

Introduction to CMOS VLSI Design

Lecture 4: DC & Transient Response

Outline

- ☐ DC Response
- ☐ Logic Levels and Noise Margins
- ☐ Transient Response
- ☐ Delay Estimation

Activity

- 1) If the width of a transistor increases, the current will
increase decrease not change
- 2) If the length of a transistor increases, the current will
increase decrease not change
- 3) If the supply voltage of a chip increases, the maximum transistor current will
increase decrease not change
- 4) If the width of a transistor increases, its gate capacitance will
increase decrease not change
- 5) If the length of a transistor increases, its gate capacitance will
increase decrease not change
- 6) If the supply voltage of a chip increases, the gate capacitance of each transistor will
increase decrease not change

Activity

- 1) If the width of a transistor increases, the current will
increase decrease not change
- 2) If the length of a transistor increases, the current will
increase **decrease** not change
- 3) If the supply voltage of a chip increases, the maximum transistor current will
increase decrease not change
- 4) If the width of a transistor increases, its gate capacitance will
increase decrease not change
- 5) If the length of a transistor increases, its gate capacitance will
increase decrease not change
- 6) If the supply voltage of a chip increases, the gate capacitance of each transistor will
increase decrease **not change**

2長度越長 電阻越大 電流越小

3供給電壓

4電晶體越長 越寬 電容也就越長 越大

DC Response

□ DC Response: V_{out} vs. V_{in} for a gate

□ Ex: Inverter

– When $V_{in} = 0$ $\rightarrow V_{out} = V_{DD}$

– When $V_{in} = V_{DD}$ $\rightarrow V_{out} = 0$

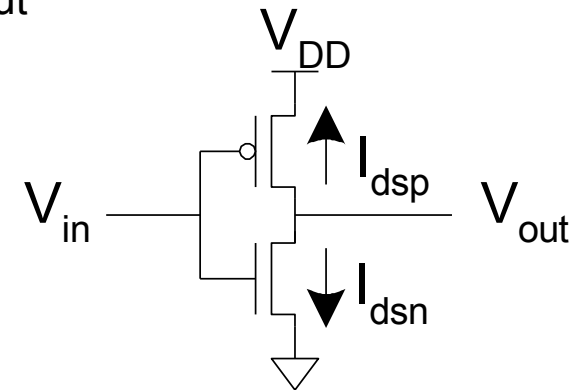
– In between, V_{out} depends on transistor size and current

– By KCL, must settle such that

$$I_{dsn} = |I_{dsp}|$$

– We could solve equations

– But graphical solution gives more insight



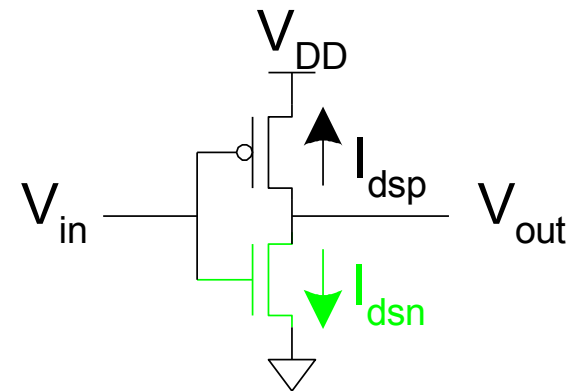
輸出 取決於
電晶體的大小 和 電流

Transistor Operation

- ❑ Current depends on region of transistor behavior
- ❑ For what V_{in} and V_{out} are nMOS and pMOS in
 - Cutoff?
 - Linear?
 - Saturation?

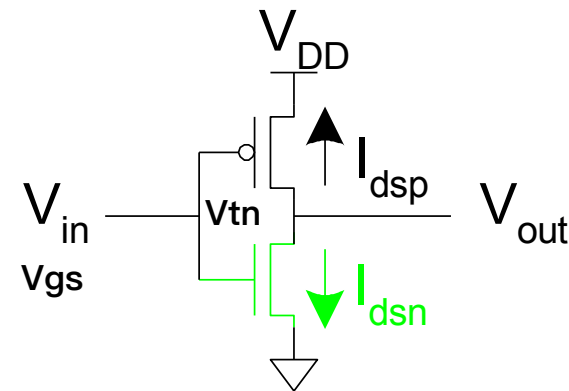
nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} <$	$V_{gsn} >$ $V_{dsn} <$	$V_{gsn} >$ $V_{dsn} >$



nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} < V_{tn}$ 這裡是要走下面的狀況 1導通	$V_{gsn} > V_{tn}$ $V_{dsn} < V_{gsn} - V_{tn}$	$V_{gsn} > V_{tn}$ $V_{dsn} > V_{gsn} - V_{tn}$

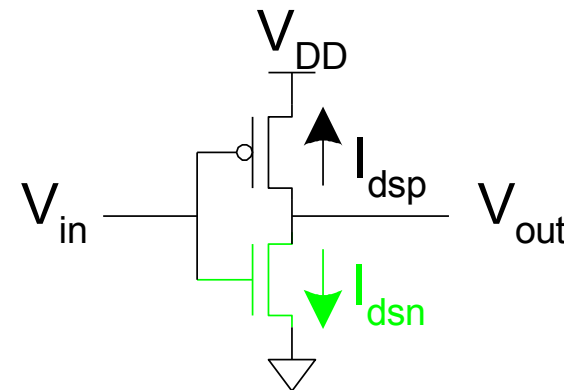


nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} < V_{tn}$	$V_{gsn} > V_{tn}$	$V_{gsn} > V_{tn}$
	$V_{dsn} < V_{gsn} - V_{tn}$	$V_{dsn} > V_{gsn} - V_{tn}$

$$V_{gsn} = V_{in}$$

$$V_{dsn} = V_{out}$$

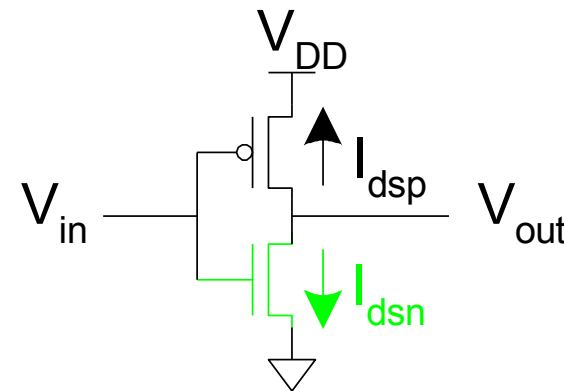


nMOS Operation

Cutoff	Linear	Saturated
$V_{gsn} < V_{tn}$ $V_{in} < V_{tn}$	$V_{gsn} > V_{tn}$ $V_{in} > V_{tn}$ $V_{dsn} < V_{gsn} - V_{tn}$ $V_{out} < V_{in} - V_{tn}$	$V_{gsn} > V_{tn}$ $V_{in} > V_{tn}$ $V_{dsn} > V_{gsn} - V_{tn}$ $V_{out} > V_{in} - V_{tn}$

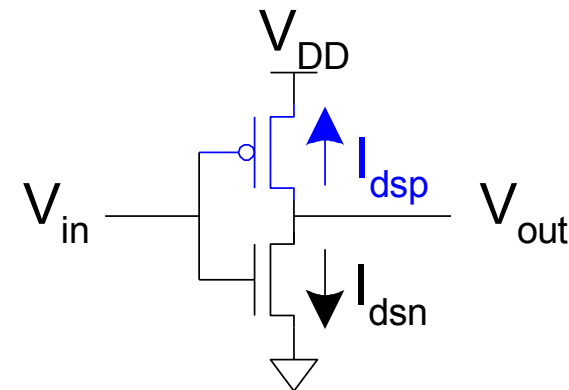
$$V_{gsn} = V_{in}$$

$$V_{dsn} = V_{out}$$



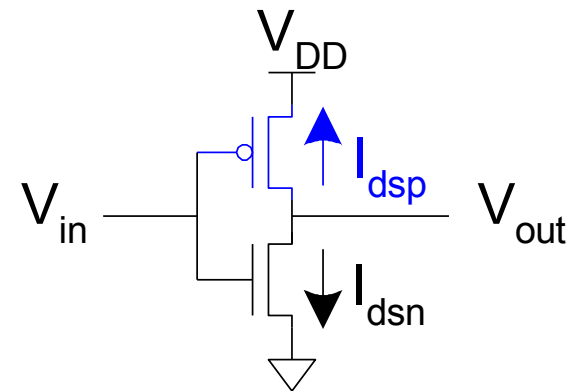
pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} >$	$V_{gsp} <$ $V_{dsp} >$	$V_{gsp} <$ $V_{dsp} <$



pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} > V_{tp}$	$V_{gsp} < V_{tp}$ $V_{dsp} > V_{gsp} - V_{tp}$	$V_{gsp} < V_{tp}$ $V_{dsp} < V_{gsp} - V_{tp}$



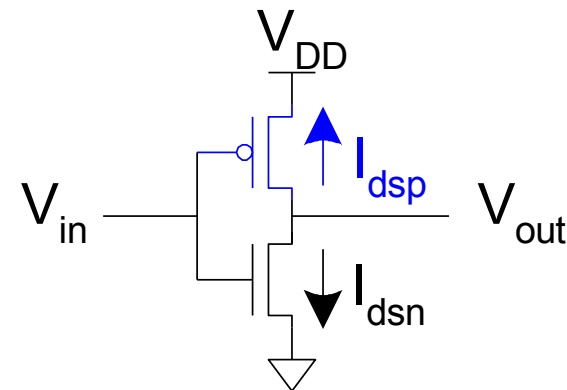
pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} > V_{tp}$	$V_{gsp} < V_{tp}$ $V_{dsp} > V_{gsp} - V_{tp}$	$V_{gsp} < V_{tp}$ $V_{dsp} < V_{gsp} - V_{tp}$

$$V_{gsp} = V_{in} - V_{DD}$$

$$V_{tp} < 0$$

$$V_{dsp} = V_{out} - V_{DD}$$



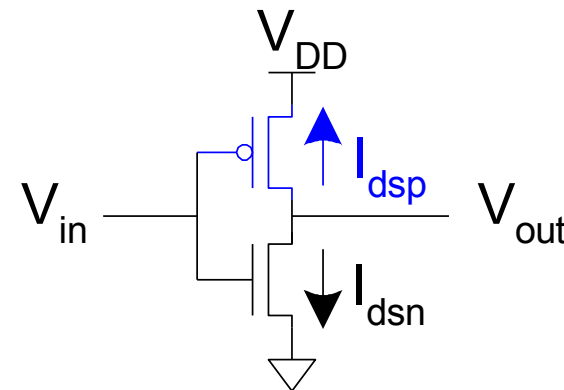
pMOS Operation

Cutoff	Linear	Saturated
$V_{gsp} > V_{tp}$ $V_{in} > V_{DD} + V_{tp}$ 這裡是要走上面的狀況 Pmos 0導通	$V_{gsp} < V_{tp}$ $V_{in} < V_{DD} + V_{tp}$ $V_{dsp} > V_{gsp} - V_{tp}$ $V_{out} > V_{in} - V_{tp}$	$V_{gsp} < V_{tp}$ $V_{in} < V_{DD} + V_{tp}$ $V_{dsp} < V_{gsp} - V_{tp}$ $V_{out} < V_{in} - V_{tp}$

$$V_{gsp} = V_{in} - V_{DD}$$

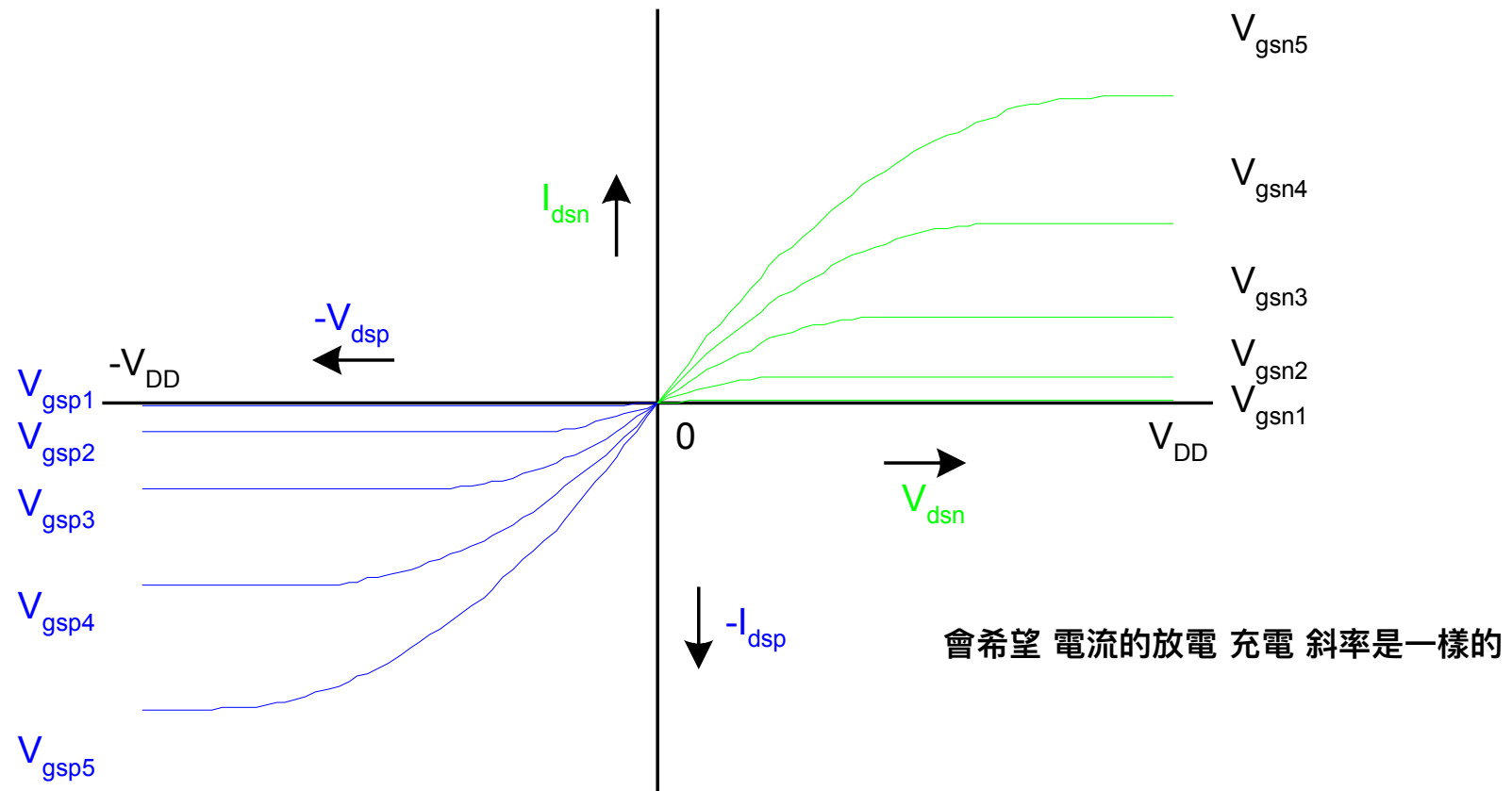
$$V_{tp} < 0$$

$$V_{dsp} = V_{out} - V_{DD}$$

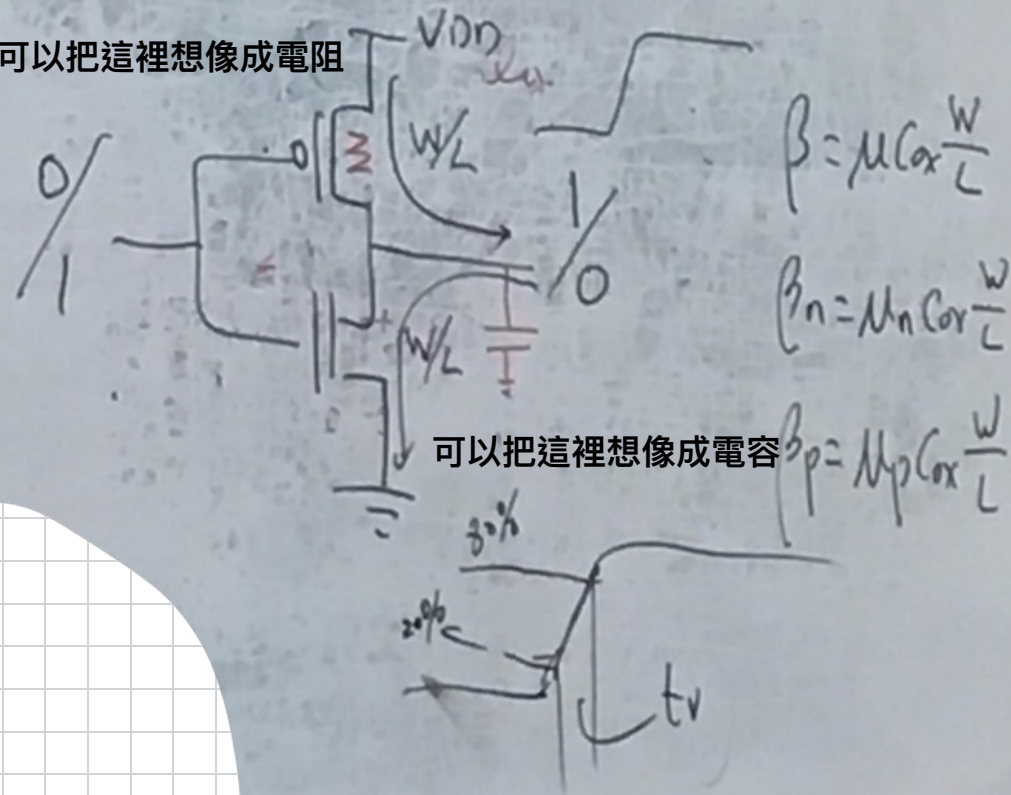


I-V Characteristics

- Make pMOS is wider than nMOS such that $\beta_n = \beta_p$



可以把這裡想像成電阻



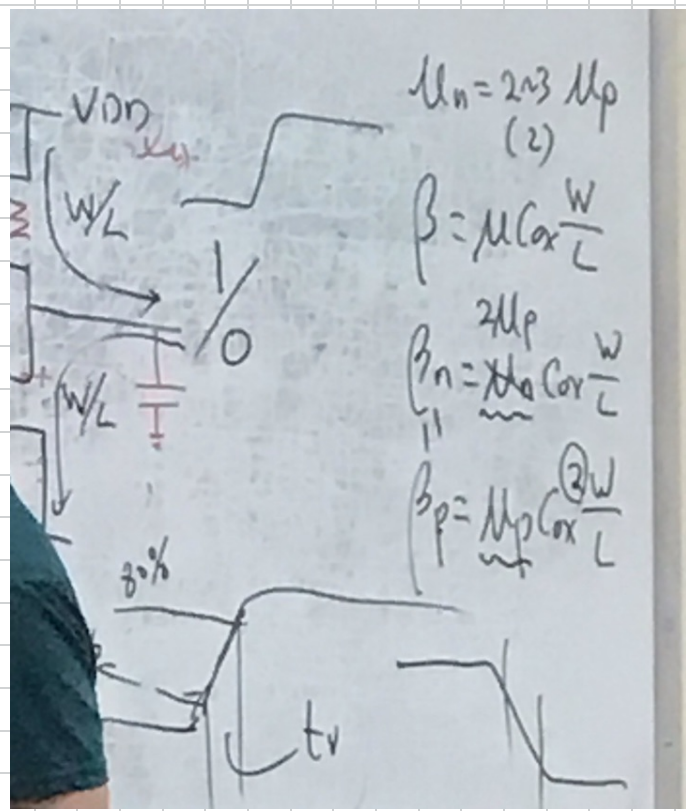
可以把這裡想像成電容

$$\mu_n = 2 \sim 3 \mu_p$$

$$\beta = \mu C_{ox} \frac{W}{L}$$

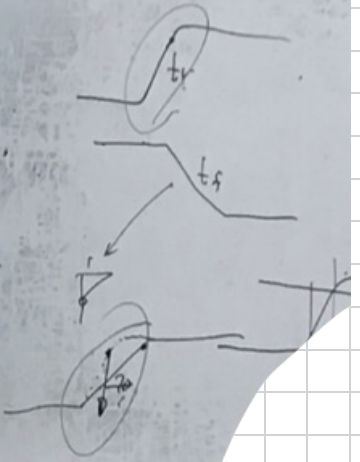
$$\beta_n = \mu_n C_{ox} \frac{W}{L}$$

$$\beta_p = \mu_p C_{ox} \frac{W}{L}$$

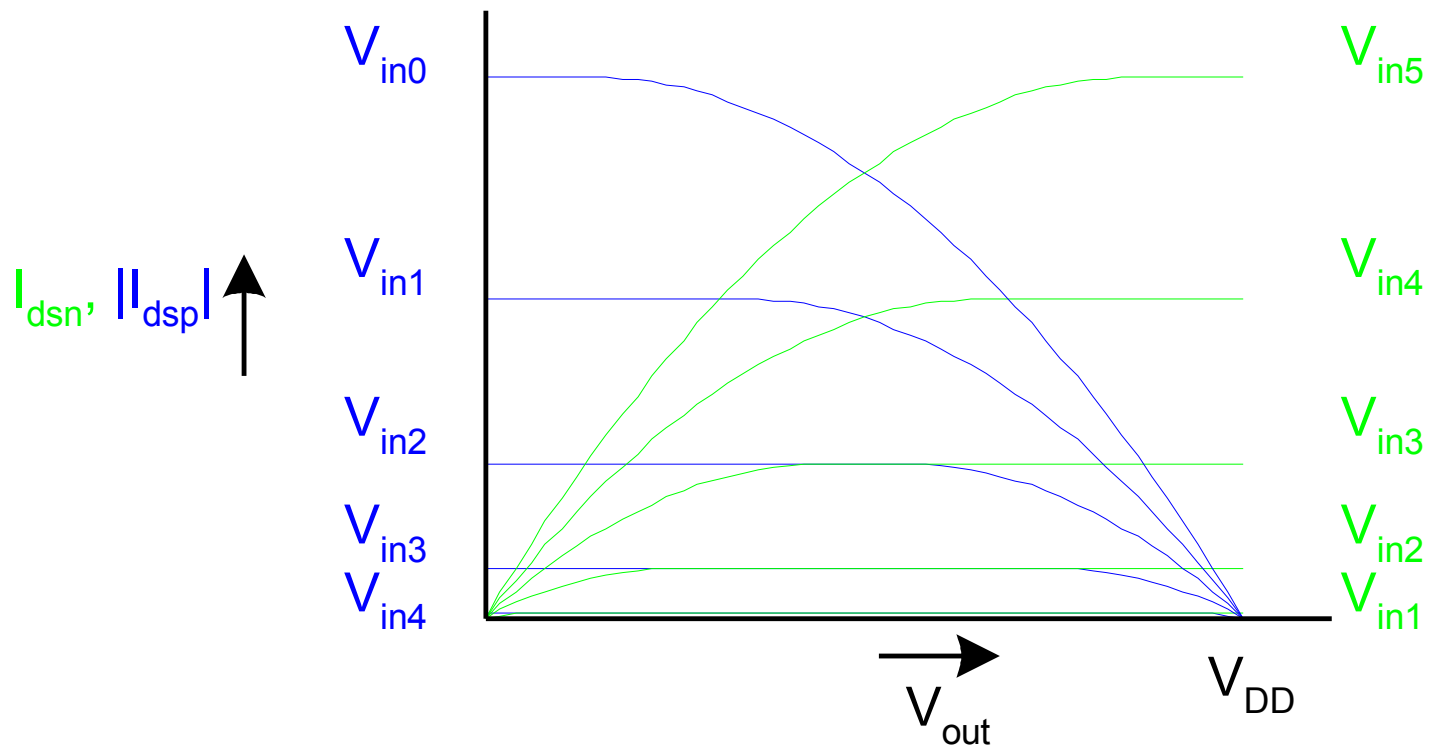


Beta 當作 導電係數

如果斜率不一樣 就會有延遲 會不好決定收取時間

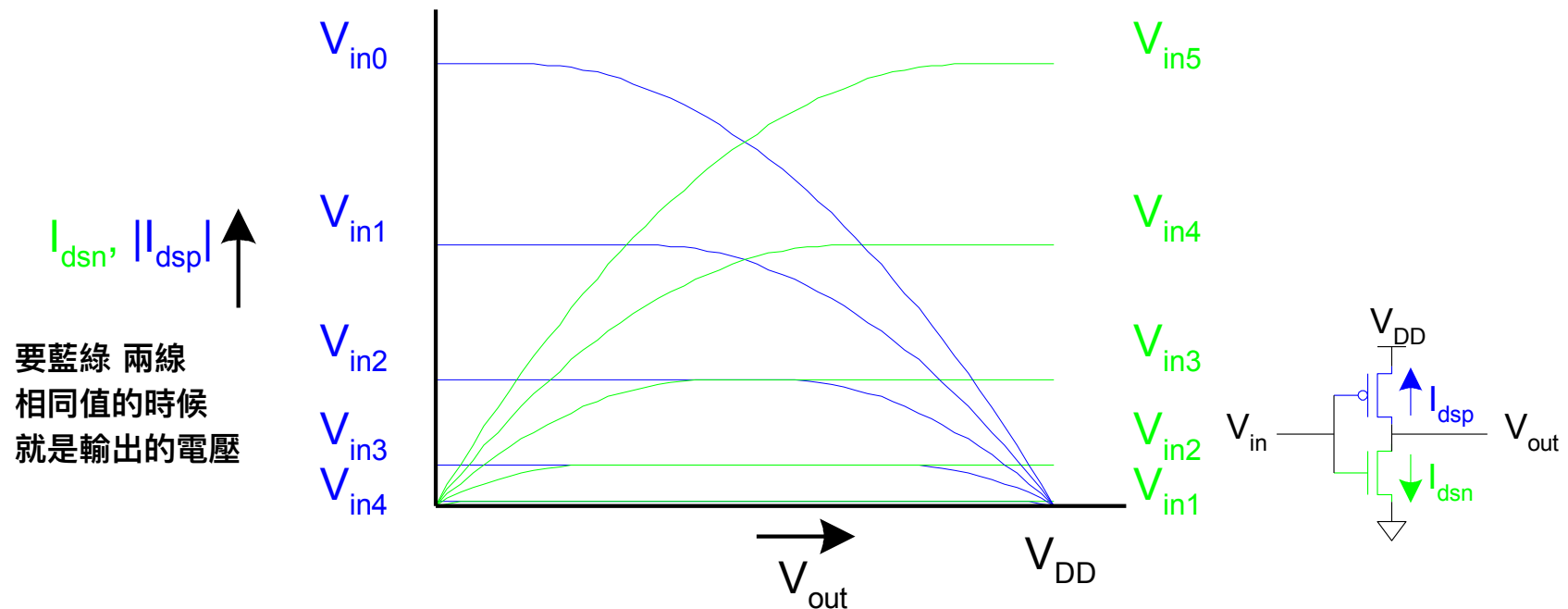


Current vs. V_{out} , V_{in}



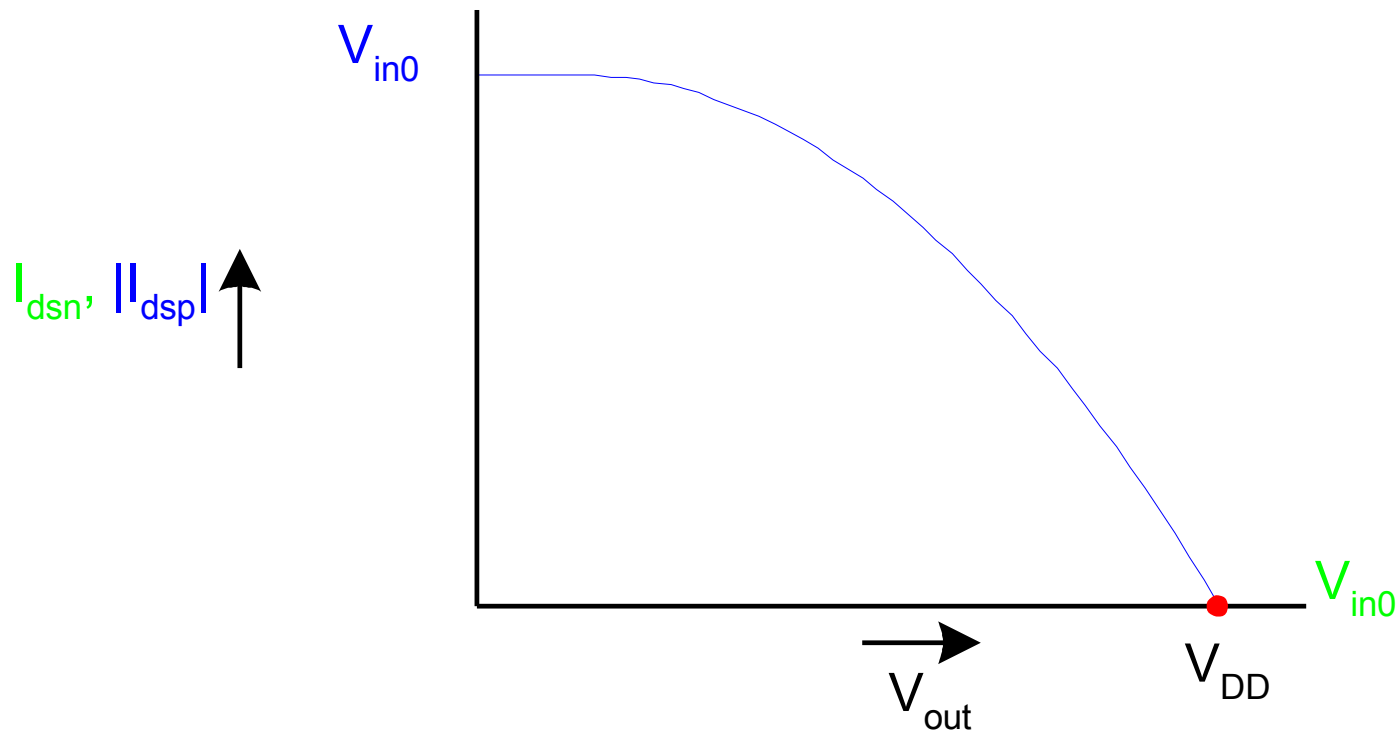
Load Line Analysis

- For a given V_{in} :
 - Plot I_{dsn} , I_{dsp} vs. V_{out}
 - V_{out} must be where |currents| are equal in



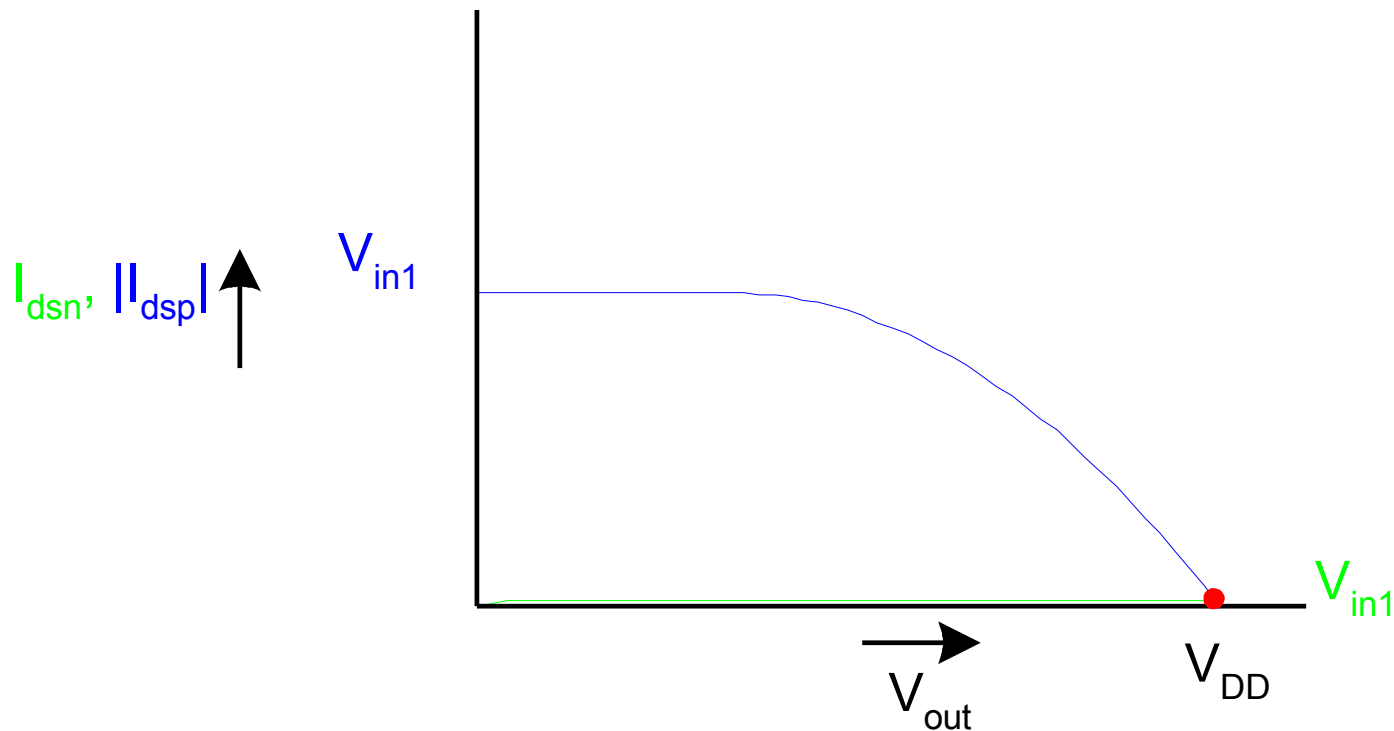
Load Line Analysis

□ $V_{in} = 0$



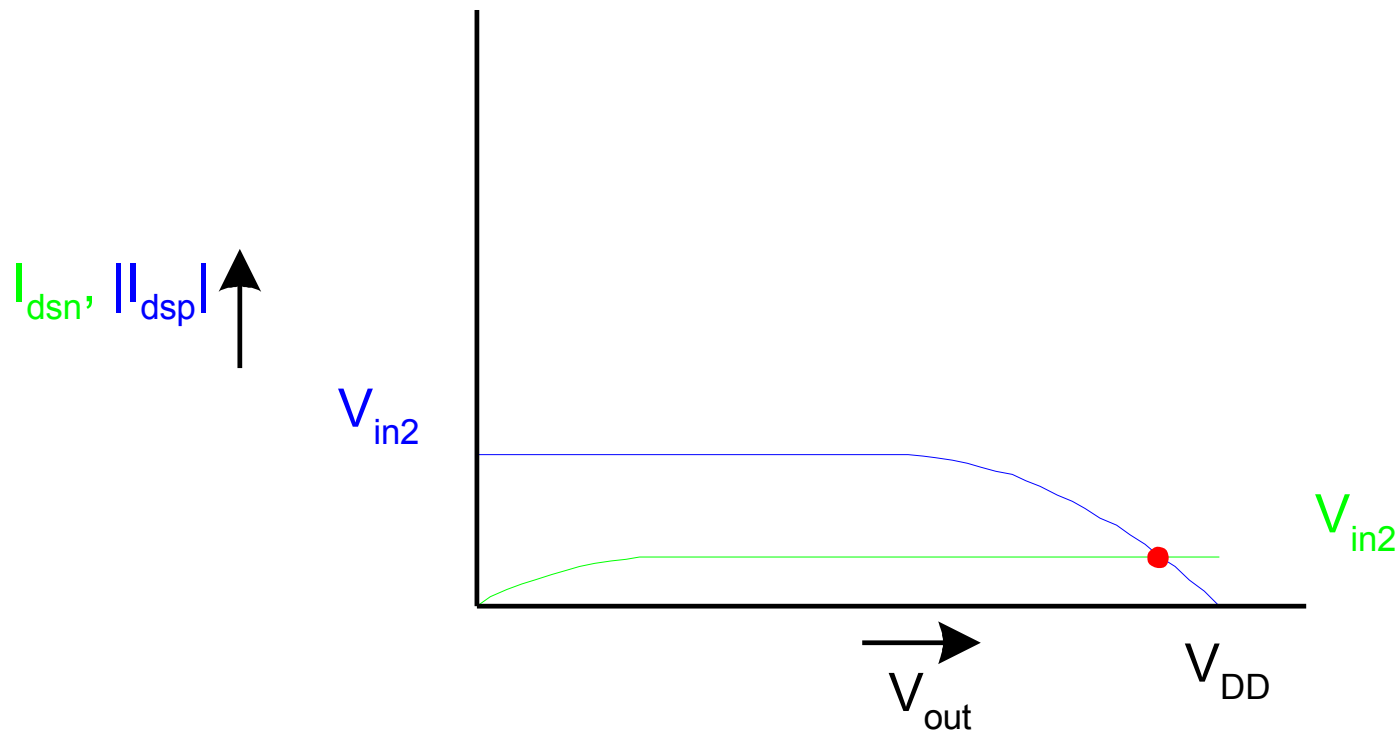
Load Line Analysis

□ $V_{in} = 0.2V_{DD}$



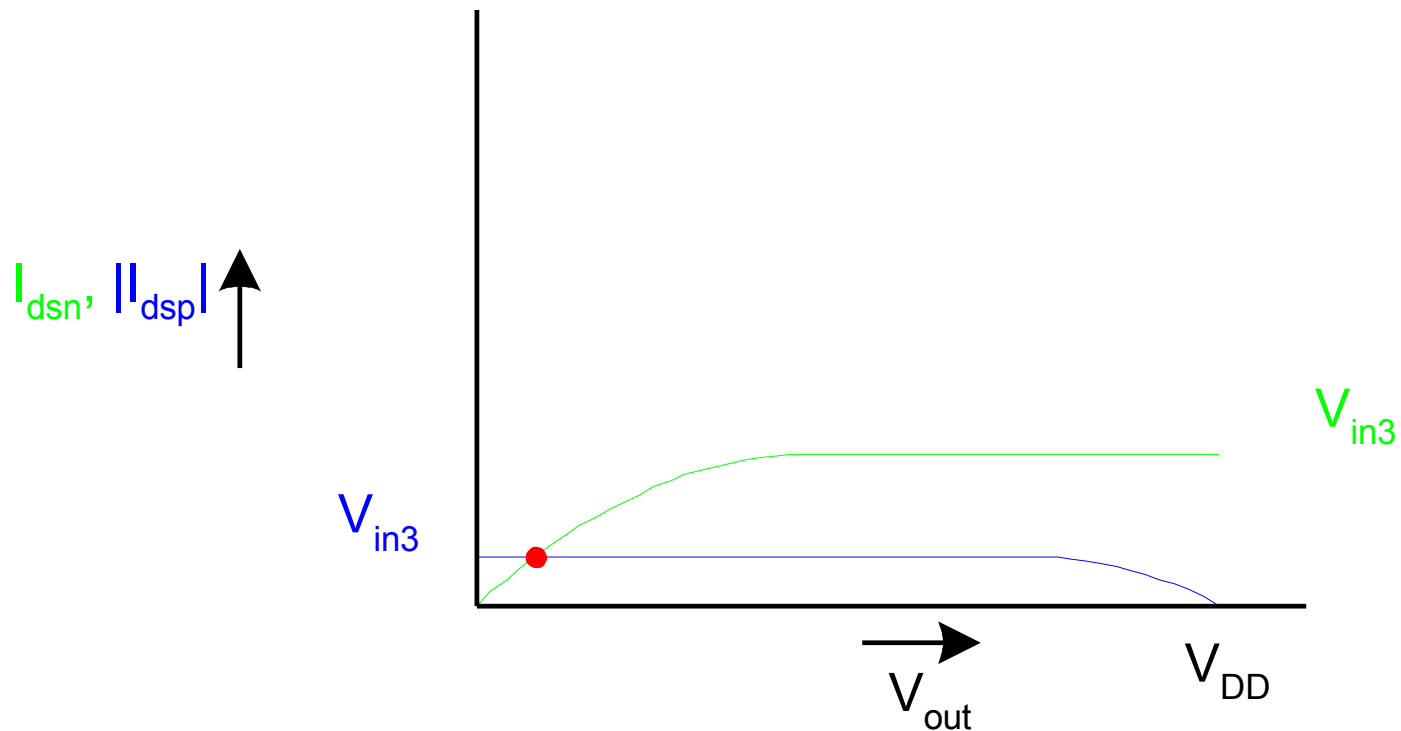
Load Line Analysis

□ $V_{in} = 0.4V_{DD}$



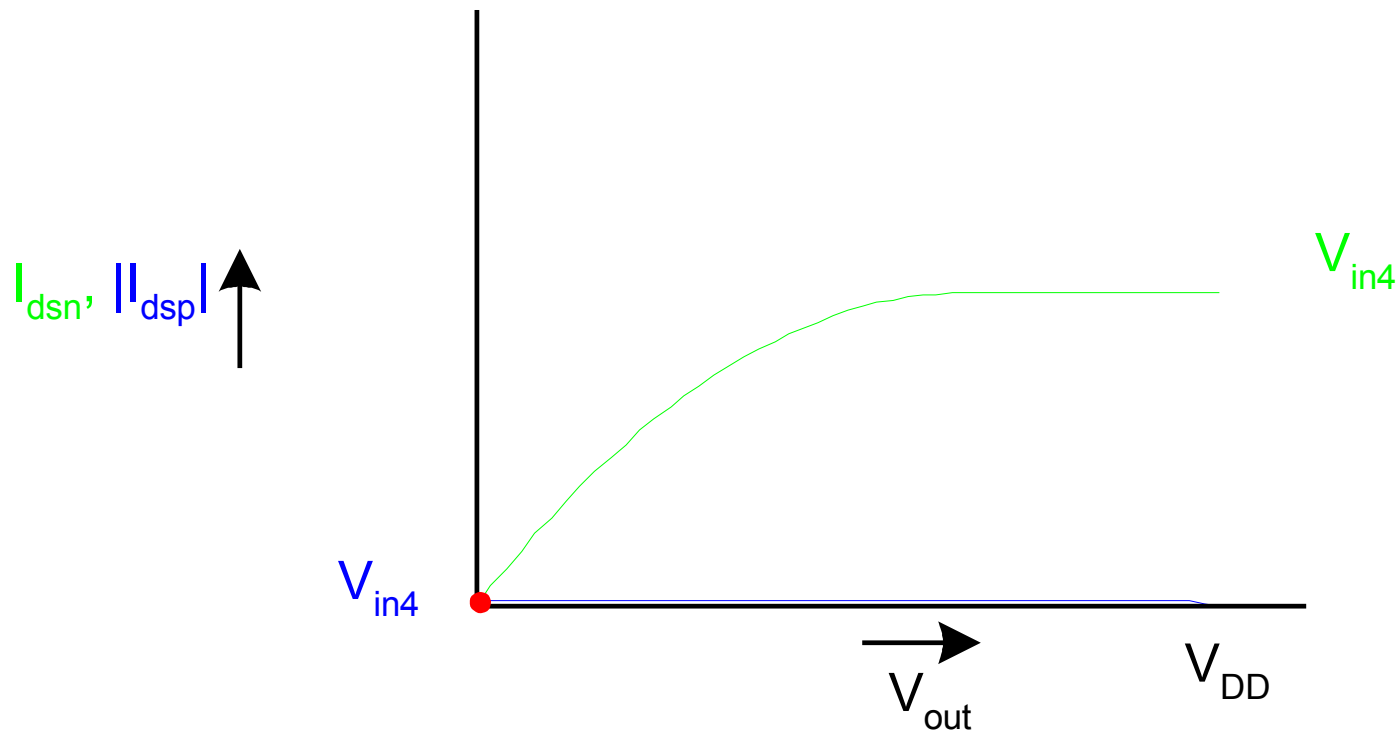
Load Line Analysis

□ $V_{in} = 0.6V_{DD}$



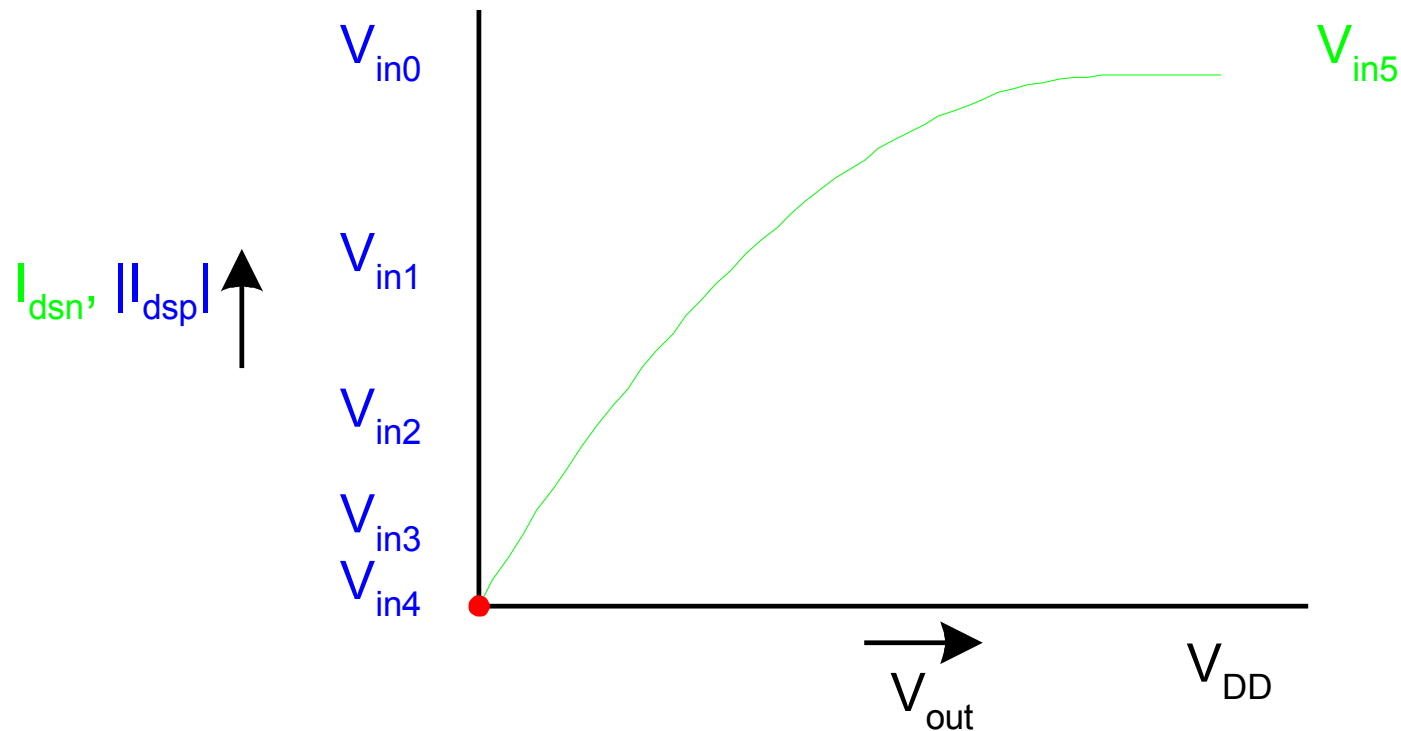
Load Line Analysis

□ $V_{in} = 0.8V_{DD}$

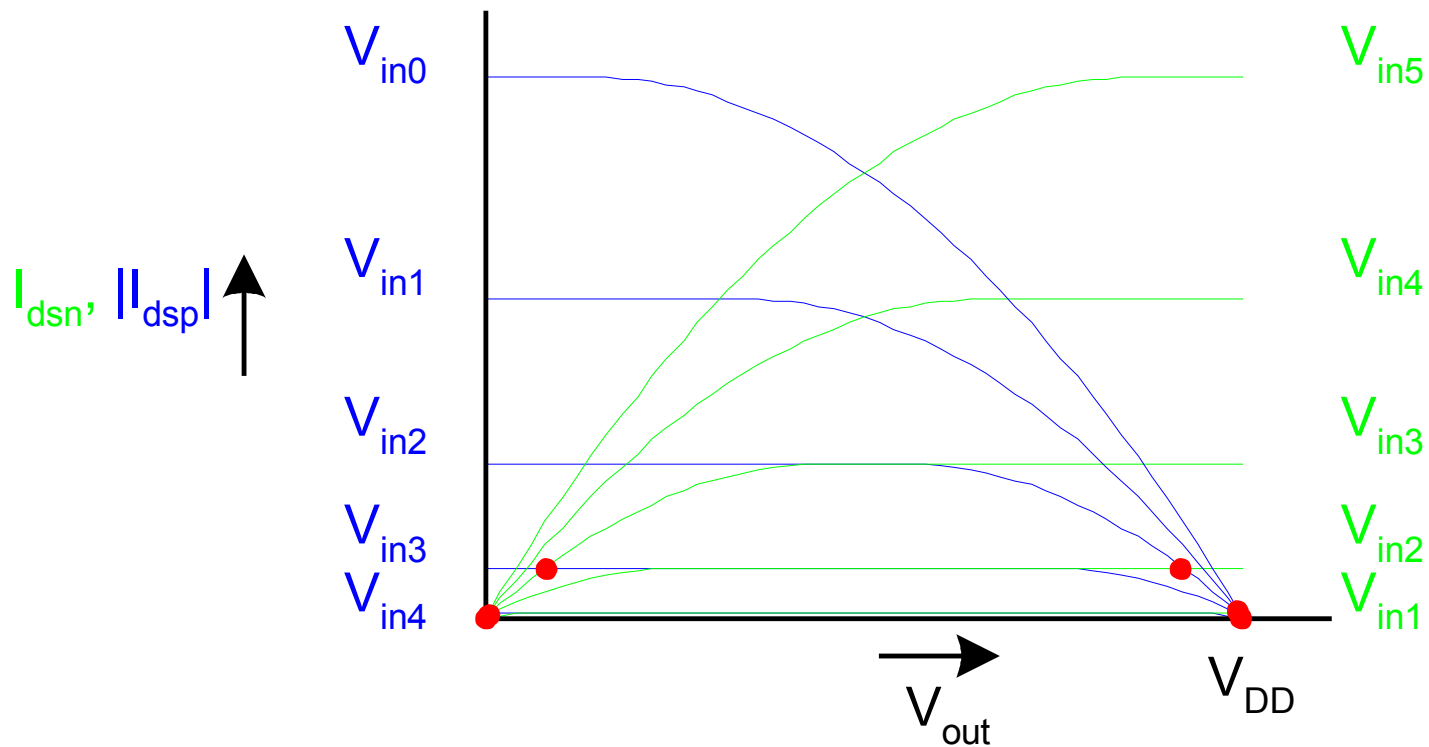


Load Line Analysis

□ $V_{in} = V_{DD}$

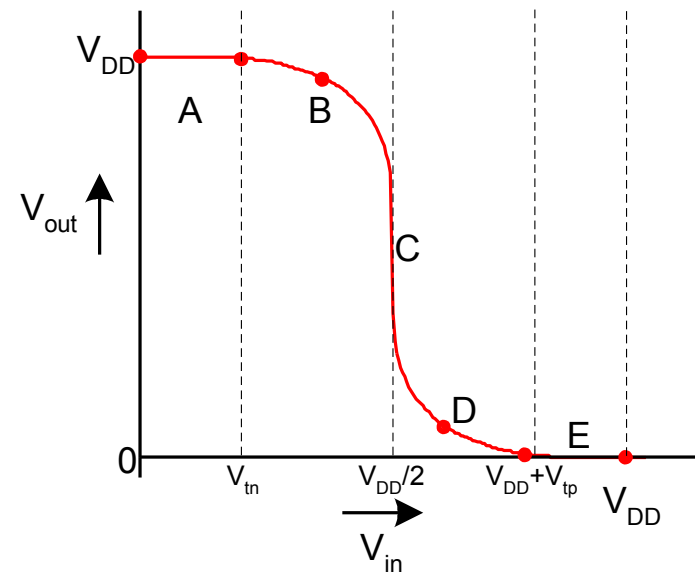
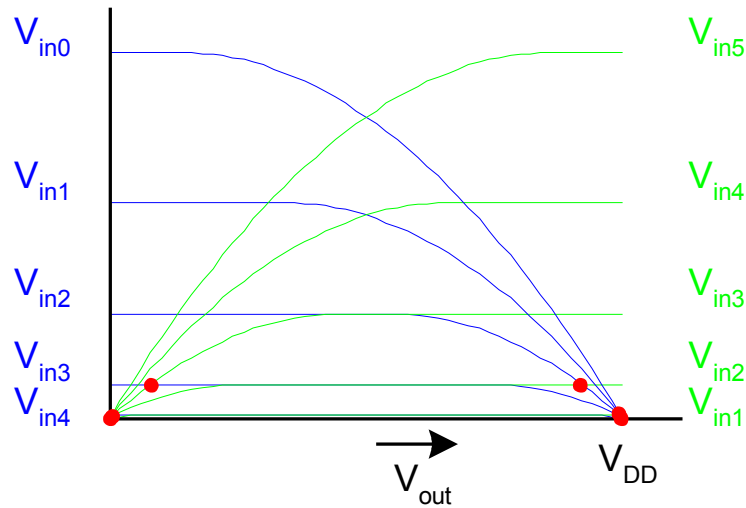


Load Line Summary



DC Transfer Curve

- Transcribe points onto V_{in} vs. V_{out} plot

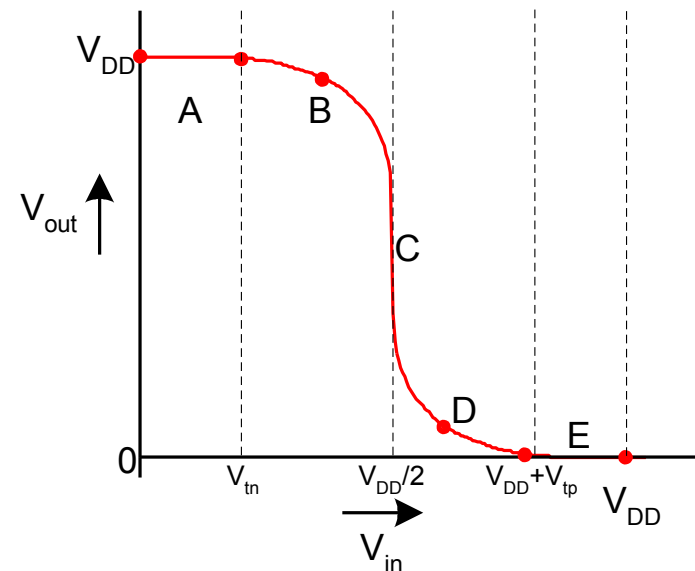


因為這是一個not 所以輸出越高 就會越接近0

Operating Regions

- ❑ Revisit transistor operating regions

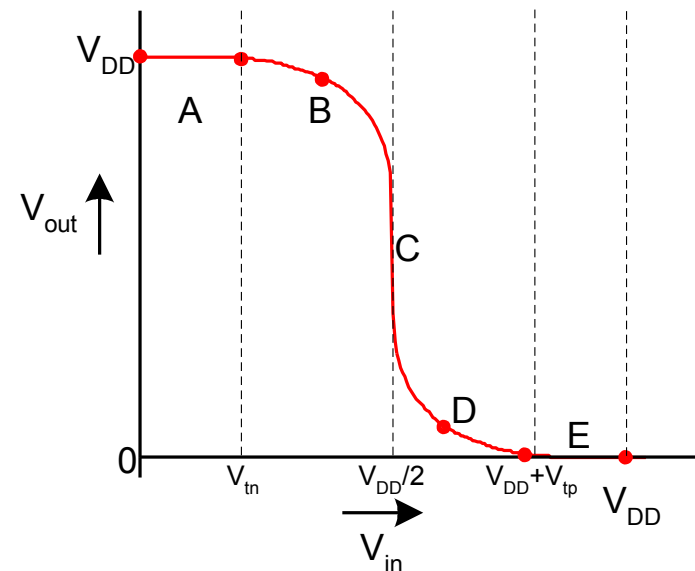
Region	nMOS	pMOS
A		
B		
C		
D		
E		



Operating Regions

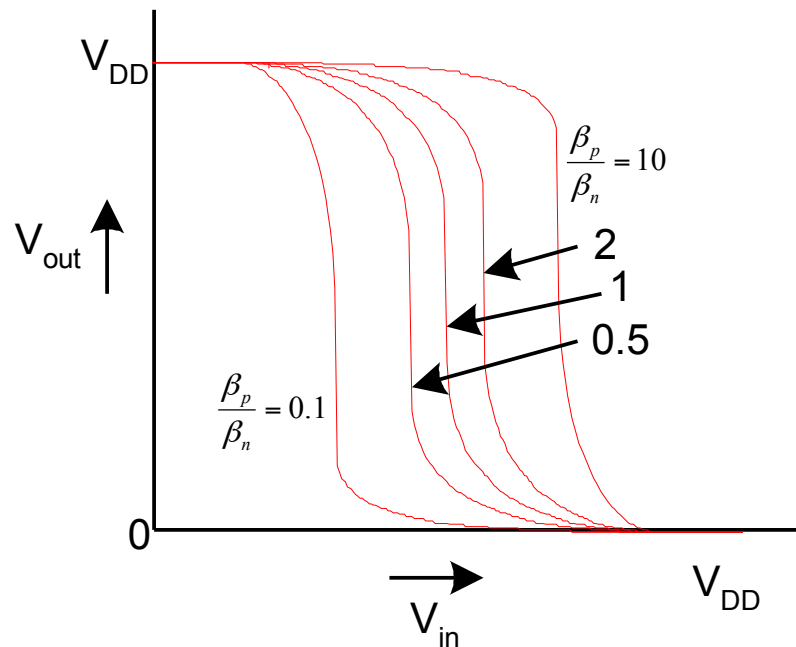
- ❑ Revisit transistor operating regions

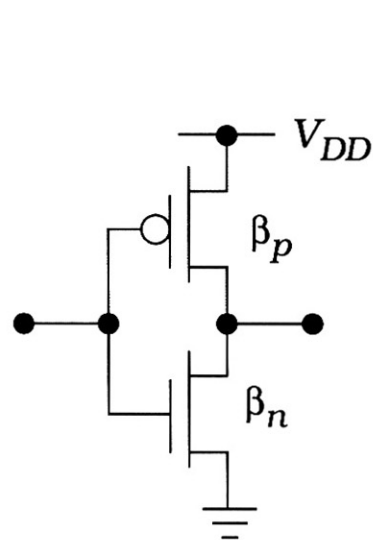
Region	nMOS	pMOS
A	Cutoff	Linear
B	Saturation	Linear
C	Saturation	Saturation
D	Linear	Saturation
E	Linear	Cutoff



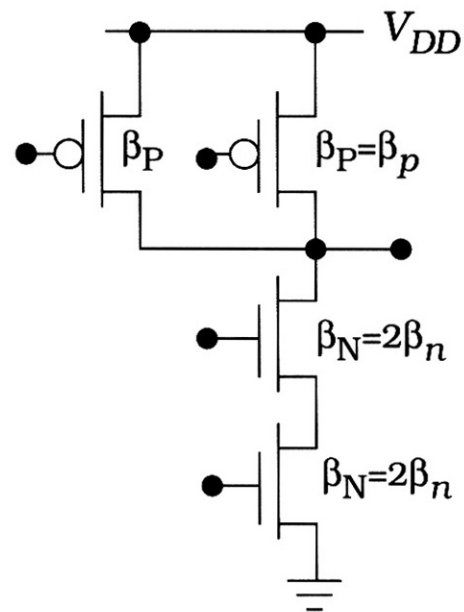
Beta Ratio

- ❑ If $\beta_p / \beta_n \neq 1$, switching point will move from $V_{DD}/2$
- ❑ Called *skewed gate*
- ❑ Other gates: collapse into equivalent inverter

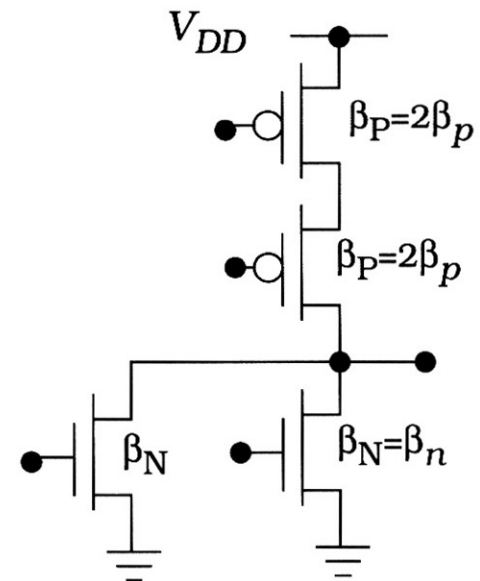




(a) Inverter

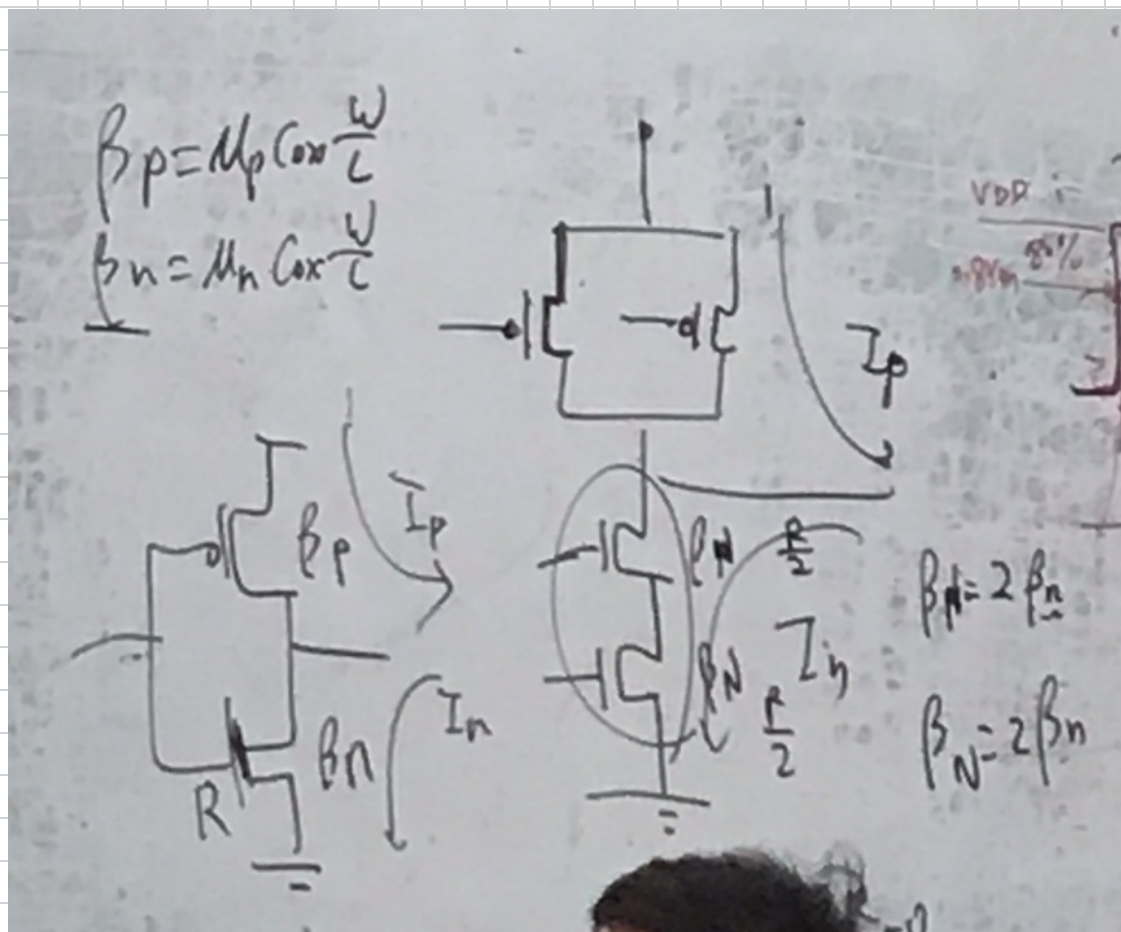


(b) NAND2



(c) NOR2

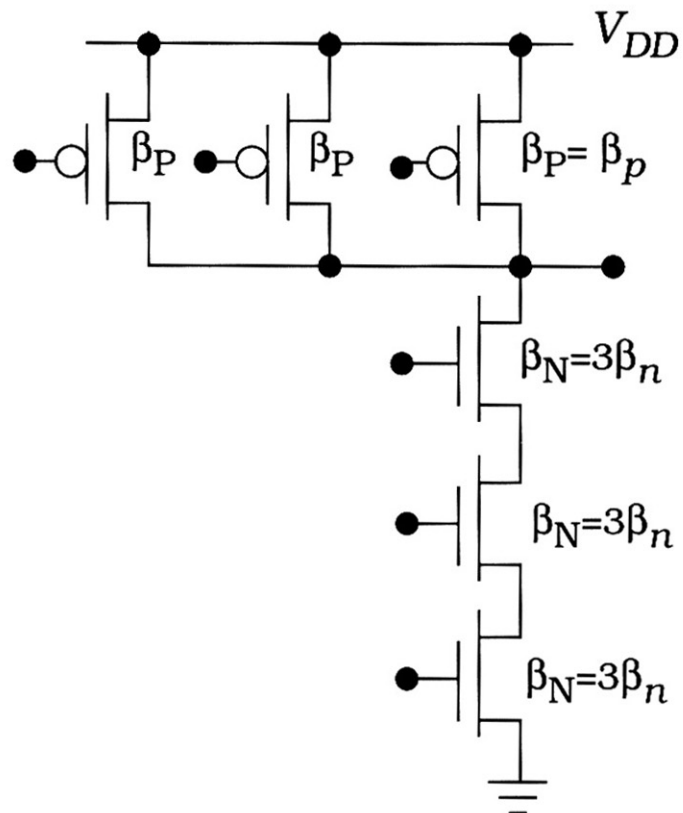
Figure 7.34 Relative FET sizing



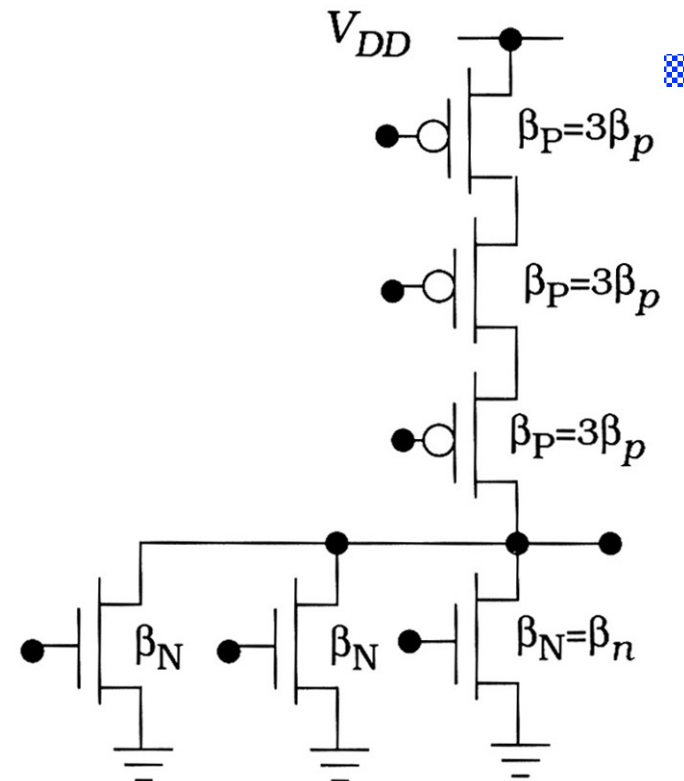
這裡是調整w 寬度 來 控制電阻

因為電阻 都要和原本的 not 的 電阻 一樣

因為 串連 電阻會加起來
所以串連的電阻 都要變成 $R/2$



(a) NAND3



(b) NOR3

Figure 7.35 Sizing for 3-input gates

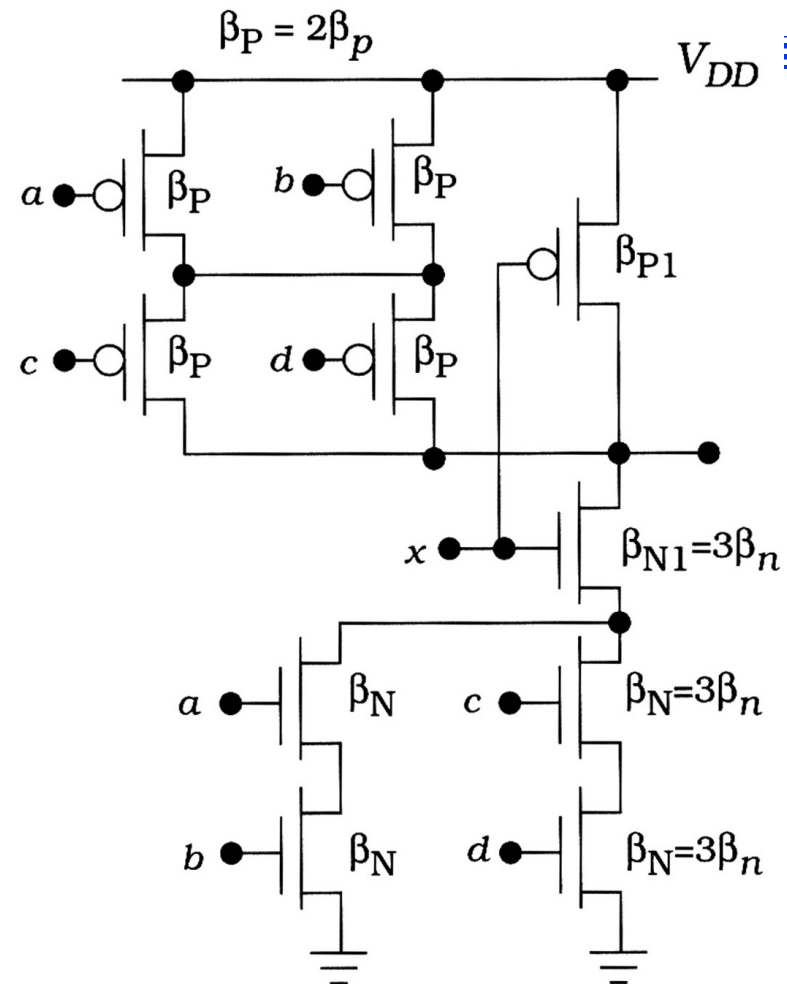
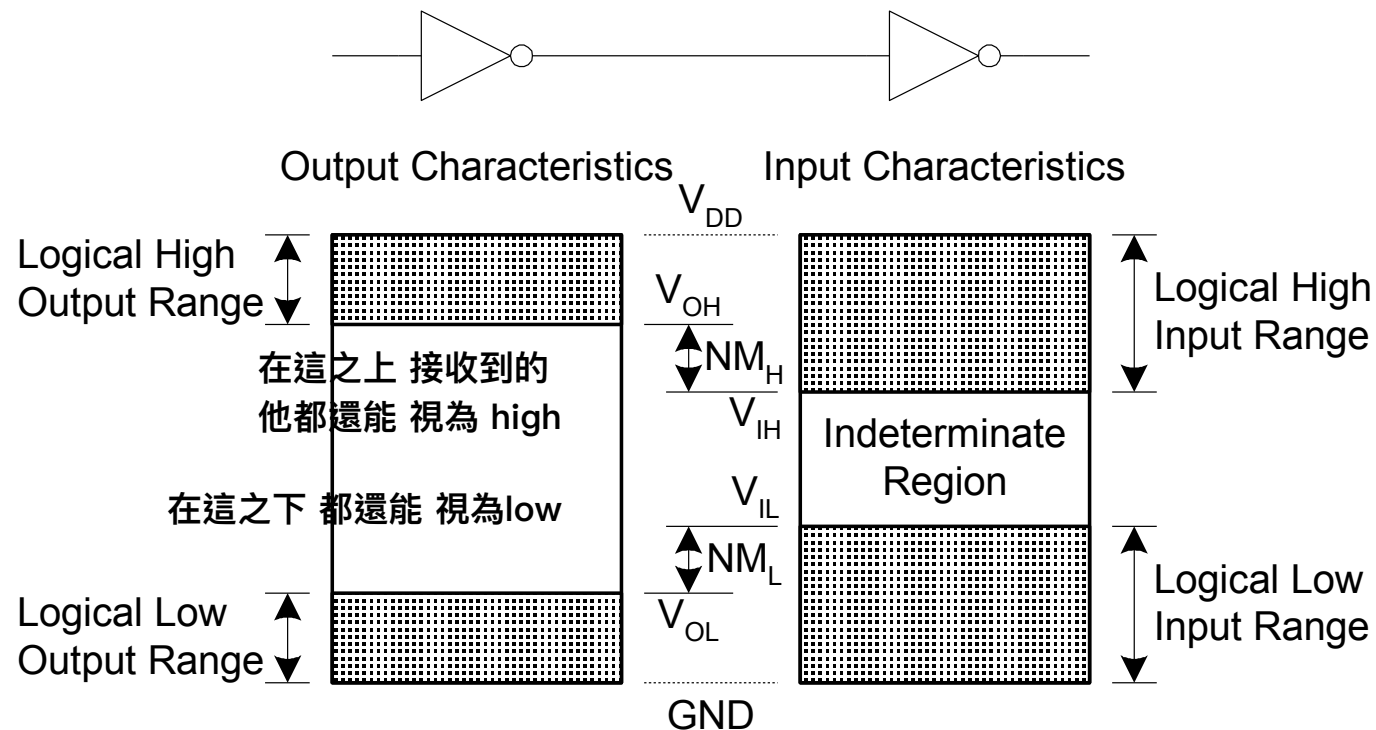


Figure 7.36 Sizing of a complex logic gate

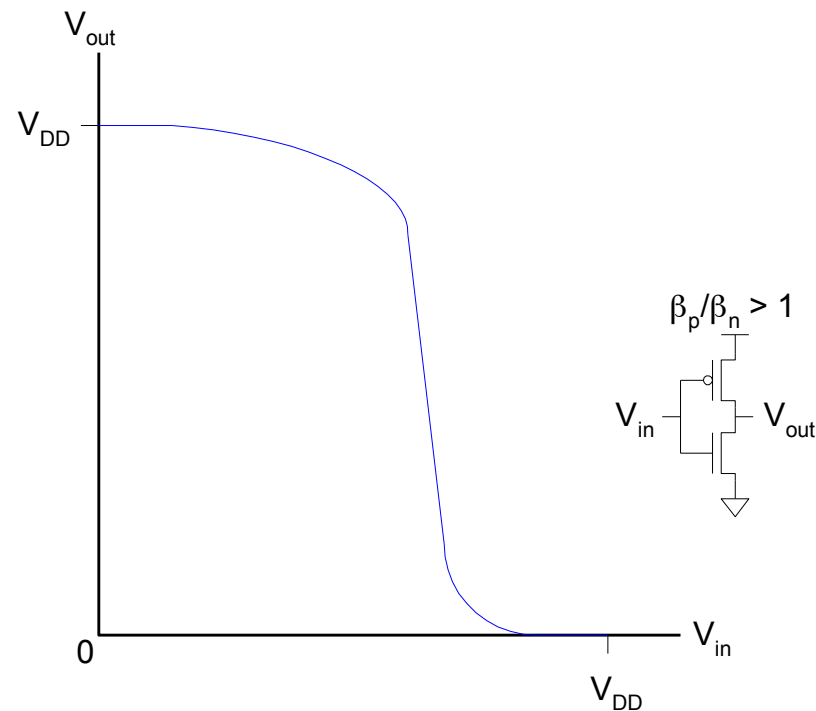
Noise Margins

- ❑ How much noise can a gate input see before it does not recognize the input?



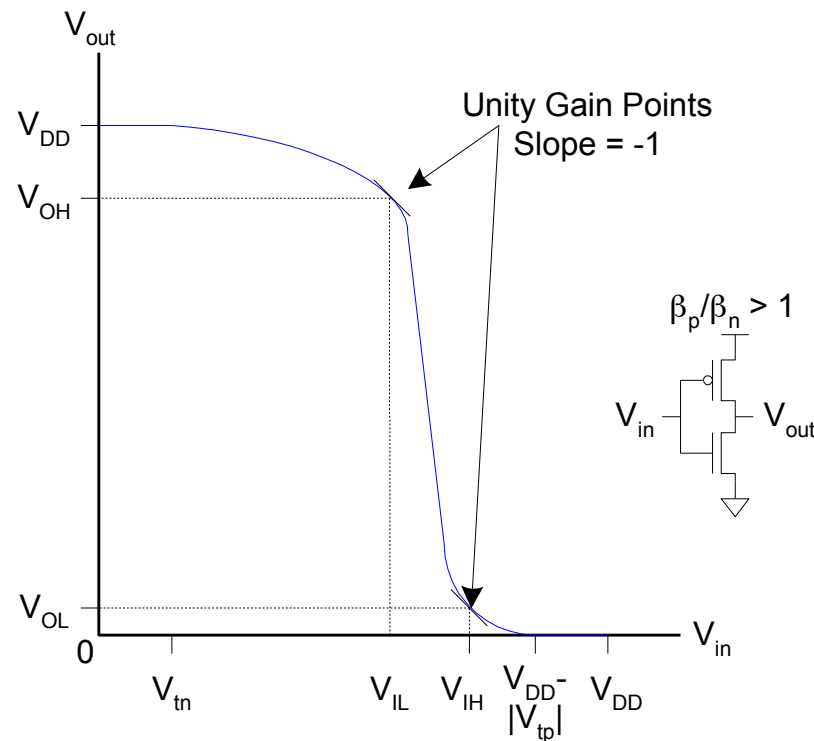
Logic Levels

- ❑ To maximize noise margins, select logic levels at



Logic Levels

- ❑ To maximize noise margins, select logic levels at
 - unity gain point of DC transfer characteristic



Transient Response

- ❑ *DC analysis* tells us V_{out} if V_{in} is constant
- ❑ *Transient analysis* tells us $V_{out}(t)$ if $V_{in}(t)$ changes
 - Requires solving differential equations
- ❑ Input is usually considered to be a step or ramp
 - From 0 to V_{DD} or vice versa

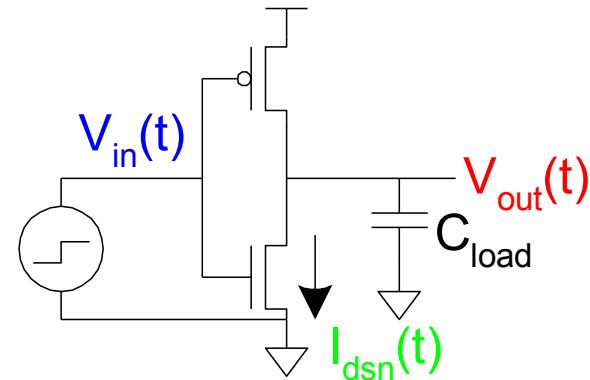
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) =$$

$$V_{out}(t < t_0) =$$

$$\frac{dV_{out}(t)}{dt} =$$



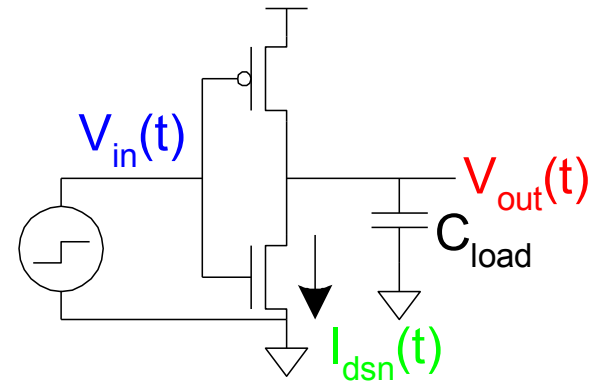
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) =$$

$$\frac{dV_{out}(t)}{dt} =$$



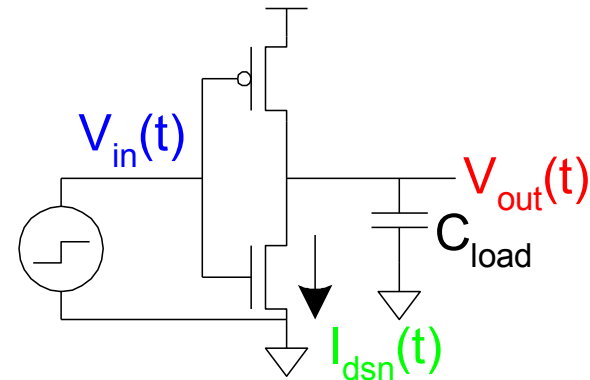
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} =$$



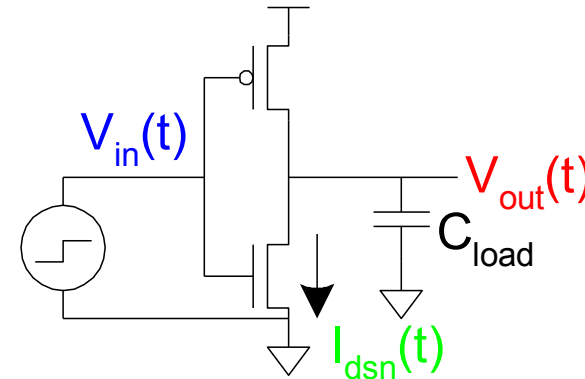
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} = -\frac{I_{dsn}(t)}{C_{load}}$$



$$I_{dsn}(t) = \begin{cases} & t \leq t_0 \\ V_{out} > V_{DD} - V_t \\ V_{out} < V_{DD} - V_t \end{cases}$$

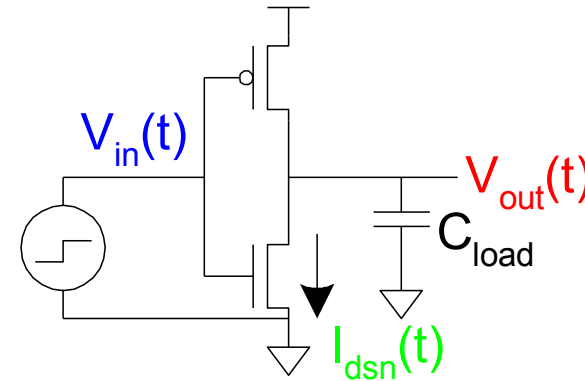
Inverter Step Response

□ Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} = -\frac{I_{dsn}(t)}{C_{load}}$$



$$I_{dsn}(t) = \begin{cases} 0 & t \leq t_0 \\ \frac{\beta}{2}(V_{DD} - V)^2 & V_{out} > V_{DD} - V_t \\ \beta\left(V_{DD} - V_t - \frac{V_{out}(t)}{2}\right)V_{out}(t) & V_{out} < V_{DD} - V_t \end{cases}$$

Inverter Step Response

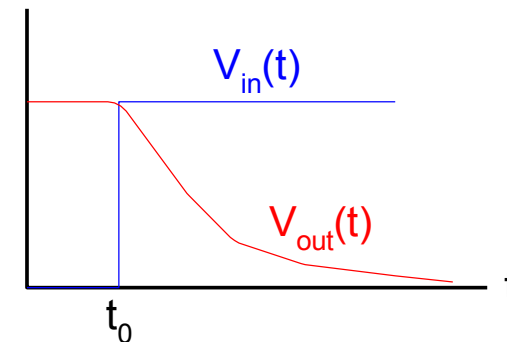
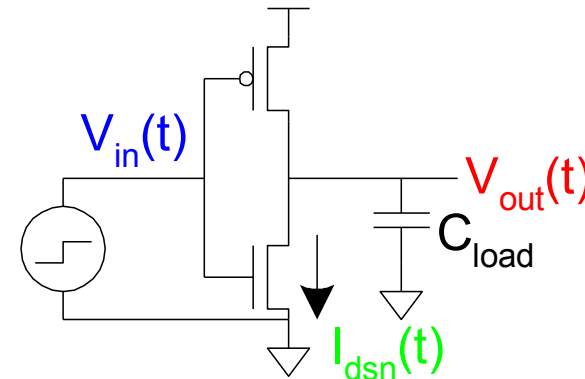
□ Ex: find step response of inverter driving load cap

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$$I_{dsn}(t) = \begin{cases} 0 & t \leq t_0 \\ \frac{\beta}{2}(V_{DD} - V)^2 & V_{out} > V_{DD} - V_t \\ \beta\left(V_{DD} - V_t - \frac{V_{out}(t)}{2}\right)V_{out}(t) & V_{out} < V_{DD} - V_t \end{cases}$$



Delay Definitions

☐ t_{pdr} :

☐ t_{pdf} :

☐ t_{pd} :

☐ t_r :

☐ t_f : *fall time*

Delay Definitions

❑ t_{pdr} : ⁽³⁾rising ⁽¹⁾propagation ⁽²⁾delay

傳遞延遲

只有到 $\frac{V_{DD}}{2}$ 的時候

– From input to rising output crossing $V_{DD}/2$

❑ t_{pdf} : falling propagation delay

– From input to falling output crossing $V_{DD}/2$

❑ t_{pd} : average propagation delay

– $t_{pd} = (t_{pdr} + t_{pdf})/2$

❑ t_r : rise time

– From output crossing $0.2 V_{DD}$ to $0.8 V_{DD}$

❑ t_f : fall time

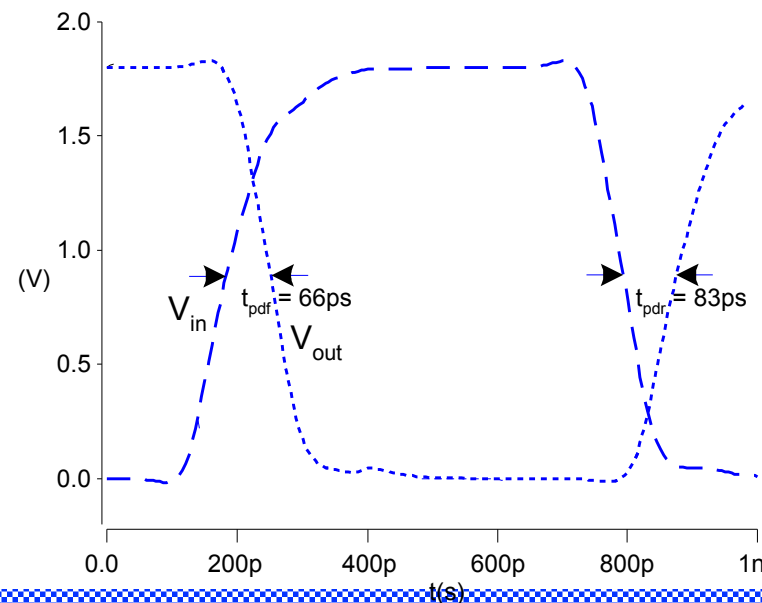
– From output crossing $0.8 V_{DD}$ to $0.2 V_{DD}$

Delay Definitions

- ❑ t_{cdr} : *rising contamination delay*
 - From input to rising output crossing $V_{DD}/2$
- ❑ t_{cdf} : *falling contamination delay*
 - From input to falling output crossing $V_{DD}/2$
- ❑ t_{cd} : *average contamination delay*
 - $t_{pd} = (t_{cdr} + t_{cdf})/2$

Simulated Inverter Delay

- ❑ Solving differential equations by hand is too hard
- ❑ SPICE simulator solves the equations numerically
 - Uses more accurate I-V models too!
- ❑ But simulations take time to write

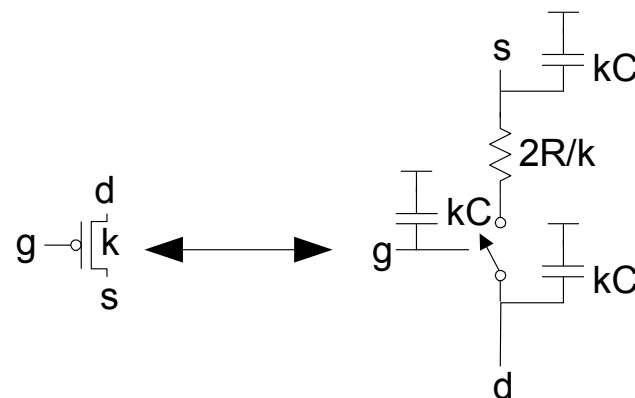
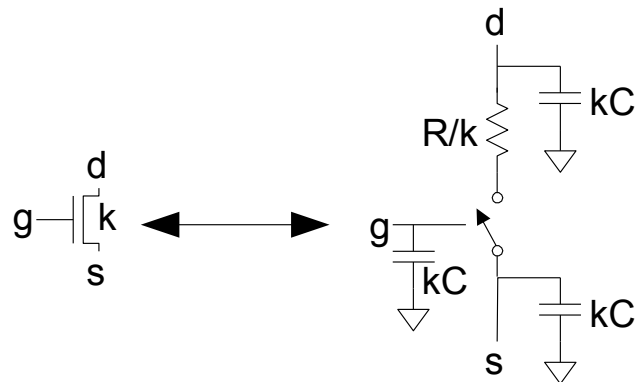


Delay Estimation

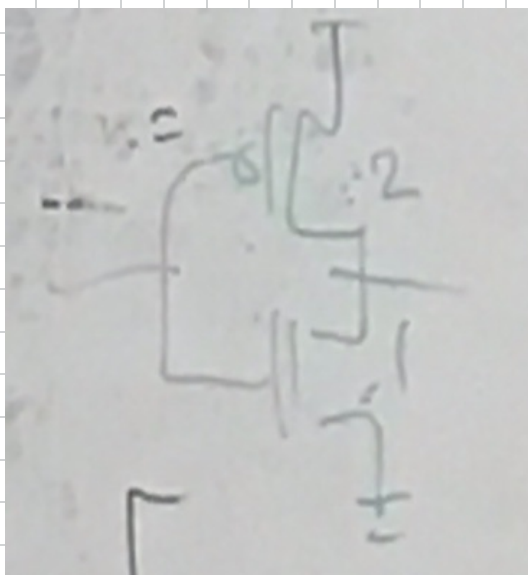
- ❑ We would like to be able to easily estimate delay
 - Not as accurate as simulation
 - But easier to ask “What if?”
- ❑ The step response usually looks like a 1st order RC response with a decaying exponential.
- ❑ Use RC delay models to estimate delay
 - C = total capacitance on output node
 - Use *effective resistance* R
 - So that $t_{pd} = RC$
- ❑ Characterize transistors by finding their effective R
 - Depends on average current as gate switches

RC Delay Models

- ❑ Use equivalent circuits for MOS transistors
 - Ideal switch + capacitance and ON resistance
 - Unit nMOS has resistance R , capacitance C
 - Unit pMOS has resistance $2R$, capacitance C
- ❑ Capacitance proportional to width
- ❑ Resistance inversely proportional to width



這裡是在算基本的擴散電容

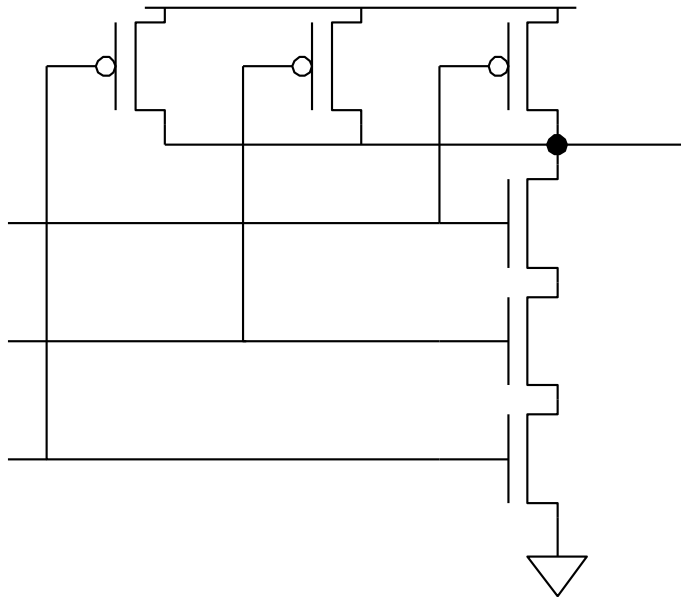


Example: 3-input NAND

- Sketch a 3-input NAND with transistor widths chosen to achieve effective rise and fall resistances equal to a unit inverter (R).

Example: 3-input NAND

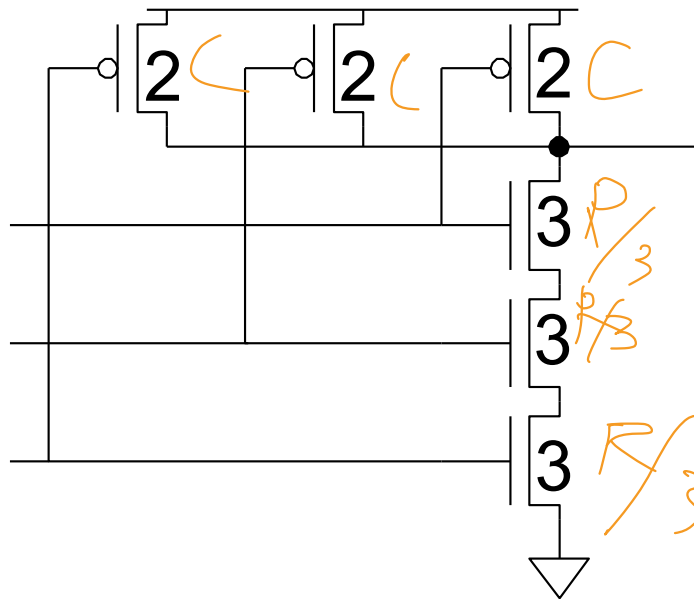
- Sketch a 3-input NAND with transistor widths chosen to achieve effective rise and fall resistances equal to a unit inverter (R).



Example: 3-input NAND

- Sketch a 3-input NAND with transistor widths chosen to achieve effective rise and fall resistances equal to a unit inverter (R).

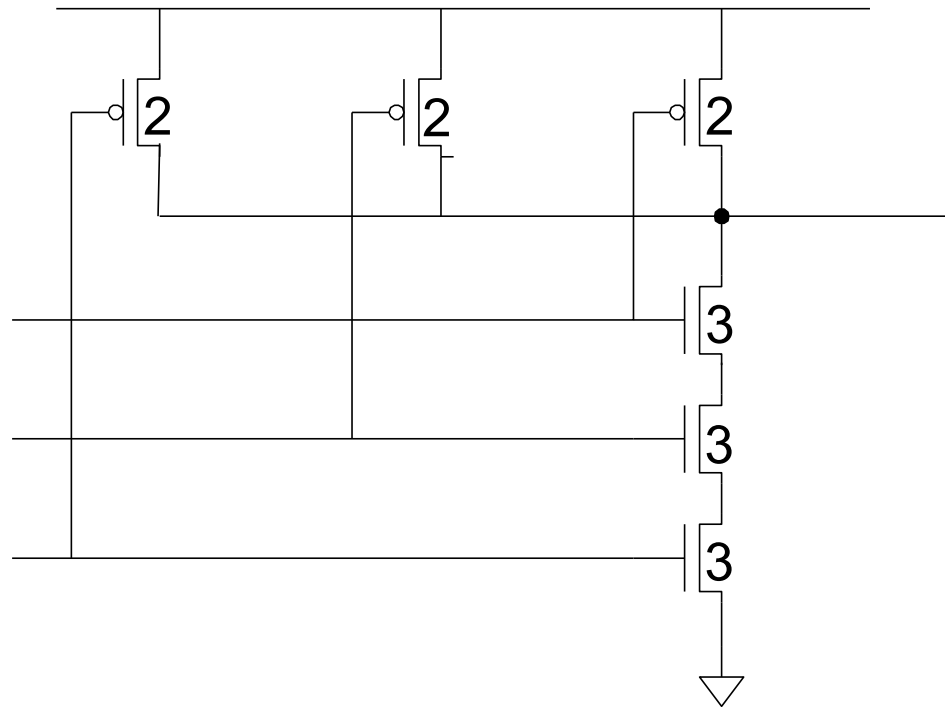
他為了把電阻變小



因為串連 電阻變三倍
把

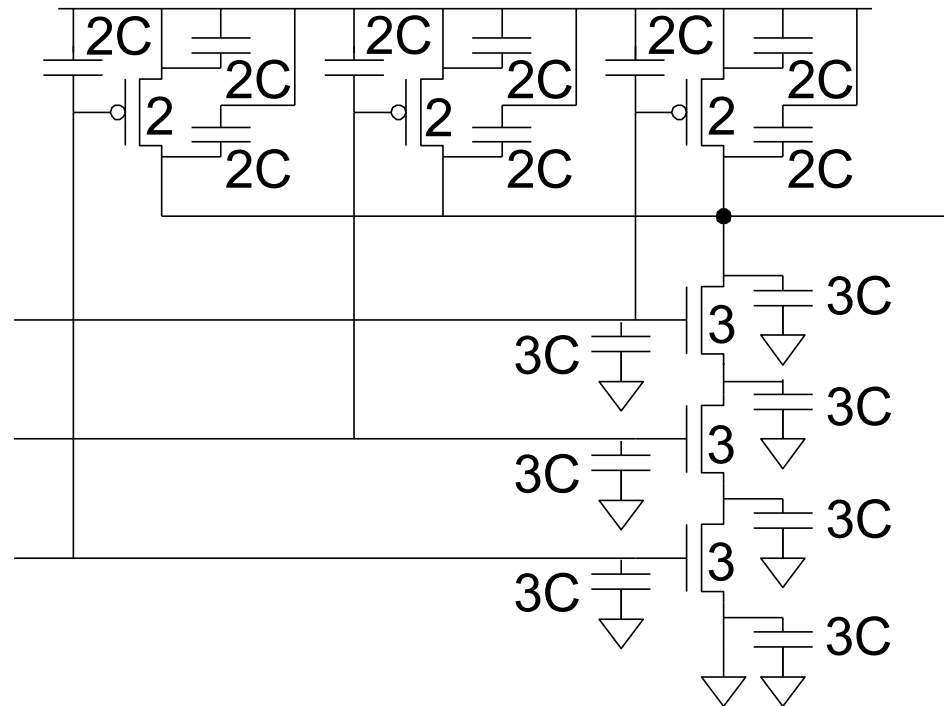
3-input NAND Caps

- ❑ Annotate the 3-input NAND gate with gate and diffusion capacitance.



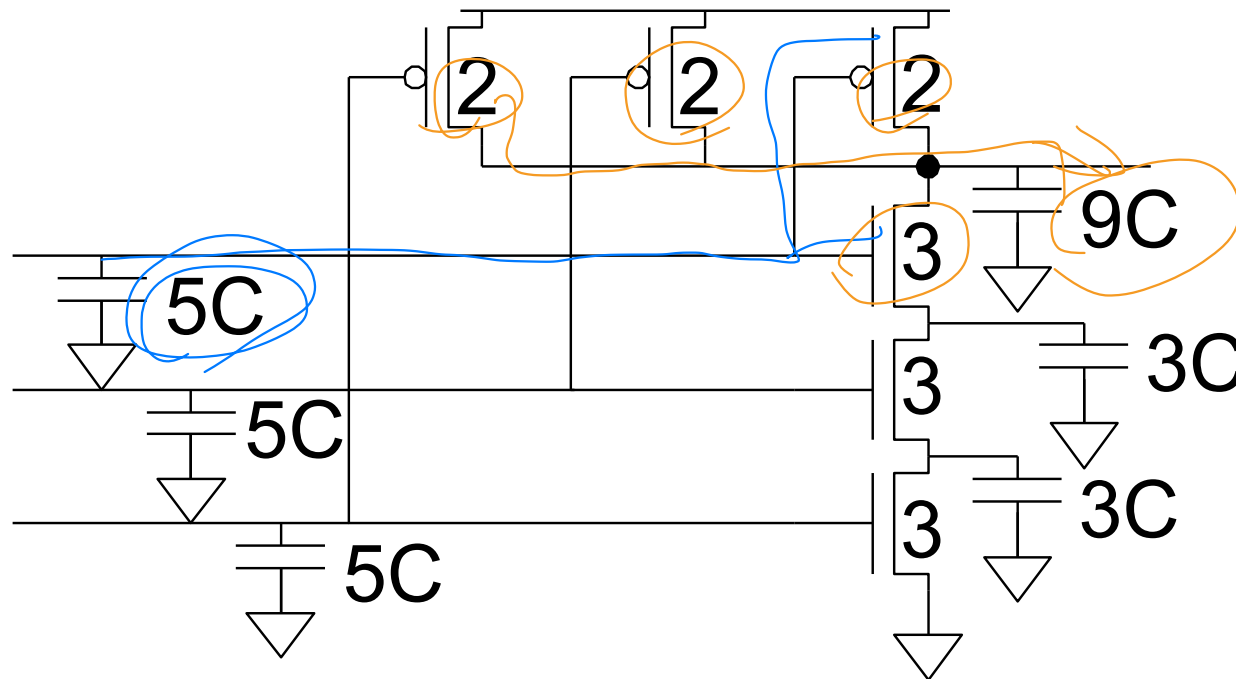
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3-input NAND Caps

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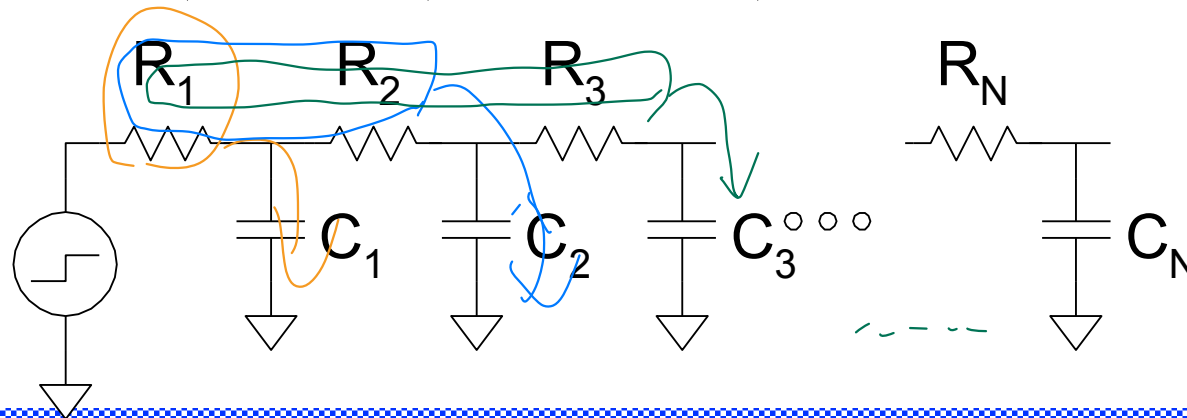


Elmore Delay

- ❑ ON transistors look like resistors
- ❑ Pullup or pulldown network modeled as *RC ladder*
- ❑ Elmore delay of RC ladder

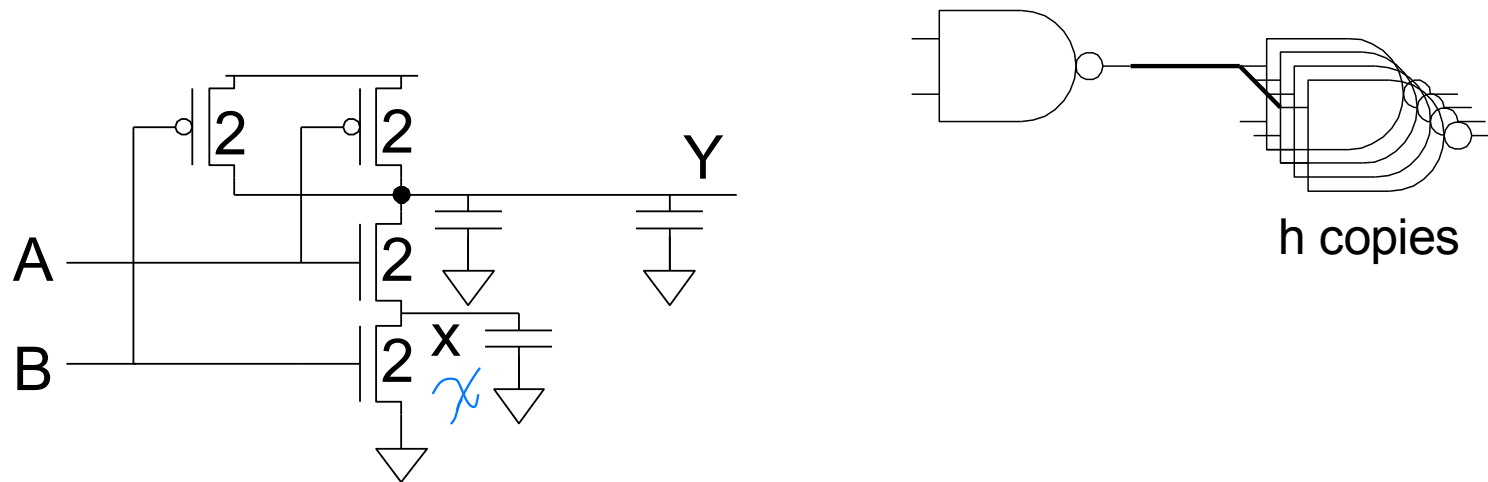
$$t_{pd} \approx \sum_{\text{nodes } i} R_{i\text{-to-source}} C_i$$

$$= R_1 C_1 + (R_1 + R_2) C_2 + \dots + (R_1 + R_2 + \dots + R_N) C_N$$



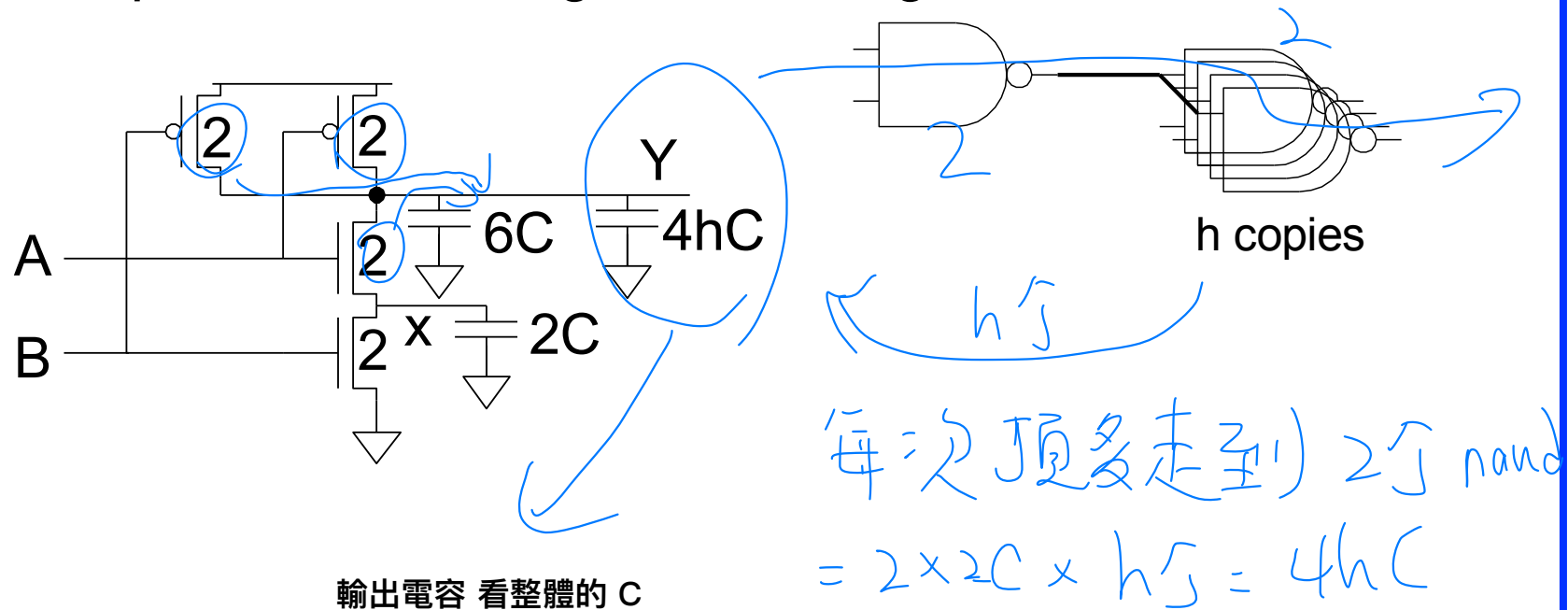
Example: 2-input NAND

- Estimate worst-case rising and falling delay of 2-input NAND driving h identical gates.



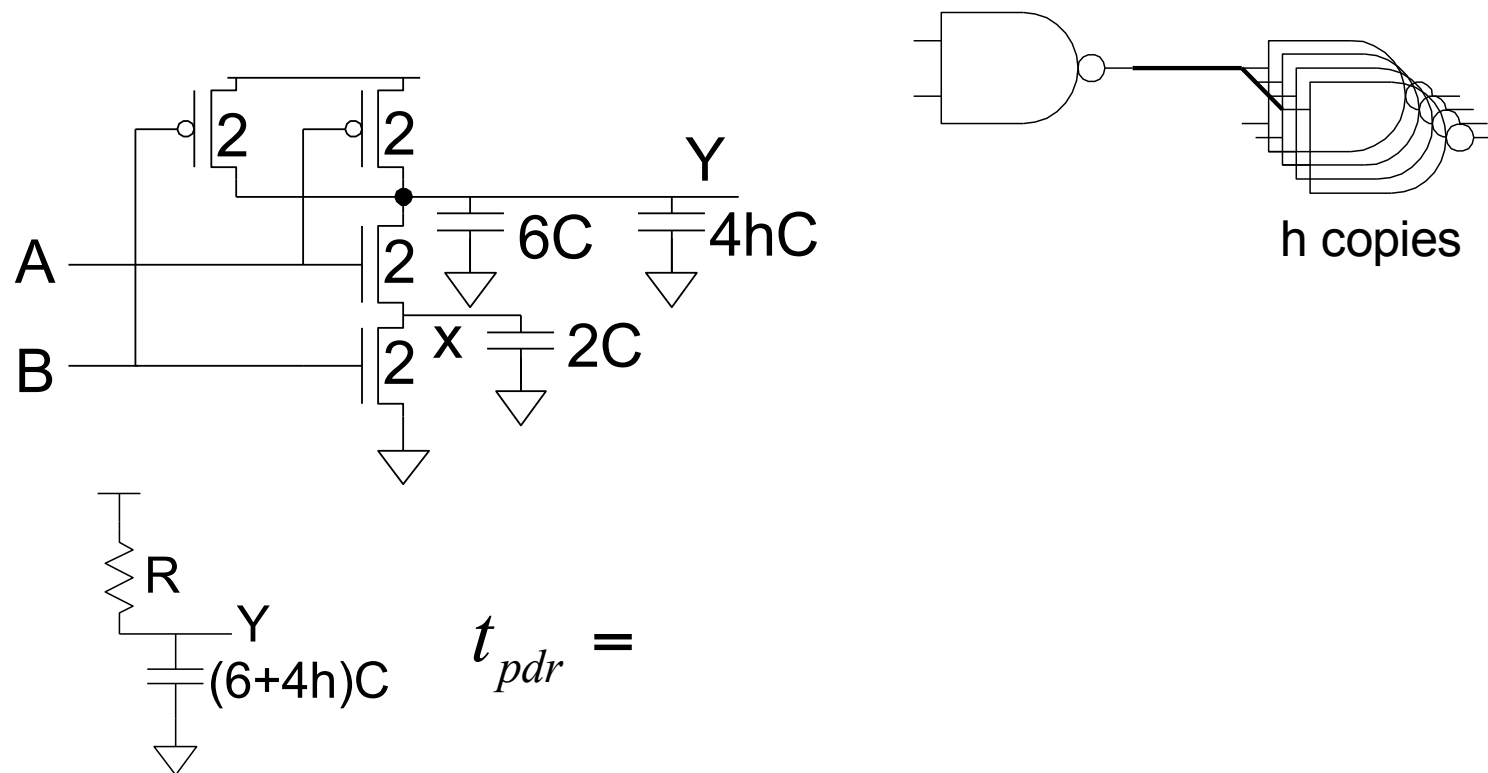
Example: 2-input NAND

- Estimate rising and falling propagation delays of a 2-input NAND driving h identical gates.



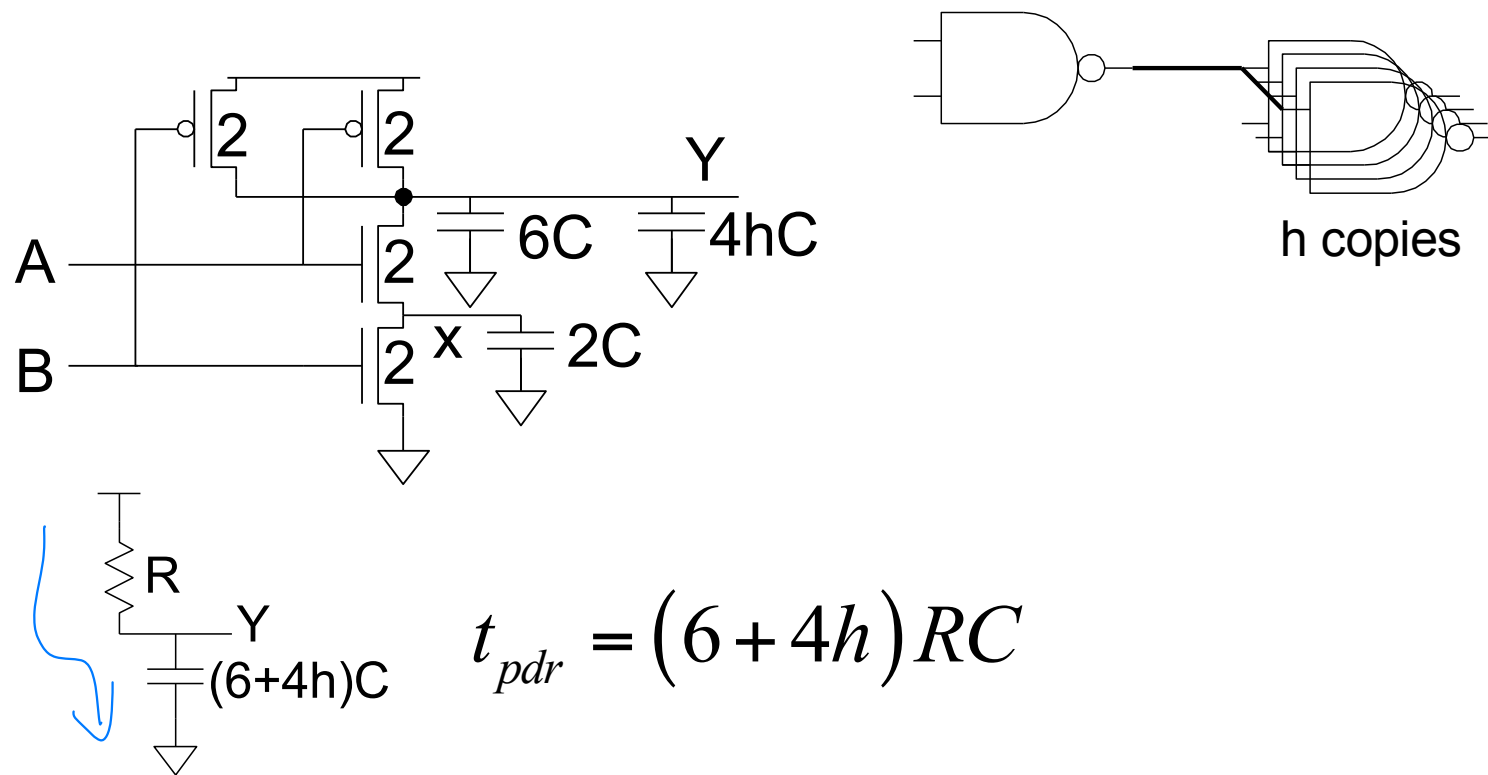
Example: 2-input NAND

- Estimate **rising** and falling propagation delays of a 2-input NAND driving h identical gates.



Example: 2-input NAND

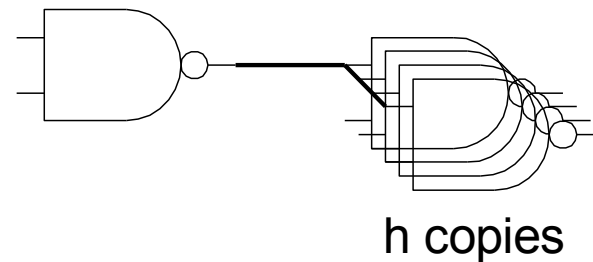
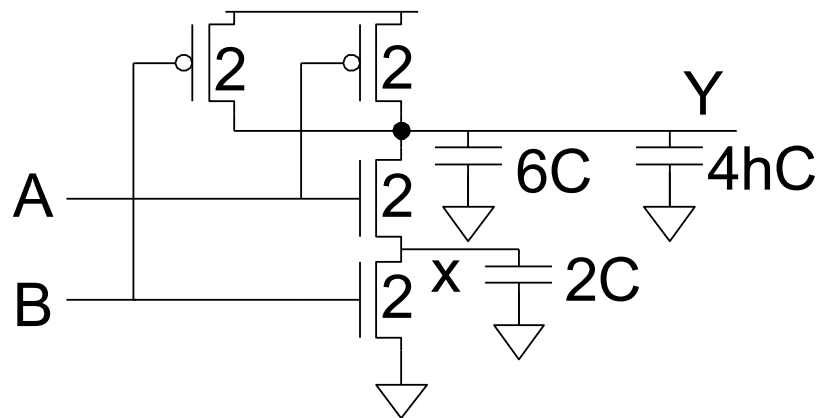
- Estimate **rising** and falling propagation delays of a 2-input NAND driving h identical gates.



充電
遅延

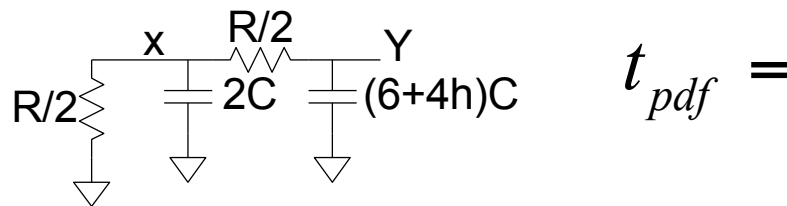
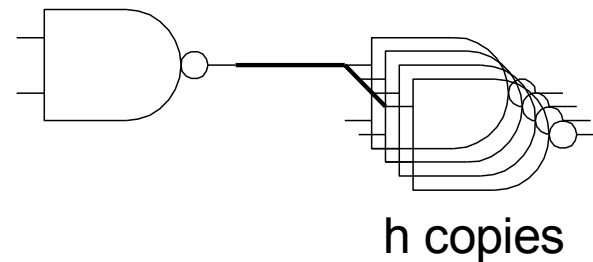
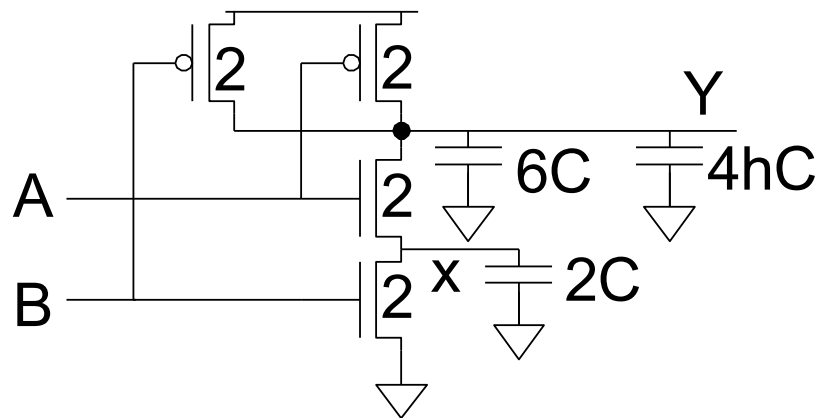
Example: 2-input NAND

- Estimate rising and **falling** propagation delays of a 2-input NAND driving h identical gates.



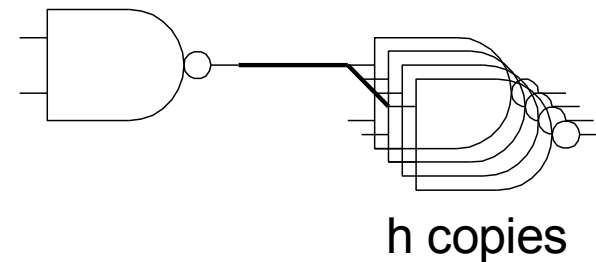
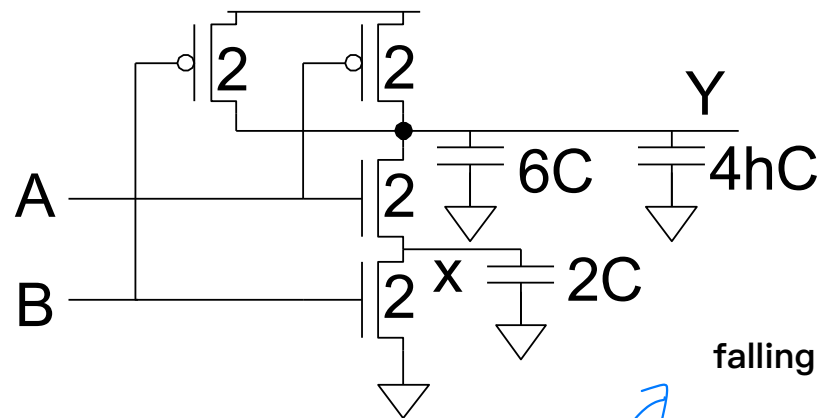
Example: 2-input NAND

- Estimate rising and falling propagation delays of a 2-input NAND driving h identical gates.

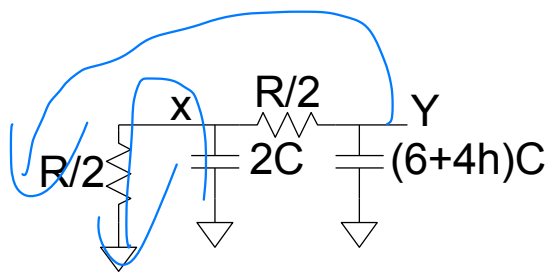


Example: 2-input NAND

- Estimate rising and **falling** propagation delays of a 2-input NAND driving h identical gates.



falling propagation delay



$$t_{pdf} = (2C)\left(\frac{R}{2}\right) + [(6 + 4h)C]\left(\frac{R}{2} + \frac{R}{2}\right)$$

$$= (7 + 4h)RC$$

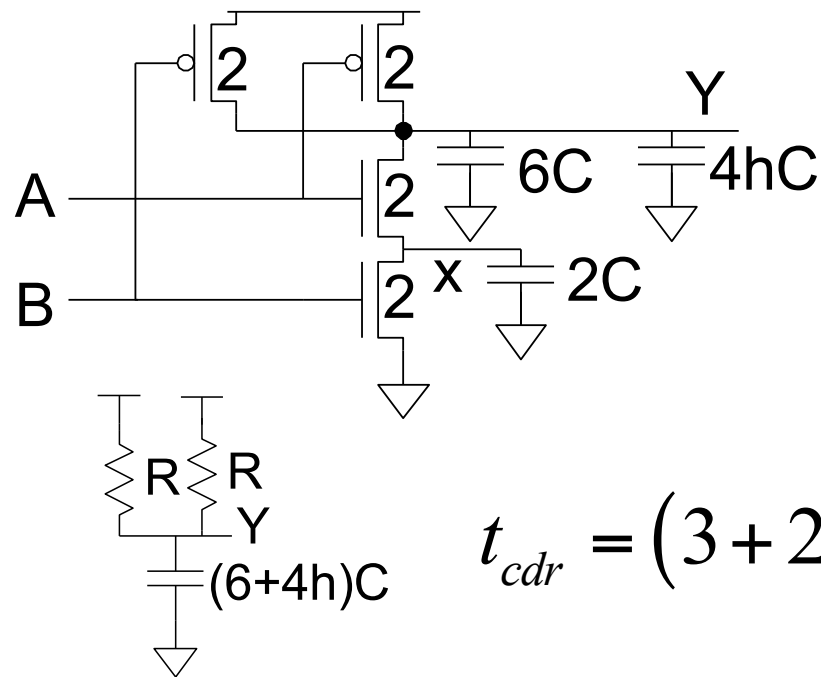
這樣放電

Delay Components

- ❑ Delay has two parts
 - *Parasitic delay* 寄生 延遲?
 - 6 or 7 RC
 - Independent of load
 - *Effort delay*
 - 4h RC
 - Proportional to load capacitance

Contamination Delay

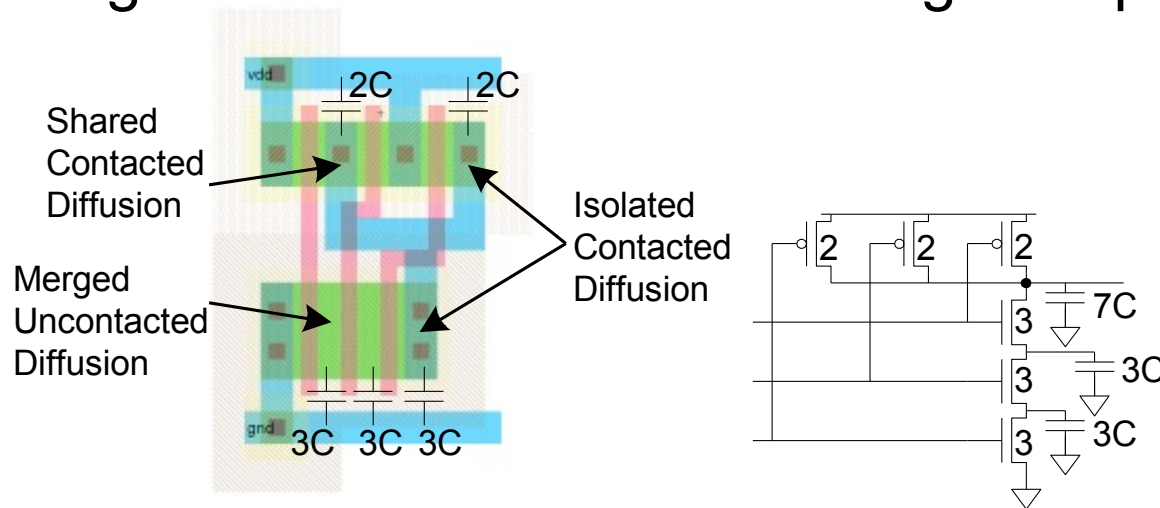
- ❑ Best-case (contamination) delay can be substantially less than propagation delay.
- ❑ Ex: If both inputs fall simultaneously



$$t_{cdr} = (3 + 2h)RC$$

Diffusion Capacitance

- ❑ we assumed contacted diffusion on every s / d.
- ❑ Good layout minimizes diffusion area
- ❑ Ex: NAND3 layout shares one diffusion contact
 - Reduces output capacitance by $2C$
 - Merged uncontacted diffusion might help too



Layout Comparison

❑ Which layout is better?

