TOPOLOGICAL DATA ANALYSIS

EXERCISES 2.1

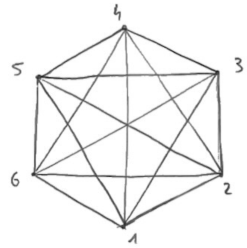
1) Let K and L be the abstract simplicial complexes whose maximal faces are, respectively;

(a) K: (124)(125)(135)(136)(146)(234)(236)(256)(345)(456)

(b) L: (014)(015)(023)(027)(035)(047)(126)(128)(148)
(156)(236)(278)(346)(348)(358)(467)(567)(578)

Prove that the geometric realizations |K| and |L| are compact surfaces, and find out which surfaces they are.

(a)

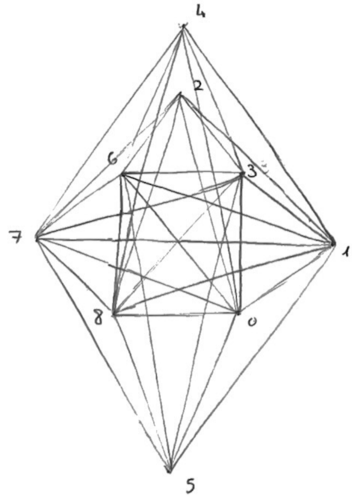


The graph is a complete graph, thus we have $|K| \cong \Delta^{n-1}$ with $n=6$ => $|K| \cong \Delta^5$

Since $\Delta^5 = \Delta(e_1, \dots, e_6) \subset \mathbb{R}^6$
$= \{x_1 e_1 + \dots + x_6 e_6 \in \mathbb{R}^6 : x_i \geq 0, \sum_i x_i = 1\}$

this shows its geometric realization is compact.

(b)



The graph can be visualized as 3 piramids with base 0,1,3,6,7,8 and vertices 2,4,5

Its geometric realization is given by the geometric simplicial complex X_L with a 2-face $\Delta(e_{i_0}, \dots, e_{i_2})$ for each 2-face of L.

Since $\Delta(e_{i_0}, \dots, e_{i_2}) = \{x_0 e_{i_0} + \dots + x_2 e_{i_2} \in \mathbb{R}^3 : x_i \geq 0, \sum_i x_i = 1\}$
this shows its geometric realization is compact.

Euler characteristic (and classification theorem for compact surfaces)

(a) $EC = 6 - 15 + 10 = 1$

$\hookrightarrow EC = 1 \Rightarrow |K| \cong \mathbb{RP}^2 \leftarrow$ projective plane

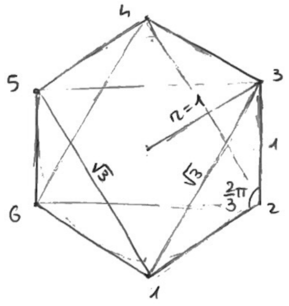
(b) It is a triangulation of the Klein bottle, $e = 0$

TOPOLOGICAL DATA ANALYSIS

EXERCISES 2.1

2) List the maximal faces of the Čech complex $C_\varepsilon(X)$ and the Vietoris-Rips complex $R_\varepsilon(X)$, depending on ε, if X is the set of vertices of a regular hexagon of radius 1.

$X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ in $\mathbb{R}^2$ such that they form a regular hexagon:



$C_\varepsilon(X)$

For $0 \leq \varepsilon < 1$: $C_\varepsilon(X)$: (1)(2)(3)(4)(5)(6)

For $1 \leq \varepsilon < \sqrt{3}$: $C_\varepsilon(X)$: (12)(16)(23)(34)(45)(56)

For $\sqrt{3} \leq \varepsilon < 2$: $C_\varepsilon(X)$: (123)(126)(156)(234)(345)(456)

For $\varepsilon \geq 2$: $C_\varepsilon(X)$: (123456)

$R_\varepsilon(X)$

For $0 \leq \varepsilon < 1$: $R_\varepsilon(X)$: (1)(2)(3)(4)(5)(6)

For $1 \leq \varepsilon < \sqrt{3}$: $R_\varepsilon(X)$: (12)(16)(23)(34)(45)(56)

For $\sqrt{3} \leq \varepsilon < 2$: $R_\varepsilon(X)$: (123)(126)(135)(156)(234)(246)(345)(456)

For $\varepsilon \geq 2$: $R_\varepsilon(X)$: (123456)