Exercise 4.1

7. Sorting the list C, O, N, Q, U, E, R in alphabetical order with insertion sort:

$$\begin{array}{lllllll}
C & O & N & Q & U & E & R \\
C\,| & \mathbf{O} & & & & & \\
C & O\,| & \mathbf{N} & & & & \\
C & N & O\,| & \mathbf{Q} & & & \\
C & N & O & Q\,| & \mathbf{U} & & \\
C & N & O & Q & U\,| & \mathbf{E} & \\
C & E & N & O & Q & U\,| & \mathbf{R} \\
C & E & N & O & Q & R & U
\end{array}$$

11. a. The largest number of inversions for $A[i]$ $(0 \le i \le n-1)$ is $n-1-i$; this happens if $A[i]$ is greater than all the elements to the right of it. Therefore, the largest number of inversions for an entire array happens for a strictly decreasing array. This largest number is given by the sum:

$$\sum_{i=0}^{n-1}(n-1-i) = (n-1)+(n-2)+\cdots+1+0 = \frac{(n-1)n}{2}.$$

The smallest number of inversions for $A[i]$ $(0 \le i \le n-1)$ is 0; this happens if $A[i]$ is smaller than or equal to all the elements to the right of it.

Therefore, the smallest number of inversions for an entire array will be 0 for nondecreasing arrays.

Exercise 4.3

2. a. The permutations of {a1, a2, a3, a4} generated by the bottom-up minimal change algorithm:

start	1
insert a2 into a1 right to left	a1a2 a2a1
insert a3 into a1a2 right to left	a1a2a3 a1a3a2 a3a1a2
insert a3 into a2a1 left to right	a3a2a1 a2a3a1 a2a1a3
insert a4 into a1a2a3 right to left	a1a2a3a4 a1a2a4a3 a1a4a2a3 a4a1a2a3
insert a4 into a1a3a2 left to right	a4a1a3a2 a1a4a3a2 a1a3a4a2 a1a3a2a4
insert a4 into a3a1a2 right to left	a3a1a2a4 a3a1a4a2 a3a4a1a2 a4a3a1a2
insert a4 into a3a2a1 left to right	a4a3a2a1 a3a4a2a1 a3a2a4a1 a3a2a1a4
insert a4 into a2a3a1 right to left	a2a3a1a4 a2a3a4a1 a2a4a3a1 a4a2a31
insert a4 into a2a1a3 left to right	a4a2a1a3 a2a4a1a3 a2a1a4a3 a2a1a3a4

b. The permutations of {a1, a2, a3, a4} generated by the Johnson-Trotter algorithm.

a1a2a3a4	a1a2a4a3	a1a4a2a3	a4a1a2a3
a4a1a3a2	a1a4a3a2	a1a3a4a2	a1a3a2a4
a3a1a2a4	a3a1a4a2	a3a4a1a2	a4a3a1a2
a4a3a2a1	a3a4a2a1	a3a2a4a1	a3a2a1a4
a2a3a1a4	a2a3a4a1	a2a4a3a1	a4a2a3a1
a4a2a1a3	a2a4a1a3	a2a1a4a3	a2a1a3a4

c. The permutations of {a1, a 2, a3, a4} generated in lexicographic order. (Read horizontally.)

a1a2a3a4	a1a2a4a3	a1a3a2a4	a1a3a4a2	a1a4a2a3	a1a4a3a2
a2a1a3a4	a2a1a4a3	a2a3a1a4	a2a3a4a1	a2a4a1a3	a2a4a3a1
a3a1a2a4	a3a1a4a2	a3a2a1a4	a3a2a4a1	a3a4a1a2	a3a4a2a1
a4a1a2a3	a4a1a3a2	a4a2a1a3	a4a2a3a1	a4a3a1a2	a4a3a2a1

4. a. For $n = 2$:

12 21

For $n = 3$ (read along the rows):

123	213
312	132
231	321

For $n = 4$ (read along the rows):

1234	2134	3124	1324	2314	3214
4231	2431	3421	4321	2341	3241
4132	1432	3412	4312	1342	3142
4123	1423	2413	4213	1243	2143

9. a. The Gray code for $n = 3$ is given at the end of the section:

$$000 \quad 001 \quad 011 \quad 010 \quad 110 \quad 111 \quad 101 \quad 100.$$

Following the $BRGC(n)$ algorithm, we obtain the binary reflected Gray code for $n = 4$ as follows:

$L1$ 000 001 011 010 110 111 101 100

$L2$ 100 101 111 110 010 011 001 000

L 0000 0001 0011 0010 0110 0111 0101 0100 1100 1101 1111 1110 1010 1011 1001 1000