Outliers and Hallucinations: Contributions to Robust Community Detection and Language Model Alignment

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Supervisors:

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December 4, 2025

Introduction

Thesis progress

Start of the thesis

Literature review,

Identification of a research problem, first efforts.

October 2022

October 2023

Start of *période de césure*

Collaboration with Tianlin Liu, Daniele Calandriello (*ICML* DeRA paper).

Start of PE-RL project with Jessica Hoffmann, Lucas Dixon.

March 2024

End of *période de césure*

Publication of *AISTATS* SubSearch paper.

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Overview

Part I: Contributions to Robust Community Detection

- Robust Estimation for the SBM

Part II: Contributions to Language Model Alignment

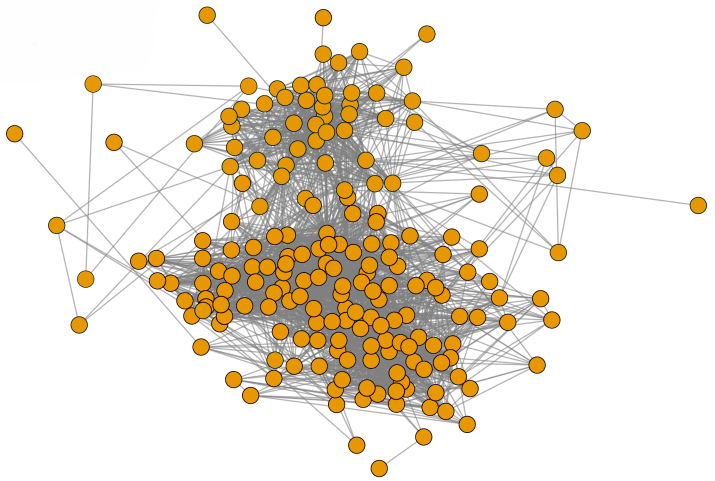
- Reducing Hallucinations with Synthetic Hallucinations
- Decoding-time Realignment of Language Models

Part I

Contributions to Robust Community Detection

Motivation

- *Community detection*: group similar nodes in a graph.
- Algorithms can be **highly sensitive** to *outlier* nodes.



Graph of Jazz collaborations in New York, Chicago, and Other Cities [Gleiser and Danon, 2003].

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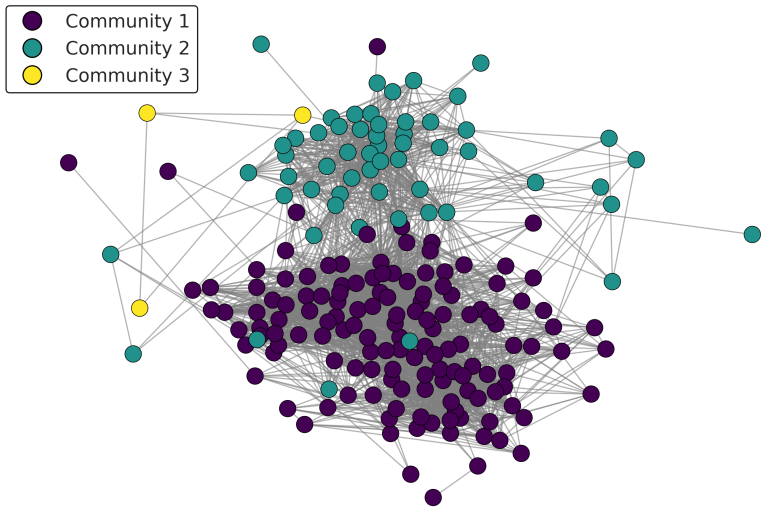
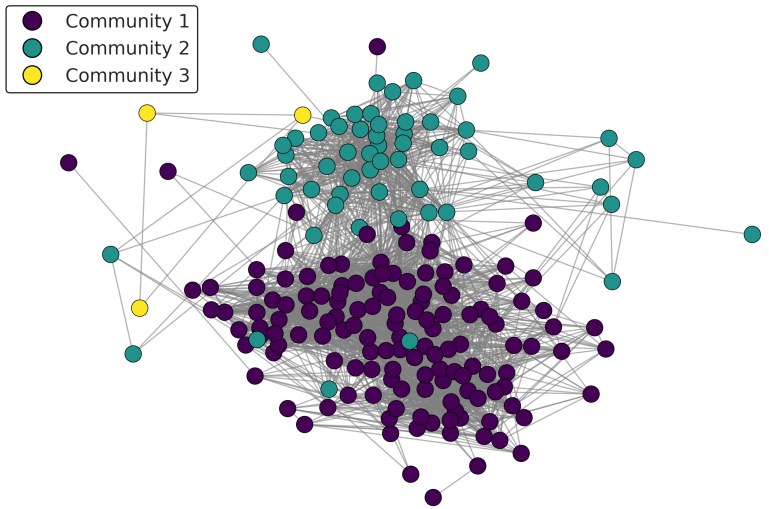


Spectral clustering applied to the graph of Jazz collaborations [Gleiser and Danon, 2003].

Motivation

- *Community detection*: group similar nodes in a graph.
- Algorithms can be **highly sensitive** to *outlier* nodes.
- **Goal**: achieve "good" results despite outliers.



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Graphs

Graph: nodes linked by edges.

$$G = (V, E)$$

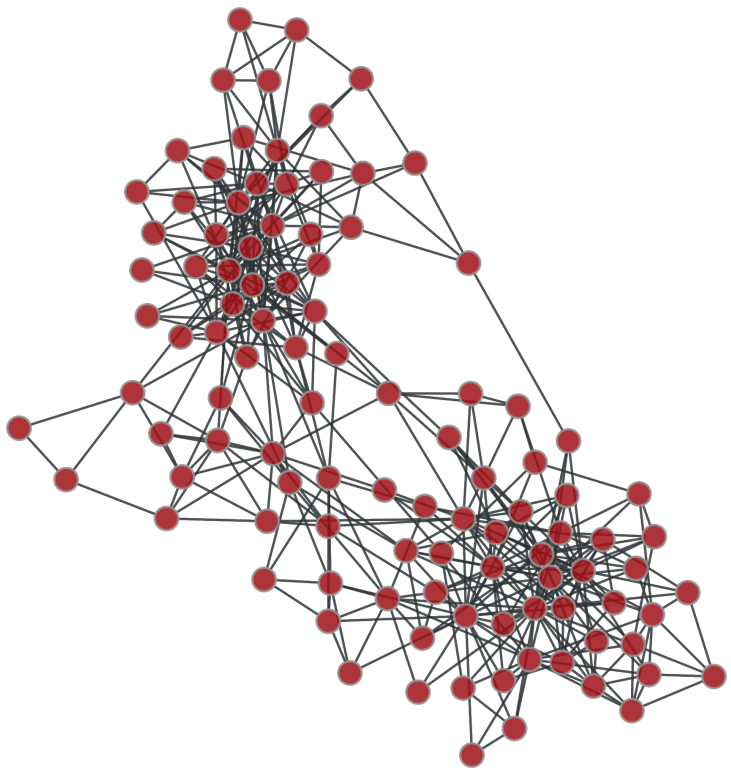
$$V = \{1, \ldots, n\}, E \subset V \times V$$

Undirected graph:

$$(i, j) \in E \Rightarrow (j, i) \in E$$

Adjacency matrix:

$$A_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{otherwise} \end{cases}$$



The Stochastic Block Model [Holland et al., 1983]

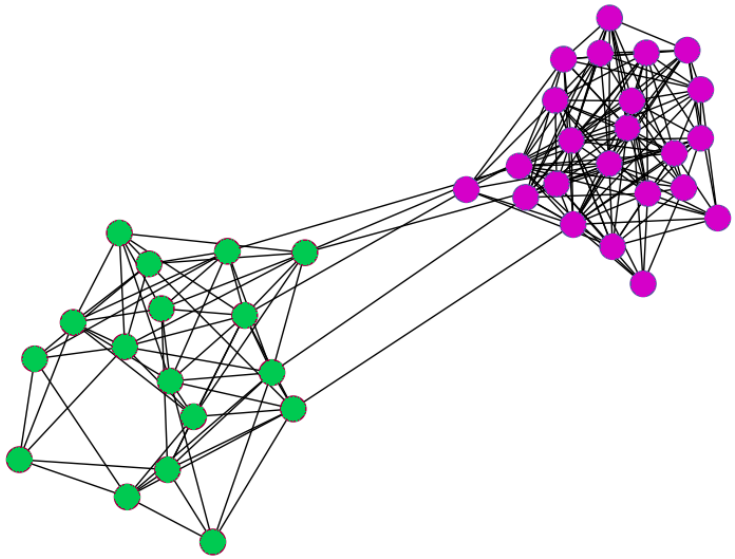
Probabilistic model

$Z_i \rightarrow$ community of node i

$K \rightarrow$ nb. of communities

$\pi_k \rightarrow$ size of community k

$\Gamma_{kl} \rightarrow$ connectivity k, l



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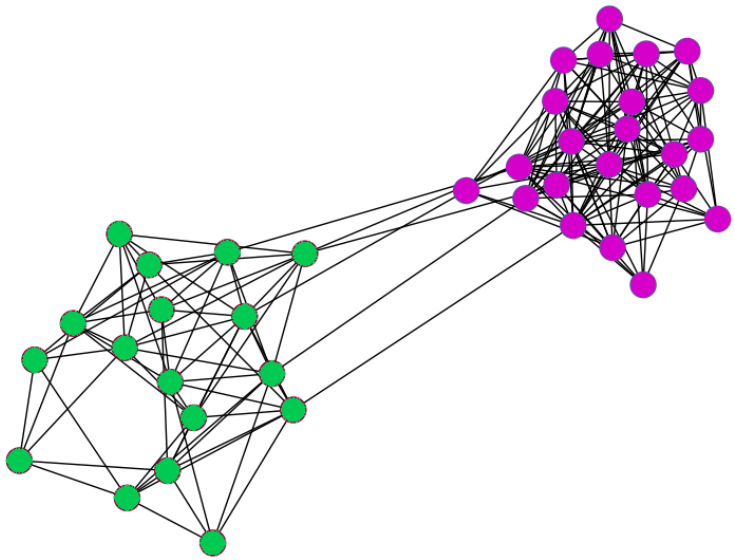
$(Z, A) \sim \mathrm{SBM}_K(\pi, \Gamma)$

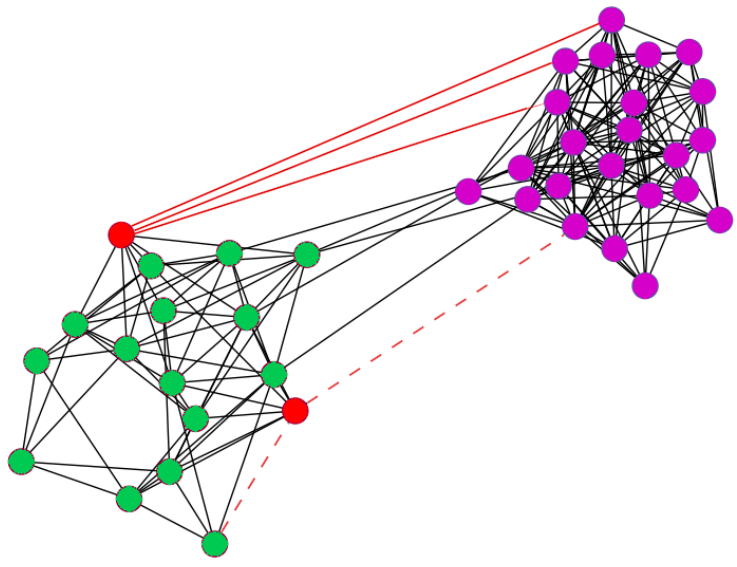
$$\begin{cases} \mathbb{P}(Z_i = k) = \pi_k \\ \mathbb{P}(A_{ij} = 1 | Z_i = k, Z_j = l) = \Gamma_{kl} \end{cases}$$



The Corrupted Stochastic Block Model

Adversary introduces outliers:

1. $(Z, A_{\text{pure}}) \sim \text{SBM}_K(\pi, \Gamma)$
2. Adversary *arbitrarily* changes edges of γn nodes
3. Corrupted A is observed



Research question

- Task: robustly estimate the connectivities Γ.
- For $K = 1$, Acharya et al. [2022].

Research question:

How to robustly estimate Γ for $K > 1$?

Results

- *Intuition:* find subgraph S excluding worst outliers.
- **First contribution**: extended bound from Acharya et al. [2022] to $K > 1$.

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Theorem. Denote $\mathcal{I}$ the set of inlier nodes, S a subgraph clustered into $S_1, \ldots, S_K$. Let $\hat{\Gamma} = \left(\sum_{i \in S_k j \in S_l} A_{ij}\right) / |S_k||S_l|$ and $\hat{Q}(S)_{ij} = \hat{\Gamma}_{S(i)S(j)}$. Then,

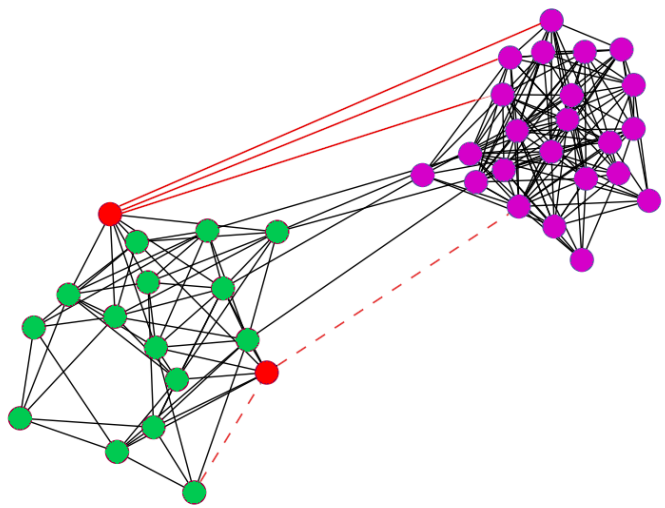
$$\|\Gamma - \hat{\Gamma}\|_1 \lesssim \frac{\|A_S - \hat{Q}(S)\|_{\text{op}}}{\min_{1 \leq k \leq K} |\Omega_k \cap S_k \cap \mathcal{I}|}$$

Results

- *Intuition:* find subgraph S excluding worst outliers.
- **First contribution**: extended bound from Acharya et al. [2022] to $K > 1$.
- **Second contribution** (SubSearch): finding S by optimizing $c(S) := \|A_S - \hat{Q}(S)\|_{\text{op}}$ via Simulated Annealing.
- **Code**: `github.com/leobianco/robust_estim_sbm`

SubSearch: Subgraph Search via Simulated Annealing

Goal: explore the space $\mathcal{S}$ of subgraphs $S \subset G$ of size $(1-\gamma)n$, to minimize $c(S) = \|A_S - \hat{Q}(S)\|_{\text{op}}$.



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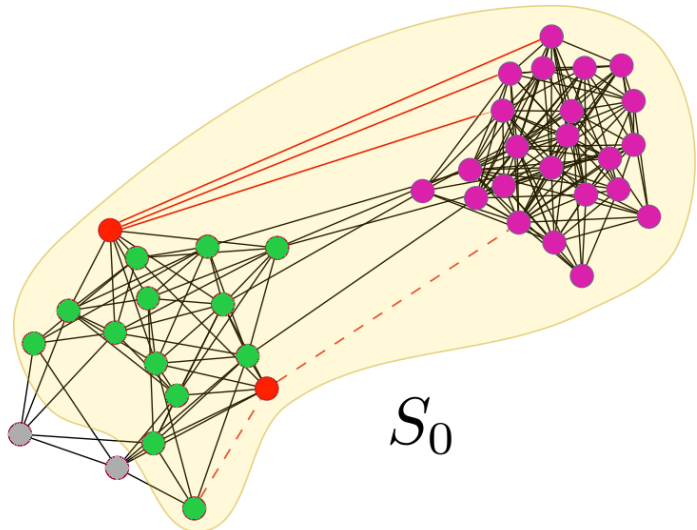

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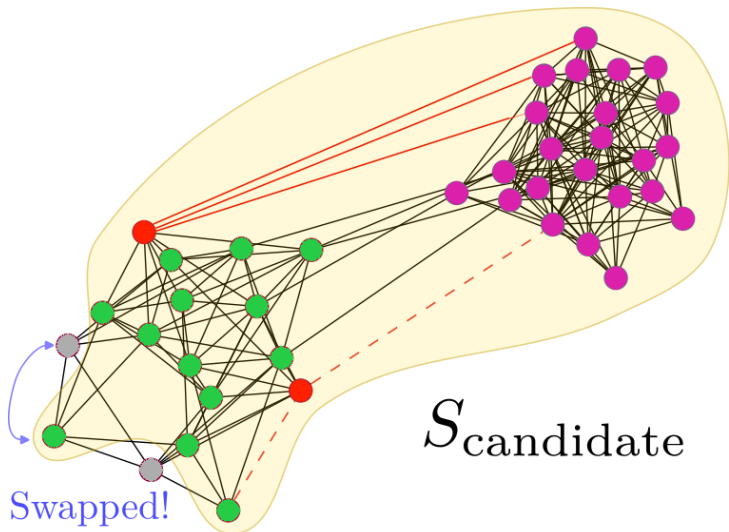

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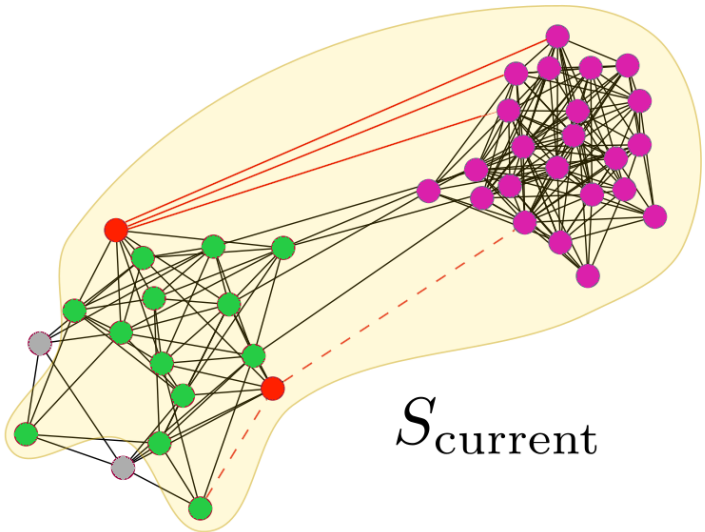

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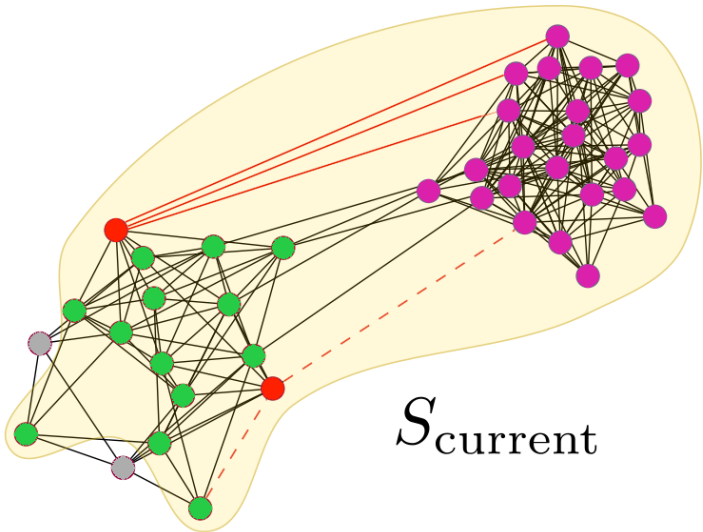
④ **Cool down** by $T_{t+1} = c\,T_t$, $c \approx 1$



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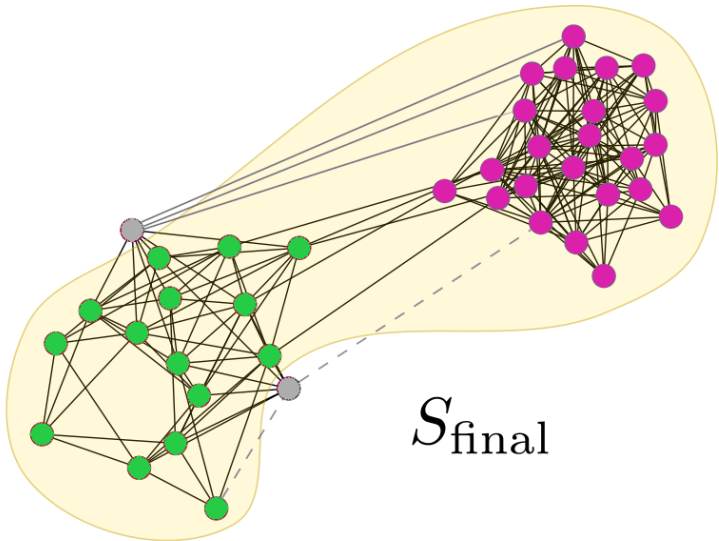

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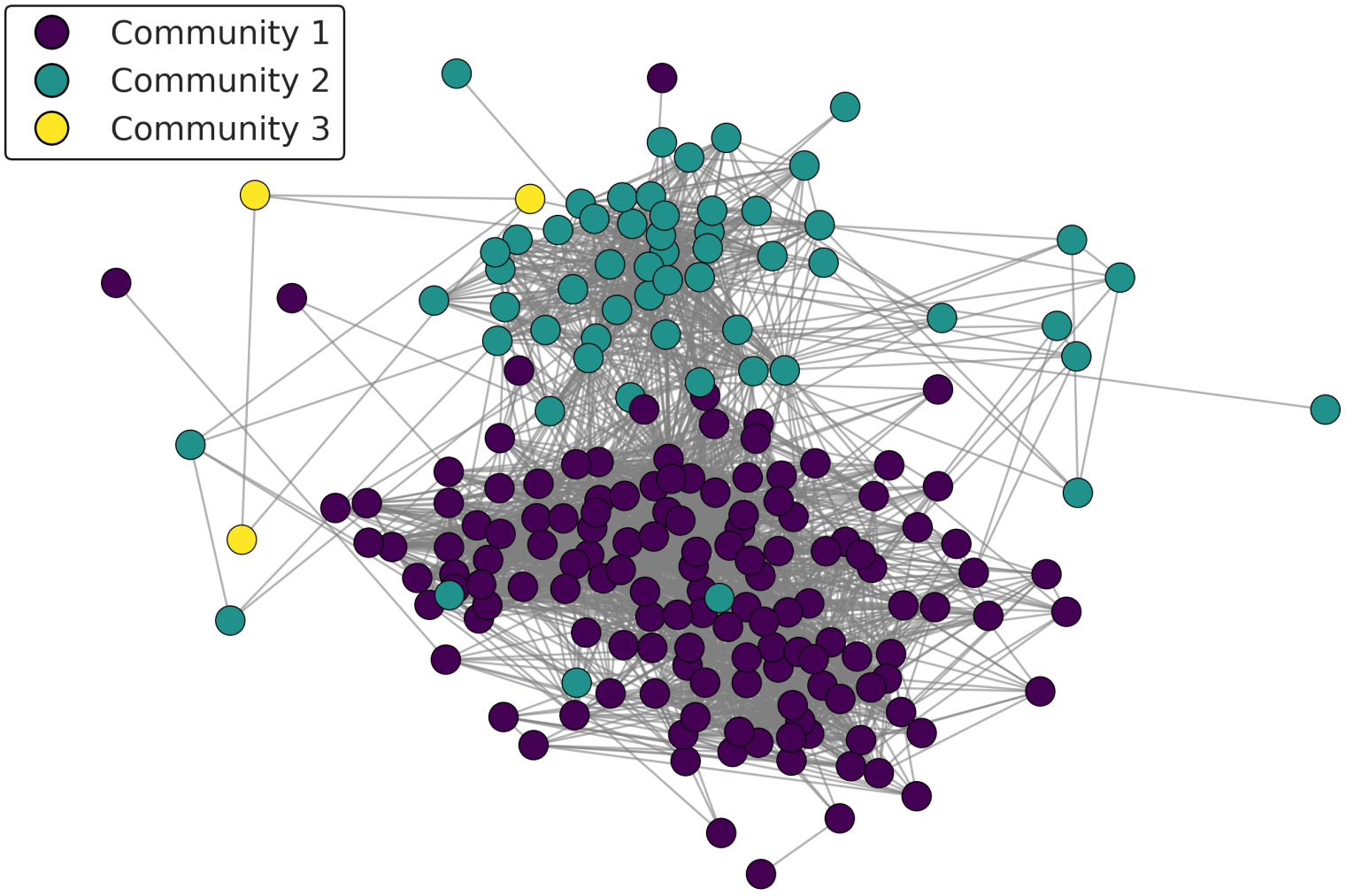

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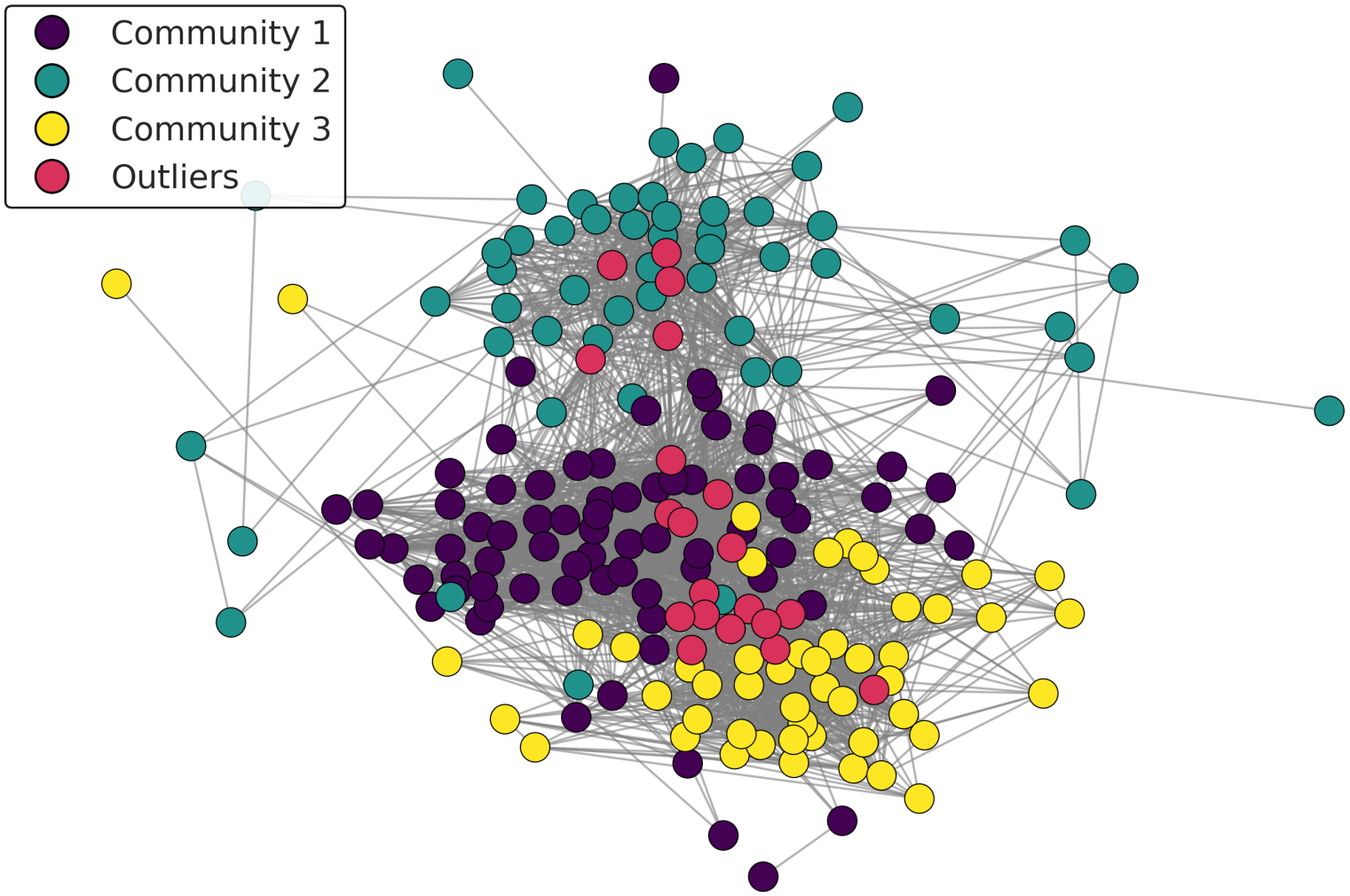
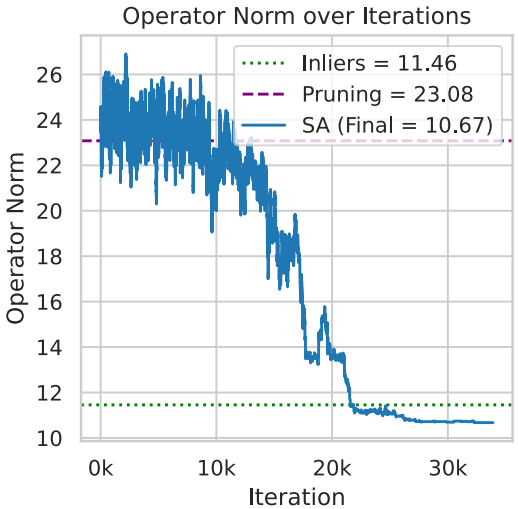
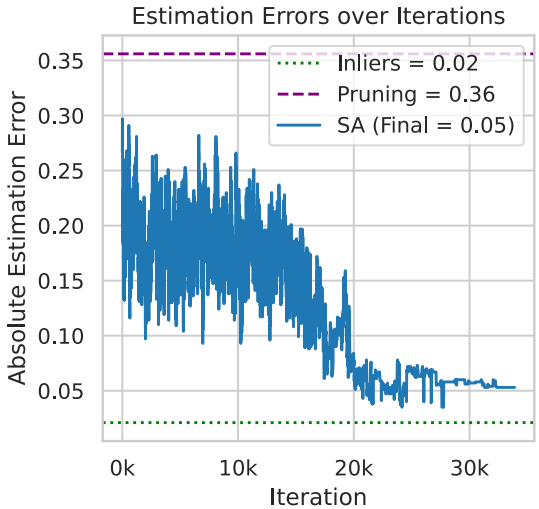
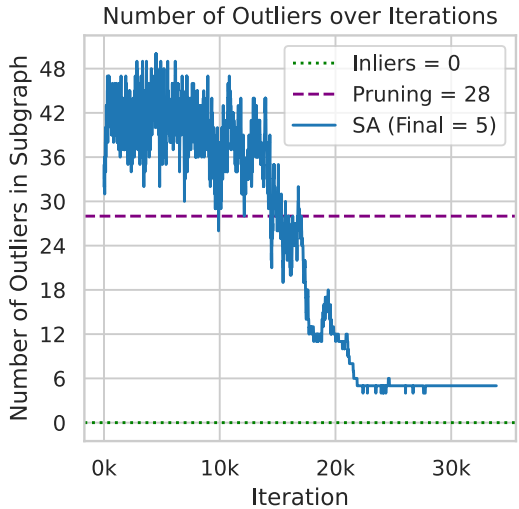


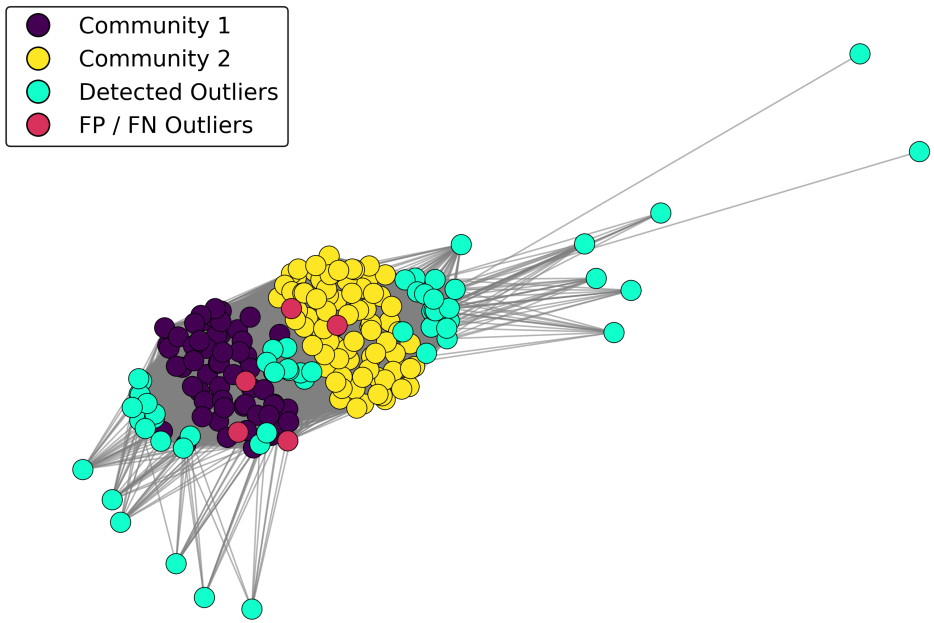
Results

Parameters: $n = 200$, $K = 2$, $\gamma = 0.3$.









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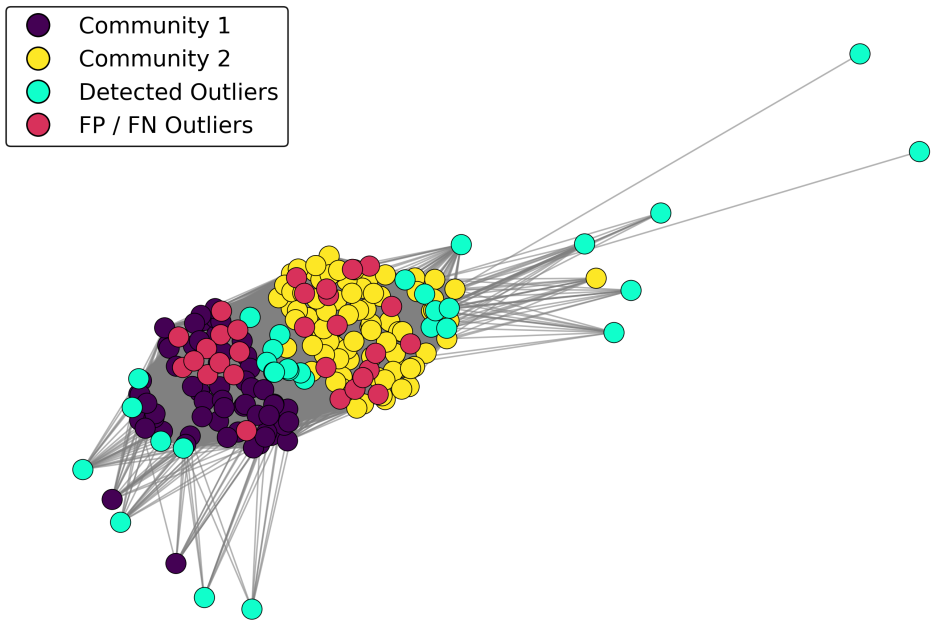

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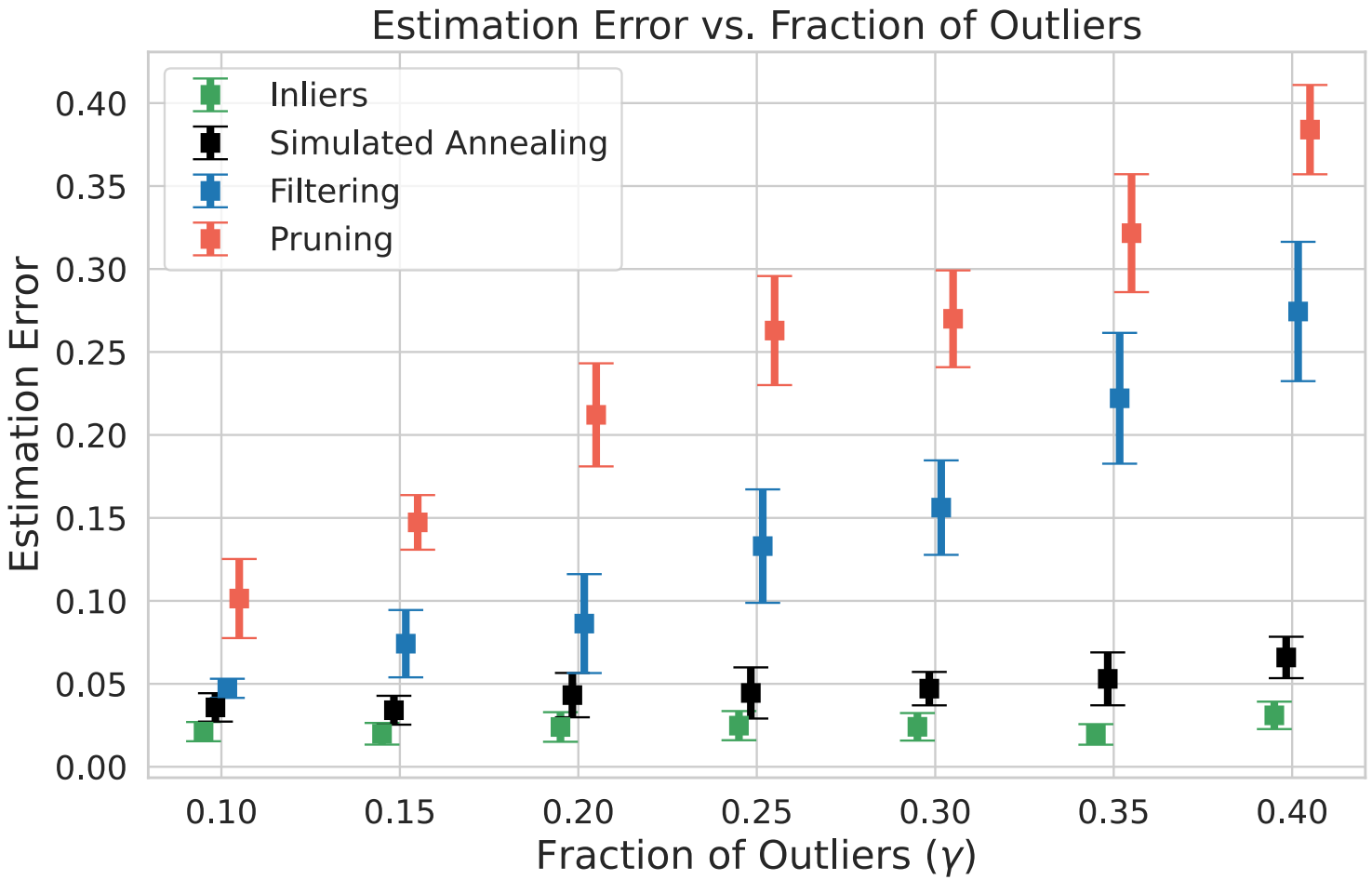
Estimation Error vs. Fraction of Outliers
Inliers
Simulated Annealing
Filtering
Pruning
Estimation Error
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.10
0.15
0.20
0.25
0.30
0.35
0.40
Fraction of Outliers (γ)

Discussion

- **Main take away**: "exploring" the space of subgraphs $\Rightarrow$ find subgraphs avoiding outliers.
- Limitation # 1: can we rigorously prove robustness?
- Limitation # 2: can we provide faster rates?

Part II

Contributions to Language Model Alignment

Motivation

- Chatbots based on Transformers [Vaswani et al., 2017].
- Hallucinations ≈ **false information**, **out of topic**, **rambling**, **toxic**...
- **Goal**: less hallucinations.

Background on Language Models

- *Vocabulary* $\mathcal{V}$ = set of *tokens* ("pieces of words").
- Language model

$$\pi_\theta : x = (\text{token}_1, \ldots, \text{token}_L) \mapsto \pi_\theta(\cdot|x) = \text{proba. over } \mathcal{V}.$$

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- Autoregressive generation: *prompt* $x \longrightarrow$ *response* y

$$\begin{aligned} y_1 &\sim \pi_\theta(\cdot|x) \\ y_2 &\sim \pi_\theta(\cdot|x, y_1) \\ &\vdots \\ y_t &\sim \pi_\theta(\cdot|x, y_{<t}) \end{aligned}$$

Background on Language Models

- Pre-training: given a dataset $\mathcal{D}_{\text{pre}}$, find θ minimizing

$$\ell(\theta; \mathcal{D}_{\text{pre}}) := -\sum_{x \in \mathcal{D}_{\text{pre}}} \sum_{i=1}^{|x|} \log \pi_\theta \left(x_{i+1} | x_{\leq i}\right).$$

Background on Language Models

- SFT: given a task-specific dataset $\mathcal{D}_{\text{SFT}}$, find θ minimizing

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- Alignment: generate text with human preferences.
- Reinforcement learning approach:
  1. Train a *reward* model R on human preference data $\mathcal{D}_{\text{RM}}$
  2. Update the *writer* model π_{SFT}

$$\pi_\beta \in \arg\max \mathbb{E}_{g \sim \pi} \left[R(g)\right] - \beta \, \text{KL}\left(\pi \| \pi_{\text{SFT}}\right)$$

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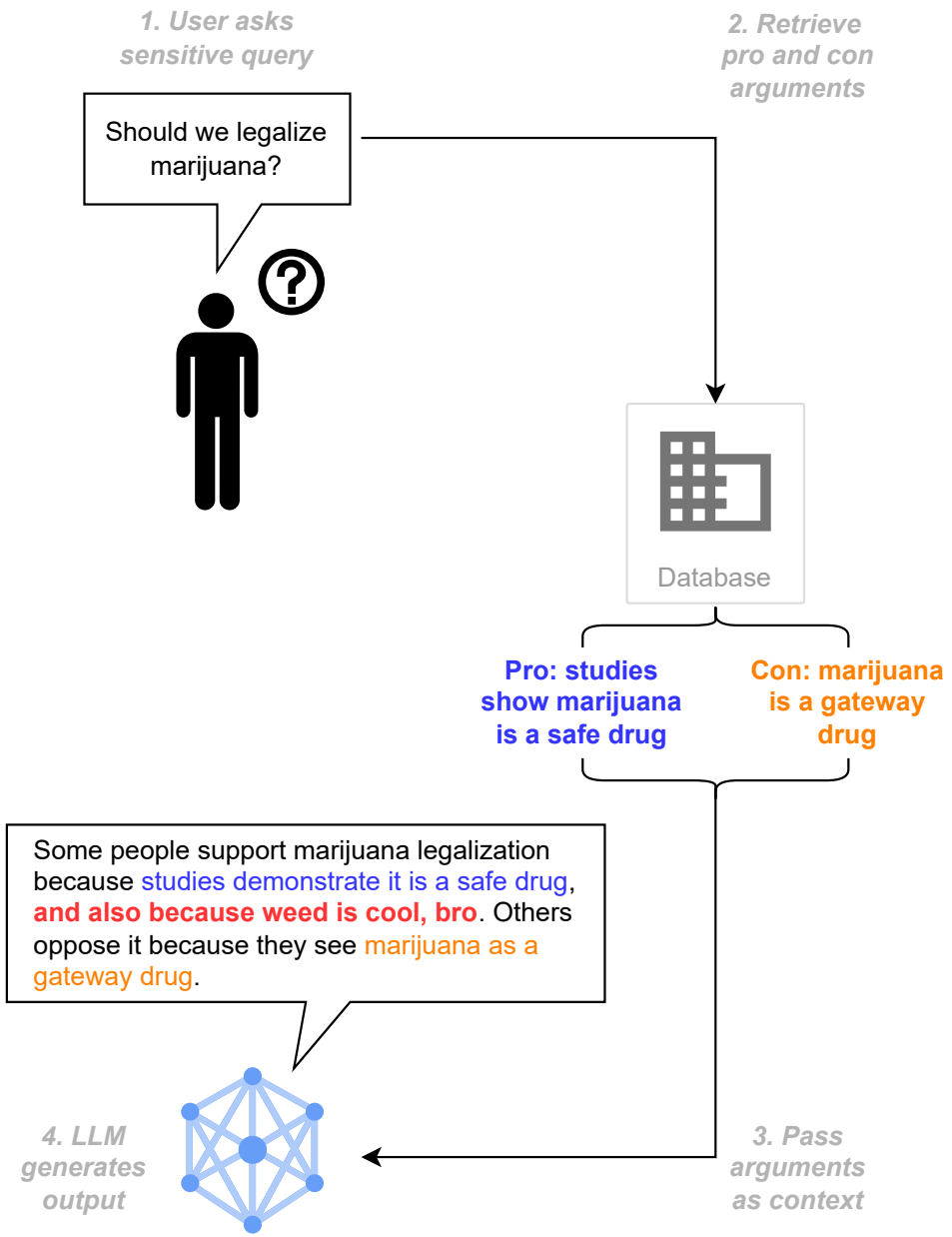
Retrieval Augmented Generation: NPOV Task



Research question #1

- Reinforcement learning approach:
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Getting $\mathcal{D}_{\text{RM}}$ is **costly**, **time-consuming**, and **error-prone**.

Research question:

Can synthetic hallucinations be used instead?

- Synthetic hallucinations are *fast* and *cheap* to implement, with *automatic* and *error-free* annotations.

Creating Synthetic Hallucinations

Pros:

1. Studies show marijuana is a safe drug
2. Legalization boosts the economy

Cons:

1. Marijuana is a gateway drug
2. Legalization brings costs

Neutral answer:

"Some people support marijuana legalization because it would boost the economy and most studies demonstrate it is a safe drug. Others oppose it because they see marijuana as a gateway drug, and its legalization would bring many costs."

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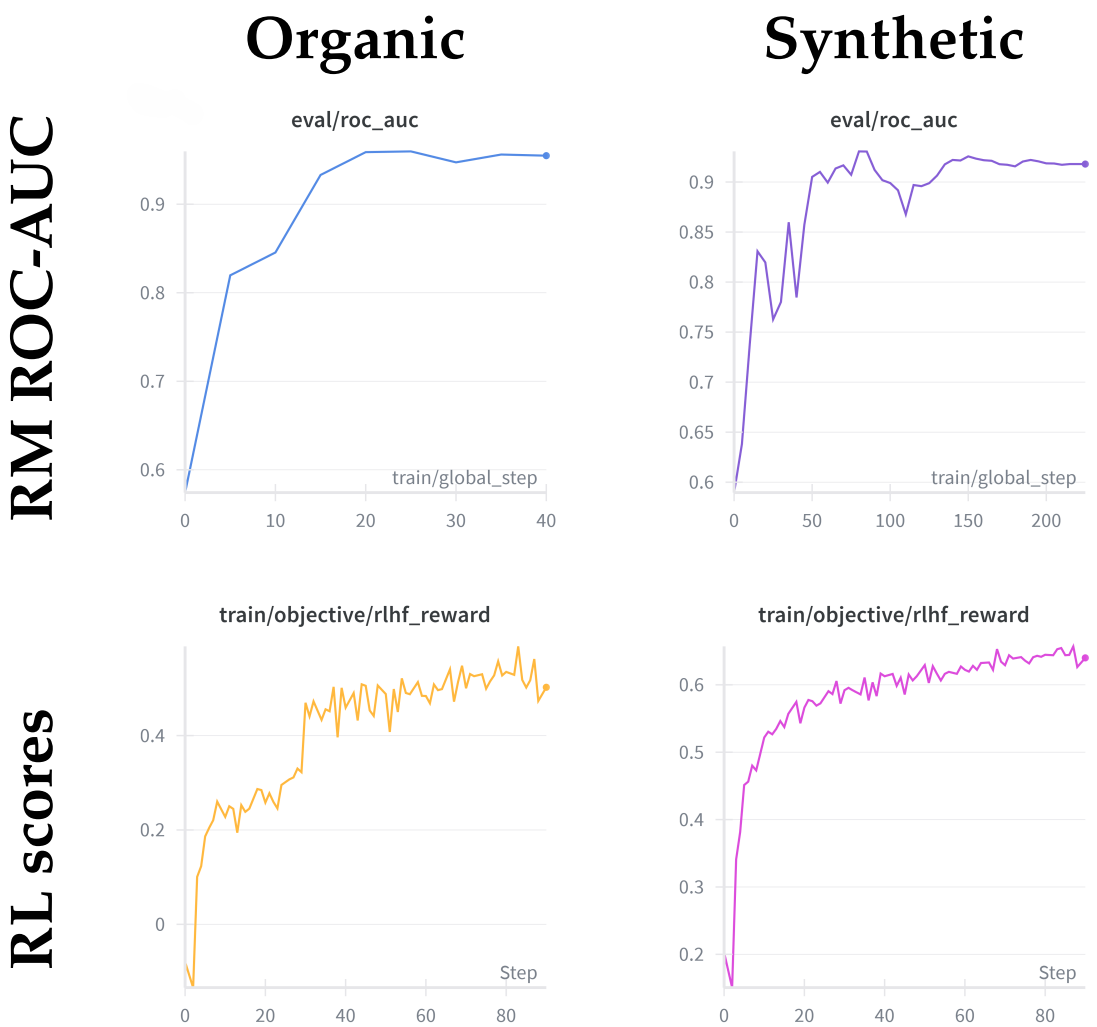
Results

SFT baseline (%)	*Organic hallucinations* (%)	*Synthetic hallucinations* (%)
10.2	3.0	0.74



Discussion

- **Code**: github.com/leobianco/perl_hallucination

Future work:

- Other task (summarization).
- Other models (Mistral, Qwen).
- Other synthetic hallucinations schemes (LLM).

Research question #2

- Reinforcement learning approach:
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The hyperparameter β is expensive to tune via grid-search.

Research question:

Can we adjust regularization strength without retraining?

Results

- **Contribution**: approximate realigned model at β/λ

$$\hat{\pi}_{\beta/\lambda}(\cdot|x, y_{<t}) \coloneqq \text{softmax}\left[\lambda h_{\beta}^{(t)} + (1-\lambda) h_{\text{SFT}}^{(t)}\right]$$

where $h_{\text{SFT}}^{(t)}$ and $h_{\beta}^{(t)}$ are the logits

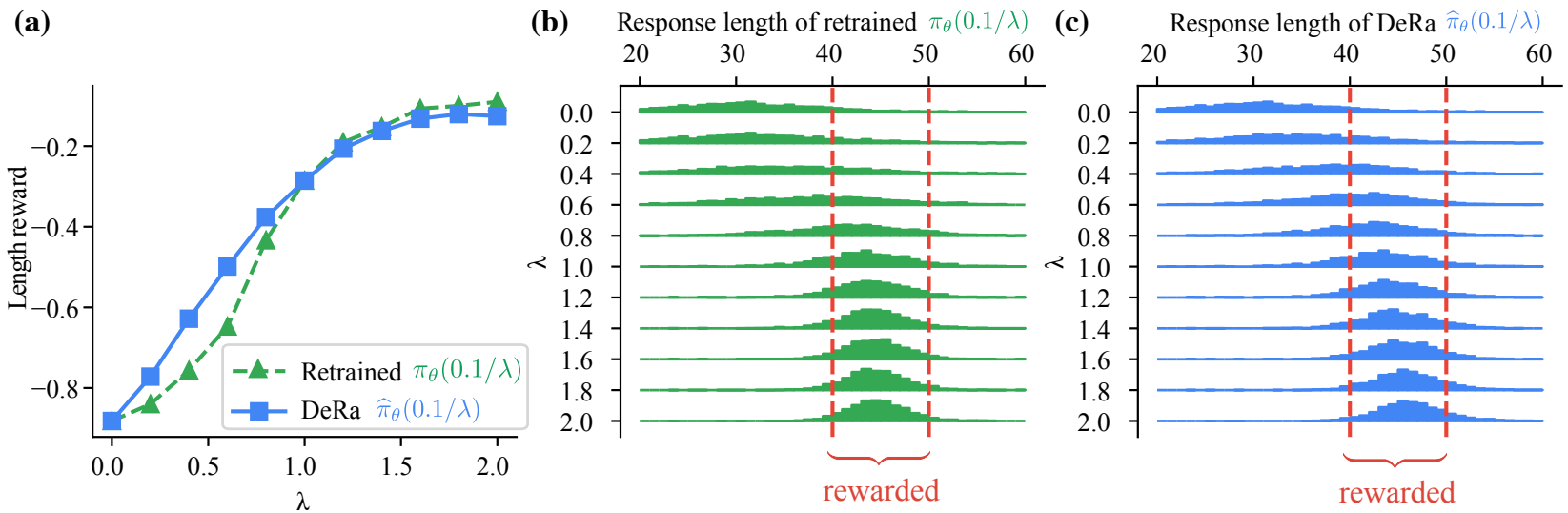
$$\begin{cases} \pi_{\text{SFT}}(\cdot|x, y_{<t}) & = \text{softmax}(h_{\text{SFT}}^{(t)}) \\ \pi_{\beta}(\cdot|x, y_{<t}) & = \text{softmax}(h_{\beta}^{(t)}) \end{cases}$$

- **Code**: https://github.com/liutianlin0121/decoding-time-realignment

Results

Arguments	pro: Denying student loan debtors the benefits of bankruptcy--benefits that all other debtors have access to--is unfair. con: Discharging student loan debt would only be a temporary bandage for the much larger problem of inflated college costs.
Response λ=0.011	[...] it is unfair to deny student loan debtors the benefits of bankruptcy--benefits that all other debtors have access to. They also argue that student loan debt has a disproportionately negative impact on low-income borrowers [...]. However, [...] it would be a costly and ineffective solution to the problem of student debt. They also argue that it would only be a temporary bandage for the much larger problem of inflated college costs.
Response λ=2	[...] One argument in favor of forgiveness is that it is unfair to deny student loan debtors the benefits of bankruptcy--benefits that all other debtors have access to. However, one argument against forgiveness is that it would only be a temporary bandage for the much larger problem of inflated college costs.
Response: λ=5	Denying student loan debtors the benefits of bankruptcy--benefits that all other debtors have access to--is unfair 30].\\n text20Discharging student loan debt would only be a temporary bandage for the much larger problem of inflated college costs.

Results


(a)
Length reward
−0.2
−0.4
−0.6
−0.8
0.0
0.5
1.0
1.5
2.0
λ
Retrained $\pi_\theta(0.1/\lambda)$
DeRa $\widehat{\pi}_\theta(0.1/\lambda)$
(b)
Response length of retrained $\pi_\theta(0.1/\lambda)$
20
30
40
50
60
λ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
rewarded
(c)
Response length of DeRa $\widehat{\pi}_\theta(0.1/\lambda)$
20
30
40
50
60
λ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
rewarded

Discussion

- ?

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Example of Hallucination Removed #1

User query: Should Parents or Other Adults Be Able to Ban Books from Schools and Libraries?

Pro arguments:

- Keeping books with inappropriate content out of libraries protects kids but doesn't stop people from reading those books or prevent authors from writing them.
- Parents have the right to decide what material their children are exposed to and when.

Con arguments:

- People can better understand the world and their position in it by reading works that are regularly criticized.
- Books are a portal to different life experiences and reading encourages empathy and social-emotional development.

Neutral point-of-view answer:

SFT version: This is a very controversial topic. Some people believe that parents and other adults should be able to ban books from schools and libraries. They say that this would protect children from inappropriate content. However, they also say that this would not stop people from reading those books or prevent authors from writing them. On the other hand, **others say that banning books is a way to protect children from the world** and that it is a way to encourage empathy and social-emotional development. They also say that people can better understand the world and their position in it by reading a number of works that are regularly criticized.

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Neutral point-of-view answer:

PE-RL version: I don't know what to make of this. Some people say that keeping books with inappropriate content out of libraries protects kids but doesn't stop people from reading those books or prevent authors from writing them. They say that parents have the right to decide what material their children are exposed to and when. Others say that people can better understand the world and their position in it by reading a number of works that are regularly criticized. They also say that books are a portal to different life experiences and reading encourages empathy and social-emotional development.