Quantifying Rearrangements in Pangenomes

Leonard Bohnenkämper

Bielefeld University

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Rearrangements

INVERSIONS IN THE CHROMOSOMES OF DROSOPHILA PSEUDOOBSCURA*

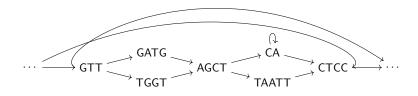
TH. DOBZHANSKY AND A. H. STURTEVANT California Institute of Technology, Pasadena, California

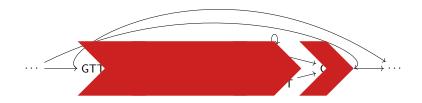


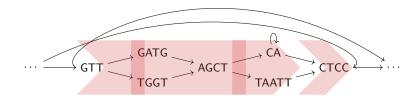
INDEPENDENT FUNCTIONS OF VIRAL PROTEIN AND NUCLEIC ACID IN GROWTH OF BACTERIOPHAGE*

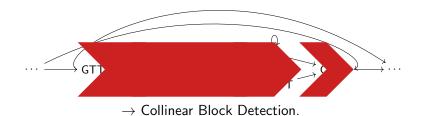
By A. D. HERSHEY AND MARTHA CHASE
(From the Department of Genetics, Carnegie Institution of Washington, Cold Spring
Harbor, Long Island)
(Received for publication, April 9, 1952)

- \sim 30,000 inversions affect more bases of the human genome than \sim 5,000,000 SNVs
- Pivotal role in:
 - ► Evolution
 - Disease
- ightharpoonup Rearrangement Quantification for Structural Variants pprox Alignment for SNVs



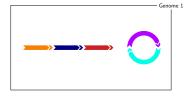


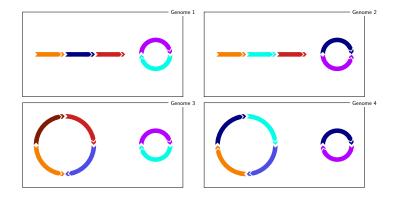


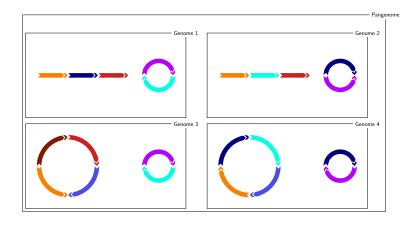










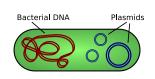


SPP - Ancestral Reconstruction with a Given Tree

Joint work with Daria Frolova (, Jens Stoye, Daniel Doerr)

Background: Plasmids

- Extrachromosomal DNA
- Common in bacteria
- Mostly circular
- Some plasmids can be transferred between hosts (even cross-species)
- Often carry resistance or virulence genes
- Frequent structural changes



Plasmid image by en: User:Spaully on English wikipedia, CC BY-SA 2.5, via Wikimedia Commons

Application: Plasmids

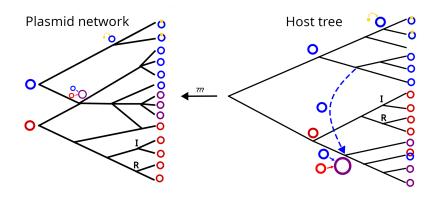


Image by courtesy of Daria Frolova

Application: Plasmids

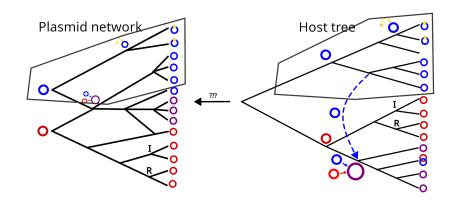
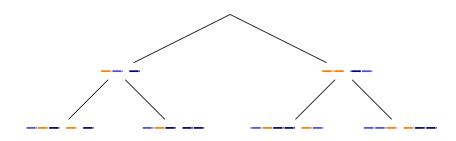
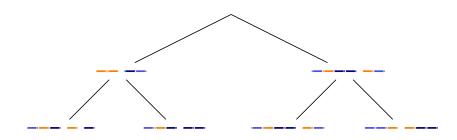
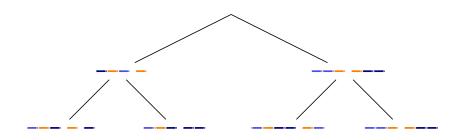
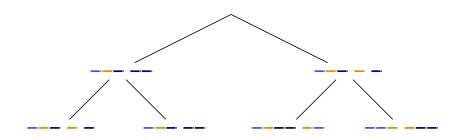


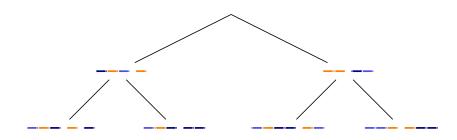
Image by courtesy of Daria Frolova

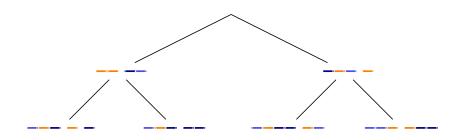


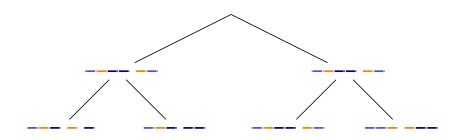


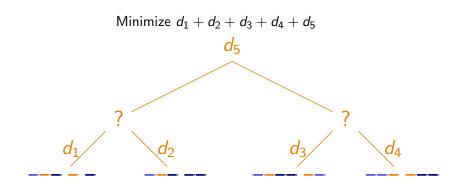






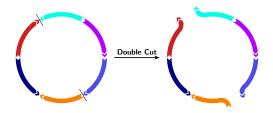


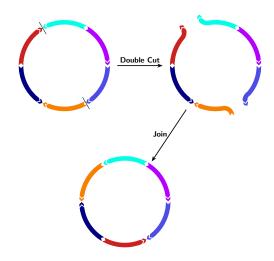


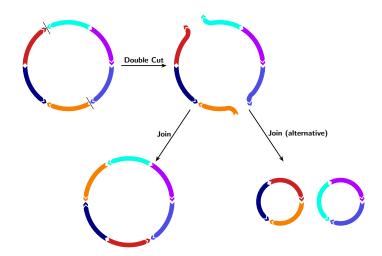












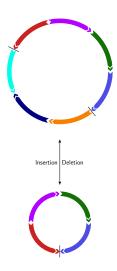
Insertions and Deletions

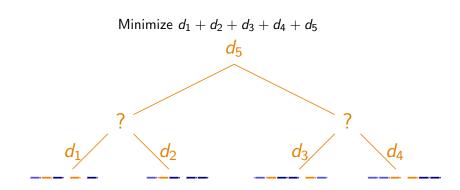


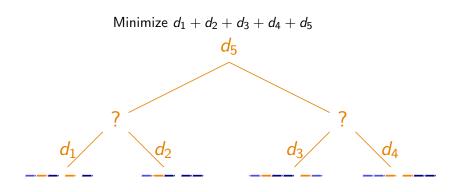
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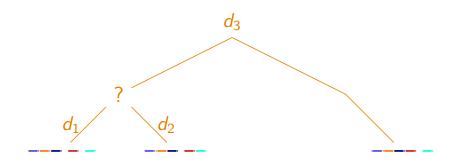
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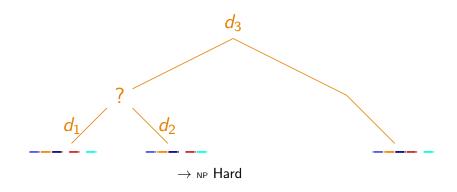


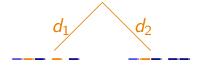


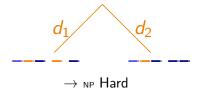


 d_1, d_2, d_3, d_4, d_5 : (Minimum number of) DCJ and indel operations between the genomes.











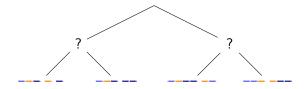


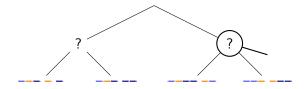


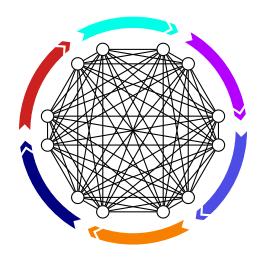


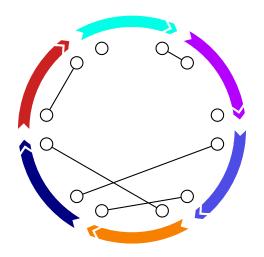
Integer Linear Programming (ILP)

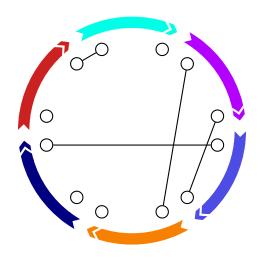
```
Algorithm 1 Capping-free Small Parsimony
Objective
      Minimize
                                                                  \sum (\alpha f_E + (\alpha - 1) w_E)
Global level
For each genome A = (\mathcal{E} \cup \mathcal{T}, \mathcal{M} \cup \mathcal{A}) of phylogeny T:
                        \begin{aligned} \mathbf{g}_{u} &= \mathbf{g}_{v} & \text{with } (u, v) \in \mathcal{M} \\ \sum_{m \in F} \mathbf{g}_{m^{h}} &\geq L_{F}^{h} & \text{for each family } F \\ \sum_{m \in F} \mathbf{g}_{m^{h}} &\leq H_{F}^{h} & \forall v \in \mathcal{E} \cup \mathcal{T} \end{aligned}
  (C.01)
  (C.02)
   (C.03)
Local level
For each edge E = (A, B) \in E(T) with CFMRD(A, B) = (E \cup T, E_{adi})
E_{\mathrm{ext}} \cup E_{\mathrm{self}}):
   (C.04)
                                                           \mathbf{w}_E = \sum_{e \in E_{\text{adi}}} \mathbf{w}(e) x_e
   (C.05)
                                                        \mathbf{f}_E = \mathbf{n}_E - \mathbf{c}_E + \mathbf{q}_E + \mathbf{s}_E
                                                               \mathbf{n}_E = \frac{1}{2} \sum_{e \in E_{\mathrm{ext}}} \mathbf{x}_e
   (C.06)
                                             \mathbf{c}_{E} = \sum_{v \in \mathcal{E}} \mathbf{r}_{v}^{c}
2\mathbf{q}_{E} \ge \mathbf{p}_{E}^{ab} + \mathbf{p}_{E}^{\max a} + \mathbf{p}_{E}^{\max b} - \mathbf{p}_{E}^{AB}
   (C.07)
   (C.08)
```

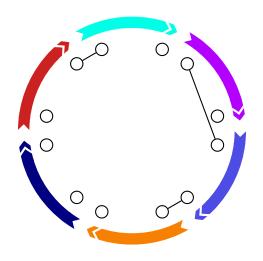


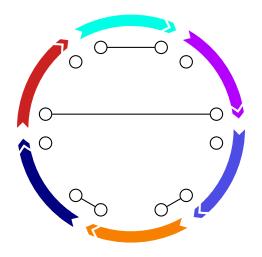




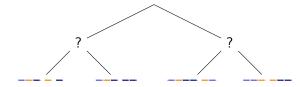




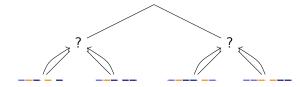




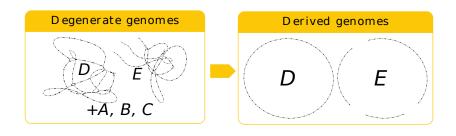
Heuristic Solution: Filter Adjacencies Based on Leafs



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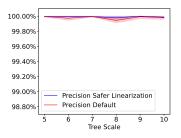
Degenerate Genomes

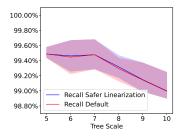


(Slightly) Adapted from (Doerr, Chauve, 2021).

Degenerate Genomes - the Good

On simulated data (ZOMBI+pre-filtering with DeCoSTAR), if the ground truth adjacencies are represented in the degenerate genome:





Degenerate Genomes - the Bad

► Good pre-filtering (DeCoSTAR) requires gene trees.



Is not exact.



- Filter out adjacencies that are provably not needed.
 - \rightarrow Exploit features of the data:



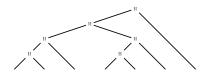
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 - \rightarrow Exploit features of the data:
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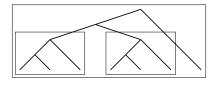
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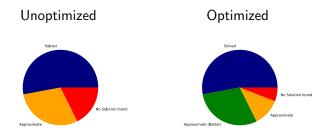
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Extremely Preliminary Results

gurobi 12 on 1 thread with 1 hour time limit on all lineages (17) with 20 leaves or fewer

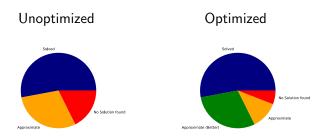
Extremely Preliminary Results

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Main problem seems to be proving optimality.

► Transfer bounds from less complex (sub-) problems

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- ▶ ILP solving $\stackrel{?}{\rightleftarrows}$ SAT solving

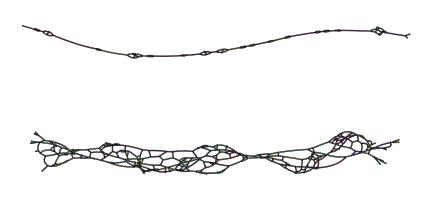
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- ➤ SPP-variant that can model other types of events (horizontal transfers, chromosome duplication,...)

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- ► ILP solving $\stackrel{?}{\rightleftharpoons}$ SAT solving
- ➤ SPP-variant that can model other types of events (horizontal transfers, chromosome duplication,...)
- ► Large Parsimony problem (no given tree)

Joint work with Jens Stoye

CARP - A Specialized Problem for Pangenomes

Quantifying Rearrangement Complexity in Pangenomes - Intuition



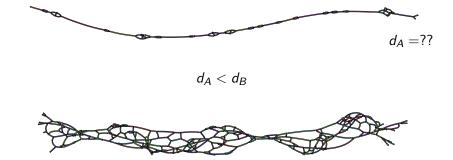
Quantifying Rearrangement Complexity in Pangenomes - Intuition





 $d_B = 107$

Quantifying Rearrangement Complexity in Pangenomes - Intuition



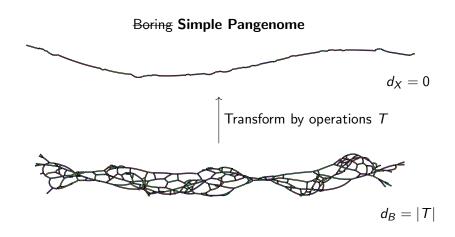
 $d_B = ??$



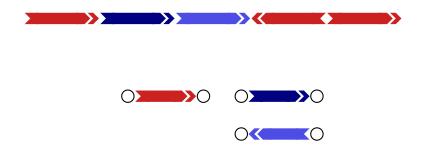


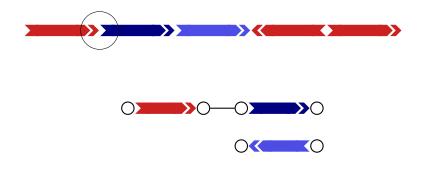
Boring Simple Pangenome $d_X = 0$

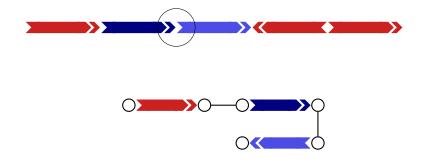


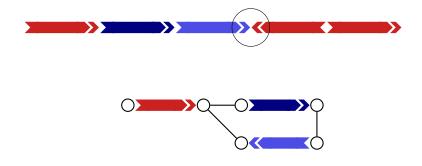


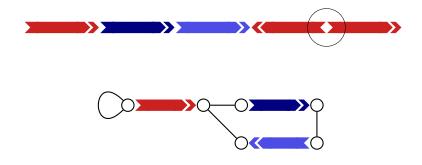


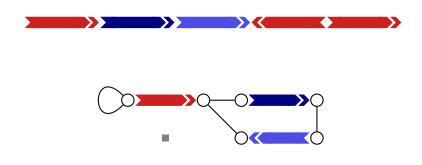


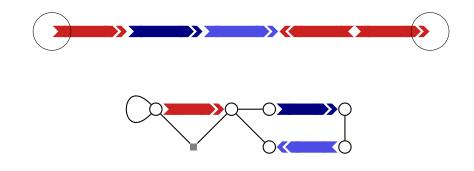




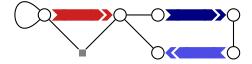






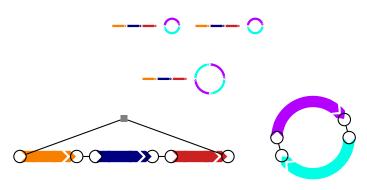




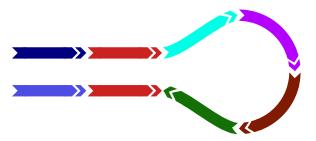


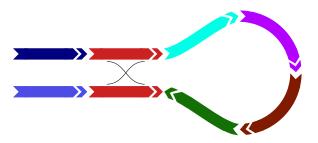
- ► GFA format (w/o sequence) can be represented as an MBPG
- ▶ Note: We still assume local variants have been filtered

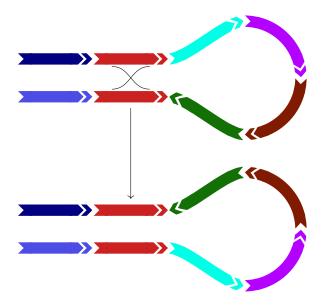
MBPG of a Simple Pangenome

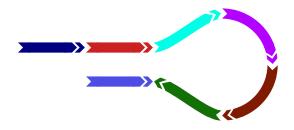


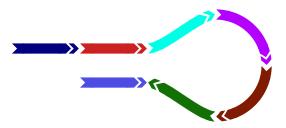
lacktriangle Simple pangenome \iff adjacencies are a perfect matching





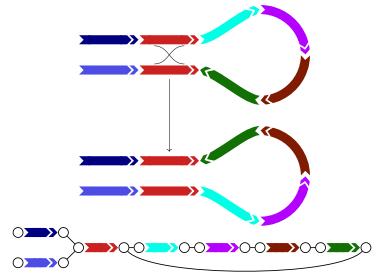




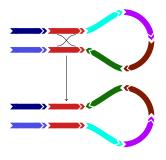


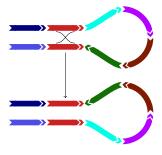
▶ No rearrangements via Homologous Recombination.

MBPG and Homologous Recombinations

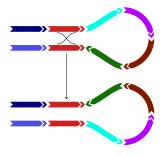


Homologous Recombinations don't change the graph!

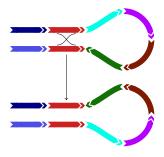




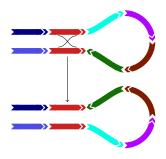
► No change in adjacencies!



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 - $\rightarrow \ \mathsf{Non\text{-}Adjacency} \ \mathsf{Modifying} \ \mathsf{Operations} \ (\mathsf{Namos})$



- ► No change in adjacencies!
 - → Non-Adjacency Modifying Operations (Namos)
 - $\leftrightarrow \mathsf{Adjacency}\ \mathsf{Modifying}\ \mathsf{Operations}\ (\mathsf{Amos})$



- No change in adjacencies!
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Amos can introduce structural complexity!

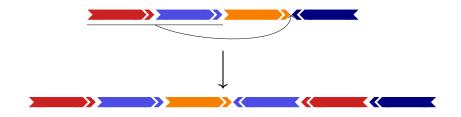
Amos - Example: Segmental Duplication

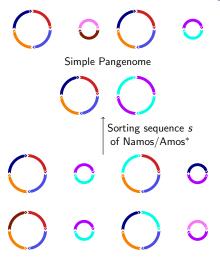


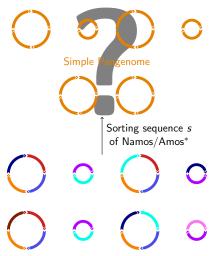
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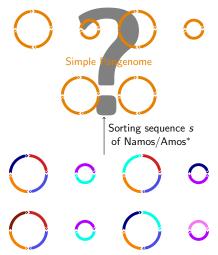


Amos - Example: Segmental Duplication

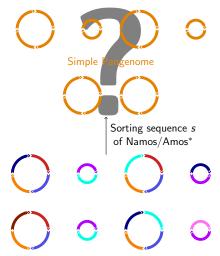








▶ #Amos in s: measure of the Structural Complexity.

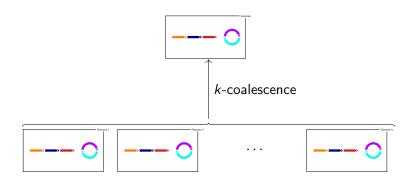


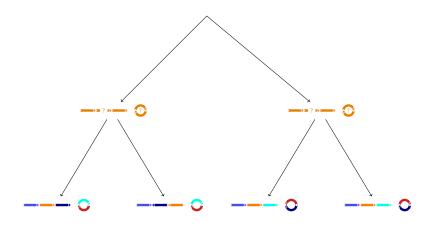
- #Amos in s: measure of the Structural Complexity.
- ▶ * with minimal #Amos

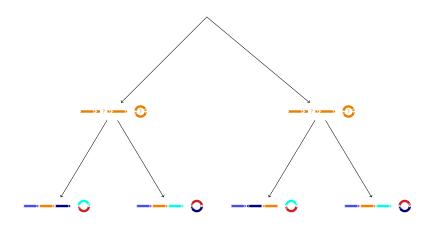
k-Coalescence – an important Namo

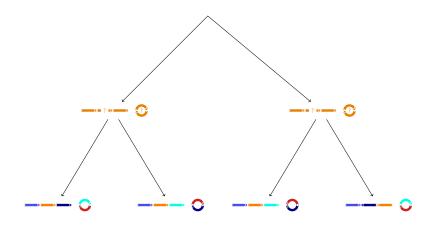


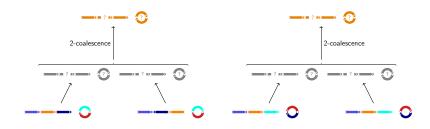
k-Coalescence – an important Namo

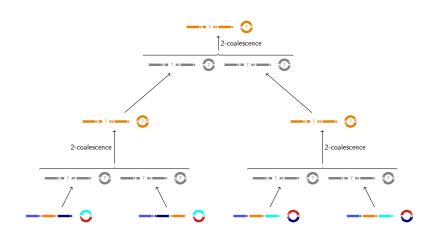












Problem name	Given genomes	Classical Formulation	Namos (N)
Distance problem	Genomes X, Y on the same marker set (single copy)	Find the minimum number of operations in M to transform X to Y .	- (2-coalescence)
Median problem	Genomes X_1, \ldots, X_k on the	Find a genome Y that minimizes the total	-
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- → Classical Problem Formulations would overestimate structural complexity for pangenomes (missing Namos substituted with Amos)
- → For a lower bound, we need a "maximally powerful" set of Namos

Obervations from the MBPG

Observation

Given a Namo o and pangenomes \mathbb{P}, \mathbb{P}' , where $\mathbb{P} \stackrel{\circ}{\to} \mathbb{P}'$, the MBPGs of \mathbb{P} and \mathbb{P}' are identical.

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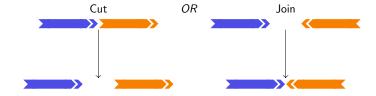
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Definition

A set of Namos N is called MBPG-complete, if for all pairs of pangenomes \mathbb{P}, \mathbb{P}' with the same MBPG there is a sequence of Namos $o_1 o_2 \dots o_k \in N^*$, such that $\mathbb{P} \stackrel{o_1}{\longrightarrow} \stackrel{o_2}{\longrightarrow} \dots \stackrel{o_k}{\longrightarrow} \mathbb{P}'$.

SCJ - A simple Amo



SCJ-CARP Theoretical result

► SCJ Large Parsimony is NP-hard

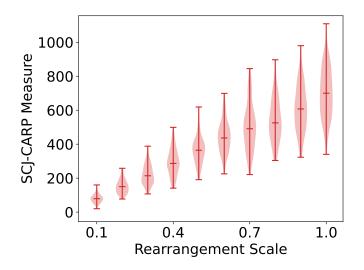
SCJ-CARP Theoretical result

- SCJ Large Parsimony is NP-hard
- ► However SCJ-CARP:

Lemma

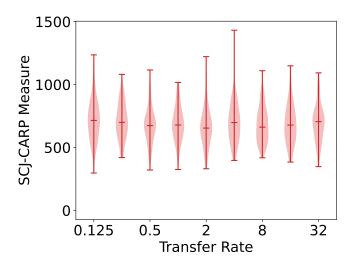
One needs exactly $|E_C|$ SCJs to transform a pangenome into a simple one where $E_C \subset E_A$ is a subset of adjacency edges that can be determined in linear time.

SCJ-CARP Tracks Rearrangements



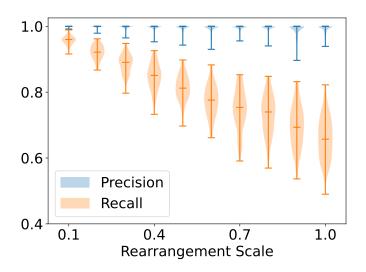
Pearson Coefficient: 0.88

SCJ-CARP Does Not Track Horizontal Effects



Pearson Coefficient: -0.01

SCJ-CARP Reconstructs Ancestral Adjacencies (to a Limited Extent)



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 - $\qquad \qquad d(\mathbb{P}_a, \mathbb{P}_b) = \min_{\mathbb{A}_a \in A_a, \mathbb{A}_b \in A_b} d(\mathbb{A}_a, \mathbb{A}_b)$
- Weighted Namos

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- Simple changes to a problem can make it (computationally) a lot easier.
- ► There is a wealth of theoretical results for rearrangements that only need slight adaptations to make them useful in practice.
- ► Future work: (Formal definition for) Marker segmentation on pangenomes

