Introduction to

The Design and Analysis of Algorithms

3rd Edition

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Greedy Technique

Let us revisit the *change-making problem* faced, at least subconsciously, by millions of cashiers all over the world: give change for a specific amount n with the least number of coins of the denominations $d_1 > d_2 > \ldots > d_m$ used in that locale. For example, the widely used coin denominations in the United States are $d_1 = 25$ (quarter), $d_2 = 10$ (dime), $d_3 = 5$ (nickel), and $d_4 = 1$ (penny). How would you give change with coins of these denominations of, say, 48 cents?

If you came up with the answer 1 quarter, 2 dimes, and 3 pennies, you followed—consciously or not—a logical strategy of making a sequence of best choices among the currently available alternatives.

Indeed, in the first step, you could have given one coin of any of the four denominations. "Greedy" thinking leads to giving one quarter because it reduces the remaining amount the most, namely, to 23 cents. In the second step, you had the same coins at your disposal, but you could not give a quarter, because it would have violated the problem's constraints. So your best selection in this step was one dime, reducing the remaining amount to 13 cents. Giving one more dime left you with 3 cents to be given with three pennies.
The approach applied in the opening paragraph to the change-making problem is called *greedy*.

The greedy **approach suggests constructing a solution through a sequence of steps, each expanding a partially constructed solution obtained so far, until a complete solution to the problem is reached. On each step—and this is the central point of this technique—the choice made must be:**

- ***feasible* (可行的)**, i.e., it has to satisfy the problem's constraints
- ***locally optimal* (局部最優), i.e., it has to be the best local choice among all feasible choices available on that step**
- ***irrevocable* (不可撤回的), i.e., once made, it cannot be changed on subsequent steps of the algorithm**