

# MATH113

# DIFFERENTIAL CALCULUS

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# 14 TIME RATES

If a quantity  $x$  is a function of time  $t$ , the time rate of change of  $x$  is given by  $\frac{dx}{dt}$ .

When two or more quantities, all functions of time  $t$ , are related by an equation, the relation between their rates of change may be obtained by differentiating both sides of the equation with respect to time  $t$ .

## Basic Time Rates:

Velocity:

$$v = \frac{ds}{dt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Discharge:

$$Q = \frac{dV}{dt}$$

Angular Speed:

$$w = \frac{d\theta}{dt}$$

Angular Acceleration:

$$\alpha = \frac{dw}{dt} = \frac{d^2\theta}{dt^2}$$

Where:

$s$  – Distance

$v$  – Velocity

$V$  – Volume

$\theta$  – Angle of rotation

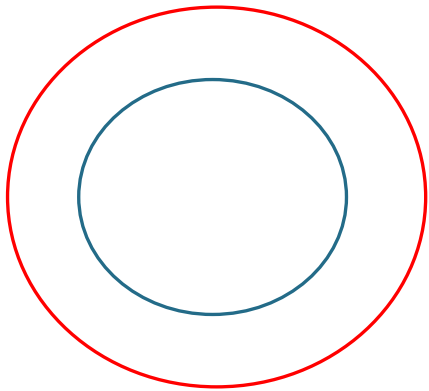
$w$  – Angular Speed

$\alpha$  – Angular Acceleration

$a$  – Acceleration

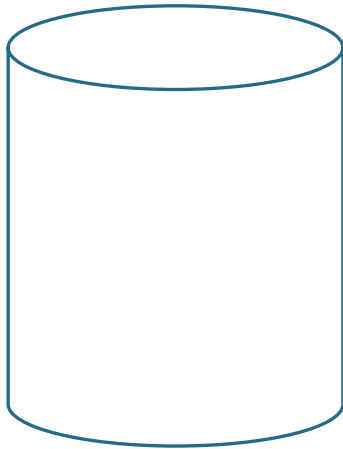
## EXAMPLE 44 Time Rates

**QUESTION:** How fast is the volume of a sphere changing when the radius is 20cm. If the radius is changing at a rate of one (1) mm per second.



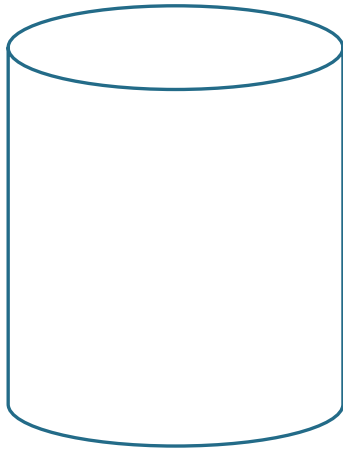
## EXAMPLE 45 Time Rates

**QUESTION:** Oil flows into a vertical cylinder tank at 500 cc/s. The oil level rises at 0.15 cm/s. Determine the diameter of the tank in cm.



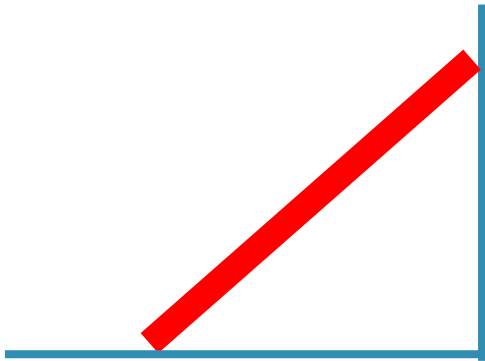
## EXAMPLE 46 Time Rates

**QUESTION:** Water flows at a rate of 2000 cc/s into a vertical cylindrical tank 120cm in diameter and 6m high. How fast is the water level rising?



## EXAMPLE 47 Time Rates

**QUESTION:** The upper end of a 3m pipe leans against the vertical wall, while the lower end is on a level concrete pavement extending to the wall. The lower end slides away at a constant rate of 2 cm/s. How fast is the upper end moving down on the wall in cm/s when the lower end is 2m away from the wall?



**EXAMPLE 48****Parametric Equations****QUESTION:**

When  $x = 3t^2 + 2t$  and  $y = 5t^3$ , determine  $\frac{dy}{dx}$ .

**EXAMPLE 49****Parametric Equations****QUESTION:**

When  $x = \tan 2t$  and  $y = \cot 2t$ , determine  $\frac{dy}{dx}$ .