M11482 PROBLEM SET 3

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*Problems marked with * are more difficult; we think...*

problem numba Problem statement

1. Topological basis for metric spaces. Show the open balls of a metric space (X, d) form a basis for the topology of the metric space.

2. Basis of a topology "spans" open sets Given a basis $\mathscr{B}$ for the topological space (X, τ), show every open set of X is a union of basis elements.

3. Metric $\implies$ Hausdorff Show that metric spaces are also Hausdorff spaces.

4. Unique Limit Points Show Hausdorff spaces have unique limits of sequences.
5. Integrals on $\mathbb{C}$ Solve the following integrals on a curve C that is a circle with center at 0 and radius 10:

a.

$$\int_C \frac{25}{z(z^2+1)} dz$$

b.

$$\int_C \frac{sin(z) + cos(z)}{z^3} dz$$

***6.** Difficult real integrals This was not gone over in class, but might be fun to figure out. The problem is doing a super hard integral on the real number-line using these methods of complex integration to make out lives easier, don't worry if you don't get it. (Hints (might give it away): The curve determines what parts of the numbers will be integrated over, watch out for boundaries, any function f(x) has a corresponding function f(z) which has the same value on the real numbers and some values that shouldn't be worried about on complex areas)

$$\int_{-\infty}^{\infty} \frac{1}{x(x-1)(x+1)} dx$$

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