Exercise 1 *Obtain the following bounds for the call prices (C) and for the put ones (P) European (E) and American (A):*

$$\max(S_n - K, 0) \leq C_n(E) \leq C_n(A);$$
$$\max(0, (1+r)^{-(N-n)}K - S_n) \leq P_n(E) \leq (1+r)^{-(N-n)}K$$

Exercise 2 *Let $\{C_n^E\}_{n=0}^N$ be the price of a European option with payoff Z_N and let $\{Z_n\}_{n=0}^N$ be the payoffs of an American option. Show that if $C_n^E \geq Z_n, n = 0, 1, ..., N-1$, then $\{C_n^A\}_{n=0}^N$ (the prices of the American option) coincide with $\{C_n^E\}_{n=0}^N$.*

Exercise 3 *Prove that, with the usual notations,*

$$\sup_{\tau \in \mathcal{T}_{0,N}} \mathbb{E}_{\mathbb{Q}} \left(\frac{(S_\tau - K)^+}{(1+r)^\tau} \right) = \mathbb{E}_{\mathbb{Q}} \left(\frac{(S_N - K)^+}{(1+r)^N} \right),$$

where $\mathbb{Q}$ is the risk neutral probability of a complete market.

Exercise 4 *Consider a market with N trading periods, a risky asset S and zero interest rate. In such a market we want to price an American option with payoffs $Z_n = d > 0$ if $n \leq N-1$ and $Z_N = S_N$ if $n = N$. Prove that its price is equal to that of a European call option on S, with strike d and maturity time $N-1$ plus the fixed amount d.*