

Exercise 1 In the Black-Scholes (BS) model compute the price and the self-financing hedging portfolios of contingent claims with payoffs:

- (1) $X = S_T^2$,
- (2) $X = S_T/S_{T_0}$, $0 \leq T_0 \leq T$.
- (3) $X = 1/S_T$.

Exercise 2 Show, in the BS model, that the price of an Asian option with floating strike (payoff $= \left(\frac{1}{T} \int_0^T S_u du - S_T \right)_+$) is given, at the initial time, by

$$C = e^{-rT} S_0 \varphi(0, 0)$$

where φ is a solution of the equation

$$r\varphi + \frac{\partial \varphi}{\partial t} - \frac{\partial \varphi}{\partial x} \left(rx + \frac{1}{T} \right) + \frac{1}{2} \frac{\partial^2 \varphi}{\partial x^2} \sigma^2 x^2 = 0$$

with boundary condition $\varphi(T, x) = (1 + x)_-$.