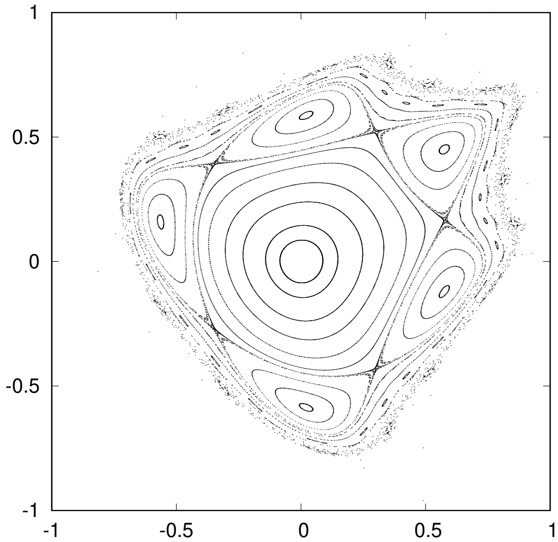
Simulation methods. Exercise 1.
Spring 2023

The conservative Hénon map can be written as

$$\left.\begin{array}{rcl} \bar{x} & = & x\cos\alpha - y\sin\alpha + x^2\sin\alpha, \\ \bar{y} & = & x\sin\alpha + y\cos\alpha - x^2\cos\alpha, \end{array}\right\}$$

where α is a given parameter. In the exercise, we choose the value $\alpha = 1.33$. The dynamics of this map can be roughly seen in the figure at the right.



The origin is an elliptic fixed point surrounded by invariant curves that are close to circles. As discussed in class, away from the origin we see a chain of islands of period five. This chain of islands surrounds an elliptic periodic orbit of period five. Note that there is also an hyperbolic orbit of period five.

The first part of the exercise is optional, and it is to produce a figure similar to the one at the right. You have several options to produce this plot: a) write a program to iterate several initial data (in a language like C or C++) and save these iterates to a file, and then use a graphical program like `gnuplot` to visualise it; b) write a program that uses graphical output to iterate and plot the points at the same time (in the computer program shown in the classroom, I used C and the library `pgplot5`). Feel free to explore any other option.

The compulsory part of the exercise is about to compute these two periodic orbits (one elliptic, one hyperbolic):

1. Show that this is an area-preserving map.
2. Compute the elliptic periodic orbit of period five mentioned before. Give all five points.
3. Compute the hyperbolic periodic orbit of period five mentioned before. Give all five points.
4. Compute the eigenvalues related to the elliptic and the hyperbolic periodic orbit. Do they depend on the starting point of the orbit? What can you tell about their product?

Send the results to me by e-mail, preferably before March 6th.