In [22]:

```
using DataFrames, CSV
using JuMP, Gurobi
using LinearAlgebra, Random, Printf
using Plots

const GRB_ENV = Gurobi.Env()
```

```
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```

Out[22]:

```
Gurobi.Env(Ptr{Nothing} @0x00007f867c9e1400, false, 0)
```

In [102…

```
arcs = CSV.read("Pb1_arcs.csv", DataFrame)
transport = CSV.read("Pb1_transport.csv", DataFrame)
demand = CSV.read("Pb1_demand.csv", DataFrame)
customers = CSV.read("Pb1_customer_OD.csv", DataFrame)
arcs_ = convert.(Int64,arcs[!,2:3])
```

Out[102…

90 rows × 2 columns

	x2	x3
	Int64	Int64
1	2	1
2	3	1
3	4	1
4	5	1
5	6	1
6	7	1
7	8	1
8	9	1
9	10	1
10	1	2
11	3	2
12	4	2
13	5	2
14	6	2
15	7	2
16	8	2
17	9	2
18	10	2
19	1	3
20	2	3
21	4	3
22	5	3
23	6	3
24	7	3
25	8	3
26	9	3
27	10	3
28	1	4

29	2	4
30	3	4
⋮	⋮	⋮

In [17]:

```
customers
```

Out[17]: 3 rows × 2 columns

	x1	x2
	Int64	Int64
1	1	9
2	4	2
3	8	2

1. Problem setup

In [107…

```
TIME_LIMIT = 90;
OPTIMALITY_GAP = 0.001;

"Solve problem using multi-cut Benders decomposition"
function solve_benders_multi(verbose::Bool=true)
    # define main problem
    MP = Model(() -> Gurobi.Optimizer(GRB_ENV));
    set_optimizer_attributes(MP, "TimeLimit" => 60, "MIPGap" => 1e-4, "Outp
    A = size(arcs, 1); K = size(customers, 1); S = size(demand, 2)
    @variable(MP, x[1:A], Bin)
    @variable(MP, θ[1:S] >= 0)
    @objective(MP, Min, sum(arcs[i,5] * x[i] for i=1:A) + sum(1/S * θ[s] fo
    lower_bound_all = []; upper_bound_all = []
    MP_time = []; SP_max_time = []; SP_time = []
    num_opt = 0; num_feas = 0
    while true
        # solve master problem
        push!(MP_time, @elapsed optimize!(MP))
        lower_bound_new = objective_value(MP)
        push!(lower_bound_all, lower_bound_new)
        x_MP = value.(MP[:x])
        # solve S subproblems
        obj_SP = zeros(S)
        SP_time_all = zeros(S)
        for s = 1:S

            SP_dual = Model(JuMP.optimizer_with_attributes(() -> Gurobi.Opt
                "OutputFlag" => 0, "DualReductions" => 0, "TimeLimit" => 6

            @variable(SP_dual, λ[1:10,1:K]);
            @variable(SP_dual, μ[1:A] <= 0);

            @objective(SP_dual, Max,
                       sum((λ[customers[k,1],k]-λ[customers[k,2],k]) * dema
                       sum(μ[i] * arcs[i,4] * x_MP[i] for i in 1:A))

            @constraint(SP_dual, [i in 1:A, k in 1:K],
                λ[arcs_[i,1],k] - λ[arcs_[i,2],k] + μ[i] <= transport[i,k]

            SP_time_all[s] = @elapsed optimize!(SP_dual)
            #obj_SP_dual = objective_value(SP_dual)
            λ_val = value.(SP_dual[:λ])
            μ_val = value.(SP_dual[:μ])
            if termination_status(SP_dual) == MOI.DUAL_INFEASIBLE # feasib
                @constraint(MP, sum((λ_val[customers[k,1],k]-λ_val[custome
                       sum(μ_val[i] * arcs[i,4] * x[i] for i in 1:A) <= 0)
                obj_SP[s] = 3200
                num_feas += 1
            elseif termination_status(SP_dual) == MOI.OPTIMAL
                @constraint(MP, θ[s] >= sum((λ_val[customers[k,1],k]-λ_val
                       sum(μ_val[i] * arcs[i,4] * x[i] for i in 1:A))
                obj_SP[s] = objective_value(SP_dual)
                num_opt += 1
            end
        end
        push!(SP_max_time, maximum(SP_time_all))
        push!(SP_time, sum(SP_time_all))
        upper_bound_new = sum(arcs[i,5] * x_MP[i] for i=1:A) + sum(1/S * o
        push!(upper_bound_all, upper_bound_new)
        verbose && @printf("UB: %.2f - LB: %.2f\n", upper_bound_all[end], 
        if sum(MP_time) + sum(SP_time) >= TIME_LIMIT ||
            (upper_bound_new-lower_bound_new)/lower_bound_new < OPTIMALITY_
            break
        end
    end
    return upper_bound_all, lower_bound_all, MP_time, SP_time, SP_max_time
end
```

Out[107… solve_benders_multi

In [108…

```
upper_bound_all, lower_bound_all, MP_time, SP_time, SP_max_time, num_feas,
```

```
UB: 3200.00 - LB: 0.00
UB: 3210.05 - LB: 10.05
UB: 3210.55 - LB: 10.55
UB: 3210.56 - LB: 10.56
UB: 3212.90 - LB: 12.90
UB: 3220.11 - LB: 20.11
UB: 3220.74 - LB: 20.74
UB: 3220.81 - LB: 20.81
UB: 3221.25 - LB: 21.25
UB: 3224.19 - LB: 24.19
UB: 3230.80 - LB: 30.80
UB: 3231.69 - LB: 31.69
UB: 3232.80 - LB: 32.80
UB: 3233.77 - LB: 33.77
UB: 3352.03 - LB: 34.25
UB: 3234.29 - LB: 34.29
UB: 3234.74 - LB: 34.74
UB: 2743.14 - LB: 34.75
UB: 3365.28 - LB: 2152.35
UB: 2756.38 - LB: 2152.85
UB: 2743.14 - LB: 2743.14
```

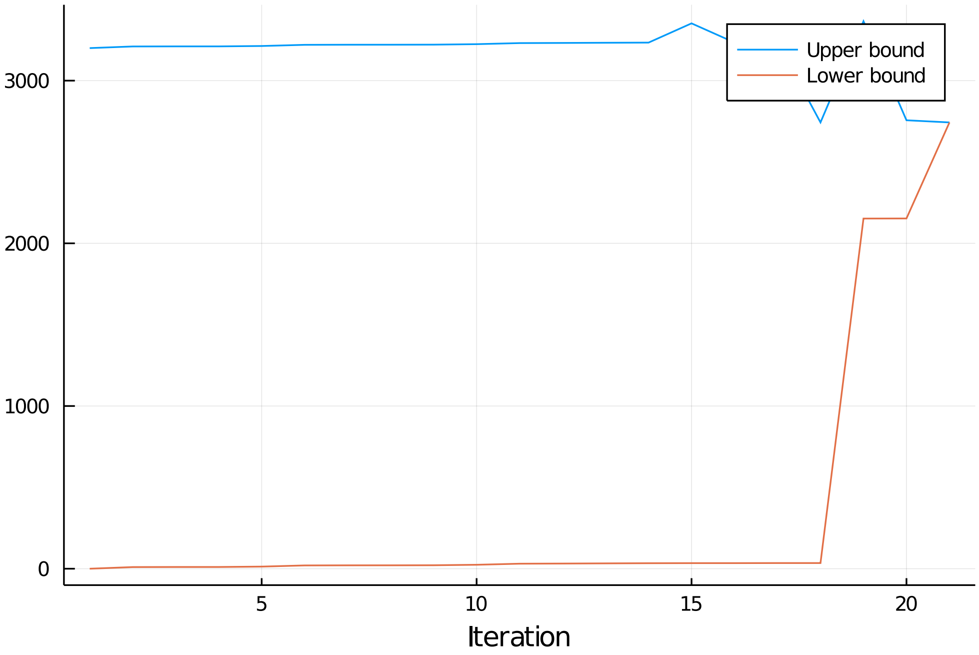
Out[108…

```
(Any[3200.0, 3210.054529448178, 3210.5504810699804, 3210.562429355936, 3212
.898361166845, 3220.1136273655734, 3220.742821014491, 3220.807068585424, 32
21.2507209222485, 3224.190843426695  …  3231.6882584851733, 3232.8006372684
135, 3233.773629273072, 3352.0328794045035, 3234.286055186306, 3234.7413244
966756, 2743.136793777782, 3365.27728915555, 2756.381203528829, 2743.136793
777782], Any[0.0, 10.054529448177977, 10.550481069980382, 10.5624293559357,
12.8983611668452l, 20.113627365573514, 20.742821014490755, 20.807068585424
7, 21.250720922248476, 24.19084342669534  …  31.688258485173435, 32.8006372
6841344, 33.77362927307213, 34.24994134409088, 34.28605518630586, 34.741324
49667572, 34.75327278263104, 2152.350838312871, 2152.854169751412, 2743.136
7937777823], Any[0.001021085, 0.002473057, 0.000699174, 0.001314428, 0.0014
64564, 0.005083717, 0.00157934, 0.002331034, 0.001769716, 0.002026514  …  0
.002012163, 0.002877235, 0.006162246, 0.003313874, 0.002469903, 0.002317671
, 0.002410215, 0.002512856, 0.002510676, 0.003399851], Any[0.005480275, 0.0
04052873, 0.004486044999999999, 0.004129187, 0.006008623, 0.004787151, 0.00
5361995, 0.0046422600000000005, 0.004833864, 0.005578787  …  0.003817553, 0
.005045758, 0.0038845250000000002, 0.0035313410000000003, 0.004731394, 0.00
4545399, 0.0036984210000000003, 0.003261647, 0.003215611, 0.004845708], Any
[0.002845987, 0.002088845, 0.002348809, 0.002199612, 0.003261186, 0.0029602
61, 0.002686592, 0.002463351, 0.002689996, 0.003098361  …  0.001990776, 0.0
02670213, 0.00246172, 0.001961768, 0.002371545, 0.002298368, 0.002084203, 0
.00166272, 0.001688792, 0.003139663], 32, 10)
```

In [109…

```
plot([upper_bound_all lower_bound_all], label=["Upper bound" "Lower bound"
```

Out[109…



In [106…

```
@show length(upper_bound_all)
@show num_opt
@show num_feas
```

```
length(upper_bound_all) = 21
num_opt = 10
num_feas = 32
```

Out[106…

```
32
```