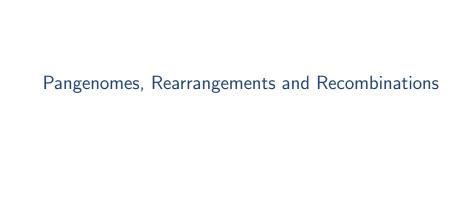
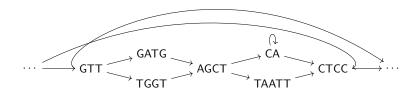
CARP - Quantifying Structural Complexity of Pangenomes

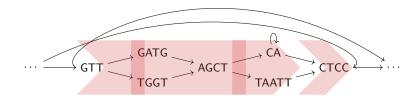
Leonard Bohnenkämper

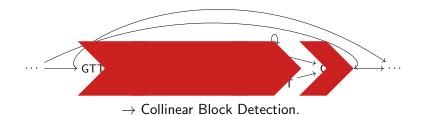
March 5th, 2025





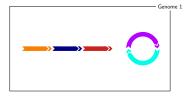


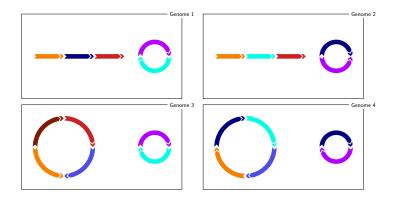


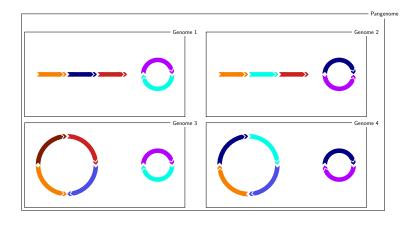


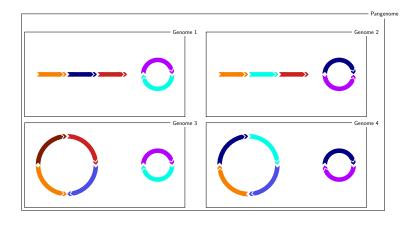


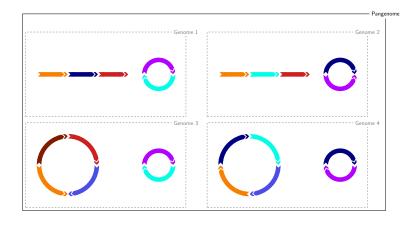






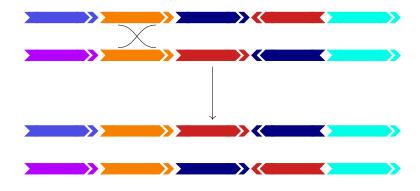


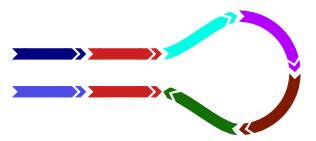


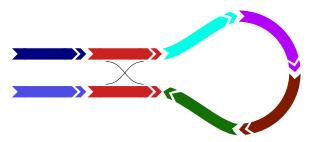


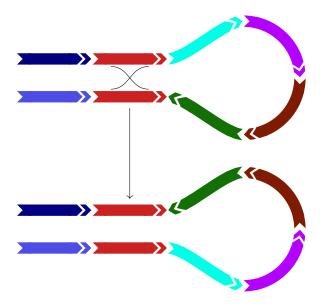


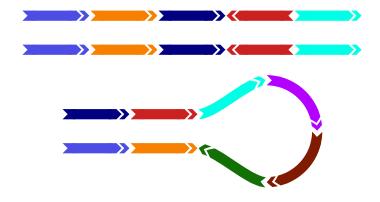


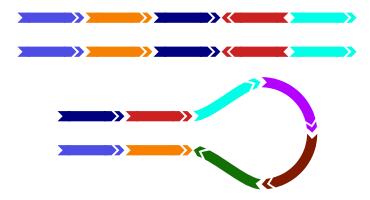








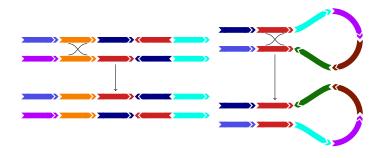


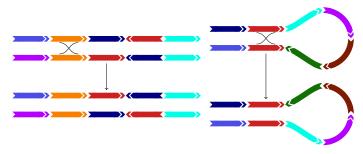


▶ No rearrangements via Homologous Recombination.

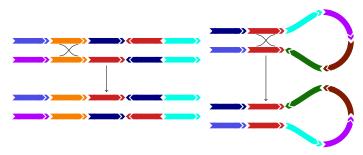
Flash Forward: Adjacencies



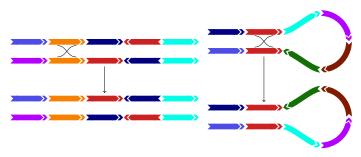




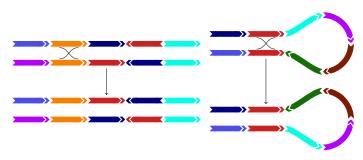
► No change in adjacencies!



- ► No change in adjacencies!
 - \rightarrow Non-Adjacency Modifying Operations (Namos)



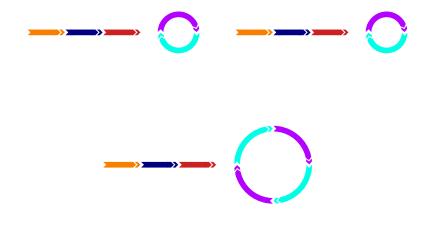
- ► No change in adjacencies!
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 - $\leftrightarrow \mathsf{Adjacency}\ \mathsf{Modifying}\ \mathsf{Operations}\ (\mathsf{Amos})$



- No change in adjacencies!
 - → Non-Adjacency Modifying Operations (Namos)
 - ⇔ Adjacency Modifying Operations (Amos)

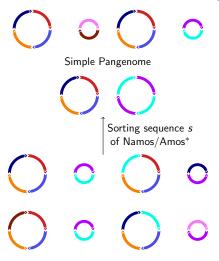
Amos can introduce structural complexity!

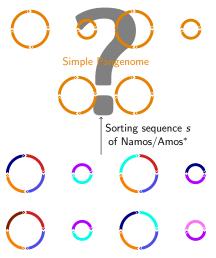
Pangenomes with Minimal Structural Complexity

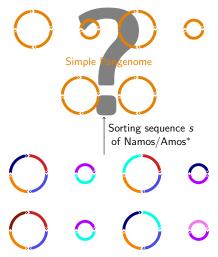


Boring Simple Pangenome

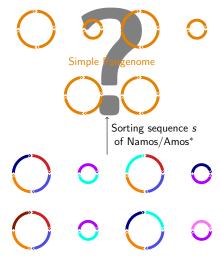








▶ #Amos in s: measure of the Structural Complexity.

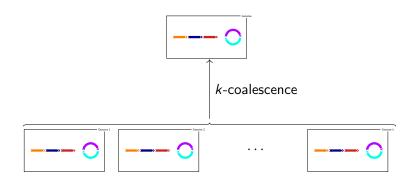


- #Amos in s: measure of the Structural Complexity.
- ▶ * with minimal #Amos

k-Coalescence – an important Namo



k-Coalescence – an important Namo



Classical Problems and GARP

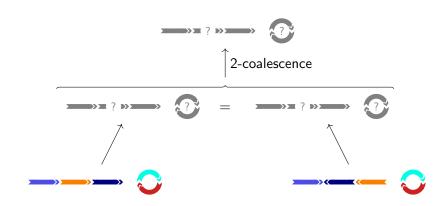
Distance Problem and GARP

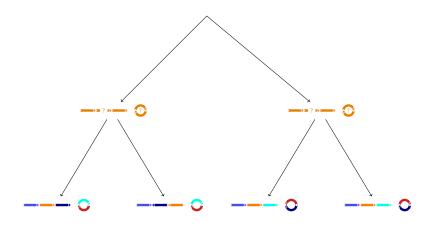


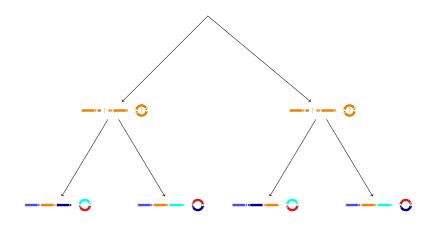
Distance Problem and GARP

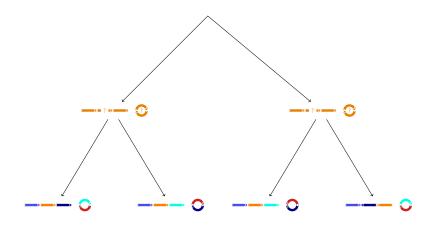


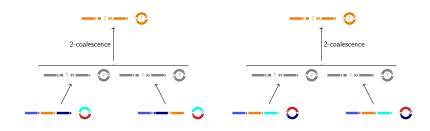
Distance Problem and GARP

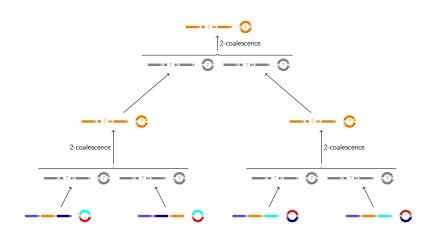












Complete Ancestral Reconstruction for Pangenomes (CARP)

Very limited interaction between genomes (only Coalescence)

- ▶ Very limited interaction between genomes (only Coalescence)
- Pangenomes

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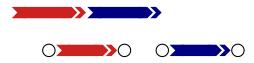
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- ▶ Pangenomes→ phylogenetic closeness→ more interaction/horizontal effects
- \rightarrow Classical Problem Formulations would overestimate structural complexity for pangenomes (missing Namos substituted with Amos)

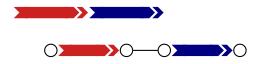
- Very limited interaction between genomes (only Coalescence)
- ▶ Pangenomes→ phylogenetic closeness→ more interaction/horizontal effects
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- ightarrow For a lower bound, we need a "maximally powerful" set of Namos

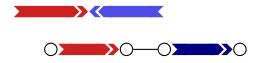


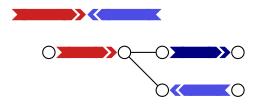


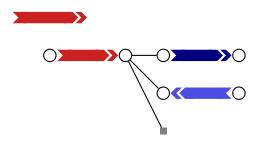


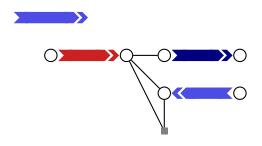


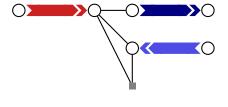




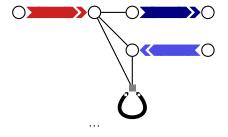




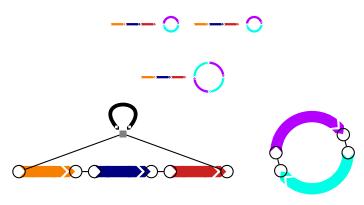




...

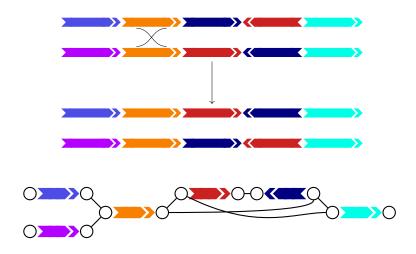


MBPG of a Simple Pangenome

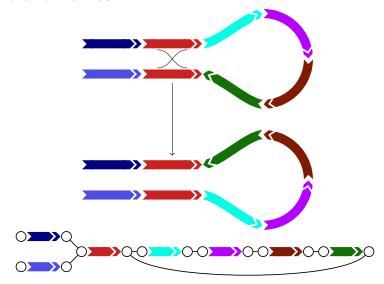


lacktriangle Simple pangenome \iff adjacencies are a perfect matching

MBPG and Namos



MBPG and Namos



► Namos don't change the graph!

Conclusions from the MBPG

Observation

Given a Namo o and pangenomes \mathbb{P}, \mathbb{P}' , where $\mathbb{P} \stackrel{\circ}{\to} \mathbb{P}'$, the MBPGs of \mathbb{P} and \mathbb{P}' are identical.

Conclusions from the MBPG

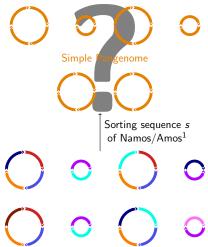
Observation

Given a Namo o and pangenomes \mathbb{P}, \mathbb{P}' , where $\mathbb{P} \stackrel{o}{\to} \mathbb{P}'$, the MBPGs of \mathbb{P} and \mathbb{P}' are identical.

Definition

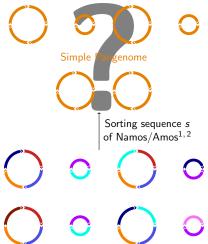
A set of Namos N is called MBPG-complete, if for all pairs of pangenomes \mathbb{P}, \mathbb{P}' with the same MBPG there is a sequence of Namos $o_1 o_2 \dots o_k \in N^*$, such that $\mathbb{P} \stackrel{o_1}{\longrightarrow} \stackrel{o_2}{\longrightarrow} \dots \stackrel{o_k}{\longrightarrow} \mathbb{P}'$.

Complete Ancestral Reconstruction for Pangenomes (CARP)



- ▶ #Amos in s: measure of the Structural Complexity.
- ▶ ¹ with minimal #Amos

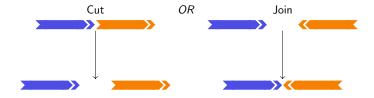
Complete Ancestral Reconstruction for Pangenomes (CARP)



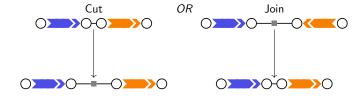
- ▶ #Amos in *s*: measure of the Structural Complexity.
- ▶ ¹ with minimal #Amos
- ▶ ² MBPG-complete set of Namos



Amos: SCJ-operations

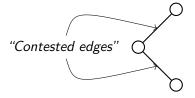


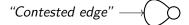
Amos: SCJ-operations

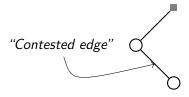




Nothing to be done for adjacencies at non-branching nodes.



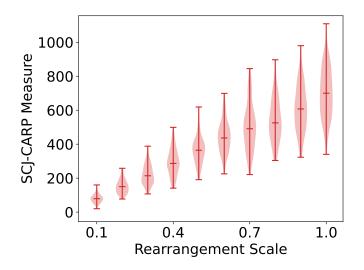




Lemma

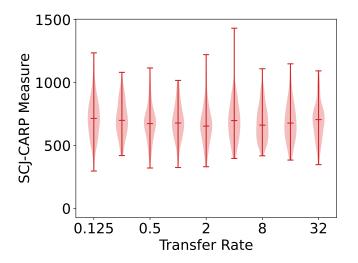
Need exactly $|E_C|$ SCJs to transform the MBPG into a simple MBPG where E_C is the set of contested adjacencies. We call $|E_C|$ the SCJ-CARP measure.

SCJ-CARP Tracks Structural Complexity



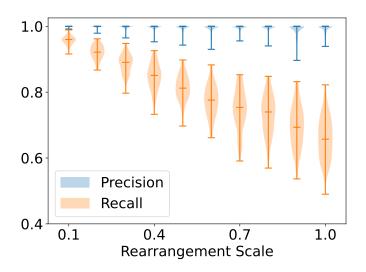
Pearson Coefficient: 0.88

SCJ-CARP Does Not Track Horizontal Effects



Pearson Coefficient: -0.01

SCJ-CARP Reconstructs Ancestral Adjacencies (to a Limited Extent)





► CARP for more complex rearrangement models (DCJ/HP/BI...)

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 - \rightarrow Characterize Pangenome by vector of CARP measures ($m_{SCJ}, m_{DCJ}, m_{HP}, m_{BI}, \ldots$).
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- (Formal definition for) Marker segmentation on pangenomes

