4.2) a) b) If a node contains one or more children in a tree then that node can be called as a parent node of those child nodes.

ParentNodes	ChildNodes
A	B, C
B	D, E
C	F
D	G, H
E	I, J
F	K
J	L, M

c) The nodes (two or more) that have the same nodes as their parents are called siblings.

ChildNodes	ParentNodes
B, C	A
D, E	B
G, H	D
I, J	E
L, M	J

B and C	siblings	because of	A
D and E	〃	〃	B
G and H	〃	〃	D
I and J	〃	〃	E
L and M	〃	〃	J

d) The depth of a node is the length of the unique path from that node to the root node of a tree. The depth of the root is zero.

Node	Depth
A	0
B	1
C	1
D	2
E	2
F	2
G	3
H	3
I	3
J	3
K	3
L	4
M	4

e) The height of a node is the length of the longest path from that node to the leaf node in that path The height of a leaf node is zero.

Node	Height
A	4
B	3
C	2
D	1
E	2
F	1
G	0
H	0
I	0
J	1
K	0
L	0
M	0

4.8) Prefix: ① Get the data of the current node
② Go to the left subtree and perform the preorder traversal
③ " right " ".

Result: - * * a b + c d e

Infix: ②
①
③

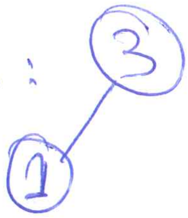
Result: ((a*b)*(c+d))-e

Postfix: ②
③
①

Result: a b * c d + * e -

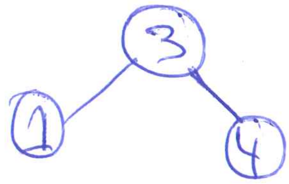
4.9) a) Step 1 (S1): 3

S2:



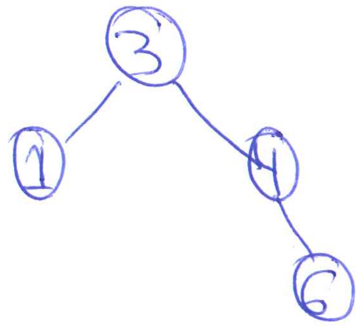
· because $1<3$, go to the left

S3:



· " $4>3$, go to the right

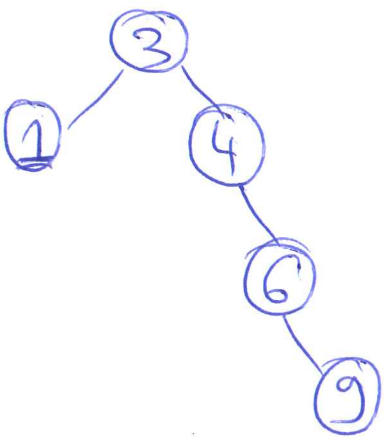
S4:



· " $6>3$ " right and

· " $6>4$ " right again

S5:

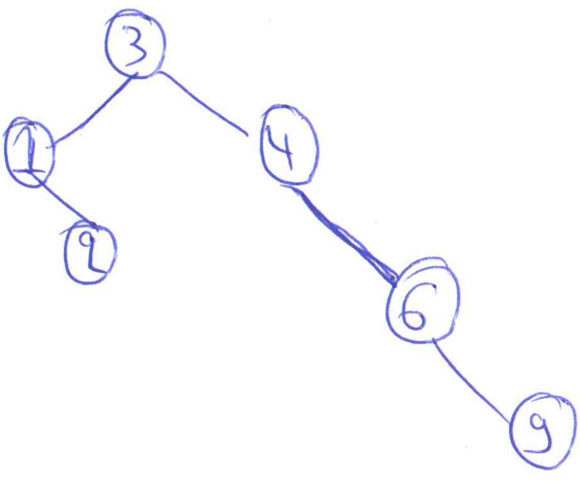


· " $9>3$ "

· " $9>4$ "

· " $9>6$ " go to the right

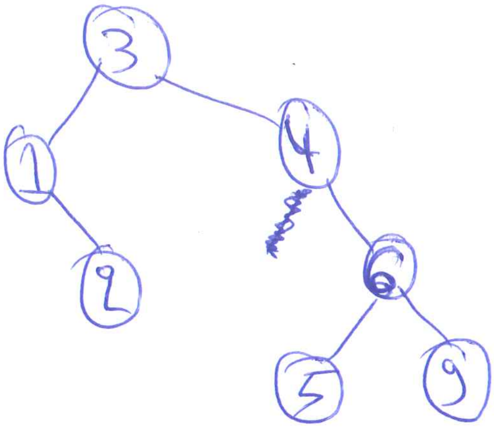
S6:



· $2<3$ go left

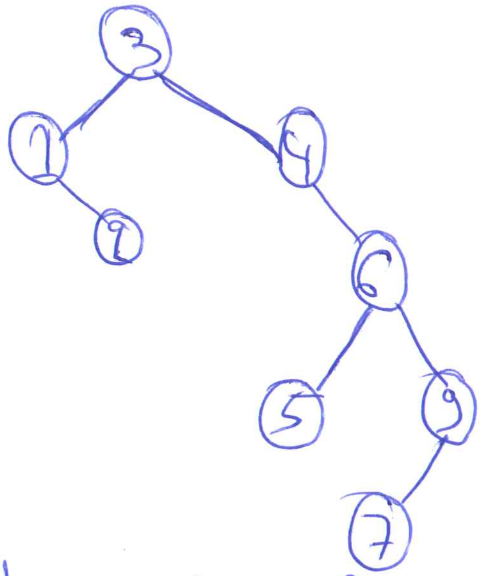
· $2>1$ go right

S7:



- $5>3$ right
- $5>4$ right
- $5<6$ left

S8:



- $7>3$ right
- $7>4$ right
- $7>6$ right
- $7<9$ left

b) We have the following possibilities:

- If the node has not children, then simply delete the node.
- If the node has the right subtree, then replace the node with the minimum of the right subtree.
- If the node has only the left subtree, then replace the node with the root of the left subtree.

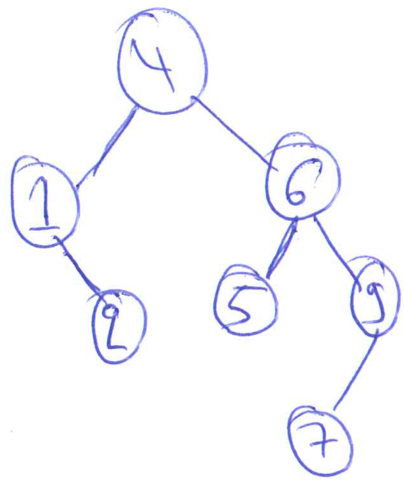
The value of the root is 3 and it has two children so:

4.9) b)

Continuation

Replace the root node with the minimum of the right subtree (here, 4) of the root node.

The result:



4.25) a) From the book and the theory slides, we know that the height of an AVL tree is at most roughly $1.44 \log(N+2) - 1.328$, but in practice it is only slightly more than $\log N$. Thus, the height of an AVL tree with N nodes is about $\log N$. It is also known that the largest unsigned integer represented by k bits is $2^k - 1$.
Hence, if $\log N = 2^k - 1$ then

$$k = \log(\log N + 1)$$

So, it requires $\log(\log N + 1)$ bits of memory to store an integer with the value $\log N$ (i.e., height).

b) Let h be the height of the smallest AVL tree that uses a k-bits counter to store its height, where $k \geq 8$. From a), we know that $h = \log N$ and $\log(\log N + 1) = k \geq 8$.
Thus, we obtain $\log(h+1) \geq 8$ and it implies $h \geq 2^8 - 1 = 255$. So, the desired AVL tree is an AVL tree with height at least 255.

4.31) Check the code on Moodle.

4.32) Check the code on Moodle.

4.33) Che the short answers on Moodle.
It has a sufficient solution there.

4.49) Check the code on Moodle.

And → We will augment the binary search tree by storing the weight of left subtree rooted at a node. So any node will keep an extra count which is equal to the number of nodes in its left subtree. This augmentation is called order statistics tree. With the help of this augmentation we can find the kth smallest element in O(logN) expected complexity for a tree. Suppose we try to find the 6th smallest element, we start at root. If root has leftWeight value 3, that means there are only 3 elements that are smaller than root. So 6th smallest element cannot be on the lest side of root. So, we try to find the element in right subtree.

While going to right subtree, we now try to find 6-4 = 2nd smallest element, because we already had 3 smaller element in root's left subtree and root itself is smaller than the right subtree. So, we call the recursive function on root right. If the value of k is less than the leftweight the we just go to the left subtree with the value k. If not then we go to the right subtree.

5.1) a) · $h(4371) = 4371 \bmod 10 = 1$

· $h(1323) = 1323$ " $= 3$

· $h(6173) = 6173$ " $= 3$

· $h(4199) = 9$

· $h(4344) = 4$

· $h(9679) = 9$

· $h(1989 = 9$

Check the short answer for a visual representation*

b) · $h(4371) = 4371 \bmod 10 = 1$

· $h(1323) = 1323 \bmod 10 = 3$

· $h(6173) = 3 \rightarrow$ collision

$(h(6173) + f(1)) \bmod 10 = (3+1) \bmod 10 = 4$

· $h(4199) = 9$

· $h(4344) = 4 \rightarrow$ collision

$(h(4344) + f(1)) = (4+1) \bmod 10$

$= 5 \bmod 10 = 5$

~~h 968~~

· $h(9679) = 9 \rightarrow$ collision

$(h(9679) + f(1)) \bmod 10 = (9+1) \bmod 10 = 0$

- $h(1989) = 1989 \bmod 10 = 9 \to$ collision

$(h(1989)+f(1)) \bmod 10 = 0 \to$ collision

$(h(1989)+f(2)) \bmod 10 = (9+2) \bmod 10 = 11 \bmod 10 = 1$

$(h(1989)+f(3)) \bmod 10 = (9+3) \bmod 10 = 2$ collision

Check the short answer for a visual representation.*

c)
- $h(4371) = 1$
- $h(1323) = 3$
- $h(6173) = 3 \to$ collision

$(h(6173)+f(1)) \bmod 10 = (3+1^2) \bmod 10 = 4$

- $h(4199) = 9$
- $h(4344) = 4 \to$ collision

$(h(4344)+f(1)) \bmod 10 = (4+1^2) \bmod 10 = 5$

- $h(9679) = 9 \to$ collision

$((h(9679)+f(1)) \bmod 10 = 0$

- $h(1989) = 9 \to$ collision

$(h(1989)+f(1)) \bmod 10 = 0 \to$ collision

» $+f(2)$ » $= 3 \to$ collision

» $+f(3)$ » $= (9+3^2) \bmod 10 = 18 \bmod 10 = 8$

Check the short ~~[illegible]~~ answer as before.*

5.1) d) • $h(4371) = 1$

• $h(1323) = 3$

• $h(6173) = 3 \to$ collision

$h_2(6173) = 7 - (6173 \bmod 7) = 7 - 6 = 1$

and

$(h(6173) + 1 \cdot h_2(6173)) \bmod 10 =$

$= (3 + 1 \cdot 1) \bmod 10 = 4 \bmod 10 = 4$

• $h(4199) = 9$

• $h(4344) = 4 \to$ collision

$h_2(4344) = 7 - (4344 \bmod 7) = 7 - 4 = 3$ and

$(h(4344) + 1 \cdot h_2(4344)) \bmod 10 = (4 + 1 \cdot 3) \bmod 10 =$

$= 7$

• $h(9679) = 9 \to$ collision

$h_2(9679) = 7 - (9679 \bmod 7) = 2$ and

$(h(9679) + 1 \cdot h_2(9679)) \bmod 10 = 1 \to$ collision

$(h(9679) + 2 \cdot h_2(9679)) \bmod 10 = 3 \to$ collision

$(h(9679) + 3 \cdot h_2(9679)) \bmod 10 = 5$

- $h(1989) = 9 \to$ collision

$h_2(1989) = 7 - (1989 \bmod 7) = 7 - 1 = 6$ and

$(h(1989) + 1 \cdot h_2(1989)) \bmod 10 = 5 \to$ collision

" $2 \cdot$ " $= 1 \to$ collision

" $3 \cdot$ " $= 7 \to$ "

" $4 \cdot$ " $= 3 \to$ "

" $5 \cdot$ " $= 9 \to$ "

" $6 \cdot$ " $= 5 \to$ "

The element 1989 cannot be inserted in the hash table.

Check the short answer as before*