TOPOLOGICAL DATA ANALYSIS

EXERCISES 2.2

1) Find the homology groups with coefficients in $\mathbb{Z}$ of the abstract simplicial complex whose maximal faces are $(12)(13)(14)(23)(25)(36)(456)$

$K = (12)(13)(14)(23)(25)(36)(456)$

$C_0(K) = \mathbb{Z}(1) \oplus \mathbb{Z}(2) \oplus \mathbb{Z}(3) \oplus \mathbb{Z}(4) \oplus \mathbb{Z}(5) \oplus \mathbb{Z}(6)$

$C_1(K) = \mathbb{Z}(12) \oplus \mathbb{Z}(13) \oplus \mathbb{Z}(14) \oplus \mathbb{Z}(23) \oplus \mathbb{Z}(25) \oplus \mathbb{Z}(36) \oplus \mathbb{Z}(45) \oplus \mathbb{Z}(46) \oplus \mathbb{Z}(56)$

$C_2(K) = \mathbb{Z}(456)$

$0 \xrightarrow{\partial_3} C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0$

$\ker(\partial_0) = \langle (1), (2), (3), (4), (5), (6) \rangle$

$\ker(\partial_1) = \langle (12)-(13)+(23),$
$(12)-(14)+(25)-(45),$
$(13)-(14)+(36)-(45)-(56),$
$(45)-(46)+(56) \rangle$

$\mathrm{Im}(\partial_1) = \langle (2)-(1), (3)-(1),$
$(4)-(1), (6)-(4), (6)-(5) \rangle$

Matrix of ∂_1:

	(12)	(13)	(14)	(23)*	(25)*	(36)*	(45)*	(46)	(56)
(1)	-1	-1	-1						
(2)	1			-1	-1				
(3)		1		1		-1			
(4)			1				-1	-1	
(5)					1		1		-1
(6)						1		1	1

$\ker(\partial_2) = 0$

$\mathrm{Im}(\partial_2) = \langle (45)-(46)+(56) \rangle$

$\mathrm{Im}(\partial_3) = 0$

Matrix of ∂_2:

	(456)
(45)	1
(46)	-1
(56)	1

$\longrightarrow H_0(K) = \ker(\partial_0)/\mathrm{Im}(\partial_1) \cong \mathbb{Z}$

$\longrightarrow H_1(K) = \ker(\partial_1)/\mathrm{Im}(\partial_2) = \langle (12)-(13)+(23), (12)-(14)+(25)-(45),$
$(13)-(14)+(36)-(45)-(56) \rangle \cong \mathbb{Z}$

$\longrightarrow H_2(K) = \ker(\partial_2)/\mathrm{Im}(\partial_3) = 0$