Fixed Effects Regression

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Observables and Unobservables Confounding Factors

- The main problem we face in estimating causal effect is that:
    - Each individual, firm, state, or country can select treatment
        - This choice could be correlated with factors that affect the outcomes of interest, which results in selection bias
- So far the key strategy to obtain causal effect was to control for **observed** confounding factors
- Yet, what if important confounding factors are unobserved?

Fixed Effects Regression: Main Idea

Fixed Effects Regression

- If unobserved confounding factors are **time-invariant**
  - If we have multiple time periods panel data or cross-sectional data
  - We can still obtain causal effect by estimating a regression that include many fixed effects in the model

- Suppose we are interested in the question whether joining union increase workers' earnings
- We might want to estimate the following regression:

$$Y_{it} = \delta + \alpha D_{it} + A_i'\gamma + X_{it}'\beta + \varepsilon_{it}$$

  - Y_{it} is outcome variable: earnings
  - D_{it} is treatment variable: union status
  - X_{it} are observed time-varying covariates: experience, education
  - A_i is unobserved but fixed confounder (time-invariant): ability or personality
  - Assume $E[\varepsilon_{it}|A_i, X_{it}] = 0$

- This regression equation implies the following potential outcomes:

$$Y_{it}^0 = \delta + A_i'\gamma + X_{it}'\beta + \varepsilon_{it}$$

$$Y_{it}^1 = Y_{it}^0 + \alpha$$

- Because A_i is unobservable, we are not able to directly include it in the regression

$$Y_{it} = \delta + \alpha D_{it} + X_{it}'\beta + \underbrace{A_i'\gamma + \varepsilon_{it}}_{u_{it}}$$

- If A_i is correlated with union status D_{it}
  - There is a correlation of D_{it} with the error term u_{it}
  - This will lead to **omitted variable bias**

Fixed Effects Regression

Example

- Address this problem by including λ_i in the regression
  - $\lambda_i = \delta + A_i'\gamma$
  - That is, we can consider λ_i as individual-specific constant term
- We estimate the following regression with individual fixed effects

$$Y_{it} = \lambda_i + \alpha D_{it} + X_{it}'\beta + \varepsilon_{it} \tag{1}$$

- Therefore, D_{it} and the error term ε_{it} would be uncorrelated
- Then, OLS estimate of α is unbiased

- In practice, there are two ways of estimating this fixed effects model:

  1. Demeaning approach (sometimes called “within estimator”)
  2. Regression with ‘N-1 dummy variables”

Demeaning Approach

1. Calculate individual averages of the outcome variable and all covariates (over time)

$$\bar{Y}_i = \bar{\lambda}_i + \alpha \bar{D}_i + \bar{X}_i' \beta + \bar{\varepsilon}_i$$

2. Subtract these averages from regression equation (1):

$$Y_{it} - \bar{Y}_i = \alpha(D_{it} - \bar{D}_i) + (X_{it} - \bar{X}_i)' \beta + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

- Note that λ_i drops out since it is time-invariant
- Therefore the error ε_{it} and the treatment D_{it} would no longer be correlated.

Regression with 'N-1 dummy variables"

$$Y_{it} = \delta + \sum_{i=2}^{N} \rho_i B_i + \alpha D_{it} + X'_{it}\beta + \varepsilon_{it}$$

- B_i is a dummy indicating individual i
- We only include $N-1$ individual dummies to avoid collinearity
- We show that this representation is actually the same as a regression with fixed effects λ_i

$$Y_{it} = \lambda_i + \alpha D_{it} + X'_{it}\beta + \varepsilon_{it}$$

Regression with 'N-1 dummy variables"

- Suppose we have three individuals in the sample so that we estimate the following regression:

$$Y_{it} = \delta + \beta_2 B_2 + \beta_3 B_3 + \alpha D_{it} + \varepsilon_{it},$$

  - $B_2 = 1$ indicates this sample is individual 2, $B_2 = 0$ otherwise
  - $B_3 = 1$ indicates this sample is individual 3, $B_3 = 0$ otherwise
  - D_{it} is a continuous treatment variable (e.g. schooling years)

Regression with ‘N-1 dummy variables”

- For individual 2, the regression can be:

$$
\begin{aligned}
Y_{2t} &= \delta + \beta_2 B_2 + \alpha D_{2t} + \varepsilon_{2t} \\
&= (\delta + \beta_2 B_2) + \alpha D_{2t} + \varepsilon_{2t}
\end{aligned}
$$

  or

$$
Y_{2t} = \lambda_2 + \alpha D_{2t} + \varepsilon_{2t}
$$

- where $\lambda_2 = \delta + \beta_2 B_2$

Regression with ‘N-1 dummy variables”

- For individual 3, the regression can be:

$$\begin{aligned} Y_{3t} &= \delta + \beta_3 B_3 + \alpha D_{3t} + \varepsilon_{3t} \\ &= (\delta + \beta_3 B_3) + \alpha D_{3t} + \varepsilon_{3t} \end{aligned}$$

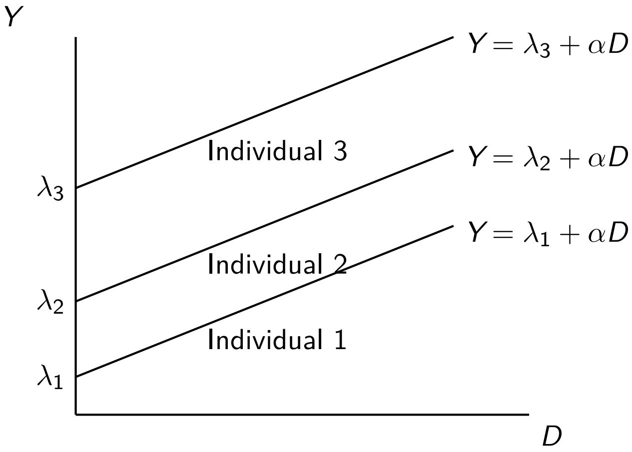
or

$$Y_{3t} = \lambda_3 + \alpha D_{3t} + \varepsilon_{3t}$$

- where $\lambda_3 = \delta + \beta_3 B_3$

Regression with 'N-1 dummy variables"

Graphical Representation



Regression with ‘N-1 dummy variables”

- Since regression with ‘N-1 dummy variables” and regression with fixed effects are the same
- Thus, when there are not many groups (e.g. state, year, county), we usually control these fixed effects by simply including many dummy variables

Fixed Effects Regression

General Form

- We can include many types of fixed effects to control for all possible time-invariant confounding factors or common time factors

$$Y_{ist} = \lambda_i + \theta_t + \kappa_s + \alpha D_{ist} + X'_{ist}\beta + \varepsilon_{ist},$$

  - λ_i is called a "individual fixed effect" or "individual effect"
    - It is the constant (fixed) effect of being in individual i
    - Example: ability or preference
  - κ_s is called a "state fixed effect" or "state effect"
    - It is the constant (fixed) effect of being in state s
    - Example: culture or geographical features
  - θ_t is called a "year fixed effect" or "year effect"
    - It is the constant (fixed) effect of being in year t
    - Example: business cycle or general time trend

Fixed Effects Regression: STATA Example

Fixed Effects Regression: STATA Example

- See fixed_effects.do
- Use cps_2014_16.dta

STATA Command: reg

Example:

```
reg incwage college i.statefip i.year, vce(robust)
```

- You can simply use **reg** by including several sets of dummy variables to get fixed effects estimation

STATA Command: areg

Syntax:

```
areg depvar [indepvars] [if] [in] [weight], absorb(
    varname) [options]
```

Example:

```
areg incwage college i.year, absorb(statefip) vce(
    robust)
```

- **areg**: Implement regressions with one level of fixed effects
- **absorb(varname)**: Specifies the categorical variable, which is to be included in the regression as if it were specified by dummy variables
- Note that **areg** can only include one fixed effect using **absorb(varname)**
- For other types of fixed effects, you need to include dummy variables by yourself

STATA Command: reghdfe

- To include many levels of fixed effects, we can use this new command **reghdfe**

```
ssc install reghdfe
```

- For more details, please visit this website: `http://scorreia.com/software/reghdfe/index.html`

STATA Command: reghdfe

Syntax:

```
reghdfe depvar [indepvars] [if] [in] [weight] ,
    absorb(absvars) [options]
```

Example:

```
reghdfe incwage college, absorb(statefip year) vce(
    robust)
```

- **reghdfe**: Implement regressions with many levels of fixed effects
- **absorb(varname)**: Specifies the categorical variable, which is to be included in the regression as if it were specified by dummy variables
- Note that **reghdfe** can include many level of fixed effects using **absorb(varname)**