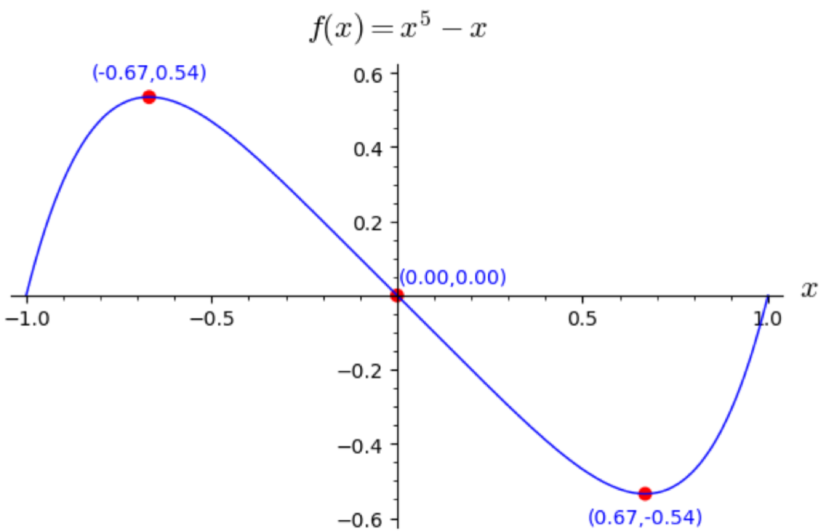
Topological Data Analysis

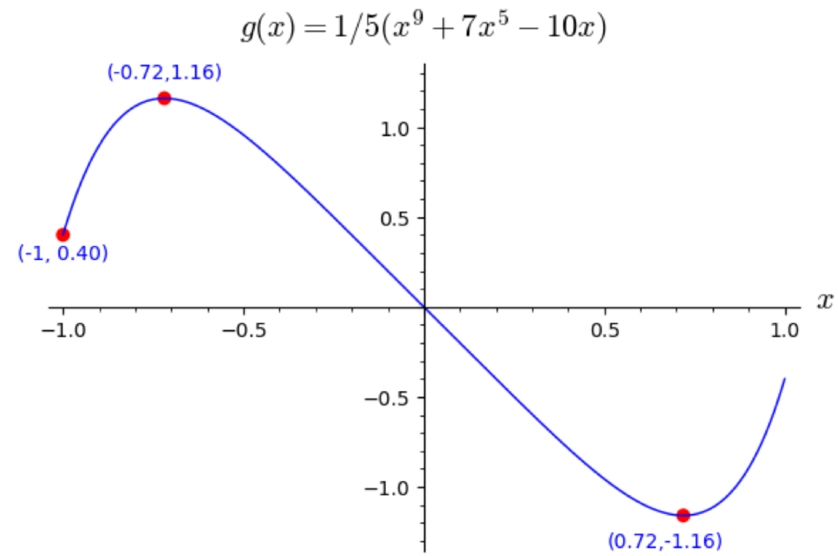
2022–2023

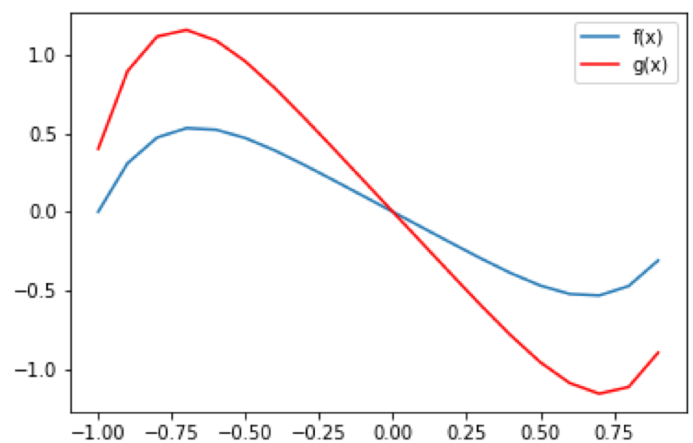
Solutions of Exercises

15 December 2022

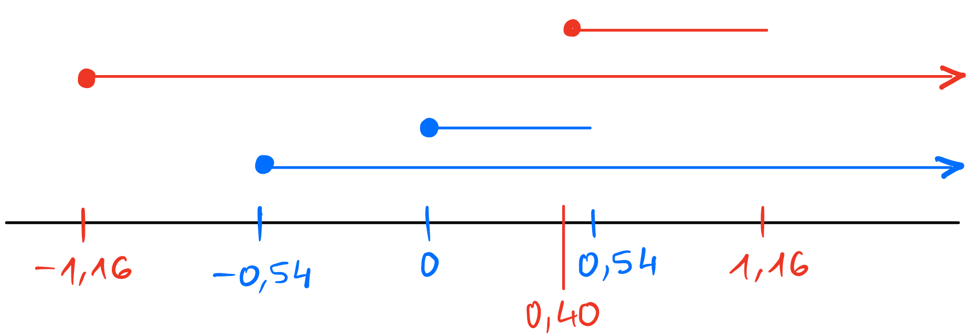
1

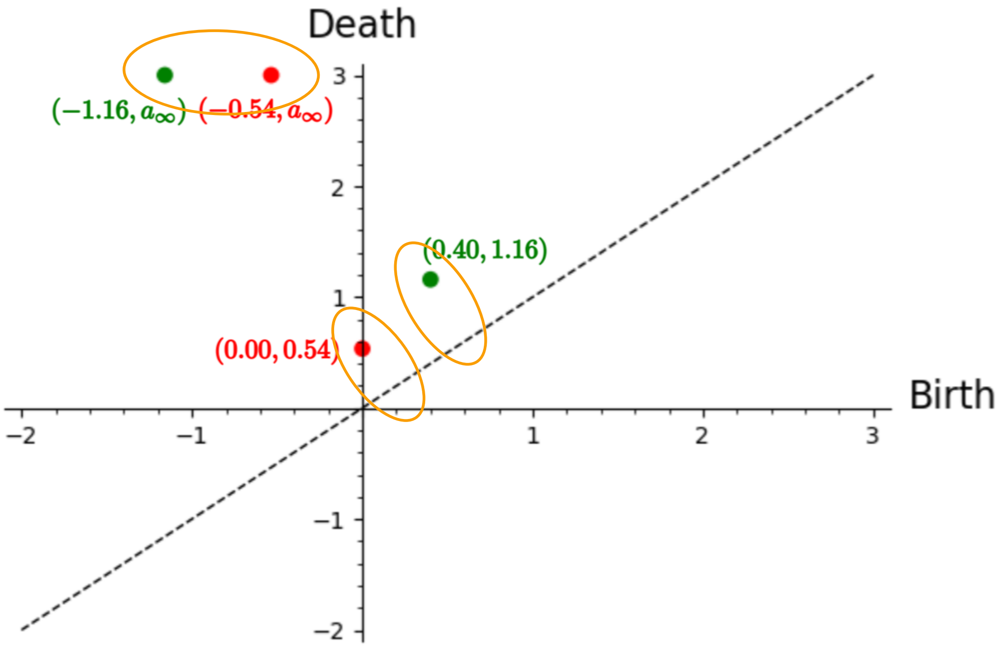






Thanks to Pere Díaz
and Flàvia Ferrús
for the pictures



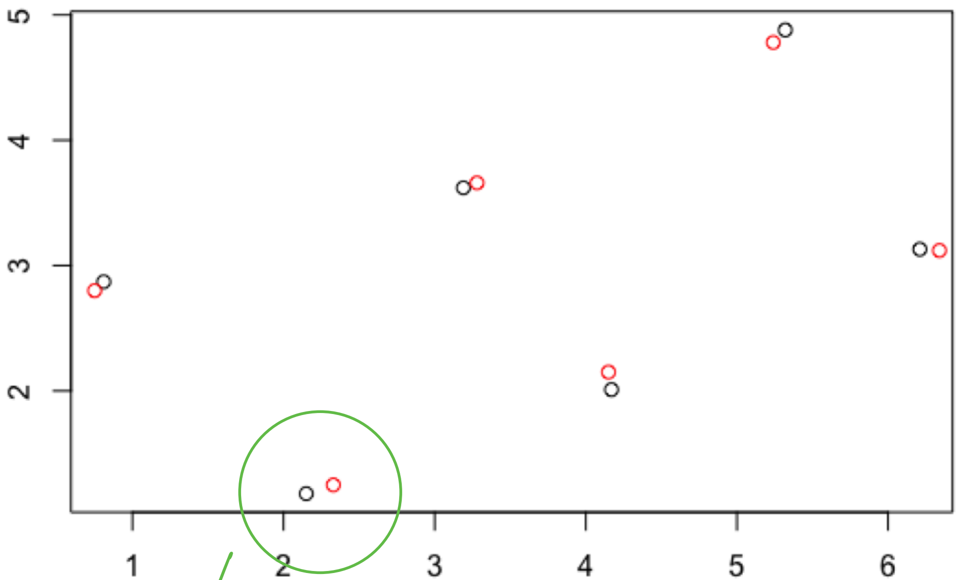


$$d_{int}(V(f), V(g)) = W_\infty(D(f), D(g)) = 0{,}6238$$

Interleaving distance between the infinite rays

$$\| f - g \|_\infty = \sup_{-1 \leq x \leq 1} \{ | f(x) - g(x) | \} = 0{,}6431$$

2



$d_H(X,Y) = 0{,}1931$

$d_{GH}(X,Y) = 0{,}1487$ — Distortion of the natural correspondence

$W_\infty(D(X), D(Y)) = 0{,}2126 < 2\, d_{GH}(X,Y)$ ✓

R packages: TDA, pracma, gromovlab

(3)

Given any compact metric space K and a one-point space $P=\{p\}$, the only possible correspondence is $C = P\times K$.

Hence

$$d_{GH}(P,K) = \frac{1}{2}\operatorname{dis}(C) = \frac{1}{2}\sup\{d^K(x,y) \mid x,y\in K\} =$$
$$= \frac{1}{2}\operatorname{diam}(K).$$

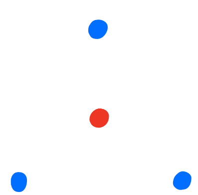
Note that this distance is attained with the distance induced on the disjoint union $P \amalg K$ by defining

$$d(p,x) = \frac{1}{2}\operatorname{diam}(K) \quad \text{for all } x\in K.$$

This satisfies the triangle inequality, since

$$d^K(x,y) \leq d(x,p) + d(p,y) = \operatorname{diam}(K) \quad \text{for all } x,y\in K.$$

Example:



$d(P,K) = \frac{1}{2}$ if $d^K(x_i,x_j) = 1$ for $i\neq j$.