Simulation methods. Exercise 4.
Spring 2023

The dissipative Hénon map can be written as

$$\left.\begin{array}{rcl} \bar{x} & = & 1 + y - ax^2, \\ \bar{y} & = & bx, \end{array}\right\}$$

where a and b are given parameters. In the exercise, we choose the classical values $a = 1.4$ and $b = 0.3$. In what follows, we will denote this map by H.

1. Show that this is an invertible map. Given any set $S \subset \mathbb{R}^2$ with positive measure, can you compute the limit of the measure of $H^{(n)}(S)$ when $n \to \infty$?
2. (Optional) Make a plot of the dynamics of H. Note that there is an attractor set for many initial conditions. Make a plot of this attracting set. Feel free to zoom into the structure of this attracting set.
3. This map has a fixed point p_0 near $(0.63, 0.19)$. Compute p_0 and its stability.
4. Compute an approximation to the Lyapunov exponent of the attractor for different (at least two) initial conditions. Explain the computational method and discuss the results.
5. (Optional) From the previous computation, it is possible to derive the full set of Lyapunov exponents of this attractor?

Delivery: "Campus Virtual" before May 29th.