Exercise set # 2.2

Due: Thursday 16/11/23

1. Consider the equation

$$x' = x - y - x(x^2 + y^2),$$
$$y' = x + y - y(x^2 + y^2).$$

Compute the Poincaré map of it with respect to the section $\Sigma = \{(x,y) \in R^2 | \ x > 0, y = 0\}$ in explicit form. Hint: use polar coordinates.

2. Let $f(x) = \lambda x + bx^2$ be a map from $\mathbb{R}$ to $\mathbb{R}$ with $|\lambda| \neq 0, 1$. Compute the Taylor expansion of a conjugation h between f and $Ax = \lambda x$, such that $h(0) = 0$ and $h'(0) = 1$, up to order 3. Do you think it is possible to find the Taylor expansion to all orders? If so, are the coefficients uniquely determined?

3. Consider the map
$$f(x,y) = (\lambda x, \lambda^2 y + x^2), \qquad 0 < \lambda < 1.$$
Prove that f cannot be linearized with a C^2 conjugation.