

FACULTY OF HEALTH SCIENCES - SCHOOL OF MEDICINE

MSc Health Statistics and Data Analytics

Association of categorical variables

Anna-Bettina Haidich
Associate Professor of Medical Statistics – Epidemiology
haidich@auth.gr







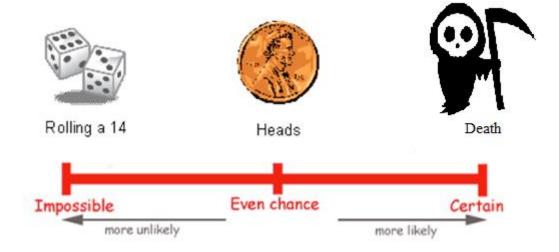
Learning objectives

- Upon completion of this lecture you will be able to:
 - Define and explain the idea of risk and its relationship with probability
 - Define and explain the idea of odds
 - Test whether two categorical variables are associated using the chi-square test or the Mc Nemar's test
 - Calculate a Risk Ratio and its confidence interval
 - Calculate an Odds Ratio and its confidence interval



Probability = Proportion = Risk

• $0 \le P \le 1$



- If the probability of an event happening is p, then the probability of the event not happening is 1 – p.
- In medicine, the term "risk" is often used in place of "proportion" or "probability"



Number of patients that died after therapy

	Death	No death	Total
Patients	30	70	100

Risk (probability) of death after therapy:

30%

The numerator **is included** in the denominator



Number of patients that died after therapy

	Death	No death	Total
Patients	30	70	100

Odds of death after therapy: (30/100)/(70/100) = 30/70 = 3/7 = 0.43

The numerator **is not included** in the denominator



Risk and Odds

The probability of something happening can be expressed as a risk or as an odds:

RISK = the probability of something happening the probability of *all* things happening

ODDS= the probability of something happening the probability of it *not* happening







What is the probability of getting a six when rolling a dice?

The numerator **is included** in the denominator



Odds

 What is the probability of getting a six when rolling a dice compared to not getting a six?

1 to 5
$$(1/5 = 20\%)$$

Numerator: Event / Denominator: No event



Numerator + Denominator: 1/6 + 5/6 = 1



Risk and Odds

Thus a risk is a proportion,

But an odds is a ratio.

An odds is a special type of ratio one in which the numerator and denominator sum to one.



Risk and Odds

- odds = p/(1 p)
- risk or probability (p)= odds/(1 + odds)
- Note:
 - $0 \le p \text{ and } 1 p \le 1$.
 - $0 \le odds \le \infty$
 - $p=0.5 \rightarrow odds = 0.5 / (1 0.5) = 1$
 - The odds always take larger values than the probability (since 1 - probability < 1)
 - Probability < 0.1 \rightarrow probability \approx odds (rare diseases)



The link between probability and odds

 The difference between risk and odds is small when the events are rare but can be large for common events

Event	Total	Risk	Odds
5	100	0.05	0.0526
50	100	0.5	1
95	100	0.95	19



Measures of association: Risk Ratio

 Measures of association quantify the relationship between two variables; usually the outcome variable and the exposure variable.

	Dis	ease	Total
Exposure	Yes	No	
Yes	а	b	a+b
No	С	d	c+d
Total	a+c	b+d	n

- Among those exposed to the risk factor, the risk of disease = a/(a + b).
- Among those not exposed, the risk of disease = c/(c + d).

$$Risk\ Ratio(RR) = rac{a/(a+b)}{c/(c+d)}$$
 H₀: RR=1 H₁: RR eq 1



Measures of association: Odds Ratio

• The Odds Ratio (OR) is the ratio between the odds of the exposure among those with the outcome and the odds of the exposure among those without the outcome.

	Dis	ease	Total
Exposure	Yes	No	
Yes	a	b	a+b
No	c 🔻	d	c+d
Total	a+c	b+d	n

The odds of exposure to the risk factor among those with the disease = a/c

The odds of exposure to the risk factor among those with no disease = b/d.

$$Odds \ Ratio(OR) = \frac{\frac{a}{c}}{\frac{b}{d}} = \frac{a \times d}{b \times c}$$

$$H_0: OR=1$$

$$H_1: OR \neq 1$$

The odds ratio (OR) is usually called the cross-product term.



Direction of results

• $0 \le RR/OR < \infty$

 RR = 1 or OR = 1 there is no difference in the risk of event between two groups

• RR > 1 or OR > 1 the event rate **is larger** in the group in the numerator (OR > RR)

 RR < 1 or OR < 1 the event rate is lower in the group in the numerator (OR < RR)



Chi-square test (Independence test)

- Test for an association between 2 categorical variables.
 - H_0 : There is no association between two categorical variables (they are independent)
 - H₁: There is an association between two categorical variables (they are dependent)
- If two categorical variables are related, it means the chance that an individual falls into a particular category for one variable depends upon the particular category they fall into for the other variable.



Assumptions of chi-square test

- Independent observations
 - Each individual is represented only once in the study. The rows (and columns) of the table are mutually exclusive, implying that each individual can belong in only one row and only one column.
 - Mc Nemar's test for dependent observations
 - e.g. cross-over trials or matched case-control studies
- Large expected frequencies.
 - A) All expected counts should be > 1.
 - B) At least 80% of expected counts should > 5.



Vaccination and Influenza Example

 Association between vaccination (yes, no) and influenza (yes, no). When we test if there is an association between these two variables, we are trying to determine if subjects being vaccinated are less likely to contract influenza. If that is the case, then we can say that vaccination and influenza are **related** or **associated**.



Chi-Square Analysis Details

The 5 Steps in a Chi-Square Test:

Step 1: Write the null and alternative hypothesis.

 H_0 : There is no association between the variables.

 H_1 : There is an association between the variables.

- Step 2: Check conditions.
 - A) All **expected counts** should be > 1.
 - B) At least 80% of **expected counts** should be > 5.
- Step 3: Calculate Test Statistic and p-value.

The test statistic measure the difference between the observed counts and the expected counts assuming independence.

$$\chi^{2} = \sum_{\text{all cells}} \frac{\text{(Observed-Expected)}^{2}}{\text{Expected}} = \sum_{i,j} \frac{(O_{ij} - E_{ij})^{2}}{E_{ij}}$$

This is called **chi-square statistic** because if the null hypothesis is true, then it has a *chi-square distribution* with (r-1)x(c-1) degrees of freedom.



Chi-Square Analysis Details

The 5 Steps in a Chi-Square Test:

- Step 4: Decide whether or not the result is statistically significant. The results are statistically significant if the p-value < α , where α is the significance level (usually α = 0.05).
- Step 5: Report the conclusion in the context of the situation.
 - The p-value <0.05, this result is statistically significant. Reject the H0. Conclude that (the two variables) are related.</p>
 - The *p-value* > 0.05, this result *is NOT statistically significant*. We cannot reject the H0. Cannot conclude that (the two variables) are related.



Vaccination & Influenza example

	Influ	Influenza		
	Yes	No	Total	
Vaccine Placebo	20 (8.3%) 80 (36.4%)	220 (91.7%) 140 (63.6%)	240 220	
Total	100 (21.7%)	360 (78.3%)	460	

- Step 1. Define the null and alternative hypotheses under study
- **H**₀: There is **no association** between the influenza vaccine and the contraction of influenza.
- **H**₁: There is **an association** between the influenza vaccine and the contraction of influenza.



Influenza example (R do the work for us)

	Influenza		Total
	Yes	No	
Vaccine	O ₁₁ = 20	O ₁₂ = 220	R ₁ = 240
Placebo	O ₂₁ = 80	O ₂₂ = 140	R ₂ = 220
Total	C ₁ = 100	C ₂ = 360	N=460

$$E_{11} = (240 \times 100) / 460 = 52.2$$
 $E_{12} = (240 \times 360) / 460 = 187.8$

$$E_{21} = (220x100)/460 = 47.8$$

$$E_{21} = (220 \times 100) / 460 = 47.8$$
 $E_{22} = (220 \times 360) / 460 = 172.2$

Step 2: All expected counts > 1

Step 3: Calculate chi-square

$$\chi^2 = \frac{\left(20 - 52.2\right)^2}{52.2} + \frac{\left(80 - 47.8\right)^2}{47.8} + \frac{\left(220 - 187.8\right)^2}{187.8} + \frac{\left(140 - 172.2\right)^2}{172.2} = 19.86 + 21.69 + 5.52 + 6.02 = 53.09$$

$$\chi^2 = \frac{460 \times (20 \times 140 - 80 \times 220)^2}{100 \times 360 \times 240 \times 220} = 53.01$$

$$\chi^2 = \frac{N(O_{11} \cdot O_{22} - O_{12} \cdot O_{21})^2}{R_1 C_1 R_2 C_2}$$



Vaccination & Influenza example (cont.)

Step 4. : Decide whether or not the result is statistically significant

 $\chi^2 = 53.01$

P<0.001 H₀ is rejected

Step 5. Interpretation

There is an association between the influenza vaccine and the contraction of influenza. The percentage contracting influenza was significantly lower in the vaccine group than in the placebo group (8.3% vs. 36.4%; P<0.001).



Chi-square test and z-test

•
$$Z^2 = \chi_1^2$$

$$z = \frac{p_1 - p_2}{\sqrt{p(1-p)(\frac{1}{n_1} + \frac{1}{n_2})}}$$

 IIIIdeliza		
Yes	No	

20 (8.3%)

Placebo 80 (36.4%) 140 (63.6%) 220

Influenza

Total 100 (21.7%) 360 (78.3%) 460

2-proportions

Vaccine

Chi-Square

H0: p1 - p2 = 0

H0: There is no association between vaccination and influenza.

Total

240

H1: $p1 - p2 \neq 0$ H1: There is an association between vaccination and influenza.

220 (91.7%)



Chi-square test and z-test

$$z = \frac{0.0833 - 0.3636}{\sqrt{0.2174 \times (1 - 0.2174) \times (\frac{1}{240} + \frac{1}{220})}} = \frac{-0.2803}{\sqrt{0.2174 \times 0.7826 \times (0.004167 + 0.004546)}}$$
$$z = \frac{-0.2803}{\sqrt{0.2174 \times 0.7826 \times 0.008713}} = \frac{-0.2803}{\sqrt{0.001482}} = \frac{-0.2803}{0.0385} \approx -7.281$$

 $z^2=(-7.281)^2\approx 53$ which is approximately the χ^2 value.

The absolute z-value 7.281 is larger than the critical $z_{0.05}$ =1.96, so the H0 is rejected.

The percentage contracting influenza was significantly lower in the vaccine group (8.3%), than in the placebo group (36.4%).



Article table

	IgG positive (n = 68)	IgG negative (n = 1543)	p value
Age, median value (IQR) Missing data	74.0 (70.3–80.0) 4	58 (42.0–71.0) 76	<0.001
Sex			0.036
Male	39 (5.4%)	681 (94.6%)	
Female	29 (3.3%)	857 (96.7%)	
Missing data	O	5	
Occupation			< 0.00 I
Farmer	50 (6.6%)	706 (93.4%)	
Other	11 (1.9%)	562 (98.1%)	
Missing data	7	275	
Contact with animals			0.020
Yes	46 (5.6%)	777 (94.4%)	
No	22 (3.1%)	687 (96.9%)	
Missing data	0	79	
Contact with goats			0.001
Yes	21 (8.8%)	219 (91.3%)	
No	47 (3.6%)	1245 (96.4%)	
Missing data	0	79	
Contact with sheep			<0.001
Yes	26 (11.6%)	199 (88.4%)	
No	42 (3.2%)	1265 (96.8%)	
Missing data	0	79	
Contact with cows			<0.001
Yes	12 (12.1%)	87 (87.9%)	
No	56 (3.9%)	1377 (96.1%)	
Missing data	0	79	
Contact with dogs			0.096
Yes	11 (2.9%)	369 (97.1%)	
No	57 (4.9%)	1095 (95.1%)	
Missing data	0	79	

Sidira T, et al.
Seroepidemiological study of
Crimean- Congo
haemorrhagic fever in
Greece, 2009–2010. Clinical
Microbiology and Infection,
Volume 18 Number 2,
February 2012





Assumptions of chi-square test

- Independent observations
 - Each individual is represented only once in the study. The rows (and columns) of the table are mutually exclusive, implying that each individual can belong in only one row and only one column.
- Large expected frequencies.
 - A) All **expected counts** should be > 1.
 - B) At least 80% of **expected counts** should > 5.

Fisher's exact test when there are small samples



Assumptions of chi-square test

- Independent observations
 - Each individual is represented only once in the study. The rows (and columns) of the table are mutually exclusive, implying that each individual can belong in only one row and only one column.
- Large expected frequencies.
 - A) All **expected counts** should be > 1.
 - B) At least 80% of **expected counts** should > 5.

Fisher's exact test when there are small samples



Example with Fisher's exact test

When expected counts <5

	Bleeding co	omplications	
Treatment regimen	Yes	No	Total
A (group 1)	1	12	13
B (group 0)	3	9	12
Total	4	21	25

- Step 1. Define the null and alternative hypotheses under study
 - H0: There is no relationship between the treatment regimens and the bleeding complications.
 - H1: There is a relationship between the treatment regimens and the bleeding complications.
- Fisher's exacte test P= 0.3217 > 0.05 do not reject H0



Vaccination & Influenza example

	Influ	Influenza		
	Yes	No	Total	
Vaccine	20 (8.3%)	220 (91.7%)	240	
Placebo	80 (36.4%)	140 (63.6%)	220	
Total	100 (21.7%)	360 (78.3%)	460	

2-proportions Chi-Square

H0: p1 - p2 = 0 H0: There is no association between vaccination and influenza.

H1: $p1 - p2 \neq 0$ H1: There is an association between vaccination and influenza.

• The table shows that the percentage contracting influenza was much lower in the vaccine group (8.3%), than in the placebo group (36.4%).

p1-p2 = 0.083-0.364 = -0.281

-The risk of influenza was 28% lower in the vaccine group than in the placebo group (P<0.001).



Vaccination & Influenza example

	Influ		
	Yes	No	Total
Vaccine	20 (8.3%)	220 (91.7%)	240
Placebo	80 (36.4%)	140 (63.6%)	220
Total	100 (21.7%)	360 (78.3%)	460

 $H_0: RR=1$

H₁: RR≠1

- -The vaccine group has 0.22 times the risk (of influenza) of the placebo group
- -The vaccine group has (0.22-1=-0.78) 78% lower risk of influenza than the placebo group 95%CI: 0.15, 0.36

reject H₀

 H_0 : OR=1

$$OR = (20*140) / (220*80) = 0.16$$

H₁: OR≠1

- -The vaccine group has 0.16 times the odds (of influenza) of the placebo group
- -The vaccine group has (0.16-1=-0.84) 84% lower odds of influenza than the placebo group

95%CI: 0.09, 0.27

reject H₀



Attendance in day care & respiratory infection

Is attendance at a day care associated with an increase in respiratory infections?

D	• 4	• •	4 •
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1/C2D	II atui v	11110	uuu
	,		

-	Yes	%	No	%	Total	%
Daycare	27	30.3	62	69.7	89	100
No daycare	53	9.6	499	90.4	552	100
Total	80	12.5	561	78.3	641	100

 H_0 : RR=1

H₁: RR≠1

-The risk of respiratory infection is 3.16 times higher in children who attended daycare than in those who didn't.

$$H_0$$
: OR=1

$$OR = (27*499) / (62*53) = 4.10$$

-The odds of respiratory infection is 4.10 times higher in children who attended daycare than in those who didn't.

Risk difference 0.303 - 0.096 = 0.207

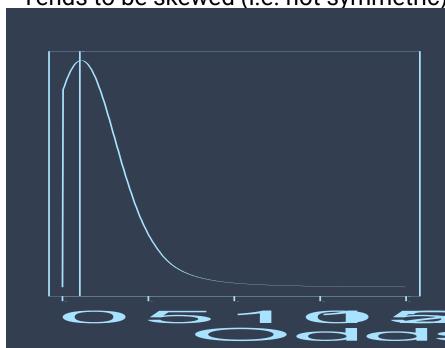
The risk of respiratory infection is 21% higher in children who attended daycare than in those who didn't.

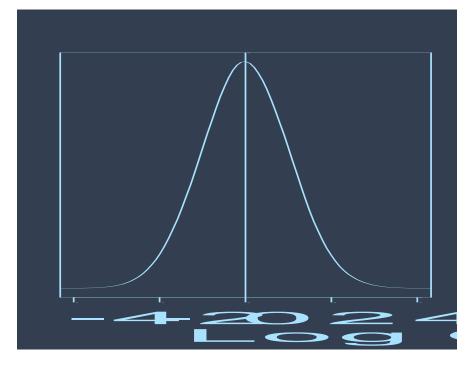


More on the Odds Ratio

Sometimes, we see the log odds ratio instead of the odds ratio.

Tends to be skewed (i.e. not symmetric)





log OR > 0: increased risk

log OR = 0: no difference in risk

log OR < 0: decreased risk

Use for the calculation of the 95% confidence intervals



95% confidence intervals OR

• (1-a)% CI $lnOR:lnOR \pm Z_a \times SE(lnOR)$

• where
$$SE(logOR) = \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$$
Respiratory

	infection		Total	
Exposure	Yes	No		
Yes	27	62	89	
No	53	499	552	
Total	80	561	641	

- $SE(logOR) = \sqrt{(1/27 + 1/53 + 1/62 + 1/499)} = 0.272$
- InOR = In(4.10) = 1.41
- 95% confidence interval of InOR = InOR ± 1.96*SE(InOR) = 1.41 ± 1.96*0.272
- Lower limit = 1.41 1.96*0.272 = 0.878
- Upper limit=1.41 + 1.96*0.272 = 1.944



95% confidence intervals OR

• Exponentiating: $e^{0.878}$ = 2.41, $e^{1.944}$ = 6.99

- H_0 : OR=1
- H₁: OR≠1

• Therefore the 95% confidence interval of the odds ratio 4.10 was 2.41 – 6.99, and this association is significant since the confidence interval does not include the value of 1 (P<0.05).



95% confidence intervals RR

Similarly,

$$SE(logRR) = \sqrt{\frac{1}{a} - \frac{1}{a+b} + \frac{1}{c} - \frac{1}{c+d}}$$

$$SE(logRR) = \sqrt{\frac{1}{27} - \frac{1}{89} + \frac{1}{53} - \frac{1}{552}} = 0.207$$

Respiratory infection			Total
Exposure	Yes	No	
Yes	27	62	89
No	53	499	552
Total	80	561	641

H₀: RR=1 H₁: RR≠1

- lnRR = ln(3.16) = 1.15
- 95% confidence interval of lnRR = lnRR ± 1.96*SE(lnRR) = 1.15 ± 1.96*0.207
- Lower limit = 1.15 1.96*0.207 = 0.745
- Upper limit=1.15 + 1.96*0.207 = 1.556
- The 95% confidence interval of the risk ratio is given by exponentiating the upper and the lower limits: $e^{0.745} = 2.11$, $e^{1.556} = 4.74$
- Therefore the 95% confidence interval of the risk ratio 3.16 was 2.11 4.74, and this
 association is significant since the confidence interval does not include the value of 1
 (P<0.05).



Which measure is best?

- RR are usually calculated for cross-sectional studies and cohort studies.
- OR are always calculated for case-control studies and often for cross-sectional studies.
- Note that in case-control studies it is not generally possible to calculate risks, and therefore RR
- OR is a consistent measure of effect, unlike the RR. (1/0.16 = 6.25 higher odds of influenza in placebo vs. vaccine group)
- OR is difficult to understand



Why do we so often see OR and not RR?

- (1) Logistic regression:
 - Allows us to look at association between two variables, adjusted for other variables. (Linear models course)

(2) Can be more globally applied. Design of study does not restrict usage.



Article table

Table 3. Associations of meta-analysis characteristics with focus on specific agent(s)

	Focus on specif	ic agent(s), n (%)	Univariable analysis		
Characteristics	Yes	No	OR	95% CI	P-value
Publication venue					
Journal	157 (48.3)	168 (51.7)	1.92	1.31 - 2.82	0.001
Cochrane	57 (32.8)	117 (67.2)	1.00		
Funding					
Industry involved	37 (75.5)	12 (24.5)	5.28	2.62-10.66	< 0.001
Not reported	60 (39.0)	94 (61.0)	1.09	0.72 - 1.66	0.676
No funding	30 (50.0)	30 (50.0)	1.71	0.97 - 3.03	0.065
No industry involved	87 (36.9)	149 (63.1)	1.00		

As shown in Table 3, in univariable analyses, metaanalysis publications focusing on specific named agent(s) were more likely to be published in journal venues rather than the *Cochrane Database of Systematic Reviews*, they were significantly more common when there was industry involvement, when harms only were addressed, and when individual patient data analyses were presented. Type of

A.-B. Haidich et al. / Journal of Clinical Epidemiology 66 (2013) 371-378



Mc Nemar's test

- Paired categorical variables
 - Before-after studies
 - Cross-over studies

treatmentabsentcdc+dColumn Totala+cb+dN=a+b+c+d	Before	present efore	а	b	a+b
Column Total a+c b+d N=a+b+c+d	treatment	eatment absent	С	d	c+d
	Column Tota	olumn Total	a+c k	o+d N=	a+b+c+d

- Matched categorical data
 - Matched case-control study

		Control		Row Total
		exposed		
Case	exposed	а	b	a+b
	unexposed	С	d	c+d
Column Total		a+c	b+d	N=a+b+c+d

After treatment

absent

present

Row Total



Mc Nemar's test

Assumptions

- The sample data have been randomly selected and consist of matched pairs.
- Large samples and b+c>25 (in the case that b+c<25, the **exact binomial probability** should be computed).



Steps for conducting the test

- Step 1. Define the null and alternative hypotheses under study
 - H_0 : The two marginal probabilities for each outcome are the same, $p_a + p_b = p_a + p_c$ and $p_c + p_d = p_b + p_d$ which means that $p_b = p_c$.
 - H_1 : The two marginal probabilities for each outcome are not the same, $p_a + p_b \neq p_a + p_c$ and $p_c + p_d \neq p_b + p_d$ which means that $p_b \neq p_c$.

		After tre	After treatment	
		present	absent	
Before treatment	present	а	b	a+b
	absent	С	d	c+d
Column Total		a+c	b+d	N=a+b+c+d



Mc Nemar's test

		Control		Row Total
		exposed	unexposed	
2	exposed	а	b	a+b
Case	unexposed	С	d	c+d
Column Total		a+c	b+d	N=a+b+c+d

Step 2. Calculate the value of the test statistic specific to H_0

$$\chi^2 = \frac{(b-c)^2}{b+c}$$

Step 3. Compare the value of the test statistic to values from the χ^2 - distribution

• If $\chi^2 > 3.84$ (which means that p-value<0.05) then H₀ is rejected.

Step 4. Interpretation



 In a survey 86 children with asthma attended a camp to learn how to self-manage their asthmatic episodes

		Knowledge at	Row Total	
		Yes	No	
Knowledge at beginning of camp	Yes	24	6	30
	No	29	27	56
Column Total		53	33	N=86

Step 1. Define the null and alternative hypotheses under study H_0 : there was no change in children's knowledge of asthma management between the beginning and completion of the health camp.

 H_1 : there was change in children's knowledge of asthma management between the beginning and completion of the health camp.



Step 2. Calculate the value of the test statistic specific to \mathbf{H}_0

$$\chi^2 = \frac{(6-29)^2}{6+29} = \frac{529}{35} = 15.11$$

Step 3. Compare the value of the test statistic to values from the χ^2 -distribution

 $X^2 = 15.11 > 3.84 \implies H_0 \text{ is rejected} \implies P < 0.05$



		Knowledge at end of camp		Row Total	
		Yes	No		
Knowledge at beginning of camp	Yes	24	6	30	
	No	29	27	56	
Column Total		53	33	N=86	

Step 4. Interpretation

- (30/86)x100=34.9% of children had appropriate knowledge at the beginning of the camp and (53/86)x100=61.6% at the end of the camp.
- The analysis show that the number of children who knew how to manage their asthma episodes appropriately increased significantly on completion of the camp.



		Knowledge at end of camp		Row Total	
		Yes	No		
Knowledge at beginning of camp	Yes	24	6	30	
	No	29	27	56	
Column Total		53	33	N=86	

Step 4. Interpretation

- (30/86)x100=34.9% of children had appropriate knowledge at the beginning of the camp and (53/86)x100=61.6% at the end of the camp.
- The analysis show that the number of children who knew how to manage their asthma episodes appropriately increased significantly on completion of the camp.



Article table

glaucoma

24-h Intraocular pressure control with evening-dosed travoprost/timolol, compared with latanoprost/timolol, fixed combinations in exfoliative

AGP Konstas¹, DG Mikropoulos¹, TA Embeslidis¹, AT Dimopoulos¹, A Papanastasiou¹, A-B Haidich² and WC Stewart³

Eye (2010) 24, 1606-1613

Patients

Patients were recruited for this prospective, singlemasked, crossover study from the Glaucoma Unit of the First University Department of Ophthalmology, AHEPA

Table 4 Adverse events recorded in the study

Adverse eventr	LTFC	T	TTFC		P-value
		Yes	No	Total	
Conjunctival hyperemia	Yes No Total	2 6 8 (20)	2 30 32 (80)	4 (10) 36 (90) 40 (100)	0.29
Stinging	Yes No Total	3 1 4 (10)	3 33 36 (90)	6 (15) 34 (85) 40 (100)	0.62
Hypertrichosis	Yes No Total	3 3 6 (15)	0 34 34 (85)	3 (7) 37 (93) 40 (100)	0.25
Skin pigmentation	Yes No Total	0 3 3 (7)	0 37 37 (93)	0 (0) 40 (100) 40 (100)	0.25



Flowchart of statistical tests

