

MATH113

DIFFERENTIAL CALCULUS

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If a quantity x is a function of time t , the time rate of change of x is given by $\frac{dx}{dt}$.

When two or more quantities, all functions of time t , are related by an equation, the relation between their rates of change may be obtained by differentiating both sides of the equation with respect to time t .

Basic Time Rates:

Velocity:

$$v = \frac{ds}{dt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Discharge:

$$Q = \frac{dV}{dt}$$

Angular Speed:

$$\omega = \frac{d\theta}{dt}$$

Angular Acceleration:

$$\alpha = \frac{dw}{dt} = \frac{d^2\theta}{dt^2}$$

Where:

s – Distance

v – Velocity

V – Volume

θ – Angle of rotation

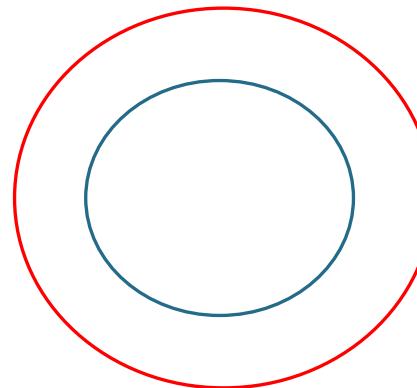
ω – Angular Speed

α – Angular Acceleration

a – Acceleration

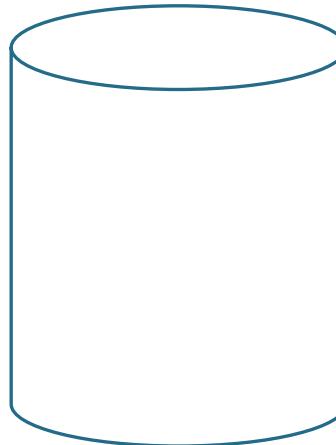
EXAMPLE 44 Time Rates

QUESTION: How fast is the volume of a sphere changing when the radius is 20cm. If the radius is changing at a rate of one (1) mm per second.



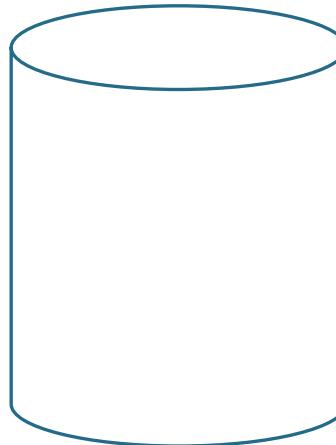
EXAMPLE 45 Time Rates

QUESTION: Oil flows into a vertical cylinder tank at 500 cc/s. The oil level rises at 0.15 cm/s. Determine the diameter of the tank in cm.



EXAMPLE 46 Time Rates

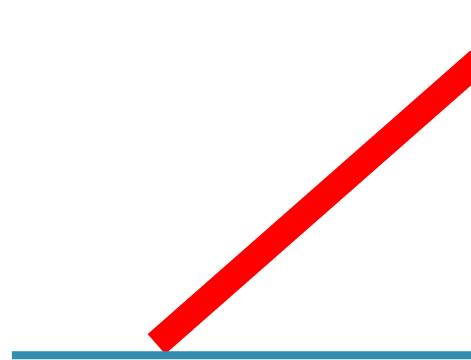
QUESTION: Water flows at a rate of 2000 cc/s into a vertical cylindrical tank 120cm in diameter and 6m high. How fast is the water level rising?



EXAMPLE 47

Time Rates

QUESTION: The upper end of a 3m pipe leans against the vertical wall, while the lower end is on a level concrete pavement extending to the wall. The lower end slides away at a constant rate of 2 cm/s. How fast is the upper end moving down on the wall in cm/s when the lower end is 2m away from the wall?



EXAMPLE 48 Parametric Equations

QUESTION:

When $x = 3t^2 + 2t$ and $y = 5t^3$, determine $\frac{dy}{dx}$.

EXAMPLE 49 Parametric Equations

QUESTION:

When $x = \tan 2t$ and $y = \cot 2t$, determine $\frac{dy}{dx}$.