M11482, PROBLEM SET 1

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*Problems marked with * are more difficult; we think...*

1 Define the order relation $<$ on $\mathbb{Z}$ using only the addition structure(s)[1] on $\mathbb{Z}$

2* Given a (totally) ordered set $(S, \preceq)$[2], we may also define the greatest lower bound property as follows:

Definition 1. Given an ordered set $(S, \preceq)$, this set has the **greatest lower bound** property if for all nonempty subsets $E \subset S$ with a lower bound (some $\beta \in S$ such that $\forall e \in E$, $\beta \leqslant e$) E has a *greatest lower bound*, β^*, such that for all lower bounds β of E, $\beta \leqslant \beta^*$.

Prove that if an ordered set has the least upper bound property it also has the greatest lower bound property.

If an ordered set has the greatest lower bound property does it necessarily have the least upper bound property?

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[1]note that this includes subtraction

[2]this denotes a set S with ordering given by $\preceq$