Topological Data Analysis

17 November 2022

Exercises

1. Prove that a morphism f of persistence modules is an isomorphism if and only if f_t is an isomorphism of vector spaces for all t.
2. Prove that two isomorphic persistence modules of finite type have the same spectrum.
3. Prove that there is a nonzero morphism $\mathbb{F}[a,b) \to \mathbb{F}[c,d)$ if and only if $c \leq a$ and $a < d \leq b$.

Please deliver through Campus Virtual as a pdf file before November 24 at 10:00.

Longer exercise (optional)

Prove that, for $a < b < c$, there is an exact sequence of persistence modules

$$0 \longrightarrow \mathbb{F}[b,c) \longrightarrow \mathbb{F}[a,c) \longrightarrow \mathbb{F}[a,b) \longrightarrow 0.$$

Prove that this exact sequence does not split; that is, $\mathbb{F}[a,c)$ is *not* isomorphic to $\mathbb{F}[a,b) \oplus \mathbb{F}[b,c)$, although $\mathbb{F}[a,c)_t \cong \mathbb{F}[a,b)_t \oplus \mathbb{F}[b,c)_t$ for all t.

Longer exercises can be delivered until December 20.