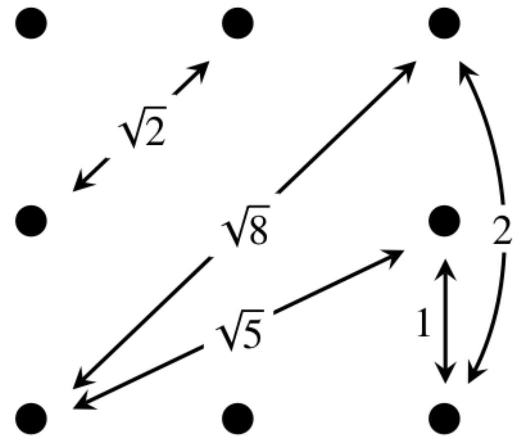
Topological Data Analysis

2022–2023

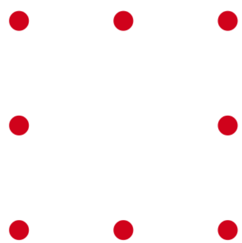
Solutions of Exercises

1 December 2022

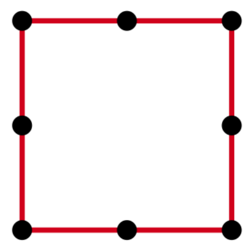
Distances between the given points are as follows:



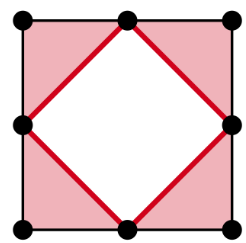
The corresponding Vietoris-Rips filtration is the following:



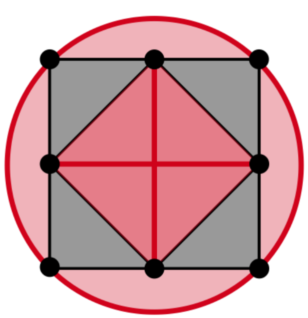
$0 \leq \varepsilon < 1$



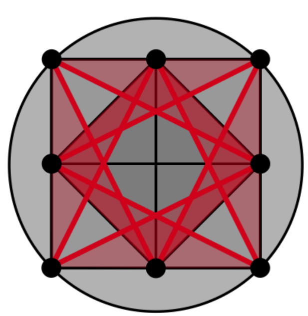
$1 \leq \varepsilon < \sqrt{2}$



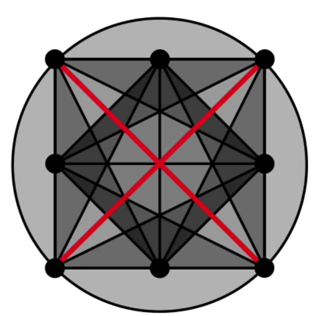
$\sqrt{2} \leq \varepsilon < 2$



$2 \leq \varepsilon < \sqrt{5}$



$\sqrt{5} \leq \varepsilon < \sqrt{8}$



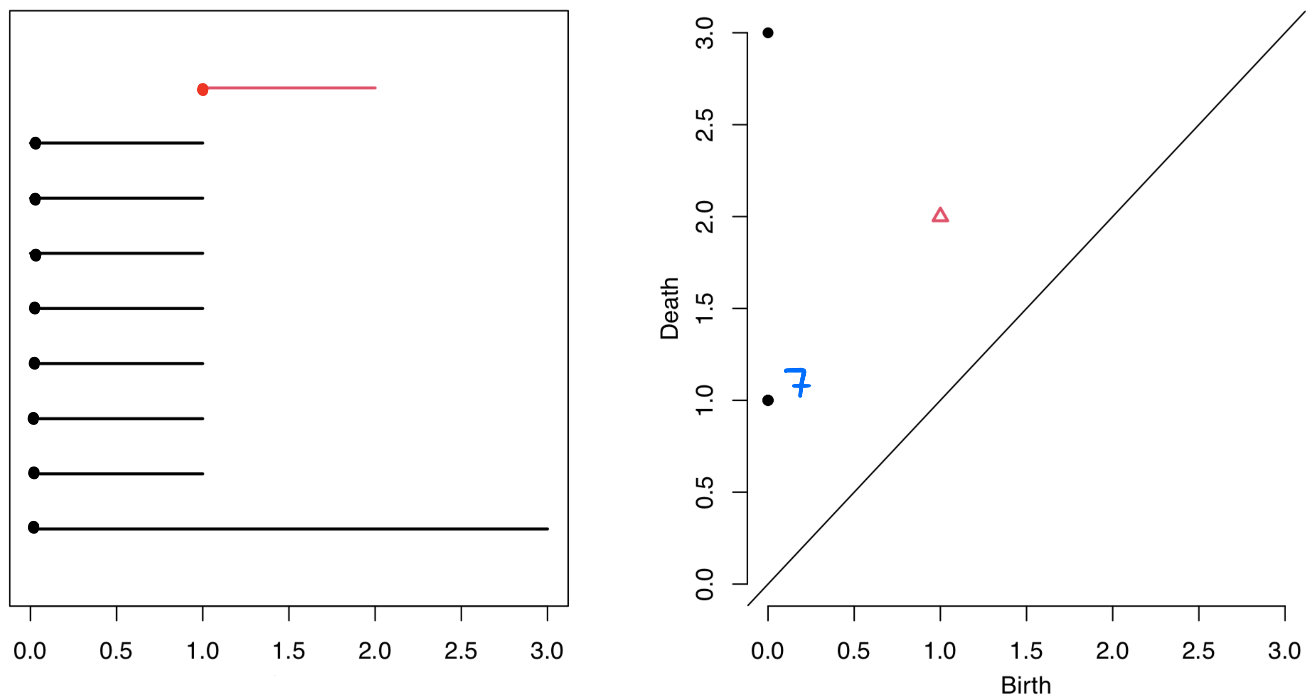
$\sqrt{8} \leq \varepsilon$

Thanks are due to Philip Pita for the pictures

Initially there are 8 zero-dimensional classes, of which only one remains after $\varepsilon = 1$.

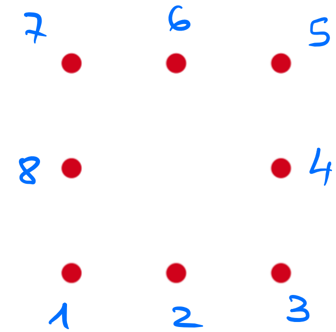
There is a 1-cycle which is born at $\varepsilon = 1$ and dies at $\varepsilon = 2$.

Software yields the following barcode and persistence diagram:



Hence we conclude that no higher dimensional homology generators occur. It is feasible to prove this fact by hand as follows.

Maximal faces:



$0 \leq \varepsilon < 1$: (1) (2) (3) (4) (5) (6) (7) (8)

$1 \leq \varepsilon < \sqrt{2}$: (12) (18) (23) (34) (45) (56) (67) (78)

$\sqrt{2} \leq \varepsilon < 2$: (128) (234) (456) (678)

$2 \leq \varepsilon < \sqrt{5}$: (128) (234) (456) (678)
(123) (178) (345) (567)
(2468)

A tetrahedron that kills the 1-cycle.
No other homology generators appear.

$\sqrt{5} \leq \varepsilon < \sqrt{8}$: (123468) (124678) (234568) (245678)

These are 5-faces that cannot enclose a 5-dimensional cavity, since the minimum number of k-faces needed to triangulate a k-sphere is that of $\partial \Delta^{k+1}$, namely $k+2$.

$\varepsilon \geq \sqrt{8}$: (12345678)