Selection Bias

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September 10, 2024

Observed Differences and Selection Bias

- In many applications, we might be interested in the ATE or the ATT
- These causal effects are defined by **comparing potential outcomes**
- But we only know:
  - Whether an individual was treated or not
  - The **observed outcome** of the individual

Observed Differences and Selection Bias

- If we do not implement any causal inference methods, we can only establish association (correlation)
- **Naive solution**: Comparing observed difference in outcome between treated and untreated individuals

$$\begin{aligned} \text{ODO} &= \mathrm{E}[\mathrm{Y}_i|D_i=1] - \mathrm{E}[\mathrm{Y}_i|D_i=0] \\ &= \underbrace{\frac{1}{N_1}\sum_{i:D_i=1}[\mathrm{Y}_i|D_i=1]}_{\text{Treated units}} - \underbrace{\frac{1}{N_0}\sum_{i:D_i=0}[\mathrm{Y}_i|D_i=0]}_{\text{Untreated units}} \end{aligned}$$

Observed Differences and Selection Bias

- The observed difference in outcome usually mix up causal effect (ATT) and selection bias

$$\underbrace{\mathrm{E}[\mathrm{Y}_i|D_i = 1] - \mathrm{E}[\mathrm{Y}_i|D_i = 0]}_{\text{ODO}}$$

$$= \mathrm{E}[\mathrm{Y}_i^1|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 1] + \mathrm{E}[\mathrm{Y}_i^0|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 0]$$

$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|D_i = 1]}_{\text{ATT}} + \boxed{\underbrace{\mathrm{E}[\mathrm{Y}_i^0|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 0]}_{\text{Selection Bias}}}$$

- **Selection Bias** implies:
  - The value of potential outcomes for two groups are different **even if both groups were under the same untreated scenario**
    - $\mathrm{E}[\mathrm{Y}_i^0|D_i = 1] \neq \mathrm{E}[\mathrm{Y}_i^0|D_i = 0]$
  - There are genuine differences between treated and untreated individuals

Observed Differences and Selection Bias

A Numerical Example

- The observed differences in outcome between treated and untreated individuals

i	D_i	Y_i^1	Y_i^0	Y_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	1	3	?	3	?
Tina	1	2	?	2	?
Mary	0	?	1	1	?
Bill	0	?	1	1	?
$E[\mathrm{Y}_i \mid D_i = 1]$				2.5	
$E[\mathrm{Y}_i \mid D_i = 0]$				1	

$$\underbrace{\mathrm{E}[\mathrm{Y}_i \mid D_i = 1] - \mathrm{E}[\mathrm{Y}_i \mid D_i = 0]}_{\text{ODO}} = 1.5$$

Observed Differences and Selection Bias

A Numerical Example

- But we are interested in causal effect (ATT):

i	D_i	Y_i^1	Y_i^0	Y_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	1	3	2	3	1
Tina	1	2	1	2	1
Mary	0	1	1	1	0
Bill	0	1	1	1	0
$E[\mathrm{Y}_i^1 \| D_i = 1]$		2.5			
$E[\mathrm{Y}_i^0 \| D_i = 1]$			1.5		

- Compare the difference in potential outcomes for treated individuals

$$\underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 1]}_{\text{ATT}} = 1$$

Observed Differences and Selection Bias

A Numerical Example

$$\underbrace{\mathrm{E}[\mathrm{Y}_i|D_i = 1] - \mathrm{E}[\mathrm{Y}_i|D_i = 0]}_{\text{ODO (1.5)}}$$

$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|D_i = 1]}_{\text{ATT (1)}} + \underbrace{\mathrm{E}[\mathrm{Y}_i^0|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 0]}_{\text{Selection Bias (0.5)}}$$

Observed Differences and Selection Bias

A Numerical Example

i	D_i	Y_i^1	Y_i^0	Y_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	1	3	2	3	1
Tina	1	2	1	2	1
Mary	0	1	1	1	0
Bill	0	1	1	1	0
$E[\mathrm{Y}_i^0 \mid D_i = 1]$			1.5		
$E[\mathrm{Y}_i^0 \mid D_i = 0]$			1		

- Here, selection bias is positive (0.5 million NT$)

$$\underbrace{\mathrm{E}[\mathrm{Y}_i^0 \mid D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0 \mid D_i = 0]}_{\text{Selection Bias}} = 0.5$$

- Those who attend graduate school could be more intelligent so they can earn more even if they did not attend graduate school

- Economists have long been concerned with selection bias arising from selection to treatment
  - Since the core of economics is analyzing how people make a choice – select a treatment
- When individuals self-select into a treatment based on observable and unobservable characteristics
  - It leads to systematic differences between the treated and untreated groups
  - Failure to account for this non-random selection process can result in biased estimates of the causal effect
- Many economics models describe this non-random selection process

Selection to Treatment

Generalized Roy Model

- Roy (1951) provides a framework for understanding how individuals self-select into different working sectors (treatments)
  - Based on their comparative advantages and expected returns.
  - It is a cornerstone of the literature in applied economics and policy evaluation
- This framework has been applied and extended to a wide range of other contexts
  - See Heckman and Taber (2008); Heckman and Vytlacil (2007a,b); Heckman and Pinto (2023)

- The Generalized Roy Model is characterized by the following equations:
  - Potential outcome in treatment 1: $\mathrm{Y}_i^1 = g^1(X) + U^1$
  - Potential outcome in treatment 0: $\mathrm{Y}_i^0 = g^0(X) + U^0$
    - X are observed factors affecting outcomes and choice
    - U are unobserved factors affecting outcomes and choice
  - Cost: $C = g_c(Z, X) + U_c$
    - Z serves as an instrumental variable
    - Could be external policy or any exogenous factors cause C change
  - Treatment choice: $D = \mathbf{1}\{\mathrm{Y}_i^1 - \mathrm{Y}_i^0 - C \geq 0\}$
    - Choose treatment 1 when $\mathrm{Y}_i^1 \geq \mathrm{Y}_i^0 + C \Rightarrow D = 1$ 唸研究所的好處
    - Choose treatment 0 when $\mathrm{Y}_i^1 < \mathrm{Y}_i^0 + C \Rightarrow D = 0$ 不唸 〃

Selection to Treatment

Generalized Roy Model

- People make different treatment choices based on their value of potential outcomes
- This self-selection behavior would result in selection bias:
  - $\mathrm{E}[\mathrm{Y}_i^0 | D_i = 1] \neq \mathrm{E}[\mathrm{Y}_i^0 | D_i = 0]$ 造成 selection bias
  - $\mathrm{E}[\mathrm{Y}_i^1 | D_i = 1] \neq \mathrm{E}[\mathrm{Y}_i^1 | D_i = 0]$
- The differences in potential outcomes between treated and untreated group reflect their observed and unobserved characteristics are different

Selection to Treatment

Generalized Roy Model

- The Roy model actually highlights possible strategies to eliminate selection bias

1. Do not allow individuals to self-select into treatment
   - Randomized Controlled Trials (RCTs)
2. Control for all possible observed confounding factors X and assume no unobserved confounding factors U
   - Matching, regression, causal machine learning
3. Exploit exogenous variation in treatment D induced by an instrumental variable Z
   - Difference-in-Differences (DID), Instrumental Variables (IV), Regression Discontinuity Design (RDD)

Causal Effect and Identification Strategy

- Identification strategy tells us what we can learn about a **causal effect** from the **observed data**
  - The main goal of identification strategy is to eliminate the **selection bias**
- Identification depends on **assumptions**, not on estimation strategies
  - Estimation strategies: OLS, MLE, GMM
  - If an effect is not identified, no estimation method will recover it
- "**What's your identification strategy?**" =
  - What are the assumptions that allow you to claim you've estimated a causal effect?

Suggested Readings

- Chapter 1 and 2, Mastering Metrics: The Path from Cause to Effect
- Chapter 2, Mostly Harmless Econometrics
- Chapter 4, Causal Inference: The Mixtape