Simulation methods. Exercise 3.
Spring 2023

Consider the following periodically perturbed pendulum

$$\ddot{x} + \omega^2 \sin x = \varepsilon \sin t.$$

Note that, for $\varepsilon = 0$, this differential equation has $x = 0$ as (elliptic) equilibrium point. We are interested in periodic orbits, of period 2π, near the origin for ε small. In what follows, we select the value $\varepsilon = 10^{-2}$.

1. Let us select $\omega = \sqrt{2}$. Compute a periodic orbit of period 2π near the origin. Compute its stability.
2. Repeat the previous point, but now for $\omega = \frac{1}{\sqrt{2}}$.
3. We are interested in periodic orbits (of period 2π) for $\frac{1}{\sqrt{2}} \leq \omega \leq \sqrt{2}$. Perform a numerical continuation w.r.t. ω starting at $\omega = \frac{1}{\sqrt{2}}$ up to $\omega = \sqrt{2}$. Perform a second numerical continuation starting at $\omega = \sqrt{2}$ down to $\omega = \frac{1}{\sqrt{2}}$. Discuss the results.

Delivery: "Campus Virtual" before April 17th.