Determine the grade of the common tangent.
② Determine the total length of first and second curve, assume one station is 20 m. long.
③ Determine the elev. of B if elev. of A = 100 m.

Solution:

① *Grade of the common tangent:*



$$H_1 = \frac{(g_1 - g_2)\,L_1}{2}$$

$$H_2 = \frac{(g_3 + g_2)\,L_2}{2}$$

$$H_1 = \frac{(g_1 + g_2)\,(g_2 - g_1)}{2\,r_1}$$

$$H_1 = \frac{g_2^2 - g_1^2}{2\,r_1}$$

$$H_1 = \frac{g_2^2 - (2)^2}{2\,(-0.5)}$$

$$H_1 = \frac{g_2^2 - (4)}{-1.0}$$

$$H_2 = \frac{(g_3 + g_2)\,(g_3 - g_2)}{2\,r_2}$$

$$H_2 = \frac{g_3^2 - g_2^2}{2\,r_2}$$

$$H_2 = \frac{(4)^2 - g_2^2}{2\,(0.5)}$$

$$H_2 = \frac{16 - g_2^2}{1}$$

$$H_1 + H_2 = 18$$

$$\frac{g_2^2 - 4}{(-1.0)} + \frac{16 - g_2^2}{1} = 18$$

$$2\,g_2^2 = -18 + 20$$

$$g_2^2 = 1$$

g_2 = ***1%*** *(grade of common tangent)*

② *Length of first and second curve, assume one station is 20 m. long:*

$$L_1 = \frac{g_2 - g_1}{r_1}$$

$$L_1 = \frac{1 - 2}{-0.5}$$

L_1 = 2 stations

L_1 = 2 (20)

L_1 = **40 m.** *(length of first curve)*

$$L_2 = \frac{g_3 - g_2}{r_2}$$

$$L_2 = \frac{4 - 1}{0.50}$$

L_2 = 6 stations

L_2 = 6 (20)

L_2 = 120 m. *(length of 2nd curve)*

Total length = 40 + 120
Total length = ***160 m.***

③ *Elev. of B:*
Total length of the curve = 40 + 120
Total length of the curve = 160 m.

$$H_1 = \frac{g_2^2 - g_1^2}{2\,r_1}$$

$$H_1 = \frac{(1)^2 - (2)^2}{2\,(-0.5)}$$

$$H_1 = \frac{1 - 4}{-1}$$

H_1 = 3 m.

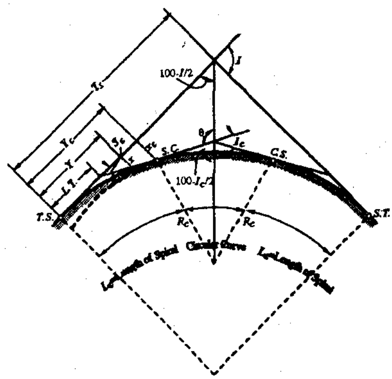
Elev. of B = 100 + 3
Elev. of B = ***103 m.***

SPIRAL CURVE

Spiral Curve

Elements of a spiral curve:

1. S.C. = spiral to curve
2. C.S. = curve to spiral
3. S.T. = spiral to tangent
4. T_s = tangent distance
5. T_c = tangent distance for the curve
6. I = angle of intersection of spiral easement curve
7. I_c = angle of intersection of simple curve
8. T.S. = tangent to spiral
9. R_c = radius of simple curve
10. D_c = degree of simple curve
11. L.T. = long tangent
12. S.T. = short tangent
13. E_s = external distance of the spiral curve
14. L.C. = long chord of spiral transition
15. X_c = offset from tangent at S.C.
16. Y_c = distance along the tangent from the T.S. to S.C.
17. X = offset from tangent at any point on the spiral
18. Y = distance along tangent at any point on the spiral
19. S_c = spiral angle at S.C.
20. i = deflection angle at any point on the spiral, it is proportional to the square of its distance.
21. L_c = length of spiral
22. L = length of spiral from T.S. to any point along the spiral

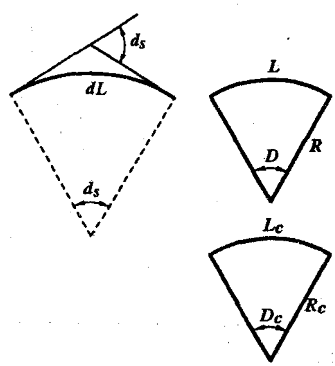


$$dL = R\,ds$$

$$ds = \frac{dL}{R}$$

$$\frac{D}{L} = \frac{D_c}{L_c}$$

But D is inversely proportional to R:



$$D = \frac{1145.916}{R} = \frac{K}{R}$$

$$\frac{K}{RL} = \frac{K}{R_c L_c}$$

$$R = \frac{R_c L_c}{L}$$

$$ds = \frac{dL\,L}{R_c L_c}$$

$$ds = \frac{L\,dL}{R_c L_c}$$

SPIRAL CURVE

$$s = \frac{L^2}{2R_c L_c}$$

At S.C.: $L = L_c$

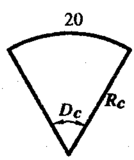
$$s_c = \frac{L_c^{\,2}}{2R_c L_c}$$

$$s_c = \frac{L_c}{2R_c}$$

$$s = \frac{L^2 (180)}{2R_c L_c \pi}$$

(spiral angle at any point on the spiral)

For metric system:



$$20 = R_c D_c$$

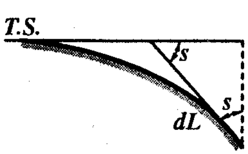
$$R_c = \frac{20}{D_c}$$

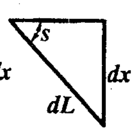
$$s_c = \frac{L_c D_c}{40}$$

(spiral angle at S.C.)

$dx = dL \sin s$

$\sin s = s$ for small angles





$$s = \frac{L^2}{2R_c L_c}$$

$$dx = s\, dL$$

$$dx = \frac{L^2\, dL}{2R_c L_c}$$

$$x = \frac{L^3}{6 R_c L_c}$$

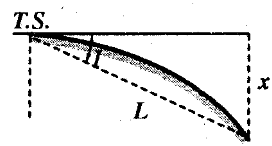
At S.C.: $L = L_c$

$$X_c = \frac{L^3}{6R_c L_c}$$

$$X_c = \frac{L_c^{\,2}}{6 R_c}$$

$$\frac{x}{X_c} = \frac{\dfrac{L^3}{6R_c L_c}}{\dfrac{L_c^{\,2}}{6R_c}}$$

$$X = \frac{L^3 X_c}{L_c^{\,3}}$$



$$\sin i = \frac{X}{L}$$

$$i = \frac{X}{L}$$

$$i = \frac{L^3}{L\, 6R_c L_c}$$

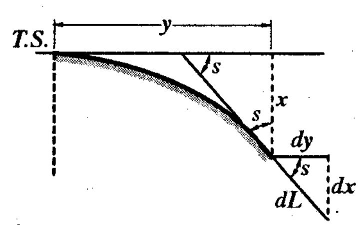
$$i = \frac{L^2}{6R_c L_c}$$

$$s = \frac{L^2}{2R_c L_c}$$

$$\frac{s}{3} = \frac{L^2}{6R_c L_c}$$

$$i = \frac{s}{3}$$

From the correction for slope:



$$c = \frac{h}{2S}$$

SPIRAL CURVE

$h = dx$
$S = dL$
$$c = \frac{(dx)^2}{2\,dL}$$
$dy = dL - c$
$$dy = dL - \frac{(dx)^2}{2\,dL}$$
$dx = dL \sin s$
$dx = s\,dL$
$$dy = dL - \frac{s^2\,dL^2}{2\,dL}$$
$$dy = dL - \frac{s^2\,dL}{2}$$

$$s = \frac{L^2}{2\,R_c\,L_c}$$

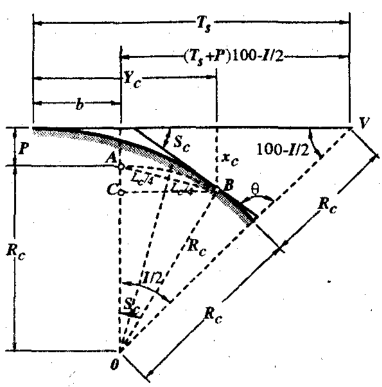
$$dy = dL - \frac{L^5}{8R_c^2\,L_c^2}$$

$$y = L - \frac{L^5}{40\,R_c^2\,L_c^2}$$

At S.C.: $L = L_c$
$$y_c = L_c - \frac{L_c^5}{40\,R_c^2\,L_c^2}$$

$$y_c = L_c - \frac{L_c^3}{40\,R_c^2}$$

From the figure shown:



$AB = R_c\,S_c$
$$AB = \frac{R_c\,L_c}{2R_c}$$
$$AB = \frac{L_c}{2}$$
$AB = b$ *(approximately)*

By ratio and proportion:
$$\frac{AG}{\frac{L_c}{2}} = \frac{\frac{L_c}{4}}{R_c}$$
$$AG = \frac{L_c^2}{8\,R_c}$$

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$AG = \left(\frac{3}{4}\right)\frac{L_c^2}{6\,R_c}$$
$$AG = \frac{L_c^2}{8\,R_c}$$
$$AG = \left(\frac{3}{4}\right)X_c$$
$p = X_c - AG$
$$p = X_c - \left(\frac{3}{4}\right)X_c$$
$$p = \frac{X_c}{4}$$

$$p = \frac{L_c^2}{24\,R_c}$$

$$T_s = b + (R_c + p)\tan\frac{1}{2}$$

$$T_s = \frac{L_c}{2} + \left(R_c + \frac{X_c}{4}\right)\tan\frac{1}{2}$$

$$\sin\frac{1}{2} = \frac{(R_c + p)\tan\frac{1}{2}}{OB}$$

$$OV = \frac{(R_c + p)\tan\frac{1}{2}}{\sin\frac{1}{2}}$$

SPIRAL CURVE

$$E_s = \frac{(R_c + p)\tan\frac{1}{2}}{\sin\frac{1}{2}} R_c$$

$$E_s = \left(R_c + \frac{X_c}{4}\right)\sec\frac{1}{2} - R_c$$

$$ø = 180 - \frac{(180 - I_c)}{2}$$

$$ø = \frac{180 + I_c}{2}$$

$$s_c + \frac{(180 - I)}{2} + \frac{(180 + I_c)}{2} = 180$$

$$2s_c + 360 - I + I_c = 360$$

$$I_c = I - 2\,s_c$$

SUMMARY OF FORMULAS FOR SPIRAL CURVE

1. $s = \frac{L^2}{2R_c L_c} \times \frac{180}{\pi}$ *(spiral angle at any point on the spiral)*
2. $s_c = \frac{D_c L_c}{40}$ *(spiral angle at S.C.)* *are basis, metric system*
3. $s_c = \frac{L_c}{2R_c} \times \frac{180}{\pi}$ *(spiral angle at S.C.)*
4. $X_c = \frac{L_c^2}{6R_c}$ *(offset distance from tangent at S.C.)*
5. $X = X_c \frac{L^3}{L_c^3}$
6. $i = \frac{s}{3}$ *(deflection angle at any point on the spiral)*
7. $y = L - \frac{L^5}{40\,R_c^2\,L_c^2}$ *(distance along tangent at any point in the spiral)*
8. $y_c = L_c - \frac{L_c^3}{40\,R_c^2}$ *(distance along tangent at S.C. from T.S.)*
9. $T_s = \frac{L_c}{2} + \left(R_c + \frac{X_c}{4}\right)\tan\frac{1}{2}$ *(tangent distance for spiral)*
10. $E_s = \left(R_c + \frac{X_c}{4}\right)\sec\frac{1}{2} - R_c$ *(external distance)*
11. $I_c = I - 2\,s_c$ *(angle of intersection of simple curve)*
12. $p = \frac{X_c}{4} - \frac{L^2}{24\,R_c}$
13. $e = \frac{0.0079\,K^2}{R}$ *(super-elevation)* *where K = kph*
14. $e = \frac{0.004\,K^2}{R}$ *(considering 75% of K to counteract the super-elevation)*
15. $L_c = \frac{0.036\,K^3}{R}$ *(desirable length of spiral)*
16. $\frac{i}{i_c} = \frac{L^2}{L_c^2}$ *(deflection angles vary as the squares of the length from the T.S.)*
17. $\frac{D}{D_c} = \frac{L}{L_c}$ *(degree of curve varies directly with the length from the T.S.)*

Problem 407:

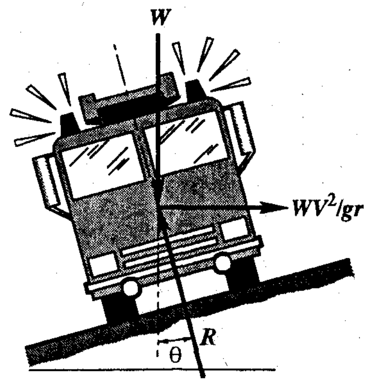
A spiral 80 m. long connects a tangent with a 6°30' circular curve. If the stationing of the T.S. is 10 + 000, and the gauge of the tract on the curve is 1.5 m.

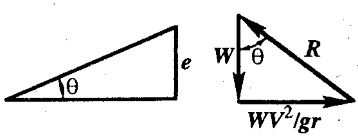
① Determine the elevation of the outer rail at the mid-point, if the velocity of the fastest train to pass over the curve is 60 kph.
② Determine the spiral angle at the first quarter point.
③ Determine the deflection angle at the end point.
④ Determine the offset from the tangent at the second quarter point.

SPIRAL CURVE

Solution:

① *Elevation of the outer rail:*





$$\tan \phi = \frac{\frac{WV^2}{gr}}{W}$$

$$\tan \phi = \frac{V^2}{gr}$$

$$V = \frac{1000\,K}{3600}$$

$$V = 0.278\,K$$

$$g = 9.8 \text{ m/sec}^2$$

$$\tan \phi = \frac{e}{1} = e$$

$$e = \frac{(0.278\,K)^2}{9.8\,r}$$

$$e = \frac{0.0079\,K^2}{r}$$

$$R = \frac{1145.916}{D}$$

$$R = \frac{1145.916}{6.5}$$

$$e = \frac{0.0079\,(60)^2\,(1.5)}{176.30}$$

$e = 0.241$ *(outer rail)*

$$e = \frac{0.241}{2}$$

$e = \mathbf{0.1205}$ *(at midpoint)*

② *Spiral angle at the first quarter point:*

$L = 20$ m.

$$s = \frac{L^2\,180}{2R_c\,L_c\,\pi}$$

$$s = \frac{(20)^2\,(180)}{2\,(176.3)\,(80)\,\pi}$$

$$s = 0.81^\circ$$

$$s = \mathbf{0^\circ 49'}$$

③ *Deflection angle at the end point:*

At the end point $L_c = 80$ m.

$$s_c = \frac{L_c\,180}{2\,R_c\,\pi}$$

$$s_c = \frac{80\,(180)}{2\,(176.3)\,\pi}$$

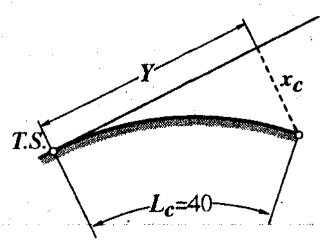
$$s_c = 13^\circ$$

$$i = \frac{s_c}{3}$$

$$i = \frac{13}{3}$$

$i = \mathbf{4.33^\circ}$ *deflection angle at the end point.*

④ *Offset from the tangent at the second quarter point:*



$L = 40$ m. *at the second quarter point*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X = X_c\,\frac{L^3}{L_c^3}$$

$$X_c = \frac{(80)^2}{6\,(176.30)}$$

$$X_c = 6.05$$

$$X = \frac{6.05\,(40)^2}{(80)^3}$$

$X = \mathbf{0.756\ m.}$

Problem 408

The tangents of a spiral curve has azimuths of 226° and 221° respectively. The minimum length of spiral is 40 m. with a minimum super-elevation of 0.10 m/m width of roadway. The maximum velocity to pass over the curve is 70 kph. Assume width of roadway to be 9 m.

① Determine the degree of simple curve.
② Determine the length of spiral at each end of simple curve.
③ Determine the super-elevation of the first 10 m. from S.C. on the spiral.

Solution:

① *Degree of simple curve:*

Use $e = \dfrac{0.004\,K^2}{R}$

$e = \dfrac{0.004\,K^2}{R}$

$0.10 = \dfrac{0.004\,(70)^2}{R}$

$R = 196$ m.

$D = \dfrac{1145.946}{R}$

$D = \dfrac{1145.916}{196}$

$D = \mathbf{5.85°}$ *(degree of curve)*

② *Length of spiral:*

$L_c = \dfrac{0.036\,K^3}{R}$

$L_c = \dfrac{0.036\,(70)^3}{196}$

$L_c = 63$ m. say ***60 m.*** *(use multiple of 10 m.)*

③ *SUper-elevation of the first 10 m. from S.C. on the spiral:*

$e_1 = \dfrac{1}{6}(0.10)$

$e_1 = 0.017$ *(at 10 m. from T.S. on the spiral)*

$e_5 = 5\,(0.017)$

$e_5 = 0.085$ *(at 10 m. from S.C. on the spiral)*

$e = 0.085\,(9)$

$e = \mathbf{0.765\ m.}$ *(super-elevation at 10 m. from S.C. on the spiral)*

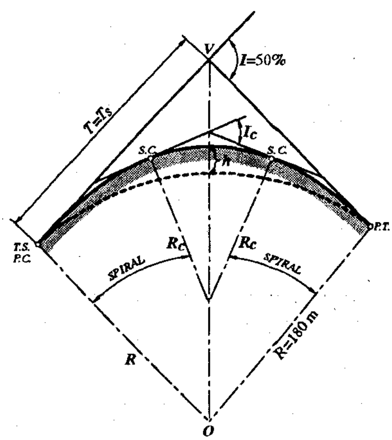
Problem 409

A simple curve having a radius of 280 m. connects two tangents intersecting at an angle of 50°. It is to be replaced by another curve having 80 m. spirals at its ends such that the point of tangency shall be the same.

① Determine the radius of the new circular curve.
② Determine the distance that the curve will nearer the vertex.
③ Determine the central angle of the circular curve.
④ Determine the deflection angle at the end point of the spiral.
⑤ Determine the offset from tangent at the end point of the spiral.
⑥ Determine the distance along the tangent at the mid-point of the spiral.

Solution:

① *Radius of the new circular curve:*



$\tan 25° = \dfrac{T_s}{280}$

$T_s = 130.57$

$T_s = \dfrac{L_c}{2} + \left(R_c + \dfrac{X_c}{4}\right)\tan\dfrac{1}{2}$

$X_c = \dfrac{L_c^{\,2}}{6\,R_c}$

$\dfrac{X_c}{4} = \dfrac{L_c^{\,2}}{24\,R_c}$

$$130.57 = \frac{80}{2} + \left[R_c + \frac{(80)^2}{24\,R_c}\right] \tan 25^\circ$$

$$90.57 = \tan 25^\circ \left[R_c + \frac{266.67}{R_c}\right]$$

$$194.22 = \frac{R_c^2 + 266.67}{R_c}$$

$$R_c^2 + 266.67 - 194.22\,R_c = 0$$

*Solving for R_c = **192.84 m.***

② *Distance that the curve will nearer the vertex:*

Old external distance:
(for old simple curve)

$$\cos 25^\circ = \frac{280}{OV}$$

$OV = 308.95$ m.
$E = 308.95 - 280$
$E = 28.95$ m.

For the new curve:
(External curve)

$$E_s = \left(R_c + \frac{X_c}{4}\right) \sec \frac{1}{2} - R_c$$

$$\frac{X_c}{4} = \frac{L_c^2}{24\,R_c}$$

③ *Central angle of the circular curve:*

$$I_c = I - 2\,s_c$$

$$s_c = \frac{L_c\,180}{2\,R_c\,\pi}$$

$$s_c = \frac{80\,(180)}{2\,(192.84)\,\pi}$$

④ *Deflection angle at the end point of the spiral:*

$$i = \frac{s_c}{3}$$

$$i = \frac{11.80}{3}$$

i = **3.96°**

⑤ *Offset from tangent at the end point of the spiral:*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\,(192.84)}$$

X_c = ***5.53 m.***

⑥ *Distance along the tangent at the mid-point of the spiral:*
when $L = 40$ m.

$$y = L - \frac{L^5}{40\,R_c^2\,L_c^2}$$

$$y = 40 - \frac{(40)^5}{40\,(192.84)^2\,(80)^2}$$

$y = 40 - 0.01$
y = ***39.99 m.***

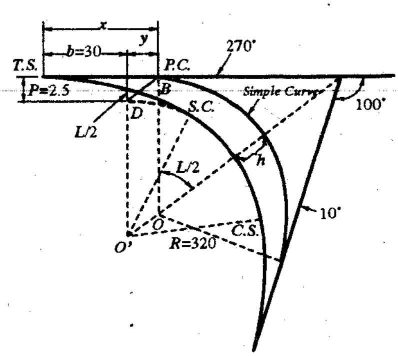
Problem 410

The two tangents of a simple curve have azimuths of 270° and 10° respectively. It has a radius of 320 m. It is required to change this curve to a spiral curve that will have values of p = 2.5 m. and b = 30 m. as shown on the figure.

① Determine the distance on which the new curve must be moved from the vertex.
② Determine the value of y.
③ Determine its distance from T.S. to the P.C. of the simple curve, if DE is parallel to h.

Solution:

① *Distance the new curve must be moved from the vertex.*



$$\cos \frac{I}{2} = \frac{P}{DE}$$

$$DE = \frac{P}{\cos \frac{I}{2}}$$

$$h = \frac{2.5}{\text{Cos } 50^\circ}$$
$h = 3.89$ m.

② *Value of y:*
$x = b + y$
$$\tan \frac{I}{2} = \frac{y}{p}$$
$y = 2.5 \tan 50^\circ$
$y = 2.98$ m.

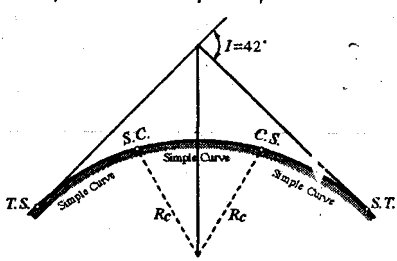
③ *Distance from T.S. to P.C.*
$x = 30 + 2.98$ m.
$x = 32.98$ m.

Problem 411

Two tangents having azimuths of 240° and 282° are connected by an 80 m. spiral curve with a 6° circular curve. The width of the roadway is 10 m. If the design velocity is 60 kph.

① Determine the super-elevation at quarter points.
② Determine the deflection angle at the end point (S.C.).
③ Determine the external distance.

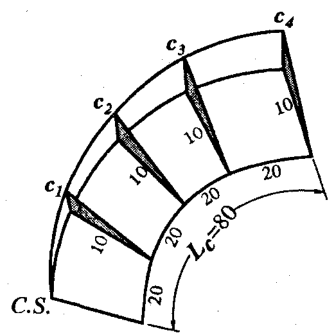
Solution:
① *Super-elevation at quarter points:*



$$R = \frac{1145.916}{D}$$
$$R = \frac{1145.916}{6} = 190.99 \text{ m.}$$
$$e = \frac{0.0079\,K^2}{R}$$
$$e = \frac{0.0079\,(60)^2}{190.99}$$
$e = 0.149$ m/m *width of roadway*

super-elevation at quarter points.
$e_4 = 0.149\,(10)$
$e_4 = 1.49$
$$e_1 = \frac{1}{4}(1.49)$$
$e_1 = 0.3725$
$$e_2 = \frac{1}{2}(1.49)$$
$e_2 = 0.745$
$$e_3 = \frac{3}{4}(1.49)$$
$e_3 = 1.118$

② *Deflection angle at the end point :*



$$S = \frac{L_c}{2\,R_c}\,\frac{180}{\pi}$$
$$S = \frac{80}{2\,(190.99)}\,\frac{180}{\pi}$$
$S = 12^\circ$
$$i = \frac{S}{3}$$
$$i = \frac{12}{3}$$
$i = 4^\circ$

③ *External distance:*
$$E_s = \left(R_c + \frac{X_c}{4}\right) \sec \frac{I}{2} - R_c$$
$I = 282 - 240$
$I = 42^\circ$

SPIRAL CURVE

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\,(190.99)}$$

$$X_c = 5.58$$

$$E_s = \left(190.99 + \frac{5.58}{4}\right) \sec 21^\circ - 190.99$$

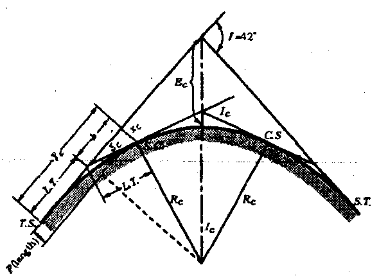
$$E_s = \mathbf{15.08\ m.}$$

Problem 412

A spiral curve was laid out in a certain portion of the Manila-Cavite Coastal Road. It has a length of spiral of 80 m. and an angle of intersection of the two tangents of 40 degrees. If the degree of curve is degrees, determine the following elements of the spiral curve to be laid out:

① Length of long and short tangent.
② External distance.
③ Length of throw.
④ Maximum velocity that a car could pass thru the curve without skidding.

Solution:



$$R_c = \frac{1145.916}{D}$$

$$R_c = \frac{1145.916}{6}$$

$$R_c = 190.99\text{ m.}$$

$$S = \frac{L_c}{2\,R_c}\,\frac{180}{\pi}$$

$$S = \frac{80\,(180)}{2\,(190.99)\,\pi}$$

$$S = 12^\circ$$

① *Length of long and short tangent:*

$$Y_c = L_c - \frac{L_c^3}{40\,R_c^2}$$

$$Y_c = 80 - \frac{(80)^3}{40\,(190.99)^2}$$

$$Y_c = 79.65\text{ m.}$$

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\,(190.99)}$$

$$X_c = 5.58\text{ m.}$$

$$\tan S = \frac{X_c}{h}$$

$$h = \frac{5.58}{\tan 12^\circ}$$

$$h = 26.25\text{ m.}$$

Long tangent (LT) $= Y_c - h$

$$LT = 79.65 - 26.25$$

$$LT = \mathbf{53.4\ m.}$$

Short tangent (ST):

$$\text{Sin } S = \frac{X_c}{ST}$$

$$ST = \frac{5.58}{\text{Sin } 12^\circ}$$

$$ST = \mathbf{26.84\ m.}$$

② *External distance:*

$$E_s = \left[R_c + \frac{X_c}{4}\right] \text{Sec } \frac{1}{2} - R_c$$

$$E_s = \left(190.99 + \frac{5.58}{4}\right) \text{Sec } 21^\circ - 190.99$$

$$E_s = \mathbf{15.08\ m.}$$

③ *Length of throw:*

$$P = \frac{X_c}{4}$$

$$P = \frac{5.58}{4}$$

$$P = \mathbf{1.395\ m.}$$

④ *Maximum velocity:*

$$L_c = \frac{0.036\,V^3}{R_c}$$

$$80 = \frac{0.036\,V^3}{190.99}$$

$$V = \mathbf{75.15\ kph}$$

SPIRAL CURVE

Problem [illegible]

As the transition curve are used to build up super elevation gradually, the radius of curvature should increase gradually from infinity to that of the circular curve, so as to enable convenient handling of the steering wheel to eliminate the shock due to the increase in centrifugal force and the radius of curvature should be inversely proportional to the length such that the rate of change of centrifugal acceleration should not cause discomfort to the driver. If a car approaches the easement curve at a speed of 90 kph and the permissible value of super elevation for the easement curve is 0.07.

① Compute the centrifugal acceleration of the car, so that it will not give discomfort to the driver.
② Compute the radius of curvature of the easement curve for a length of transition curve of 120 m. to limit the centrifugal acceleration.
③ Compute the lateral friction on the easement curve.

Solution:

① *Centrifugal acceleration:*

$$C = \frac{80}{75 + V}$$

$$C = \frac{80}{75 + 90}$$

$$C = \mathbf{0.484\ m/sec^2}$$

② *Radius of curvature:*

$$L_c = \frac{0.0215\ V^3}{CR}$$

$$120 = \frac{0.0215\ (90)^3}{0.484\ R}$$

$$R = \mathbf{269.86\ m.}$$

③ *Lateral friction on the easement curve:*

$$R = \frac{V^2}{127\ (f + e)}$$

$$269.86 = \frac{(90)^2}{127\ (f + 0.07)}$$

$$f = \mathbf{0.166}$$

Problem [illegible]

If a vehicle traveling along a tangent meets the circular arc, the full impact of centrifugal side thrust is suddenly experienced. because of this the steering angle must be changed instantaneously, otherwise the vehicle will not be able to follow the circular path. Obviously no driver can be expected to keep up to such a situation at every curve and the vehicle will be subjected to shock and the passengers subjected to sway. The situation can be greatly improved by providing transition curves. If a car approaches the transition curve at a rate of centrifugal acceleration of 0.50 m/sec³ with a degree of curve of 5° on the central curve.

① Compute the velocity of the car as it approaches the transition curve.
② Compute the spiral angle of the transition curve at the S.C.
③ Compute the length of the short tangent on the transition curve.

Solution:

① *Velocity of car.*

$$C = \frac{80}{75 + V}$$

$$0.50 = \frac{80}{75 + V}$$

$$V = \mathbf{85\ kph}$$

② *Spiral angle at the S.C.*

$$R_c = \frac{1145.916}{D}$$

$$R_c = \frac{1145.916}{5}$$

$$R_c = 229.18\ m.$$

$$S_c = \frac{L_c\ 180}{2\ R_c\ \pi}$$

$$L_c = \frac{0.0215\ V^3}{C\ R_c}$$

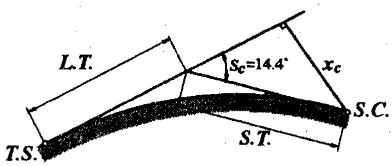
$$L_c = \frac{0.0215\ (85)^3}{0.50\ (229.18)}$$

$$L_c = 115.23\ m,$$

SPIRAL CURVE

$$S_c = \frac{L_c\,180}{2\,R_c\,\pi}$$
$$S_c = \frac{115.23\,(180)}{2\,(229.18)\,\pi}$$
$$S_c = \mathbf{14.40^\circ}$$

③ *Length of short tangent:*



$$X_c = \frac{L_c^2}{6\,R_c}$$
$$X_c = \frac{(115.23)^2}{6\,(229.18)}$$
$$X_c = 9.66$$

$$\text{Sin } 14.4^\circ = \frac{9.66}{S.T.}$$
$$S.T. = \mathbf{38.84\ m.}$$

Problem 415:

An easement spiral curve has a design speed of 100 kph. The radius of the central curve is 360 m, with a permissible super elevation of 0.07.

① Compute the centrifugal acceleration so as not to cause discomfort to the driver in m/sec^3.
② Compute the length of the transition curve to limit the centrifugal acceleration.
③ Compute the length of the short tangent of the transition curve.

Solution:

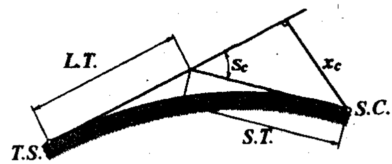
① *Centrifugal acceleration:*

$$C = \frac{80}{75 + V}$$
$$C = \frac{80}{75 + 100}$$
$$C = \mathbf{0.457\ m/sec^3}$$

② *Length of transition curve to limit centrifugal acceleration:*

$$L_c = \frac{0.0215\,V^3}{C\,R}$$
$$L_c = \frac{0.0215\,(100)^3}{0.457\,(360)}$$
$$L_c = \mathbf{130.68\ m.}$$

③ *Length of short tangent of transition curve:*



$$X_c = \frac{L_c^2}{6\,R_c}$$
$$X_c = \frac{(130.68)^2}{6\,(360)}$$
$$X_c = 7.91\text{ m.}$$

$$S_c = \frac{L_c\,(180)}{2\,R_c\,\pi}$$
$$S_c = \frac{130.68\,(180)}{2\,(360)\,\pi}$$
$$S_c = 10.4^\circ$$
$$\text{Sin } 10.4^\circ = \frac{X_c}{S.T.}$$
$$S.T. = \frac{7.91}{\text{Sin } 10.4^\circ}$$
$$S.T. = \mathbf{43.82\ m.}$$

Problem 416:

The length of a spiral curve is 80 m. with a radius of 280 m. at the central curve.

① Determine the offset distance from the tangent on the first quadrant point of the spiral.
② Compute the length of throw for the spiral curve.
③ What is the max. velocity that a car could pass thru the easement curve.

Solution:

① *Offset distance on the first quarter point:*

$$x = \frac{L^3}{6\,R_c\,L_c}$$

$$L = \frac{1}{4}(80)$$

$L = 20$ m.

$L_c = 80$ m.

$$x = \frac{(20)^3}{6\,(280)\,(80)}$$

$x = \mathbf{0.06\ m.}$

② *Length of throw:*

$$x_c = \frac{L_c^2}{6\,R_c}$$

$$x_c = \frac{(80)^2}{6\,(280)}$$

$x_c = 3.81$ m.

$$\text{Length of throw} = \frac{x_c}{4}$$

$$\text{Length of throw} = \frac{3.81}{4}$$

Length of throw = **0.95**

③ *Max. velocity:*

$$L_c = \frac{0.036\,K^3}{R_c}$$

$$80 = \frac{0.036\,K^3}{280}$$

$K = \mathbf{85.37\ kph}$

Problem 417:

The central curve of an easement curve is on a 5° curve. Spiral easement curve has a length of throw equal to 1.02 m. at the T.S.

① Compute the required length of the spiral curve.
② Determine the velocity of the car passing thru this curve so that it will not exceed the minimum centrifugal acceleration of 0.50 m/sec.
③ What is the length of the long tangent of a spiral easement curve if the distance along the tangent up to S.C. is 73.60 m. long.

Solution:

① *Length of spiral curve:*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$P = \frac{X_c}{4}$$

$$P = \frac{L_c^2}{24\,R_c}$$

$$R_c = \frac{1145.916}{5°}$$

$R_c = 229.18$ m.

$$1.02 = \frac{L_c^2}{24\,(229.18)}$$

$L_c = \mathbf{74.90\ m.}$

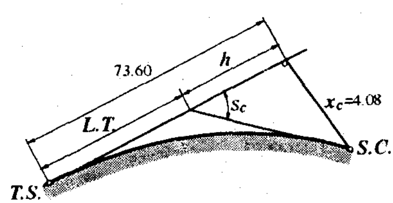
② *Velocity of car so as not to exceed the min. centrifugal acceleration:*

$$L_c = \frac{0.0215\,V^3}{CR}$$

$$74.90 = \frac{0.0215\,V^3}{0.50\,(229.18)}$$

$V = \mathbf{73.63\ kph}$

③ *Length of long tangent:*



$$S_c = \frac{L_c\,180}{2\,R_c\,\pi}$$

$$S_c = \frac{74.90\,(180)}{2\,(229.18)\,(\pi)}$$

$S_c = 9.36°$

$X_c = 4\,(1.02)$

$X_c = 4.08$ m.

$$\tan 9.36° = \frac{X_c}{h}$$

$$\tan 9.36° = \frac{4.08}{h}$$

$h = 24.75$

Long tangent = 73.60 - 24.75

Long tangent = **48.85 m.**

SPIRAL CURVE

Problem [illegible]

A car is approaching an easement curve having a rate of centrifugal acceleration of 0.5161 m/sec³ so as not to cause discomfort to the passengers. The length of spiral curve is 80 m. long.

① Compute the velocity of the approaching car in kph.
② Compute the required radius of the central curve of the easement curve to limit the centrifugal acceleration.
③ Compute the length of the long tangent of the spiral curve if the distance along the tangent from T.S. to S.C. is 79.30 m. long.

Solution:

① *Velocity of approaching car:*

$$C = \frac{80}{75 + V}$$

$$0.5161 = \frac{80}{75 + V}$$

$$V = \boldsymbol{80\ kph}$$

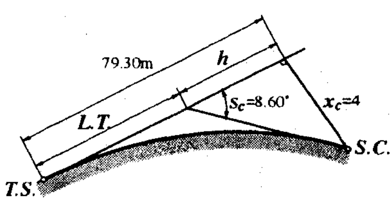
② *Radius of central curve:*

$$L_c = \frac{0.0215\ V^3}{C\ R_c}$$

$$80 = \frac{0.215\ (80)^3}{0.5161\ R_c}$$

$$R_c = \boldsymbol{266.61\ m.}$$

③ *Length of long tangent:*



$$S_c = \frac{L_c\,180}{2\,R_c\,\pi}$$

$$S_c = \frac{80\ (180)}{2\ (266.61)\ \pi}$$

$$S_c = 8.60°$$

$$X_c = \frac{L_c^{\,2}}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\ (266.61)}$$

$$X_c = 4\ m.$$

$$\tan 8.6° = \frac{4}{h}$$

$$h = 26.45$$

$$LT = 79.30 - 26.45$$

$$LT = \boldsymbol{52.85\ m.}$$

Problem 419

The spiral angle at the S.C. of a spiral easement curve is equal to 11.46°, with a radius of 200 m. for the central curve.

① Compute the length of throw at the T.S.
② Compute the length of the long tangent of the spiral easement curve if the distance along the tangent from the T.S. to the S.C. is 79.20 m.
③ Compute the value of the centrifugal acceleration in meters/sec³.

Solution:

① *Length of throw:*

$$S_c = \frac{L_c\,180}{2\,R_c\,\pi}$$

$$11.46° = \frac{L_c\,(180)}{2\ (200)\ \pi}$$

$$L_c = 80\ m.$$

$$X_c = \frac{L_c^{\,2}}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\ (200)}$$

$$X_c = 5.33$$

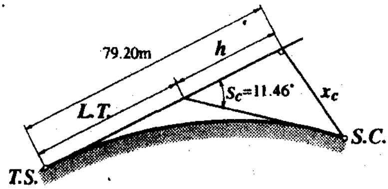
$$P = \frac{X_c}{4}$$

$$P = \frac{5.33}{4}$$

$$P = \boldsymbol{1.33}$$

SPIRAL CURVE

② *Length of long tangent:*



$$\tan 11.46° = \frac{X_c}{h}$$

$$h = \frac{5.33}{\tan 11.46°}$$

$$h = 26.29 \text{ m.}$$

Long tangent = 27.20 - 26.29

Long tangent = ***52.91 m.***

③ *Centrifugal acceleration:*

$$L_c = \frac{0.036\ V^3}{R_c}$$

$$80 = \frac{0.036\ V^3}{200}$$

$$V = 76.31 \text{ kph}$$

$$L_c = \frac{0.0215\ V^3}{CR}$$

$$80 = \frac{0.0215\ (76.31)^3}{C\ (200)}$$

$$C = \boldsymbol{0.597\ m/sec^3}$$

Problem 420:

The design speed of a car passing thru an easement curve is equal to 80 kph. The radius of the central curve of the spiral curve is equal to 260 m. long.

① Compute the value of the rate of centrifugal acceleration in m/sec for this speed.
② Compute the length of the spiral curve based on the centrifugal acceleration.
③ Compute for the length of throw.

Solution:

① *Min. value of centrifugal acceleration:*

$$C = \frac{80}{75 + V}$$

$$C = \frac{80}{75 + 80}$$

$$C = \boldsymbol{0.516\ m/s^3}$$

② *Length of spiral curve:*

$$L_c = \frac{0.0215\ V^3}{CR}$$

$$L_c = \frac{0.0215\ (80)^3}{0.516\ (260)}$$

$$L_c = \boldsymbol{82.05\ m.}$$

③ *Length of throw:*

$$P = \frac{X_c}{4}$$

$$X_c = \frac{L_c^{\,2}}{6\ R_c}$$

$$X_c = \frac{(82.05)^2}{6\ (260)}$$

$$X_c = 4.32$$

$$P = \frac{4.32}{4}$$

$$P = \boldsymbol{1.08\ m.}$$

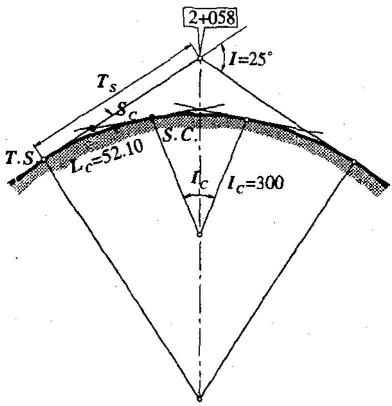
420-A CE Board May 2010

The tangents of a spiral curve forms an angle of intersection of 25° at station 2 + 058. Design speed is 80 km/hr. For a radius of central curve of 300 m. and a length of spiral of 52.10 m..

① Find the stationing at the point where the spiral starts.
② Find the stationing of the start of central curve.
③ Find the length of central curve.

Solution:

① Stationing at the point where the spiral starts.



$$T_s = \frac{L_c}{2} + \left(R_c + \frac{L_c^2}{24\,R_c}\right)\tan\frac{I}{2}$$

$$T_s = \frac{52.10}{2} + \left[300 + \frac{(52.10)^2}{24(300)}\right]\tan\frac{25}{2}$$

$$T_s = 92.64 \text{ m.}$$

Stationing @ T.S. = (2 + 058) – 92.64
Stationing @ T.S. = ***1 + 965.36***

② Stationing of the start of central curve.

Sta. @ S.C. = Sta. @ T.S. + L_c
Sta. @ S.C. = (1 + 965.36) + 52.10
Sta. @ S.C. = ***2 + 017.46***

③ Length of central curve.

$$S_c = \frac{L_c}{2\,R_c} \times \frac{180}{\pi}$$

$$S_c = \frac{52.10}{2(300)} \times \frac{180}{\pi}$$

$$S_c = 4.975^\circ$$

$I_c = I - 2\,S_c$
$I_c = 25 - 2(4.975)$
$I_c = 15.05^\circ$

Length of central curve:

$$S = R_c\,I_c\,\frac{\pi}{180}$$

$$S = 300(15.05^\circ)\,\frac{\pi}{180}$$

$$S = \mathbf{78.8\ m.}$$

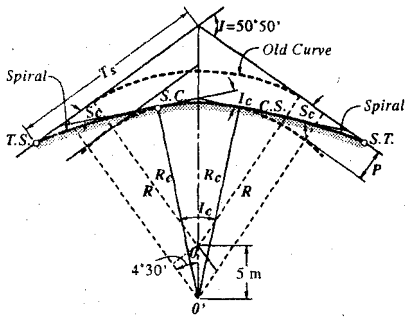
Problem 420-B

A simple curve having a degree of curve equal to 4°30' has central angle of 50°50'. It is required to replace the simple curve to another circular curve by connecting a transition curve (spiral) at each ends by maintaining the radius of the old curve and the center of the new central curve is moved away by 5 m. from the intersection point.

① Determine the central angle of the new circular curve.
② Compute the tangent distance Ts of the spiral curve.
③ What is the maximum velocity that a car could pass thru the curve without skidding?

Solution:

① Central angle of the new circular curve.



$R = R_c$

$R = \dfrac{1145.916}{4.5°}$

$R = 254.65$

$O'A = 5 \cos 25°25'$

$O'A = 4.52$ m.

$P = \dfrac{L_c^{\,2}}{24 R_c}$

$L_c^{\,2} = P\,(24)\,R_c$

$L_c^{\,2} = 4.52(24)(254.65)$

$L_c = 166.2$ m. (length of spiral)

$S_c = \dfrac{Lc\,180°}{2 R_c \pi}$

$S_c = \dfrac{166.2\,(180°)}{2\,(254.65)\,\pi}$

$S_c = 18°42'$

$I_c = I - 2\,S_c$

$I_c = 50°50' - 2(18°42')$

$I_c =$ ***13°26'*** *(central angle of the new circular curve)*

② Tangent distance Ts of the spiral curve.

$\dfrac{X_c}{4} = P = 4.52$

$T_s = \dfrac{L_c}{2} + \left(R_c + \dfrac{X_c}{4}\right) \tan \dfrac{I}{2}$

$T_s = \dfrac{166.2}{2} + (254.65 + 4.52) \tan 25°25'$

$T_s =$ ***206.26*** (tangent distance)

③ Maximum velocity:

$L_c = \dfrac{0.036\,V^3}{R_c}$

$166.2 = \dfrac{0.036\,V^3}{254.65}$

$V =$ ***105.54 kph***

Problem 420-C

A simple curve having a radius of 600 m. has an angle of intersection of its tangents equal to 40°30'. This curve is to be replaced by one of smaller radius so as to admit a 100 m. spiral at each end. The deviation of the new curve from the old curve at their midpoint is 0.50 m. towards the intersection of the tangents.

① Determine the radius of the central curve.
② Determine its central angle.
③ If the stationing of the intersection of the tangents is 10 + 820.94, determine the stationing of the T.S. of the spiral curve.

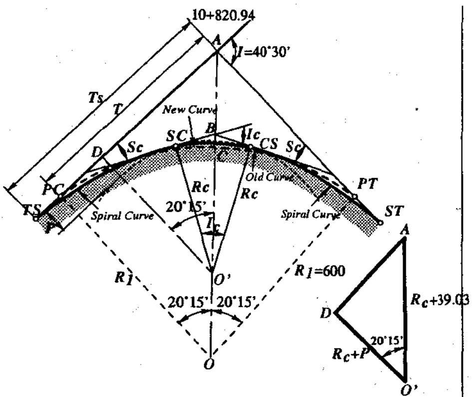
Solution:

① Radius of the central curve.

$\cos 20°15 = \dfrac{600}{OA}$

$OA = 639.53$ m.
$BC = 0.50$ m.
$AC = 639.53 - 600$
$AC = 39.53$
$AB = 39.53 - 0.50$
$AB = 39.03$ m.

COMPOUND CURVES



$$P = \frac{L_c^2}{24 R_c}$$

$$P = \frac{(100)^2}{24 R_c}$$

$$P = \frac{416.67}{R_c}$$

$$O'A = R_c + 39.03$$

Considering Triangle ADO'

$$\cos 20^\circ 15' = \frac{R_c + P}{R_c + 39.03}$$

$$\cos 20^\circ 15' = \frac{R_c + \frac{416.67}{R_c}}{R_c + 39.03}$$

$$\cos 20^\circ 15' = \frac{R_c^2 + 416.67}{R_c (R_c + 39.03)}$$

$$0.94 R_c^2 + 36.62 R_c = R_c^2 + 416.67$$

$$0.06 R_c^2 - 610.33 R_c + 6944.50 = 0$$

$$R_c = \frac{610.33 \pm 587.13}{2}$$

R_c = ***598.73 m.*** (radius of the central curve)

② Central angle.

$$\tan 20^\circ 15' = \frac{T}{600}$$

$$T_s = \frac{L_c}{2} + \left(R_c + \frac{X_c}{4}\right) \tan 20^\circ 15'$$

$$\frac{X_c}{4} = P$$

$$P = \frac{L_c^2}{24 R_c}$$

$$P = \frac{(100)^2}{24 (598.73)}$$

$$P = 0.70$$

$$T_s = \frac{100}{2} + (598.73 + 0.7) \tan 20^\circ 15'$$

$$T_s = 271.14 \text{ m.}$$

$$S_c = \frac{L_c \, 180^\circ}{2 R_c \pi}$$

$$S_c = \frac{100(180)}{2 (298.73) \pi}$$

$$S_c = 4.78^\circ$$

$I_c = I - 2 S_c$

$I_c = 40^\circ 30' - 2(4.78^\circ)$

I_c = ***30°56' 24"***

(central angle of new curve)

③ Stationing of the T.S. of the spiral curve.

Sta. of T.S. = (10 + 820.94) – (271.14)

Sta. of T.S. = ***10 + 549.80***

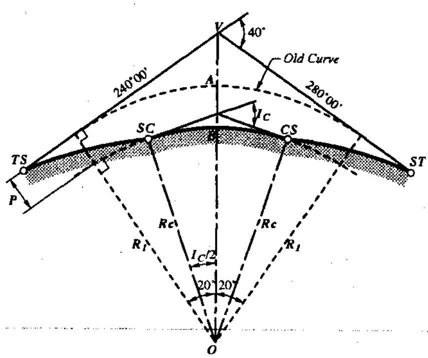
Problem 420-D

A simple curve having a degree of curve equal to 6° is connected by two tangents having an azimuth of 240° and 280° respectively. It is required to replace this curve by introducing a transition curve 80 m. long at each end of a new central curve which is to be shifted at its midpoint away from the intersection of the tangents.

① Determine the radius of the new central curve if the center of the old curve is retained.
② Determine the distance which the new curve is shifted away from the intersection of the tangents.
③ Compute the length of throw.

Solution:

① Radius of central curve:



$$P = \frac{L_c^2}{24 R_c}$$

$$R_1 - R_c = P$$

$$R_1 = \frac{1145.916}{D}$$

$$R_1 = \frac{1145.916}{6}$$

$$R_1 = 190.99 \text{ m.}$$

$$190.00 - R_c = \frac{(80)^2}{24 R_c}$$

$$4583.76 R_c - 24 R_c^2 = 6400$$

$$R_c^2 - 190.99 R_c + 266.67 = 0$$

$$R_c = \frac{190.99 \pm 188.18}{2}$$

$$R_c = \mathbf{189.59 \text{ m.}}$$

② Distance which the new curve is shifted away from the intersection of the tangents.

$h = R_1 - R_c$

$h = 190.00 - 189.59$

$h = \mathbf{1.40 \text{ m.}}$ *(amount the new curve is shifted away from the intersection of the tangents)*

③ Length of throw:

$$P = \frac{(L_c)^2}{24 R_c}$$

$$P = \frac{(80)^2}{24 (189.59)}$$

$$P = \mathbf{1.41 \text{ m.}}$$

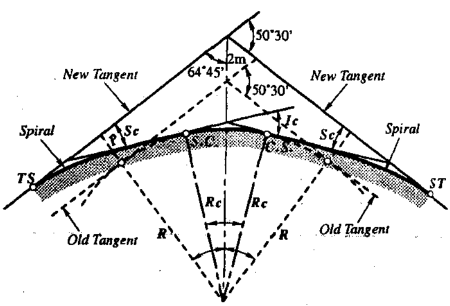
Problem 420-E

A simple curve having a radius of 200 m. has a central angle of 50°30'. It is required to be replaced by another curve by connecting spiral (transition curve) at its ends by maintaining the radius of the old curve and its center but the tangents are moved outwards to allow transition. Part of the original curve is retained. The new intersection of the tangents is moved outward by 2 meters from its original position along the line connecting the intersection of tangents and the center of the curve.

① Determine the length of the transition curve (spiral) at each end of the central curve.
② Compute the spiral angle.
③ Compute the central angle of the central curve from the S.C. to C.S.

Solution:

① Length of the transition curve:



$P = 2 \sin 64°45'$
$P = 1.81$ m.

$$P = \frac{(L_c)^2}{24 R_c}$$

$L_c^2 = P(24) R_c$
$L_c^2 = 1.81(24)(200)$
$L_c =$ ***92.95 m.*** *(length of spiral)*

② Spiral angle.

$$S_c = \frac{L_c \, 180°}{2 R_c \pi}$$

$$S_c = \frac{92.95(180°)}{2(200)\pi}$$

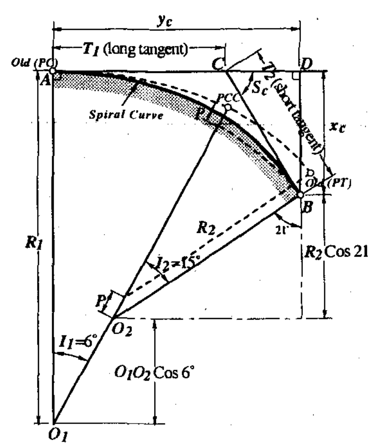
$S_c =$ ***13°19'***

③ Central angle:

$I_c = I - 2 S_c$
$I_c = 50°30' - 2(13°19')$
$I_c =$ ***23°52'*** *(central angle of the new curve)*

Problem 420-F

From the given compound curve, it is required to replace it with a transition (spiral) curve 100 m. long starting at A and ends up at B. The degree of curve of the first curve is 4° while that of the second curve is 10°. Central angles are 6° and 15° respectively for first and second curve.



① Determine the radius of central curve.
② Determine the length of short tangent CB.
③ Determine the length of long tangent AC.

Solution:

① Radius of central curve.

$$R_1 = \frac{1145.916}{4}$$
$$R_1 = 286.48 \text{ m.}$$

$$R_2 = \frac{1145.916}{10}$$
$$R_2 = 114.59 \text{ m.}$$

$$O_1 O_2 = R_1 - R_2 - P$$
$$O_1 O_2 = 286.48 - 114.59 - P$$
$$O_1 O_2 = 171.89 - \frac{L_c^2}{24 R_c}$$

$$R_1 = X_c + R_2 \cos 21^\circ + O_1 O_2 \cos 6^\circ$$

$$R_1 = \frac{L_c^2}{6 R_c} + R_2 \cos 21^\circ + \left(171 - \frac{L_c^2}{24 R_c}\right) \cos 6^\circ$$

$$286.48 = \frac{(100)^2}{6 R_c} + 114.59 \cos 21^\circ + \left(171.89 - \frac{(100)^2}{24 R_c}\right) \cos 6^\circ$$

$$8.55 = \frac{(100)^2}{6 R_c} - \frac{(100)^2}{24 R_c} \cos 6^\circ$$
$$8.55 = \frac{(100)^2}{6 R_c}\left(1 - \frac{\cos 6^\circ}{4}\right)$$
$$R_c = \boldsymbol{146.46 \text{ m.}}$$

② Length of short tangent CB.

$$X_c = \frac{L_c^2}{6 R_c}$$
$$X_c = \frac{(100)^2}{6(146.46)}$$
$$X_c = 11.38 \text{ m.}$$

$$Y_c = L_c - \frac{L_c^3}{40 R_c^2}$$
$$Y_c = 100 - \frac{(100)^3}{40 (146.46)^2}$$
$$Y_c = 98.93 \text{ m.}$$

$$S_c = \frac{L_c}{2 R_c} \frac{180}{\pi}$$
$$S_c = \frac{100}{2(146.46)} \frac{180}{\pi}$$
$$S_c = 19.6^\circ$$

$$\sin S_c = \frac{X_c}{CB}$$
$$CB = \frac{11.38}{\sin 19.6^\circ}$$
$$CB = \boldsymbol{33.92 \text{ m.}}$$

③ Length of long tangent AC.

$$\tan S_c = \frac{X_c}{CD}$$
$$CD = \frac{11.38}{\tan 19.6^\circ}$$
$$CD = 31.96 \text{ m.}$$

$$AC = Y_c - CD$$
$$AC = 98.83 - 31.96$$
$$AC = \boldsymbol{66.87 \text{ m.}}$$

EARTHWORKS

Derive the prismoidal correction formula for a triangular end areas using the prismoidal formula.

$$V_p = \frac{L}{6}(A_1 + 4A_m + A_2)$$

Solution:

V_c = prismoidal correction

$$V_c = V_E - V_p$$

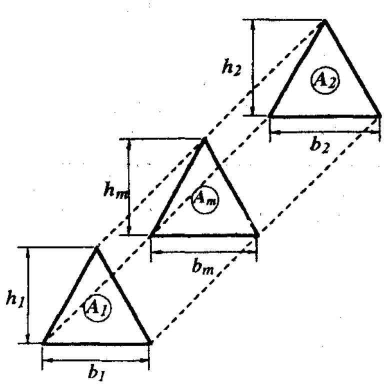
$$V_c = \frac{(A_1 + A_2)}{2}L - \frac{L}{6}(A_1 + 4A_m + A_2)$$

$$V_c = \frac{(3A_1 + 3A_2 - A_1 - 4A_m - A_2)\,L}{6}$$

$$V_c = \frac{2\,(A_1 + A_2 - 2A_m)\,L}{6}$$

$$V_c = \frac{L\,(A_1 + A_2 - 2A_m)}{3}$$

Let us consider the triangular prismoidal shown below:



$$V_c = \frac{L}{3}(A_1 - 2A_m + A_2)$$

$$A_1 = \frac{b_1\,h_1}{2}$$

$$A_2 = \frac{b_2\,h_2}{2}$$

$$A_m = \frac{b_m\,h_m}{2}$$

$$V_c = \frac{L}{3}\left[\frac{b_1\,h_1}{2} - \frac{2\,b_m\,h_m}{2} + \frac{b_2\,h_2}{2}\right]$$

$$b_m = \frac{b_1 + b_2}{2}$$

$$h_m = \frac{h_1 + h_2}{2}$$

$$V_c = \frac{L}{3}\left[\frac{b_1\,h_1}{2} - \frac{(b_1 + b_2)}{2}\,\frac{(h_1 + h_2)}{2} + \frac{b_2 + h_2}{2}\right]$$

$$V_c = \frac{L}{3}\left[\frac{b_1h_1}{2} - \frac{b_1h_1}{4} - \frac{b_2h_1}{4} - \frac{b_1h_2}{4} - \frac{b_2h_2}{4} + \frac{b_2h_2}{2}\right]$$

$$V_c = \frac{L}{3}\left[\frac{b_1h_1 - b_1h_2 - b_2h_1 + b_2h_2}{4}\right]$$

$$V_c = \frac{L}{12}\left[b_1\,(h_1 - h_2) - b_2\,(h_1 - h_2)\right]$$

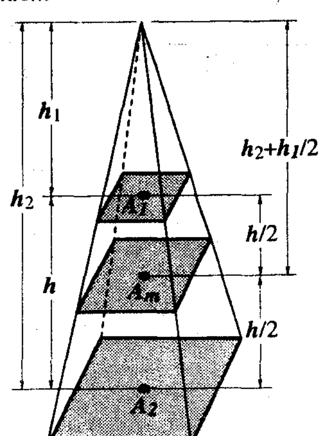
$$V_c = \frac{L}{12}(b_1 - b_2)\,(h_1 - h_2)$$

(prismoidal correction to be subtracted algebraically from the volume by end area method.)

Derive the Prismoidal Formula for determining volumes of regular solid.

$$V = \frac{h}{6}(A_1 + 4A_m + A_2)$$

Solution:



$$V = \frac{h}{A}A_2 - \frac{h_1 A_1}{3}$$

$$\frac{A_1}{A_2} = \frac{h_1^2}{h_2^2}$$

$$\frac{\sqrt{A_1}}{\sqrt{A_2}} = \frac{h_1}{h_2} \quad \frac{\sqrt{A_2}}{\sqrt{A_1}} = \frac{h_2}{h_1}$$

$$A_2 = \frac{A_1 h_2^2}{h_1^2}$$

$$V = \frac{h_2 A_1 h_2^2}{3 h_1^2} - \frac{h_1 A_1}{3}$$

$$V = \frac{h_2^3 A_1 - h_1^3 A_1}{3 h_1^2}$$

$$V = \frac{A_1}{3 h_1^2}(h_2^3 - h_1^3)$$

$$V = \frac{A_1}{3 h_1^2}(h_2 - h_1)(h_2^2 + h_2 h_1 + h_1^2)$$

$$V = \frac{A_1 h}{3 h_1^2}(h_2^2 + h_2 h_1 + h_1^2)$$

$$V = \frac{A h\, h_2^2}{3 h_1^2} + \frac{A_1 h\, h_2}{3 h_1} + \frac{A_1 h}{3}$$

$$V = \frac{A_1 h}{3}\left(\frac{A_2}{A_1}\right) + \frac{A_1 h}{3}\frac{\sqrt{A_2}}{A_1} + \frac{A_1 h}{3}$$

$V = \frac{h}{3}(A_2 + \sqrt{A_1 A_2} + A_1)$ *(frustum of a pyramid)*

$$\frac{A_m}{A_1} = \frac{\frac{(h_2 + h_1)^2}{2}}{h_1^2}$$

$$4 A_m = \frac{A_1}{h_1^2}(h_2^2 + 2 h_2 h_1 + h_1^2)$$

$$4 A_m = A_1 + \frac{A_1 h_2^2}{h_1^2} + \frac{2 A_1 h_2}{h_1}$$

$$4 A_m = A_1 + A_2 + 2 A_1 \frac{\sqrt{A_2}}{\sqrt{A_1}}$$

$$4 A_m = A_1 + A_2 + 2\sqrt{A_2 A_1}$$

$$2\sqrt{A_1 A_2} = 4 A_m - A_1 - A_2$$

$$\sqrt{A_1 A_2} = 2 A_m - \frac{A_1}{2} - \frac{A_2}{2}$$

$$V = \frac{h}{3}\left(A_1 + 2 A_m - \frac{A_1}{2} - \frac{A_2}{2} + A_2\right)$$

$$V = \frac{h}{3}\left[2 A_1 + 4 A_m - \frac{A_1}{2} - A_2 + 2 A_2\right]$$

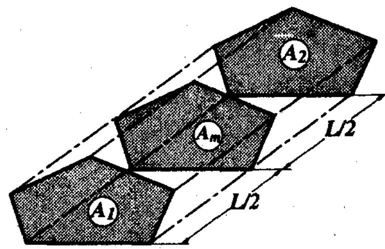
$V = \frac{h}{6}(A_1 + 4 A_m + A_2)$ *Prismoidal Formula*

VOLUME OF EARTHWORK

(1) *End area*

$$V = \frac{(A_1 + A_2) L}{2}$$

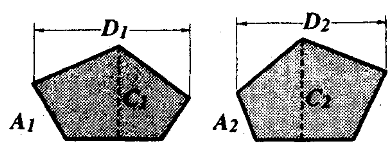
(2) *Prismoidal Formula*



$$V_p = \frac{L}{6}(A_1 + 4 A_m + A_2)$$

A_m = area of mid-section

(3) *Volume with Prismoidal Correction:*
(Applicable only to three level section)



$V = V_E - V_{cp}$

V_E = volume by end area

V_{cp} = prismoidal correction

$$V_{cp} = \frac{L}{12}(C_1 - C_2)(D_1 - D_2)$$

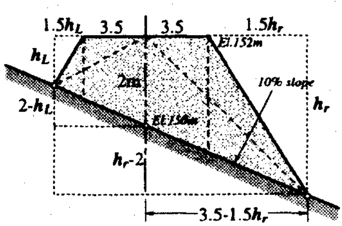
Problem 421:

The cross section notes of the ground surface at sta. 1 + 200 of a road survey, shows that the ground is sloping at a 10% grade downward to the right. The elevation of the ground along the center line of the proposed road at this station is 150 m. and that of the finished subgrade is 152 m. Width of subgrade is 7.00 m. with side slopes of 1.5 : 1.

① Compute the distance of the right slope stake from the center of the road.
② Compute the distance of the left slope stake from the center of the road.
③ Compute the difference in elevation of the right slope stake and the left slope stake.

Solution:

① *Distance of right slope stake from center of the road:*



$$\frac{h_r - 2}{3.5 + 1.5\,h_r} = \frac{10}{100} = 0.10$$

$h_r - 2 = 0.35 + 0.15\,h_r$

$0.85\,h_r = 2.35$

$h_r = 2.76$

Distance of right slope stake = $3.5 + 1.5\,h_r$
Distance of right slope stake = $3.5 + 1.5(2.76)$
Distance of right slope stake = ***7.64 m.***

② *Distance of left slope stake from center of the road:*

$$\frac{2 - h_L}{1.5\,h_L + 3.5} = \frac{10}{100}$$

$2 - h_L = 0.15\,h_L + 0.35$

$1.15\,h_L = 1.65$

$h_L = 1.43$

Distance of left slope stake = $1.5\,h_L + 3.5$
Distance of left slope stake = $1.5\,(1.43) + 3.5$
Distance of left slope stake = ***5.65 m.***

③ *Diff. in elevation of right and left slope stake:*

Elev. of left slope stake = 152 - 1.43
Elev. of left slope stake = 150.57 m.
Elev. of right slope stake = 152 - 2.76
Elev. of right slope stake = 149.24 m.

Diff. in elev. = 150.57 - 149.24
Diff. in elev. = ***1.33 m.***

Problem 422:

Given the cross section notes of an earthwork between station 10 + 100 to 10 + 200. Assume both stations to have the same side slope and width of the base.

STA. 10 + 100

Left	Center	Right
$\frac{6.45}{+2.3}$	$\frac{0}{+1.5}$	$\frac{4.5}{+1.0}$

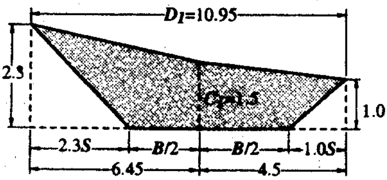
STA. 10 + 200

$\frac{6.0}{+2.0}$	$\frac{0}{x}$	$\frac{6.9}{+2.6}$

① Compute the side slope of both sections.
② Compute the value of x at station 10 + 200 if it has a cross sectional area of 14.64 m².
③ Compute the volume between stations 10 + 100 and 10 + 200 using end area method with prismoidal correction.

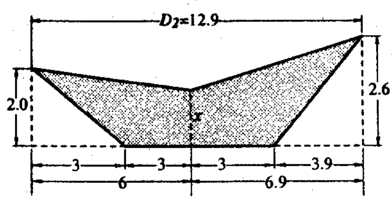
Solution:

① *Width of base:*



$$\frac{B}{2} + 2.3\,S = 6.45$$
$$\frac{B}{2} + 1.0\,S = 4.5$$
$$1.3\,S = 1.95$$
$$S = 1.5$$
$$\frac{B}{2} + 1.0(1.5) = 4.5$$
$$B = \boldsymbol{6\ m.}$$

② *Value of x:*



$$A = \frac{2(3)}{2} + \frac{6x}{2} + \frac{6.9(x)}{2} + \frac{3(2.6)}{2}$$
$$A = 14.64$$
$$x = \boldsymbol{1.2}$$

③ *Volume between sta. 10 + 100 and 10 + 200:*

$$A_1 = \frac{2.3(3)}{2} + \frac{1.5(6.45)}{2} + \frac{4.5(1.5)}{2} + \frac{3(1)}{2}$$
$$A_1 = 13.1625\ m^2$$
$$A_2 = 14.64\ m^2$$
$$V_E = \frac{(A_1 + A_2)\,L}{2}$$
$$V_E = \frac{(13.1625 + 14.64)\,(100)}{2}$$
$$V_E = 1390.125\ m^3$$

Prismoidal correction:

$$V_p = \frac{L}{12}(C_1 - C_2)\,(D_1 - D_2)$$
$$V_p = \frac{100}{12}\,[(10.95 - 12.9)\,(1.5 - 1.2)]$$
$$V_p = -\,4.875\ m^3$$
$$V_{cp} = V_E - V_p$$
$$V_{cp} = 1390.125 - (-\,4.875)$$
$$V_{cp} = \boldsymbol{1395\ m^3}$$

Problem 423:

From the given cross section of an earthworks between A (20 + 200) and B (20 + 220) assuming both have same slope and width of base.

STA. A

$$\frac{6.60}{+2.4} \qquad \frac{0}{+2.0} \qquad \frac{4.8}{+1.2}$$

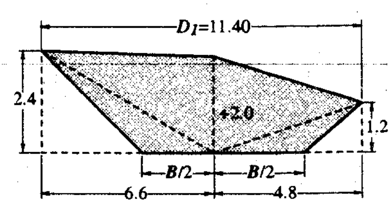
STA. B

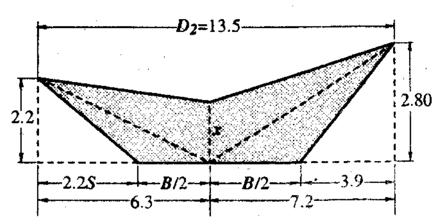
$$\frac{6.3}{+2.2} \qquad \frac{0}{?} \qquad \frac{7.2}{+2.80}$$

① Compute the width of the base.
② Compute the value of cut at station B if it has an area of 16.82 m^2.
③ Compute the volume between A and B with Prismoidal Correction.

Solution:

① *Width of base:*





$$6.3 = 2.2S + \frac{B}{2}$$

$$7.2 = 2.8S + \frac{B}{2}$$

$$0.9 = 0.6\,S$$
$$S = 1.5$$

$$6.3 = 2.2(1.5) + \frac{B}{2}$$

$B = 6$ *m.*

② *Value of cut at station B:*

$$\frac{2.2(3)}{2} + \frac{6.3x}{2} + \frac{7.2(x)}{2} + \frac{2.8(3)}{2} = 16.82$$
$$6.75x = 9.32$$

$x = 1.38$ *m.*

③ *Volume using Prismoidal correction:*

$$A_1 = \frac{2.4(3)}{2} + \frac{6.6(2)}{2} + \frac{4.8(2)}{2} + \frac{1.2(3)}{2}$$
$$A_1 = 16.80 \text{ m}^2$$
$$V_E = \frac{(A_1 + A_2)\,L}{2}$$
$$V_E = \frac{(16.80 + 16.82)(20)}{2}$$
$$V_E = 336.20$$
$$V_p = \frac{L}{12}(C_1 - C_2)\,(D_1 - D_2)$$
$$V_p = \frac{20}{12}(2 - 1.38)(11.40 - 13.5)$$
$$V_p = -2.17$$
$$V = V_E - V_p$$
$$V = 336.20 - (2.17)$$

$V = 338.37$ *cu.m.*

Problem 424:

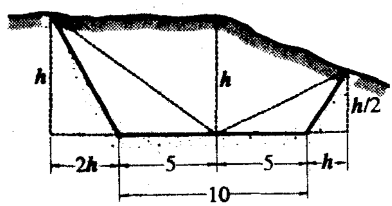
At station 1 + 100 of a portion of a highway stretch has an area of 100 sq. meters in cut while that of station 1 + 200 the area is 240 sq. meters in cut. At station 1 + 100, the ground surface to the left of the center line is flat and the height of the right slope stake above the grade line is one half that of the left, while that of station 1 + 200, the height of the right slope stake is 3 times higher than that of the left slope stake. The center cut at station 1 + 200. The width of the roadway is 10 m. with a side slope of 2 : 1.

① Determine the height of cut at the center of sta. 1 + 100.
② Determine the height of the right slope stake at sta. 1 + 200.
③ Determine the volume between sta. 1 + 100 and 1 + 200 by applying prismoidal correction.

Solution:

① *Height of cut at the center of sta. 1 + 100:*

Station 1 + 100



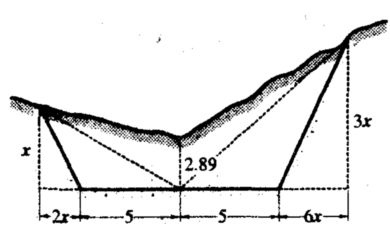
$A = 100$ sq.m.

$$\frac{5h}{2} + \frac{h(5+2h)}{2} + \frac{h(5+h)}{2} + \frac{5\left(\frac{h}{2}\right)}{2} = 100$$
$$5h + 5h + 2h^2 + 5h + h^2 + 2.5h = 200$$
$$3h^2 + 17.5h - 200 = 0$$
$$h = \frac{-17.5 - \sqrt{(17.5)^2 - 4(3)(-200)}}{2(3)}$$
$$h = \frac{-17.5 - \sqrt{2706}}{6}$$
$$h = \frac{-17.5 + 52.2}{6}$$

$h = 5.78$ *m.*

② *Height of the right slope stake at sta. 1 + 200:*

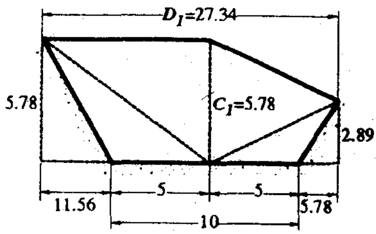
Station 1 + 200

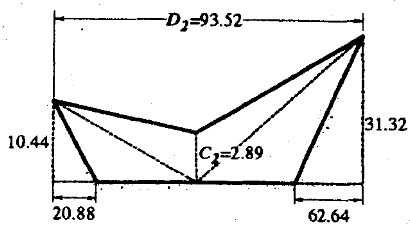


$A = 240$ sq.m.

$$\frac{5x}{2}+\frac{2.89\,(5+2x)}{2}+\frac{2.89\,(5+6x)}{2}+\frac{5\,(3x)}{2}=240$$
$$5x+14.45+5.78x+14.45+17.34x+15x=480$$
$$43.12x=451.1$$
$$x=10.44\text{ m.}$$
$$3x=\boldsymbol{31.32\text{ m.}}$$

③ *Volume between sta. 1 + 100 and 1 + 200:*





Volume by end area:

$$V_E=\frac{(A_1+A_2)\,L}{2}$$
$$V_E=\frac{(100+240)\,(100)}{2}$$
$$V_E=17{,}000\text{ cu.m.}$$
$$C_p=\frac{L}{12}(C_1-C_2)\,(D_1-D_2)$$
$$C_p=\frac{100}{12}(5.78-2.89)(27.34-93.52)$$
$$C_p=\frac{100}{12}(2.89)\,(-66.18)$$
$$C_p=-15.94\text{ cu.m.}$$

Corrected volume:

$$V=V_E-V_p$$
$$V=17{,}000-(-1594)$$
$$V=\boldsymbol{18{,}594\text{ cu.m.}}$$

Problem 425:

In a certain portion of road to be constructed the following data were taken.

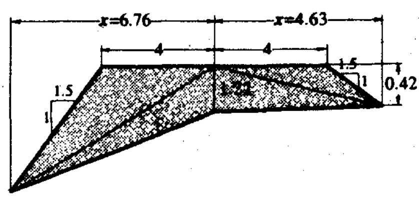
	1 + 040	
$\frac{x}{-1.84}$	$\frac{0}{-1.22}$	$\frac{x}{-0.42}$
	1 + 100	
$\frac{x}{1.098}$	$\frac{0}{+3.05}$	$\frac{x}{+0.50}$

Base for cut = 9 m Sideslopes = 1:1
Base for fill = 8 m Sideslopes = 1.5:1

① Compute the area of station 1 + 040.
② Find the area of station 1 + 100.
③ Determine the diff. in volume of cut and fill using end area method.

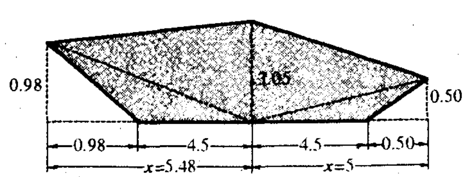
Solution:

① *Area of station 1 + 040:*



$$A_{fill}=\frac{1}{2}(4)\,1.84+\frac{1}{2}(1.22)\,6.76$$
$$+\frac{1}{2}(1.22)\,4.63+\frac{1}{2}(4)\,0.42$$
$$A_{fill}=\boldsymbol{11.47\text{ m}^2}$$

② *Area of station 1 + 100:*

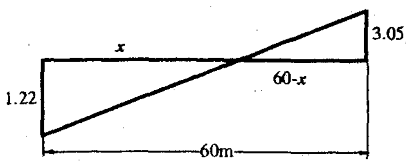


$$A_{cut} = \frac{1}{2}(4.5)\,0.98 + \frac{1}{2}(3.05)\,5.48$$

$$+\frac{1}{2}(3.05\,5 + \frac{1}{2}(4.5)\,0.5$$

$$A_{cut} = \mathbf{19.31\ m^2}$$

③ *Diff. in volume of cut and fill using end area method:*



$$\frac{1.22}{x} = \frac{1.22 + 3.05}{60}$$

$$x = 17.14$$

$$60 - x = 42.86$$

$$V_{cut} = \frac{0 + 19.31}{2}(42.86)$$

$$V_{cut} = 413.18\ m^3$$

$$V_{fill} = \frac{11.47 + 0}{2}(17.14)$$

$$V_{fill} = 98.3\ m^3$$

$$Diff. = \mathbf{315.51\ m^3}$$

Problem 426:

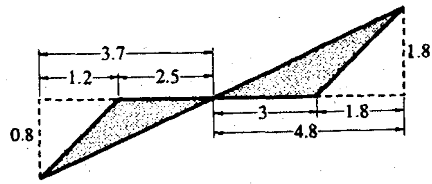
Given the following section of an earthworks for a proposed road construction on a hilly portion of the route. The width of the road base for cut is 6 m. for allowance of drainage canals and 5 m. for fill. Sides slopes for cut is 1:1 and for fill is 1.5:1.

$\frac{3.7}{-0.8}$	0	$\frac{x}{+1.8}$

① Compute the value of x.
② Compute the area in fill.
③ Compute the area in cut.

Solution:

① *Value of x:*



$$x = 3 + 1.8$$

$$x = \mathbf{4.8}$$

② *Area of fill:*

$$\text{Area of fill} = \frac{2.5\,(0.8)}{2}$$

$$\text{Area of fill} = \mathbf{1.0\ m^2}$$

③ *Area of cut:*

$$\text{Area of cut} = \frac{3\,(1.8)}{2}$$

$$\text{Area of cut} = \mathbf{2.7\ m^2}$$

Problem 427:

The following is a set of notes of an earthworks of a road construction which is undertaken by the Bureau of Public Works.

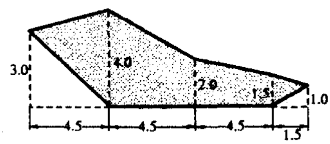
Station	Cross section				
1 + 020	$\frac{9.0}{+3.0}$	$\frac{4.5}{+4}$	+2.0	$\frac{4.5}{+1.5}$	$\frac{6.0}{+1.0}$
1 + 040	$\frac{7.5}{+2.0}$	$\frac{4.5}{+5.0}$	+4.0	$\frac{4.5}{+2.0}$	$\frac{9.0}{+3.0}$

The base of the road way is 9 m. which conforms with the BPWH standards.
Side slope is 1.5:1.

① Compute the cross sectional area at sta. 1 + 020.
② Compute the cross sectional area at sta. 1 + 040.
③ Compute the volume between the two stations using end area method.

Solution:

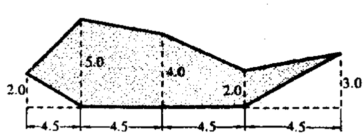
① *Area section f + 020:*



$$A_1 = \frac{4(4.5)}{2} + \frac{(4+2)(4.5)}{2} + \frac{(2+1.5)(4.5)}{2} + \frac{1.5(1.5)}{2}$$

$$A_1 = 31.50\ m^2$$

② *Area of section 1 + 040:*



$$A_2 = \frac{5(3)}{2} + \frac{(5+4)(4.5)}{2} + \frac{(4+2)(4.5)}{2} + \frac{2(4.5)}{2}$$

$$A_2 = 45.75\ m^2$$

③ *Volume between stations:*

$$V = \frac{(A_1 + A_2)\,L}{2}$$

$$V = \frac{(31.50 + 45.75)(20)}{2}$$

$$V = 772.5\ m^3$$

Problem 428:

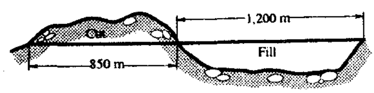
In determining the position of the balance line in the profile diagram, a horizontal grade line is drawn such that the length of the cut is 850 m. and that of fill is 1200 m. The profile area between the ground line and the grade line in the cut is 7800 sq.m. while that of fill is 8500 sq.m. If the road bed is 10 m. wide for cut and 8 meters wide for fill and if the side slope for cut is 1.5 : 1 while that for fill is 2 : 1.

Assume a level section with an average value of cut and fill for each stretch.

① Determine the volume of cut.
② Determine the volume of fill.
③ If the shrinkage factor is 1.2, determine the volume borrow or waste.

Solution:

Average depth of cut:



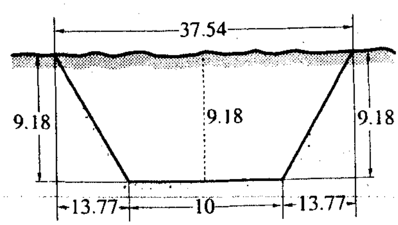
$$C = \frac{7800}{850}$$

$C = 9.18$ m.

Average depth of fill:

$$f = \frac{8500}{1200}$$

$f = 7.08$

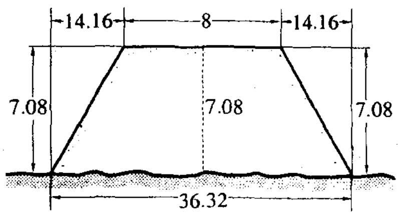


Side slope = 1.5 : 1 Cut

$$A = \frac{(10 + 37.54)(9.18)}{2}$$

$A = 218.21$ sq.m.

① *Volume of cut:*
V_c = 218.21 (850)
V_c = ***185,500 cu.m.***



Side slope = 2 : 1 Fill

$$A = \frac{(8 + 36.32)(7.08)}{2}$$
A = ***156.89 sq.m.***

② *Volume of fill:*
V_f = 156.89 (1200)
V_f = ***188,000 cu.m.***

③ *Volume of borrow:*
Vol. of borrow = 188000 (1.2) - 185500
Vol. of borrow = ***40100 cu.m.***

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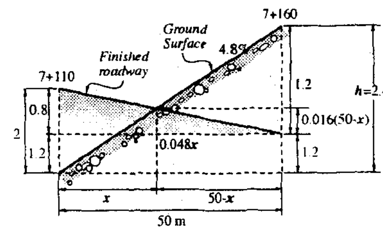
The center height of the road at sta. 7 + 110 is 2 m. fill while at sta. 7 + 160 it is 1.2 m. cut. From sta. 7 + 110 to the other station the ground makes a uniform slope of 4.8%.

① Compute the slope of the new road.
② Find the distance in meters from station 7 + 110 in which the fill is extended.
③ Compute the stationing of the point where the fill is extended.

Solution:



① Slope of the new road:
$$\text{Slope} = \frac{0.8}{50} = 0.\boldsymbol{016}$$

② Distance in which the fill is extended:
0.048x = 1.2 + 0.016(50 - x)
0.064x = 2
x = ***31.25***

③ Stationing of the point where the fill is extended:
Sta. = (7 + 110) + (31.25)
Sta. = ***71 + 141.25***

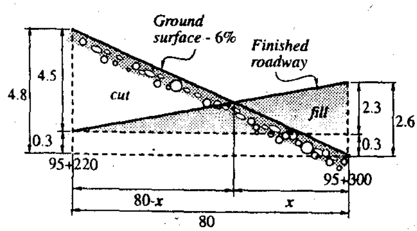
Problem 428-B:

At station 95 + 220, the center height of the road is 4.5 m. cut, while at station 95 + 300, it is 2.6 m. fill. The ground between station 95 + 220 to the other station has a uniform slope of – 6%.

① What is the grade of the road?
② How far in meters, from station 95 + 300 toward station 95 + 220 will the filling extend?
③ At what station will the filling extend.

Solution:

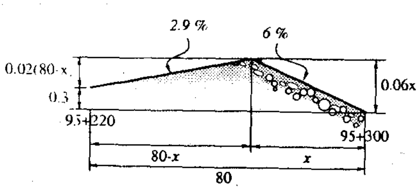
① Grade of road



$$\text{Slope of road} = \frac{2.3}{80}$$

Slope of road = 0.02875 say ***0.029***

② Distance from 95 + 300 where filling will extend:



$$0.029(80 - x) + 0.30 = 0.06x$$
$$0.089x = 0.029(80) + 0.30$$
$$x = \mathbf{29.44\ m.}$$

③ Station where filling extend:
(95 + 300) – (29.44) = ***95 + 270.56***

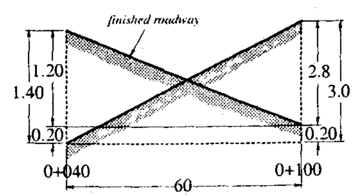
Problem 429:

From station 0 + 040, with center height of 1.40 m. fill, the ground line makes a uniform slope of 5% to station 0 + 100, whose center height is 2.80 m. cut. Assume both sections to be level sections with side slopes of 2 : 1 for fill and 1.5 : 1 for cut.

① Find the grade of the finished road surface.
② Find the area at each station.
③ By end area method, find the amount of cut and fill.
④ Between these two stations, is it borrow or waste?
Roadway for fill is 9.00 m. and for cut it is 10.00 m.

Solution:

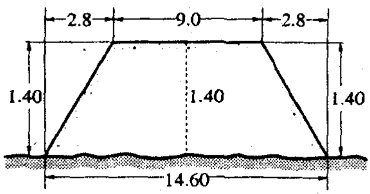
① *Slope of roadway:*



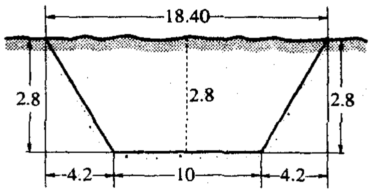
$$\textit{Slope of roadway} = \frac{-1.20}{60}$$

Slope of roadway = ***- 2%*** *(downward)*

② *Area at each station:*



Station 0 + 040



Station 0 +100

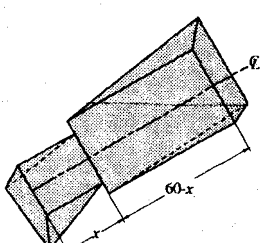
$$A = \frac{(14.60 + 9)(1.40)}{2}$$

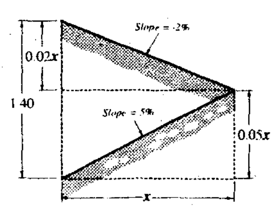
$A =$ **16.52 sq.m.** *(fill)*

$$A = \frac{(10 + 18.40)(2.8)}{2}$$

$A =$ **39.76 sq.m.** *(cut)*

③ *Volumes of cut and fill:*





$1.40 - 0.02\,x = 0.05\,x$

$0.07\,x = 1.40$

$x = 20$

$60 - x = 40$

$$\text{Vol. of fill} = \frac{L}{2}(A_1 + A_2)$$

$$\text{Vol. of fill} = \frac{20}{2}(16.52 + 0)$$

Vol. of fill = **165.20 cu.m.**

$$\text{Vol. of cut} = \frac{L}{2}(A_1 + A_2)$$

$$\text{Vol. of cut} = \frac{40}{2}(39.76 + 0)$$

Vol. of cut = **795.20 cu.m.**

④ *Since the volume of cut is excessive than the volume of fill, it is then necessary to throw the excess volume of cut as waste by an amount equal to*

795.20 - 165.20 = **63.00 cu.m.**

Problem 430:

The following data are the cross section notes at station 0 + 020 and 0 + 040. The natural ground slope is almost even.

Base width	Side slope
Cut = 9 m.	Cut = 1 : 1
Fill = 8 m.	Fill = 1.5 : 1

Station 0 + 020

$\frac{?}{?}$	$\frac{+3.0}{4.5}$	$\frac{+1.5}{0}$	$\frac{0}{0}$

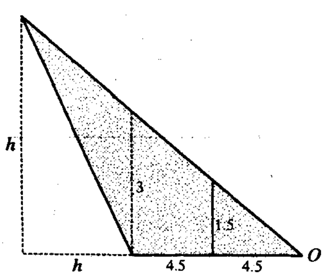
Station 0 + 040

$\frac{?}{?}$	$\frac{-2.0}{4}$	$\frac{-1.0}{0}$	$\frac{0}{0}$

① Compute the area of section 0 + 020.
② Compute the area of section 0 + 040.
③ Compute the volume of borrow or waste from station 0 + 020 and 0 + 040 assuming shrinkage factor of 1.20.

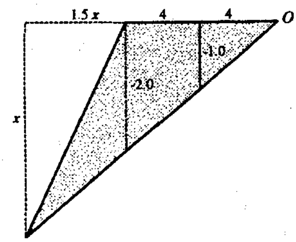
Solution:

① *Area of section 0 + 020:*



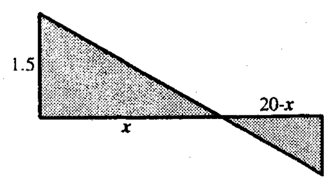
$$\frac{1.5}{4.5} = \frac{h}{h+9}$$
$$4.5h = 1.5h + 13.5$$
$$h = 4.5$$
$$A = \frac{4.5\,(9)}{2}$$
$$A = \mathbf{20.25\ m^2}\ \text{(cut)}$$

② *Area of section 0 + 040:*



$$\frac{2}{8} = \frac{x}{1.5x + 8}$$
$$8x = 3x + 16$$
$$5x = 16$$
$$x = 3.2$$
$$A = \frac{3.2\,(8)}{2}$$
$$A = \mathbf{12.8\ m^2}\ \text{(fill)}$$

③ *Volume of borrow or waste:*



$$\frac{1.5}{x} = \frac{1.0}{20 - x}$$
$$x = 30 - 1.5x$$
$$2.5x = 30$$
$$x = 12$$
$$20 - x = 8$$
$$\text{Vol. of cut} = \frac{(A_1 + A_2)\,L}{2}$$
$$\text{Vol. of cut} = \frac{(20.25 + 0)\,(12)}{2}$$
$$\text{Vol. of cut} = 121.5\ m^3$$
$$\text{Vol. of fill} = \frac{(A_1 + A_2)\,L\,(1.20)}{2}$$
$$\text{Vol. of fill} = \frac{(12.8 + 0)(8)(1.20)}{2}$$
$$\text{Vol. of fill} = 61.44\ m^3$$

$$\text{Vol. of waste} = 121.5 - 61.44$$
$$\text{Vol. of waste} = \mathbf{60.06\ m^3}$$

Problem 431:

In a 20 meter road stretch, the following cross-section of the existing ground and corresponding subgrade cross-section notes were taken.

Existing Ground Cross Sections

Sections	Left			Center	Right		
10 + 280	$\frac{0}{16.5}$	$\frac{-2}{9}$	$\frac{-1}{5}$	$\frac{0}{0}$	$\frac{1}{4}$	$\frac{2}{12}$	$\frac{1}{18}$
10 + 300	$\frac{-3}{13.5}$	$\frac{-2}{10}$	$\frac{1}{7}$	$\frac{1}{0}$	$\frac{1}{5}$	$\frac{0}{9}$	$\frac{-1.5}{17}$

Subgrade Cross Sections

Sections	Left			Center	Right		
10 + 280	$\frac{0}{16.5}$	$\frac{-5.5}{7}$	$\frac{-5}{6}$	$\frac{-5}{0}$	$\frac{-5}{6}$	$\frac{-5.5}{7}$	$\frac{1}{18}$
10 + 300	$\frac{-3}{13.5}$	$\frac{-7.5}{7}$	$\frac{-7}{6}$	$\frac{-7}{0}$	$\frac{-7}{6}$	$\frac{-7.5}{7}$	$\frac{-1.5}{17}$

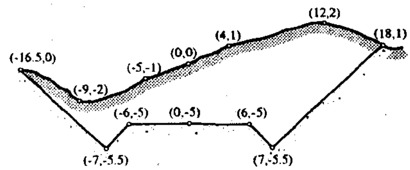
① Compute the cross sectional area at station 10 + 280.
② Compute the cross sectional area at station 10 + 300.
③ Compute the volume between the two stations.

Solution:

① *Area of station 10 + 280:*

$$A_1 = \frac{1}{2}\left[\frac{x_1}{y_1}\frac{x_2}{y_2}\frac{x_3}{y_3}\frac{x_4}{y_4}\frac{x_5}{y_5}\frac{x_6}{y_6}\frac{x_7}{y_7}\frac{x_8}{y_8}\frac{x_9}{y_9}\frac{x_{10}}{y_{10}}\frac{x_{11}}{y_{11}}\frac{x_1}{y_1}\right]$$

$$A_1 = \frac{1}{2}\left[\frac{6}{-5}\frac{7}{-5.5}\frac{18}{1}\frac{12}{2}\frac{4}{1}\frac{0}{0}\frac{-5}{-1}\frac{-9}{-9}\frac{-16.5}{0}\frac{-7}{-5.5}\frac{-6}{-5}\frac{6}{-5}\right]$$



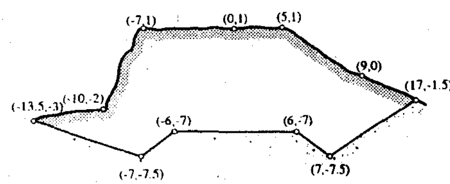
$$A_1 = \frac{1}{2}\,[6(-5.5) + 7(1) + 18(2) + 12(1) + 4(0) + 0(-1) + (-5)(-2) + (-16.5)(-5.5) + (-7)(-5) + (-6)(-5)]$$

$$-\,[-5(7) + 18(-5.5) + 1(12) + 2(4) + 1(0) + 0(-5) + (-1)(-9) + (-2)(-16.5) + 0(-7) + (-6)(-5.5) + (-5)(6)]$$

$$A_1 = \frac{1}{2}\,[(247.75) - (-69)]$$

A_1 = ***158.375 sq.m.***

② *Area at station 10 + 300:*



$$A_2 = \frac{1}{2}\left[\frac{x_1}{y_1}\frac{x_2}{y_2}\frac{x_3}{y_3}\frac{x_4}{y_4}\frac{x_5}{y_5}\frac{x_6}{y_6}\frac{x_7}{y_7}\frac{x_8}{y_8}\frac{x_9}{y_9}\frac{x_{10}}{y_{10}}\frac{x_1}{y_1}\right]$$

$$A_2 = \frac{1}{2}\left[\frac{6}{-7}\frac{7}{-7.5}\frac{17}{-1.5}\frac{9}{0}\frac{5}{1}\frac{-7}{1}\frac{-10}{-2}\frac{-13.5}{-3}\frac{-7}{-7.5}\frac{-6}{-7}\frac{6}{-7}\right]$$

$$A_2 = \frac{1}{2}[6(-7.5) + 7(-1.5) + (17(0) + 9(1) + 5(1) + (-7)(-2) + (-10)(-3) + (-13.5)(-7.5) + (-7)(-7) + (-6)(-7)] - [7(-7) + (17)(-7.5) + 9(-1.5) + 0(5) + 1(-7) + 1(-10) + (-2)(-13.5) + (-3)(-7) + (-6)(-7.5) + (-7)(6)]$$

$$A_2 = \frac{1}{2}[(194.75) - (-156)]$$

$$A_2 = \textbf{175.375 sq.m.}$$

③ *Volume between two stations:*

$$V = \frac{L}{2}(A_1 + A_2)$$

$$V = \frac{20}{2}(158.375 + 175.375)$$

$$V = \textbf{3,337.50 cu.m.}$$

Problem 432:

From a road plan, the following cross-section of the existing ground and the corresponding cross-section notes for a 40 m. stretch was taken.

Existing Ground Cross Sections

Sections	Left				Center	Right		
10 + 040			$\frac{0}{10}$	$\frac{0}{7.5}$	0		$\frac{0.82}{8.3}$	$\frac{1}{10}$
10 + 060	$\frac{0}{10}$	$\frac{-0.80}{8.1}$	$\frac{-2.0}{5.0}$	$\frac{-1.5}{2.0}$	0	$\frac{0.5}{3}$	$\frac{1.0}{5.0}$	$\frac{2.5}{10}$
10 + 080	$\frac{0.5}{10}$	$\frac{0}{9}$	$\frac{-1.5}{6.0}$	$\frac{-1.5}{2.0}$	0	$\frac{0.5}{5.0}$	$\frac{0.15}{8.5}$	$\frac{0}{10}$

Subgrade Cross Sections

Sections	Left			Center	Right		
10 + 040	$\frac{0}{7.5}$	$\frac{-2.5}{5.0}$	$\frac{-2.0}{3.0}$	- 2.0	$\frac{-2.0}{3.0}$	$\frac{-2.5}{5.0}$	$\frac{0.82}{8.3}$
10 + 060		$\frac{-0.8}{8.1}$	$\frac{0.5}{3.0}$	+0.5	$\frac{0.5}{3.0}$	$\frac{0}{5.0}$	$\frac{1.4}{6.4}$
10 + 080		$\frac{0}{9.0}$	$\frac{1.5}{3.0}$	+1.5	$\frac{1.5}{3.0}$	$\frac{0.15}{8.5}$	

① Compute the area of cut at station 10 + 040.

② Compute the volume of cut at station 10 + 060.

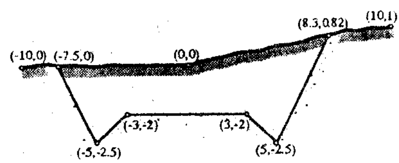
③ Compute the volume of borrow or waste from station 10 + 040 to 10 + 080 considering shrinkage factor of 25%.

Solution:

① *Area of cut at station 10 + 040:*

$$A_1 = \frac{1}{2}\left[\frac{x_1}{y_1}\ \frac{x_2}{y_2}\ \frac{x_3}{y_3}\ \frac{x_4}{y_4}\ \frac{x_5}{y_5}\ \frac{x_6}{y_6}\ \frac{x_7}{y_7}\ \frac{x_1}{y_1}\right]$$

$$2A_1 = \left[\frac{0}{0}\ \frac{-7.5}{0}\ \frac{-5}{-2.5}\ \frac{-3}{-2}\ \frac{3}{-2}\ \frac{5}{-2.5}\ \frac{8.3}{0.82}\ \frac{0}{0}\right]$$



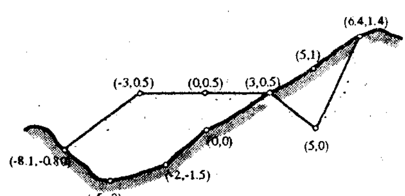
$$2A_1 = [0(0) + (-7.5)(-2.5) + (-5)(-2) + (-3)(-2) + 3(-2.5) + 5(0.82) + 8.3(0)] - [0(-7.5) + 0(-5) + (-2.5)(-3) + (-2)(3) + (-2)(5) + (-2.5)(8.3) + (0.82)(0)]$$

$$2A_1 = [(31.35) - (-29.25)]$$

$$2A_1 = 60.60$$

$$A_1 = \textbf{30.30 sq.m.}\ \textit{(cut)}$$

② *Area of cut at sta. 10 + 060:*



$$A_2 = \frac{1}{2}\left[\frac{x_1}{y_1}\ \frac{x_2}{y_2}\ \frac{x_3}{y_3}\ \frac{x_4}{y_4}\ \frac{x_5}{y_5}\ \frac{x_6}{y_6}\ \frac{x_1}{y_1}\right]$$

$$2A_2 = \left[\frac{0}{0}\ \frac{3}{0.5}\ \frac{-3}{0.5}\ \frac{-8.1}{-0.8}\ \frac{-5}{-2}\ \frac{-2}{-1.5}\ \frac{0}{0}\right]$$

$$2A_2 = [0(0.5) + 3(0.5) + (-3)(-0.80) + (-8.1)(-2) + (-5)(-1.5) + (-2)(0)] - [0(3) + 0.5(-3) + (0.5)(-8.1) + (-0.80)(-5) + (-2)(-2) + (-1.5)(0)]$$

$$2A_2 = 27.60 - 2.45$$
$$A_2 = 12.575 \text{ sq.m. (fill)}$$

Area of cut:

$$A_3 = \frac{1}{2}\left[\frac{x_1}{y_1}\frac{x_2}{y_2}\frac{x_3}{y_3}\frac{x_4}{y_4}\frac{x_1}{y_1}\right]$$

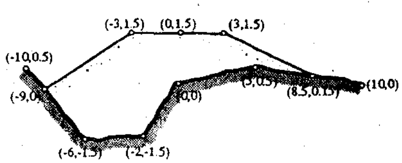
$$2A_3 = \left[\frac{3}{0.5}\frac{5}{0}\frac{6.4}{1.4}\frac{5}{1}\frac{3}{0.5}\right]$$

$$2A_3 = [3(0) + 5(1.4) + 6.4(1) + 5(0.5)] - [0.5(5) + 0(6.4) + 1.4(5) + 1(3)]$$

$$2A_3 = 15.9 - 12.5$$
$$2A_3 = 3.4$$
$$A_3 = \textit{1.70 sq.m. (cut)}$$

③ *Volume of borrow or waste:*
Considering station 1 + 080

$$A_4 = \frac{1}{2}\left[\frac{0}{0}\frac{5}{0.5}\frac{8.5}{0.15}\frac{3}{1.5}\frac{-3}{1.5}\frac{-9}{0}\frac{-6}{-1.5}\frac{-2}{-1.5}\frac{0}{0}\right]$$



$$2A_4 = [(0(5) + 5(0.15) + 8.5(1.5) + 3(1.5) + (0)(-3) + (-9)(-1.5) + (-6)(-1.5) + (0)(-2)] - [0(5) + 0.5(8.5) + 3(0.15) + (1.5)(-3) + 1.5(-9) + 0(-6) + (-2)(-1.5) + (0)(-1.5)]$$

$$2A_4 = 40.5 + 10.3$$
$$A_4 = 25.40 \text{ sq.m. (fill)}$$

Volume of cut from station 10 + 040 to 10 + 060

$$V_1 = \frac{L}{2}(A_1 + A_2)$$
$$V_1 = \frac{20\,(30.30 + 1.70)}{2}$$
$$V_1 = 320 \text{ cu.m. (cut)}$$

Volume of cut from station 10 + 060 to 10 + 080

$$V_2 = \frac{L}{2}(A_1 + A_2)$$
$$V_2 = \frac{20\,(1.70 + 0)}{2}$$
$$V_2 = 17 \text{ cu.m. (cut)}$$

Volume of fill from station 10 + 040 10 + 060

$$V_1 = \frac{L}{2}(A_1 + A_2)$$
$$V_1 = \frac{20\,(0 + 12.575)}{2}$$
$$V_1 = 125.75 \text{ cu.m. (fill)}$$

Volume of fill from station 10 + 060 to 10 + 080

$$V_2 = \frac{L}{2}(A_1 + A_2)$$
$$V_2 = \frac{20\,(12.575 + 25.40)}{2}$$
$$V_2 = 379.75 \text{ cu.m. (fill)}$$

Total volume of cut = 320 + 17
Total volume of cut = 337 cu.m.

Total volume of fill = 125.75 + 379.75
Total volume of fill = 505.50 cu.m.

Volume of fill required from sta. 10 + 040 to 10 + 080
Vol. of fill = 505.50 (1.25)
Vol. of fill = 631.875 m^3

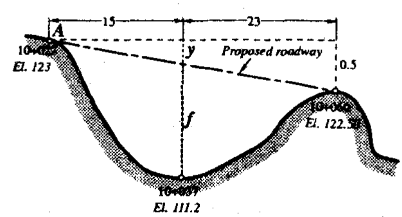
Therefore there is a need of borrow since vol. of fill is greater than that of the volume of cut.

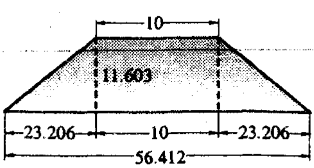
Vol. of borrow = 631.875 - 337
Vol. of borrow = ***294.875 m^3***

Problem 433:

The centerline of a proposed road cross section crosses a small valley between station 10 + 022 (elevation 123.00 m.) and station 10 + 060 (elevation 122.50 m.). The stationing at the bottom of the valley is 10 + 037 (elev. 111.2 m.). The grade line of the proposed road passes the ground points at the edges of the valley (sta. 10 + 022) and (10 + 060) and the section at any of these stations are three level sections. Width of road base = 10 m. with sideslope of 2:1. Assume that the sides of the valley slope directly to the lowest point from the edges.

① Find the cross sectional area of fill at station 10 + 037.
② Compute the volume of fill from station (10 + 022) to (10 + 037)
③ Compute the volume of fill from station (10 + 037) to (10 + 060).





Solution:

① *Area of fill at 10 + 037*

$$\frac{y}{15} = \frac{0.5}{38}$$

$$y = 0.197$$

$$f + y = 123 - 111.2$$

$$f = 123 - 111.2 - 0.197$$

$$f = 11.603$$

$$A = \frac{(10 + 56.412)}{2}(11.603)$$

$$A = 385.29$$

② *Vol. of fill from (10 + 022) to (10 + 037)*

$$V = \frac{(A_1 + 0)}{2}(15)$$

$$V = \frac{[385.29 + 0](15)}{2}$$

$$V = 2890 \text{ m}^3$$

③ *Vol. of fill from (10 + 037) to (10 + 060)*

$$V = \frac{(385.29 + 0)(23)}{2}$$

$$V = 4431 \text{ m}^3$$

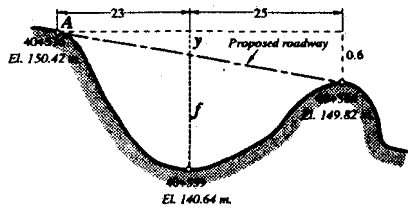
Problem 434:

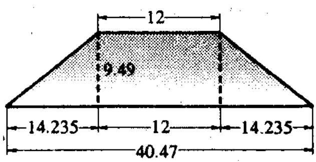
The location survey of the proposed road passes through a rough terrain, and crosses a small valley between two points along the center line of the proposed road. One of the points is at station 40 + 536.00 and at elevation (150.42 m.), the other point is at station 40 + 584.00 and at elevation (149.82 m.). The lowest point at the bottom of the valley is 23 m. from the highest point and has an elevation of its bottom equal to 140.64 m. The road passes through these three points. All sections on this proposed roadway are three level sections having a width of roadway equal to 12 m. with side slope of 1.5 : 1. Assume shrinkage factor to be 1.30.

① Compute the cross sectional area at station 40 + 559.
② Compute the volume of fill needed starting from the highest point of road to the lowest point of the valley.
③ Compute the volume of fill needed from station 40 + 559 to 40 + 584.

Solution:

① *Cross sectional area at 40 + 559:*





$$\frac{y}{23}=\frac{0.6}{48}$$

$y = 0.29$

$f = 150.42 - 0.29 - 140.64$

$f = 9.49$ m.

$$Area=\frac{(12+40.47)(9.49)}{2}$$

$Area = \mathbf{248.97\ m^2}$

② *Volume of fill from 40 + 536 to 40 + 559:*

$$V=\frac{(0+248.97)(23)(1.30)}{2}$$

$V = \mathbf{3722.10\ m^3}$

③ *Volume of fill from (40 + 559 to 40 + 584):*

$$V=\frac{(0+248.97)(25)(1.30)}{2}$$

$V = \mathbf{4045.76\ m^3}$

Problem 435:

Given the following cross section notes of an earthwork on a rolling terrain.

STA. 5 + 000

$$\frac{+10}{31}\qquad\frac{+5}{0}\qquad\frac{+3}{13.5}$$

STA. 5 + 020

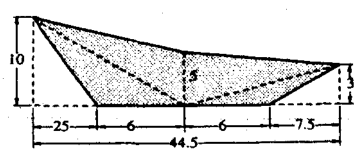
$$\frac{+14}{41}\qquad\frac{+7}{0}\qquad\frac{+4.5}{17.25}$$

The width of the road is 12 m. and the side slope is 2.5 : 1.

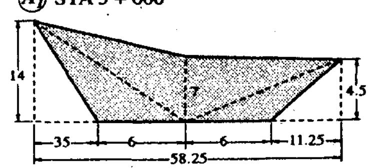
① Compute the volume using Prismoidal formula.
② Compute the volume using End area with Prismoidal Correction.
③ Compute the volume using End area with curvature correction if the road is on a 6° curve which turns to the right with the given cross sections.

Solution:

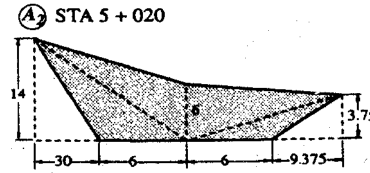
① *Volume using Prismoidal Formula:*



A_1 STA 5 + 000



A_2 STA 5 + 020



Use average values of dimensions of A_1 and A_2 A_m (mid-section)

$$A_1 = \frac{10(6)}{2} + \frac{5(31)}{2} + \frac{5(13.5)}{2} + \frac{6(3)}{2}$$

$A_1 = 150.25\ m^2$

$$A_2 = \frac{6(14)}{2} + \frac{7(41)}{2} + \frac{7(17.25)}{2} + \frac{6(4.5)}{2}$$

$A_2 = 259.375\ m^2$

$$A_m = \frac{12(6)}{2} + \frac{6(36)}{2} + \frac{6(15.375)}{2} + \frac{3.75(6)}{2}$$

$A_m = 201.375\ m^2$

$$Vol. = \frac{L}{6}(A_1 + 4A_m + A_2)$$

$$Vol. = \frac{20}{6}[150.25 + 4(201.375) + 259.375]$$

$Vol. = \mathbf{4050.42\ cu.m.}$

② *Volume by End area with Prismoidal Correction:*

$Vol. = V_E - V_p$

$$V_E = \frac{(A_1 + A_2)\,L}{2}$$

$$V_E = \frac{(150.25 + 259.375)\,(20)}{2}$$

$V_E = 4096.25\ m^3$

$$V_p = \frac{L}{12}(C_1 - C_2)(D_1 - D_2)$$

$$V_p = \frac{20}{12}[(5 - 7)(44.5 - 58.25)]$$

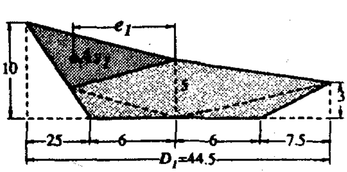
$V_p = 45.83\ m^3$

$Vol. = V_E - V_p$

$Vol. = 4096.25 - 45.83$

$Vol. = \mathbf{4050.42\ m^3}$

③ *Volume with curvature correction:*



$$As_1 = A_1 - \left[\frac{5\,(13.5)}{2} + \frac{3\,(6)}{2}\right](2)$$

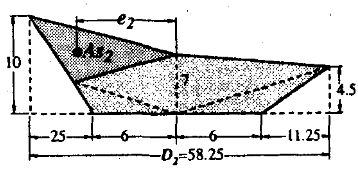
$As_1 = 150.25 - 85.5$

$As_1 = 64.75\ m^2$

$$e_1 = \frac{1}{3}D_1$$

$$e_1 = \frac{44.5}{3}$$

$e_1 = 14.83$ *(positive the excess area is away from the center of curve)*



$$As_2 = A_2 - \left[\frac{7\,(17.25)}{2} + \frac{6\,(4.5)}{2}\right](2)$$

$As_2 = 259.375 - 147.75$

$As_2 = 111.625\ m^2$

$$e_2 = \frac{D_2}{3}$$

$$e_2 = \frac{58.25}{3}$$

$e_2 = 19.42$ *(positive)*

$Vol. = V_E + V_c$

$$V_c = \frac{L}{2R}(As_1\, e_1 + As_2\, e_2)$$

$$R = \frac{1145.916}{D}$$

$$R = \frac{1145.916}{6}$$

$R = 190.99$ m.

$$V_c = \frac{20}{2\,(190.99)}[64.75(14.83) + 111.625(19.42)]$$

$V_c = 163.78\ m^3$

$$V_E = \frac{(A_1 + A_2)\,L}{2}$$

$$V_E = \frac{(150.25 = 259.375)\,(20)}{2}$$

$V_E = 4096.25\ m^3$

$Vol. = V_E + V_c$

$Vol. = 4096.25 + 163.78$

$Vol. = \mathbf{4260.03\ m^3}$

Problem 436:

A highway fill stretches between stations 5 + 040 and 5 + 140 with a uniform ground slope. It has a side slope of 2 : 1 and width of roadway is 12 m.

STA. 5 + 040

$$\frac{?}{?} \qquad \frac{-4}{6} \qquad \frac{-5}{0} \qquad \frac{-6}{6} \qquad \frac{?}{?}$$

STA. 5 + 140

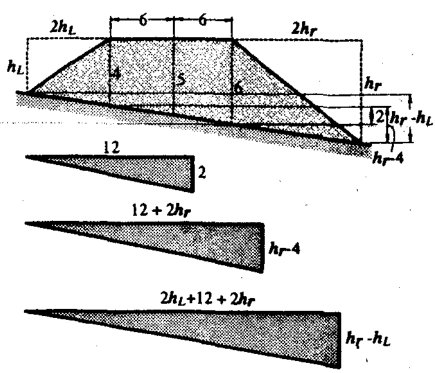
$$\frac{?}{?} \qquad \frac{-6}{6} \qquad \frac{-7}{0} \qquad \frac{-8}{6} \qquad \frac{?}{?}$$

① Compute the volume between the two stations using Prismoidal formula.
② Compute the Prismoidal correction between the two stations in cu.m.
③ Compute the curvature correction between the two stations if the road is on a 5 degree curve which turns to the right of the cross sections in cu.m.

Solution:

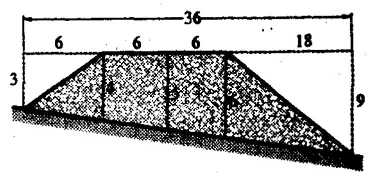
① *Volume using Prismoidal formula:*

STA 5 + 040



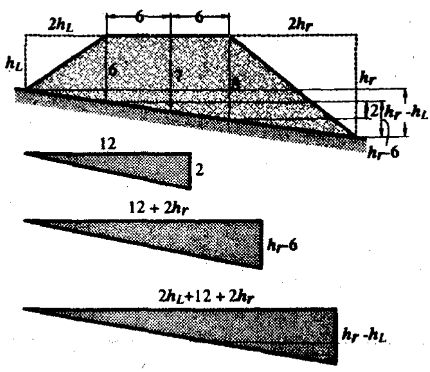
$$\frac{h_r - 4}{12 + 2h_r} = \frac{2}{12}$$
$$6h_r - 24 = 12 + 2h_r$$
$$4h_r = 36$$
$$h_r = 9 \text{ m.}$$

$$\frac{h_r - h_l}{2h_L + 12 + 2h_r} = \frac{2}{12}$$
$$\frac{9 - h_l}{2h_L + 12 + 18} = \frac{1}{6}$$
$$54 - 6h_L = 2\,h_L + 30$$
$$8h_L = 24$$
$$h_L = 3 \text{ m.}$$

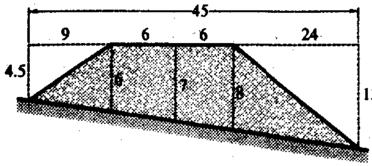


$$A_1 = \frac{(3+9)(36)}{2} - \frac{6(3)}{2} - \frac{18(9)}{2}$$
$$A_1 = 126 \text{ m}^2$$

STA 5 + 140

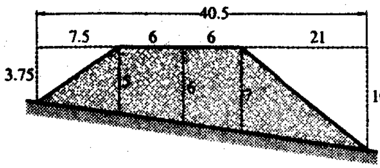


$$\frac{h_r - 6}{12 + 2h_r} = \frac{2}{12}$$
$$6h_r - 36 = 12 + 2h_r$$
$$4h_r = 48$$
$$h_r = 12$$
$$\frac{h_r - h_l}{2h_L + 12 + 2h_r} = \frac{2}{12}$$
$$\frac{12 - h_l}{2h_L + 12 + 24} = \frac{1}{6}$$
$$72 - 6h_L = 2h_L + 36$$
$$8h_L = 36$$
$$h_L = 4.5$$



$$A_2 = \frac{(4.5+12)(45)}{2} - \frac{4.5(9)}{2} - \frac{24(12)}{2}$$

$$A_2 = 206.75\ m^2$$



A_m (mid - section)

$$A_m = \frac{(3.75+10.5)(40.5)}{2} - \frac{7.5(3.75)}{2} - \frac{21(10.5)}{2}$$

$$A_m = 164.25\ m^2$$

$$Vol. = \frac{L}{6}(A_1 + 4A_m + A_2)$$

$$Vol. = \frac{100}{6}[126 + 4(164.25) + 206.75]$$

$$\textbf{Vol. = 16495.83 m}^3$$

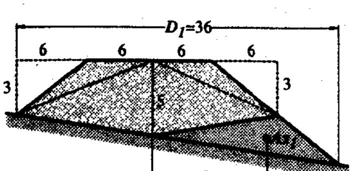
② *Prismoidal correction:*

$$V_p = \frac{L}{12}(C_1 - C_2)(D_1 - D_2)$$

$$V_p = \frac{100}{12}[(5-7)(36-45)]$$

$$\mathbf{V_p = 150\ m^3}$$

③ *Curvature correction:*



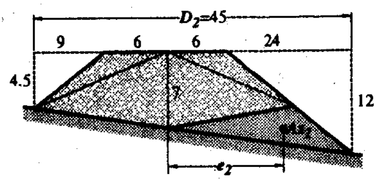
$$As_1 = 126 - \left[\frac{6(3)}{2} + \frac{5(12)}{2}\right](2)$$

$$As_1 = 48\ m^2$$

$$e_1 = \frac{1}{3}(D_1)$$

$$e_1 = \frac{1}{3}(36)$$

$e_1 = -12$ *(neg. towards the center of curve)*



$$As_2 = 206.75 - \left[\frac{6(4.5)}{2} + \frac{7(15)}{2}\right](2)$$

$$As_2 = 74.75\ m^2$$

$$e_2 = \frac{1}{3}D_2$$

$$e_2 = \frac{1}{3}(45)$$

$e_2 = -15$ *(neg. towards the center of curve)*

$$R = \frac{1145.916}{D}$$

$$R = \frac{1145.916}{5} = 229\ m.$$

$$V_c = \frac{L}{2R}(As_1\, e_1 + As_2\, e_2)$$

$$V_c = \frac{100}{2(229)}[(48)(-12) + 74.75(-15)]$$

$$\mathbf{V_c = -370.58\ m^3}$$

Problem 437:

From the given cross section of the proposed barangay road.

STA. 1 + 020

$\frac{6.45}{+2.3}$	+1.5	$\frac{4.5}{+1.0}$

STA. 1 + 040

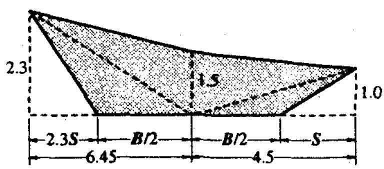
$\frac{6.0}{+2.0}$	+1.2	$\frac{6.9}{+2.6}$

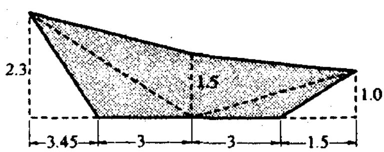
EARTHWORKS

① Compute the volume between the two stations using Prismoidal formula.
② Compute the volume between the two stations using end area with Prismoidal correction.
③ Compute the volume between the two stations if the road is on a curve which turns to the left with the given cross sections if it has a radius of 200 m.

Solution:

① *Volume by Prismoidal Formula:*





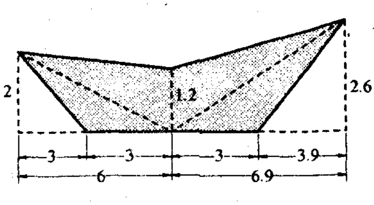
STATION 1+020

STA 1 + 020

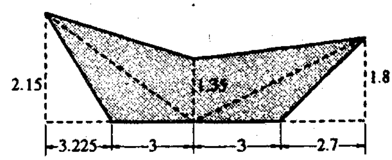
$$\frac{B}{2} + 2.3S = 6.45$$
$$\frac{B}{2} + S = 4.5$$
$$1.3S = 1.95$$
$$S = 1.5$$
$$\frac{B}{2} + 1.5 = 4.5$$
$$B = 6 \text{ m.}$$

$$A_1 = \frac{2.3(3)}{2} + \frac{1.5(6.45)}{2} + \frac{1.5(4.5)}{2} + \frac{3(1)}{2}$$
$$A_1 = 13.1625 \text{ m}^2$$

STA 1 + 040



$$A_2 = \frac{3(2)}{2} + \frac{1.2(6)}{2} + \frac{1.2(6.9)}{2} + \frac{3(2.6)}{2}$$
$$A_2 = 14.64 \text{ m}^2$$

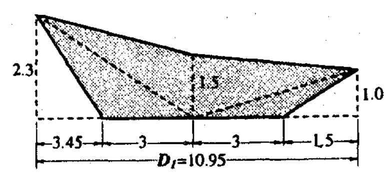


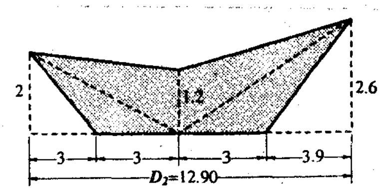
A_m section

Note: Use average dimensions of sta. 1 + 020 and 1 + 040

$$A_m = \frac{2.15(3)}{2} + \frac{1.35(6.225)}{2} + \frac{1.35(5.7)}{2} + \frac{3(1.8)}{2}.$$
$$A_m = 13.974 \text{ m}^2$$
$$Vol. = \frac{L}{6}(A_1 + 4Am + A_2)$$
$$Vol. = \frac{20}{6}[(13.1625) + 4(13.974) + 14.64]$$
$$Vol. = \mathbf{279\ m^3}$$

② *Volume by end area with Prismoidal correction:*



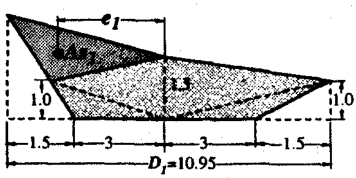


$$V_E = \frac{(A_1 + A_2)\,L}{2}$$
$$V_E = \frac{(13.1625 + 14.64)\,(20)}{2}$$
$$V_E = 278.025 \text{ m}^3$$

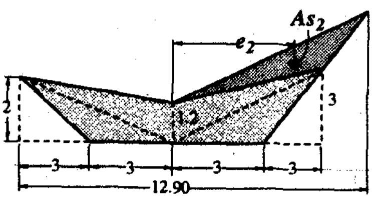
$$V_p = \frac{L}{12}(C_1 - C_2)(D_1 - D_2)$$
$$V_p = \frac{20}{12}[(1.5 - 1.2)(10.95 - 12.90)]$$
$$V_p = -0.975 \text{ m}^3$$

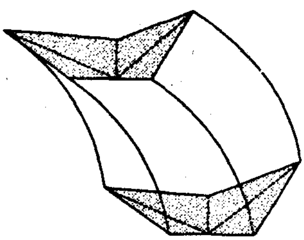
$$V_{cp} = V_E - V_p$$
$$V_{cp} = 278.025 - (-0.975)$$
$$V_{cp} = \boldsymbol{279\ m^3}$$

③ *Volume by end area with curvature correction:*



$$As_1 = 13.1625 - \left[\frac{(3)(1)}{2} + \frac{1.5(4.5)}{2}\right](2)$$
$$As_1 = 3.4125$$
$$e_1 = \frac{1}{3}(10.95)$$
$e_1 = -3.65$ *(neg. towards the center of curve)*





$$As_2 = 14.64 - [\frac{(3)(2)}{2} + \frac{1.2(6)}{2}](2)$$
$$As_2 = 1.44$$

$$e_2 = \frac{1}{3}(12.90)$$
$e_2 = +4.3$ *(positive away from center of curve)*

$$V_c = \frac{L}{2R}(As_1\, e_1 + As_2\, e_2)$$
$$V_c = \frac{20}{2(200)}[3.4125(-3.65) + 1.44(4.3)]$$
$$V_c = -0.313 \text{ m}^3$$

$$V = V_E + V_c$$
$$V = 278.025 + (-0.313)$$
$$V = \boldsymbol{277.712\ m^3}$$

Problem 438:

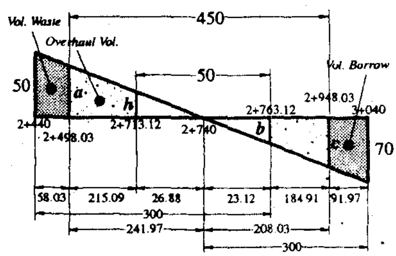
The earthworks data of a proposed highway is shown on the tabulated data.
Length of economical haul = 450 m.
Stationing of limits of economical haul
= 2 + 498.03 and 2 + 948.03
Stationing of limits of free haul
= 2 + 713.12 and 2 + 763.12
Free haul distance is 50 m.
Assume the ground surface to be uniformly sloping.

STATION	AREA	
	CUT	FILL
2 + 440	50 m²	
2 + 740	0	Balancing Point
3 + 040		70 m²

① Compute the overhaul volume.
② Compute the volume of waste.
③ Compute the volume of borrow.

Solution:

① *Overhaul volume:*



$$\frac{h}{26.88}=\frac{50}{300}$$
$h = 4.48$
$$\frac{a}{241.97}=\frac{50}{300}$$
$a = 40.33$
$$\text{Overhaul volume} = \frac{(4.48+40.33)(215.09)}{2}$$
Overhaul volume = **4819.10 m³**

② Volume of waste:
$$V=\frac{(40.33+50)(58.03)}{2}$$
$V = \mathbf{2620.92\ m^3}$

③ Volume of borrow:
$$\frac{C}{208.03}=\frac{70}{300}$$
$C = 48.54$
$$\text{Vol. of borrow} = \frac{(48.54+70)(91.97)}{2}$$
Vol. of borrow = **5451.06 m³**

Problem 439:

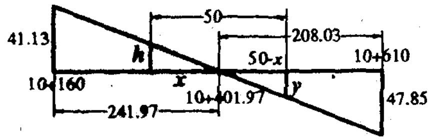
The given data off a proposed Manila - Cavite Coastal road is tabulated below. The free haul distance is 50 m. and the cost of borrow is P420 per cu.m., cost of excavation is P350 per cu.m. and the cost of haul is P21 per meter station. The ground surface is assume to be uniformly sloping.

STATION	AREA	
	CUT	FILL
10 + 160	41.13 m²	
10 + 401.97	0	(balancing point)
10 + 610		47.85 m²

① Compute the limit of economical haul.
② Compute the free haul volume.
③ Compute the overhaul volume.

Solution:

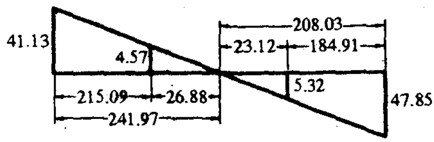
① *Limit of economical haul:*



$$LEH=\frac{C_b C}{C_h}+FHD$$
$$LEH=\frac{420(20)}{21}+50$$
$LEH = \mathbf{450\ m.}$

② *Free haul volume:*
$$\frac{h}{x}=\frac{41.13}{241.97}$$
$h = 0.17x$
$$\frac{47.85}{208.03}=\frac{y}{50-x}$$
$y = 0.23(50-x)$
$$\frac{hx}{2}=\frac{y(50-x)}{2}$$
$$\frac{0.17x^2}{2}=\frac{0.23(50-x)^2}{2}$$
$0.86x = 50 - x$
$x = 26.88$
$50 - x = 23.12$
$$\text{Free haul volume} = \frac{hx}{2}$$
$h = 0.17(26.88)$
$h = 4.57$
$y = 0.23(23.12)$
$y = 5.32$
$$\text{Free haul vol.} = \frac{4.57(26.88)}{2}$$
Free haul vol. = **61.42 cu.m.**

③ *Overhaul volume:*



$$V_1=\frac{(41.13+4.57)(215.09)}{2}$$
$V_1 = \mathbf{4915\ m^3}$

Problem 440:

The following data are results of the earthwork computations of areas, free haul distance and limits of economical haul by analytical solution instead of graphical solution (mass diagrams). The cross sectional area at station 1 + 460 is 40 sq.m. in fill and at station 2 + 060 the cross sectional area is 60 sq.m. in cut. The balancing point is at station 1 + 760 where area is equal to zero. Assume the ground surface to be sloping upward uniformly from station 1 + 460 to 1 + 760 and then with slightly steeper slope to 2 + 060. Assume free haul distance = 50 m. and limit of economical haul = 450 m.

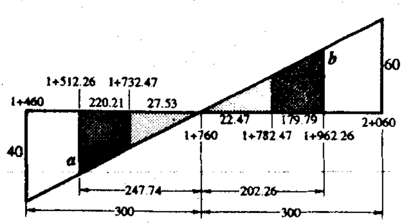
Stationing of the limits of free haul distance
= (1 + 732.47) and (1 + 782.47)

Stationing the limits of economical haul
= (1 + 512.26) and (1 + 962.26)

① Determine the overhaul volume.
② Determine the volume of waste.
③ Determine the volume of borrow.

Solution:

① *Overhaul volume:*

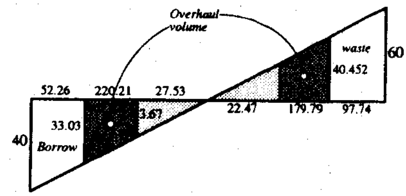


$$\frac{x}{27.53} = \frac{40}{300}$$
$x = 3.67$

$$\frac{y}{22.47} = \frac{60}{300}$$
$y = 4.494$

$$\frac{a}{220.21} = \frac{40}{300}$$
$a = 29.36$

$$\frac{b}{179.79} = \frac{60}{300}$$
$b = 35.958$



Overhaul volume
$$= \frac{(4.494 + 40.452)}{2}(179.79)$$
$= \mathbf{4040\ cu.m.}$

② *Volume of waste:*
$$V = \frac{(40.452 + 60)}{2}(97.74)$$
$V = \mathbf{4909}$

③ *Volume of borrow:*
$$V = \frac{(40 + 33.032)}{2}(52.26)$$
$V = \mathbf{1908\ cu.m.}$

Problem 441:

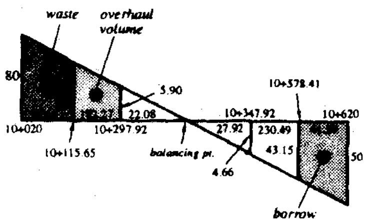
Here under shows a table of quantities of earthworks of a proposed Highway to connect Bogo City and Danao City. The length of the free haul distance is specified to be 50 m. long and the limit of economical haul is 462.76 long. Assume the ground surface to be sloping uniformly.

Station	End Areas		Remarks
	Cut	Fill	
10 + 020	80.00		Initial point
10 + 115.65	54.57		Limit of economical haul
10 + 297.92	5.90		Limit of free haul
10 + 320	0		Balancing point
10 + 347.92		4.66	Limit of free haul
10 + 578.41		43.15	Limit of economical haul
10 + 620		50.00	End point

① Compute the overhaul volume.
② Compute the volume of borrow.
③ Compute the volume of waste.

Solution:

① *Overhaul volume:*



$$Overhaul\ volume = \frac{(54.57 + 5.90)(182.27)}{2}$$

$$Overhaul\ volume = \mathbf{5510.93\ m^3}$$

② *Volume of borrow:*

$$Volume\ of\ borrow = \frac{(50 + 43.15)(41.59)}{2}$$

$$Volume\ of\ borrow = \mathbf{1937.05\ m^3}$$

③ *Volume of waste:*

$$Volume\ of\ waste = \frac{(80 + 54.57)(95.65)}{2}$$

$$Volume\ of\ waste = \mathbf{6435.81\ m^3}$$

Problem 442:

The profile of the ground surface along which the center line of the roadway is sloping uniformly at a certain grade. At sta. 5 + 400 the cross sectional area is 20.89 m^2 in fill and the finished roadway slopes upward producing a cross sectional area of 28.6 m^2 in cut at station 5 + 850. The stationing of the balancing point is 5 + 650.

Free haul distance = 50 m.
Cost of haul = P0.20 per meter station
Cost of borrow = P4 per cu.m.

① Compute the limit of economical haul.
② Compute the stationing of the limits of freehaul distance.
③ Compute the freehaul volume.

Solution:

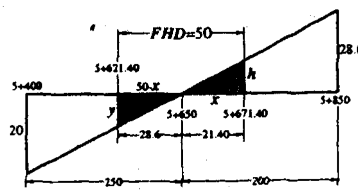
① *Limit of economical haul:*

$$LEH = \frac{C_b C}{C_h} + FHD$$

$$LEH = \frac{4(20)}{0.20} + 50$$

$$LEH = \mathbf{450\ m.}$$

② *Stationing of limits of freehaul distance:*



$$\frac{h}{x} = \frac{28.6}{200}$$

$$h = 0.143\,x$$

$$\frac{y}{50 - x} = \frac{20}{250}$$

$$y = 0.08\,(50 - x)$$

Vol. of excavation = Vol. of embankment

$$\frac{hx}{2} = \frac{y(50 - x)}{2}$$

$$\frac{0.143\,x\,(x)}{2} = \frac{0.08\,(50 - x)(50 - x)}{2}$$

$$0.143\,x^2 = 0.08\,(50 - x)^2$$

$$0.378\,x = 0.283\,(50 - x)$$

$$50 - x = 133697\,x$$

$$x = 21.40 \text{ m.}$$

Limits of freehaul distance
= (5 + 650) + 21.40
= 5 + 671.40

= (5 + 671.40) - 50
= 5 + 621.40

Limits of freehaul distance
= ***5 + 671.40*** and ***5 + 621.40***

③ *Freehaul volume:*

$$\text{Freehaul vol.} = \frac{hx}{2}$$

$h = 0.143\,(21.4)$

$h = 3.06$

$$\text{Freehaul vol.} = \frac{3.06\,(21.4)}{2}$$

$$\text{Freehaul vol.} = \mathbf{32.74\ m^3}$$

Problem 443:

The grading works of the portion of the proposed expansion of the North expressway shows the following notes. Free haul distance = 50 m. Limit of economical haul = 450 m.

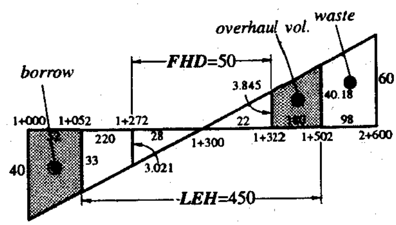
Limits of free haul distance is from sta. 1 + 272 to 1 + 322.
Limits of economical haul is from sta. 1 + 052 and 1 + 502

STATION	END AREAS (m^2) CUT	FILL
1 + 000		40
1 + 052		33
1 + 272		3.021
1 + 300	0	(Balancing point)
1 + 322	3.845	
1 + 502	40.18	
1 + 600	60	

Assume the ground surface to be sloping uniformly from one end to the other end.

① Determine the overhaul volume.
② Determine the volume of borrow.
③ Determine the volume of waste.

Solution:
① *Overhaul volume:*



$$\text{Overhaul volume} = \frac{(3.845 + 40.18)(180)}{2}$$

$$\text{Overhaul volume} = \mathbf{3962.25}$$

② $$\text{Volume of waste} = \frac{(40.18 + 60)(98)}{2}$$

$$\text{Volume of waste} = \mathbf{4908.82\ cu.m.}$$

③ $$\text{Volume of borrow} = \frac{(33 + 40)\,52}{2}$$

$$\text{Volume of borrow} = \mathbf{1898\ cu.m.}$$

Problem 444:

The following data represents a single summit mass diagram of a proposed expansion of the Tolosa Expressway.

STATIONS	VOLUME CUT (m^3)	FILL (m^3)
10 + 000	+200	
10 + 040	+100	
10 + 080	+150	
10 + 120	+140	
10 + 160	+110	
10 + 200	+190	
10 + 240	+50	
10 + 280		- 40
10 + 320		- 120
10 + 360		- 90
10 + 400		- 80
10 + 440		- 200
10 + 480		- 220
10 + 520		- 110
10 + 560		- 320
10 + 600		- 280

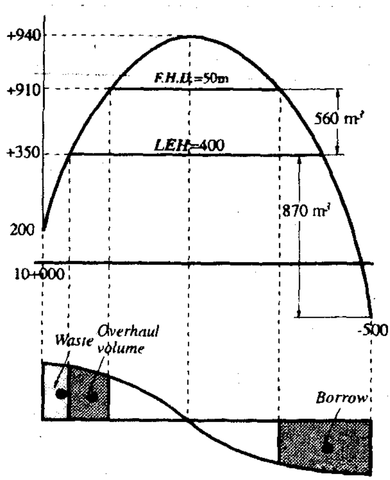
EARTHWORKS

Free haul distance = 50 m.
Limit of Economical Haul = 400 m.
Mass ordinate of initial limit of free haul distance ---------- +910 m^3
Mass coordinate of initial limit of economical haul ----- +350 m^3

① Compute the volume of waste in cu.m.
② Compute the volume of overhaul in cu.m.
③ Compute the volume of borrow in cu.m.

Solution:

STATION	VOLUME	Mass Ordinates
10 + 000	+200	+200
10 + 040	+100	+300
10 + 080	+150	+450
10 + 120	+140	+590
10 + 160	+110	+700
10 + 200	+190	+890
10 + 240	+50	+940
10 + 280	- 40	+900
10 + 320	- 120	+780
10 + 360	- 90	+690
10 + 400	- 80	+610
10 + 440	- 200	+410
10 + 480	- 220	+190
10 + 520	- 110	+80
10 + 560	- 320	- 240
10 + 600	- 280	- 520



① *Volume of waste:*
Vol. of waste = 350 - 200
Vol. of waste = ***150 m^3***

② *Overhaul volume:*
Overhaul volume = 910 - 350
Overhaul volume = ***560 m^3***

③ *Volume of borrow:*
Volume of borrow = 350 + 520
Volume of borrow = ***870 m^3***

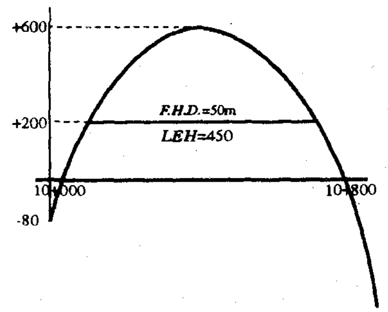
Problem 445:

A single summit mass diagram from station 0 + 100 to 0 + 800 of a proposed extension of Santander Highway. Has the following technical data.
Mass ordinate of the initial limit of economical distance = +600 m^3
Free haul distance = 50 m.
Limit of economical haul = 450 m.
Cost of borrow = P500 per cu.m.
Mass ordinate of station 0 + 100 = - 80 m^3
Mass ordinate of station 0 + 800 = - 130 m^3

① Compute the overhaul volume in cu.m.
② Compute the length of overhaul in meters if the total cost of hauling is P105,750.00
③ Compute the total cost of borrow.

Solution:

① *Overhaul volume:*



Overhaul volume = 600 - 200
Overhaul volume = ***400 m^3***

② *Length of overhaul:*

$$LEH = \frac{C_b C}{C_h} + FHD$$

$$450 = \frac{500\,(20)}{C_h} + 50$$

C_h = P25 per cu.m. / meter station

Total cost of haul = P105750.00

$$105750 = \frac{25\,(L)\,(400)}{20}$$

L = ***211.50 m.***

③ *Total cost of borrow:*

Vol. of borrow = 200 + 130

Vol. of borrow = 330 m^3

Cost of borrow = 330 (500)

Cost of borrow = ***P165,000***

Problem 446:

The cost of borrow per cu.m. is P500 and the cost of haul per meter station is P25. Cost of excavation is approximately P650 per cu.m. The free haul distance is 50 m. long and the length of overhaul is equal to 201.40 m. If the mass ordinate of the initial point of the free haul distance is +800 m^3 and the mass ordinates of the summit mass diagram from 10 + 000 to 10 + 600 are - 60 m^3 and - 140 m^3 respectively.

① Compute the length of economical haul.
② Compute the mass ordinate of the initial point of the limit of economical haul if the total cost of hauling is P171190.
③ Compute the total cost of waste.

Solution:

① *Limit of economical haul:*

$$LEH = \frac{C_b C}{C_h} + FHD$$

$$LEH = \frac{500\,(20)}{25} + 50$$

LEH = ***450 m.***

② *Mass ordinate of inital point of limit of economical haul:*

Cost of haul:

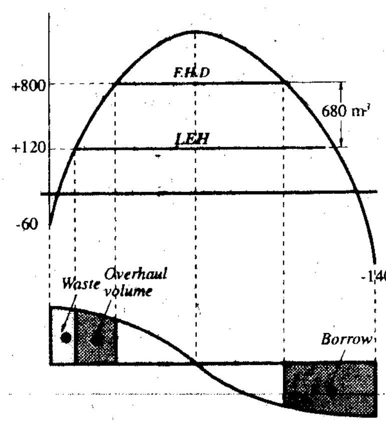
$$171190 = \frac{25\,(201.40)\ \text{Vol. of overhaul}}{20}$$

Vol. of overhaul = 680 m^3

Mass ordinate of inital point of limit of economical haul

= 800 - 680

= ***120 m³***



③ *Cost of waste:*

Cost = 650 (120 + 60)

Cost = ***P117,000***

Problem 446-A:

The following are the data on a simple summit mass diagram.

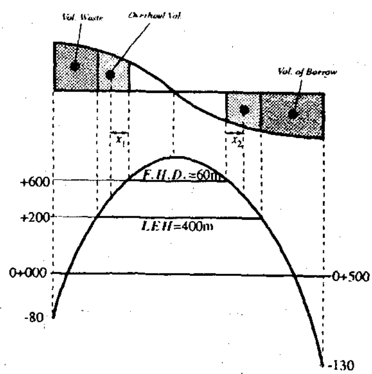
STA	MASS ORDINATE (m^3)
0 + 000	- 80
0 + 500	- 130

Initial point of limit of freehaul distance = +600
Initial limit of economic haul = +200
Freehaul distance = 60 m.
Limit of economical distance = 400 m.
Cost of haul = P120 per cu.m per meter station.

① Determine the volume of waste in m^3.
② *Determine the volume of borrow in m^3.*
③ Determine the overhaul volume in cu.m.
④ Determine the length of overhaul if the total cost of hauling is P192,000.

Solution:

① Volume of waste:



Volume of waste = 200 + 80
Volume of waste = ***280 m³***

② Volume of borrow:
Volume of borrow = 200 + 130
Volume of borrow = ***330 m³***

③ Overhaul volume:
Overhaul volume = 600 – 200
Overhaul volume = ***400 m³***

④ Length of overhaul:

$$192{,}000 = \frac{120(400)\,x}{20}$$

x = ***80 m.***

Problem 446-B:

Using the following notes on cuts and fills and a shrinkage factor of 1.25.

① Find the mass ordinate at station 20 + 040.
② Find the mass ordinate at station 20 + 120.
③ Find the mass ordinate at station 20 + 180.

STATIONS	VOLUMES	
	CUT(m^3)	FILL(m^3)
20 + 000		60
20 + 020		70
20 + 040		30
20 + 060	110	
20 + 080	50	
20 + 100	50	
20 + 120		40
20 + 140		60
20 + 160	20	
20 + 180	30	

Solution:

① Mass ordinate at station 20 + 040:

STATIONS	VOLUMES		MASS ORDINATES
	CUT (m^3)	CORRECTED FILL (m^3)	
20 + 000		1.25(60) = - 75	20 + 000
20 + 020		1.25(70) = - 87.5	20 + 020
20 + 040		1.25(30) = - 37.5	20 + 040
20 + 060	110	+ 110	20 + 060
20 + 080	50	+ 80	20 + 080
20 + 100	50	+ 50	20 + 100
20 + 120		1.25(40) = - 50	20 + 120
20 + 140		1.25(60) = - 75	20 + 140
20 + 160	20	+ 20	20 + 160
20 + 180	30	+ 30	20 + 180

Mass ordinate at station 20 + 040 = ***- 200***

② Mass ordinate at station 20 + 120 = ***- 10***

③ Mass ordinate at station 20 + 180: = ***- 35***

Problem 446-C:

The grading works of a proposed National Road shows the following data of an earthworks:

Free haul distance = 50 m.
Cost of borrow = P5 per cu.m.
Cost of haul = P0.25 per meter station
Stationing of one limit of Free Haul
= 2 + 763.12
Stationing of one limit of Economical Haul
= 2 + 948.03

Assume the ground surface has a uniform slope from cut to fill.

STATION	AREA	
	CUT (m^2)	FILL (m^2)
2 + 440	51 m^2	
2 + 740	0	Balancing Point
3 + 040		69 m^2

① Compute the length of economical haul.
② Compute the overhaul volume.
③ Compute the volume of borrow.
④ Compute the volume of waste.

Solution:

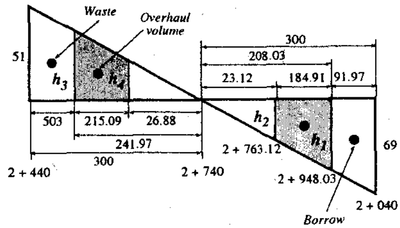
① Limit of economical haul:

$$LEH = \frac{C_b 20}{C_h} + FHD$$

$$LEH = \frac{5(20)}{0.25} + 50$$

$LEH = \textbf{450 m.}$

② Overhaul volume:



$$\frac{h_1}{208.03} = \frac{69}{300}$$
$h_1 = 47.85$
$$\frac{h_2}{23.12} = \frac{69}{300}$$
$h_2 = 5.32$
$$\frac{h_3}{241.97} = \frac{51}{300}$$
$h_3 = 41.13$
$$\frac{h_4}{26.88} = \frac{51}{300}$$
$h_4 = 4.57$

$$\text{Overhaul volume} = \frac{(h_3 + h_4)}{2}(215.09)$$

$$\text{Overhaul volume} = \frac{(41.13 + 4.57)}{2}(215.09)$$

Overhaul volume = ***4915 m³***

Check:

$$Vol. = \frac{(h_2 + h_1)}{2}(184.91)$$

$$Vol. = \frac{(5.32 + 47.85)}{2}(184.91)$$

Vol. = ***4915 m³***

③ Volume of borrow:

$$\text{Vol. of borrow} = \frac{(47.85 + 69)}{2}(91.97)$$

Vol. of borrow = ***5373.35 m³***

④ Volume of waste:

$$\text{Vol. of waste} = \frac{(51 + 41.13)}{2}(58.03)$$

Vol. of waste = ***2673.15 m³***

TRAFFIC ENGINEERING

Highway Safety and Accident Analysis

1. **Accident rates for 100 million vehicle miles of travel (HMVM) for a segment of a highway:**

$$R = \frac{A\,(100{,}000{,}000)}{ADT \times N \times 365 \times L}$$

R = the accident rate for 100 million vehicle miles
A = the number of accidents during period of analysis
ADT = average daily traffic
N = time period in years
L = length of segment in miles

2. **Accident rates per million entering vehicles (MEV) for an intersection:**

$$R = \frac{A\,(1{,}000{,}000)}{ADT \times N \times 365}$$

R = the accident rate for one million entering vehicles
ADT = the average daily traffic entering the intersection from all legs
N = time period in years

3. **Severity ratio**

$$= \frac{\text{fatal + injury accidents}}{\text{fatal + injury + property damage}}$$

4. **Space mean speed of a vehicle:**

$$\mu_s = \frac{nd}{\sum t_i}$$

$\sum t_i$ = sum of all time observations
n = no. of vehicles
d = length of a segment of the road
μ_s = space mean speed

5. **Space mean speed of a vehicle:**

$$\mu_t = \frac{\sum \frac{d}{t_i}}{n}$$

d = length of a segment of the road
t_i = time of observation
n = no. of vehicles

6. **Rate of flow:**

$$q = K\mu_s$$

q = rate of flow in vehicles/hour
K = density in vehicles/hour/mile
μ_s = space mean speed

7.

Spacing of vehicles:

No. of vehicles per km

$$= \frac{\text{Vol. of traffic in vehicles/hour}}{\text{ave. speed of car in km/hr}}$$

Average density = no. of vehicles per km/

$$\text{Spacing of vehicles} = \frac{1000}{\text{ave. density}}$$

Note: 1 km = 1000 m.

8.

Capacity of a single lane in vehicles per hour:

$$S = Vt + L$$

S = ave. center to center spacing of cars in meters
V = ave. speed of cars in meters
t = reaction time in seconds
L = Length of one car in meters

$$C = \frac{1000\,(V)}{S}$$

C = Capacity of a single lane in vehicles/hour

9.

Min. time headway:

$$H_t = \frac{3600}{C}$$

H_t = time headway in sec.
C = capacity in sec.

$$C = \frac{1000V}{S}$$

V = average velocity in kph
S = spacing between cars
S = Vt + L
t = reaction time in sec.
L = length of one car in meters

10.

Time mean speed:

$$\mu_t = \frac{\Sigma\mu_i}{n}$$

μ_t = time mean speed

$\Sigma\mu_i$ = sum of all spot speeds (kph)

n = no. of vehicles

11.

Space mean speed:

$$\mu_s = \frac{n}{\Sigma \frac{1}{\mu_1}}$$

$\Sigma \frac{1}{\mu_1}$ = sum of the reciprocal of spot speeds

n = no. of vehicles

μ_s = space mean speed

12.

Density of traffic:

$$K = \frac{q}{\mu_s}$$

K = density of traffic in vehicles/km

q = flow of traffic in vehicles/hr

μ_s = space mean speed in kph

13.

Density of traffic:

$$K = \frac{R_1}{\text{Ave. length of vehicles}}$$

K = density of traffic in vehicles/km

$$R_1 = \frac{\text{sum of vehicle lengths}}{\text{length of roadway section}}$$

14.

Variance about the space means speed:

$$\mu_t + \mu_s = \frac{\sigma_s^2}{\mu_s}$$

σ_s^2 = variance about the space mean speed

μ_t = time meas speed

μ_s = space mean speed

15.

5 min. peak hour factor:

P.H.F. = peak hour factor

$$P.H.F. = \frac{\text{sum of flow rate in one hour}}{\text{max. peak flow rate x } \frac{60}{5}}$$

447. CE Board Nov. 1998

Data on a traffic accident recorded on a certain intersection for the past 5 years has an accident rate of 4160 per million entering vehicles (ARMV). If the average daily traffic entering the intersection is 504, find the total number of accidents during the 5 year period.

Solution:

$$R = \frac{A\,(1000000)}{ADT\,(N)(365)}$$

$$4160 = \frac{A\,(1000000)}{504(5)(365)}$$

A = **3826** (number of accidents)

448. CE Board Nov. 2000

Data on a traffic accident recorded on a certain intersection for the past 4 years has an accident rate of P9200 per million entering vehicles (ARMV). If the total number of accidents is 802, find the average daily traffic entering the intersection during the 4 year period.

Solution:

$$R = \frac{A\,(1{,}000{,}000)}{ADT\,(365)\,N}$$

$$9200 = \frac{802(1{,}000{,}000)}{ADT\,(365)\,(4)}$$

ADT = **59.71**

449. Problem:

The accident rate at a sharp highway curve was 1240 per million passing vehicles. In the last 5 years, there had been 2607 accidents. What was the average daily traffic?

Solution:

$$R = \frac{A\,(1000000)}{ADT\,(365)\,N}$$

$$1240 = \frac{2607\,(1000000)}{ADT\,(365)}$$

ADT = **1152**

450. Problem:

Find the accident rate at a road intersection per million entering vehicles if the average daily traffic is 348 and 1742 accidents had occurred in the last 4 years.

Solution:

$$R = \frac{A\,(1000000)}{ADT\,(365)\,N}$$

$$R = \frac{1742\,(1000000)}{348\,(365)(4)}$$

$R = \mathbf{3429}$

451. Problem:

Data on a traffic accident recorded for the past 5 years on a certain stretch of a two lane highway is tabulated as follows.

Year	Property Damage	Injury	Fatal
1960	110	42	4
1961	210	54	2
1962	182	60	5
1963	240	74	7
1964	175	94	6

Compute the severity ratio.

Solution:

$$\text{Severity ratio} = \frac{\text{fatal + injury}}{\text{fatal + injury + property damage}}$$

$$\text{Severity ratio} = \frac{24 + 324}{917 + 23 + 324}$$

Severity ratio = **0.275**

452. Problem:

Based on the following record of road accidents find the severity ratio for a period of 5 years.

Year	Property Damage	Injury	Fatal
1992	205	56	2
1993	178	48	3
1994	152	41	3
1995	190	60	5
1996	236	88	8
	961	293	21

Solution:

$$\text{Severity ratio} = \frac{\text{fatal + injury}}{\text{fatal + injury + property damage}}$$

$$\text{Severity ratio} = \frac{21 + 293}{21 + 293 + 961}$$

Severity ratio = **0.246**

453. Problem:

Based on the record of road accidents, find the number of fatal accidents in 1996, if the severity ratio for a period of 5 years was 0.24863.

Year	Property Damage	Injury	Fatal
1992	205	56	2
1993	178	48	3
1994	152	41	3
1995	190	60	5
1996	236	88	x
	961	293	13 + x

Solution:

$$\text{Severity ratio} = \frac{\text{fatal + injury}}{\text{fatal + injury + property damage}}$$

$$0.24863 = \frac{13 + x + 293}{13 + x + 293 + 961}$$

$315.01 + 0.24863x = 306 + x$

$0.75137x = 9.01$

$x = \mathbf{12}$

454. Problem:

Using the road accident record shown below, determine the total no. of fatal accidents in 1993 and 1994 if the severity ratio for a period of 5 yrs. was 0.26.

Year	Property Damage	Injury	Fatal
1990	215	61	4
1991	188	53	5
1992	163	46	5
1993	198	65	x
1994	242	93	y
	1006	318	14 + x + y

Solution:

$$\text{Severity ratio} = \frac{\text{Injury} + \text{Fatal}}{\text{Injury} + \text{Fatal} + \text{Prop. damage}}$$

$$0.26 = \frac{318 + 14 + (x + y)}{318 + (14 + x + y) + 1006}$$

$$0.26 = \frac{332 + (x + y)}{1338 + (x + y)}$$

$$347.88 + 0.26\,(x + y) = 332 + (x + y)$$

$$0.74\,(x + y) = 347.88 - 332$$

$x + y = 21.46$ say ***21***

455. Problem:

How many vehicles pass thru a certain point in a highway every hour if the density is 48 vehicles/mile and space mean speed is 50 kph

Solution:

$q = K\,\mu_s$

$$q = \mu_s = \frac{50000\,(3.28)}{5280}$$

$\mu_s = 31.06$ mph

$q = K\,\mu_s$

$q = 48\,(31.06)$

$q =$ ***1490***

456. Problem:

Data on traffic passing thru an intersection indicates that vehicles moved at a space mean speed of 40 mph where the density is 22 vehicles per hour per mile. Compute the rate of flow in vehicles per hour.

Solution:

$q = K\,\mu_S$

$q = 22(40)$

$q =$ ***880*** *vehicles per hour (rate of flow)*

457. Problem:

Compute the rate of flow in vehicles per hour if the space mean speed is 30 mph and the density is 14 vehicles per km.

Solution:

$K = 14$ *vehicles per km*

$\mu_S = 30$ mph

$$\mu_S = \frac{30\,(5280)}{3.281\,(1000)}$$

$\mu_S = 48.28$ kph

$q = K\,\mu_S$

$q = 14\,(48.28)$

$q =$ ***675.92 vehicles/hour***

458. CE Board May 2000

The rate of flow at a point in the highway is 1200 vehicles per hour. Find the space mean speed if the density is 25 vehicles per mile.

Solution:

$q = K\,\mu_S$

$1200 = 25\,\mu_S$

$\mu_S =$ ***48 mph***

459. Problem:

Determine the approximate spacing of vehicles center to center in a certain lane if the average speed of the cars using that particular lane is 40 kph and the volume of traffic is 800 vehicles per hour.

Solution:

$$\text{No. of vehicles per hour} = \frac{800}{40}$$

$$\text{No. of vehicles per hour} = 20 \quad \text{(density)}$$

$$\text{Spacing of vehicles} = \frac{1000}{20}$$

$$\text{Spacing of vehicles} = \mathbf{50\ m.}$$

460. Problem:

The spacing of vehicles in a two lane highway is equal to 80 m. center to center. If the average speed of the car is 50 kph, determine the volume of traffic in vehicles per hour.

Solution:

$$80 = \frac{1000}{\text{Density}}$$

$$\text{Density} = \frac{1000}{80}$$

$$\text{Density} = 12.5$$

$$\text{Vol. of traffic} = 12.5\,(50)$$

$$\text{Vol. of traffic} = \mathbf{625}$$

461. Problem:

The average spacing of vehicles in a single highway is 50 m. center to center. The volume of traffic is 600 vehicles per hour. Determine the average speed of the cars using this lane in kph.

Solution:

$$\text{Density} = \frac{1000}{50}$$

$$\text{Density} = 20$$

$$\text{Speed of car} = \frac{600}{20}$$

$$\text{Speed of car} = \mathbf{30\ kph}$$

462. Problem:

Determine the appropriate spacing of vehicles center to center in a certain lane if the average speed of the cars using that particular lane is 40 kph and the volume of traffic is 800 vehicles per hour.

Solution:

$$\text{No. of vehicles per km} = \frac{800}{40}$$

$$\text{No. of vehicles per km} = 20 \quad \text{(average density)}$$

$$\text{Spacing of vehicles} = \frac{1000}{20}$$

$$\text{Spacing of vehicles} = \mathbf{50\ m.} \quad \text{center to center}$$

463. Problem:

Compute the average speed in kph that a passenger car should travel in a certain freeway if the spacing of the cars moving in the same lane is 40 m. center to center. Volume of traffic at this instant is 2000 vehicles per hour.

Solution:

$$\text{Spacing of vehicles} = \frac{1000}{\text{No. of vehicles/km}}$$

$$40 = \frac{1000}{\text{No. of vehicles/km}}$$

$$\text{No. of vehicles per km} = \frac{1000}{40}$$

$$\text{No. of vehicles per km} = 25$$

$$\text{Velocity} = \frac{\text{vehicles/hr}}{\text{vehicles/km}}$$

$$\text{Velocity} = \text{km/hr}$$

$$\text{Velocity} = \frac{2000}{25}$$

$$\text{Velocity} = \mathbf{80\ kph}$$

464. Problem:

The speed of a car moving on a single lane is 60 kph. If the length of the car is 4.2 in. and the value of the reaction time is 0.7 sec.

① Compute the average center to center of cars in meters.
② Compute the capacity of the single lane in vehicles per hour.
③ Compute the average density of traffic in vehicles per km.

Solution:

① *Average center to center spacing of cars:*

$$S = Vt + L$$

$$S = \frac{60000}{3600}(0.7) + 4.2$$

$$S = \mathbf{15.87\ m.}$$

② *Capacity of single lane in vehicles/hr:*

$$C = \frac{60000}{15.87}$$

$$C = \mathbf{3781\ vehicles/hr}$$

③ *Average density in vehicles/km:*

$$\text{Vel.} = \frac{\text{Vehicles/hr}}{\text{Vehicles/km}} = \frac{km}{hr}$$

$$\text{Vel.} = \frac{3781}{\text{Density}}$$

$$\text{Density} = \mathbf{63\ vehicles/hr.}$$

465. Problem:

In an observation post shows that 5 vehicles passes through the post at intervals of 8 sec, 9 sec, 10 sec, 11 sec and 13 sec. respectively. The speeds of the vehicles were 80 kph, 76 kph, 70 kph, 80 kph and 50 kph respectively.

① Compute the time mean speed.
② Compute the space mean speed if the distance travel by the vehicles is 250 m.
③ If the density of traffic is 20 vehicles per km, compute the rate of flow of traffic in vehicles/hour.

Solution:

① *Time mean speed:*

$$\mu_t = \frac{80 + 76 + 70 + 60 + 50}{5}$$

$$\mu_t = \mathbf{67.2\ kph}$$

② *Space mean speed:*

$$\mu_s = \frac{n\,d}{\sum t}$$

$$\mu_s = \frac{5\,(250)}{8 + 9 + 10 + 11 + 13}$$

$$\mu_s = 24.51\ m/s$$

$$\mu_s = \frac{24.51\,(3600)}{1000}$$

$$\mu_s = \mathbf{88.24\ kph}$$

③ *Rate of flow:*

$q = K\,\mu_s$

$q = 20\,(88.24)$

$q =$ ***1765 vehicles/hour***

466. CE Board Nov. 2004

Two sets of students are collecting traffic data at two sections A and B of a highway 200 m. apart. Observation at A shows that 5 vehicles passes that section at intervals of 8.18 sec, 9.09 sec, 10.23 sec, 11.68 sec, and 13.64 sec. respectively. If the speed of the vehicles were 80, 72, 64, 56 and 48 kph respectively.

① Compute the density of traffic in vehicles per km.
② Compute the time mean speed in kph.
③ Compute the space mean speed in kph.

Solution:

① *Density of traffic:*

$$K = \frac{5}{200}(100)$$

$K =$ ***25 vehicles/km.***

② *Time mean speed:*

$$\mu_t = \frac{80 + 72 + 64 + 56 + 48}{5}$$

$\mu_t =$ ***64 kph***

③ *Space mean speed:*

$$\mu_s = \frac{n\,d}{\sum t}$$

$$\mu_s = \frac{5\,(200)}{8.18 + 9.09 + 10.23 + 11.68 + 13.64}$$

$\mu_s = 18.93$ m/s

$$\mu_s = \frac{18.93}{1000}(3600)$$

$\mu_s =$ ***68.16 kph***

467. Problem:

From the data of a highway traffic observation team, shows the distances each vehicle have traveled every 3 seconds on a portion of the highway.

Vehicle	Distance
1	88 m.
2	86 m.
3	83 m.
4	82 m.

① Compute the space mean speed of the traffic.
② Compute the flow of traffic.
③ Compute the density of traffic.

Solution:

① *Space mean speed:*

$$\mu_s = \frac{\sum S}{n\,t}$$

$$\mu_s = \frac{88 + 86 + 83 + 82}{4\,(3)}$$

$\mu_s = 28.25$ m/s

$\mu_s =$ ***101.7 kph***

② *Flow of traffic:*

$$q = \frac{4\,(3600)}{3}$$

$q =$ ***4800 vehicles/hr.***

③ *Density of traffic:*

$q = \mu_s K$

$4800 = 101.7\,K$

$K =$ ***47 vehicles/km***

468. Problem:

The following travel times were observed for four vehicles traversing a one km. segment of highway.

Vehicle	Time (min.)
1	1.6
2	1.2
3	1.5
4	1.7

Compute the space mean speed in kph.

Solution:

Space mean speed $\mu_s = \dfrac{nd}{\Sigma t}$

n = 4 vehicles
d = one km

$$\mu_s = \frac{4(1)}{1.6 + 1.2 + 1.5 + 1.7}$$

$\mu_s = 0.667$ km/min.

$\mu_s = 0.667(60)$

$\mu_s = 40$ **kph**

469. Problem:

From the given data of 5 vehicles passing through a 1 km segment of Edsa.

Vehicle	Time (sec.)
1	96
2	72
3	90
4	102
5	108

Compute the time mean speed in kph.

Solution:

$\mu_t = \dfrac{\Sigma\, d/t}{n}$ $\quad n = 5$, $\quad d = 1$ km

$$\mu_t = \frac{\frac{1}{96} + \frac{1}{72} + \frac{1}{90} + \frac{1}{102} + \frac{1}{108}}{5}$$

$\mu_t = 0.0108$ km/sec.

$\mu_t = 0.0108(3600)$

$\mu_t = 39.23$ **kph**

470. Problem:

There are 5 vehicles passing through an intersection of two highway in a period of 20 sec.

Vehicle	Spot speed (kph)
1	34.20 kph
2	42.40 kph
3	46.30 kph
4	41.10 kph
5	43.40 kph

Compute the space mean speed in kph.

Solution:

$$\mu_s = \frac{n}{\Sigma \frac{1}{\mu_1}}$$

$$\mu_s = \frac{5}{\frac{1}{34.20} + \frac{1}{42.40} + \frac{1}{46.30} + \frac{1}{41.10} + \frac{1}{43.40}}$$

$\mu_s = 41.05$ **kph**

471. Problem:

From the following data of a freeway surveillance, there are 5 vehicles counted for a length of 200 m. and the following distance "S" are the distance that each vehicle have traveled when observed on the two photographs taken 2 seconds apart. Compute the flow of traffic if the density of flow is 25 vehicles per km. Express in km/hr.

Vehicle	Distance "S" (m.)
1	24.4 m.
2	18.8 m.
3	24.7 m.
4	26.9 m.
5	22.9 m.

Solution:

space mean speed $\mu_S = \frac{\sum S}{n\,t}$

$$\mu_S = \frac{24.4 + 18.8 + 24.7 + 26.9 + 22.9}{5(2)}$$

$\mu_S = 11.77$ m/sec.

$$\mu_S = \frac{11.77(3600)}{1000}$$

$\mu_S = 42.37$ kph

Flow of traffic:

$q = \mu_S\,k$

k = density of flow in vehicles/km

$q = 42.37(25)$

$q =$ ***1059 vehicles/hour***

472. Problem:

Compute the space mean speed from the following data of an observation of four vehicles with the corresponding distance that each have traveled when observed on two photographs taken 3 seconds apart.

Vehicle	Distance (meter)
1	25.6 m.
2	19.8 m.
3	24.2 m.
4	23.6 m.

Solution:

$$\mu_S = \frac{\sum S}{n\,t}$$

$$\mu_S = \frac{25.6 + 19.8 + 24.2 + 23.6}{4(3)}$$

$\mu_S = 7.77$ m/s

$$\mu_S = \frac{7.77(3600)}{1000}$$

$\mu_S =$ ***27.96 kph***

473. Problem:

There are 5 vehicles passing thru the intersection every 4 sec. and the foll. data of a freeway surveillance indicates the distances which each vehicle has traveled every 4 sec.

Vehicle	S (m)
1	96 m.
2	98 m.
3	97 m.
4	95 m.
5	94 m.

① Compute the flow of traffic in vehicles/hour.
② Compute the space mean speed in kph.
③ Compute the density of traffic in vehicles per km.

Solution:

① *Flow of traffic in vehicles per hour:*

$$q = \frac{5\,(3600)}{4}$$

$q =$ ***4500***

② *Space mean speed:*

$$\mu_s = \frac{\sum S}{N\,t}$$

$$\mu_s = \frac{96 + 98 + 97 + 95 + 94}{5\,(4)}$$

$\mu_s = 24$ m/s

$\mu_s =$ ***86.4 kph***

③ *Density of traffic in vehicles per km.*

$q = \mu_s\,K$

$4500 = 86.4\,K$

$K =$ ***52 vehicles/km***

474. Problem:

From the foll. data of a freeway surveillance, there are 4 vehicles counted for a length of 200 m. and the foll. distances each had traveled when observed on the photographs taken every 2 seconds apart.

Vehicle	S (m)
1	24.4 m.
2	23.6 m.
3	25.2 m.
4	24.0 m.

① Compute the density of flow in vehicles/km.
② Compute the space mean speed in kph.
③ Compute the flow of traffic in vehicles/hour.

Solution:

① *Density in vehicles/km.*

$$K = \frac{4}{0.200}$$

K = ***20 vehicles/km.***

② *Space mean speed:*

$$\mu_s = \frac{\Sigma S}{Nt}$$

$$\mu_s = \frac{(24.4) + (23.6) + (25.2) + (24)}{4\,(2)}$$

$$\mu_s = 12.15 \text{ m/s}$$

$$\mu_s = \frac{12.15\,(3600)}{1000}$$

μ_s = ***43.74 kph***

③ *Flow of traffic:*

$$q = \mu_s K$$

$$q = 43.74\,(20)$$

q = ***875***

475. Problem:

The traffic intensity of a certain freeway is 0.833. The arrival rate of vehicles at a certain intersection (stop sign) is 200 vehicles per hour.

① Compute the service rate in vehicles/hr.
② Compute the average waiting time at the stop sign per vehicle in seconds.
③ Compute the total delay time (queue time plus service time) in min.

Solution:

① *Traffic intensity:*

$$P = \frac{\lambda}{\mu}$$

λ = arrival rate

λ = 200 vehicles/hr.

$$0.833 = \frac{200}{\mu}$$

μ = ***240 vehicles/hr*** *(service rate)*

② *Average waiting time:*

$$\mu = \frac{3600}{t}$$

$$240 = \frac{3600}{t}$$

t = ***15 sec.***

③ *Total delay time:*

$$t = \frac{1}{\mu - \lambda}$$

$$t = \frac{1}{240 - 200}$$

t = 0.025 hrs.

t = ***1.5 min.***

476. Problem:

Vehicles arrive at a stop sign at an average rate of 300 per hour. Average waiting time at the stop sign is 10 seconds per vehicle. Assuming both arrivals and departures are exponentially distributed.

① Compute the traffic intensity.
② What is the average delay time per vehicle.
③ What is the total delay time spent in the system.

Solution:

① *Traffic intensity:*

$$\mu = \frac{3600}{10}$$

$\mu = 360$ vehicles/hr. *(service rate)*

$$P = \frac{\lambda}{\mu}$$

$\lambda = 300$ *(arrival rate)*

$$P = \frac{300}{360}$$

$P = \mathbf{0.833}$ *(traffic intensity)*

② *Average delay time per vehicle:*

$$\omega = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$\omega = \frac{300}{360(360 - 300)}$$

$\omega = 0.014$ hrs.

$\omega = \mathbf{0.83\ min.}$

③ *Total delay time:*

$$t = \frac{1}{\mu - \lambda}$$

$$t = \frac{1}{360 - 300}$$

$t = 0.017$ hrs.

$t = \mathbf{1.02\ min.}$

477. Problem:

A stop sign is installed at the corner of EDSA and Ortigas Avenue where vehicles during rush hour arrive at an average rate of 300 per hour. If the traffic intensity is 0.80.

① Compute for the average waiting time in seconds at the stop sign.
② What is the average delay time per vehicle at this instant in seconds.
③ Compute the total delay time in seconds.

Solution:

① *Average waiting time:*

$$P = \frac{\lambda}{\mu}$$

$$0.80 = \frac{300}{\mu}$$

$\mu = 375$

$$\textit{Average waiting time} = \frac{3600}{375}$$

Average waiting time = **9.6 sec.**

② *Average delay time:*

$$\omega = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$\omega = \frac{300}{375(375 - 300)}$$

$\omega = 0.0107$ hrs.

$\omega = \mathbf{38.4\ sec.}$

③ *Total delay time:*

$$t = \frac{1}{\mu - \lambda}$$

$$t = \frac{1}{375 - 300}$$

$t = 0.0133$ hrs.

$t = \mathbf{48\ sec.}$

478. Problem:

Scheduled maintenance will close two of the four westbound lanes of a freeway during one weekday for the period from 9:00 A.M. to 4:00 P.M. The demand on the two lanes are as follows:

Time	Demand (Vehicles/hour)
9 - 10 AM	4000
10 - 11 AM	3500
11 - 12 Noon	2500
12 - 1 PM	2000
1 - 2 PM	2000
2 - 3 PM	2000
3 - 4 PM	2000

If the estimated capacity of the 4 lanes with 2 lanes open is 2960 vehicles/hour.

① Compute the time when queue occurs.
② Compute the max. queue (no. of vehicles)
③ Compute the max. queue length in meters if one car is assumed to be 5 m. long.

Solution:

① *Time when queue occurs*

Time	Cummulative Demand	Cummulative Capacity
10:00	4000	2960
11:00	7500	5920
12:00	10000	8880
1:00	12000	11840
2:00	14000	14800
3:00	16000	17760
4:00	18000	20720

Time	Demand-Capacity	
10:00	4000 - 2960	= 1040
11:00	7500 - 5920	= 1580
12:00	10000 - 8880	= 1120
1:00	12000 - 11840	= 160
2:00	14000 - 14800	= - 800
3:00	16000 - 17760	= - 1760
4:00	18000 - 20720	= - 2720

Max. queue occurs at ***11:00 A.M.***

② *Max. queue (no. of vehicles)*

Max. queue = ***1580 vehicles***

③ *Max. queue length:*

$$\text{No. of vehicles per lane} = \frac{1580}{4}$$

No. of vehicles per lane = 395

Total length of queue = 395(5)

Total length of queue = ***1975 m.***

479. Problem:

A bridge has been constructed between the mainland and an island. The total cost (excluding tool fees) to travel across the bridge is expressed as C = 50 + 0.5 V where V is the number of vehicles per hour and C is the cost per vehicle in cents. The demand for travel across the bridge is V = 2900- 10 C.

① Determine the volume of traffic across the bridge in vehicles/hour.
② If a total of 25 centavos is added, what is the volume of traffic across the bridge.
③ A toll both is to be added, thus reducing the travel time to cross the bridge. The new cost function is C = 50 + 0.20 V. Determine the volume of traffic that would cross the bridge.

Solution:

① *Volume of traffic across the bridge:*

$V = 2900 - 10\,C$
$C = 50 + 0.5\,V$
$V = 2900 - 10\,(50 + 0.5\,V)$
$V = 2900 - 500 - 5\,V$
$6\,V = 2400$
$V =$ ***400 vehicles/hour***

② *Volume of traffic across the bridge if 25 centavos is added to the toll:*

$C = 50 + 0.5\,V$
$C = 50 + 0.5\,(400)$
$C = 250$ cents

New $C = 250 + 25$
New $C = 275$ cents

$V = 2900 - 10\,C$
$V = 2900 - 10\,(275)$
$V =$ **150 vehicles/hour**

③ Vol. of traffic if $C = 50 + 0.20\,V$:
$V = 2900 - 10\,C$
$V = 2900 - 10\,(50 + 0.20\,V)$
$V = 2900 - 500 - 2\,V$
$3\,V = 2400$
$V =$ **800 vehicles/hour**

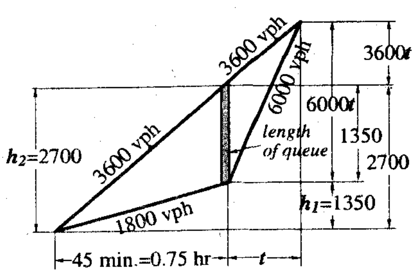
480. Problem:

A freeway has three lanes in each direction and has a max. flow of 6000 vph. It is operating at 3600 vph at t = 0. A collision occurs, blocking the two lanes, and restricting the flow of the third lane to 1800 vph. The freeways constant speed is 60 mph and its 3 lane jam density is 60 vpm. The incident is completely cleared in 45 minutes and traffic returns to normal as soon as the back-up is dissipated.

① Determine the length of queue.
② How long does it take to dissipate the back up?
③ What is the average delay per vehicle.

Solution:

① Length of queue:



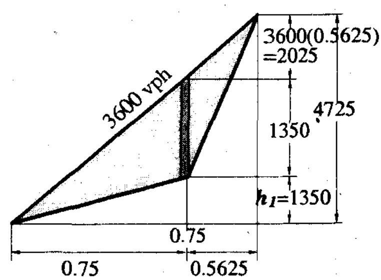
$h_1 = 1800\,(0.75)$
$h_1 = 1350$
$h_2 = 3600\,(0.75)$
$h_2 = 2700$

Length of queue $= 2700 - 1350$
Length of queue = **1350 cars**

② Time to dissipate the back up:
$1350 + 6000\,t = 2700 + 3600\,t$
$2400\,t = 1350$
$t = 0.5625$ hrs.
$t = 33.75$ min.

Total time to dissipate
$= 45 + 33.75$
= **78.75 minutes** after the accident

③ Average delay per vehicle:



Total delay = area of shaded section

$$\text{Total delay time} = \frac{1350\,(0.75)}{2} + \frac{1350\,(0.5625)}{2}$$

Total delay time = 885.94 vehicle-hour

$$\text{Average delay per vehicle} = \frac{885.94}{4725}$$

Average delay per vehicle = 0.1875 hrs.
Average delay per vehicle = **11.25 minutes**

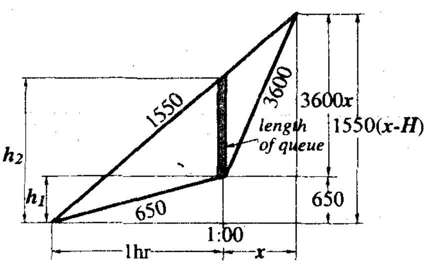
481. Problem:

A truck overturned at 12:00 noon near 50 km on the north bound road in the province of Cebu, completely blocking that highway. Fortunately the incident site is just beyond an overpass, between an off-ramp and an on-ramp. This means that most vehicles will see the blockage and exit at the off ramp, avoiding a long back up and long delay. This also makes the detour of through vehicles simply a matter of using these ramps to go around the incident site. After the trucks mishap, the ramp capacities were govern by a stop sign at the end of the off-ramp and the priority given to cross traffic, which did not have a stop sign. The ramps service rate for detouring traffic was approximately 650 vph. At exactly 11:00 PM. the highway was reopened to through traffic with capacity of 3600 vph. The flow rate at this time of the day is 1550 vph.

① What was the longest vehicle queue?
② At approximately what time does the queue dissipate?
③ Compute the average delay per vehicle.

Solution:

① *Longest vehicle queue:*



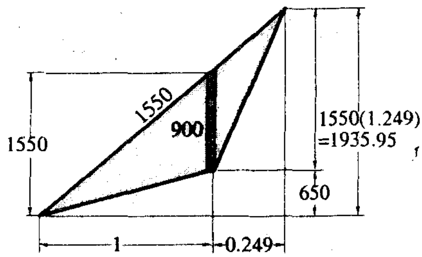
$h_1 = 650\,(1)$
$h_1 = 650$
$h_2 = 1550\,(1)$
$h_2 = 1550$

Longest vehicle queue = 1550 - 650
Longest vehicle queue = **900 vehicles**

② *Time the queue dissipate:*
$3600\,x + 650 = 1550\,(x + 1)$
$2050\,x = 1500 - 650$
$x = 0.4146$ hrs.
$x = 24.9$ min.

*Time queue will dissipate is **1:24.9 PM***

③ *Average delay per vehicle:*



Longest vehicle delay = area of shaded section

$$Area = \frac{900\,(1)}{2} + \frac{900\,(0.249)}{2}$$

Area = 562.05 veh-hr.
Longest vehicle delay = 562.05 veh-hrs

$$\text{Average delay per vehicle} = \frac{562.05}{1935.95}$$

Average delay per vehicle = 0.29 hrs.
Average delay per vehicle = **17.42 min/vehicle**

482. Problem:

The intersection of EDSA and Ortigas Avenue may not have qualified as a hazardous intersection, but many drivers perceive it as unsafe. A team of observers spent 40 hours at the intersection and collected the following information.

① 94 total conflicts, with 54 being of rear-end conflict type.
② Average hourly approach volume = 1205 vehicles.
③ Total time to collision (TTC) severity = 190 for the 94 conflicts.
④ Total risk of collision (ROC) severity = 201 for the 94 conflicts.

① Determine the average hourly conflict (AHC) per thousand entering vehicles.
② Determine the overall average conflict severity (OACS).
③ Determine the total conflict severity (TCS.)

Solution:

① *Average hourly conflict per thousand entering vehicles (AHC):*

$$AHC = \frac{\text{Total no. of conflict}}{\text{Number of observation hours}}$$

$$AHC = \frac{94}{40}$$

$$AHC = 2.35$$

$$\text{AHC per thousand entering vehicles} = \frac{2.35\,(1000)}{1205}$$

$$\text{AHC per thousand entering vehicles} = \mathbf{1.95}$$

② *Total conflict severity (TCS):*

$$TCS = TTC + ROC$$

$$TCS = 190 + 201$$

$$TCS = \mathbf{391}$$

③ *Overall average conflict severity (OACS):*

$$OACS = \frac{TCS}{\text{Total conflict}}$$

$$OACS = \frac{391}{94}$$

$$OACS = \mathbf{4.16}$$

483. Problem:

The Manila City Engineers staff believes that installing stop signs at a previously uncontrolled intersection will reduce crashes (accidents) by 26 percent. In the base year (last year) there were 11 right angle collision at the intersection whose approach volume was 3273 vehicles per day. Ten years from now, the approach average daily traffic (ADT) is forecasted to be 4000 vehicles per day.

① How many crashes will be prevented 10 yrs from now if the stop signs are installed?
② If we extend the period of analysis to 15 yrs, the traffic growth rate will increase by 2% and the number of right angle crashes will also increase by 2%. Determine the total number of crashes prevented on the 3rd year.
③ If the fatal crash is only 0.50%, determine the number of crashes prevented after one year.

Solution:

① *No. of crashes prevented 10 yrs. from now:*

$$N = (EC)\,(CRF)\,\frac{\text{Forecast } ADT}{\text{base } ADT}$$

EC = expected no. of crashes over a specified time

CRF = crash reduction factor

ADT = average daily traffic

$$N = (11)\,(0.26)\left(\frac{4000}{3273}\right)$$

$$N = \mathbf{3.5\ per\ year}$$

② *Total number of crashes prevented on the 3rd year:*

$$N = (EC)\,(CRF)\,(1 + r)^n$$

$$N = (11)\,(0.26)\,(1.02)^3$$

$$N = \mathbf{3.035}$$

③ *Fatal benefit after 1 year:*

$$\text{Crashes prevented} = (EC)\,(CRP)\,(1 + r)^n$$

Crash prevented after one year

$$= 11\,(0.26)\,(1.02)^1$$

$$= 2.91 \text{ crashes}$$

$$\text{Fatal benefit} = 2.917\,(0.005)\,(2854500)$$

$$\text{Fatal benefit} = \mathbf{P41{,}633.88}$$

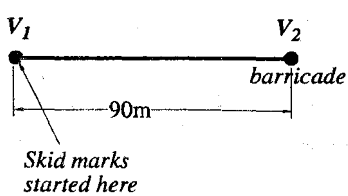
484. Problem:

A teenage driver hit a barricade traveling at about 40 kph. The road along which the driver is passing thru is posted with an 88 kph speed limit. The road is straight and level. The first warning sign of the barricade was located 300 m. before the barricade and the second sign was 180 m. before the barricade. The skid marks from the car begin 90 m. before the barricade. According to the weather service, the road was met but visibility was good. Coefficient of frcition between road and tires is 0.30.

① If the velocity of impact was 40 kph, what was the initial velocity before the 90 m. of skidding?
② What would have been the response time from the first warning sign to the initiation of the skid marks, if he was driving at the speed limit?
③ Determine the response time to the second warning sign.

Solution:

① *Initial velocity:*



$V_2^2 = V_1^2 - 2\,g\,(\mu)\,S$

$V_2 = \dfrac{40000}{3600}$

$V_2 = 11.11$ m/s

$(11.11)^2 = V_1^2 - 2(9.81)(0.30)(90)$

$V_1 = 25.56$ m/s

$V_1 = \dfrac{25.56\,(3600)}{1000}$

$V_1 = \mathbf{92\ kph}$

② *Response time if he was driving at speed limit:*

Distance the car traveled from the first warning sign

$= 300 - 90$

$= 210$ m.

Speed limit $= \dfrac{88000}{3600}$

Speed limit $= 24.44$ m/s

Response time $= \dfrac{210}{24.44}$

Response time = ***8.59 seconds***

③ *Response time to the second sign:*

speed $= 25.56$ m/s

$t = \dfrac{90}{25.56}$

$t =$ ***3.52 sec.***

485. Problem:

A man is driving along a local road at 88 kph. This man hates to wait for anything, even the train that he sees heading for the grade crossing a head. There is no gate at this grade crossing, only a cross buck sign and a bell. He decided to try to beat the train to the crossing. Although he can guess at these values, the train is 150 m. from the crossing and moving at 80 kph when the man first sees it. At that time he was 240 m. from the crossing.

① Assuming he has a reaction time of 0.6 seconds, how far from the crossing will he be when he begins to accelerate?
② If the car can accelerate at the rate of 8.5 m/s² but has a max. speed of 140 kph, how fast will it be going when it reaches the crossing?
③ How much time did he beat the train?

Solution:

① *Distance from crossing when he begins to accelerate:*

$$\text{Velocity of car} = \frac{88000}{3600}$$

Velocity of car = 24.44 m/s

$D = 240 - 24.44\,(0.6)$

$D = \mathbf{225.34\ m.}$

② *Velocity of car when it reaches the crossing:*

$V_2^2 - V_1^2 + 2\,a\,S$

$$V_1 = \frac{140000}{3600}$$

$V_1 = 38.89$ m/s

$V_2^2 = (38.89)^2 + 2\,(8.5)(225.34)$

$V_2 = 73.10$ m/s

$V_2 = \mathbf{263.2\ kph}$

③ *Time he beat the train:*

$V_2 = V_1 + a\,t$

$73.10 = 38.89 + 8.5\,t$

$t = 4.02$ sec.

$$\text{Velocity of train} = \frac{80000}{3600}$$

Velocity of train = 22.22 m/s

$$t = \frac{150}{22.22}$$

$t = 6.75$ sec.

Time he beat the train = 6.75 - 4.02

Time he beat the train = **2.73 sec.**

486. Problem:

The below shows a set of observations made at regular intervals of 15 seconds on a certain intersection. The field data indicates a count made of the number of vehicles stopped on a given approach each 15 seconds. The assumption is that each vehicle counted in this way will wait 15 seconds. The total approach volume of vehicles during the 10 min. period was 41.

Approach street : Rizal Avenue
Cross street : Recto Avenue
Approach Direction : West Bound
Time : 5:30 P.M. to 5:40 P.M.
City of Manila:

	No. of stopped Vehicles			
Time (min.)	0 sec.	15 sec.	30 sec.	45 sec.
0	0	4	7	8
1	8	2	0	2
2	2	4	5	6
3	7	4	0	1
4	2	3	5	7
5	8	0	0	1
6	4	6	5	2
7	0	1	1	3
8	4	6	8	0
9	1	2	3	4
	36	32	34	34

① Compute the total number of vehicles in queue.
② Compute the total delay in minutes.
③ Compute the average stopped delay in seconds/vehicle.

Solution:

① *Total vehicles in queue:*

Total vehicles in queue = 36 + 32 + 34 + 34

Total vehicles in queue = ***136***

② *Total delay:*

Total delay = 136 (15)

Total delay = 2040"

Total delay = ***34 min.***

③ *Average stopped delay:*

$$\text{Average stopped delay} = \frac{2040}{41}$$

Average stopped delay = **49.8 sec/vehicle**

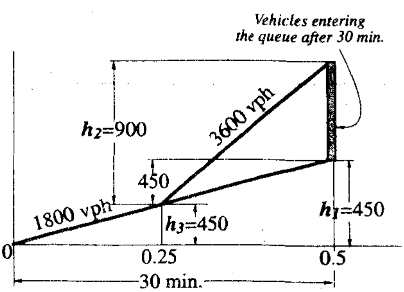
487. Problem:

An incident response program called ERUF is stationed along the freeway during peak traffic hours. Traffic is monitored by detectors, so that the locations of an incident that causes the traffic to back up is known within a few minutes. Traffic backs up quickly but the ERUP vehicle was able to reach the scene of the incident is 15 min. and in 15 additional minutes, is able to push disabled cars to the shoulder, increasing the flow from 1800 vph to 3600 vph. Because this new service rate is the same as the deterministic arrival rate, the shock wave will not move farther back. The wrecker and ambulance come and clear things fully in 30 minutes, so that the freeway is opened up for full flow.

① Determine the no. of vehicles entering the queue after 30 min.
② What is the reduction in delay for cars on the freeway if it has a max. flow of 6000 vph.
③ At P1500 per vehicle hour, what is the value of the time savings?

Solution:

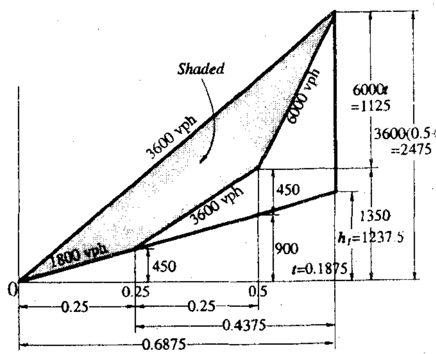
① *No. of vehicles entering the queue after 30 min.*



$h_1 = 1800\,(0.5)$
$h_1 = 900$
$h_3 = 1800\,(0.25)$
$h_3 = 450$
$h_2 = 3600\,(0.25)$
$h_2 = 900$

Vehicles entering the queue after 30 min.
= 900 - 450
= ***450 vehicles***

② *Reduction in delay for cars on the freeway:*



$6000\,t + 1350 = 3600\,(0.5 + t)$
$2400\,t = 450$
$t = 0.1875$ hrs.
$t = 11.25$ min.
$h = 1800\,(0.50 + 0.1875)$
$h = 1237.5$

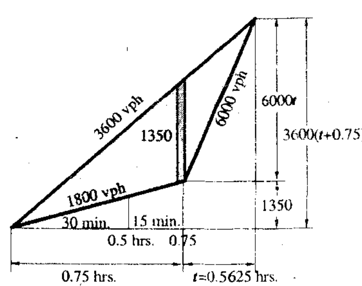
Reduction of delay with ERUP
= area of shaded section

Reduction of delay

$$= \frac{2475\,(0.6875)}{2} - \frac{450\,(0.25)}{2} - \frac{(450 + 1350)(0.25)}{2} - \frac{(1350 + 2475)(0.1875)}{2}$$

Reduction of delay = ***183.74 vehicle-hours***

③ *Value of time savings:*



$6000\,t + 1350 = 3600\,(t + 0.75)$
$2400\,t = 1350$
$t = 0.5625$ hrs.
$t = 33.75$ min.

$$\text{Total delay} = \frac{1350\,(0.75)}{2} + 1350\,\frac{(0.5625)}{2}$$

Total delay = 886
Savings = (886 - 183.94)(1500)
Savings = ***P1,053,090***

488. Problem:

A driver traveling in his 4.8 m. SUV at a speed limit of 48 kph was arrested for running a red light at an intersection that is 18 m. wide. The driver claimed innocence, on the grounds that the traffic signals were not set properly. The yellow light was on for the standard 4 seconds. The SUV drivers reaction time is assumed to be 1.5 sec. Comfortable deceleration is at a rate of 3 m/s^2.

① Compute the min. distance needed to stop as soon as he sees the yellow traffic signal.
② Compute the distance the driver can travel in the 4 seconds that the yellow light was on at a constant speed of 48 kph.
③ How long is the dilemma zone this intersection approach?

Solution:

① *Min. distance:*

$V_2 = V_1 - a\,t$

$$V_1 = \frac{48000}{3600}$$

$V_1 = 13.33$ m/s

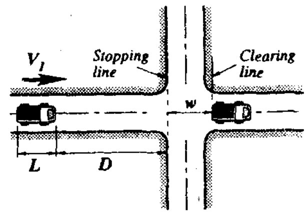
$0 = 13.33 - 3\,t_2$
$t_2 = 4.44$ sec. *time to stop*

$$D = V_1\,t_1 + V_1\,t_2 - \frac{1}{2}\,a\,t^2$$

$$D = 13.33\,(1.5) + 13.33\,(4.44) - \frac{1}{2}(3)(4.44)^2$$

$D = \boldsymbol{49.61\ m.}$

② *Distance the driver can travel in 4 secs.*



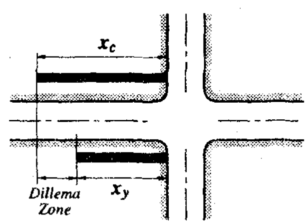
Clearing the intersection requires going a distance of $D + L + W$ before the light turns red.

$$x_y = V_1\,t_1 + V_1\,(4 - t_1) + \frac{1}{2}\,a\,(4 - t_1)^2$$

$$x_y = 13.33\,(1.5) + 13.33\,(4 - 1.5) + \frac{1}{2}(0)(4 - 1.5)^2$$

$x_y = \boldsymbol{53.32\ m.}$

③ *Length of dillema zone:*



If the driver was any closer to the intersection than the distance 49.61 m. just calculated, he must be able to drive $W + L = 18 + 4.8 = 22.8$ m. farther than 49.61 to clear the intersection.

$X_c = 49.61 + 22.8$
$X_c = 72.41$ m.
$X_y < X_c = 53.32 < 72.41$ *a dillema zone exists.*

Length of dillema zone = 72.41 - 53.32
Length of dillema zone = ***19.09 m.***

489. Problem:

A man is driving at a speed of 48 kph and is approaching an intersection which is 18 m. wide. The length of his car is 4.8 m. The yellow light was on for the standard 4 seconds. If the drivers reaction time is 1.5 sec. and he decelerates at a rate of 3 m/s^2 as soon as he sees the yellow light signal was on.

① Compute the min. stopping distance.
② Determine the length of time that the red light was on for the vehicle to clear the intersection.
③ If all red clearance interval is 2 sec. long, determine the speed at which a vehicle can clear the intersection.

Solution:

① *Min. stopping distance:*

$$D = V_1\, t_1 + V_2\, t_2 - \frac{1}{2} a\, t_2^2$$

$$V_2 = V_1 - a\, t_2$$

$$V_1 = \frac{48000}{3600}$$

$$V_1 = 13.33 \text{ m/s}$$

$$0 = 13.33 - 3\,(t_2)$$

$$t_2 = 4.44 \text{ sec.}$$

$$D = 13.33\,(1.5) + 13.33\,(4.44) - \frac{1}{2}(3)(4.44)^2$$

$$D = \mathbf{49.61\ m.}$$

② *Length of time the red light was on for the vehicle to clear the intersection:*

t = time to clear the intersection
- time of yellow light

Distance to clear the intersection
= 49.61 + 18 + 4.8
= 72.41 m.

$$\text{time to clear the intersection} = \frac{72.41}{13.33}$$

time to clear the intersection = 5.43 sec.

Time the red light was on = 5.43 - 4
Time the red light was on = ***1.43 secs.***

③ *Speed at which a vehicle can clear the intersection:*

$$V_2 = V_1 - a\, t_2$$

$$0 = V_1 - (3)\, t_2$$

$$t_2 = \frac{V_1}{3}$$

$$\text{Max. stopping distance} = V_1\, t_1 + V_1\, t_2 - \frac{1}{2} a\, t_2^2$$

$$X_s = V_1\,(1.5) + \frac{V_1\,(V_1)}{3} - \frac{1}{2}(3)\left(\frac{V_1^2}{9}\right)$$

$$X_s = 1.5\, V_1 + \frac{V_1^2}{3} - \frac{1}{2}\frac{V_1^2}{3}$$

$$X_s = 1.5\, V_1 + \frac{V_1^2}{6}$$

Distance to clear the intersection = X_s + 18 + 4.8
Distance to clear the intersection = X_s + 22.8

$$X_c = 1.5\, V_1 + \frac{V_1^2}{6} + 22.8$$

when t_{red} = 2 secs. *(all red clearance interval)*

$$X_c = (t_y + t_{red})\, V_1$$

$$X_c = (4 + 2)\, V_1$$

$$6\, V_1 = 1.5\, V_1 + \frac{V_1^2}{6} + 22.8$$

$$\frac{V_1^2}{6} = 4.5\, V_1 - 22.8$$

$$V_1^2 - 27\, V_1 + 136.8 = 0$$

$$V_1 = 26.98 \text{ m/s}$$

$$V_1 = \frac{26.98\,(3600)}{1000}$$

$$V_1 = \mathbf{97.14\ kph}$$

490. Problem:

Compute the peak hour factor (PHF) if the hourly volume of traffic is 1800 vehicles per hour and the highest 5 min. volume is 250.

Solution:

For one hour = 60 min.

$\frac{60}{5} = 12$ *(there are 12-5 min in one hour)*

$$PHF = \frac{1800}{250(12)}$$

$PHF = \mathbf{0.60}$

491. Problem:

The highest 10 minute volume of traffic is 500 vehicles. If the peak hour factor is 0.60, what is the volume in vehicles/hour?

Solution:

$$0.60 = \frac{Vol.}{500\left(\frac{60}{10}\right)}$$

$Vol. = \mathbf{1800}$

492. Problem:

What is the peak hour factor (PHF) if the volume of the traffic is 1500 vehicles per hour and the highest 5 minute volume is 210.

Solution:

$$PHF = \frac{1500}{210\left(\frac{60}{5}\right)}$$

$PHF = \mathbf{0.595}$

493. Problem:

The peak hour factor (PHF) = 0.60 and the volume of traffic is 2400 vehicles/hr. What is the highest 6 min. volume of traffic?

Solution:

$$0.60 = \frac{2400}{x\left(\frac{60}{6}\right)}$$

$x = \mathbf{400}$

494. Problem:

The table shows a 15 minute volume counts during the peak hour on an approach of an intersection.

Time	Volume of Traffic
6:00 - 6:15 PM	375
6:15 - 6:30 PM	380
6:30 - 6:45 PM	412
6:45 - 7:00 PM	390

① Determine the peak hour volume.
② Determine the peak hour factor.
③ Determine the design hourly volume (DHV) of the approach.

Solution:

① *Peak hour volume:*

$Vol. = 375 + 380 + 412 + 390$

$Vol. = \mathbf{1557}$

② *Peak hour factor:*

$$PHF = \frac{\text{Vol. during peak hour}}{\frac{60}{15} \times (\text{Vol. during peak 15 min. within peak hour})}$$

$$PHF = \frac{1557}{\frac{60}{15}(412)}$$

$PHF = \mathbf{0.945}$

③ *Design hourly volume:*

$$DHV = \frac{\text{Peak-hour Vol.}}{\text{Peak-hour factor}}$$

$$DHV = \frac{1557}{0.945}$$

$DHV = \mathbf{1648}$

495. CE Board May 2005

The peak hour factor for a traffic during rush hour is equal to 0.60 with a highest 5 min. volume of 250 vehicles. The space mean speed of the traffic is 90 kph.

① Compute the flow of traffic in vehicles/hour.
② Compute the density of traffic in vehicles/km.
③ Compute the max. spacing of the cars in meters.

Solution:

① *Flow of traffic:*

$$PHF = \frac{\text{Flow of traffic}}{\text{traffic}\left(\frac{60}{15}\right)}$$

$$0.60 = \frac{\text{Flow of traffic}}{250\left(\frac{60}{15}\right)}$$

q = ***1800 vehicles/hour***

② *Density of traffic in vehicles per km.*

$$q = \mu_s K$$

$$1800 = 90\,K$$

K = ***20 vehicles/km.***

③ *Spacing of vehicles:*

$$\text{Spacing} = \frac{1000}{20}$$

Spacing = ***50 m.*** *center to center*

496. Problem:

A car traveling 70 kph requires 48 m. to stop after the brakes have been applied. What average coefficient of friction was developed between the tires and the pavement?

Solution:

$$S = \frac{V^2}{2g\,f}$$

$$V = \frac{70000}{3600}$$

$$V = 19.44$$

$$48 = \frac{(19.44)^2}{2\,(9.81)\,f}$$

f = ***0.40***

497. Problem:

Find the length of the skid mark if the average skid resistance is 0.15 and the velocity of the car when the brakes were applied was 40 kph.

Solution:

$$V = \frac{40000}{3600}$$

V = 11.11 m/s

$$S = \frac{V^2}{2g\,f}$$

$$S = \frac{(11.11)^2}{2\,(9.81)\,(0.15)}$$

S = ***42 m.***

498. Problem:

Applying full brakes at a speed of 60 kph, the car traveled 40 m. until it stopped. Determine the average skid resistance.

Solution:

$$S = \frac{V^2}{2g\,f}$$

$$V = \frac{60000}{3600}$$

V = 16.67 m/s

$$40 = \frac{(16.67)^2}{2\,(9.81)\,f}$$

f = ***0.35***

499. Problem:

Full brakes were applied when the cars speed was 60 kph. If the average skid resistance is 0.24, find the length of the skid mark.

Solution:

$$S = \frac{V^2}{2g\,f}$$

$$V = \frac{60000}{3600}$$

$V = 16.67$ m/s

$$S = \frac{(16.67)^2}{2\,(9.81)(0.24)}$$

$S = $ ***59 m.***

500. Problem:

The brakes are suddenly applied to stop a car is running at 48 kph. Determine the braking distance if the coeff. of friction between the tires and the road surface is 0.35.

Solution:

$$S = \frac{V^2}{2g\,f}$$

$$V = \frac{48000}{3600}$$

$V = 13.33$

$$S = \frac{(13.33)^2}{2\,(9.81)\,(0.35)}$$

$S = $ ***25.9 m.***

501. Problem:

A car runs 48 m. from the the brakes were suddenly applied until it stopped. The road grade is 2% up hill and the coeff. of friction between the tires and the road surface Is 0.35.

① What was the speed of the car in kph, just before the application of the brakes.

② Compute the speed if the road grade is 2% downhill.

Solution:

① *Speed of the car in kph:*

$$S = \frac{V^2}{2g\,(f + G)}$$

$$48 = \frac{V^2}{2\,(9.81)\,(0.35 + 0.02)}$$

$V = 18.67$ m/s

$$V = \frac{18.67\,(3600)}{1000}$$

$V = $ ***67.21 kph***

② *Speed if the road grade is 2% downhill:*

$$S = \frac{V^2}{2g\,(f - G)}$$

$$48 = \frac{V^2}{2\,(9.81)\,(0.35 - 0.02)}$$

$V = 17.63$ m/s

$$V = \frac{17.63\,(3600)}{1000}$$

$V = $ ***63.46 kph***

502. Problem:

A bus is running at a speed of 50 mph downhill on a grade of - 2%. The coeff. of friction between the road surface and the tire is 0.34. After suddenly applying full brakes, how far will the bus travel until it stops?

Solution:

① $V = 50$ mph

$$V = \frac{50\,(5280)}{3.28\,(3600)}$$

$V = 22.36$ m/s

$$S = \frac{V^2}{2g\,(f - G)}$$

$$S = \frac{(22.36)^2}{2\,(9.81)\,(0.34 - 0.02)}$$

$S = $ ***79.7 m.***

② *When it is moving uphill:*

$$S = \frac{(22.36)^2}{2\,(9.81)\,(0.34 + 0.02)}$$

$S = $ ***70.8 m.***

503. Problem:

Brakes are suddenly applied on a car that is traveling down hill at a speed of 80 kph on a grade of - 4%. Find the braking distance if the coeff. of friction between the tires and the road surface is 0.3.

Solution:

$$V = \frac{80000}{3600}$$

$$V = 22.22 \text{ m/s}$$

$$S = \frac{V^2}{2g\,(f - G)}$$

$$S = \frac{(22.22)^2}{2\,(9.81)\,(0.3 - 0.04)}$$

$$S = \boldsymbol{96.8 \text{ m.}}$$

504. Problem:

A car is traveling at 40 mph on an uphill grade of 5%. If the brakes are suddenly applied, it will travel 56 m. then stop. What is the coif. of friction between the tires and the road surface?

Solution:

$$V = \frac{40\,(5280)}{3.28\,(3600)}$$

$$V = 17.89 \text{ m/s}$$

$$S = \frac{V^2}{2g\,(f + G)}$$

$$56 = \frac{(17.89)^2}{2\,(9.81)\,(f + 0.05)}$$

$$f = \boldsymbol{0.24}$$

505. Problem:

A car traveling at 40 mph on a downhill grade of - 5%. If the brakes are suddenly applied it will travel 70 m. then stop. Determine the coeff. of friction between the road surface and the tires.

Solution:

$$V = \frac{40\,(5280)}{3.28\,(3600)}$$

$$V = 17.89 \text{ m/s}$$

$$S = \frac{V^2}{2g\,(f - G)}$$

$$70 = \frac{(17.89)^2}{2\,(9.81)\,(f - 0.05)}$$

$$f - 0.05 = 0.23$$

$$f = \boldsymbol{0.28}$$

506. Problem:

The driver of a truck running at 100 kph starts applying the brakes with increasing force such that $a = ct$ where $c = -2.5 \text{ m/s}^3$, t = time in sec. In how many seconds after the application of the brakes will the car stop?

Solution:

$$V = V_o + at$$

$$V = V_o + \int_0^t ct\,dt$$

$$V_o = \frac{100000}{3600} = 27.78 \text{ m/s}$$

when $V = 0$ *(stops)*

$$0 = 27.78 + \left(-\frac{ct^2}{2}\right)$$

$$27.78 = \frac{2.5\,t^2}{2}$$

$$t = \boldsymbol{4.71} \textbf{ sec.}$$

507. Problem:

The brakes were fully applied when the car speed was 40 kph. The skid mark on a level pavement was 8.5 m. long. Determine the efficiency of the cars brakes if the average skid resistance is 0.85.

Solution:

$V = \frac{40000}{3600}$
$V = 11.11$ m/s

$S = \frac{V^2}{2gf}$
$8.5 = \frac{(11.11)^2}{2\,(9.81)\,f}$
$f = 0.74$ *(average skid resistance)*

$Eff = \frac{0.74}{0.85}(100)$
$Eff = \mathbf{87.09\%}$

508. Problem:

When a car is traveling at 60 kph, after the brakes are suddenly applied, the car will still travel 30 m. before it stops. What is the coefficient of friction between the tires and the road surface?

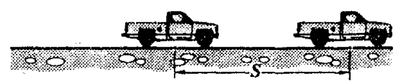
Solution:

$S = \frac{V^2}{2gf}$
$V = \frac{60000}{3600}$
$V = 16.68$ m/s
$30 = \frac{(16.68)^2}{2\,(9.81)\,f}$
$f = \mathbf{0.47}$

509. Problem:

The driver of a car traveling 70 kph requires 48 meters to stop after the brakes have been applied. What average coeff. of friction was developed between the tires and the pavement.

Solution:



$S = \frac{V^2}{2\,g\,f}$
$V = \frac{70000}{3600}$
$V = 19.44$ m/s
$48 = \frac{(19.44)^2}{2(9.81)\,f}$
$f = \mathbf{0.40}$

510. Problem:

Find the total distance that a car traveled from the time the driver saw the hazard when he was traveling at 75 kph. Perception time is 2 sec. and the average skid resistance is 0.60. Assume that the car has an efficiency of 80%.

Solution:

$f = 0.60\,(80)$
$f = 0.48$
$V = \frac{75000}{3600}$
$V = 20.83$ m/s

$S = Vt + \frac{V^2}{2gf}$
$S = 20.83\,(2) + \frac{(20.83)^2}{2\,(9.81)(0.48)}$
$S = \mathbf{87.70\ m.}$

511. Problem:

A truck was traveling downhill at 50 kph. The brakes are suddenly applied and the truck stopped in a distance of 32 m. If the coeff. of friction between the tires and the road surface is 0.4, what is the grade of the road?

Solution:

$$S = \frac{V^2}{2g\,(f + G)}$$

$$V = \frac{50000}{3600}$$

$$V = 13.89 \text{ m/s}$$

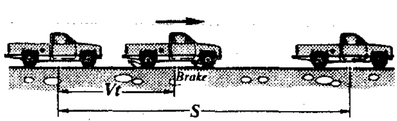
$$32 = \frac{(13.89)^2}{2\,(9.81)\,(0.4 + G)}$$

$$G = -\mathbf{0.09}$$

512. Problem:

Compute the total distance that a car had traveled from the time the driver sees a hazard when he is traveling at a speed of 75 kph. Perception time is 2 seconds, and the average skid resistance is equal to 0.60. Assume the car has an efficiency of 80%.

Solution:



$$f = 0.60(0.80)$$

$$f = 0.48$$

$$V = \frac{75000}{3600} = 20.83 \text{ m/s}$$

$$S = Vt + \frac{V^2}{2gf}$$

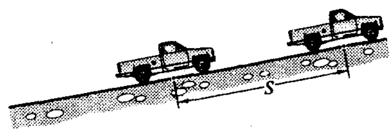
$$S = 20.83(2) + \frac{(20.83)^2}{2(9.81)(0.48)}$$

$$S = \mathbf{87.73\ m.}$$

513. CE Board May 2001

A vehicle is moving at a speed of 80 kph along an incline surface having a slope of 4%. If the coefficient of friction is 0.30, compute the braking distance.

Solution:



$$S = \frac{V^2}{2g\,(f + G)}$$

$$V = \frac{80000}{3600} = 22.22 \text{ m/s}$$

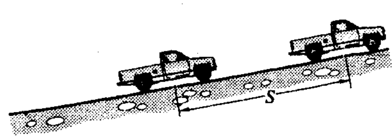
$$S = \frac{(22.22)^2}{2(9.81)(0.3 + 0.04)}$$

$$S = \mathbf{74.01\ m.}$$

514. Problem:

A car traveling at 80 kph requires 48 m. to stop after the brakes have been applied. Determine the slope of the road surface if the average coefficient of friction is 0.50.

Solution:



$$S = \frac{V^2}{2g\,(f + G)}$$

$$V = \frac{80000}{3600}$$

$$V = 22.22 \text{ m/s}$$

$$48 = \frac{(22.22)^2}{2(9.81)(0.50 + G)}$$

$$0.50 + G = 0.524$$

$$G = 0.024$$

$G = \mathbf{2.4\%}$ *(upward)*

515. CE Board Nov. 2000

A vehicle moving at 60 kph along an incline surface was stopped by applying brakes and the braking distance was 30 m. If the coefficient of friction is 0.50, compute the slope of the inclined surface.

Solution:

$$S = \frac{V^2}{2g\,(f + G)}$$

$$V = \frac{60000}{3600}$$

$V = 16.67$ m/s

$$30 = \frac{(16.67)^2}{2(9.81)(0.5 + G)}$$

$G = -\,0.0279$

$G = \mathbf{-\,2.79\%}$

516. Problem:

A truck was traveling downhill at 50 kph. The brakes are suddenly applied and the truck stopped in a distance of 32 m. If the coeff. of friction between the tires and the road surface is 0.40, what is the grade of the road?

Solution:

$$V = \frac{50000}{3600}$$

$V = 13.89$ m/s

$$S = \frac{V^2}{2g\,(f + G)}$$

$$32 = \frac{(13.89)^2}{2\,(9.81)\,(0.40 + G)}$$

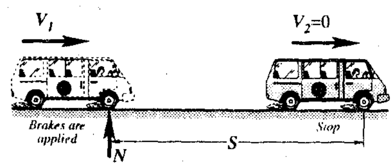
$0.40 + G = 0.317$

$G = -\,0.0927$ say $\mathbf{-\,9.27\%}$

517. Problem:

A vehicle moving a 40 kph was stopped by applying brakes and the length of the skid mark was 12.2 m. If the average skid resistance of the level pavement is known to be 0.70, determine the brake efficiency of the test vehicle.

Solution:



$$V = \frac{40(1000)}{3600} = 1.11 \text{ m/sec.}$$

$$S = \frac{V^2}{2g\,(f' + G)}$$

$$12.2 = \frac{(11.11)^2}{2(9.81)(f' + 0)}$$

$f' = 0.516$

$$\text{Brake efficiency} = \frac{f'}{f} \times 100$$

$$\text{Brake efficiency} = \frac{0.516}{0.70} \times 100$$

Brake efficiency = **73.7%**

518. Problem:

A truck driver approached a hazard at a speed of 58 mph. What was the distance traveled during perception reaction time if the PIEV (perception, identification, emotion and volition) time is 2.6 sec.

Solution:

$D = Vt$

$$V = \frac{58\,(5280)}{3.28\,(3600)}$$

$V = 25.93$ m/s

$D = 25.93\,(2.6)$

$D = \mathbf{67.43\ m.}$

519. Problem:

A bus driver approaches a hazard at a speed of 80 kph. What is the distance traveled during the perception-reaction time if the PIEV (perception, identifica-tion, emotion and volition) time is 2.4 sec.

Solution:

$d = Vt$

$d = \frac{80(1000)}{3600}(2.4)$

$d = \mathbf{53.33\ meters}$

520. Problem:

A bus driver approaches a hazard at a speed of 80 mph and travels a distance of 53.33 m. during a given perception-reaction time. Compute the drivers (PIEV) perception, identification, emotion and volition time.

Solution:

$D = Vt$

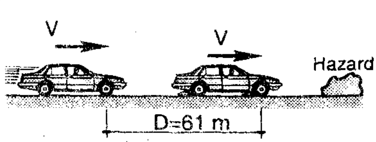
$53.33 = \frac{80(5280)}{3600(3.28)}t$

$t = \mathbf{1.49\ sec.}$ *(PIEV)*

521. CE Board May 2000

A car driver approached a hazard and traveled a distance of 61 m. during the perception-reaction time of 2.8 s. What was the car's speed of approach in mph?

Solution:



$D = Vt$

$61 = V(2.8)$

$V = 24.79$ m/sec.

$V = \frac{21.79\,(3.281)(3600)}{5280}$

$V = 48.7$ *say* ***49 mph***

522. Problem:

A truck driver approached a hazard at a speed of 52 mph. What was the distance traveled during the perception-reaction time if the PIEV (perception, identification, emotion and volition) time was 2.6 sec.

Solution:

$V = \frac{52\,(5280)}{3.28\,(3600)}$

$V = 23.25$ m/s

$S = Vt$

$S = 23.25\,(2.6)$

$S = \mathbf{60.45\ m.}$

523. Problem:

A car driver traveling at a speed of 65 mph approached a hazard and traveled 72.2 m. during the perception-reaction time. What was the drivers PIEV (perception, identification, emotion and volition) time in seconds.

Solution:

$V = \frac{65\,(5280)}{3.28\,(3600)}$

$V = 29.07$ m/s

$S = Vt$

$t = \frac{72.2}{29.07}$

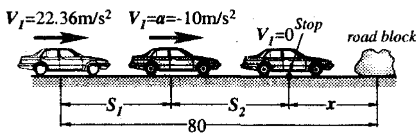
$t = \mathbf{2.5\ sec.}$

524. CE Board Nov. 1998

A car was traveling at a speed of 50 mph. The driver saw a road block 80 m. ahead and stepped on the brake causing the car to decelerate uniformly at 10 m/sec^2. Find the distance from the road block to the point where the car stopped. Assume perception-reaction time is 2 sec.

Solution:

$$V_1 = \frac{50\,(5280)}{3.28\,(3600)}$$
$V_1 = 22.36$ m/s



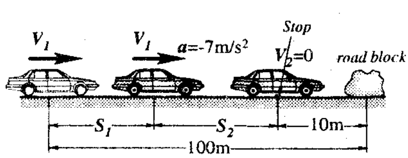
$S_1 = V_1\,t$
$S_1 = 22.36\,(2)$
$S_1 = 44.72$

$V_2^2 = V_1^2 - 2a\,S_2$
$0 = (22.36)^2 - 2\,(10)\,S_2$
$S_2 = 25$ m.
$x = 80 - 44.72 - 25$
$x = \mathbf{10.28}$

525. Problem:

While driving at 45 mph, the driver noticed a road block 100 m. away. He applied the brake and the car decelerated uniformly at 7 m/s^2. If the car stopped 10 m. from the road block, what was the perception-reaction time of the driver?

Solution:



$$V_1 = \frac{45\,(5280)}{3.28\,(3600)}$$
$V_1 = 20.12$ m/s

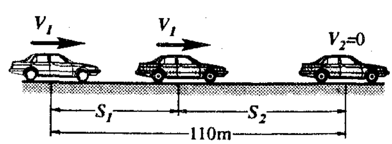
$V_2^2 = V_1^2 - 2a\,S_2$
$0 = (20.12)^2 - 2\,(7)\,S_2$
$S_2 = 28.92$
$S_1 = 100 - 28.92 - 10$
$S_1 = 61.08$ m.

$S_1 = V_1\,t$
$61.08 = 20.12\,t$
$t = \mathbf{3.04}$ **sec.**

526. Problem:

A car driver noticed a road block 110 m. away while driving at a speed of 50 mph. He applied the brakes and the car decelerated uniformly. The car stopped almost touching the road block. If the perception reaction time is 3 sec., determine the rate of deceleration in m/s^2.

Solution:



$$V_1 = \frac{50\,(5280)}{3.28\,(3600)}$$
$V_1 = 22.36$ m/s

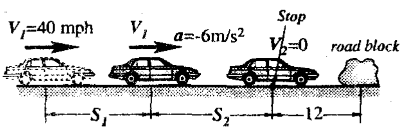
$S_1 = V_1\,t$
$S_1 = 22.36\,(3)$
$S_1 = 67.08$ m.
$S_2 = 110 - 67.08$
$S_2 = 42.92$ m.

$V_2^2 = V_1^2 - 2a\,S_2$
$0 = (22.36)^2 - 2a\,(42.92)$
$a = \mathbf{5.8\ m/s^2}$

527. Problem:

A driver noticed a road block while traveling at 40 mph. He applied the brakes causing the car to accelerate uniformly at 6 m/sec². The car stopped 12 m. from the road block. How far was the block when the driver saw it first? Assume a perception time of 3 sec.

Solution:



$$V_1 = \frac{40\,(5280)}{3.28\,(3600)} = 17.89 \text{ m/s}$$

$S_1 = V_1\,t$
$S_1 = 17.89\,(3)$
$S_1 = 53.67$ m.

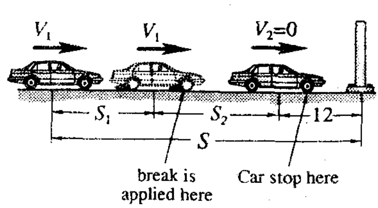
$V_2^2 = V_1^2 - 2a\,S_2$
$0 = (17.89)^2 - 2\,(6)\,S_2$
$S_2 = 26.67$ m.

Total $S = S_1 + S_2 + 12$
Total $S = 53.67 + 26.67 + 12$
Total $S =$ ***92.4 m.***

528. CE Board Nov. 2001

A driver traveling at 50 mph sees a wall at a certain distance ahead. The driver applies the brakes immediately (perception time is 3 seconds) and begins slowing the vehicle at 6 m/sec² (decelerating). If the distance from the stopping point to the wall is 12 m., how far was the car from the wall upon perception?

Solution:



$$V_1 = \frac{50(5280)}{3600(3.28)}$$

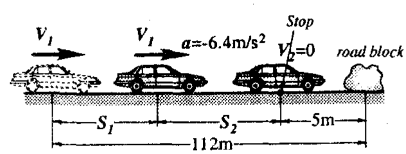
$V_1 = 22.36$ m/s
$S_1 = V_1\,t$
$S_1 = 22.36(3)$
$S_1 = 67.08$ m.
$V_2^2 = V_1^2 - 2aS_2$
$(0)^2 = (22.36)^2 - 2(6)\,S_2$
$S_2 = 41.66$ m.

Distance from wall upon perception:
$S = 67.08 + 41.66 + 12$
$S =$ ***120.74 m.***

529. Problem:

While traveling at a constant speed a driver saw a road block 112 m. away. He applied the brakes and the car decelerated uniformly at 6.4 m/s². It stopped 5 m. from the block. Find the initial constant speed of the vehicle in mph, if the drivers perception reaction time is 2.2 seconds.

Solution:



$S_1 = V_1\,t$
$S_1 = V_1\,(2.2)$

$V_2^2 = V_1^2 - 2a\,S_2$
$0 = V_1^2 - 2\,(6.4)\,S_2$

$$S_2 = \frac{V_1^2}{12.8}$$

$S_1 + S_2 + 5 = 112$

$$2.2\,V_1 + \frac{V_1^2}{12.8} + 5 = 112$$

$28.16\,V_1 + V_1^2 + 64 = 1433.6$
$V_1^2 + 28.16\,V_1 - 1369.6 = 0$
$V_1 = 25.52$ m/s

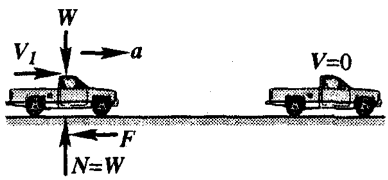
$$V_1 = \frac{25.52\,(3.28)(3600)}{5280}$$

$V_1 =$ ***57.06 mph***

530. Problem:

A vehicle travelling at 40 kph was stopped within 1.8 sec. after the application of brakes. Determine the average skid resistance.

Solution:



$V_2 = V_1 \pm at$

$0 = \frac{40\,(1000)}{3600} - a\,(1.8)$

$a = 6.17\ \text{m/sec}^2$

$F = m\,a$

$f\,W = \frac{W}{g}\,a$

$f = \frac{a}{g}$

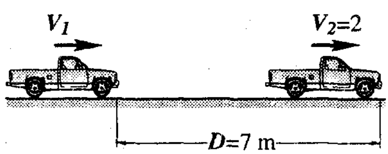
$f = \frac{6.17}{9.81}$

$f = \mathbf{0.63}$

531. Problem:

A vehicle was stopped in 1.4 sec. by fully jamming the brakes and the skid mark measured 7 m. Determine the average skid resistance on the level pavement surface.

Solution:



$V_2 = V_1 - at$

$0 = V_1 - at$

$V_1 = at$

$V_2^2 = V_1^2 \pm 2aS$

$0 = (at)^2 \pm 2aS$

$a^2 + t^2 = 2aS$

$a = \frac{2S}{t^2}$

$F = ma$

$fW = \frac{W}{g}\,a$

$f = \frac{a}{g}$

$f = \frac{2S}{t^2\,g}$

$f = \frac{2(7)}{(1.4)^2\,(9.81)}$

$f = \mathbf{0.728}$

532. Problem:

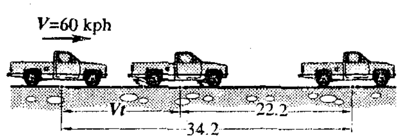
① A vehicle moving at 60 kph was stopped by applying brakes and the length of the skid was 22.2 m. If the distance from the point where it stops to the vehicle position when the driver initially reacted was 34.2 m, find the perception time.

② A vehicle moving at an initial speed of 60 kph was stopped within 3 sec. (braking time) after the application of brakes. Compute the average coefficient of frictional or skid resistance.

③ Compute the total distance that a car had traveled from the time the driver sees a hazard when he is traveling at a speed of 60 kph. Perception time is 2.5 sec. and average skid resistance is 0.60. Brake efficiency is 85%.

Solution:

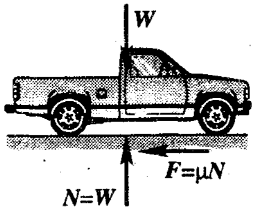
① *Perception time:*



$\frac{60000}{3600}\,(t) + 22.2 = 34.2$

$t = \mathbf{0.72\ sec.}$

② *Skid resistance:*



$V_2 = V_1 \pm a\,t$

$0 = \dfrac{60000}{3600} - a\,(3)$

$a = 5.56 \text{ m/sec}^2$

$F = ma$

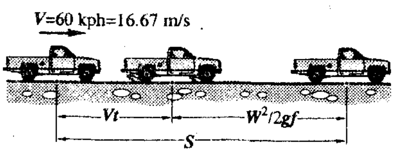
$\mu N = \dfrac{W}{g} a$

$\mu W = \dfrac{W}{g} a$

$\mu = \dfrac{a}{g} = \dfrac{5.56}{9.81}$

$\mu = \mathbf{0.57}$

③ *Total distance the car had traveled:*



$S = V\,t + \dfrac{V^2}{2g\,f}$

$f = 0.60\ (0.85)$

$f = 0.51$

$S = 16.67\ (2.5) + \dfrac{(16.67)^2}{2\,(9.81)(0.51)}$

$S = \mathbf{69.45\ m.}$

533. Problem:

A driver traveling at 50 mph is 80 m. from a wall ahead. If the driver applies the brakes immediately and begins slowing the vehicle at 10 m/sec^2 (decelerating), find the (PIEV) time when the distance of the car from the wall when it stop is 10.28 m.

Solution:

$V = \dfrac{50(5280)}{3600\,(3.28)}$

$V = 22.36$ m/sec.

$S = 80 - 10.28 = 69.72$ m.

$S_1 = 22.36t$

$V_2^2 = V_1^2 - 2a\,S_2$

$0 = (22.36)^2 - 2(10)\,S_2$

$S_2 = 25$ m.

$S_1 = 69.72 - 25 = 44.72$ m.

$S_1 = V\,t$

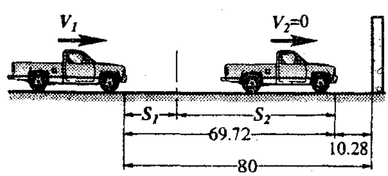
$44.72 = 22.36\,t$

$t = \mathbf{2\ sec.}$ *(PIEV) time*

534. Problem:

A driver traveling at 50 mph is 80 m. from a wall ahead. If the driver applies the brakes immediately. (PIEV) time is 2 seconds and begins slowing the vehicle. If the distance from the stopping point to the wall is 10.28 m., find the deceleration of the car.

Solution:



$V_1 = \dfrac{50(5280)}{3600(3.28)}$

$V_1 = 22.36$ m/s

$S_1 = V_1\,t$

$S_1 = 22.36(2)$

$S_1 = 44.72$ m.

$S_2 = 69.72 - 44.72$

$S_2 = 25$ m.

$V_2^2 = V_1^2 - 2a\,S_2$

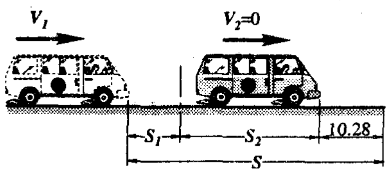
$0 = (22.36)^2 - 2\,a(25)$

$a = \mathbf{10\ m/sec^2}$ *(deceleration)*

535. Problem:

A driver traveling at 50 mph sees a hazard (boulder) at a certain distance ahead. The driver then applies the brakes immediately (PIEV) time is 2 seconds and begins slowing the vehicle at 10 m/sec^2 (decelerating). If the distance from the stopping point to avoid hitting the boulder is 10.28 m., how far was the car from the boulder upon perception?

Solution:



$S_1 = V_1 t$

$V_1 = \frac{50(5280)}{3600(3.28)}$

$V_1 = 22.36$ m/s

$S_1 = 22.36(2)$

$S_1 = 44.72$ m.

$V_2^2 = V_1^2 - 2a\,S_2$

$0 = (22.36)^2 - 2(10)S_2$

$S_2 = 25$ m.

Distance of car from boulder upon perception

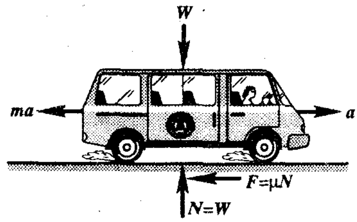
$S = 44.72 + 25 + 10.28$

$S = \mathbf{80\ m.}$

536. Problem:

A vehicle moving at an initial speed of 30 mph was stopped within 2 sec. (braking time) after the application of brakes. Compute the average coefficient of frictional or skid resistance.

Solution:



$V_1 = \frac{30(5280)}{3600\,(3.28)}$

$V_1 = 13.41$ m/s

$V_2 = V_1 - a\,t$

$0 = 13.41 - a(2)$

$a = 6.71$ m/s^2

$F = m\,a$

$\mu N = m\,a$

$\mu W = \frac{W}{g} a$

$\mu = \frac{a}{g}$

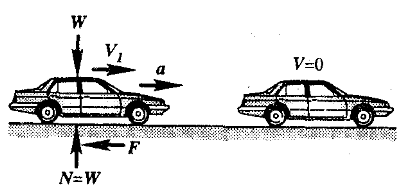
$\mu = \frac{6.71}{9.81}$

$\mu = \mathbf{0.68}$

537. Problem:

A vehicle moving at 30 mph was stopped after the application of the brakes. If the skid resistance is 0.68, compute the stopping time (braking time).

Solution:



$V_1 = \frac{30(5280)}{3600(3.28)}$

$V_1 = 13.41$ m/s

$\mu N = m a$

$\mu W = \frac{W}{g} a$

$0.68 = \frac{a}{9.81}$

$a = 6.67 \text{ m/s}^2$

$V_2 = V_1 - a t$

$0 = 13.41 - 6.67t$

$t =$ **2.01 seconds** *(breaking time)*

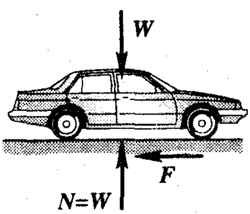
538. Problem:

2.5 seconds after the brakes were applied, a car stopped. Find the average coefficient of resistance if the initial speed was 35 mph.

Solution:

$V = \frac{35\,(5280)}{3.28\,(3600)}$

$V = 15.65$ m/s



$N = W$

$F = \mu N$

$F = m a$

$\mu N = \frac{W}{g} a$

$\mu W = \frac{W}{g} a$

$\mu = \frac{a}{g}$

$V_2 = V_1 + at$

$0 = 15.65 \pm a\,(2.5)$

$a = 6.26 \text{ m/s}^2$

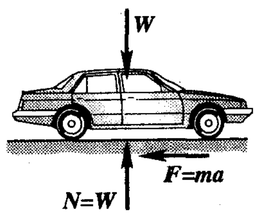
$\mu = \frac{6.26}{9.81}$

$\mu = \mathbf{0.638}$

539. Problem:

The driver applied the brakes on a car that is running at a speed of 38 mph. In how many seconds will the car stop if the average skid resistance is 0.70.

Solution:



$V_1 = \frac{38\,(5280)}{3.28\,(3600)}$

$V_1 = 16.99$

$N = W$

$F = \frac{W}{g} a$

$F = \mu N$

$\frac{W}{g} a = \mu W$

$\mu = \frac{a}{g}$

$0.70 = \frac{a}{g}$

$a = 6.867 \text{ m/s}^2$

$V_2 = V_1 + at$

$0 = 16.99 \pm 6.867\,t$

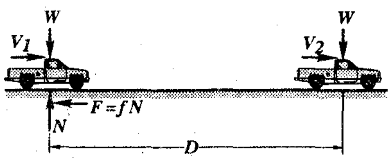
$t =$ **2.47 sec.**

540. Problem:

Compute the braking distance for a car moving at an initial velocity of 60 kph and a final velocity of 40 kph.

Slope of roadway is +5%
Coefficient of friction between road pavement and tires = 0.15
Perception-reaction time is $= \frac{3}{4}$ sec.

Solution:



$$V_1 = \frac{60000}{3600}$$
$$V_1 = 16.67 \text{ m/s}$$
$$V_2 = \frac{40000}{3600}$$
$$V_2 = 11.11 \text{ m/s}$$

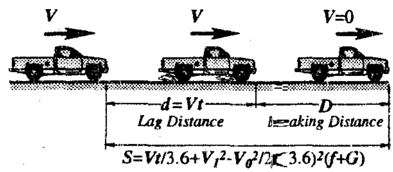
$$\textit{Braking distance} = \frac{(V_1^2 - V_2^2)}{2\,(g)\,(f \pm G)}$$
$$\textit{Braking distance} = \frac{(16.67)^2 - (11.11)^2}{2\,(9.81)(0.15 + 0.05)}$$
$$\textit{Braking distance} = \mathbf{39.33\ m.}$$

541. Problem:

Compute the total stopping distance that a car moves during the accident based on the following data:

Initial velocity of car = 60 kph
Final velocity when it stops = 0 kph
Coeff. of friction between tires and pavement = 0.15
Slope of roadway = - 2%
Perception-reaction time = 0.75 sec.

Solution:



$$V_1 = \frac{60000}{3600} = 16.67 \text{ m/s}$$

$$S = V_1\,t + \frac{V_1^2 - V_2^2}{2\,g\,(f - G)}$$

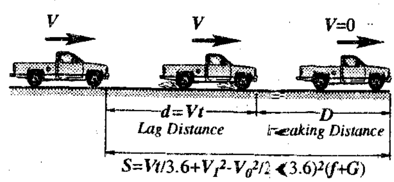
$$S = 16.67\,(0.75) + \frac{(16.67)^2 - 0}{2\,(9.81)\,(0.15 - 0.02)}$$

$$S = \mathbf{121.41\ m.}$$

542. Problem:

Compute the intermediate sight distance for a freeway with a design speed of 80 kph if the perception time is assumed to be 2.5 seconds with a skid resistance of 0.70. Assume brake efficiency to be 60%.

Solution:



$$V = \frac{80000}{3600} = 22.22 \text{ m/s}$$

$$S = V\,t + \frac{V^2}{2\,(g)\,f}$$

$f = 0.70\ (0.6)$ *(60% brake efficiency)*
$f = 0.42$

$$S = 22.22\,(2.5) + \frac{(22.22)^2}{2\,(9.81)\,(0.42)}$$

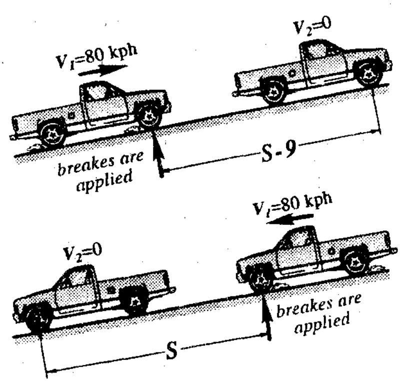
$S = 115.48$ m. *(stopping sight distance)*

Intermediate sight distance
= *twice the stopping sight distance*
= 2 (115.48)
= ***230.96 m.***

543. Problem:

The driver of a vehicle traveling at 80 kph up a grade requires 9 m. less to stop after he applies the brakes than the driver traveling at the same initial speed down the same grade. If the coefficient of friction between the tires and pavement is 0.50, what is the percent grade and what is the braking distance down the grade.

Solution:



$$V = \frac{80000}{3600} = 22.22 \text{ m/s}$$

$$D = \frac{V^2}{2\,(9.81)\,(f + G)}$$

$$D - 9 = \frac{(22.22)^2}{2\,(9.81)\,(0.5 + G)}$$

$$D - 9 = \frac{25.17}{0.5 + G}$$

$$D = \frac{(22.22)^2}{2\,(9.81)\,(0.5 - G)}$$

$$D = \frac{25.17}{0.5 - G}$$

$$D - 9 = \frac{25.17}{0.5 + G}$$

$$D = \frac{25.17}{0.5 + G} + 9$$

$$\frac{25.17}{0.5 - G} = \frac{25.17}{0.5 + G} + 9$$

$$25.17\,(0.5 + G) = 25.17\,(0.5 - G) + 9\,(0.25 - G^2)$$

$$12.585 + 25.17G = 12.585 - 25.17G + 2.25 - 9G^2$$

$$9G^2 + 50.34G - 2.25 = 0$$

$$G^2 + 5.59G - 0.25 = 0$$

$$G = 0.044$$

$$G = \boldsymbol{4.4\%}$$

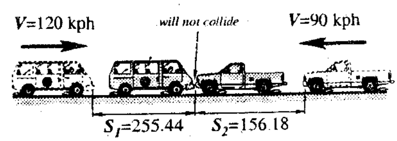
$$D = \frac{25.17}{0.5 + 0.44} + 9$$

$$D = \boldsymbol{55.27 \text{ m.}}$$

544. Problem:

Two cars are approaching each other from the opposite directions at a speed of 120 kph and 90 kph respectively. Assuming a reaction time of 2.0 seconds and a coefficient of friction of 0.60 with a brake efficiency of 50%. Compute the minimum sight distance required to avoid a head on collision of the two cars.

Solution:



$$V = \frac{120000}{3600} = 33.33$$

$$V = \frac{90000}{3600} = 25$$

For the 120 kph car:

$$f = 0.50\,(0.6)$$

$$f = 0.30$$

$$S_1 = V\,t + \frac{V^2}{2\,g\,(f + G)}$$

$$S_1 = (33.33)\,(2) + \frac{(33.33)^2}{2\,(9.81)\,(0.3)}$$

$$S_1 = \boldsymbol{255.44 \text{ m.}}$$

For the 90 kph car:

$$S_2 = (25)\,(2) + \frac{(25)^2}{2\,(9.81)\,(0.3)}$$

$$S_2 = 156.18 \text{ m.}$$

Sight distance to avoid head on collision

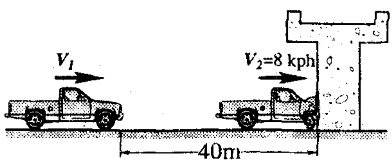
= 156.18 + 255.44

= ***411.62 m.***

545. Problem:

A Pajero Intercooler skidded into an intersection of Magsaysay Avenue and Quirino Avenue and struck a bystander and continued until it hit one of the column support of the skyway. Based on the damage to the front of the car, the police report estimated that the car was doing 8 kph at the moment of impact on the column support. The length of the skid marks was recorded to be 40 m. The road has a downhill grade of - 5%. A test car skidded 14 m. on the same section of the road when the brake is applied from a speed of 40 kph to the halt. Determine the probable speed of the car involved in the accident when the brakes were applied in kph.

Solution:



$$V = \frac{40000}{3600}$$
$$V = 11.11 \text{ m/s}$$

$$S = \frac{V^2}{2(g)(f - G)}$$
$$14 = \frac{(11.11)^2}{2(9.81)(f - 0.05)}$$
$$f - 0.05 = 0.449$$
$$f = 0.50 \quad \textit{(coeff. of friction)}$$

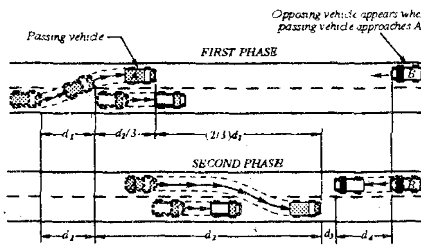
$$V_2 = \frac{8000}{3600}$$
$$V_2 = 2.22 \text{ m/s}$$
$$S = \frac{V_1^2 - V_2^2}{2g(f - G)}$$
$$40 = \frac{V_1^2 - (2.22)^2}{2(9.81)(0.50 - 0.05)}$$
$$V_1^2 - 64 = 4576.95$$
$$V_1 = \mathbf{68.12\ kph}$$

546. Problem:

Compute the passing sight distance for the following data:

Speed of the passing car = 96 kph
Speed of the overtaken vehicle = 88 kph
Time of initial maneuver = 4.3 sec.
Average acceleration = 2.37 kph/sec.
Time passing vehicle occupies the left lane = 10.4 sec.
Distance between the passing vehicle at the end of its maneuver and the opposing vehicle = 76 m.

Solution:



$$m = 96 - 88 = 8 \text{ kph}$$
$$d_1 = \frac{t_1}{3.6}\left(V - m + \frac{at_1}{2}\right)$$
$$d_1 = \frac{4.3}{3.6}\left[96 - 8 + \frac{2.37(4.3)}{2}\right]$$
$$d_1 = 111.20 \text{ m.}$$
$$d_2 = \frac{V t_2}{3.6}$$
$$d_2 = \frac{96(10.4)}{3.6} = 277.33 \text{ m.}$$
$$d_3 = 76 \text{ m.}$$

$$d_4 = \frac{2}{3}(d_2)$$
$$d_4 = \frac{2}{3}(277.33)$$
$$d_4 = 1.84.89 \text{ m.}$$

Total passing sight distance
$$= d_1 + d_2 + d_3 + d_4$$
$$= 111.20 + 277.33 + 76 + 184.89$$
$$= \mathbf{649.42\ m.}$$

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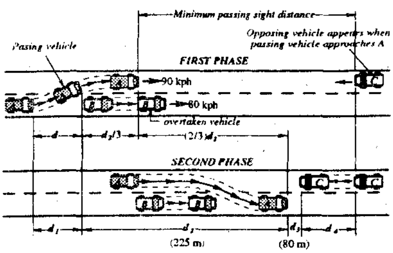
547. Problem:

Compute the minimum passing sight distance for the following data.

Speed of the passing car = 90 kph
Speed of the overtaken vehicle= 80 kph
Time of initial maneuver = 4 sec.
Average acceleration = 2.4 kph/sec.
Time passing vehicle occupies the left lane = 9 sec.
Distance between the passing vehicle at the end of its maneuver and the opposing vehicle = 80 m.

Solution:



Min. passing sight distance $= \frac{2}{3} d_2 + d_3 + d_4$

but $d_4 = \frac{2}{3} d_2$

Min. passing sight distance $= \frac{4}{3} d_2 + d_3$

d_3 = *distance between passing vehicle at the end of its maneuver and opposing vehicle.*

$d_2 = Vt$

$d_2 = \frac{90(1000)(9)}{3600}$

$d_2 = 225$ m.

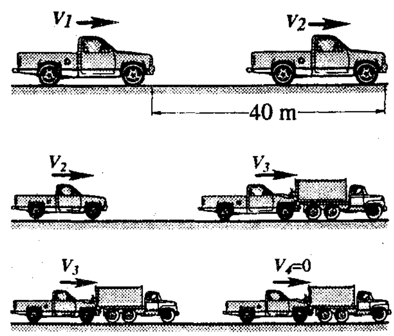
Min. passing distance $= \frac{4}{3}(225) + 80$

Min. passing distance = 380 m.

548. Problem:

A vehicle travel a distance of 40 m. before colliding with another parked vehicles, the weight of which is 75 percent of the former. After collision, if both vehicles skid through 14 m. before stopping, compute the initial speed of the moving vehicle. Assume friction coefficient of 0.62.

Solution:



After collision:

$$-(W_a + W_b) f S_2 = \frac{(W_a + W_b)}{2g}(V_4^2 - V_3^2)$$

$$-f S_2 = \frac{0 - V_3^2}{2g}$$

$V_3^2 = 2gf S_2$

$V_3^2 = 1(9.81)(0.62)(14)$

$V_3 = 13.05$ m/s

Momentum before impact = momentum after impact

$$\frac{W_a V_2}{g} = \frac{(W_a + W_b)}{g} V_3$$

$W_b = 0.75\ W_a$

$$\frac{W_a V_2}{g} = \frac{(W_a + 0.75 W_a)}{g} V_3$$

$V_2 = 1.75 V_3$

$V_2 = 1.75\ (13.05)$

$V_2 = 22.84$ m/s

Before collision

$$-W_a f S_1 = \frac{W_a}{2g}(V_2^2 - V_1^2)$$

$$-0.62(40) = \frac{V_2^2 - V_1^2}{2g}$$

$$-0.62(40) = \frac{(22.84)^2 - V_1^2}{2(9.81)}$$

$$(22.84)^2 - V_1^2 = -486.576$$

$$V_1 = 31.75 \text{ m/s}$$

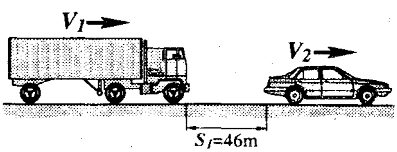
$$V_1 = \frac{31.75(3600)}{1000}$$

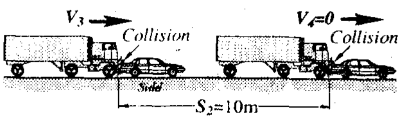
$$\boldsymbol{V_1 = 114.31 \text{ kph}}$$

549. Problem:

A cargo truck having a weight of 4000 lb. skids through a distance of 46 m. before colliding with a parked Toyota land cruiser having a weight of 2000 lb. After collision both vehicles skid through a distance equal to 10 m. before stopping. If the coefficient of friction between tires and pavement is 0.6, compute the initial speed of the cargo truck.

Solution:





After Collision:

$$-\frac{(W_a + W_b)}{g} f S_2 = \frac{(W_a + W_b)}{2g}(V_4^2 - V_3^2)$$

$$-f S_2 = \frac{0 - V_3^2}{2g}$$

$$V_3^2 = 2g f S_2$$

$$V_3^2 = 2(9.81)(0.6)(10)$$

$$V_3 = 10.85 \text{ m.s}$$

Momentum before impact = momentum after impact

$$\frac{W_a V_2}{g} = \frac{(W_a + W_b)}{g} V_3$$

$$\frac{4000 V_2}{g} = \frac{(4000 + 2000)}{g} V_3$$

$$V_2 = \frac{10.85(6000)}{4000}$$

$$V_2 = 16.275 \text{ m/s}$$

Before collision:

$$-W_a f S_1 = \frac{W_a(V_2^2 - V_1^2)}{2g}$$

$$-f S_1 = \frac{V_2^2 - V_1^2}{2g}$$

$$-0.60(46) = \frac{(16.275)^2 - V_1^2}{2(9.81)}$$

$$(16.275)^2 - V_1^2 = -541.512$$

$$V_1 = 28.40 \text{ m/s}$$

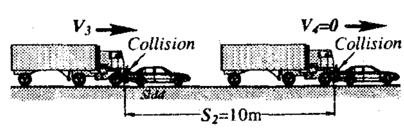
$$V_1 = \frac{28.40(3600)}{1000}$$

$$\boldsymbol{V_1 = 102.23 \text{ kph}}$$

550. Problem:

A cargo truck of weight 6000 lb. hits a Mercedes Benz having a weight of 1600 lb. and both the vehicles skid together through a distance of 5 m. before coming to stop. Compute the initial speed of the cargo truck if it does not apply brakes before collision. Ass. coeff. of friction = 0.60.

Solution:



After coalition:

$$-(W_1 + W_2) f S_2 = \frac{(W_1 + W_2)}{2g}(V_4^2 - V_3^2)$$

$$-f S_2 = \frac{V_4^2 - V_3^2}{2g}$$

$$-0.60(5) = \frac{0 - V_3^2}{2g}$$

$V_3 = 7.67$ m/s *(vel. of impact)*

Momentum before impact = momentum after impact

$$\frac{W_1 + W_2}{g} = \frac{(W_1 + W_2)}{g} V_3$$

$$\frac{6000 V_2}{g} = \frac{(6000 + 1600) V_3}{g}$$

$$6000 V_2 = 7600 V_3$$

$$V_2 = \frac{7600 (7.67)}{6000}$$

$V_2 = 9.72$ m/s

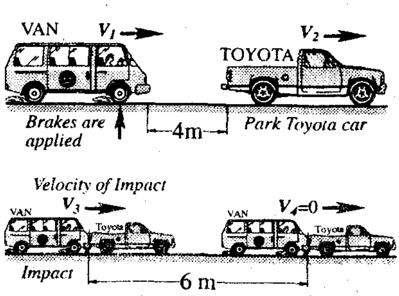
$$V_2 = \frac{9.72 (3600)}{1000}$$

$V_2 = \mathbf{34.99\ kph}$

551. Problem:

A van having a weight of 8000 lb. hits a parked Toyota car of weight 2000 lb. and both vehicles skid together through a distance of 6 m. before coming to stop. Compute the velocity of impact if the van applies brakes and skids through a distance of 4 m. before collision. Assume coefficient of friction is 0.50.

Solution:



After collision:

$$-FS_2 = \frac{(W_1 + W_2)}{2g}(V_4^2 - V_3^2)$$

$$F = (W_1 + W_2) f$$

$$-(W_1 + W_2) f S_2 = \frac{(W_1 + W_2)}{2g}(V_4^2 - V_3^2)$$

$$-f S_2 = \frac{V_4^2 - V_3^2}{2g}$$

$$-0.50(6) = \frac{0 - V_3^2}{2g}$$

$V_3 = 7.67$ m/s *(velocity at impact)*

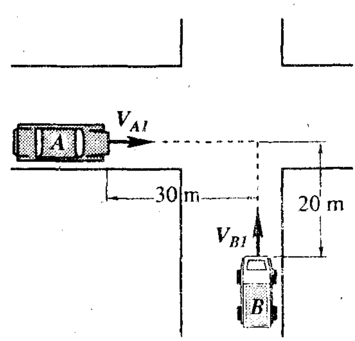
$$V_3 = \frac{7.67 (3000)}{1000}$$

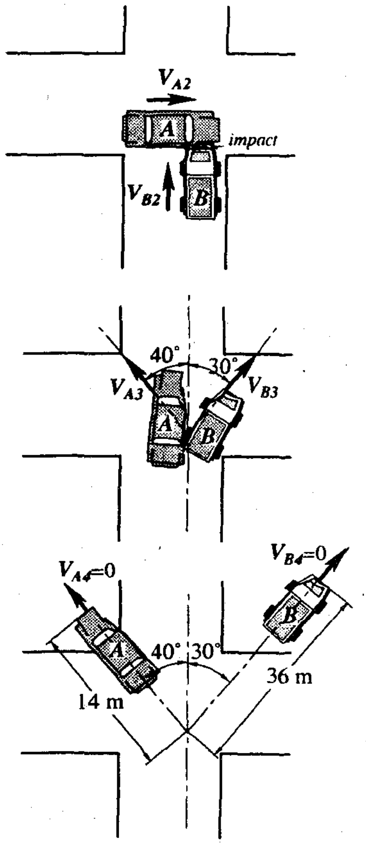
$V_3 = \mathbf{27.61\ kph}$

552. Problem:

Two cars A and B of equal weight are approaching an intersection of two perpendicular roads, A from the west and B from the south and collide with each other. The initial skid distances of A and B before collision are 30 m. and 20 m. respectively. After the collision, the skid distance of A and B are 14 m. and 36 m. respectively. A skids a direction of N. 40' W while B skids a direction of N. 30' E. If the average skid resistance of the pavement is found to be 0.60, compute the speed of A in kph before he applies the brake.

Solution:





After collision:

Work done in skidding = change in kinetic energy
Positive work - negative work
= change in kinetic energy

$$0 - W_A\,(f)\,(14) = \frac{W_A\,(V_{A_4}{}^2 - V_{A_3}{}^2)}{2g}$$

$$-0.60\,(14) = \frac{0 - V_{A_3}{}^2}{2(9.81)}$$

$V_{A_3} = 12.84$ m/s

For B:

$$0 - W_B\,(0.60)(36) = \frac{W_B\,(V_{B_4}{}^2 - V_{B_3}{}^2)}{2g}$$

$$-0.60\,(36) = -\frac{V_{B_3}{}^2}{2g}$$

$V_{B_3} = 20.59$ m/s

At collision:

(Along the West-East direction only)
momentum before impact = momentum after impact

$$\frac{W_A\,V_{A_2}}{g} + \frac{W_A\,(0)}{g} = \frac{W_A}{g}\,(-V_{A_3}\,\text{Sin } 40^\circ) + \frac{W_B}{g}\,V_{B_3}\,\text{Sin } 30^\circ$$

$$W_A\,(V_{A_2}) = W_A\,(-12.84\,\text{Sin } 40^\circ) + W_B\,(20.59\,\text{Sin } 30^\circ)$$

$W_A = W_B$

$V_{A_2} = -8.25 + 10.295$

$V_{A_2} = 2.045$ m/s

$$-W_A\,(0.60)(30) = \frac{W_A\,(V_{A_2}{}^2 - V_{A_1}{}^2)}{2g}$$

$$-60.30\,(30) = \frac{(2.045)^2 - V_{A_1}{}^2}{2(9.81)}$$

$4.182 - V_{A_1}{}^2 = -353.16$

$V_{A_1}{}^2 = 357.342$

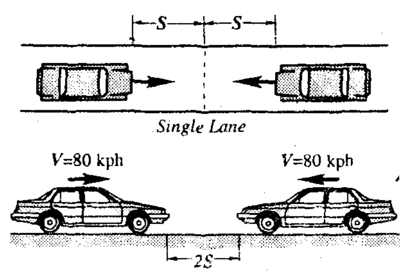
$V_{A_1} = 18.90$ m/s

$$V_{A_1} = \frac{18.90\,(3600)}{1000} = \mathbf{68.50\ kph}$$

553. Problem:

Compute the minimum required sight distance to avoid a collision for two-way traffic with single lane with a car approaching from the opposite directions if both cars are moving at a speed of 80 kph. Total perception and reaction time is 2.5 sec. Coefficient of friction is 0.40 and brake efficiency is 50%.

Solution:



$$V = \frac{80000}{3600} = 22.22 \text{ m/s}$$

$$f = 0.4\,(0.5)$$

$$f = 0.20$$

$$S = V\,t + \frac{V^2}{2\,g\,(f + G)}$$

$$S = 22.22\,(2.5) + \frac{(22.22)^2}{2\,(9.81)\,(0.4 + 0)(0.5)}$$

$$S = 55.56 + 125.85$$

$$S = 181.41 \text{ m.}$$

Sight distance to avoid collision

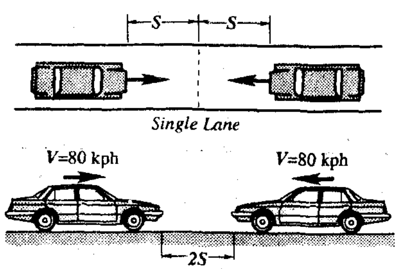
= 2 (181.41)

= ***362.82 m.***

554. Problem:

Compute the required safe stopping sight distance for a two way traffic in a single lane to avoid collision with a car approaching from the opposite direction if both cars are moving at a speed of 80 kph. Total perception - reaction time of the driver is 2 sec. Coefficient of friction between the tires and the pavement is 0.50. Slope of roadway is +2%.

Solution:



$$V = \frac{80000}{3600} = 22.22 \text{ m/s}$$

$$S = V\,t + \frac{V^2}{2\,g\,(f + G)}$$

$$S = (22.22)\,(2) + \frac{(22.22)^2}{2\,(9.81)\,(0.5 + 0.02)}$$

$$S = 92.85 \text{ m.}$$

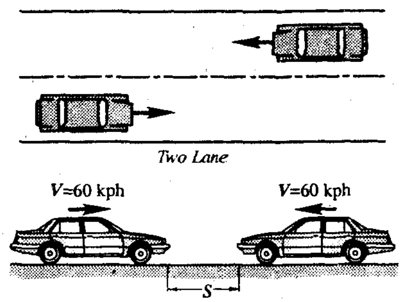
Safe stopping sight distance = 2 (92.85)

Safe stopping sight distance = ***185.70 m.***

555. Problem:

Compute the required length of the safe stopping sight distance in a two-way traffic in a two-way traffic in a two lane road if the design speed is 60 kph. Perception and reaction time of the driver is 2.6 sec. and the coefficient of friction between tires and pavement is 0.40. Assume the slope of roadway to be horizontal.

Solution:



$$V = \frac{60000}{3600}$$

$$V = 16.67 \text{ m/s}$$

$$S = V\,t + \frac{V^2}{2\,g\,(f + G)}$$

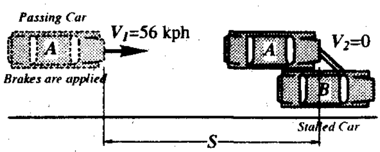
$$S = 16.67\,(2.6) + \frac{(16.67)^2}{2\,(9.81)\,(0.4 + 0)}$$

S = ***78.73 m.*** *(stopping distance)*

556. Problem:

Vehicles often travel city streets adjacent to parking lanes at 56 kph or faster. At his speed and setting detection through response-initiation time for an alert driver at 2 sec. and $f = 0.50$, how far must the driver be away from a suddenly opened car door to avoid striking it?

Solution:



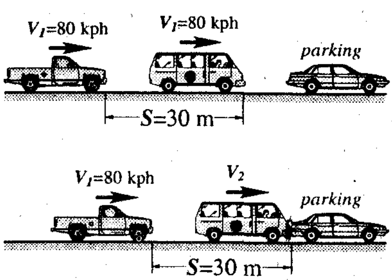
$$V = \frac{56000}{3600}$$
$$V = 15.56 \text{ m/s}$$

$$S = 15.56\,(2) + \frac{(15.56)^2}{2\,(9.81)\,(0.50 + 0)}$$
$$S = \mathbf{55.78\ m.}$$

557. Problem:

One vehicle is following another on a two-lane two highway at night according to the safe driving rule of thumb of one car length spacing for each 16 kph of speed. If both vehicles are traveling at 80 kph and the lead car crashes at that speed into the rear of an unlighted parked truck, at what speed will the following vehicle hit the wreckage? Assume a car length is 6 m., reaction time is 0.5 sec. and a coefficient of friction is 0.65.

Solution:



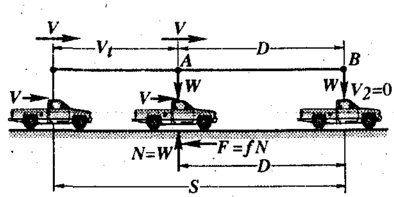
$$V_1 = \frac{80000}{3600}$$
$$V_1 = 22.22 \text{ m/s}$$

$$S = V_1\,t + \frac{V_1^2 - V_2^2}{2\,(g)\,(f + G)}$$
$$S = \frac{80}{16}\,(6)$$
$$S = 30 \text{ m.}$$
$$30 = 22.22\,(0.5) + \frac{(22.22)^2 - V_2^2}{2\,(9.81)\,(0.65 + 0)}$$
$$(22.22)^2 - V_2^2 = 240.90$$
$$V_2 = 15.9 \text{ m/s}$$
$$V_2 = \frac{15.9}{1000}\,(3600)$$
$$V_2 = \mathbf{57.2\ kph}$$

558. Problem:

Compute the heading light sight distance for a freeway with a design speed of 75 kph. Assume time of perception to be 3 sec. and skid resistance to be 0.60. Use 80% brake efficiency.

Solution:



$$V = \frac{75000}{3600}$$
$$V = 20.83 \text{ m/s}$$

$$S = V\,t + \frac{V^2}{2\,g\,f}$$
$$S = 20.83\,(3) + \frac{(20.83)^2}{2\,(9.81)\,(0.48)}$$
$$S = \mathbf{108.59\ m.}$$

559. Problem:

A car having a gross weight of 50 kN is moving at a certain design speed along a highway curve. Neglecting friction between the tires and the pavement, find the force that will tend to pull the car away from the center of the curve if the curve has an impact factor of 0.30.

Solution:

$$\text{Impact factor} = \frac{\text{Centrifugal force}}{\text{weight of vehicle}}$$

$$0.30 = \frac{F}{50}$$

$$F = \mathbf{15\ kN}$$

560. Problem:

A car is running at max. allowable speed along a highway curve with an impact factor of 0.20. If the weight of the car and its load is 40 kN, what is the centrifugal force on it. Neglect friction between the tires and the pavement.

Solution:

$$\text{Centrifugal force} = \frac{W}{g}\frac{V^2}{r}$$

$$\text{Impact factor} = \frac{V^2}{g\,r}$$

$$\text{Centrifugal force} = 40\ (0.2)$$

$$\text{Centrifugal force} = \mathbf{8\ kN}$$

561. Problem:

A car weighing 1200 kg runs at 50 kph around an unbanked curve with a radius of 70 m. What force of friction on the tires is required to keep the car on the circular track?

Solution:

$$F = \frac{W}{g}\frac{V^2}{r}$$

$$V = \frac{50000}{3600} = 13.89\text{ m/s}$$

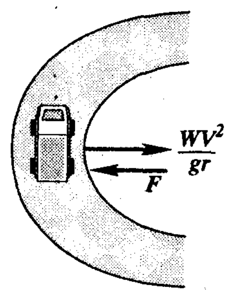
$$F = \frac{1200\ (1389)^2}{70}$$

$$F = \mathbf{3307\ N}$$

562. CE Board May 2003

A car weighing 1000 kg runs at 60 kph around an unbanked circular curve with a radius of 100 m. What force of friction on the tires should be to prevent the car from sliding?

Solution:



$$F = \frac{W}{g}\frac{V^2}{r}$$

$$V = \frac{60000}{3600} = 16.67\text{ m/s}$$

$$F = \frac{1000(9.81)(16.67)^2}{9.81\ (100)}$$

$$F = \mathbf{2778}$$

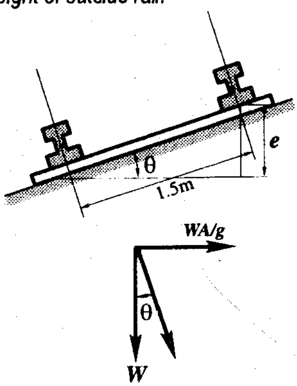
563. Problem:

Because some railroad track follows a river bed, tight curves are required. The tightest curve has a radius of 213.41 m. The center to center distance between rails is called the effective gage and is equal to 1.5 m.

① To what height must the outside rail be raised if an equilibrium speed of 64 kph is to be maintained?
② What is the comfortable speed along the river bed?
③ What is the length of the transition spiral with the 125 mm elevation if the equilibrium speed is used?

Solution:

① *Height of outside rail:*



$$\tan\theta = \frac{W\,a}{g\,W}$$

$$\tan\theta = \frac{a}{g}$$

$$a = \frac{V^2}{R}$$

$$\tan\theta = \frac{V^2}{g\,R}$$

$$\text{Sin}\,\theta = \frac{e}{1.5} = \tan\theta \text{ for small angles}$$

$$\frac{e}{1.5} = \frac{V^2}{g\,R}$$

$$V = \frac{64000}{3600} = 17.78 \text{ m/s}$$

$$\frac{e}{1.5} = \frac{(17.78)^2}{9.81\,(213.41)}$$

$e = 0.226$ m. = ***226 m.***

② *Comfortable speed:*
when $e = 75$ mm *(3 inches)*

$$\frac{e}{1.5} = \frac{V^2}{g\,R}$$

$$\frac{0.075}{1.5} = \frac{V^2}{9.81\,(213.41)}$$

$V = 10.23$ m/s = ***36.8 kph***

③ *Length of spiral:*
$e = 125$ mm = 5 inches

Note:
Length of spiral in feet is usually 62 times "e" in inches.

$L_s = 62\,(5)$
$L_s = 310$ ft.
$L_s =$ ***94.51 m.***

564. Problem:

The design speed for a horizontal curve of a hill road is limited to 60 kph.

① Compute the required super elevation to be provided at horizontal curve of a hill road having a radius of curve of 200 m.
② What is the min. radius of horizontal curve in a hilly road that is required if the lateral friction factor is taken as 0.15 and a super elevation of 0.068 m.
③ What is the allowable rate of change of centrifugal acceleration for a horizontal curve of a hill road.

Solution:

① *Required super elevation for a hill road:*

$$e = \frac{V^2}{225\,R}$$

$$e = \frac{(60)^2}{225\,(200)}$$

$e =$ ***0.08***

② *Min. radius of horizontal curve in hill road:*

$$R = \frac{V^2}{(e + f)\,(127)}$$

$$R = \frac{0.008\,(65)^2}{0.068 + 0.15}$$

$R =$ ***155 m.***

③ *Allowable rate of change of centrifugal acceleration of horizontal curve in hill road:*

$$C = \frac{80}{V + 75}$$

$$C = \frac{80}{65 + 75}$$

$C =$ ***0.571 m/sec³***

565. Problem:

A car weighing 60 kN moves a certain speed around a circular curve causing a centrifugal force which tends to pull the car away from the center of the curve at a magnitude equal to 15 kN.

① Compute the equivalent centrifugal ratio.
② What is the max. speed that this car could maneuver on the curve so that it will not overturn if the curve has a radius of 280 m.
③ What would be the design super elevation per meter to prevent sliding or overturning if the friction factor is 0.12.

Solution:

① *Centrifugal ratio:*

$$\text{Centrifugal ratio} = \frac{F}{W}$$

$$\text{Centrifugal ratio} = \frac{15}{60}$$

$$\text{Centrifugal ratio} = \mathbf{0.25}$$

② *Max. speed:*

$$\text{Centrifugal ratio} = \frac{V^2}{g\,r}$$

$$0.25 = \frac{V^2}{9.81\,(280)}$$

$$V = 26.20 \text{ m/s}$$

$$V = \frac{26.20\,(3600)}{1000}$$

$$V = \mathbf{94.34\ kph}$$

③ *Super elevation:*

$$R = \frac{V^2}{127\,(e+f)}$$

$$280 = \frac{(94.34)^2}{127\,(e+0.12)}$$

$$e = \mathbf{0.13\ m/m}$$

566. Problem:

A two lane highway having a width of 3.6 m. on each lane with a design speed of 100 kph has a 400 m. radius horizontal curve connecting tangents with bearings of N. 75° E. and S. 78° E.

① Determine the super elevation rate if the friction factor is 0.12.
② Compute the length of spiral if the difference in grade between the centerline and edge of the traveled way is limited to 1/200.
③ Compute the spiral angle of the S.C.

Solution:

① *Super elevation:*

$$R = \frac{V^2}{127\,(e+f)}$$

$$400 = \frac{(100)^2}{127\,(e+0.12)}$$

$$e = \mathbf{0.08}$$

② *Length of spiral:*

$$\frac{D\,e}{L} = \frac{1}{200}$$

$$\frac{3.6\,(0.08)}{L} = \frac{1}{200}$$

$$L = \mathbf{57.6\ m.}$$

③ *Spiral angle:*

$$S_c = \frac{L_c}{2\,R_c}\,\frac{180}{\pi}$$

$$S_c = \frac{57.6}{2\,(400)}\,\frac{180}{\pi}$$

$$S_c = \mathbf{4.13^{\circ}}$$

567. CE Board May 2005

An easement curve has a length of spiral equal to 60 m. having a central curve of a radius of 400 m. The design velocity of the car allowed to pass thru this portion is 100 kph.

① Compute the rate of increase of centripetal acceleration.
② If the friction factor is equal to 0.14, compute the super elevation rate in m/m width of roadway.
③ Compute the width of one lane of roadway if the difference in grade between the center line and the edge of the roadway is 1/220.

Solution:

① *Centripetal acceleration:*

$$L = \frac{0.0215\ V^3}{R\,C}$$

$$60 = \frac{0.0215\ (100)^3}{400\ C}$$

$$C = \mathbf{0.896\ m/sec^2}$$

② *Super elevation rate:*

$$R = \frac{V^2}{127\ (e + f)}$$

$$400 = \frac{(100)^2}{127\ (e + 0.14)}$$

$$e = \mathbf{0.06\ m/m}$$

③ *Width of one lane:*

$$\frac{D\,e}{L} = \frac{1}{220}$$

$$\frac{D\ (0.06)}{60} = \frac{1}{220}$$

$$D = \mathbf{4.55\ m.}$$

568. Problem:

A car having a weight of 40 kN is moving at a certain speed around a given curve. Neglecting friction between the tire and pavement and assuming a centrifugal ratio of 0.30.

① Compute the force that will tend to pull the car away from the center of the curve in kN.
② If the degree of curve is 4°, compute the max. speed in kph that a car could move around the curve.
③ Compute the value of the super elevation to be actually provided for this speed if the skid resistance is 0.12.

Solution:

① *Force that will tend to pull the car away from center of curve:*

$$\text{Centrifugal ratio} = \frac{F}{W}$$

$$0.30 = \frac{F}{40}$$

$$F = \mathbf{12\ kN}$$

② *Max. speed:*

$$\text{Centrifugal ratio} = \frac{V^2}{g\,r}$$

$$r = \frac{1145.916}{4}$$

$$r = 286.48$$

$$0.30 = \frac{V^2}{9.81\ (286.48)}$$

$$V = 29.04\ m/s$$

$$V = \mathbf{104.53\ kph}$$

③ *Super elevation:*

$$R = \frac{V^2}{127\ (e + f)}$$

$$286.48 = \frac{(104.53)^2}{127\ (e + 0.12)}$$

$$e = \mathbf{0.18}$$

569. Problem:

A two lane spiral easement curve having a spiral length of 60 m. has a super elevation of 0.06 and a difference of grade of 1/250 between the center line and the road edge. Radius of central curve is 420 m.

① Compute the width required for each lane.
② Compute the velocity of a car moving along this curve if it has a friction factor of 0.15.
③ Compute its centrifugal ratio.

Solution:

① *Width for each lane:*

$$\frac{D\,e}{L_c}=\frac{1}{250}$$

$$\frac{D\,(0.06)}{60}=\frac{1}{250}$$

$$D=\mathbf{4\ m.}$$

② *Velocity of car:*

$$R=\frac{V^2}{127\,(e+f)}$$

$$420=\frac{V^2}{127\,(0.06+0.15)}$$

$$V=\mathbf{105.84\ kph}$$

③ *Centrifugal ratio:*

$$\text{Centrifugal ratio}=\frac{V^2}{g\,r}$$

$$V=\frac{105.84\,(1000)}{3600}$$

$$V=29.4\ \text{m/s}$$

$$\text{Centrifugal ratio}=\frac{(29.4)^2}{9.81\,(420)}$$

$$\text{Centrifugal ratio}=\mathbf{0.21}$$

570. Problem:

The design speed of curve portion of the Manila-Cavite circumferential road is only 80 kph. The radius of the curve portion is 350 m.

① Allowing a skid resistance or friction factor of 0.12, what must be the super elevation rate in m/m to avoid overturning?
② Compute the impact factor to be considered in the design.
③ Compute the angle of embankment to prevent overturning.

Solution:

① *Super elevation:*

$$R=\frac{V^2}{127\,(e+f)}$$

$$350=\frac{(80)^2}{127\,(e+0.12)}$$

$$e=\mathbf{0.024\ m/m}$$

② *Impact factor:*

$$\text{Impact factor}=\frac{V^2}{g\,R}$$

$$V=\frac{80000}{3600}$$

$$V=22.22\ \text{m/s}$$

$$\text{Impact factor}=\frac{(22.22)^2}{2\,(350)}$$

$$\text{Impact factor}=\mathbf{0.705}$$

③ *Angle of embankment:*

$$\tan\theta=\frac{V^2}{g\,r}$$

$$\tan\theta=0.705$$

$$\theta=\mathbf{35.20^\circ}$$

571. Problem:

An old dilapidated road having a curve portion is to be improved to accommodate a design speed of 90 kph. Super elevation is 0.08 and skid resistance is 0.10.

① Compute the radius curve to accommodate the design speed of 90 kph.
② Compute the degree of curve.
③ Compute the impact factor due to this speed.

Solution:

① *Radius of curve:*

$$R = \frac{V^2}{127\,(e+f)}$$

$$R = \frac{(90)^2}{127\,(0.08+0.10)}$$

$$R = \mathbf{354.33\ m.}$$

② *Degree of curve:*

$$D = \frac{1145.916}{354.33}$$

$$D = 3.23^\circ$$

③ *Impact factor:*

$$\text{Impact factor} = \frac{V^2}{g\,R}$$

$$V = \frac{90000}{3600}$$

$$V = 25\ \text{m/s}$$

$$\text{Impact factor} = \frac{(25)^2}{9.81\,(354.33)}$$

$$\text{Impact factor} = \mathbf{0.18}$$

572. Problem:

Compute the impact factor for a horizontal curve radius of 400 m. if the design speed is 120 kph.

Solution:

Centrifugal ratio or impact factor

$$= \frac{\text{Centrifugal force}}{\text{Weight of vehicle}}$$

$$\text{Impact factor} = \frac{\frac{WV^2}{gr}}{W}$$

$$\text{Impact factor} = \frac{V^2}{gr}$$

$$V = \frac{120(1000)}{3600}$$

$$V = 33.33\ \text{m/s}$$

$$\text{Impact factor} = \frac{(33.33)^2}{9.81(400)}$$

$$\text{Impact factor} = \mathbf{0.283}$$

573. CE Board May 2002

Determine the max. speed in kph that a car could move around a curve having a radius of 500 m. if the impact factor of that curve is 0.15. Neglect the friction between fires and pavement.

Solution:

$$\text{Impact factor} = \frac{V^2}{g\,r}$$

$$0.15 = \frac{V^2}{9.81\,(500)}$$

$$V = 27.12\ \text{m/s}$$

$$V = \frac{27.12(3600)}{1000}$$

$$V = \mathbf{97.63\ kph}$$

574. CE Board Nov. 2005

Determine the max. speed in kph that a car could run around a 5 degrees curve if the impact factor of the curve is 0.14. Neglect the friction between the tires and the pavement.

Solution:

$$Impact\ factor = \frac{V^2}{g\,r}$$

$$r = \frac{1145.916}{D^{\circ}}$$

$$r = \frac{1145.916}{5}$$

$$r = 229.18$$

$$0.14 = \frac{V^2}{9.81\,(229.18)}$$

$$V = 17.74 \text{ m/s}$$

$$V = \frac{17.74\,(3600)}{1000}$$

$$V = \mathbf{63.9\ kph}$$

575. Problem:

A car having a weight of 40 kN is moving at a certain speed around a given curve. Neglecting friction between the tire and the pavement and assuming centrifugal ratio of 0.30

① Compute the force that will tend to pull the car away from the center of the curve.
② If the degree of curve is 4°, compute the max. speed in kph that a car could move around the curve.
③ Compute the value of the super elevation to be actually provided for this speed if the skid resistance is equal to 0.12.

Solution:

① *Force to pull the car away:*

$$Centrifugal\ ratio = \frac{F}{W}$$

$$0.30 = \frac{F}{40}$$

$$F = \mathbf{12\ kN}$$

② *Max. speed that a car could move around the curve:*

$$Centrifugal\ ratio = \frac{V^2}{gr}$$

$$r = \frac{1145.916}{D}$$

$$r = \frac{1145.916}{4}$$

$$r = 286.48 \text{ m.}$$

$$0.60 = \frac{V^2}{9.81(286.48)}$$

$$V = 29.04 \text{ m/s}$$

$$V = \mathbf{104.53\ kph}$$

③ *Super elevation:*

$$R = \frac{V^2}{127(e+f)}$$

$$286.48 = \frac{(104.53)^2}{127(e+0.12)}$$

$$e + 0.12 = 0.30$$

$$e = \mathbf{0.18\ m/m}$$

576. CE Board Nov. 1999

A horizontal curve has a design speed of 50 mph. If $e = 0.10$ and $f = 0.16$, find the degree of curve.

Solution:

$$R = \frac{V^2}{127\,(e+f)}$$

$$V = 50 \text{ mph}$$

$$V = \frac{50\,(5280)}{3.28\,(1000)}$$

$$V = 80.49 \text{ kph}$$

$$R = \frac{(80.49)^2}{127\,(0.10 + 0.16)}$$
$R = 196.19$ m.
$$D = \frac{1145.916}{R}$$
$$D = \frac{1145.916}{196.16}$$
$D = \mathbf{5.84^\circ}$

577. Problem:

A simple and horizontal curve road has a degree of curve of 3.2°. Determine the design speed on this curve in mph if e = 0.08 and f = 0.12.

Solution:

$$R = \frac{1145.916}{3.2}$$
$R = 358.1$ m.
$$R = \frac{V^2}{127\,(e + f)}$$
$$358.1 = \frac{V^2}{127\,(0.08 + 0.12)}$$
$V = 95.37$ kph
$$V = \frac{95.37\,(1000)\,(3.28)}{5280}$$
$V = \mathbf{59.2\ mph}$

578. Problem:

The degree of curve of a simple curve is 5°. Compute the desired superelevation required if the design speed of a car passing thru the curve is 80 kph and the skid resistance is equal to 0.12.

Solution:

$$R = \frac{1145.916}{D^\circ}$$
$$R = \frac{1145.916}{5}$$
$R = 229.18$ m.
$$R = \frac{V^2}{127\,(e + f)}$$
$$229.18 = \frac{(80)^2}{127\,(e + 0.12)}$$
$e + 0.12 = 0.22$
$e = \mathbf{0.10\ m/m}$

579. CE Board May 2004

A curve road 74 m. in radius has a super elevation of 0.12 and a design speed of 80 kph. Determine the coefficient of friction between the tires and the pavement.

Solution:

$$R = \frac{V^2}{127(e + f)}$$
$$74 = \frac{(80)^2}{127\,(0.12 + f)}$$
$0.12 + f = 0.68$
$f = \mathbf{0.56}$

580. Problem:

A 4° horizontal curve road is design for a speed of 50 mph. Super elevation is 0.06. Determine the coefficient of lateral friction.

Solution:

$$V = \frac{50\,(5280)}{3.28\,(1000)}$$
$V = 80.48$ kph
$$R = \frac{1145.916}{4}$$
$R = 286.48$
$$R = \frac{V^2}{127\,(e + f)}$$
$$286.48 = \frac{(80.48)^2}{127\,(0.06 + f)}$$
$f = \mathbf{0.118}$

Spiral Curve

581. Problem:

Find the length of the transition curve at the ends of a central curve having a radius of 195 m. to be laid such that cars traveling at a speed of 70 kph will not skid or overturn.

Solution:

$$L_c = \frac{0.036\,K^3}{R_c}$$

$$L_c = \frac{0.036\,(70)^3}{195}$$

$$L_c = 63.3\text{ m.}$$

582. Problem:

Determine the length of the spiral curve designed for a max. car speed of 90 kph if the degree of the central curve is 5°. Use arc basis.

Solution:

$$R = \frac{1145.916}{5}$$

$$R = 229.18\text{ m.}$$

$$L_c = \frac{0.036\,K^3}{R_c}$$

$$L_c = \frac{0.036\,(90)^3}{229.18}$$

$$L_c = 114.51\text{ m.}$$

583. Problem:

Determine the length of the spiral curve designed for a max. car speed of 90 kph if the degree of the central curve is 5°. Use arc basis.

Solution:

$$R = \frac{1145.916}{5}$$

$$R = 229.18\text{ m.}$$

$$L_c = \frac{0.036\,K^3}{R_c}$$

$$L_c = \frac{0.036\,(90)^3}{229.18}$$

$$L_c = 114.51\text{ m.}$$

584. Problem:

At what max. speed in kph could a car pass through a spiral easement curve without overturning if the radius of the central curve is 180 m. and the super elevation is 0.12 per meter of roadway?

Solution:

$$e = \frac{0.079\,K^2}{R}$$

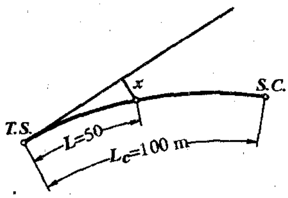
$$0.12 = \frac{0.079\,K^2}{180}$$

$$K = 52.3\text{ kph}$$

585. Problem:

A spiral easement curve has a length of 100 m. with a central curve having a radius of 300 m. Determine the offset distance from the tangent to the 2nd quarter point of the spiral.

Solution:



$$x = \frac{L^3}{6\,R_c\,L_c}$$

$$x = \frac{(50)^3}{6(300)(100)}$$

$x = 0.69$ **m.**

586. Problem:

What is the max. speed in kph at which a car could pass through an easement curve having a length of spiral of 120 m. if the radius of the central curve is 260 m.?

Solution:

$$L = \frac{0.036\,K^3}{R}$$

$$120 = \frac{0.036\,K^3}{260}$$

$K = 95.3$ **kph**

587. Problem:

What is the offset distance from the tangent to the second quarter point of a spiral curve that a has a length of 80 m. and a central radius of 300 m.

Solution:

$$x = \frac{L^3}{6\,R_c\,L_c}$$

$$x = \frac{(40)^3}{6\,(300)(80)}$$

$x = 0.444$ m.

$x = 44.4$ **cm.**

588. Problem:

A spiral 100 m. long connects a tangent with a 4 degree circular curve. Find the spiral angle at the S.C.

Solution:

$$R = \frac{1145.916}{4}$$

$$R = 286.48$$

$$S_c = \frac{L_c}{2\,R_c}\,\frac{180}{\pi}$$

$$S_c = \frac{100}{2\,(286.48)}\left(\frac{180}{\pi}\right)$$

$S_c = 10°$

589. Problem:

Find the degree of curve of a central simple curve if has a spiral curve of 100 m. long on two sides on which a car traveling at 75 kph will not skid. Use arc basis.

Solution:

$$L = \frac{0.036\,K^3}{R}$$

$$100 = \frac{0.036\,(75)^3}{R}$$

$$R = 151.875$$

$$D = \frac{1145.916}{151.875}$$

$D = 7.55°$

590. Problem:

The radius of the central angle of a spiral easement curve is equal to 230 m. The central curve has a central angle of 36°.

① Compute for the offset distance at the S.C. if the external distance is 13.20 m.
② Compute the length of the spiral curve.
③ Compute the max. design speed of the curve.

Solution:

① *Offset distance at S.C.*

$$E_s = (R_c + P)\sec\frac{I}{2} - R_c$$

$$13.20 = (230 + P)\ \text{Sec}\ 18° - 230$$

$$P = 1.30$$

$$X_c = 4\,(1.3)$$

$$X_c = \mathbf{5.2}$$

② *Length of spiral curve:*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$5.2 = \frac{L_c^2}{6\,(230)}$$

$$L_c = \mathbf{84.7\ m.}$$

③ *Design speed:*

$$L = \frac{0.036\,K^3}{R_c}$$

$$84.7 = \frac{0.036\,K^3}{20}$$

$$K = \mathbf{81.5\ kph}$$

591. Problem:

A 80 m. spiral easement curve has a 6° curve for its central curve.

① Determine the radius of the central curve.
② Compute the length of throw of the spiral curve.
③ If the central angle of the central curve is 42°, compute the external distance of the central curve of a spiral easement curve.

Solution:

① *Radius of curve:*

$$R = \frac{1145.916}{6}$$

$$R = \mathbf{190.99\ m.}$$

② *Length of throw:*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\,(190.99)}$$

$$X_c = 5.58$$

$$P = \frac{X_c}{4}$$

$$P = \frac{5.88}{4}$$

$$P = \mathbf{1.395\ m.}$$

③ *External distance:*

$$E_s = (R_c + P)\ \text{Sec}\frac{I}{2} - R_c$$

$$E_s = (190.99 + 1.395)\ \text{Sec}\frac{42°}{2} - 190.99$$

$$E_s = \mathbf{15.08\ m.}$$

592. Problem:

The radius of the interior curve of a spiral easement curve is 190 m. The central angle of the interior curve is 42° and its external distance is 15.98 m.

① Compute the length of throw of the spiral curve.
② Compute the length of the spiral.
③ Compute the max. velocity that a car could pass thru the easement curve.

Solution:

① *Length of throw:*

$$E_s = (R_c + P)\ \text{Sec}\frac{I}{2} - R_c$$

$$15.98 = (190 + P)\ \text{Sec}\frac{42°}{2} - 190$$

$$190 + P = 192.30$$

$$P = \mathbf{2.30\ m.}$$

② *Length of spiral:*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$\frac{X_c}{4} = P$$

$$X_c = 4\,(2.3)$$

$$X_c = 9.2$$

$$9.2 = \frac{L_c^2}{6\,(190)}$$

$$L_c = \mathbf{102.41\ m.}$$

③ *Max. speed:*

$$L_c = \frac{0.036\,K^3}{R_c}$$

$$102.41 = \frac{0.036\,K^3}{190}$$

$$K = \mathbf{81.46\ kph}$$

593. Problem:

The central curve of the spiral easement curve has a 5° curve with an angle of intersection of the tangents equal to 40°. If the length of the spiral curve is 80 m. long.

① Compute the offset distance at the S.C. of the spiral easement curve.
② Compute the length of throw.
③ Compute the external distance of the central curve.

Solution:

① *Offset distance at S.C.*

$$R_c = \frac{1145.916}{5^\circ}$$

$$R_c = 229.18\text{ m.}$$

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\,(229.18)}$$

$$X_c = \mathbf{4.65}$$

② *Length of throw:*

$$P = \frac{X_c}{4}$$

$$P = \frac{4.65}{4}$$

$$P = \mathbf{1.16}$$

③ *External distance:*

$$E_s = (R_c + P)\,\text{Sec}\,\frac{I}{2} - R_c$$

$$E_s = (229.18 + 1.16)\,\text{Sec}\,20^\circ - 229.18$$

$$E_s = \mathbf{15.94\ m.}$$

594. Problem:

The length of a spiral is 80 m. with a radius of the central curve equal to 200 m.

① Compute the spiral angle at the end point (S.C.)
② Compute the offset distance at the end point (S.C.)
③ Compute the length of throw.

Solution:

① *Spiral angle at S.C.*

$$S_c = \frac{L_c\,(180)}{2\,\pi\,R}$$

$$S_c = \frac{80\,(180)}{2\,\pi\,(200)}$$

$$S_c = \mathbf{11.46^\circ}$$

② *Offset distance at end point (S.C.)*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X_c = \frac{(80)^2}{6\,(200)}$$

$$X_c = \mathbf{5.33}$$

③ *Length of throw:*

$$P = \frac{X_c}{4}$$

$$P = \frac{5.33}{4}$$

$$P = \mathbf{1.33}$$

595. Problem:

The radius of the central curve of spiral easement curve is equal to 360 m. The design speed passing through the curve is 100 kph.

① Compute the length of the spiral curve if centripetal acceleration is limited to $0.60\ m/sec^2$.
② Compute the offset distance at the S.C.
③ Compute the length of throw of the spiral.

Solution:

① *Length of spiral:*

$$L_c = \frac{0.0215\,V^3}{RC}$$

$$L_c = \frac{0.0215\,(100)^3}{360\,(0.6)}$$

$$L_c = \mathbf{99.54\ m.}$$

② *Offset distance at S.C.*

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$X_c = \frac{(99.54)^2}{6\,(360)}$$

$$X_c = \mathbf{4.59}$$

③ *Length of throw:*

$$P = \frac{1}{4}X_c$$

$$P = \frac{4.59}{4}$$

$$P = \mathbf{1.15}$$

596. Problem:

In a spiral easement curve the length of throw of a spiral curve is 1.333 m. It has a radius of its central curve of 200 m.

① Compute the length of spiral.
② Compute the max. velocity in kph that a car could pass thru the spiral easement curve.
③ Compute the spiral angle at the S.C.

Solution:

① *Length of spiral:*

$$P = \frac{X_c}{4}$$

$$X_c = 4\,(1.333)$$

$$X_c = 5.332$$

$$X_c = \frac{L_c^2}{6\,R_c}$$

$$5.332 = \frac{L_c^2}{6\,(200)}$$

$$L_c = \mathbf{80\ m.}$$

② *Max. velocity:*

$$L_c = \frac{0.036\,V^3}{R_c}$$

$$80 = \frac{0.036\,V^3}{200}$$

$$V = \mathbf{76.31\ kph}$$

③ *Spiral angle at S.C.*

$$S_c = \frac{L_c\,180}{2\,\pi\,R}$$

$$S_c = \frac{80\,(180)}{2\,\pi\,(200)}$$

$$S_c = \mathbf{11.46^\circ}$$

597. Problem:

The centripetal acceleration of an easement curve having a central curve of radius 340 m. is equal to 0.79 m/sec^2. If the design speed of the curve is 100 kph.

① Determine the length of the spiral.
② Compute the length of throw.
③ Compute the external distance of the spiral curve if the central curve has a central angle of 40°.

Solution:

① *Length of spiral:*

$$L_c = \frac{0.0215\ V^3}{R\,C}$$

$$L_c = \frac{0.0215\ (100)^3}{340\ (0.79)}$$

$$L_c = \mathbf{80.04\ m.}$$

② *Length of throw:*

$$X_c = \frac{L_c^{\ 2}}{6\ R_c}$$

$$X_c = \frac{(80.04)^2}{6\ (340)}$$

$$X_c = 3.14$$

$$P = \frac{1}{4} X_c$$

$$P = \frac{3.14}{4}$$

$$P = \mathbf{0.785\ m.}$$

③ *External distance:*

$$E_s = (\,R_c + P)\ \text{Sec}\ \frac{I}{2} - R_c$$

$$E_s = (340 + 0.785)\ \text{Sec}\ 20° - 340$$

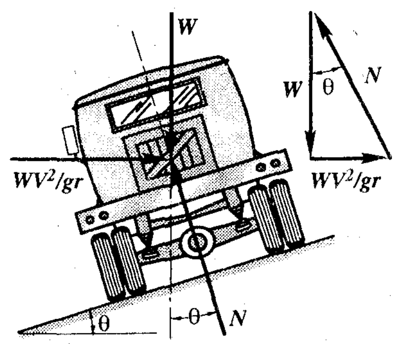
$$E_s = \mathbf{22.66\ m.}$$

Embankment of Curve

598. Problem:

A highway curve having a radius of 400 ft. is banked so that there will be no lateral pressure on the car's wheel at a speed of 48 kph. What is the angle of elevation of the embankment ?

Solution:



$$r = \frac{400}{3.281}$$

$$R = 121.95\ \text{m.}$$

$$V = 48\ \text{kph}$$

$$V = \frac{48000}{3600}$$

$$V = 13.33\ \text{m/s}$$

$$\tan ø = \frac{WV^2}{gr\,W}$$

$$\tan ø = \frac{V^2}{gr}$$

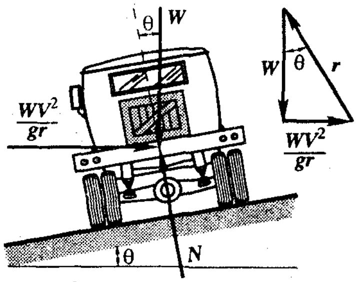
$$\tan ø = \frac{(\,13.33)^2}{9.81\,(\,121.95)}$$

$$ø = \mathbf{8.45°}$$

599. CE Board Nov. 1999

A highway curve has a radius of 122 m. find the angle of super elevation in degrees so that there will be no lateral pressure between the tires and the roadway at a speed of 30 mph.

Solution:



$$\tan\theta = \frac{WV^2}{grW}$$

$$\tan\theta = \frac{V^2}{g}$$

$$V = \frac{30\,(5280)}{3.28\,(3600)}$$

$V = 13.71$ m/s

$$\tan\theta = \frac{(13.71)^2}{9.81\,(122)}$$

$\theta = \boldsymbol{8.92°}$

600. Problem:

A highway curve is super elevated at 7°. Find the radius of the curve if there is no lateral pressure on the wheels of a car at a speed of 40 mph.

Solution:

$$\tan\theta = \frac{V^2}{gr}$$

$$V = \frac{40\,(5280)}{3.28\,(3600)}$$

$V = 17.89$ m/s

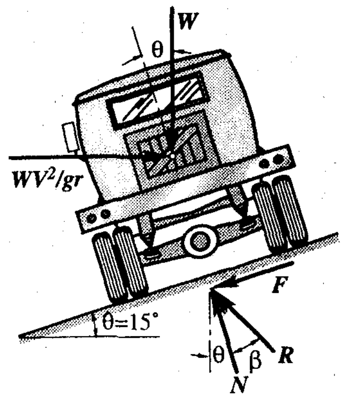
$$\tan 7° = \frac{(17.89)^2}{9.81\,r}$$

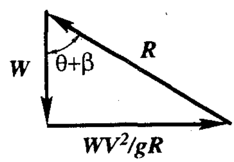
$r = \boldsymbol{265.6\ m}$

601. Problem:

A car runs on a 15° banked track on a curve having a radius of 120 m. The coefficient of friction between the tires and the track is 0.3. Determine the max. speed in kph at which the car can run without skidding.

Solution:





$$\tan(\theta + ß) = \frac{WV^2}{gr\,W}$$

$$\tan(\theta + ß) = \frac{V^2}{gr}$$

$\tan ß = 0.30$

$ß = 16.7°$

$$\tan(15 + 16.7) = \frac{V^2}{9.81\,(120)}$$

$V = 26.96$ m/s

$$V = \frac{26.96\,(3600)}{1000}$$

$V = \boldsymbol{97\ kph}$

602. Problem:

The super elevation of a highway curve is 6°. At what max. speed can a car ran on it such that there is no lateral pressure on the wheels. The radius of the curve is 150 m.

Solution:

$$\tan\theta = \frac{V^2}{gr}$$

$$\tan 6° = \frac{V^2}{9.81\,(150)}$$

$V = 12.44$ m/s

$$V = \frac{12.44\,(3600)}{1000}$$

$V = 44.77$ kph

$$V = \frac{44.77\,(1000)\,(3.28)}{5280}$$

V = 27.81 mph

603. CE Board Nov. 2005

The super elevation of a highway curve is 6°. At what max. speed can a car ran on it such that there is no lateral pressure on the wheels. The radius of the curve is 150 m.

Solution:

$$\tan\theta = \frac{V^2}{gr}$$

$$\tan 6° = \frac{V^2}{9.81\,(150)}$$

$V = 12.44$ m/s

$$V = \frac{12.44\,(3600)}{1000}$$

$V = 44.77$ kph

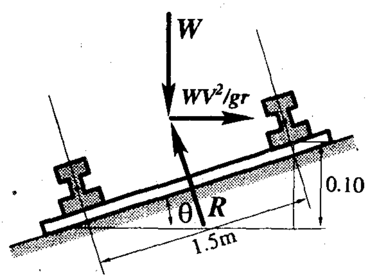
$$V = \frac{44.77\,(1000)\,(3.28)}{5280}$$

V = 27.81 mph

604. Problem:

The rails of a railway curve are 1.5 apart. The outer rail is elevated at 10 cm. higher than the inner rail. There is no lateral pressure on wheels at a max. speed of 45 mph. Determine the radius of the railway curve.

Solution:



$$\tan\theta = \frac{V^2}{gr}$$

Sin θ = tan θ for small angles

$$V = \frac{45\,(5280)}{3.28\,(3600)}$$

$V = 20.12$ m/s

$$\frac{0.10}{1.5} = \frac{(20.12)^2}{9.81\,r}$$

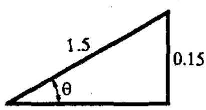
r = ***619.10 m.***

605. Problem:

A railway curve has a radius of 420 m. The rails are 1.5 m. apart and the outer rail is elevated at 15 cm. At what max. speed will there be no lateral pressure on the wheels in mph.

Solution:

For small angle $\tan\theta = \sin\theta$



$$\tan\theta = \frac{V^2}{gr}$$

$$\frac{0.15}{1.5} = \frac{V^2}{(9.81)(420)}$$

$$V = 20.30 \text{ m/s}$$

$$V = \frac{20.30\,(3.28)}{5280}(3600)$$

$$V = \mathbf{45.39\ mph}$$

606. Problem:

At a max. speed of 50 mph, there is no lateral pressure on the wheels of a train. What is the angle of super elevation of the outer rail if the radius of the railway curve is 287.5 m.

Solution:

$$\tan\theta = \frac{V^2}{gr}$$

$$V = \frac{50\,(5280)}{3.28\,(3600)}$$

$$V = 22.36 \text{ m/s}$$

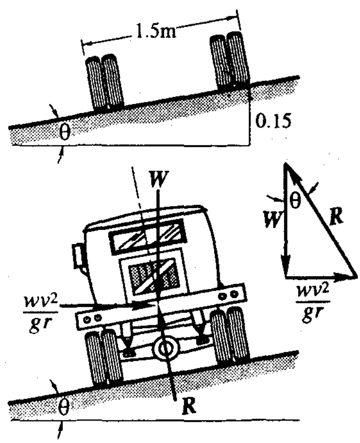
$$\tan\theta = \frac{(22.36)^2}{9.81\,(287.5)}$$

$$\theta = \mathbf{10.05^\circ}$$

607. CE Board Nov. 2001

A railway curve having a radius of 420 m. has a distance between their rails of 1.5 m. and the outer rail is 15 cm. higher than the inner rail. Find the max. speed that a train could move along this curve so that there will be no lateral pressure on the wheels and the rails.

Solution:



For small angles $\sin\theta = \tan\theta$

$$\sin\theta = \frac{0.15}{1.5}$$

$$\tan\theta = \frac{0.15}{1.5}$$

$$\tan\theta = \frac{W\,V^2}{gr\,W}$$

$$\tan\theta = \frac{V^2}{gr}$$

$$\frac{0.15}{1.5} = \frac{V^2}{9.81(420)}$$

$$V = 20.30 \text{ m/s}$$

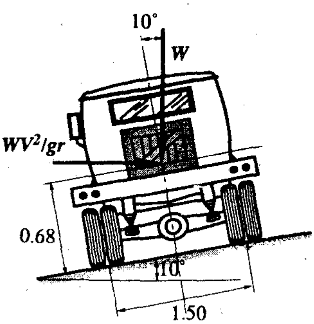
$$V = \frac{20.30(3600)}{1000}$$

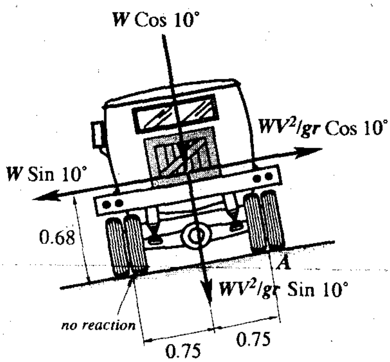
$$V = \mathbf{73.70\ kph}$$

608. Problem:

If friction is great enough to prevent skidding, a vehicle would impend to overturn at a speed of "V" kph on a highway curve having a radius of 150 m. The vehicles center of gravity is 0.68 m. above the road and its tread is 1.5 m. Determine the value of "V" if the angle of super elevation is 10°.

Solution:





$\Sigma M_A = 0$

$W \text{Sin } 10° (0.68) + W \text{Cos } 10° (0.75)$

$+ \dfrac{W V^2 \text{ Sin } 10°}{g r} (0.75) = \dfrac{W V^2 \text{ Cos } 10°}{g r} (0.68)$

$0.118 + 0.739 + 0.0000885\ V^2 = 0.000455\ V^2$

$V = 48.35$ m/s

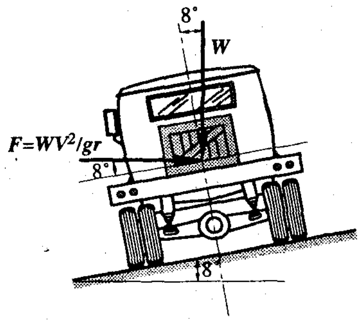
$V = \dfrac{48.35}{1000}(3600)$

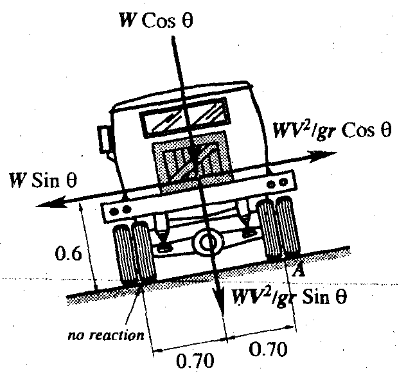
$V =$ ***174 kph***

609. Problem:

The angle of super elevation of a highway curve is 8°. A vehicle impends to overturn on this curve. When traveling at 120 kph. Its center of gravity is 0.6 m. above the road and its tread is 1.4 m. Find the radius of curvature of the road. Assume no skidding.

Solution:





$\Sigma M_A = 0$

$W \text{Cos } \theta\ (0.70) + \dfrac{W V^2}{g r} \text{Sin } \theta\ (0.70)$

$+ W \text{Sin } \theta\ (0.6) = \dfrac{W V^2}{g r} (0.60) \text{ Cos } \theta$

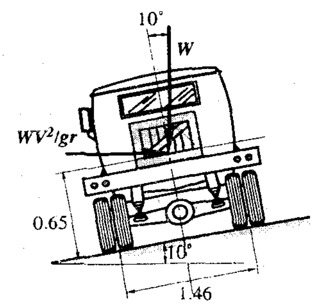
$V = \dfrac{120\ (1000)}{3600}$

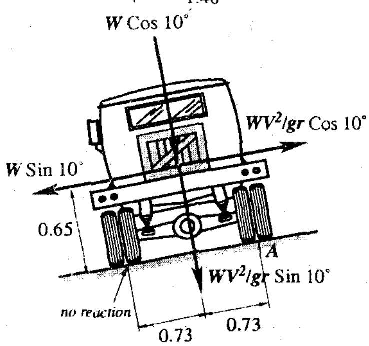
$V =$ ***33.33 m/s***

610. Problem:

A vehicle impends to overturn on a super elevation highway curve at a speed of 60 mph. The height of its center of gravity is 0.65 m. and its tread is 1.46 m. The super elevation is 10°. Compute the radius of the curve assuming no skidding occurs.

Solution:





$$V = \frac{60\,(52.80)}{3.28\,(3600)} = 26.83 \text{ m/s}$$

$\Sigma M_A = 0$

$W \text{Sin } 10°\,(0.65) + W \text{Cos } 10°\,(0.73)$

$$+ \frac{WV^2}{gr} \text{Sin } 10°\,(0.73) = \frac{WV^2}{gr} \text{Cos } 10°\,(0.65)$$

$$0.113 + 0.719 + \frac{(26.83)^2 \text{ Sin } 10°\,(0.73)}{9.81\,r}$$

$$= \frac{(26.83)^2 \text{ Cos } 10°\,(0.65)}{9.81\,r}$$

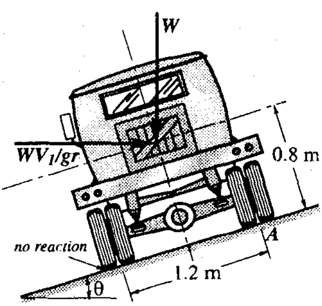
$$\frac{37.67}{r} = 0.832$$

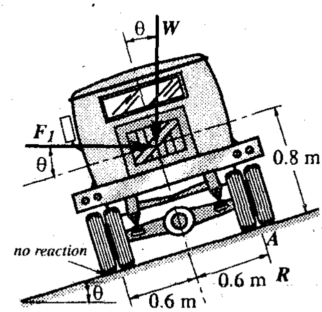
$r = \mathbf{45.28}$

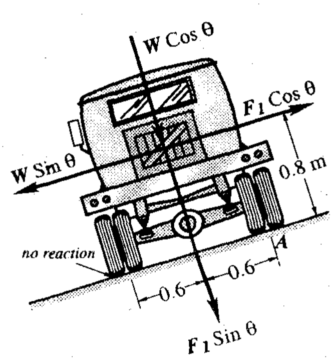
611. Problem:

A highway curve having a radius of 120 m. has an angle of embankment equal to 9.31°. A car has a weight of 15 kN and center of gravity is located 0.80 m. above the roadway. Distance between front wheels is 1. 2m. If friction is great enough to prevent skidding, at what speed would overturning impend ?

Solution:







$\Sigma MA = 0$

$F_1 \text{ Cos } ø\ (0.8) = W \text{ Sin } ø\ (0.8) + W \text{ Cos } ø\ (0.6) + F_1 \text{ Sin } ø\ (0.6)$

$F_1 \text{ Cos } 9.31^\circ\ (0.8) = 15000 \text{ Sin } 9.31^\circ\ (0.8) + 1500 \text{ Cos } 9.31^\circ\ (0.6) + F_1 \text{ Sin } 9.31^\circ\ (0.6)$

$0.79F_1 = 1941.31 + 8881.45 + 0.097F_1$

$0.693\ F_1 = 10822.76$

$F_1 = 15617$

$F_1 = 25$

$$F_1 = \frac{WV_1^2}{gr}$$

$$15617.26 = \frac{1500\ V_1^2}{9.81\ (120)}$$

$V_1 = 35.01$ m/sec.

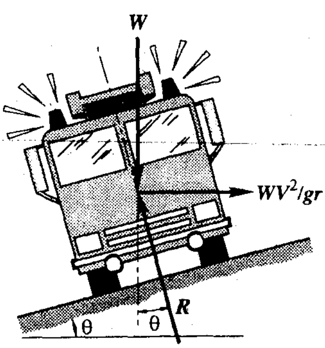
$$V_1 = \frac{35.02\ (3600)}{1000}$$

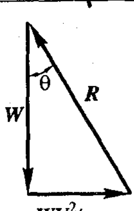
$V_1 =$ ***126 kph***

612. Problem:

A highway curve has a radius of 265 m. and an angle of elevation of embankment of 7°. Find the max. speed that the car could move in these curve so that there will be no lateral pressure between the tires and the roadway.

Solution:





$$\tan \theta = \frac{V^2}{g\,r}$$

$$\tan 7^\circ = \frac{V^2}{9.81(265)}$$

$V = 17.87$ m/s

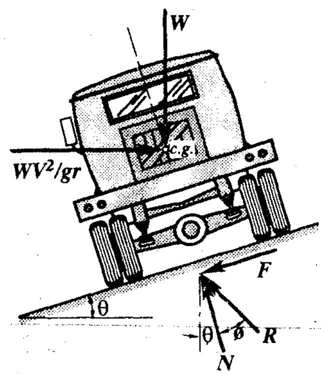
$$V = \frac{17.87(3.28)(3600)}{5280}$$

$V =$ ***39.96 mph***

613. Problem:

The rated speed of a highway curve of 100 m. radius is 65 kph. If the coefficient of friction between the tires and the road is 0.60, what is the maximum speed at which a car can round the curve without skidding?

Solution:



$$V = \frac{65\ (1000)}{3600}$$

$V = 81.06$ m/s.

$$\tan \theta = \frac{V^2}{gr}$$

$$\tan \theta = \frac{(18.06)^2}{9.81(100)}$$

$\theta = 18.4^\circ$

$\tan ø = 0.60$

$ø^\circ = 31^\circ$

$$\tan(\phi + \theta) = \frac{V^2}{gr}$$

$$\tan 49.4^\circ = \frac{V^2}{gr}$$

$$V^2 = 9.81\,(100)\tan 49.4^\circ$$

$$V = 33.83 \text{ m/s}$$

$$V = \frac{33.83}{1000}\,(3600)$$

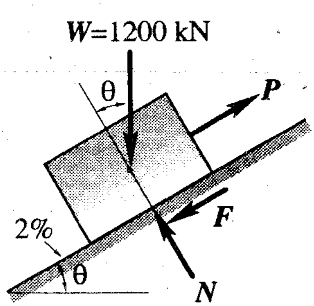
$$V = \mathbf{121.79\ kph.}$$

Train Resistance

614. Problem:

A power of 4000 kW is to pull a train up on a 2% grade. The train resistance is 5 N per kN. If the train weighs 12000 kN, determine its max. speed in kph.

Solution:



$$P = F + W \sin\theta$$

$$P = 0.005\,(12000) + 12000\,(0.02)$$

$$P = 300 \text{ kN}$$

$$Power = P\,V$$

$$4000 = 300\,V$$

$$V = 13.33 \text{ m/s}$$

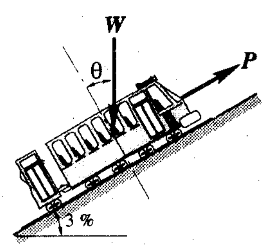
$$V = \frac{13.33\,(3600)}{1000}$$

$$V = \mathbf{48\ kph}$$

615. Problem:

A power of 3500 kW is to pull a train weighing 10000 kN up on a 3% grade. The train resistance is 4 N/kN. Determine the max. speed of the train in m/s.

Solution:



$$P = W \sin\theta + \frac{4}{1000}\,(3500)$$

$$P = 1000\,(0.03) + 0.004\,(10000)$$

$$P = 340 \text{ kN}$$

$$Power = PV$$

$$3500 = 340\,V$$

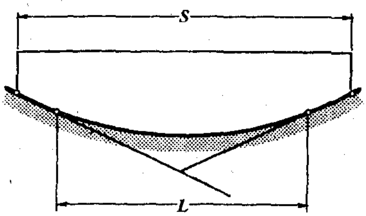
$$V = \mathbf{10.29\ m/s}$$

SIGHT DISTANCE

A SIGHT DISTANCE: For Sag Curve

Metric System:

S > L



$$L = 2S - \frac{(122 + 3.5S)}{A}$$

where:

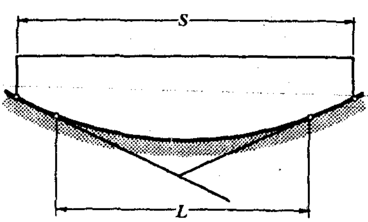
L = length of curve in meters
S = sight distance in meters
$A = g_1 - g_2$

$$L = \frac{V^2 A}{395}$$

V = velocity of car that could pass thru the curve in kph.

English System

S > L



$$L = 2S - \frac{(400 + 3.5S)}{A}$$

where:

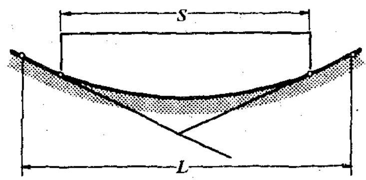
L = length of curve in feet
S = sight distance in feet
$A = g_1 - g_2$

$$L = \frac{V^2 A}{46.50}$$

V = velocity of car that could pass thru the curve in mph.

Metric System:

S < L



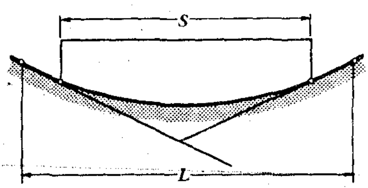
$$L = \frac{A S^2}{122 + 3.5S}$$

$$L = \frac{V^2 A}{395}$$

L = length of curve in meters
S = length of sight distance in meters
V = velocity of car that could pass thru the curve in kph.

English System

S < L



$$L = \frac{A S^2}{400 + 3.5S}$$

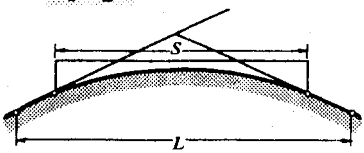
$$L = \frac{V^2 A}{46.50}$$

L = length of curve in feet
S = length of sight distance in feet
V = velocity of car that could pass thru the curve in mph.

B SIGHT DISTANCE: For Summit Curve

Metric or English System

S < L



$$L = \frac{AS^2}{100\left(\sqrt{2h_1} + \sqrt{2h_2}\right)^2}$$

where:

L = length of crest of vertical summit in (m) or ft.

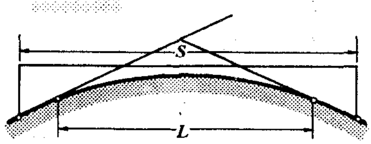
S = sight distance (m) or ft.

h_1 = height of eye of average driver above roadway (m) or ft.

h_2 = height of object sighted above roadway (m) or ft.

A = algebraic difference in grades in percent

S > L



$$L = 2S - \left[\frac{100\left(\sqrt{2h_1} + \sqrt{2h_2}\right)^2}{A}\right]$$

Metric or English System

S < L

$$L = \frac{AS^2}{1400}$$

S > L

$$L = 2S - \frac{1400}{A}$$

where:

L = length of curve in feet

S = stopping sight distance in feet

$A = g_1 - g_2$

616. Problem:

A vertical sag curve has tangent grades of - 1.5% and + 3.5%. Compute the following to have a minimum visibility of 89 m.

① Length of sight distance in meters.
② Length of curve in meters.
③ Max. velocity of a car that could pass thru the vertical sag curve in kph.

Solution:

① *Sight distance:*

$S = 2(89)$

$S = \mathbf{178\ m.}$

② *Length of curve:*

Assume: $S < L$

$$L = \frac{AS^2}{122 + 3.5S}$$

$A = 3.5 + 1.5$

$A = 5$

$$L = \frac{5(178)^2}{122 + 3.5(178)}$$

$L = \mathbf{212.64\ m.}$ *ok as assumed*

③ *Max. velocity of car:*

$$L = \frac{AV^2}{395}$$

$$212.64 = \frac{V^2(5)}{395}$$

$V = \mathbf{129.6\ kph}$

617. Problem:

A descending curve has a downward grade of - 1.4% and an upward grade of +3.6%. The length of curve is 220 m. long.

① Compute the length of minimum visibility of the curve in meters.
② Compute the max. design speed of the car passing thru the sag curve in kph.
③ What is the stationing of the lowest point of the curve if the P.C. is at station 12 + 120.60?

Solution:

① *Minimum visibility of curve:*
Assume: $S < L$

$$L = \frac{AS^2}{122 + 3.5S}$$
$$A = 3.6 - (-1.4)$$
$$A = 5$$

$$220 = \frac{5S^2}{122 + 3.5S}$$
$$26840 + 770S = 5S^2$$
$$S^2 - 154S - 5368 = 0$$
$$S = \frac{154 \pm 212.57}{2}$$
$$S = 183.29 \text{ m.}$$

$$\text{Min. visibility} = \frac{183.29}{2}$$
$$\text{Min. visibility} = \mathbf{91.64\ m.}$$

② *Max. design speed:*
$$L = \frac{AV^2}{395}$$
$$220 = \frac{5V^2}{395}$$
$$V = \mathbf{131.8\ kph}$$

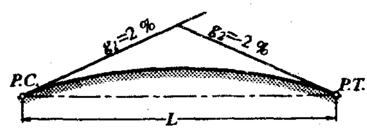
③ *Stationing of lowest point of curve:*
$$S = \frac{g_1 L}{g_1 - g_2}$$
$$S = \frac{-0.014(220)}{-0.014 - 0.036}$$
$$S = 61.60 \text{ m.}$$

$$\text{Stationing} = (12 + 120.60) + (61.60)$$
$$\text{Stationing} = \mathbf{12 + 182.20}$$

618. Problem:

50 mph is the design speed on a vertical summit curve connecting a +2% grade with a - 2% grade on a highway. Using a perception-reaction time of 2.5 sec. and a coeff. of friction of 0.30, determine the min. length of the vertical curve.

Solution:



$$V = \frac{50(5280)}{3.28(3600)}$$
$$V = 22.36 \text{ m/s}$$
$$S = V_t + \frac{V^2}{2g(f+G)}$$
$$S = 22.36(2.5) + \frac{(22.36)^2}{2(9.81)(0.3 + 0.02)}$$
$$S = 135.53 \text{ m.}$$
$$S = 444.54 \text{ ft.}$$

$$L = \frac{AS^2}{1400}$$
$$A = g_1 - g_2$$
$$A = 2 - (-2)$$
$$A = 4$$
$$L = \frac{4(444.54)^2}{1400}$$
$$L = 564.62 \text{ ft.}$$
$$L = \mathbf{172.14\ m.}$$

619. Problem:

The radius of the summit curve having a safe stopping distance of 130 m. is equal to 3558 m. The tangent grades of the summit curve is +2.6% and - 1.8%. If the height of the observer above the road surface is equal to 0.15.

① Compute the height of the object above the road surface that the observer could see at the other side of the curve.
② Compute the length of the summit curve.
③ Compute the height difference from the beginning of the curve to a point 30 m. horizontally from it.

Solution:

① *Height of object above the road surface:*

$$R = \frac{S^2}{2\,(\sqrt{h_1} + \sqrt{h_2})^2}$$

$$3558 = \frac{(130)^2}{2\,(\sqrt{1.5} + \sqrt{h_2})^2}$$

$$(\sqrt{1.5} + \sqrt{h_2})^2 = 2.375$$

$$\sqrt{1.5} + \sqrt{h_2} = 1.54$$

$$\sqrt{h_2} = 0.316$$

$$h_2 = \mathbf{0.10\ m.}$$

② *Length of summit curve:*

$$L = \frac{R_v\,(g_1 - g_2)}{100}$$

$$L = \frac{3558\,(2.6 + 1.8)}{100}$$

$$L = \mathbf{156.55\ m.}$$

③ *Height difference from the P.C. at a horizontal distance of 30 m. from P.C.*

$$y = g_1\,x + \frac{x^2}{2\,R}$$

$$y = 0.026\,(30) + \frac{(30)^2}{2(3558)}$$

$$y = \mathbf{0.654\ m.}$$

620. Problem:

The radius of the sag vertical curve is equal to 1532 m. The sag curve has tangent grades of - 2.8% and +2.2%.

① Compute the max. speed that a car could pass thru the sag curve.
② Compute the length of the sag vertical curve.
③ Compute the length of the sight distance.

Solution:

① *Speed of car:*

$$R = \frac{V^2}{6.5}$$

$$1532 = \frac{V^2}{6.5}$$

$$V = \mathbf{99.79\ kph}$$

② *Length of curve:*

$$L = \frac{A\,V^2}{395}$$

$$A = 2.2 + 2.8$$

$$A = 5$$

$$L = \frac{5\,(99.79)^2}{395}$$

$$L = \mathbf{126.05\ m.}$$

③ *Length of sight distance:*

$$L = \frac{A\,S^2}{122 + 3.5S}$$

$$126.05 = \frac{5\,S^2}{122 + 3.5S}$$

$$15378.10 + 441.175\,S = 5\,S^2$$

$$S^2 - 88.235\,S - 3075.62 = 0$$

$$S = \mathbf{114.98\ m.}$$

621. Problem:

A vertical summit curve has tangent grades of +5% and - 3.8%. The horizontal distance from the P.C. of the curve to the vertex of the summit curve is 113.64 m.

① Compute the length of the summit curve.
② Compute the radius of the summit curve.
③ Compute the tangent of the summit curve.

Solution:

① *Length of summit curve:*

$$S_1 = \frac{g_1\,L}{g_1 - g_2}$$

$$113.64 = \frac{0.05\,L}{0.05 + 0.038}$$

$$L = \mathbf{200\ m.}$$

② *Radius of summit curve:*

$$L = \frac{R_v\,(g_1 - g_2)}{100}$$

$$200 = \frac{R_v\,(5 + 3.8)}{100}$$

$$R_v = \mathbf{2272.73\ m.}$$

③ *Tangent length:*

$$T = \frac{R_v}{2}\frac{(g_1 - g_2)}{100}$$

$$T = \frac{2272.73\,(5 + 3.8)}{200}$$

$T =$ ***100 m.***

622. Problem:

The sight distance of a sag vertical curve is equal to 115 m. If the tangent grades of the curve are - 2% and +3%.

① Compute the length of the curve.
② Compute the design speed of the vertical sag curve.
③ Compute the minimum radius of the sag vertical curve.

Solution:

① *Length of curve:*

$$L = \frac{A\,S^2}{122 + 3.5S}$$

$A = g_2 - g_1$

$A = 3 - (-2)$

$A = 5$

$$L = \frac{5\,(115)^2}{122 + 3.5\,(115)}$$

$L =$ ***126.07 m.***

② *Design speed:*

$$L = \frac{A\,V^2}{395}$$

$$126.07 = \frac{5\,V^2}{395}$$

$V =$ ***99.80 kph***

③ *Min. radius of sag vertical curve:*

$$R_{min.} = \frac{V^2}{6.5}$$

$$R_{min.} = \frac{(99.80)^2}{6.5}$$

$R_{min.} =$ ***1532.31 m.***

623. Problem:

A vertical sag curve have tangent grades of - 1.8% and +3.2%. If the minimum radius of the sag curve is 1500 m. long

① Compute the design speed of the vertical sag curve.
② Compute the length of the vertical sag curve.
③ Compute the length of sight distance of the vertical sag curve.

Solution:

① *Design speed of vertical sag curve:*

$$R_{min.} = \frac{V^2}{6.5}$$

$$1500 = \frac{V^2}{6.5}$$

$V =$ ***98.74 kph***

② *Length of vertical sag curve:*

$$L = \frac{A\,V^2}{395}$$

$A = g_2 - g_1$

$A = 3.2 - (-1.8) = 5$

$$L = \frac{5\,(98.74)^2}{395} = \mathbf{123.42\ m.}$$

③ *Length of sight distance of vertical sag curve:*

$$L = \frac{A\,S^2}{122 + 3.5S}$$

$$123.42 = \frac{5\,S^2}{122 + 3.5S}$$

$15057.24 + 431.97\,S = 5\,S^2$

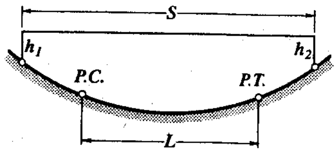
$S^2 - 86.394\,S - 3011.448 = 0$

$S =$ ***113.04 m.***

624. Problem:

Compute the max. velocity that a car could pass thru a sag parabolic curve having a sight distance of 500 ft. when the tangent grades are - 1.5% and +2.5%. Express in mph.

Solution:



Assume S > L

$$L = 2S - \frac{(400 + 3.5S)}{A}$$

$A = g_2 - g_1$

$A = 2.5 - (-1.5) = 4$

$$L = 2(500) - \frac{[400 + 3.5(500)]}{4}$$

$L = 462.5$ ft.

$S > L$ ok

$$L = \frac{V^2 A}{46.5} \text{ (Relation of L, V and A)}$$

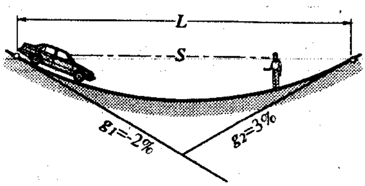
$$462.5 = \frac{V^2 (4)}{46.5}$$

$V = $ ***73.33 mph***

625. Problem:

A vertical sag parabolic curve has tangent grades of -2% and +3%. Compute the length of curve if it has a sight distance of 178 m.

Solution:



Assume S < L

$A = g_2 - g_1$

$A = 3 - (-2)$

$A = 5$

$$L = \frac{AS^2}{122 + 3.5\,S}$$

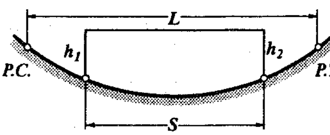
$$L = \frac{5\,(178)^2}{122 + 3.5\,(178)}$$

$L = $ ***212.64 m.***

626. Problem:

Compute the max. velocity of a car that could pass thru a vertical sag curve having tangent grades of - 1.5% and +3.5%, if it has a sight distance of 182.93 m.

Solution:



$$L = \frac{AS^2}{400 + 3.5S}$$

$A = 3.5 - (-1.5)$

$A = 5$

$S = 182.93\,(3.28)$

$S = 600$ ft.

$$L = \frac{5(600)^2}{400 + 3.5(600)}$$

$L = 720$ ft.

$$L = \frac{V^2 A}{46.50}$$

$$720 = \frac{V^2 (5)}{46.50}$$

$V = $ ***81.83 mph***

627. Problem:

Compute the capacity of a single lane in vehicles per hour if the speed of the car moving in the single lane is 50 kph. Length of car is 4.8 m. with a reaction time 0.8 sec.

Solution:

Spacing of cars = V t + L

$$S = \frac{50000}{3600}(0.8) + 4.8$$

$S = 15.91$ m.

$$\text{Capacity of single lane} = \frac{50000}{15.91}$$

Capacity of single lane = ***3142 vehicles/hour***

628. Problem:

Compute the length of a vertical summit curve having tangent grades of +3% and - 3% if the passing sight distance is 160 m.

Solution:

Assume: $S < L$

$$L = \frac{AS^2}{3000}$$
$$A = 3 - (-3)$$
$$A = 6$$

$$S = 160\,(3.28)$$
$$S = 524.8 \text{ ft.}$$

$$L = \frac{6\,(524.8)^2}{3000}$$
$$L = 550.8 \text{ ft.}$$
$$L = \mathbf{167.94\ m.}$$

629. Problem:

Find the passing sight distance of a 220 m. long vertical summit curve having tangent grades of +2% and - 3%.

Solution:

$$A = 2 - (-3)$$
$$A = 5$$

$S < L$

$$L = 220\,(3.28)$$
$$L = 721.6$$

$$L = \frac{AS^2}{3000}$$
$$L = \frac{5\,(721.6)^2}{3000}$$
$$L = 867.84 \text{ ft.}$$
$$L = \mathbf{264.59\ m.}$$

630. Problem:

A vertical curve connecting a +2% with a - 2% is 175 m. long. Compute the passing sight distance.

Solution:

$S > L$

$$L = 175\,(3.28)$$
$$L = 574 \text{ ft.}$$
$$A = 2 - (-2)$$
$$A = 4$$
$$L = 2\,S - \frac{3000}{A}$$
$$574 = 2\,S - \frac{3000}{4}$$
$$S = 662 \text{ ft.}$$
$$S = \mathbf{201.83\ m.}$$

631. Problem:

Determine the sight distance of a vertical parabolic sag curve 225 m. long and connecting a - 2% grade with a 3% grade.

Solution:

Assume: $S < L$

$$L = \frac{AS^2}{122 + 3.5S}$$
$$A = 3 - (-2)$$
$$A = 5$$

$$225 = \frac{5\,S^2}{122 + 3.5S}$$
$$27450 + 787.5\,S = 5\,S^2$$
$$S^2 - 157.5\,S - 5490 = 0$$
$$S = \mathbf{186.9\ m.}$$

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632. Problem:

A vertical parabolic sag curve connecting a - 1.7% grade with a +2.3% grade. Compute the max. speed in mph that a car could travel on this curve if the sight distance is 150 m.

Solution:

Assume: $S > L$

$$L = 2S - \frac{(122 + 3.5S)}{A}$$
$$A = 2.3 - (-1.7)$$
$$A = 4$$
$$L = 2(150) - \frac{[122 + 3.5(150)]}{4}$$
$$L = 138.25 \text{ m.} < S = 150$$
$$L = \frac{V^2 A}{395}$$
$$138.25 = \frac{V^2 (4)}{395}$$
$$V = 116.84 \text{ kph}$$
$$V = \frac{116.84(1000)(3.28)}{5280}$$
$$V = \mathbf{72.6\ mph}$$

633. Problem:

A vertical parabolic sag curve has tangent grades of - 2% and +3%. If the sight distance is 179.4 m. at what max. speed can a car pass thru the curve in mph.

Solution:

Assume: $S < L$

$$L = \frac{AS^2}{122 + 3.5S}$$
$$A = 3 - (-2)$$
$$A = 5$$
$$L = \frac{5(179.4)^2}{122 + 3.5(179.4)}$$
$$L = 214.59 \text{ m.}$$
$$L = \frac{V^2 A}{395}$$
$$214.59 = \frac{V^2 (5)}{395}$$
$$V = 130.20 \text{ kph}$$
$$V = \frac{130.20(1000)(3.28)}{5280}$$
$$V = \mathbf{80.9\ mph}$$

634. Problem:

A vertical summit curve connects with a +2% grade with a - 2% grade. If the stopping sight distance is 130 m. long, compute the length of the vertical summit curve.

Solution:

Assume: $S < L$

$$L = \frac{AS^2}{1400}$$
$$A = 2 - (-2) = 4$$
$$S = 130(3.28)$$
$$S = 426.4 \text{ ft.}$$
$$L = \frac{(426.4)^2 (4)}{1400}$$
$$L = 519.48 \text{ ft.} = \mathbf{158.4\ m.}$$

635. Problem:

A vertical summit curve connects a +2.2% with a - 1.8% grade. If the stopping distance is 100 m. Find the length of curve.

Solution:

$S > L$

$$L = 2S - \frac{1400}{A}$$
$$A = g_1 - g_2$$
$$A = 2.2 - (-1.8) = 4$$
$$S = 100(3.28)$$
$$S = 328 \text{ ft.}$$
$$L = 2(328) - \frac{1400}{4}$$
$$L = 306 \text{ ft.} = \mathit{93.3\ m.}$$

636. Problem:

The Land Transportation Office (LTO) requires that cars must switch on their head lights when traveling at night time to avoid accidents. Car specifications requires head light to have an angle of tilt of 1° above the longitudinal axis of the car and at a height of 0.90 m. above the pavement. If a car passes through a vertical sag parabolic curve 153 m. long having grade tangents of - 4.4% and +3.2% respectively.

① What max. speed could a car maneuver on this curve to prevent sliding if the coefficient of friction between tires and pavement is 0.15?
② Compute the max. head lamp sight distance to avoid collision along the curve.
③ If a driver approaching this curve sees an object a head of him, find the time taken from the instant the object is visible to the driver to the instant the brakes are applied effectively.

Solution:

① *Max. speed:*

$A = g_2 - g_1$

$A = 3.2 - (-4.4)$

$A = 7.6$

$L = \frac{AV^2}{395}$

$153 = \frac{7.6\,V^2}{395}$

$V = \mathbf{89.17\ kph}$

② *Max. head lamp sight distance:*

$S\theta + h = \frac{(g_2 - g_1)\,L}{2}$

$\theta = \frac{1\,(\pi)}{180}$

$\theta = 0.01745$

$S\,(0.01745) + 0.90 = \frac{(0.022 + 0.028)\,(153)}{2}$

$S = \mathbf{167.62\ m.}$

③ *Perception time:*

$V = \frac{89170}{3600}$

$V = 24.77$ m/s

$S = Vt + \frac{V^2}{2gf}$

$167.62 = 24.77\,t + \frac{(24.77)^2}{2\,(9.81)(0.15)}$

$t = \mathbf{1.65\ sec.}$

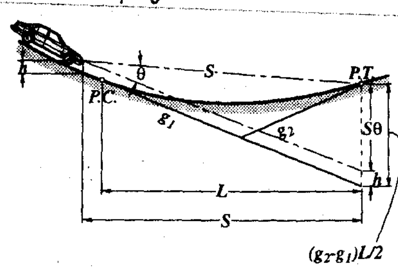
637. Problem:

A cargo truck approaches a sag parabolic curve at a speed of 100 kph. The length of the curve is 180 m. long with grade tangents of - 3% and +2% respectively. The intersection of the grade tangents is a 10 + 430 with an elevation of 240.60 m. The river has to switch on the beam lights at night time travel with the beam light making an angle of tilt of 0.85° above the longitudinal axis of the car. The drivers perception reaction time is 0.78 sec.

① Assuming a coeff. of friction of 0.18, compute the length of the head lamp sight distance.
② How high was the head lamp above the pavement at this instant?
③ What is the max. design speed that a car could maneuver on this curve?

Solution:

① *Head lamp sight distance:*



$S = Vt + \frac{V^2}{2gf}$

$V = \frac{100000}{3600}$

$V = 27.78$ m/s

$$S = 27.78\,(0.78) + \frac{(27.78)^2}{2\,(9.81)(0.18)}$$
$$S = \mathbf{240.19\ m.}$$

② *Height of head lamp:*

$$S\phi + h = \frac{(g_2 - g_1)\,L}{2}$$
$$\phi = \frac{0.85\,(\pi)}{180}$$
$$\phi = 0.0148$$
$$240.19\,(0.0148) + h = \frac{(0.02 + 0.03)(180)}{2}$$
$$h = \mathbf{0.945\ m.}$$

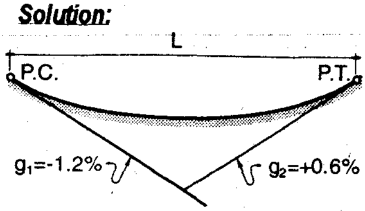
③ *Max. design speed:*

$$L = \frac{AV^2}{395}$$
$$A = 2 - (-3)$$
$$A = 5$$
$$180 = \frac{5\,V^2}{395}$$
$$V = \mathbf{119.2\ kph}$$

638. CE Board May 2000

A parabolic curve has a descending grade of -1.2% which meets an ascending grade of +0.60%. The allowable change of grade per 20 m. station is 0.18. Find the length of the curve.

Solution:



$$n = \frac{g_2 - g_1}{r}$$
$$n = \frac{0.6 - (-1.2)}{0.18}$$
$$n = 10 \text{ stations}$$

Length of curve = 10(20)
Length of curve = **200 m.**

PAVEMENTS

1. Rigid Pavement: (Olders Theory)

❶ **Without dowels or tie bars:**

The critical section is at the edge of a contraction joint, it will crack approximately 45° with the edges.

$$M = Wx$$
$$f = \frac{6M}{bd^2}$$
$$M = Wx$$
$$b = 2x$$
$$d = t$$
$$f = \frac{6Wx}{2x\,t^2}$$
$$t = \sqrt{\frac{3W}{f}}$$ *(thickness of pavement at edge and at center)*

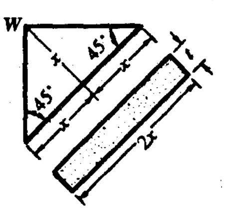
f = allowable tensile stress of concrete in
W = wheel load in lb. or kg.

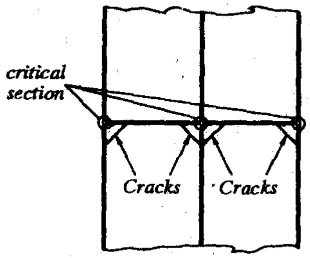
❷ **With dowels or tie bars:**

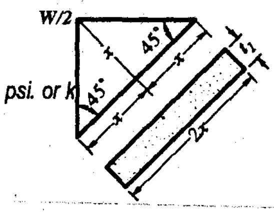
Purpose of dowel is to transmit the stresses due to the load from the adjacent pavement.

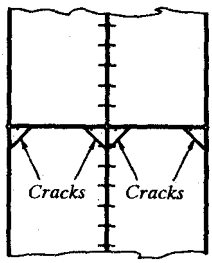
At the edge of pavement:

$$M = \frac{Wx}{2}$$
$$f = \frac{6\left(\frac{W}{2}\right)x}{2x\,t_1^{\,2}}$$
$$t_1 = \sqrt{\frac{3W}{2f}}$$









At the center of the pavement:

$$M = \frac{Wx}{4}$$

$$f = \frac{6\left(\frac{Wx}{4}\right)}{2x\, t_2^2}$$

$$t_2 = \sqrt{\frac{3W}{4f}} \quad \textit{(thickness at the center)}$$

2. Flexible Pavements:

$$A_2 = \frac{W}{f}$$

$$A_1 = \pi r^2$$

By ratio and proportion:

$$\frac{A_1}{r^2} = \frac{A_2}{(t+r)^2}$$

$$\frac{\pi r^2}{r^2} = \frac{\frac{W}{f}}{(t+r)^2}$$

$$(t + r^2) = \frac{1}{\pi}\,\frac{W}{f}$$

$$t + r = 0.564\sqrt{\frac{W}{f}}$$

$$t = 0.564\sqrt{\frac{W}{f}} - r$$

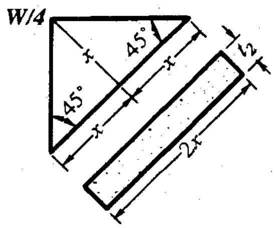
$$t_1 = 0.564\sqrt{\frac{W}{f_1}} - r$$

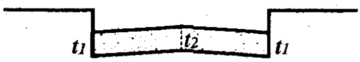
3. Thickness of pavement using Mcleods Method

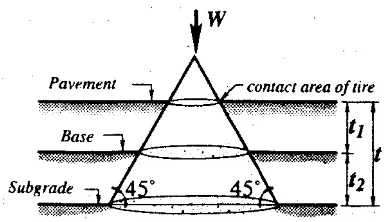
$$T = K \log_{10}\frac{P}{S}$$

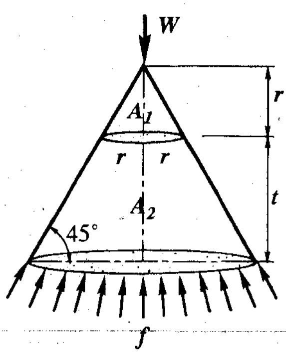
P = wheel load
S = subgrade pressure
K = constant value from table

TRANSPORTATION ENGINEERING









4. Thickness of pavement using U.S. Corps of Engineers

$$t = \sqrt{W}\left[\frac{1.75}{GBR} - \frac{1}{p\pi}\right]^{1/2}$$

t =thickness of pavement in cm.
W = wheel load in kg
CBR = California Bearing Ratio
p = tire pressure in kg/cm^2

5. Thickness by expansion pressure method:

$$t = \frac{\text{expansion pressure}}{\text{average pavement density}}$$

t = thickness of pavement

6. Stiffness factor of pavement:

$$S.F. = \left(\frac{E_B}{E_p}\right)^{1/3}$$

E_B = modulus of elasticity of subgrade
E_p = modulus of elasticity of pavement
S.F. = stiffness factor

7. Bulk specific gravity of a core of compacted asphalt concrete pavement

$$d = \frac{A}{D - E - \frac{(D - A)}{F}}$$

d = bulk sp.gr. of core
A = weight of dry specimen in air
D = weight of specimen plus paraffin coating in air
E = weight of specimen plus paraffin coating in water
F = bulk specific gravity of paraffin

8. Absolute specific gravity of composite aggregates

$$G = \frac{100}{\frac{P_c}{G_c} + \frac{P_f}{G_f}}$$

G = absolute sp.gr. of composite aggregates
P_c = percentages of course material by wt.
G_c = sp.gr. of course material
P_f = percentages of fine materials by weight
G_f = sp.gr. of fine material

9. Relation of porosity, absolute sp.gr. of composite aggregates, and bulk sp.gr.

$$n = \frac{G - d}{G}$$

n = porosity
G = absolute sp.gr.
d = bulk sp.gr.

10. Bulk sp.gr. of specimen

$$d = \frac{w_a}{w_a - w_w}$$

d = bulk sp.gr.
w_a = weight of specimen in air
w_w = weight of specimen in water

11. Percentage of Voids

$$V = \frac{(G - d)}{G} \times 100$$

V = percentage of voids
G = theoritical or absolute sp.gr.
d = bulk sp.gr.

12. Bulk sp.gr. of total aggregate

$$G_{sb} = \frac{P_1 + P_2}{\frac{P_1}{d_1} + \frac{P_2}{d_2}}$$

G_{sb} = bulk sp.gr. of total aggregate
P_1 = percentage of total weight of coarse aggregate
P_2 = percentage of total weight of fine aggregate
d_1 = bulk sp.gr. of coarse aggregate
d_2 = bulk sp.gr. of fine aggregate

13. Effective sp.gr. of aggregate

$$G_{se} = \frac{P_{mm} - P_b}{\frac{P_{mm}}{G_{mm}} - \frac{P_b}{G_b}}$$

G_{se} = effective sp.gr. of aggregate
P_{mm} = total loose mixture
G_{mm} = max. sp.gr. of paving mixture
P_b = asphalt (percentage by total weight)
G_b = sp.gr. of asphalt

14. Asphalt absorption

$$P_{ba} = 100\left(\frac{G_{se} - G_{sb}}{G_{sb}\,G_{se}}\right)G_b$$

P_{be} = asphalt absorption
G_{se} = effective sp.gr. of aggregate
G_{sb} = bulk sp.gr. of aggregate
G_b = sp.gr. of asphalt

15. Effective asphalt content

$$P_{be} = P_b - \frac{P_{ba}\,P_s}{100}$$

P_{be} = effective asphalt content
P_b = % weight of fine aggregates
P_s = sum of % weight of fine and coarse aggregates
P_{ba} = asphalt absorption

16. Air voids in a pavement mixture

$$V_A = \frac{(G_{mm} - G_{mb})}{G_{mm}}100$$

V_A = air voids
G_{mm} = max. sp.gr. of paving mixture
G_{mb} = bulk sp.gr. of compacted mix

17. Percentage of Voids in Mineral aggregates

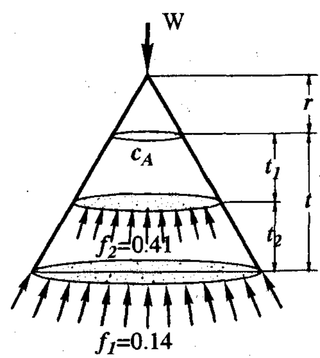
$$VMA = 100 - \frac{G_{mb}\,P_s}{G_{sb}}$$

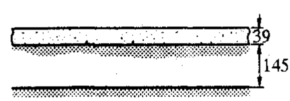
VMA = percentage of voids in mineral aggregates
G_{mb} = bulk sp.gr. of compacted mix
G_{sb} = bulk sp.gr. of aggregates
P_s = sum of % weight of fine and course aggregates

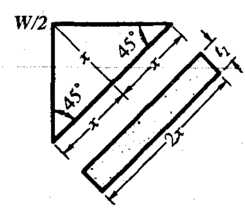
639. Problem:

A flexible pavement carries a static wheel load of 53.5 kN. The circular contact area of the tire is 85806 mm^2 and the transmitted load is distributed across a wide area of the subgrade at an angle of 45˚. The subgrade bearing value is 0.14 MPa, while that of the base is 0.41 MPa. Design the thickness of pavement and that of the base.

Solution:







Flexible Pavement:

$$A_1 = \frac{W}{f_1}$$

$$A = \pi r^2$$

$$\frac{A}{r^2} = \frac{A_1}{(t + r^2)}$$

$$\frac{\pi r^2}{r^2} = \frac{W/f_1}{(t + r)^2}$$

$$(t + r)^2 = 0.564\sqrt{\frac{W}{f_1}} - r$$

$$t = 0.564\sqrt{\frac{53500}{0.14}} - 165$$

$t = 184$ mm

$A = \pi r^2$

$85806 = \pi r^2$

$r = 165$ mm

$$t_1 = 0.564\sqrt{\frac{W}{f_2}} - r$$

$$t_1 = 0.564\sqrt{\frac{53500}{0.41}} - 165$$

$t_1 =$ ***39 mm*** *(thickness of pavement)*

$t_2 = t - t_1$

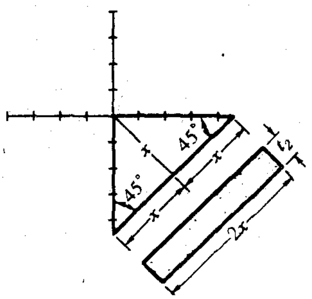
$t_2 = 184 - 39$

$t_2' =$ ***145 mm*** *(thickness of base)*

640. Problem:

A rigid pavement is to be used to carry a wheel load of 53.5 kN, design the thickness of the pavement. The allowable tensile stress of concrete is 1.38 MPa. Sufficient dowels are used across the joints.

Solution:



At the edge: (with dowels)

$$M = \frac{W}{2}x$$

$$f = \frac{6M}{bd^2}$$

$$f = \frac{6\left(\frac{W}{2}\right)x}{2x\, t_1^2}$$

$$t_1 = \sqrt{\frac{3W}{2f}}$$

$$t_1 = \sqrt{\frac{3\,(53500)}{2(1.38)}}$$

$t_1 =$ *241 mm*

At the center:

$$M = \frac{W}{4}x$$

$$f = \frac{6M}{bd^2}$$

$$f = \frac{6\,(W/4)x}{2xt_2^2}$$

$$t_2 = \sqrt{\frac{3W}{4f}}$$

$$t_2 = \sqrt{\frac{3(53500)}{4(1.38)}}$$

$t =$ ***171 mm*** *(at the center)*

641. Problem:

Determine the thickness of a rigid pavement of the proposed Nagtahan road to carry a max. wheel load of 60 kN. Neglect effect of dowels. $fc' = 20$ MPa. Allowable tensile stress of concrete pavement is $0.06\,fc'$.

Solution:

$$t = \sqrt{\frac{2W}{f}}$$

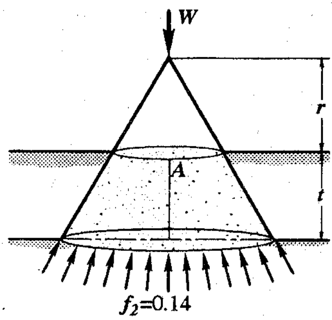
$$t = \sqrt{\frac{3(60000)}{0.60(20)}}$$

$t =$ ***387.3 mm***

642. Problem:

A 53.5 KN wheel load has a max. tire pressure of 0.62 MPa. This pressure is to be uniformly distributed over the area of tire contact on the roadway. Assuming the subgrade pressure is not to exceed 0.14 MPa, determine the required thickness of flexible pavement structure, according to the principle of the cone pressure distribution.

Solution:



$$t = 0.564\sqrt{\frac{w}{p}} - r$$

$$A = \frac{w}{p}$$

$$A = \frac{53\,500}{0.62}$$

$$A = 86\,290\ \text{mm}^2$$

$$\pi r^2 = 86\,290$$

$$r = 165.73$$

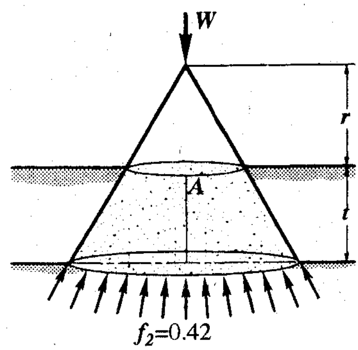
$$t = 0.564\sqrt{\frac{53\,500}{0.14}} - 165.73$$

t = ***182.92 mm*** *(thickness of pavement)*

643. Problem:

A flexible pavement having a thickness of 46 mm carries a static wheel load of "W". The circular contact are of time has an equivalent radius of 150 mm. If the load "W" is assumed to be transmitted across a wide area of subgrade at an angle of 45°, compute the value of the wheel load "W" if the bearing stress of the base is 0.42 MPa.

Solution:



$$t = 0.564\sqrt{\frac{W}{f}} - r$$

$$46 = 0.564\sqrt{\frac{W}{0.42}} - 150$$

W = 50723 N

W = ***50.72 kN***

644. Problem:

Determine the pavement thickness in cm using an expansion pressure of 0.15 kg/cm² and a pavement density of 0.0025 kg/cm³. Use the expansion pressure method.

Solution:

$$t = \frac{\text{expansion pressure}}{\text{average pavement density}}$$

$$t = \frac{0.15}{0.0025}$$

t = ***60 cm.***

645. Problem:

Determine the thickness of the different types of pavement using the following data:

① Rigid pavement with a wheel load capacity of 54 kN if the allowable tensile stress of concrete is 1.6 MPa. Neglect the effect of dowels.

② Flexible pavement with a wheel load of 54 kN wirh an allowable bearing pressure on the base of the pavement equal to 0.15 MPa using the principle of cone distributation where the load assumed to be transmitted across a wide area of subgrade at an angle of 45° and that the equivalent radius of the contact area of the tires is equal to 165 mm.

③ Pavement subjected to an expansion pressure of 0.50 kg/cm² with an average pavement density of 0.05 kg/cm³. Express in mm.

Solution:

① *Rigid pavement:*

$$t = \sqrt{\frac{3\,W}{f}}$$

$$t = \sqrt{\frac{3\,(54000)}{1.6}}$$

$t = 318$ *mm*

② *Flexible pavement:*

$$t = 0.564\sqrt{\frac{W}{f}} - r$$

$$t = 0.564\sqrt{\frac{54000}{0.15}} - 165$$

$t = 173.4$ *mm*

③ *Expansion pressure method:*

$$t = \frac{\text{expansion pressure}}{\text{Ave. density of pavement}}$$

$$t = \frac{0.50}{0.05}$$

$t = 10$ cm.

$t = 100$ *mm.*

646. Problem:

A cement concrete pavement has a thickness of 18 cm. and has two lanes of 7 meters with a longitudinal joint. Design the spacing of the tie bar and the length of bar.

Allowable working stress is steel in tension = 1600 kg/cm²

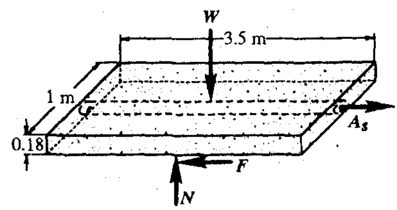
Unit weight of concrete = 2400 kg/cu.m

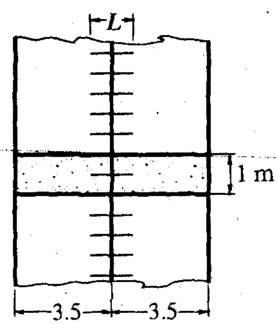
Coefficient of friction between pavement and subgrade = 1.5

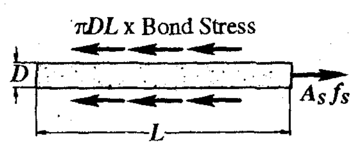
Allowable bond stress in concrete = 24 kg/cm²

Use 16 mm diam. steel bars.

Solution:







Consider one meter length of slab.

$W = 0.18\,(3.5)(1)(2400)$

$W = 1512$ kg = N

$F = \mu N$

$F = 1.5(1512)$

$F = 2268$ kg

$As\,fs = F$

$As\,(1600) = 2268$

$As = 1.42$ sq.cm/meter

$As = \frac{\pi}{4}(1.6)^2$

$As = 1.256$ sq.cm.

$$Spacing = \frac{1256}{1.42}$$

Spacing = 0.88 m *Use 80 cm on centers*

Note: The length of bar must be at least twice the computed value.

Length of bars:

$As\,fs = (\pi DL)$ *(Bondstress)*

$1.42\,(1600) = \pi\,(1.6)(24)L$

$L = 18.83$ say 19 mm

Use $L = 2(19)$

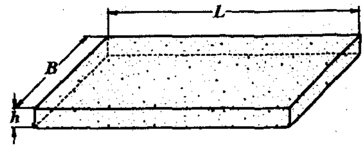
$L =$ ***38 cm***

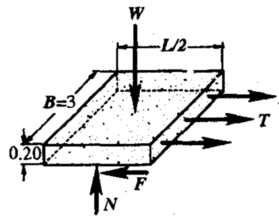
647. Problem:

Determine the spacing between contraction joints for a 3.0 m. slab having a thickness of 20 cm. Coefficient of friction between concrete and subgrade is 1.5 and unit wt. of concrete 2400 kg/cu.m. Allowable tensile stress of concrete is 0.8 kg/cm^2 and that of steel is 800 kg/cm^2. Unit weight of steel is 7500 kg/cu.m steel bars having a diameter of 1.6 cm. Total reinforcement is 4 kg/m^2 and is equally distributed in both directions. For plain cement concrete (without dowels).

$b = 3$ m $\qquad b = 300$ cm.

Solution:





Consider only half of the section (Using Principles of Mechanics)

$$W = \left(\frac{L}{2}\right)\frac{(20)}{100}(3)(2400)$$

$W = 720\,L$ kg.

$N = 729L$ kg.

$F = \mu N$

$F = 1.5(720L)$

$F = 1080L$ kg.

$T = 300(20)(0.8)$

$T = 4800$ kg.

$T = F$

$4800 = 1080L$

$L = 4.44$ m.

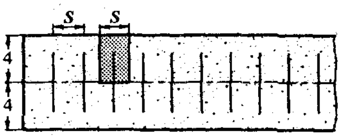
$$L = \frac{2f \times 10^4}{uD} = \frac{2(0.8)(10)^4}{1.5(2400)}$$

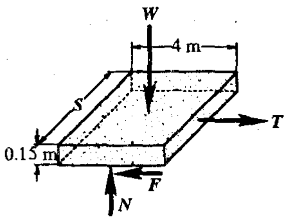
$L =$ ***4.44 m.***

648. Problem:

A concrete pavement 8m wide and 150 mm thick is to be provided with a center longitudinal joint using 12 mm ø bars. The unit weight of concrete is 2,400 kg/m^3. Coefficient of friction of the slab on the subgrade is 2.0. Assuming an allowable working stress in tension for steel bars at 138 MPa, determine the spacing of the longitudinal bars in mm.

Solution:





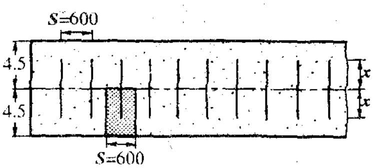
$W = 0.15\,(4)\,(S)\,(2400)\,(9.81) = 14\,126.4\,S$
$T = fs\,As$
$T = 138\,\frac{\pi}{4}\,(12)^2$
$T = 15\,600$ N
$N = W = 14\,126.4\,S$

$F = \mu N$
$F = 2\,(14\,126.4)\,S = 28\,252.8\,S$
$T = F$
$15\,600 = 28\,252.8\,S$
$S = 0.552$ m
$S =$ ***552 mm***

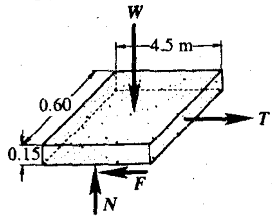
649. Problem:

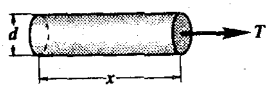
A 12 mm ø bars is used as the longitudinal bars of a concrete pavement. It is spaced at 600 mm on centers. The width of roadway is meters and the coefficient of friction of the slab on the subgrade is 2.0. Thickness of slab is 150 mm. If the allowable bond stress is 0.83 MPa, determine the length of to longitudinal bars.

Solution:



Solution:





$W = 0.60\,(0.15)\,(4.5)\,(2\,400)\,9.81$
$W = 9\,535.32$
$W = N = 9\,535.32$

$F = \mu N$
$F = 2\,(9\,535.32)$
$T = F$
$T = 19\,070.64$
$T = \pi dx\,U$
$19\,070.64 = \pi\,(12)\,x\,(.83)$
$x = 609.5$ mm
$2x =$ ***1 219 mm*** *(length of bars)*

650. Problem:

The width of expansion joint gap is 24 mm in a cement concrete pavement. If the laying temperature is 12° C and the maximum slab temperature is 50°C, calculate the spacing between the expansion joints. Assume coefficient of thermal expansion of concrete to be 9.5 x 10^{-6} per C°. The expansion joint gap should be twice the allowable expansion in concrete.

Solution:

Expansion in concrete $= \frac{24}{2}$
Expansion in concrete = 12 mm
Expansion in concrete = 0.012
$\Delta = 0.012$ m.
$\Delta = KL\,(T_2 - T_1)$
$0.012 = 9.5 \times 10^{-6}\,(50 - 12)\,L$
$L =$ ***33.24 m.***
(Spacing between expansion joints)

651. Problem:

Compute the modulus of subgrade reaction if a force of 5000 lb. is applied under a circular plate having a radius of 9 in. produces a deflection of 0.12 inch under the plate.

Solution:

$F = SA$

$5000 = S(\pi)(9)^2$

$S = 19.65$ psi

$$\text{Modulus subgrade reaction} = \frac{\text{Stress}}{\text{Deflection}}$$

$$\text{Modulus subgrade reaction} = \frac{19.65}{0.12}$$

Modulus of subgrade reaction = ***163.75 psi***

652. Problem:

① Upon completion of grading operations a subgrade was tested for bearing capacity by loading on large bearing plates. Compute the modulus of subgrade reaction if a force of 5000 lb. is applied under a circular plate having a radius of 9 inches produces a deflection of 0.12 inch under the plate.

② The soil sample was obtained from the project site after completion of grading operations and the CBR test was conducted at field density. The sample with the same surcharged imposed upon it is then subjected to a penetration test by a piston plunger 5 cm. in diameter at a certain speed. The CBR value of a standard crushed rock for 2.5 mm penetration is 78.68 kg/cm². Compute the CBR of the soil sample when subjected to a load of 58 kg it produces a penetration of 2.5 mm.

③ From the result of the Proctor Compaction test after the completion of grading operations it indicates that the materials compacted on the roadway will have a void ratio of 0.52. The undisturbed sample of the material taken from the borrow pit has a void ratio 0.72. What shrinkage factor should be used in computing borrow and embankment quantities.

Solution:

① *Modulus of subgrade reaction:*

$F = SA$

$5000 = S(\pi)(9)^2$

$S = 19.65$ psi

$$\text{Modulus of subgrade reaction} = \frac{\text{Stress}}{\text{Deflection}}$$

$$\text{Modulus of subgrade reaction} = \frac{19.65}{0.12}$$

Modulus of subgrade reaction = ***163.75 psi***

② *CBR of soil sample:*

$$\text{Stress} = \frac{P}{A}$$

$$\text{Stress} = \frac{58}{\frac{\pi}{4}(5)^2}$$

$\text{Stress} = 2.95$ kg/cm²

$$CBR = \frac{2.95}{78.68}(100)$$

$CBR = \mathbf{3.75\%}$

③ *Shrinkage Factor:*

$$S.F. = \frac{(e_1 - e_2)\,100}{(1 + e_1)}$$

$$S.F. = \frac{(0.72 - 0.52)(100)}{(1 + 0.72)}$$

$S.F. = \mathbf{11.63\%}$

653. Problem:

The soil sample was obtained from the project site and the CBR test was conducted at field density. The sample with the same subgrade imposed upon it is then subjected to a penetration test by a piston plunger 5 cm. diam. moving at a certain speed. The CBR value of a standard crushed rock for 2.5 mm penetration is 70.45 kg/cm^2. Compute the CBR of the soil sample when subjected to a load of 55.33 kg it produces a penetration of 2.5 mm.

Solution:

$$Stress = \frac{P}{A}$$

$$Stress = \frac{55.33}{\frac{\pi}{4}(5)^2}$$

$$Stress = 2.82 \text{ kg/cm}^2$$

$$CBR = \frac{2.82}{70.45} \times 100$$

$$CBR = \mathbf{4\%}$$

654. Problem:

The standard CBR value of a standard crushed rock for 5 mm penetration is 105.68 kg/cm^2. Compute the CBR of the soil sample whose result is as follows:

Load applied = 75.2 kg
Penetration = 5 mm
Dia. of piston plunger = 4 cm.

Solution:

$$Stress = \frac{P}{A}$$

$$Stress = \frac{75.2}{\frac{\pi}{4}(4)^2}$$

$$Stress = 5.98 \text{ kg/cm}^2$$

$$CBR = \frac{5.98}{105.68} \times 100$$

$$CBR = \mathbf{5.62\%}$$

655. Problem:

Compute the CBR of a soil sample, if the sample is subjected to a load on a piston plunger 2 inches in diam. and produces a penetration of 0.10 inch. The CBR value of a standard crushed rock for a 0.10 inch penetration is 1000 psi and for the 0.20 inch penetration is 1500 psi. The load applied is 1600 lb.

Solution:

$$Stress = \frac{P}{A}$$

$$Stress = \frac{1600}{\frac{\pi}{4}(2)^2}$$

$$Stress = 50.9\%$$

$$CBR = \frac{509.30}{1000} \times 100$$

$$CBR = \mathbf{50.9\%}$$

656. Problem:

The CBR value of a standard crushed rock for a 0.20 inch penetration is equal to 1500 psi. A soil sample was tested by applying a load of a piston plunger which produces a penetration of 0.20 inch if the diameter of the piston plunger is equal to 1.98 inches. If the computed CBR of the soil sample is 47%, compute the load applied to the plunger.

Solution:

$$CBR = \frac{Stress}{1500} \times 100$$

$$47 = \frac{Stress\,(100)}{1500}$$

$$Stress = 705 \text{ psi.}$$

$$Stress = \frac{P}{A}$$

$$Stress = \frac{P}{\frac{\pi}{4}(1.98)^2}$$

$$P = \mathbf{2171\ lb.}$$

657. Problem:

The computed CBR of a soil sample which was tested by applying a load of 75.3 kg on a piston plunger, which penetrates 5 mm is equal to 3.62%. The CBR value of a standard crushed rock for a 5 mm penetration is 105.68 kg/cm^2. Compute the diam. of the piston plunger.

Solution:

$$CBR = \frac{Stress}{105.68} \times 100$$

$$3.62 = \frac{Stress\ (100)}{105.68}$$

$$Stress = 3.83 \text{ kg/cm}^2$$

$$Stress = \frac{P}{A}$$

$$3.83 = \frac{75.3}{\frac{\pi}{4} D^2}$$

$$D = \mathbf{5\ cm.}$$

658. CE Board May 2003

Compute the modulus of elasticity of the sub-grade if the modulus of elasticity of the pavement is 120 MPa with a stiffness factor of 0.50.

Solution:

$$Stiffness\ factor = \left(\frac{E_s}{E_p}\right)^{1/3}$$

$$0.5 = \left(\frac{E_s}{120}\right)^{1/3}$$

$$0.125 = \frac{E_s}{120}$$

$$E_s = \mathbf{15\ MPa}$$

659. Problem:

A sheet asphalt mixture is to be made in the batching plant of the Olongapo City Engr. office using the following percentages by weight of total mix.

Materials	Percent
Sand (sp.gr. = 2.68)	80%
Filler (sp.gr. = 2.70)	12%
Asphalt Cement (sp.gr. = 1.01)	8 %

A compacted test specimen weighing 1140 grams in air was found to weigh 645 grams when suspended in water.

① Compute the absolute specific gravity of the bituminous mixture.
② Compute the bulk specific gravity of the compacted specimen.
③ Compute the porosity of the compacted specimen.

Solution:

① *Absolute sp.gr. of the bituminous mixture:*

$$G = \frac{100}{\frac{P_s}{G_s} + \frac{P_f}{G_f} + \frac{P_a}{G_a}}$$

$$G = \frac{100}{\frac{80}{2.68} + \frac{12}{2.70} + \frac{8}{1.01}}$$

$$G = \mathbf{2.368}$$

② *Bulk specific gravity:*

$$d = \frac{W_a}{W_a - W_w}$$

$$d = \frac{1140}{1140 - 645}$$

$$d = \mathbf{2.303}$$

③ *Porosity:*

$$Porosity = \frac{(G - d)\ 100}{G}$$

$$Porosity = \frac{(2.368 - 2.303)(100)}{2.368}$$

$$Porosity = \mathbf{2.74\%}$$

660. Problem:

A sheet asphalt mixture is made using the following percentages by weight of the total mix.

Materials	Sp.gr.	Percent by weight
Sand	2.60	78
Filler	2.70	14
Asphalt Cement	1.02	8

① Compute the absolute sp.gr. of the compacted specimen.
② If the test specimen weighs 1130 gr. in air and was found to weigh 635 gr when suspended in water, compute the bulk sp.gr. of the compacted specimen.
③ Compute the porosity of the compacted specimen.

Solution:

① *Absolute sp.gr.*

$$G = \frac{100}{\frac{P_s}{G_s} + \frac{P_f}{G_f} + \frac{P_a}{G_a}}$$

$$G = \frac{100}{\frac{78}{2.60} + \frac{14}{2.70} + \frac{8}{1.02}}$$

$$G = \mathbf{2.324}$$

② *Bulk specific gravity:*

$$d = \frac{W_a}{W_a - W_w}$$

$$d = \frac{1130}{1130 - 635}$$

$$d = \mathbf{2.283}$$

③ *Porosity:*

$$Porosity = \frac{(G - d)(100)}{G}$$

$$Porosity = \frac{(2.324 - 2.283)(100)}{2.324}$$

$$Porosity = \mathbf{1.76\%}$$

661. Problem:

A plant mix is to be made using the foll. percentages by weight of the total mix.

Sand (sp.gr. = 2.68) 79%
Filler (sp.gr. = 2.70) 14%
Asphalt Cement (sp.gr. = 1.01) 7%
Wt. of the compacted specimen in air = 1265.5 gr
Wt. of the compacted specimen when suspended in water = 720 gr.

① Determine the absolute sp.gr. of the total compacted specimen.
② Determine the bulk sp.gr. of the compacted specimen.
③ Determine the porosity of the compacted specimen.

Solution:

① *Absolute sp.gr.*

$$G = \frac{100}{\frac{P_s}{G_s} + \frac{P_f}{G_f} + \frac{P_a}{G_a}}$$

$$G = \frac{100}{\frac{79}{2.68} + \frac{14}{2.70} + \frac{7}{1.01}}$$

$$G = \mathbf{2.404}$$

② *Bulk specific gravity:*

$$d = \frac{W_a}{W_a - W_w}$$

$$d = \frac{1265.5}{1265.5 - 720}$$

$$d = \mathbf{2.32}$$

③ *Porosity:*

$$Porosity = \frac{(G - d)\,100}{G}$$

$$Porosity = \frac{(2.404 - 2.32)(100)}{2.404}$$

$$Porosity = \mathbf{3.49\%}$$

662. Problem:

A core or compacted asphalt concrete pavement was tested for specific gravity. The following weights were obtained.

Weight of dry specimen in air = 2007.5 grams
Weight of specimen plus paraffin coating in air = 2036.5 grams
Weight of specimen plus paraffin coating in water = 1135.0 grams
Bulk specific gravity of the paraffin = 0.903

Solution:

$$d = \frac{A}{D - E - \frac{(D - A)}{F}}$$

d = bulk specific gravity of the core
A = weight of dry specimen in air
D = weight of specimen plus paraffin coating in air
E = weight of specimen plus paraffin coating in air
F = bulk specific gravity of paraffin

$$d = \frac{2007.5}{(2036.5 - 1135) - \frac{(2036.5 - 2007.5)}{0.903}}$$

$d = 2.309$
$d = \mathbf{2.31}$ *(bulk specific gravity of core)*

663. Problem:

During a working day of 8 hr. a particular hot plant produces enough asphalt concrete to lay 11500 m^2 of wearing course 75 mm thick, compacted. Mix proportions, by weight, sp.gr. are as follows:

Materials	Specific gravity	Percent by Weight
Asphalt cement	1.02	6
Limestone dust	2.75	8
Sand	2.66	41
Crushed stone	2.77	45

Solution:

$$G = \frac{100}{\frac{P_a}{G_a} + \frac{P_d}{G_d} + \frac{P_s}{G_s} + \frac{P_c}{G_c}}$$

$$G = \frac{100}{\frac{6}{1.02} + \frac{8}{2.75} + \frac{41}{2.66} + \frac{45}{2.77}}$$

$G = 2.47$

$W = V \times D$
$W = 11500\,(0.075)(2.47)(9.81)$
$W = 20898.98$ kN
(required wt. of surfacing)

No. of batches required $= \frac{20898.98}{90}$

No. of batches required = ***232.21 batches***

664. Problem:

The dry mass of a sample of aggregates is 1980 g. The mass in a saturated dry condition is 2000 g. The volume of aggregates excluding the volume of absorbed water is 730 cm^3.

① Compute the apparent specific gravity of the sample aggregates.
② Calculate the percentage absorption.
③ Calculate the bulk sp.gr. of the sample aggregates.

Solution:

① *Apparent sp.gr.:*
$W = V \times D$
$1980 = 730\,(1)\,G_s$
$G_s = \mathbf{2.71}$

② *Percentage absorption:*

$$\%\ absorption = \frac{(2000 - 1980)}{1980} \times 100$$

$\%\ absorption = 0.01 = \mathbf{1\%}$

③ *Bulk specific gravity:*
Wt. of water = 2000 - 1980
Wt. of water = 20 g

$Vol.\ of\ absorption\ water = \frac{20}{1} = 20\ cm^3$

$Bulk\ vol. = 20 + 730 = 750\ cm^3$

$W = Vol. \times D \times G_B$
$1980 = 750\ (1)\ G_B$
$G_B = \mathbf{2.64}$

665. Problem:

The following data of a particular asphalt concrete mixture. Compute the percentage of voids in the laboratory molded specimen.

Materials	Specific gravity	Percent by Weight
Limestone dust	2.80	17.0
Sand	2.60	73.0
Asphalt cement	1.02	10

A cylindrical specimen of the mixture was molded in the laboratory and weighted in air and water as follows.
Weight of dry specimen in air = 110 grams
Weight of saturated surface - dry specimen in air = 114 grams
Weight of saturated specimen in water = 60 grams

Solution:

$$G = \frac{100}{\frac{P_d}{G_d} + \frac{P_s}{G_s} + \frac{P_a}{G_a}}$$

$$G = \frac{100}{\frac{17}{2.80} + \frac{73}{2.60} + \frac{10}{1.02}}$$

$G = 2.28$

$$d = \frac{A}{B - C}$$

$$d = \frac{100}{114 - 60}$$

$d = 2.04$ *(bulk sp.gr.)*

$$V = \frac{(G - d)}{G} \times 100$$

$$V = \frac{(2.28 - 2.04)}{2.28} 100$$

$V = \mathbf{10.53\%}$ *(percentage of voids in the laboratory molded specimen)*

666. Problem:

From the given data shown in the table for a mix design for asphalt concrete.

Materials	Bulk sp.gr.	Sp.gr.	% of weight
Asphalt cement		1.03	5.3
Fine aggregates	2.689		47.3
Coarse aggregates	2.716		47.4

Max. specific gravity of paving mixture
$G_{mm} = 2.535$
Bulk specific gravity of compacted mixture
$G_{mb} = 2.442$
Compute the effective specific gravity of aggregate

Solution:

$$G = \frac{P_{mm} - P_b}{\frac{P_{mm}}{G_{mm}} - \frac{P_b}{G_b}}$$

P_{mm} = total loose mixture
G_{mm} = max. sp.gr. of paving mixture
G_b = sp.gr. of asphalt
P_b = asphalt *(percentage by total wt. of mixture)*

$$G_{se} = \frac{100 - 5.3}{\frac{100}{2.535} - \frac{5.3}{1.03}}$$

$G_{se} = \mathbf{2.761}$

667. Problem:

A tabulation shown are materials and its properties which are used in a compacted paving mixture:

Materials	Sp.gr.	Bulk sp.gr.	% of weight
Coarse aggregate		2.69	46.2
Fine aggregate		2.72	46.0
Asphalt cement	1.2		7.8

Compute the asphalt absorption of the aggregate expressed as percentage by weight of aggregate. Max. sp.gr. of paving mixture $G_{mm} = 2.54$.

Solution:

$$P_{ba} = 100\ \frac{(G_{se} - G_{sb})G_b}{G_{sb}\,G_{se}}$$

G_{se} = *effective sp.gr. of aggregate*
G_{sb} = *bulk sp.gr. of aggregate*
G_b = *sp.gr. of asphalt*
P_{ba} = *absorbed asphalt*

$$G_{se} = \frac{P_{mm} - P_b}{\frac{P_{mm}}{G_{mm}} - \frac{P_b}{G_b}}$$

$$G_{se} = \frac{100 - 7.8}{\frac{100}{2.54} - \frac{7.8}{1.02}}$$

$$G_{se} = 2.906$$

$$G_{sb} = \frac{P_1 + P_2}{\frac{P_1}{G_1} + \frac{P_2}{G_2}}$$

$$G_{sb} = \frac{46.2 + 46.0}{\frac{46.2}{2.69} + \frac{46}{2.72}}$$

$$G_{sb} = 2.705$$

$$P_{ba} = 100\ \frac{(2.906 - 2.705)}{2.705\,(2.906)}(1.02)$$

$$P_{ba} = \mathbf{2.61}$$

668. Problem:

Compute the effective asphalt content of a paving mixture which are as follows:

Materials	Bulk sp.gr.	Sp.gr.	% of weight
Asphalt cement		1.03	5.3
Fine aggregates	2.689		47.3
Coarse aggregates	2.716		47.4

Max. sp.gr. of paving mixture
G_{mm} = 2.535

Solution:

$$G_{sb} = \frac{P_1 + P_2}{\frac{P_1}{G_1} + \frac{P_2}{G_2}}$$

$$G_{sb} = \frac{47.3 + 47.4}{\frac{47.3}{2.689} + \frac{47.4}{2.716}}$$

G_{sb} = 2.702 *(bulk sp.gr. of aggregate)*

$$G_{se} = \frac{P_{mm} - P_b}{\frac{P_{mm}}{G_{mm}} - \frac{P_b}{G_b}}$$

$$G_{se} = \frac{100 - 5.3}{\frac{100}{2.535} - \frac{5.3}{1.03}}$$

G_{se} = 2.761 *(effective sp.gr. of aggregate)*

Asphalt absorption of the aggregate:

$$P_{ba} = 100\ \frac{(G_{se} - G_{sb})}{G_{sb}\,G_{se}}\,G_b$$

$$P_{ba} = 100\ \frac{(2.761 - 2.702)}{2.761\,(2.702)}(1.03)$$

P_{ba} = 0.81% *by wt. of aggregate*

Effective asphalt content:

$$P_{be} = P_b - \frac{P_{ba}\,P_s}{100}$$

$$P_s = 47.3 + 47.4$$

$$P_s = 94.7$$

$$P_{be} = 5.3 - \frac{0.81(94.7)}{100}$$

P_{be} = **4.53** *(effective asphalt content)*

669. Problem:

The following ingredients are used in the preparation of an asphalt concrete paving mixture.

Materials	Percentage of Total Mix by Weight	Specific Gravity
Asphalt cement	7.0	1.030
Mineral filler	7.0	3.100
Fine aggregate	30.0	2.690(bulk sp.gr
Coarse aggregate	56	2.611(bulk sp.gr

Max. specific gravity of the paving mixture
G_{mm} = 2.478
Bulk specific gravity of the compacted paving mixture sample G_{mb} = 2.384
Compute the percentage of voids in the compacted mineral aggregates.

Solution:

$$G_{sb} = \frac{P_1 + P_2 + P_3}{\frac{P_1}{g_1} + \frac{P_2}{g_2} + \frac{P_3}{g_3}}$$

$$G_{sb} = \frac{56 + 30 + 7}{\frac{56}{2.611} + \frac{30}{2.690} + \frac{7}{3.10}}$$

$$G_{sb} = 2.668$$

VMA = % of voids in the mineral aggregates

$$VMA = 100 - \frac{G_{mb} P_s}{G_{sb}}$$

$$P_s = 30 + 56$$

$$P_s = 86$$

$$VMA = 100 - \frac{2.384\,(86)}{2.668}$$

$VMA =$ ***23.154%*** *of voids in the mineral aggregates*

670. Problem:

Compute the percentage of voids filled with asphalt if the maximum sp.gr. of paving mixture G_{mm} = 2.535. Bulk sp.gr. of compacted mix G_{mb} = 2.442. Percentage weight of asphalt cement is 5.3 while that of fine aggregates and coarse aggregates are 47.3 and 47.4 respectively. The bulk of sp.gr. of aggregates G_{sb} = 2.703 and the effective specific gravity of aggregate G_{se} = 2.761.

Solution:

$$\text{Air voids } (V_A) = \frac{(G_{mm} - G_{mb})}{G_{mm}} 100$$

$$V_A = \frac{(2.535 - 2.442)}{2.535} 100$$

$V_A = 3.67$ *(air voids)*

Percentage of voids in the mineral aggregate:

$$VMA = 100 - \frac{G_{mb} P_s}{G_{sb}}$$

$$VMA = 100 - \frac{2.442\,(47.3 + 47.4)}{2.703}$$

$$VMA = 14.44$$

Percentage of voids filled with asphalt:

$$VFA = \frac{100\,(VMA - V_a)}{VMA}$$

$$VFA = \frac{100\,(14.44 - 3.6)}{14.44}$$

$VFA =$ ***74.58***

671. Problem:

The proportions by weight and specific gravities of each of the constituents of a particular sheet asphalt paving mixture are as follows:

Materials	Specific Gravity	Percent by Weight
Asphalt cement	1.04	10.0
Limestone dust	2.82	16.5
Sand	2.66	73.5

A cylindrical specimen of the mixture was molded in the laboratory and weighted in air and in water with the following results.
Weight of dry specimen in air = 111.95 grams
Weight of saturated, surface-dry specimen in air = 112.09 grams
Weight of saturated specimen in water = 61.20
Compute the bulk specific gravity of the compacted specimen.

Solution:

$$\text{Bulk sp.gr.} = \frac{A}{B - C}$$

A = weight of dry specimen in air
A = 111.95 grams
B = weight of saturated, surface-dry specimen in air
B = 112.09 grams
C = weight of saturated specimen in water
C = 61.20 grams

$$d = \frac{A}{B - C}$$

$$d = \frac{111.95}{112.09 - 61.20}$$

$d =$ ***2.20*** *(bulk sp.gr. of compacted specimen)*

672. Problem:

The dry mass of a sample of aggregates is 1206 grams. The mass is a saturated dry condition is 1226.8 grams. The volume of the aggregates, excluding the volume of absorbed water is 440.6 cm^3. Calculate the bulk specific gravity of the sample of aggregates.

Solution:

Wt. of water = 1226.8 - 1206

Wt. of water = 20.8 kg

$$\text{Vol. of absorbed water} = \frac{\text{Vol. of water}}{\text{Density of water}}$$

$$\text{Vol. of absorbed water} = \frac{20.8}{1}$$

Vol. of absorbed water = 20.8 cm^3

Bulk volume = 20.8 + 440.6

Bulk volume = 461.40 cm^3

Bulk specific gravity : G_B

$$G_B = \frac{M_D}{V_B\,W}$$

V_B = total volume of aggregates including vol. of absorbed water

$$G_B = \frac{1206}{461.40(1)}$$

G_B = **2.614**

673. Problem:

A turnout has a frog number of 9 with a length of heel spread equal to 336.11 mm.

① Compute the length of heel.

② If the length of the toe is equal to 1820 mm, compute the total length of the turnout.

③ Compute the angle subtended by the heel spread.

Solution:

① Length of heel:

$$\text{Frog no.} = \frac{\text{length of heel}}{\text{heel spread}}$$

$$9 = \frac{\text{length of heel}}{336.11}$$

length of heel = **3025 mm**

② Total length of turnout:

= 3025 + 1820

= **4845 mm**

③ Angle subtended by heel spread:

$$\text{Frog no.} = \frac{1}{2}\text{Cot}\frac{\theta}{2}$$

$$9 = \frac{1}{2}\text{Cot}\frac{\theta}{2}$$

$$18 = \text{Cot}\frac{\theta}{2}$$

$$\frac{\theta}{2} = 3.18^\circ$$

θ = **6°22'**

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A turnout has a length of heel equal to 3025 mm and a heel spread of 336.11. If the length of the toe is 1925 mm, compute the frog number of the turnout.

Solution:

$$\text{Frog no.} = \frac{\text{Heel length}}{\text{Heel spread}}$$

$$\text{Frog no.} = \frac{3025}{336.11}$$

Frog no. = **9**

675. Problem:

Fatal crashes are those that result in at least one death, while crashes that result in injuries but no deaths are classified as personal injury but crashes that result neither death nor injuries but involved damage to property are classified as property damage. This method of summarizing crashes is commonly used to make comparisons at different locations by assigning a weight scale to each crash based on its severity. A typical weighing scale have been used which is as follows.

Fatality = 12
Personal injury = 3
Property damage only = 1

If one fatal crash, 3 personal injuries and 5 property damage crashes occurred during a year at a particular site, compute its severity number.

Solution:

Severity number = 12 (1) + 3 (3) + 1 (5)
Severity number = **26**

676. Problem:

The density of traffic in a certain observation point on a highway was recorded to be 30 vehicles per km. If the space mean speed of the vehicle is 50 kph, how many vehicles will be passing every 30 seconds.

Solution:

$$K = \frac{q}{\mu_s}$$

$$30 = \frac{q}{50}$$

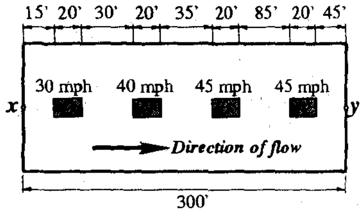
q = 1500 vehicles/hr.

$$\textit{No. of vehicles per sec} = \frac{1500\,(30)}{3600}$$

No. of vehicles per sec = ***12.5 vehicles***

677. Problem:

The figure shows vehicles traveling at constant speeds on two lane highway between sections x and y with their position and speeds obtained at an instant of time by photography. An observer located at point x observes 4 vehicles passing through point x during a period of T seconds. The velocities of the vehicles as measured are 45, 45, 40 and 30 mph respectively.



① Compute the density of traffic.
② Compute the time mean speed.
③ Compute the space mean speed.

Solution:

① *Density of traffic:*

$$K = \frac{4}{\frac{300}{5280}}$$

K = ***70.4 vehicles per mile***

② *Time mean speed:*

$$\mu_t = \frac{30 + 40 + 45 + 45}{4}$$

μ_t = ***40 mph***

③ *Space mean speed:*

$$\mu_s = \frac{n\,d}{\Sigma t}$$

$$\Sigma t = \frac{300}{\frac{30\,(5280)}{3600}} + \frac{300}{\frac{40\,(5280)}{3600}} + \frac{300}{\frac{45\,(5280)}{3600}} + \frac{300}{\frac{45\,(5280)}{3600}}$$

Σt = 6.82 + 5.11 + 4.55 + 4.55
Σt = 21.03 sec.

$$\mu_s = \frac{4\,(300)}{21.03} = 57.06 \text{ fps.}$$

$$\mu_s = \frac{57.06\,(3600)}{5280}$$

μ_s = ***38.9 mph***

MISCELLANEOUS

678. Problem:

Two sets of students are collecting traffic data at two sections x and y of a highway 600 ft. apart. Observation at x show that five vehicles passed that section at intervals of 8.18, 9.09, 10.23, 11.68 and 13.64 sec. respectively. If the speeds of the vehicles were 50, 45, 40, 35 and 30 mph respectively.

① Compute the density of traffic on the highway.
② Compute the time mean speed.
③ Compute the space mean speed.

Solution:

① *Density of traffic:*

$$K = \frac{5}{\frac{600}{5280}}$$

K = ***44 vehicles per mile***

② *Time mean speed:*

$$\mu_t = \frac{50 + 45 + 40 + 35 + 30}{5}$$

μ_t = ***40 mph***

③ *Space mean speed:*

$$\mu_s = \frac{n\,d}{\Sigma t}$$

$$\mu_s = \frac{5\,(600)}{8.18 + 9.09 + 10.23 + 11.68 + 13.64}$$

$\mu_s = 56.8$ fps.

$$\mu_s = \frac{56.8\,(3600)}{5280}$$

μ_s = ***38.7 mph***

679. Problem:

A traffic engineer urgently needs to determine the AADT on a rural primary road that has the volume distribution characteristics shown in the given table. She collected the data shown below on a Tuesday during the month of May.

Hour	Volume of Traffic	Hourly Expansion Factor (HEF
7:00 - 8:00 AM	400	29
8:00 - 9:00 AM	535	22.05
9:00 - 10:00 AM	650	18.80
10:00 - 11:00 AM	710	17.10
11:00 - 12:00 noon	650	18.52

DEF (daily expansion factor) for Tuesday = 7.727

MEF (monthly expansion factor) for May = 1.394

① Compute the 24 hr volume of traffic for Tuesday.
② Compute the average volume of traffic for the week.
③ Compute the average annual daily traffic for the month of May.

Solution:

① *24 hr. volume of traffic for Tuesday:*

24 hr Vol.

$$= \frac{400(29)+535(22.05)+650(18.80)+710(17.10)+650(18.52)}{5}$$

= ***11,959***

② *Seven-day Volume of traffic:*

7 day Vol. of traffic = 11959 (7.727)

7 day Vol. of traffic = ***92407.19***

③ *Average annual daily traffic:*

AADT = (MEF) (ADT)

$$AADT = (1.394)\,\frac{92407.19}{7}$$

AADT = ***18402***

680. Problem:

The radius of the summit curve having a safe stopping distance of 130 m. is equal to 3558 m. The tangent grades of the summit curve is +2.6% and - 1.8%. If the height of the observer above the road surface is equal to 0.15.

① Compute the height of the object above the road surface that the observer could see at the other side of the curve.
② Compute the length of the summit curve.
③ Compute the height difference from the beginning of the curve to a point 30 m. horizontally from it.

Solution:

① *Height of object above the road surface:*

$$R = \frac{S^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$$

$$3558 = \frac{(130)^2}{2(\sqrt{1.5} + \sqrt{h_2})^2}$$

$$(\sqrt{1.5} + \sqrt{h_2})^2 = 2.375$$

$$\sqrt{1.5} + \sqrt{h_2} = 1.54$$

$$\sqrt{h_2} = 0.316$$

$$h_2 = \mathbf{0.10\ m.}$$

② *Length of summit curve:*

$$L = \frac{R_v(g_1 - g_2)}{100}$$

$$L = \frac{3558\,(2.6 + 1.8)}{100}$$

$$L = \mathbf{156.55\ m.}$$

③ *Height difference from the P.C. at a horizontal distance of 30 m. from P.C..*

$$y = g_1 x + \frac{x^2}{2R}$$

$$y = 0.026\,(30) + \frac{(30)^2}{2(3558)}$$

$$y = \mathbf{0.654\ m.}$$

681. Problem:

The radius of the sag vertical curve is equal to 1532 m. The sag curve has tangent grades of - 2.8% and +2.2%.

① Compute the max. speed that a car could pass thru the sag curve.
② Compute the length of the sag vertical curve.
③ Compute the length of the sight distance.

Solution:

① *Speed of car:*

$$R = \frac{V^2}{6.5}$$

$$1532 = \frac{V^2}{6.5}$$

$V = 99.79$ kph

② *Length of curve:*

$$L = \frac{A V^2}{395}$$

$$A = 2.2 + 2.8 = 5$$

$$L = \frac{5\,(99.79)^2}{395}$$

$$L = \mathbf{126.05\ m.}$$

③ *Length of sight distance:*

$$L = \frac{A S^2}{122 + 3.5S}$$

$$126.05 = \frac{5S^2}{122 + 3.5S}$$

$$15378.10 + 441.175\,S = 5\,S^2$$

$$S^2 - 88.235\,S - 3075.62 = 0$$

$$S = \mathbf{114.98\ m.}$$

682. Problem:

A vertical summit curve has tangent grades of +5% and - 3.8%. The horizontal distance from the P.C. of the curve to the vertex of the summit curve is 113.64 m.

① Compute the length of the summit curve.
② Compute the radius of the summit curve.
③ Compute the tangent of the summit curve.

Solution:

① *Length of summit curve:*

$$S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$113.64 = \frac{0.05\,L}{0.05 + 0.038}$$

$$L = \mathbf{200\ m.}$$

② *Radius of summit curve:*

$$L = \frac{R_v(g_1 - g_2)}{100}$$

$$200 = \frac{R_v\,(5 + 3.8)}{100}$$

$$R_v = \mathbf{2272.73\ m.}$$

③ *Tangent length:*

$$T = \frac{R_v}{2}\,\frac{(g_1 - g_2)}{100}$$

$$T = \frac{2272.73\,(5 + 3.8)}{200}$$

$$T = \mathbf{100\ m.}$$

MISCELLANEOUS

683. Problem:

The sight distance of a sag vertical curve is equal to 115 m. If the tangent grades of the curve are - 2% and +3%.

① Compute the length of the curve.
② Compute the design speed of the vertical sag curve.
③ Compute the minimum radius of the sag vertical curve.

Solution:

① *Length of curve:*

$$L = \frac{A\,S^2}{122 + 3.5S}$$
$$A = g_2 - g_1$$
$$A = 3 - (-2) = 5$$
$$L = \frac{5\,(115)^2}{122 + 3.5\,(115)}$$
$$L = \mathbf{126.07\ m.}$$

② *Design speed:*

$$L = \frac{A\,V^2}{395}$$
$$126.07 = \frac{5\,V^2}{395}$$
$$V = \mathbf{99.80\ kph}$$

③ *Min. radius of sag vertical curve:*

$$R_{min.} = \frac{V^2}{6.5}$$
$$R_{min.} = \frac{(99.80)^2}{6.5}$$
$$R_{min.} = \mathbf{1532.31\ m.}$$

684. Problem:

A vertical sag curve have tangent grades of - 1.8% and +3.2%. If the minimum radius of the sag curve is 1500 m. long

① Compute the design speed of the vertical sag curve.
② Compute the length of the vertical sag curve.
③ Compute the length of sight distance of the vertical sag curve

Solution:

① *Design speed of vertical sag curve:*

$$R_{min.} = \frac{V^2}{6.5}$$
$$1500 = \frac{V^2}{6.5}$$
$$V = \mathbf{98.74\ kph}$$

② *Length of vertical sag curve:*

$$L = \frac{A\,V^2}{395}$$
$$A = g_2 - g_1$$
$$A = 3.2 - (-1.8)$$
$$A = 5$$
$$L = \frac{5\,(98.74)^2}{395}$$
$$L = \mathbf{123.42\ m.}$$

③ *Length of sight distance of vertical sag curve:*

$$L = \frac{A\,S^2}{122 + 3.5S}$$
$$123.42 = \frac{5\,S^2}{122 + 3.5S}$$
$$15057.24 + 431.97\,S = 5\,S^2$$
$$S^2 - 86.394\,S - 3011.448 = 0$$
$$S = \mathbf{113.04\ m.}$$

685. Problem:

A sample of compacted asphaltic concrete was cut from the roadway for laboratory analysis. The sample was found to weigh 1020 grams in air. Since the sample was quite porous, it was completely coated with paraffin having a sp.gr. of 0.85. The coated sample weighed 1035.3 grams in air. The weight of the coated sample in water was 575.3 grams.

① Determine the volume of paraffin.
② Determine the unit weight of pavement sample in grams per cu.cm.
③ If this asphaltic concrete costs P10000 per ton in place, determine the cost per station for surfacing 8 m. wide and 200 mm. thick with side slope of 1:1.

Solution:

① *Volume of paraffin:*

Wt. of paraffin = 1035.3 - 1020

Wt. of paraffin = 15.3 gr.

$$Volume\ of\ paraffin = \frac{15.3}{0.85\,(1)}$$

Volume of paraffin = ***18 cu.cm.***

② *Unit weight of pavement sample in grams per cu.cm.*

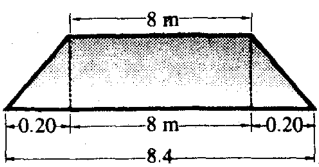
$$Vol.\ of\ asphaltic\ sample = \frac{1035.3 - 575.3}{(1)} - 18$$

Vol. of asphaltic sample = 442 cu.cm.

$$Unit\ weight = \frac{1020}{442}$$

Unit weight = ***2.31 gr/cc.***

③ *Cost per station:*



$$Volume = \frac{(8 + 8.4)(0.20)(20)}{2}$$

Volume = 32.8 cu.m.

W = Vol. x Density

Unit wt. = 2.31 gr/cc

$$Unit\ wt. = \frac{2.31\,(9.81)(100)^3}{1000}$$

Unit wt. = 22.66 kN/m^3

Weight = 32.8 (22.66)

Weight = 743.248 kN

1 ton = 1000 kg

1 ton = 9810 N

1 ton = 9.81 kN

$$Weight = \frac{743.248}{9.81}$$

Weight = 75.76 tons

Total cost = 10000 (75.76)

Total cost = ***P757,643.22***

686. Problem:

A trial mixture of asphaltic concrete contains the following materials. Percentages are by weight of total mix.

Coarse aggregates	45%	sp.gr. = 2.60
Fine aggregates	45%	sp.gr. = 2.70
Mineral filler	5%	sp.gr. = 2.76
Asphalt cement	5%	sp.gr. = 1.00

The compacted sample weighed 1190 gr. in air and 672 gr. in water.

① Compute the total volume of the mixture plus air.
② Compute the percent air voids.
③ What percentage of the total volume do the aggregates occupy?

Solution:

① *Total volume of mixture plus air:*

$$Vol. = \frac{W}{G_s\,\gamma_w}$$

$$Vol. = \frac{1190 - 672}{(1)\,(1)}$$

Vol. = ***518 cu.cm.***

② *Total percent of air voids:*

$$Vol.\ of\ coarse\ aggregate = \frac{0.45\,(1190)}{0.60\,(1)}$$

Vol. of coarse aggregate = 205.96 cu.cm.

$$Vol.\ of\ fine\ aggregate = \frac{0.45\,(1190)}{2.70\,(1)}$$

Vol. of fine aggregate = 198.33 cu.cm.

$$Vol.\ of\ mineral\ filler = \frac{0.05\,(1190)}{2.76\,(1)}$$

Vol. of mineral filler = 21.56 cu.cm.

$$Vol.\ of\ asphalt\ cement = \frac{0.05\,(1190)}{(1)\,(1)}$$

Vol. of asphalt cement = 59.50 cu.cm.

V = 485.35 cu.cm.

Vol. of air = 518 - 485.35

Vol. of air = 32.65 cu.cm.

$$Percent\ of\ air\ voids = \frac{32.65\,(100)}{518}$$

Percent of air voids = ***6.3%***

③ *Percentage of total volume occupied by aggregates:*
Vol. of aggregates = 518 - 32.65 - 59.50
Vol. of aggregates = 425.85 cu.cm.

$$\text{Percentage volume} = \frac{425.85\,(100)}{518}$$

Percentage volume = ***82.2%***

687. Problem:

A 1 ft^3 compacted asphaltic concrete has a unit weight of 151 pcf. The asphalt content is 5% (dry weight of aggregate basis). The following aggregates were used.

Aggregate	Percent by weight	Specific gravity
Crushed Stone	61	2.80
Coarse Sand	29	2.70
Find Sand	7	2.71
Filler	3	2.73
	100	

Specific gravity of asphalt = 1.0

① Determine the volume of aggregate.
② Determine the volume of asphalt.
③ Determine the percent air voids.
④ Determine the percent voids filled with asphalt.
⑤ To what density must the mix above be compacted to provide a mix with a percent voids filled with asphalt of 75%?

Solution:

① *Volume of aggregate:*
Absolute sp.gr. of aggregates

$$= \frac{100}{\frac{61}{2.80} + \frac{29}{2.7} + \frac{7}{2.71} + \frac{3}{2.73}}$$

Absolute sp.gr. of aggregates = 2.76

$$\text{Wt. of aggregate per cu.ft.} = \frac{151}{1.05}$$

Wt. of aggregate per cu.ft. = 143.81 lb

$$\text{Vol. of aggregate} = \frac{143.81}{2.76\,(62.4)}$$

Vol. of aggregate = ***0.835 ft³***

② *Volume of asphalt:*
Wt. of asphalt = 151 - 143.81
Wt. of asphalt = 7.19 lb

$$\text{Vol. of asphalt} = \frac{7.19}{(1)\,(62.4)}$$

Vol. of asphalt = ***0.115 ft³***

③ *Percent air voids:*
Vol. of solids = 0.835 + 0.115
Vol. of solids = 0.95 ft^3
Vol. of air = 1 - 0.95
Vol. of air = 0.05 ft^3

$$\text{Percent air voids} = \frac{0.05\,(100)}{1}$$

Percent air voids = ***5%***

④ *Percent voids filled with asphalt:*

$$\text{Percent aggregate voids} = \frac{(1 - 0.835)\,100}{1}$$

Percent aggregate voids = 16.5%
Percent voids filled with asphalt

$$= \frac{0.115\,(100)}{0.165} = \mathbf{69.7\%}$$

⑤ *Density to which the mix be compacted to provide a mix with a 75% voids filled with asphalt:*

$$\text{Volume of voids} = \frac{0.115}{0.75}$$

Volume of voids = 0.153 ft^3

Total volume of mix = 0.153 + 0.835
Total volume of mix = 0.988 ft^3

$$\text{Density} = \frac{151}{0.988}$$

Density = ***153 pcf.***

688. Problem:

A pavement on a horizontal curve of a new highway is to be aligned along a rolling terrain with the following data:

1. National highway design speed = 80 kph
2. Normal pavement width = 7 m.
3. Number of lanes = 2
4. Wheel base of truck = 6.1 m.
5. Radius of curve = 250 m.

① Compute the width of the mechanical widening required.
② Compute the width of the psychological widening required.
③ Find the pavement width required on the curve.

Solution:

① *Width of mechanical widening required:*

$$W_m = \frac{n L^2}{2R}$$

n = no. of lanes
L = length of wheel base in meters
R = radius of curve

$$W_m = \frac{2\,(6.1)^2}{2\,(250)}$$

W_m = ***0.149 m.***

② *Width of the psychological widening required:*

$$W_p = \frac{V}{9.5\sqrt{R}}$$

$$W_p = \frac{80}{9.5\sqrt{250}}$$

W_p = ***0.533 m.***

③ *Pavement width required:*
Pavement width = 7 + 0.149 + 0.533
Pavement width = ***7.682 m.***

689. Problem:

A bridge has been constructed between the mainland and an island. The total cost (excluding tools) to travel across the bridge is expressed as C = 50 + 0.5 V where V is the number of vehicles/hour and C is the cost/vehicle in cents. The demand for travel across the bridge.

① Determine the volume of traffic across the bridge.
② If a toll of 25 cents is added, what is the volume across the bridge.
③ A tool both is to be added, thus reducing the travel time to cross the bridge. The new cost function is C = 50 + 0.20 V. Determine the volume of traffic that would cross the bridge.

Solution:

① *Volume of traffic across the bridge:*
$V = 2900 - 10\,C$
$C = 50 + 0.5\,V$
$V = 2900 - 10\,(50 + 0.5\,V)$
$V = 2900 - 500 - 5\,V$
$6\,V = 2400$
V = ***400 vehicles/hr.***

② *Volume across the bridge:*
$C = 50 + 0.5\,(400)$
$C = 50 + 200$
C = 250 cents
New $C = 250 + 25$
New C = 275 cents

$V = 2900 - 10\,C$
$V = 2900 - 10\,(275)$
V = ***150 vehicles/hr.***

③ *Volume of traffic:*
$V = 2900 - 10\,C$
$V = 2900 - 10\,(50 + 0.20\,V)$
$V = 2900 - 500 - 2\,V$
$3\,V = 2400$
V = ***800 vehicles/hr.***

690. Problem:

A tool bridge carries 10000 vehicles/day. The current toll is P3.00 per vehicle. Studies have shown that for each increase in toll of 50 cents, the traffic volume will decrease by 1000 vehicles per day. It is desired to increase the toll to a point where revenue will be maximized.

① Determine the toll charge to maximize revenue.
② Determine the traffic volume per day after the toll increase.
③ Determine the total revenue increase with the new tool.

Solution:

① *Total charge to max. revenue:*
x = the increase in toll in centavos per vehicle

$300 + x$ = new toll per vehicle

$$10000 - \frac{1000\,x}{50} = \text{volume of vehicles per day}$$

$$R = (300 + x)(10000 - 20\,x)$$
$$\frac{dR}{dx} = (300 + x)(-20) + (10000 - 20\,x)(1) = 0$$
$$20\,(300 + x) = 10000 - 20\,x$$
$$6000 + 20\,x = 10000 - 20\,x$$
$$40\,x = 4000$$
$$x = 100 \text{ centavos}$$

Total charge = 300 + 100
Total charge = **400** centavos
Total charge = ***P4.00***

② *Traffic volume after toll increase:*
$$V = 10000 - 20\,x$$
$$V = 10000 - 20\,(100)$$
V = ***8000 vehicles***

③ *Total revenue increase with new too;*
$$R = 8000\,(4) - 10000\,(3)$$
$$R = 32000 - 30000$$
R = ***P2000***

691. Problem:

① The number of all crashes recorded at an intersection in a year was 23, and the average 24 - hour volume entering from all approaches was 6500. Determine the crash rate per million entering vehicles (RMEV).

② It is observed that 40 crashes occurred on a 17.5 mile section of a highway in one year. The average daily traffic (ADT) on the section was 5000 vehicles. Determine the rate of total crashes per 100 million vehicles - miles, ($RMVM_T$).

③ There are 60 crashes occurring in a 20 mile section of a highway in one year. The average daily traffic on the section was 6000 vehicles. Determine the rate of fatal crashes per 100 million vehicle - miles, if 5% of the crashes involved fatalities, ($RMVM_F$).

Solution:

① *Crash rate per million entering vehicles:*
$$RMEV = \frac{A\,(1000000)}{ADT(365)}$$
$$RMEV = \frac{23\,(1000000)}{6500\,(365)}$$
RMEV = ***9.69 crashes/million entering vehicles***

② *Rate of total crashes per 100 million vehicle miles:*
$$RMVM_T = \frac{A\,(100000000)}{ADT\,(L)\,(365)}$$
$$RMVM_T = \frac{40\,(100000000)}{5000\,(365)\,(17.5)} = \textbf{125.24 crashes}$$

③ *Rate of fatal crashes per 100 million vehicles - miles:*
$$RMVM_F = \frac{A\,(100000000) \times \%}{ADT\,(365)\,L}$$
$$RMVM_F = \frac{60\,(100000000)\,(0.05)}{6000\,(365)\,20} = \textbf{6.85 crashes}$$

692. Problem:

An urban arterial street segment 0.30 km long has an average annual daily traffic (AADT) of 15400 vehicles per day. The nationwide average crash experience is 375 per 100 million vehicles per km for a 3 year period of which 120 involved death and injury and 255 caused property damage only. In identifying hazardous locations, consider that a single death or injury crash is equivalent to 3 property damage crashes.

① Calculate the traffic base in vehicles per km (TB).
② Calculate the 3 year average crash rate (AVR).
③ One method of summarizing crashes is to use comparisons at different locations by assigning a weight scale to each crash base on its severity. A typical weighing scale have been used which is as follows. Fatality = 12, personal injury = 3, property damage only = 1. If one fatal crash, 3 personal injuries and 5 property damage crashes occurred during a year at a particular site, compute its severity number.

Solution:

① *Traffic base (TB):*

$$TB = \frac{L\,(AADT)\,(N)\,(365)}{100000000}$$

$$TB = \frac{0.30\,(15400)\,(3)\,(365)}{100000000}$$

$TB =$ ***0.051 vehicles/km.***

② *3 year average crash rate (AVR):*

AVR = 3 (death + injuries) + property damage only

$AVR = 3\,(120) + 255$

$AVR =$ ***615***

③ *Severity number:*

$SN = 1\,(12) + 3\,(3) + 5\,(1)$

$SN =$ ***26***

693. Problem:

The density of traffic in an expressway was recorded to be 16.05 vehicles per mile per lane and the free flow speed of 70 mph. If the average speed of a driver is 60 mph.

① Compute the jam density in vehicles per mile per lane.
② If the average space between the front bumper of a following car an rear bumper of the leading car must account for the number of vehicles in a lane (112.35 per mile) and length of each vehicle is 15 ft., compute the space taken up by the vehicles in feet of each lane-mile.
③ Compute the average spacing between the vehicles.

Solution:

① *Jam density:*

$$D_j = \frac{S_f\,D}{S_f - S}$$

$$D_j = \frac{70\,(16.05)}{70 - 60}$$

$D_j =$ ***112.35 vehicles per mile***

② *Space taken up by the vehicle in feet of each lane - mile:*

Space taken up = 112.35 (15)

Space taken up = ***1685.25 ft.***

③ *Average spacing between vehicles:*

$$S = \frac{5280 - 1685.25}{112.35}$$

$S =$ ***32 ft.***

694. Problem:

The length of an average vehicle passing thru an express way is 14 ft. If the average spacing between vehicles is 16 ft.

① Compute the jam density in vehicles per mile per lane.
② If the free flow speed is 104.85 mph, compute the speed at max. flow.
③ Compute the maximum flow in vehicles per hour.

Solution:

① *Jam Density:*

$$D_j = \frac{5280}{14 + 16}$$

$D_j = 176$

② *Speed at max. flow:*

$$S_{max} = \frac{S_f}{2}$$

$$S_{max} = \frac{104.85}{2}$$

$S_{max} =$ ***52.43 mph***

③ *Max. flow:*

$$q = \frac{S_f\,D_j}{4}$$

$$q = \frac{104.85\,(176)}{4}$$

$q =$ ***4613 vph***

695. Problem:

The tabulated data are the results of a 60 second experiment to obtain headway calculations. At mid block on the Magsaysay avenue, an observer noting at what time after an arbitrary start time vehicles cross a mark on the pavement in the lane nearest the rear curb. We could envision the measurements being made by installing a sensor in the pavement that would send signals to a remote data collection device. The first vehicle crossed the mark at 6.52 sec. after the arbitrary start time. The second vehicle crossed the mark at 11.26 sec. after the arbitrary start time. The observer in this example used the front bumper of each vehicle crossing the mark on the pavement as the definition of the event being observed. Alternative definitions could be rear bumper, front tires, or rearmost tires. After 60 sec., the observer ceases making observations having seen eight vehicles.

① Compute the time headway between the two vehicles 1 and 2.

② Compute the average headway for 8 vehicles observed during the 60 sec. period.

③ Compute the traffic flow rate in vehicles per hour.

Vehicle No.	Crossed Line at (seconds)
1	6.52
2	11.26
3	14.59
4	19.33
5	28.30
6	39.93
7	43.76
8	58.16

Solution:

① *Time headway between vehicles 1 and 2:*

Vehicle No.	Crossed Line (sec)	Time headway (sec)
1	6.52	
2	11.26	11.26 - 6.52 = 4.74
3	14.59	14.59 - 11.26 = 3.33
4	19.33	19.33 - 14.59 = 4.74
5	28.30	28.30 - 19.33 = 8.79
6	39.93	39.33 - 28.30 = 11.63
7	43.76	43.76 - 39.93 = 3.83
8	58.16	58.16 - 43.76 = 14.40
		51.64

Time headway between vehicle 1 and 2

$= 11.26 - 6.52$

$= 47.74$ sec.

② *Average headway for 8 vehicles:*

$$\text{Average headway} = \frac{51.64}{7}$$

Average headway = 7.38 sec.

③ *Traffic flow rate in vehicles/hr.*

$$\text{Flow rate } q = \frac{1}{\text{average headway}}$$

$$q = \frac{1\,(3600)}{7.38}$$

q = **488 vehicles per hour**

696. Problem:

You are going to travel form Manila to San Francisco (1200 miles). There are two choices: Flight A is non stop and takes 3 hrs, while flight B requires that you change planes in Japan, with a total time to San Francisco of 4 hrs. The time to change planes in Japan is 40 minutes. The difference in flight distance is negligible. Assume that it takes 30 minutes to drive the 20 miles from your home to the airport and you must arrive at the airport 90 minutes early to check in. When you land in San Francisco, it takes 20 minutes to get your luggage and an additional 40 minutes to the down town hotel 30 miles from the airport. The airfare for the non stop flight A is $500 where as the whereas the ticket for flight B with the transfer at Japan is $360. Assume all other costs are the same at $60 and that you value your time at $50 per hour.

① Which flight is cheaper?
② By how much cheaper?
③ At what value of time (VOT) would the two flights be equally expensive?

Solution:

① *Which flight is cheaper.*

For A:

$t = 3 + 30$ min. + 90 min. + 20 min. + 40 min.

$t = 6$ hrs.

Cost of flight A = \$500 + \$60 + 6 (\$50)

Cost of flight A = \$860

For B:

$t = 4 + 30$ min. + 90 min. + 20 min. + 40 min.

$t = 7$ hrs.

Cost of flight B = \$360 + \$60 + 7 (\$50)

Cost of flight B = \$770

Fight B is cheaper

② *How much cheaper:*

Flight B is cheaper by 860 - 770 = <u>*\$90*</u>

③ *VOT when both flights would be equally expensive:*

500 + 60 + 6 (*VOT*) = 360 + 60 + 7 (*VOT*)

VOT = 560 - 420

VOT = ***\$140***

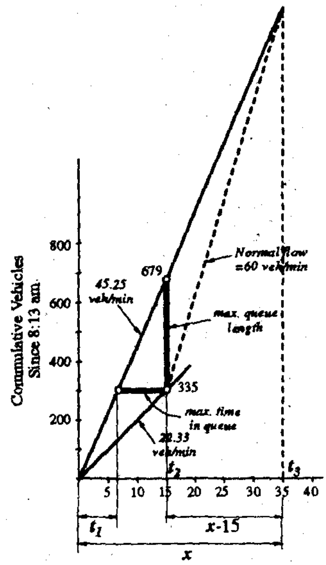
697. Problem:

Because a minor collision, one the two lanes of a certain freeway is blocked at 8:13 AM. The normal freeway capacity is 3600 vehicles per hour (60 veh/min) is reduced to 1340 vehicles per hour (22.33 veh/min). The flow rate on the freeway at this time of day is 2715 vehicles per hour (45.25 veh/min). The blockage is removed after 15 minutes.

① What is the max. length of the queue?
② What was the longest time any single vehicle was in the queue?
③ At what time did the queue clear?

Solution:

① *Max. length of the queue:*



Arrival Curve:

After $x = 15$ min, $y_2 = 15\,(45.25)$

$y_2 = 679$ vehicles

Departure Curve:

After $x = 15$ min, $y_1 = 15\,(22.33)$

$y_1 = 335$ vehicles

Max. queue length = 679 - 335

Max. queue length = ***344 vehicles***

② *Max. time in queue:*

when $x = 15$ min, $y = 15\,(22.33)$

$y = 335$ vehicles

Time the 335th vehicle enter the queue t_1

$= \dfrac{335}{45.25} = 7.4$ min.

Max. time in queue = 15 - 7.4 min.

Max. time in queue = ***7.6 min.***

③ *Time the queue cleared:*

x = time the queue was cleared.

Equation for arrival curve $y_3 = 45.25\,x$

Equation for departure curve = $y_4 + 335$

where $y_4 = 60\,(x - 15)$

MISCELLANEOUS

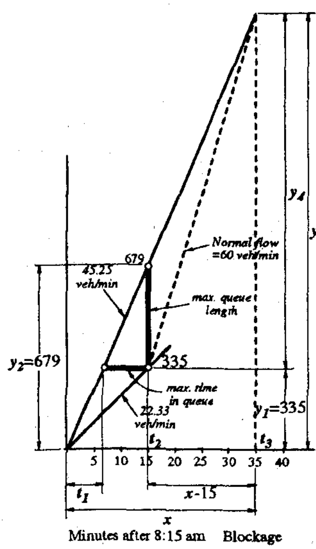
$y_3 = y_4 + 335$

$45.25\,x = 60\,(x - 15) + 335$

$14.75\,x - 900 + 335 = 0$

$x = 38.3$ minutes

*Time the queue was cleared is **8:51.3 AM***



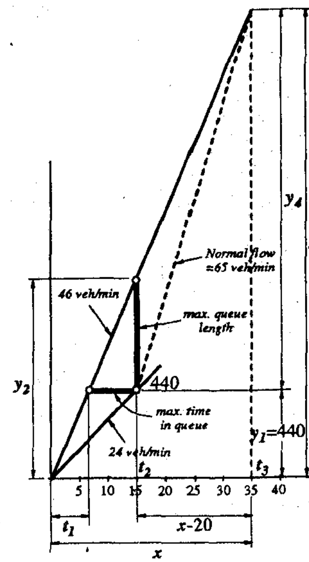
698. Problem:

A traffic accident occurred along the North Diversion road at 6:00 AM. The flow rate on the expressway at this time of day was 46 vehicles per min. The normal freeway capacity is 65 vehicles per minute but it was reduced to 24 vehicles per min. due to the traffic accident. The traffic was cleared by the MMDA after 20 minutes.

① Determine the max. length of the queue before the blockage was removed?
② Determine the time that the vehicles waited for the long queue before the blockage was cleared.
③ What time was the queue cleared.

Solution:

① *Max. length of queue:*



$y_1 = 24\,(20)$

$y_1 = 480$ vehicles

$y_2 = 46\,(20)$

$y_2 = 920$ vehicles

Max. queue = 920 - 480

Max. queue = ***440 vehicles***

② *Time the vehicles waited for the queue:*

$t_1 = \dfrac{440}{46}$

$t_1 = 9.56$ min.

Time the vehicles waited for the queue
= 20 - 9.56
= ***10.44 m.***

③ *Time the queue was cleared:*

$y_3 = 46\,x$

$y_4 = 65\,(x - 20)$

$y_3 = y_4 + 440$

$46\,x = 65\,(x - 20) + 440$

$19\,x - 1300 + 440 = 0$

$x = 45.26$ min.

Time the traffic was cleared = ***6:45.26 AM***

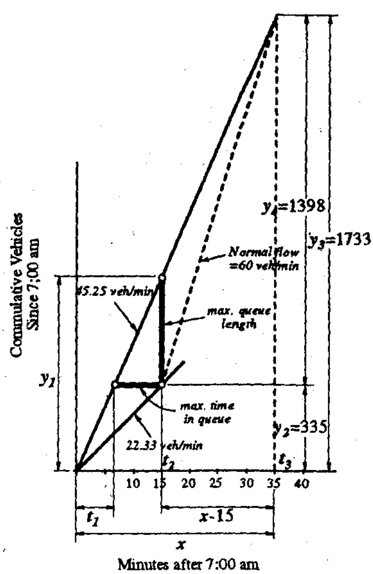
699. Problem:

Due to traffic accident, one of the two lanes of the South Super Highway is blocked at 7:00 AM. Because of the traffic, the normal capacity of freeway which is 60 vehicles per min was reduced to 22.33 veh/min. The flow rate at this particular day is 45.25 veh/min. The traffic blockage was removed after 15 min.

① What was the total delay to traffic in veh-hrs. because of the lane blockage?
② If a vehicle entered the queue at 7:12 AM, how many vehicles would be ahead of it in the queue.
③ How long would the driver have to wait in the queue?

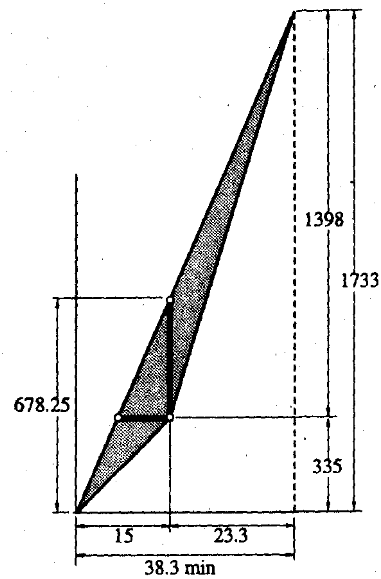
Solution:

① *Total delay to traffic because of lane blockage:*



$y_1 = 45.25\,(15)$
$y_1 = 678.25$ vehicles
$y_2 = 22.33\,(15)$
$y_2 = 335$ vehicles

$y_3 = 45.25\,x$
$y_4 = 60\,(x - 15)$
$y_3 = 335 + y_4$
$45.25\,x = 335 + 60\,(x - 15)$
$x = 38.3$ min.
$y_4 = 60\,(38.3 - 15)$
$y_4 = 1398$



The area of the shaded section represents the total delay on the freeway because of the traffic accident.

$$Area = \frac{1733\,(38.3)}{2} - \frac{335\,(15)}{2} - 335\,(23.3) - \frac{23.3\,(1398)}{2}$$

Area = 6582.25 veh-min.

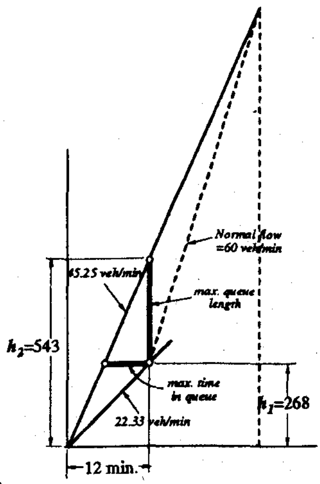
$$Area = \frac{6582.25}{60}$$

Area = 109.70 veh-hrs.

Total delay = ***109.70 veh-hrs.***

MISCELLANEOUS

② *No. of vehicles ahead of it in the queue:*

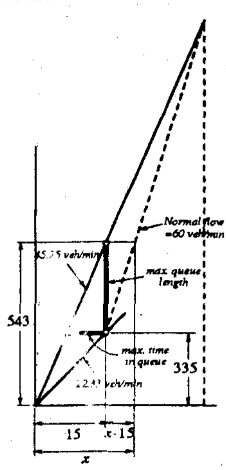


From 7:00 AM to 7:12 AM
$t = 12$ min.

$h_1 = 22.33\ (12)$
$h_1 = 268$ vehicles
$h_2 = 45.25\ (12)$
$h_2 = 543$ vehicles

Length of queue at 7:12 AM = 543 - 268
Length of queue at 7:12 AM = 275 vehicles
No. of vehicles ahead of it in the queue
= ***275 vehicles***

③ *Time the driver have to wait in the queue:*



$60\ (x - 15) + 335 = 543$
$x = 18.47$ min.
time the 7:12 AM arrival will leave the queue.

Waiting time for this vehicle is 18.47 - 12
= ***6.47 min.***

700. Problem:

A man is driving his 4.2 m. long automobile at 80 kph, when the traffic signal in front of him changes to yellow. He is 40 m. from the intersection when he applies the brakes after a 1 second reaction time.

① If the car can decelerate at a rate of 4.6 m/s^2, at what velocity will he be moving when he reaches the intersection?
② If the yellow light is 4 seconds long, where will the man be when the light turns red?
③ Assuming he will continue through at the speed found in part "b" when the light changes to red. How long will the light have been red when he clears the intersection having a width of 15 m?

Solution:

① *Velocity of car when it reaches the intersection:*

$V_2^2 = V_1^2 - 2\,a\,S$

$V_1 = \frac{80000}{3600}$

$V_1 = 22.22$ m/s
$V_2^2 = (22.22)^2 - 2\ (4.6)\ (40)$
$V_2 = 11.21$ m/s

$V_2 = \frac{11.21\ (3600)}{1000}$

$V_2 =$ **40.4 kph**

② *Location of man when the light turns red:*

$V_2 = V_1 - a\,t$
$V_2 = 22.22 - (4.6)\ (4 - 1)$
$V_2 = 8.42$ m/s *(his velocity when the light turns red)*

Distance he will have traveled:

$V_2^2 = V_1^2 - 2\,a\,S$
$(8.42)^2 = (22.22)^2 - 2\ (4.6)\ S$
$S = 45.96$ m.

He is ***45.96 m.*** *after entering the intersecting*

③ *Time the light have been red when he clears the intersection:*

$t = \frac{D}{V}$

$D = (15 - 5.96) + 4.2$

$D = 13.24$ m.

$t = \frac{13.24}{8.42}$

$t = \mathbf{1.54}$ **sec.**

701. Problem:

The following data for a rural primary road shows the volume of traffic daily and monthly.

Day of week	Volume	Month	Average Daily Traffic
Sunday	7895	Jan.	1350
Monday	10714	Feb.	1200
Tuesday	9722	March	1450
Wednesday	11413	April	1600
Thursday	10714	May	1700
Friday	13125	June	2500
Saturday	11539	July	4100
	Total = 75122	Aug	4550
		Sept.	3750
		Oct.	2500
		Nov.	2000
		Dec.	1750
			Total = 28450

① Compute the daily expansion factor (DEF) for Wednesday.
② Compute the monthly expansion factor (MEF) for the month of August.
③ Compute the average annual daily traffic (AADT) for the month of May.

Solution:

① *Daily expansion factor (DEF) for Wednesday:*

$DEF = \frac{75122}{11413}$

$DEF = \mathbf{6.582}$

② *Monthly expansion factor (MEF) for August:*

$MEF = \frac{28450}{4550\,(12)}$

$MEF = \mathbf{0.521}$

③ *Average Annual daily traffic for month of May:*

$AADT = MEF \times ADT$

$MEF = \frac{28450}{12\,(1700)}$

$MEF = 1.394$

$AADT = 1.394\,(1700)$

$AADT = \mathbf{2369.80}$

702. Problem:

For 10 years, the Ceres Bus Co. has kept is base adult fare at P50 from Cebu to Santander. Over the past years, however, operating costs have risen steadily while ridership levels have been stagnant. The result has been an increasing operating deficit that must be subsidized by income and property taxes. Some local government officials want a portion of that rising deficit shifted back onto the users of the bus service, which means raising the bus fares. The Ceres Bus Co. is worried that most bus users have low income and they can least afford a fare increase. It seems reasonable to expect that a fare increase will cause some Ceres Bus patrons to cease using its bus service. If the fare is increase to P75. Assume the bus route carries an average of 1000 passengers per day. Assume shrinkage ratio elasticity of - 0.33.

① How many riders will be driven away due to increase in fares?
② What would be difference in its revenue due to the increase in fare?
③ Assuming half of the passengers on a typical day rode during the AM and PM peak periods where the peak and off-peak head ways were 30 min., what would be the ridership if the headways were increased to 60 min. between buses? Assume a peak headway elasticity of - 0.37, and an off-peak headway elasticity of - 0.46.

Solution:

① No. or riders driven away due to increase in fares:

Shrinkage ratio elasticity $= \left(\frac{Q_1 - Q_0}{P_1 - P_0}\right)\frac{P_0}{Q_0}$

$-0.33 = \frac{(Q_1 - 1000)(50)}{(75 - 50)(1000)}$

$Q_1 - 1000 = -165$

$Q_1 = 835$

No. of riders driven away = 1000 - 835

No. of riders driven away = **165 passengers**

② Difference in revenues daily:

50 (1000) = 50000

75 (835) = 62625

Diff. = 62625 - 50000

Diff. = **P12,625**

③ No. of ridership if headways were increased to 60 min. between buses:

Peak headway elasticity $= \frac{(Q_1 - Q_0)\, S_0}{(S_1 - S_0)\, Q_0}$

$-0.37 = \frac{(Q_1 - 500)(30)}{(60 - 30)(500)}$

$Q_1 = 315$

Off peak headway elasticity $= \frac{(Q_1 - 500)(30)}{(60 - 30)(500)}$

Off peak headway elasticity = - 0.46

$Q_1 = 270$

Total no. of ridership = 315 + 270

Total no. of ridership = **585**

703. Problem:

A bus achieves 7 miles per gallon of diesel fuel. Each bus has capacity of 138700 BTU per gallon.

① How many passengers on the average will the bus have to carry to achieve 3729 BTU per passenger mile?

② Assuming that the average bus has a space for 30 persons and that 30 persons were carried, what would the energy intensity of the bus be?

③ What are the energy intensities of the bus with 7 riders including the driver if the fuel economy of the bus is equal to 28.7 miles per gallon.

Solution:

① No. of passengers on the average:

$N = \frac{138700}{7\,(3729)}$

N = **5.3 passengers**

② Energy intensity:

$x = \frac{138700}{7\,(30)}$

x = **660.5 BTU/passenger-mile**

③ Energy intensity:

$x = \frac{138700}{28.7\,(7)}$

x = **690.4 BTU/passenger-mile**

704. Problem:

The following traffic counts were made during a study period of one hour from 9 AM to 10:00 AM as shown on the 15 minute volume counts.

Time	Volume of Traffic
9:00 - 9:15	465
9:15 - 9:30	480
9:30 - 9:45	510
9:45: - 10:00	490

① Determine the peak hour volume.

② Determine the peak hour factor.

③ Determine the design hourly volume (DHV).

Solution:

① Peak hour volume:

Peak hour volume = 465 + 480 + 510 + 490

Peak hour volume = **1945**

② Peak hour factor:

Peak hour factor $= \frac{1945}{\frac{60}{15}(510)}$

Peak hour factor = **0.953**

③ Design hourly volume:

Design hourly volume $= \frac{1945}{0.953}$

Design hourly volume = **2041**

705. Problem:

The soil sample was obtained from the project site and the CBR test was conducted at the field. The sample with the same subgrade imposed upon it is then subjected to a penetration test by a piston plunger 5 cm. diameter moving a certain speed. The CBR value of a standard crushed rock for 2.5 mm penetration is 70.45 kg/cm^2. The sample was subjected to a load of 83 kg. and it produces a penetration of 2.5 mm.

① Compute the CBR value of the soil sample.
② Using this soil for a subgrade of a pavement, determine the thickness of the pavement when a wheel load of 3000 kg with a tire pressure of 8 kg/cm^2 is imposed on the subgrade. Use U.S. Corps of Engineers formula.
③ If the pavement is made up of concrete, determine the spacing between construction joints for an 8 m. width of a two lane roadway if the coefficient of friction between the subgrade and pavement is 0.15. Allowable tensile stress of concrete is 0.8 kg/cm^2. Assume weight of concrete slab to be 2400 kg/m^3.

Solution:

① *CBR of soil sample:*

$$Stress = \frac{P}{A}$$

$$Stress = \frac{83}{\frac{\pi}{4}(5)^2}$$

$$Stress = 4.23 \text{ kg/cm}^2$$

$$CBR = \frac{4.23}{70.45}(100)$$

$$CBR = \mathbf{6\%}$$

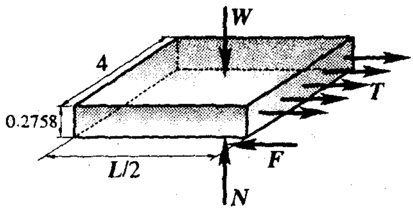
② *Thickness of pavement (US Corps of Engineers):*

$$t = \sqrt{W}\left[\frac{1.75}{CBR} - \frac{1}{P\pi}\right]^{1/2}$$

$$t = \sqrt{3000}\left[\frac{1.75}{6} - \frac{1}{8\pi}\right]^{1/2}$$

$$t = \mathbf{27.58\ cm.}$$

③ *Spacing between construction joints:*



$$W = \frac{L}{2}(0.2758)(4)(2400)$$

$$W = 1323.84\,L$$

$$N = 1323.84\,L$$

$$F = \mu N$$

$$F = 1.5\,(1323.84\,L)$$

$$F = 1985.76\,L$$

$$T = F$$

$$T = A_c f_t$$

$$27.58\,(400)\,(0.8) = T$$

$$T = 8825.6 \text{ kg}$$

$$8825.6 = 1985.76\,L$$

$$L = \mathbf{4.44\ m.}$$

706. Problem:

Design the thickness of a pavement to carry a wheel load of 30 kN based on the following conditions and type of pavement.

① A rigid pavement with an allowable tensile stress of concrete equal to fc' with a specified compressive strength of concrete of 28.5 MPa. Neglect effect of dowels. Use Olders Theory.
② A flexible pavement with an allowable subgrade pressure of 0.14 MPa and the max. time pressure equal to 0.82 MPa. This pressure is assumed to be uniformly distributed over the area of tire contact on the roadway according to the principle of cone pressure distribution.
③ A pavement with a maximum CBR value of 6% for the subgrade soil supporting this load. The tire pressure is equal to 4 kg/cm^2. Use U.S. Corps o Engineers Formula.

MISCELLANEOUS

Solution:

① *Rigid pavement:*

$$t = \sqrt{\frac{3\,W}{f}}$$

$$t = \sqrt{\frac{3\,(30000)}{0.06\,(28.5)}}$$

$t = \mathbf{229.42\ mm.}$

② *Thickness of flexible pavement:*

$$t = 0.564\sqrt{\frac{W}{f}}$$

$W = AP$

$30000 = \pi r^2 (0.82)$

$r = 107.91$ mm.

$$t = 0.564\sqrt{\frac{30000}{0.14}} - 107.91$$

$t = \mathbf{153.17\ mm}$

③ *Thickness of pavement based on US Corps of Engineers:*

$$t = \sqrt{W}\left[\frac{1.75}{CBR} - \frac{1}{P\pi}\right]^{1/2}$$

$$W = \frac{30000}{9.81}$$

$W = 3058$ kg

$$t = \sqrt{3058}\left[\frac{1.75}{6} - \frac{1}{4\pi}\right]^{1/2}$$

$t = 25.46$ cm.

$t = \mathbf{254.6\ mm.}$

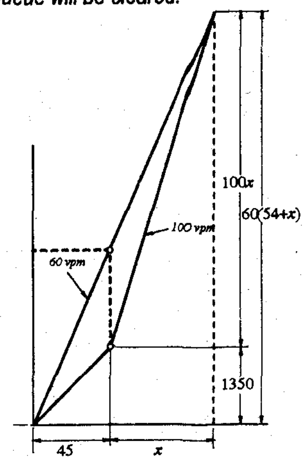
707. Problem:

A gaper's block occurs when traffic in one direction slows to look at an incident on the opposite side of the roadways median. The gapers block reduces roadway capacity from 100 vehicles per min. to 30 vehicles per min. for 45 minutes. If the arrival rate stays at 60 vehicles per min.

① How long after the gapers block starts will the queue be cleared?
② What is the total vehicle delay to traffic because of the gapers block in vehicles-min?
③ What is the average delay per vehicle?

Solution:

① *Time after gapers block starts until the queue will be cleared:*

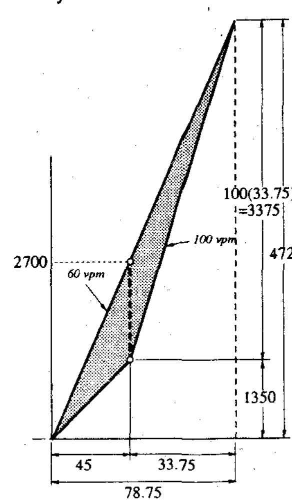


$30(45) = 1350$ cars
$60\,(45) = 2700$ cars
$100\,x + 1350 = 60\,(45 + x)$
$40\,x = 1350$
$x = 33.75$ min.

Time after gapers block starts until queue will be cleared = 45 + 33.75 = ***78.75 min.***

② *Total delay of traffic because of gaper's block:*

Total delay of traffic = area of shaded section



$$Area = \frac{(2700 - 1350)(45)}{2} + \frac{(2700 - 1350)(33.75)}{2}$$

Area = ***53156.25 veh-min.***

③ *Average delay per vehicle:*

$$\text{Average delay per vehicle} = \frac{53156.25}{4725}$$

Average delay per vehicle
*= **11.25 min/vehicle***

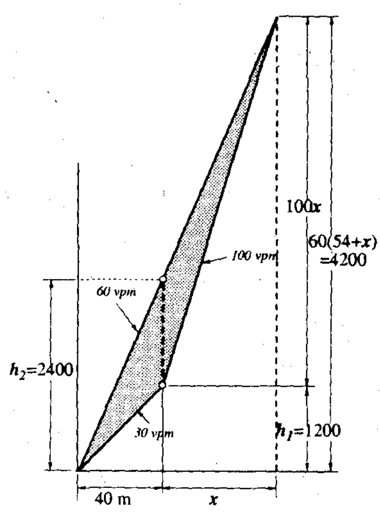
708. Problem:

The super highway is design to have a roadway capacity of 100 vpm. But due to some resurfacing of the portion of the highway, the roadway capacity was reduce to 30 vpm. The traffic arrival rate is 60 vpm. The traffic then resume after 40 min.

① Compute the maximum length of the queue.
② What is the average delay per vehicle due to some resurfacing of the portion of the highway.
③ What is the longest time any vehicle spent in the queue?

Solution:

① *Max. length of queue:*



$h_1 = 30\,(40)$
$h_1 = 1200$ cars
$h_2 = 60\,(40)$
$h_2 = 2400$ cars

Max. length of queue = 2400 - 1200
Max. length of queue = ***1200 cars***

② *Average delay per vehicle:*
Total vehicle delay = area of shaded section
$100\,x + 1200 = 60\,(x + 40)$
$40\,x = 1200$
$x = 30$ min.

Total vehicle delay

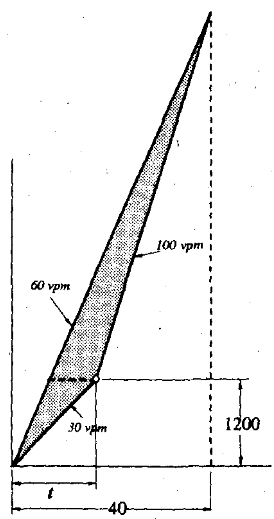
$$= \frac{(2400 - 1200)(40)}{2} + \frac{(2400 - 1200)(30)}{2}$$

Total vehicle delay = 42000 veh-min.

$$\text{Average delay per vehicle} = \frac{42000}{4200}$$

Average delay per vehicle = ***10 min.***

③ *Longest time any vehicle spent in the queue:*



$60\,t = 1200$
$t = 20$ min.

Longest time any vehicle spent in the queue
= 40 - 20
= ***20 min.***

709. CE Board May 2009

A closed traverse has the following data.

LINES	BEARING	DISTANCE
A-B	N. 30° 30' E	46.50 m.
B-C	S. 20° 30' E	36.50 m.
C-A	- - -	- - -

① Find the distance of line C-A.
② Find the bearing of line C-A.
③ Find the area of the closed traverse.

Solution:

① Distance CA:

Lines	Bearing	Distance	Lat	Dep
AB	N 30°30' E	46.50	+40.07	+23.60
BC	S 20°30' E	36.50	- 34.19	+12.78
CA	- - -	- - -	- 5.88	- 36.38

Distance AC = $\sqrt{(5.88)^2 + (36.38)^2}$

AC = ***36.85 m.***

② Bearing CA:

$$\tan \text{bearing} = \frac{\text{Dep}}{\text{Lat}}$$

$$\tan \text{bearing} = \frac{36.38}{5.88}$$

Bearing = ***S. 80°49' W.***

③ Bearing CA:

Lines	Lat	Dep	DMD	Double Area
AB	+40.07	+23.60	+23.60	23.60(40.07)
				= +945.65
BC	- 34.19	+12.78	+59.98	59.98(-34.19)
				= 2050.72
CA	- 5.88	- 36.38	+36.38	36.38(-5.88)
				= 213.91

2A = + 945.65 – 2050.72 – 213.91

A = - 659.49

Area of transverse ABC = ***659.49 sq.m.***

710. CE Board May 2010

Given the data of a closed traverse:

LINES	LAT.	DEP.	DMD	2A
AB	446.56	30.731	30.731	1372.324
BC	y	75.451	x	2158.023
CD	- 58.328	- - -	148.621	-8668.766
DA	- 2.090	- 42.439	- - -	z

① Find the value of x.
② Find the value of y.
③ Find the value of z.

Solution:

① Value of x:

LINES	DEP	DMD
AB	30.731	30.731
BC	75.451	x

30.731 + 30.731 + 75.451 = x

x = ***136.913***

② Value of y:

	LAT	DMD	2A
BC	y	136.913	2158.023

2A = DMD x LAT.

2158.023 = 136.913 (y)

y = ***15.762***

Check:

44.656 + 15.762 = - 58.328 – 2.090

60.418 = - 60.418 ok

③ Value of z:

LINES	LAT.	DEP.	DMD	2A
AB	446.56	30.731	30.731	1372.324
BC	15.762	75.451	136.913	2158.023
CD	- 58.328	- 63.743	148.621	-8668.766
DA	- 2.090	- 42.439	42.439	z
	0	0		

2A = DMD x LAT

z = 42.439 (- 2.090)

z = ***- 88.698***

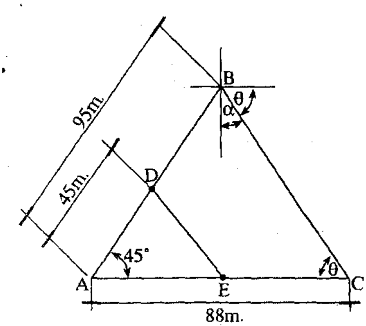
711. CE Board Nov. 2009

A lot is bounded by 3 straight sides A, B, C. AB is N. 45° E. 95 m. long and AC is due East, 88 m. long. From point D, 43 m. from A on side AB, a dividing line runs to E which is on side CA. The area ADE is to be 1/7 of the total area of the lot.

① Determine the distance DE.
② Determine the bearing of side BC.
③ Determine the distance AE.

Solution:

① *Distance DE:*



$$\frac{(AE)(43)\ \text{Sin}\ 45^\circ}{2} = \frac{1}{7}\ \frac{(95)(88)\ \text{Sin}\ 45^\circ}{2}$$

AE = 27.77 m.

Using Cosine Law:

$(DE)^2 = (43)^2 + (27.77)^2 - 2(43)(27.77)\ \text{Cos}45^\circ$

DE = ***30.52 m.***

② *Bearing of line BC:*

$(BC)^2 = (95)^2 + (88)^2 - 2(95)(88)\ \text{Cos}\ 45^\circ$

BC = 70.33 m.

Using Sine Law:

$$\frac{95}{\text{Sin}\ \theta} = \frac{70.33}{\text{Sin}\ 45^\circ}$$

$\theta = 72^\circ 46'$

$\alpha = 90^\circ - 72^\circ 46'$

$\alpha = 17^\circ 14'$

Bearing of BC = ***S. 17°14' E.***

③ *Distance AE:*

AE = ***27.77 m.***

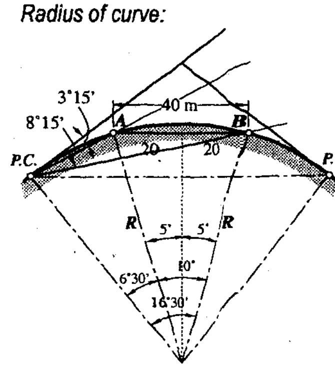
712. CE Board Nov. 2006

The deflection angles of two intermediate points A and B of a simple curve are 3°15' and 8°15' respectively. If the chord distance between A and B is 40 m.

① Find the radius of the curve.
② Find the length of curve from P.C. to A.
③ Find the length of chord from P.C. to B.

Solution:

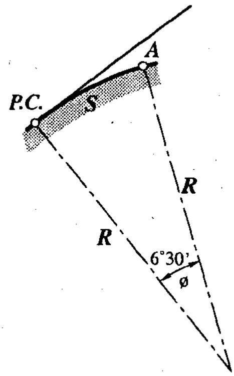
① *Radius of curve:*



$$\text{Sin}\ 5^\circ = \frac{20}{R}$$

R = ***229.47 m.***

② *Length of curve from P.C. to A:*



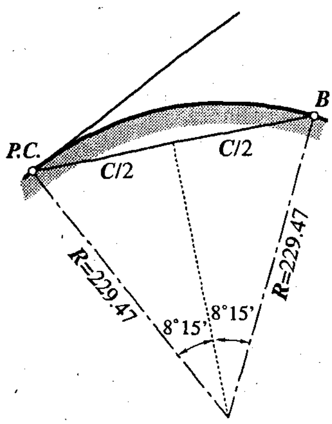
$S = R\,ø$

$$S = \frac{229.47\,(6°30')\,\pi}{180}$$

$$S = \frac{229.47\,(6.5)\,\pi}{180°}$$

S = ***26.03 m.***

③ *Length of chord from PC to B:*



$$\text{Sin } 8°15' = \frac{C}{2R}$$

C = 2(229.47) Sin 8°15'

C = ***65.86 m.***

713. Problem

An unsymmetrical parabolic curve has a forward tangent of – 8% and a back tangent of + 5%. The length of the curve on the left side of the curve is 40 m. long while that on the right side is 60 m. long. P.C. is at station 6 + 780 and has an elevation of 110 m.

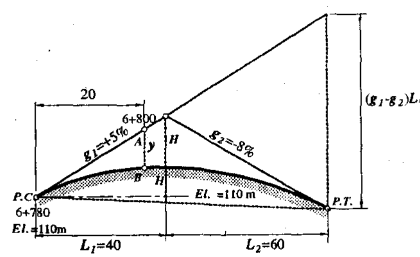
① Compute the elevation of the current at station 6 + 800.

② Compute the stationing of the highest point on the curve.

③ Compute the elevation of the highest point on the curve.

Solution:

① *Elevation at sta. 6 + 800*



$$\frac{2H}{L_1} = \frac{L_2\,(g_1 - g_2)}{L_1 + L_2}$$

$$\frac{2H}{40} = \frac{60\,(0.05 + 0.08)}{40 + 60}$$

H = 1.56

$$\frac{y}{(20)^2} = \frac{1.56}{(40)^2}$$

y = 0.39

Elevation of the curve at sta. 6 + 800

Elev. A = 110 + 0.05(20) – 0.39

Elev. A = ***110.61 m.***

② *Stationing of highest point of the curve: Check the location of the highest point of the curve.*

When $\frac{L_1 g_1}{2} < H$ it is on the 40 m. side

$$S_1 = \frac{g_1 L_1^2}{2H}$$

When $\frac{L_1 g_1}{2} > H$ it is on the 60 m. side

$$S_2 = \frac{g_2 L_2^2}{2H}$$

$$\frac{L_1 g_1}{2} = \frac{40(0.05}{2} = 1.0 < H = 1.56$$

Use:

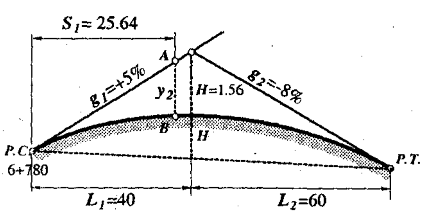
$$S_1 = \frac{g_1 L_1^2}{2H} \text{ from P.C.}$$

$$S_1 = \frac{0.05\,(40)^2}{2\,(1.56)}$$

$$S_1 = 25.64 \text{ m.}$$

Stationing of highest point of curve:
Sta. (6 + 780) + (25.64)
Sta. = ***6 + 805.64***

③ *Elevation of highest point of curve:*



$$\frac{y_2}{(25.64)^2} = \frac{1.56}{(40)^2}$$

$$y_2 = 0.64$$

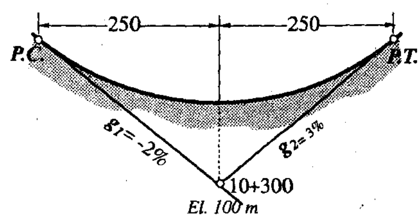
Elevation B = 110 + 0.05(25.64) – 0.64
Elevation B = ***110.642 m.***

714. CE Board Nov. 2007

A parabolic sag curve has a grade of the back tangent of - 2% and forward tangent of + 3%. The stationing of the P.C. is at 10 + 300 with an elevation of 100 m..

① Compute the length of curve if the rate of change of grade is restricted to 0.20% in 20 m.
② Compute the elevation of the P.T.
③ Compute the stationing of the P.T.

Solution:



① Length of curve if the rate of change of grade is 0.20% in 20 m.

$$r = \frac{g_1 - g_2}{n}$$

$$0.2 = \left| \frac{-2-3}{n} \right|$$

n = 25 stations

Length of curve = 25(20)
Length of curve = ***500 m***

② Elevation of the P.T.
Elev. P.T. = 100 - 0.02(250) + 0.03(250)
Elev. P.T. = ***102.50 m.***

③ Sationing of the P.T.
Sta. of P.T. = Sta. of P.C. + 500
Sta. of P.T. = (10 + 300) + 500
Sta. of P.T. = ***10 + 800***

MISCELLANEOUS

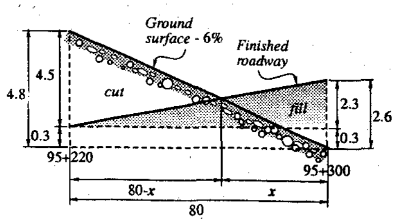
715. CE Board May 2009

At station 95 + 220, the center height of the road is 4.5 m. cut, while at station 95 + 300, it is 2.6 m. fill. The ground between station 95 + 220 to the other station has a uniform slope of – 6%.

① What is the grade of the road?
② How far in meters, from station 95 + 300 toward station 95 + 220 will the filling extend?
③ At what station will the filling extend.

Solution:

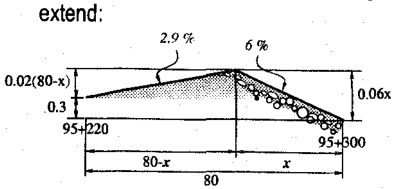
① Grade of road



$$\text{Slope of road} = \frac{2.3}{80}$$

Slope of road = 0.02875 say ***0.029***

② Distance from 95 + 300 where filling will extend:



$0.029(80 – x) + 0.30 = 0.06x$
$0.089x = 0.029(80) + 0.30$
$x =$ ***29.44 m.***

③ Station where filling extend:
$(95 + 300) – (29.44) =$ ***95 + 270.56***

716. Problem

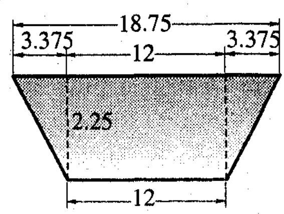
At sta. A 12 + 200
Section: Level and trapezoidal
Base width: 12 m.
Side slope: 1.5:1
Depth of cut: 2.25 m.

At sta. B 12 + 220
Section: Level and trapezoidal
Base width: 12 m.
Side slope: 2:1
Depth of cut = 1.8 m.

① Determine the volume of cut in cu.m. from Sta A to Sta B.
② If the volume of cut from A to B is 650 cu.m., find the depth at section A?
③ If the volume of cut from A to B is 650 cu.m., what is the top width of the section A.

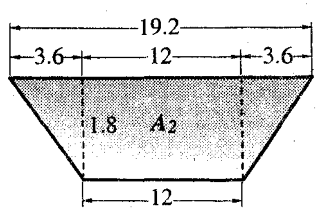
Solution:

① Volume of cut in cu.m. from Sta A to Sta B.



$$A_1 = \frac{(18.75 + 12)(2.25)}{2}$$

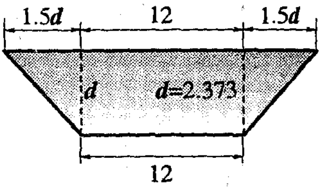
$A_1 = 34.59 \text{ m}^2$



$$A_2 = \frac{(19.2 + 12)(1.8)}{2}$$

$A_2 = 28.08$

② Depth at section A.



$$A_1 = \frac{(3d + 12 + 12)\,d}{2}$$

$$A_1 = 1.5d^2 + 12d$$

$$V = \frac{(A_1 + 28.08)\,(20)}{2}$$

$A_1 = 36.92$

$36.92 = 1.5d^2 + 12d$

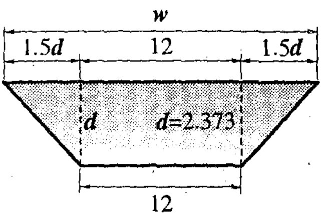
$d = 2.373$

$$V = \frac{(A_1 + A_2)\,L}{2}$$

$$V = \frac{(34.59 + 28.08)\,(20)}{2}$$

$V = \mathbf{626.7\ m^3}$

③ Top width of the section A.



$W = 1.5d\,(2) + 12$

$W = B(2.373) + 12$

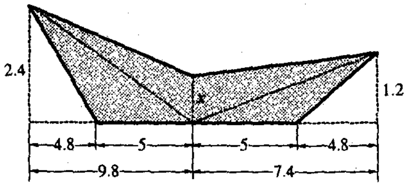
$W = \mathbf{19.12\ m.}$

717. Problem

Given a side slope of 2:1, a road width of 10 m. and a cross-sectional area of 31.7 sq.m., find the value of x in the following cross-section notes.

$$\frac{9.8}{+2.4} \quad \frac{0}{x} \quad \frac{7.4}{+1.2}$$

Solution:



$$31.7 = \frac{9.8(x)}{2} + \frac{7.4(x)}{2} + \frac{5(2.4)}{2} + \frac{5(1.2)}{2}$$

$8.6x = 22.7$

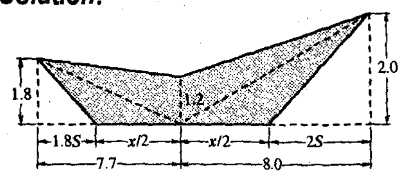
$x = \mathbf{2.64\ m.}$

718. Problem

Determine the side slope from the following cross section notes of an earthwork.

$$\frac{7.7}{+1.8} \quad \frac{0}{1.2} \quad \frac{8.0}{+2}$$

Solution:



$$1.8S + \frac{x}{2} = 7.7$$

$$2S + \frac{x}{2} = 8$$

$0.6S = 0.3$

$$S = \frac{3}{2}$$

$S = 1.5$

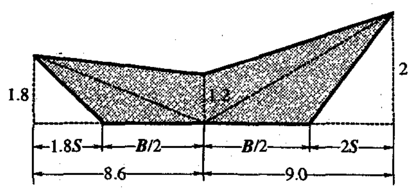
Side slope = ***1.5:1***

719. Problem

Determine the side slope from the following cross-section notes of an earthwork.

$$\frac{8.6}{+1.8} \quad \frac{0}{+1.2} \quad \frac{9.0}{+2.0}$$

Solution:



$$8.6 = 1.8S + \frac{B}{2}$$

$$9 = 2S + \frac{B}{2}$$

$0.4 + -0.2S$

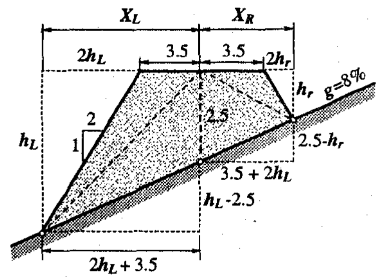
$S = 2$

Side slope is ***2:1***

720. CE Board May 2007

The cross section notes of the ground surface at a given station of a road survey shows that the ground is sloping at an 8% grade upward to the right. The difference in elevation between the ground surface and the finished subgrade at the center line of the proposed road is 2.5 m. Width of subgrade is 7 m. with sideslope of 2:1.

① Determine the area of the cross section.
② Compute the distance of the left slope stake from the center of the road.
③ Compute the distance of the right slope stake from the center of the road.



Solution:

① Area of cross section:

$$\frac{h_L - 2.5}{2h_L + 3.5} = \frac{8}{100}$$

$100h_L - 250 = 16h_L + 28$

$84h_L = 278$

$h_L = 3.31$ m.

$$\frac{2.5 - h_r}{3.5 + 2h_r} = \frac{8}{100}$$

$250 - 100h_r = 28 + 16h_r$

$116h_r = 222$

$h_r = 1.91$ m.

$$A = \frac{1}{2}(3.5)(3.31) + \frac{1}{2}(2.5)(2 \times 3.31 + 3.5)$$
$$+ \frac{1}{2}(2.5)(3.5 + 2 \times 1.91) + \frac{1}{2}(3.5)(1.91)$$

$A = 30.935$

$A = \boldsymbol{31\ m^2}$

② Distance of the left slope stake from the center of the road.

$x_L = 2h_L + 3.5$

$x_L = 2(3.31) + 3.5$

$x_L =$ ***10.12 m.***

③ Distance of the right slope stake from the center of the road.

$x_R = 3.5 + 2h_r$

$x_R = 3.5 + 2(1.91)$

$x_R =$ ***7.32 m.***

721. CE Board Nov. 2008

Given the following data of the cross section of an earthworks.

Sta. 2 + 100

$$\frac{2.75}{9.5} \qquad \frac{1.5}{0} \qquad \frac{0.5}{5.0}$$

Sta. 2 + 120

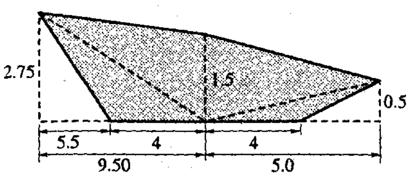
$$\frac{2.25}{9.0} \qquad \frac{1.0}{0} \qquad \frac{0.8}{5.6}$$

Width of base is 8 m.

① Compute the area of station 2 +100.
② Compute the area of station 2 + 120.
③ Compute the volume between stations using average area method.

Solution:

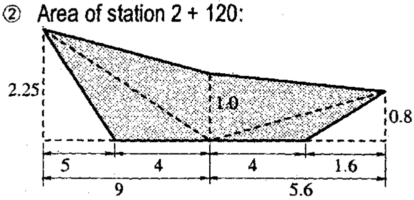
① Area of station 2 +100:



$$A_1 = \frac{4(2.75)}{2} + \frac{1.5(9.5)}{2} + \frac{1.5(5)}{2} + \frac{4(0.5)}{2}$$

$A_1 = \mathbf{17.375\ m^2}$

② Area of station 2 + 120:



$$A_2 = \frac{4(2.25)}{2} + \frac{1(9)}{2} + \frac{1(5.6)}{2} + \frac{4(0.80)}{2}$$

$A_2 = \mathbf{13.4\ m^2}$

③ Volume between stations:

$$V = \frac{(A_1 + A_2)}{2} L$$

$$V = \frac{(17.375 + 13.4)}{2}(20) = \mathbf{307.75\ cu.m.}$$

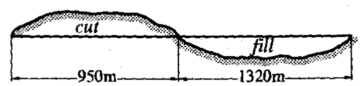
722. Problem

The longitudinal ground profile diagram and the grade line shows that the length of the cut is 950 m. and that of the fill is 1320 m. The road bed is 10 m. wide for cut and 8 m. wide for fill. The side slope is 1:1 for cut and 2:1 for fill. The profile areas between the ground line and the grade line are 8100 sq.m. for cut and 9240 sq.m. for fill.

① Find the volume of cut.
② Find the volume of fill.
③ If the shrinkage factor is 1.30, find the volume of borrow or waste.

Solution:

① Volume of cut:

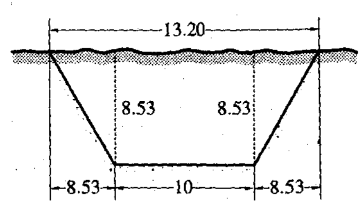


Average depth of cut:

$$C = \frac{1800}{950} = 8.53\text{ m.}$$

Average depth of fill:

$$f = \frac{9240}{1320} = 7\text{ m.}$$

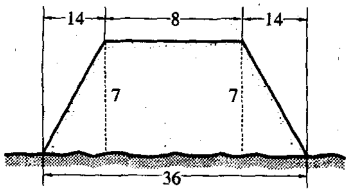


$$A_{cut} = \frac{(27.06 + 10)(8.53)}{2}$$

A_{rea} cut = 158.06 m^2

Volume of cut = 158.06(950)
Volume of cut = **150,157.86**

② Volume of fill:



$$A_{rea}\ \text{fill} = \frac{(36+8)(7)}{2}$$

A_{rea} fill = 154 m^2
Volume of fill = 154 (1320)
Volume of fill =***203,280***

③ Volume of borrow:
Vol. of fill required = 203,280(1.3)
Vol. of fill required = 264,264 m^3
Vol. of borrow = 264,264 – 150,157.86
Vol. of borrow = ***114,106.14 m³***

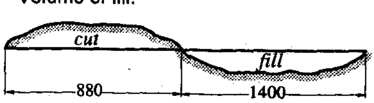
723. Problem

The longitudinal ground profile diagram and the grade line shows a length of cut and 880 m. and a length of fill of 1400 m. The profile areas are 8432 sq.m. for cut and 9240 sq.m. for fill. The widths of the road bed are 10 m. for cut and 8 m. for fill. The side slopes are 1:1 for cut and 2:1 for fill.

① Find the volume of fill.
② Find the volume of cut.
③ Find the volume of waste or borrow if the shrinkage factor is 1.25.

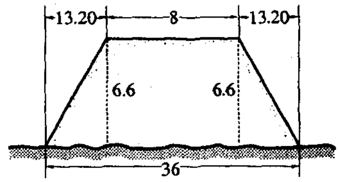
Solution:

① Volume of fill:



$$\text{Depth of cut} = \frac{8432}{880} = 9.64$$

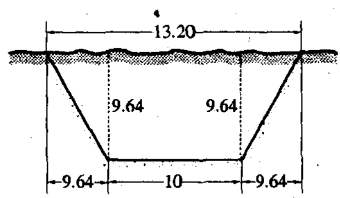
$$\text{Depth of fill} = \frac{9240}{1400} = 6.6$$



$$\text{Area of fill} = \frac{(34.40+8)}{2}(6.6)$$

Area of fill = 139.92 m^2
Vol. of fill = 139.92(1400)
Vol. of fill = ***195,888 m³***

② Volume of cut:



$$\text{Area of cut} = \frac{(29.28+10)}{2}(9.64)$$

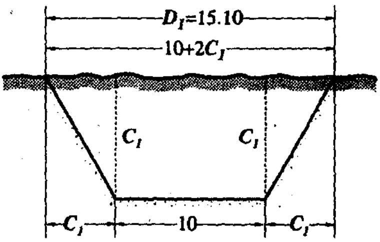
Area of cut = 189.33 m^2
Vol. of cut = 189.33(880)
Vol. of cut = ***166,610 m³***

③ Volume of borrow:
Vol. of borrow = 195,888(1.25) – 166,610
Vol. of borrow = ***78,250 m³***

724. Problem

The areas in cut of two irregular sections 40 m. apart are 32 sq.m. and 68 sq.m., respectively. The base width is 10 m. and the side slope is 1:1. Find the corrected volume of cut in cu.m. using the prismoidal correction formula.

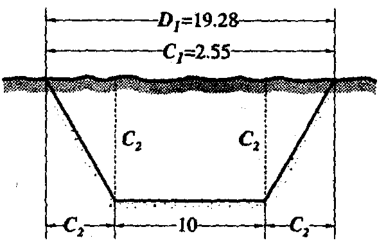
Solution:



$$A = \frac{(10 + 2C_1 + 10)}{2} C_1$$

$$32 = (10 + C_1)\, C_1$$

$$C_1^2 + 10C_1 - 32 = 0$$



$$A = \frac{(10 + 2C_2 + 10)}{2} C_2$$

$$68 = (10 + C_2)\, C_2$$

$$C_2^2 + 10C_2 - 68 = 0$$

$$C_2 = 4.64$$

$$V_c = \frac{L}{12}(D_1 - D_2)(C_1 - C_2)$$

$$V_c = \frac{40}{12}(15.10 - 19.28)(2.55 - 4.64)$$

$$V_c = 29.12 \text{ m}^3$$

$$V_E = \frac{(A_1 + A_2)\, L}{2}$$

$$V_E = \frac{(32 + 68)\, 40}{2}$$

$$V_E = 2000$$

$$V_{c_r} = V_E - V_c$$

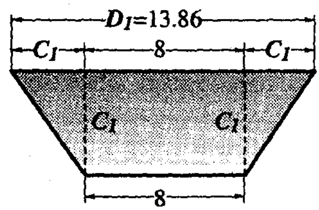
$$V_{c_r} = 2000 - 29.12$$

$$V_{c_r} = \boldsymbol{1970.88 \text{ m}^3}$$

725. Problem

Two irregular sections 50 m. apart have areas in cut of 32 sq.m. and 68 sq.m. respectively. Side slope is 1:1 and base width = 8 m. Using the Prismoidal correction formula, find the corrected volume of cut in cu.m.

Solution:



$$A_1 = \frac{(8 + 2c_1 + 8)\, C_1}{2}$$

$$32 = \frac{(16 + 2C_1)\, C_1}{2}$$

$$2C_1^2 + 16C_2 - 64 = 0$$

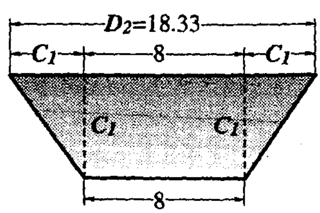
$$C_1^2 + 8C_1 - 32 = 0$$

$$C_1 = \frac{-8 \pm 13.86}{2}$$

$$C_1 = 2.93$$

$$D_1 = 8 + 2(2.93)$$

$$D_1 = 13.86$$



$$A_2 = \frac{(2\,C_2 + 8 + 8)\, C_2}{2}$$

$$68 = \frac{(2\,C_2 + 16)\, C_2}{2}$$

$$2C_2^2 + 16C_2 - 136 = -0$$

$$C_2^2 + 8C_2 - 68 = 0$$

$$C_2 = 5.17$$

$D_2 = 2(5.17) + 8$

$D_2 = 18.33$

$$V_c = \frac{L}{12}(C_1 - C_2)(D_1 - D_2)$$

$$V_c = \frac{50}{12}(2.93 - 5.17)(13.86 - 18.33)$$

$V_c = 41.72$

$$C_{\text{corrected vol.}} = \frac{(A_1 + A_2)L}{2} - V_c$$

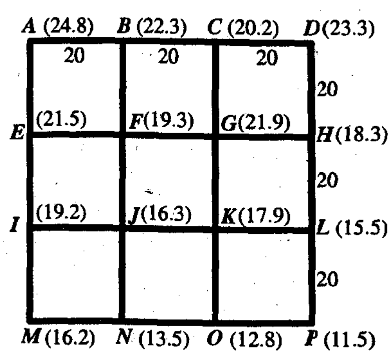
$$V_{cor} = \frac{(32 + 68)(50)}{2} - 41.72$$

$V_{cor} =$ ***2458.28.***

726. Problem

A square piece of land 60 m. x 60 m. is to be leveled down to 5 m. above elevation zero. To determine the volume of earth to be removed by the Borrow Pit Method the land is divided into 9 squares whose corners are arranged as follows with the corresponding elevations, in meters, above elevation zero. Find the volume of cut in cu.m. by unit area basis.

A = 29.8	B = 27.3	C = 25.2	D = 28.3
E = 26.5	F = 24.3	G = 26.9	H = 23.3
I = 24.2	J = 21.3	K = 22.9	L = 20.5
M = 21.2	N = 18.5	O = 117.8	P = 16.5



Solution:

$$V = \frac{A}{4}\left[\sum h_1 + 2\sum h_2 + 3\sum h_3 + 4\sum h_4\right]$$

$\sum h_1 = 24.8 + 23.3 + 16.2 + 1.5$

$\sum h_2 = 22.3 + 20.2 + 18.3 + 15.5 + 12.8$
$+ 13.5 + 19.2 + 21.5$

$\sum h_2 = 143.3$

$\sum h_3 = 0$

$\sum h_4 = 19.3 + 21.9 + 16.3 + 17.9$

$\sum h_4 = 75.4$

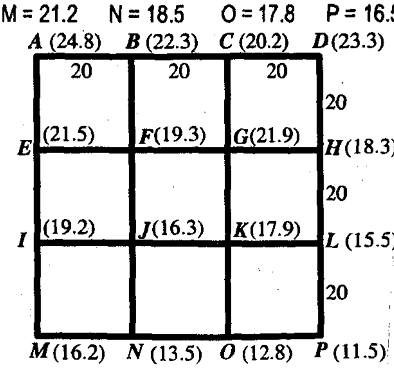
$$V = \frac{20(20)}{4}\left[75.8 + 2(143.3) + 3(0)\ 4(75.4)\right]$$

$V =$ ***66400 m³***

727. Problem

A square piece of land, 90 m x 90 m, is to be leveled down to 5 m above elevation zero. To determine the volume of earth to be removed by the Borrow Pit Method, the land is divide into 9 squares whose corners are arrange as follows with the corresponding elevations, in m., above elevation zero. Find the volume of cut, in cu.m., by unit area basis.

A = 29.8	B = 27.3	C = 25.2	D = 28.3
E = 26.5	F = 24.3	G = 26.9	H = 23.3
I = 24.2	J = 21.3	K = 22.9	L = 20.5
M = 21.2	N = 18.5	O = 17.8	P = 16.5



Solution:

$V = \frac{A}{4}\left[\sum h_1 + 2\sum h_2 + 3\sum h_3 + 4\sum h_4\right]$

$\sum h_1 = 24.8 + 23.3 + 16.2 + 1.5$

$\sum h_2 = 22.3 + 20.2 + 18.3 + 15.5 + 12.8 + 13.5 + 19.2 + 21.5$

$\sum h_2 = 143.3$

$\sum h_3 = 0$

$\sum h_4 = 19.3 + 21.9 + 16.3 + 17.9$

$\sum h_4 = 75.4$

$V = \frac{30(30)}{4}\left[75.8 + 2(143.3) + 0 + 4(75.4)\right]$

$V = \mathbf{149{,}400\ m^3}$

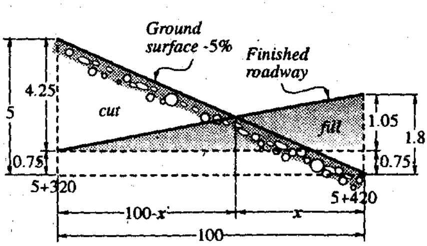
728. Problem

Center height of the road at STA 5 + 320 is 4.25 cut. At STA 5 + 420 it is 1.80 m. fill. The ground slopes uniformly at - 5% from STA 5 + 320.

① Find the grade of the finished road.
② How far in meters from STA 5 + 420 toward STA 5 + 320 will the filling extend?
③ How far, in meters from STA 5 + 320 toward STA 5 + 420 will the excavation extend?

Solution:

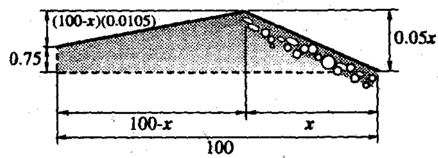
① Grade of the finished road.



Slope of finished roadway $= \frac{1.05}{100}$

Slope of finished roadway = ***0.0105***

② Distance from 5 + 420 where filling extend:



$(100 - x)(0.0105) + 0.75 = 0.05x$

$0.0605x = 0.75 + 100(0.0105)$

$x = \mathbf{29.75\ m.}$

(from 5 + 420 where filling will extend)

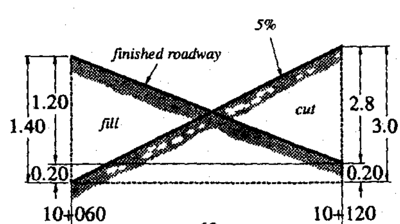
③ Distance from 5 + 320 where excavation will extend

$100 - x = \mathbf{70.25\ m.}$ *from 5 + 320 where excavation will extend*

729. Problem

From STA 10 + 060, with center height of 1.4 m fill, the ground makes a uniform slope of 5% to STA 10 + 120 whose center height is 2.8 m. cut.

① Find the grade of the finished road.
② If the roadway for fill is 9 m. wide and the side slope is 2:1, find the cross-sectional area of fill at STA 10 + 060 assuming that it is a level section.
③ If the roadway for cut is 10 m. and the side slope is 1.5:1, find the cross-sectional area for cut at STA 10 + 120 assuming that it is a level section.



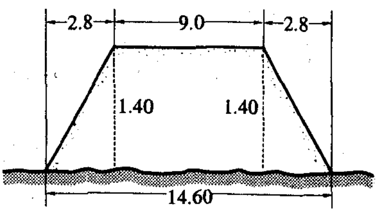
Solution:

① Grade of finished roadway:

$$S = \frac{-1.2}{60}(100)$$

$S = \textbf{-2\%}$

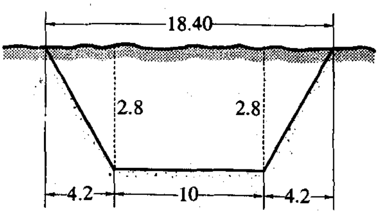
② Cross sectional area of fill at sta. 10 + 060



$$\text{Area} = \frac{(14.6 + 9)(1.4)}{2}$$

Area = ***16.52 m²***

③ Area at sta. 10 + 120



$$\text{Area} = \frac{(18.4 + 10)(2.8)}{2}$$

Area = ***39.76 m²***

MASS DIAGRAM

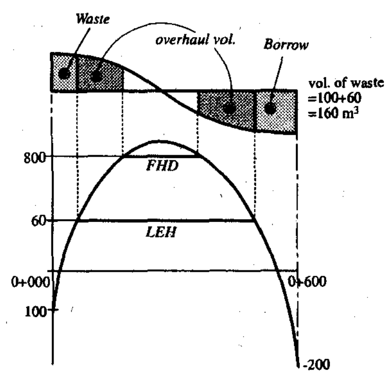
730. Problem

Using the following data of a mass diagram,

Length of economical haul = 450 m.
Free haul distance = 50 m.
Mass ordinate at initial point of mass diagram (sta) = - 100 m^3
Mass ordinate where length of economical haul intersects the mass diagram = 60 m^3
Mass ordinate where the free haul intersects the mass diagram = 800 m^3
Mass ordinate at the final station (0+600) = - 200 m^3

① Find the volume of waste.
② Find overhaul volume.
③ Find volume of borrow.

Solution:



① Volume of waste.

volume of waste = 100 + 60

Volume of waste = ***160 m³***

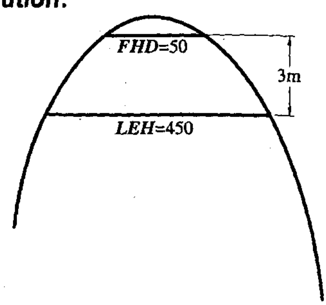
② Overhaul volume.
$V = 800 - 60$
$V = \mathbf{740\ m^3}$

③ Volume of borrow.
$V = 200 + 60$
$V = \mathbf{260\ m^3}$

731. Problem

The length of economical haul intersecting horizontally the summit mass diagram is 450 m. The free haul distance is 50 m. The vertical distance between the free haul distance and the limit of economical haul is 3 cm. The scale of the diagram is 1 cm. = 100 m³. Determine the volume of overhaul in m³.

Solution:

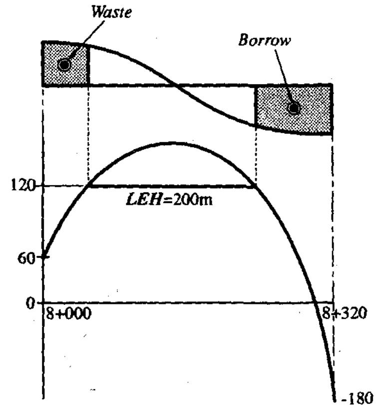


Vol. of overhaul = 3(100)
Vol. of overhaul = ***300 m³***

732. Problem

The mass diagram for an earthwork starts from sta (8 + 000) where the mass ordinate is + 60 cu.m. and ends at sta. (8 + 320) where the mass ordinate is - 180 cu.m. The length of economical haul, plotted horizontally, intersects the summit mass diagram at a point where the mass ordinate is + 120 cu.m. The length of economical haul is 200 m.

① Determine the volume of borrow in m³.
② Determine the volume of waste in m³.
③ Determine the volume of overhaul if the length of the free haul plotted horizontally intersects the mass diagram at a point where the mass ordinate is 300 cu.m. length of free haul distance is 50.



Solution:

① Volume of borrow in m³.
Volume of borrow = 120 + 180
Volume of borrow = ***300 m³***

② Volume of waste.
$V = 120 - 60$
$V = \mathbf{60\ m^3}$

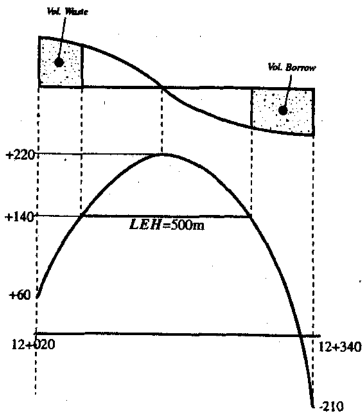
③ Volume of over haul:
$V = 300 - 120$
$V = \mathbf{180\ m^3}$

733. Problem

The mass diagram for an earthwork starts from STA (12 + 010) where the mass ordinate is + 60 cu.m. and ends at STA (12 + 340) where the mass ordinate is - 210 cu.m. The length of economical haul, plotted horizontally, intersects the summit mass diagram at a point where the mass ordinate is + 140 cu.m.

① What is the volume of the borrow, in cu.m.?
② What is the volume of waste.
③ If the summit has a mass ordinate of +240, what is the volume of over haul.

Solution:



① Vol. of borrow:
Vol. of borrow = 140 + 210
Vol. of borrow = ***350 m³***

② Volume of waste:
Vol. of waste = 140 – 60
Vol. of waste = ***80 m³***

③ Vol. of overhaul:
Overhaul volume = 240 – 140
Overhaul volume = ***100 m³***

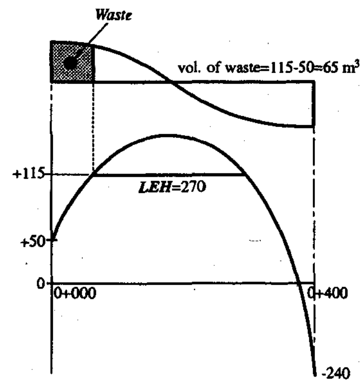
734. Problem

The following are the data on a single-summit mass diagram.

STA	Mass Ordinate (m^3)
0 + 000	+ 50
0 + 400	- 240

Length of economical haul = 270 m.
Initial point of limit of economical haul is + 115
What is the volume of waste in cu.m.?

Solution:



Volume of waste = 115 - 50
Volume of waste = ***65 m³***

735. Problem

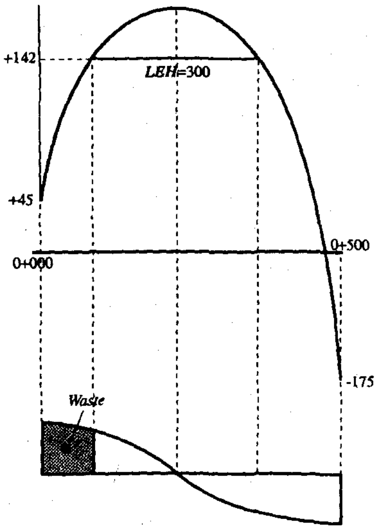
Using the following data on a single summit mass diagram:

STA	MASS ORDINATE (m^3)
0 + 000	+ 45
0 + 500	- 175

Initial point of limit of economic haul = + 142
Length of economic haul = 300 m.

① Find the volume of waste .
② Find the volume of borrow.

Solution:



Volume of waste = 142 - 45
Volume of waste = 97 cu.m.

Volume of borrow = 175 + 142
Volume of borrow = ***317 cu.m.***

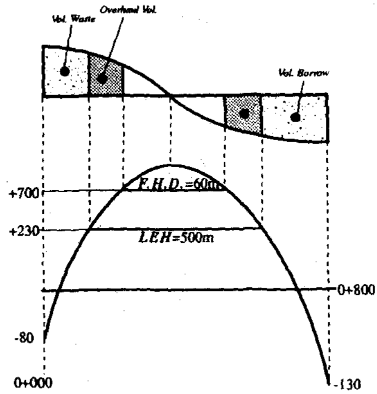
736. Problem

A single-summit mass diagram has the following data:

STA	MASS ORDINATE (m^3)
0 + 000	- 80
0 + 800	- 130

Initial pointof freehaul distance = + 700
Initial point of economic haul = + 230
Freehaul distance = 60 m.
Limit economical haul = 400

① Determine the overhaul volume, in cu.m.
② Determine the volume of waste.
③ Determine the volume of borrow.



Solution:

① Overhaul volume:
Overhaul volume = 700 – 230
Overhaul volume = ***470 m^3***

② Volume of waste:
Volume of waste = 230 + 80
Volume of waste = ***310 m^3***

③ Volume of borrow:
Volume of borrow = 230 + 130
Volume of borrow = ***360 m^3***