Exercise 7.1

3. Assuming that the set of possible list values is {a, b, c, d}, sort the following list in alphabetical order by the distribution-counting algorithm: b, c, d, c, b, a, a, b

Sol:

3. Input: A: $b,\ c,\ d,\ c,\ b,\ a,\ a,\ b$

Frequencies

a	b	c	d
2	3	2	1

Distribution values

a	b	c	d
2	5	7	8

	$D[a..d]$				$S[0..7]$							
$A[7]=b$	2	**5**	7	8					b			
$A[6]=a$	**2**	4	7	8		a						
$A[5]=a$	**1**	4	7	8	a							
$A[4]=b$	0	**4**	7	8				b				
$A[3]=c$	0	3	**7**	8							c	
$A[2]=d$	0	3	6	**8**								d
$A[1]=c$	0	3	**6**	7						c		
$A[0]=b$	0	**3**	5	7			b					

Exercise 7.3

1. For the input 40, 60, 37, 83, 42, 18 and hash function h(K) = K mod 11

   a. construct the open hash table.

Sol:

The list of keys: 40, 60, 37, 83, 42, 19

The hash function: $h(K) = K \bmod 11$

The hash addresses:

K	40	60	37	83	42	19
$h(K)$	7	5	4	6	9	8

The open hash table:

0	1	2	3	4	5	6	7	8	9	10
				↓	↓	↓	↓	↓	↓	
				37	60	83	40	19	42	

2. For the input 40, 60, 37, 83, 42, 18 and hash function h(K) = K mod 11

   a. construct the closed hash table.

Sol:

The list of keys: 40, 60, 37, 83, 42, 19

The hash function: $h(K) = K \bmod 11$

The hash addresses:

K	40	60	37	83	42	19
$h(K)$	7	5	4	6	9	8

The open hash table:

0	1	2	3	4	5	6	7	8	9	10
							40			
					60		40			
				37	60		40			
				37	60	83	40			
				37	60	83	40		42	
				37	60	83	40	19	42	

Exercise 8.2

1. a. Apply the bottom-up dynamic programming algorithm to the following instance of the knapsack problem:

item	weight	value
1	3	\$25
2	2	\$20
3	1	\$15
4	4	\$40
5	5	\$50

, capacity $W = 6$.

Sol:

		capacity j						
	i	0	1	2	3	4	5	6
	0	0	0	0	0	0	0	0
$w_1 = 3,\ v_1 = 25$	1	0	0	0	25	25	25	25
$w_2 = 2,\ v_2 = 20$	2	0	0	20	25	25	45	45
$w_3 = 1,\ v_3 = 15$	3	0	15	20	35	40	45	60
$w_4 = 4,\ v_4 = 40$	4	0	15	20	35	40	55	60
$w_5 = 5,\ v_5 = 50$	5	0	15	20	35	40	55	65

The maximal value of a feasible subset is $F[5, 6] = 65$. The optimal subset is {item 3, item 5}.

6. Apply the memory function method to the instance of the knapsack problem given in Problem 1. Indicate the entries of the dynamic programming table that are (i) never computed by the memory function method on this instance, (ii) retrieved without a recomputation.

Sol:

In the table below, the cells marked by a minus are the ones for which no entry is computed for the instance in question; the only nontrivial entry that is retrieved without recomputation is $(2, 1)$.

		capacity j						
	i	0	1	2	3	4	5	6
	0	0	0	0	0	0	0	0
$w_1 = 3,\ v_1 = 25$	1	0	0	0	25	25	25	25
$w_2 = 2,\ v_2 = 20$	2	0	0	20	-	-	45	45
$w_3 = 1,\ v_3 = 15$	3	0	15	20	-	-	-	60
$w_4 = 4,\ v_4 = 40$	4	0	15	-	-	-	-	60
$w_5 = 5,\ v_5 = 50$	5	0	-	-	-	-	-	65

Exercise 8.4

1. Apply Warshall's algorithm to find the transitive closure of the digraph defined by the following adjacency matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Sol:.

Applying Warshall's algorithm yields the following sequence of matrices:

$$R^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \mathbf{1} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^{(4)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \mathbf{1} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = T$$

7. Solve the all-pairs shortest-path problem for the digraph with the weight matrix:

$$\begin{bmatrix} 0 & 3 & \infty & 2 & 6 \\ 5 & 0 & 4 & 2 & \infty \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & 1 & 0 & 4 \\ 5 & \infty & \infty & \infty & 0 \end{bmatrix}$$

Sol:

Applying Floyd's algorithm to the given weight matrix generates the following sequence of matrices:

$$D^{(0)} = \begin{bmatrix} 0 & 3 & \infty & 2 & 6 \\ 5 & 0 & 4 & 2 & \infty \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & \infty & 0 & 4 \\ 5 & \infty & \infty & \infty & 0 \end{bmatrix} \qquad D^{(1)} = \begin{bmatrix} 0 & 3 & \infty & 2 & 6 \\ 5 & 0 & 4 & 2 & 11 \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & \infty & 0 & 4 \\ 5 & 8 & \infty & & 0 \end{bmatrix}$$

In this manner, find up to D(5).

Exercise 9.4

1. a. Construct a Huffman code for the following data:

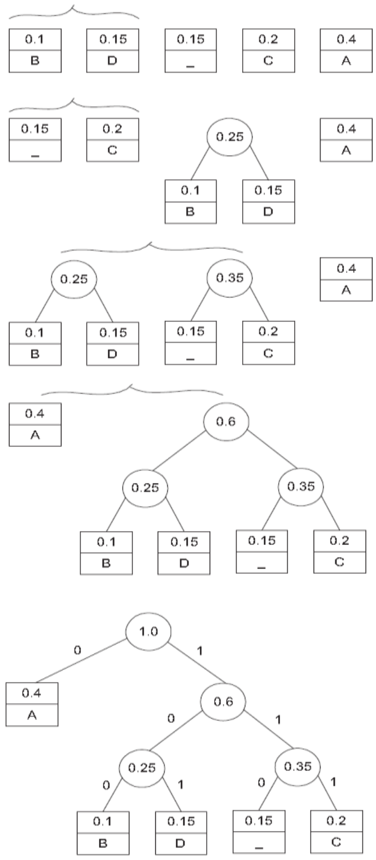
character	A	B	C	D	_
probability	0.4	0.1	0.2	0.15	0.15

b. Encode the text ABACABAD using the code of question a.

c. Decode the text whose encoding is 100010111001010 in the code of question a.

Sol:

a.



character	A	B	C	D	_
probability	0.4	0.1	0.2	0.15	0.15
codeword	0	100	111	101	110

b. The text ABACABAD will be encoded as 0100011101000101.

c. With the code of part a, 100010111001010 will be decoded as

$$\underset{B}{100}|\underset{A}{0}|\underset{D}{101}|\underset{\_}{110}|\underset{A}{0}|\underset{D}{101}|\underset{A}{0}$$