1. Let $T \in \mathcal{S}'$. Show that

   (a) $\widehat{T}^{(k)} = [(-2\pi i t)^k T]^\wedge$.

   (b) $\widehat{T^{(k)}} = (2\pi i \xi)^k \widehat{T}$.

2. Given $a \in \mathbb{R}$ let $\tau_a T$ be defined by $\langle \tau_a T, \varphi \rangle = \langle T, \tau_{-a}\varphi \rangle$, $\varphi \in \mathcal{S}$, and let $M_a T = e^{2\pi i a t} T$. Show that
$$\widehat{\tau_a T} = M_{-a}\widehat{T}\,, \qquad [M_a T]^\wedge = \tau_a \widehat{T}.$$

3. Let $H(x) = \chi_{(0,+\infty)}(x)$ (Heaviside function). Prove that $H' = \delta_0$.

4. Let T be a distribution with $T' \equiv 0$. Show that there exists a constant C such that
$$\langle T, \varphi \rangle = C \int \varphi \quad \text{for all } \varphi \in \mathbb{C}_c^\infty.$$
   (Hint: notice that $\phi \in \mathcal{C}_c^\infty(\mathbb{R})$ is of the form $\phi = \varphi'$, $\varphi \in \mathcal{C}_c^\infty(\mathbb{R})$, if and only if $\int \psi = 0$).

5. (a) Let $f \in \mathcal{C}^1(\mathbb{R} \setminus \{a\})$ be such that $f(a^+) - f(a^-) = s$. Prove that $T_f' = T_{f'} + s\delta_a$.

   (b) More generally, if $f \in \mathcal{C}^1(\mathbb{R} \setminus \{a_n\}_n)$, with $\lim_n |a_n| = \infty$, and each a_n is a jump discontinuty of size s_n, then $T_f' = T_{f'} + \sum_n s_n \delta_{a_n}$.

   (c) Let f be the T-periodic function with value $f(x) = x/T$ in $[0, T)$. Prove that, in the sense of distributions $f' = 1/T - \sum\limits_{n\in\mathbb{Z}} \delta_{nT}$.

   (d) Let $\{\alpha_n\}_n$ be such that $\lim_n |\alpha_n| = \infty$ slowly, meaning that $|\alpha_n| = O(|n|^k)$, for some $k \geq 1$. Let $a \in \mathbb{R}$. Prove that $\sum_n \alpha_n \delta_{na}$ is a tempered distribution and that its Fourier transform is $\sum_n \alpha_n e^{-2\pi i a n\, t}$.

6. Given a tempered distribution $T \in \mathcal{S}^*$ and $f \in \mathcal{S}$, define the convolution $T * f = f * T$ as
$$(T * f)(t) = \langle T, \tau_t \tilde{f} \rangle,$$
   where $\tilde{f}(s) = f(-s)$. Prove that:

   (a) $(\delta_a * f)(t) = \tau_a f(t)$.

   (b) $(\delta_0' * f)(t) = f'(t)$.

   (c) $\widehat{(T * f)} = \widehat{T} \cdot \widehat{f}$.