Reducing Polarization in Social Media

A Thesis

by

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Table of Contents

Chapter 1

Introduction and the Theory Behind Polarization

1.1 Introduction

Polarization describes the division of people into two contrasting groups or sets of opinions or beliefs. The term is used in various domains such as politics and social studies. For example political polarization refers to the divergence of political attitudes to ideological extremes. Social studies use this term to describe the segregation within a society in terms of income inequality or social and class status. Currently social media have a big role as a source of news and information and a lot of the related discussions of people have gone online. Polarization is linked with harmful effects such as intensifying stereotypes and creating echo chambers. In echo chambers individuals get their news only from like-minded people as they share and reinforce one another's opinions. Additionally the fact that people tend to ignore opposing views in combination with algorithmic personalization results a significant increase of polarization.

1.2 Social and Psychological Factors

Individuals experience discomfort when given data that actively challenge their opinions. In the field of psychology, cognitive dissonance occurs when a person holds two or more contradictory beliefs, ideas, or values and experiences psychological stress because of that. In simple terms dissonance is defined as a the lack of agreement. Individuals want to reduce the discomfort that is caused from cognitive dissonance. Reduction occurs by strengthening opinions that come in agreement with their own and downplaying everything that challenges them. This leads individuals to a selective exposure on information [1]. Selective exposure is also demonstrated in groups. Furthermore people assign themselves with social identities. The self-categorization theory stems from the social identity theory, which holds that conformity stems from psychological processes. Accordingly, proponents of the self-categorization model hold that group polarization occurs because individuals identify with a particular group and conform to a prototypical group position that is more extreme than the group mean. It is shown that groups of people tend to make decisions that are more extreme than the initial inclination of its members [2].

1.3 Polarization online

Online entities such as news or social media platforms are aware of their users opinions and aim to maximize their satisfaction. As discussed above, platforms will present content in a way that minimizes psychological stress. This leads to media bias. Media bias is the bias or perceived bias of journalists and news producers within the mass media in the selection of many events and stories that are reported and how they are covered. When this happens online, personalization of the content creates algorithmic bias. Algorithmic bias describes systematic and repeatable errors in a computer system that create unfair outcomes, such as privileging one arbitrary group of users over others. Bias can emerge due to many factors like the design of the algorithm. Due to personalization we don't see the same content and this is the main reason for the formation of filter bubbles.

1.4 Filter Bubbles

Filter bubbles are the echo chambers of social media. In news media, an echo chamber is a metaphorical description of a situation in which beliefs and opinions are strengthened by communication and repetition inside a closed system. It is important to distinguish the difference between echo chambers and filter bubbles. This two concepts are almost identical, however, filter bubbles are a result of algorithms that choose content based on previous online behaviour, as with search histories or online shopping activity.

Chapter 2

RelatedWork

2.1 Measuring the Polarization of a Network

At first we have to measure the opinion polarization in a social Network. The actions and information of a user can give us insights about his opinions on a topic e.g. accounts a user follows, content they repost, comments they make , etc. Using this information we can measure the polarization. Assume a graph $G = (V, E)$ representing a network that is connected and undirected. Z will be the vector of expressed opinions for the whole network. Each value Z_i of the vector will represent a node and can be computed with the opinion-formation model of Friedkin and Johnsen. The length of the opinion vector $||z||^2$ measures the polarization of the network and $\pi(z) = \frac{||z||^2}{n}$ is defined as the polarization index of the network, where n is the number of nodes in the graph so the polarization index can be independent of the network size. There is a direct link between this opinion model and random walks. Given the graph $G = (V, E)$ we can construct the augmented graph $H(V \cup X, E \cup R)$. For each vertex of V we will add a new vertex on X and a directed edge (v_i, x_i) in R. The node x_i corresponds to the internal opinion of the node v_i. In the model we follow z_j or else the expressed opinion of a user that can be computed by the probability of $P(x_i|v_j)$. This probability represents that a random walk on the augmented graph that started from the node V_j ended at the node X_i or else how much likely the probability of user V_j adopting the opinion of user V_i. This probability depends in the structure of the graph. Two problems are introduced, the $ModerateInternal$ and the $ModerateExpressed$. When moderating opinions a small set of nodes T_s is being set to zero, in each problem, as their names suggests, internal or external opinions are set to zero. Two algorithms are proposed for the $ModerateInternalproblem$. A greedy algorithm that finds the set T_s of nodes iteratively according to the biggest decrease it causes and the Binary Orthogonal Matching Pursuit (BOMP) algorithm. For the $ModerateExpressed$ problem the same greedy algorithm is used. [3]

2.2 Polarization and Disagreement

Another way of looking at polarization is by combining it with disagreement. The main problem of minimising polarization and disagreement lies in the opinions of each user and how targeted ads and recommendations influence their opinions. Considering the disagreement in combination with polarization a network can choose how to respond in different situations. Their recommendation system could choose between keeping the disagreement low or exposing users to radically different opinions. There are situations that this optimisation can reduce the overall polarization-disagreement in the network by recommending edges in different parts of the network than the ones that agree with the human confirmation bias. Given a social network $G = (V, E, w)$ and initial opinions $s : V \to [0, 1]$ the equilibrium vector according to the Friedkin-Johnsen model is defined as $z^* = (I + L)^1 s$ where L is the laplacian matrix of the graph and I the identity matrix. Disagreement of $d(u, v)$ of edge (u, v) is defined as the squared difference between the opinions of u, v at equilibrium: $d(u, v) = w_{uv}(Z_u^* - Z_v*)^2$. The total disagreement is defined as $D_{G,s} = \sum_{(u,v)\epsilon E} d(u, v)$. Let $\bar{z} = z^* - \frac{z*^T \vec{1}}{n} \vec{1}$ polarization is measured as a deviation from the average with the standard definition of variance as $P_{G,s} = \sum_{u\epsilon V} \bar{z}_u^2 = \bar{z}^T \bar{z}$ The polarization-disagreement index is defined as follows $I_{G,s} = P_{G,s} + D_{G,s}$. The objective is to minimize this index. Muco and Tsourakakis have shown that minimising

$\bar{z}^T\bar{z} + \bar{z}^T L\bar{z}$ is the same to minimising the polarization-disagreement index. Here, L is a matrix among the set of valid combinatorial Laplacians of connected graphs.[4]

2.3 Quantifying and minimizing Risk of Conflict in Social Networks

We know for a fact that opinions are formed through social interactions and in every interaction conflict arises. Online networks offer public access to social disputes on controversial matters that allows the study and moderation of them. The majority of studies are based in the Friedkin-Johnsen model. The main problem is with the Friedkin-Johnsen model metrics. The external opinion of a user, which by definition is hard to measure, combined with the internal opinion which is impossible to be measured. Another problem occurs in the editing of the social graph. We edit the social graph in a way that minimises the conflict of a certain social issue. This can lead to an increased conflict of one or more social issues inside the network. Chen, Lijffijt and De Bie still use the Friedkin-Johnsen model to evaluate the network conflict but the quantifications depend only on the network topology in a way that the conflict can be reduced over all issues. Worst-case(WCR) conflict risk and average-case conflict risk(ACR) are defined to represent two separate problems, how the network can be minimised in the worst case or in the average case scenario by altering the social graph. These problems consider the measures of internal conflict, external conflict, and controversy. Internal conflict (ic) measures the difference of the internal and the expressed opinion of a user. $ic = \sum_i (z_i - s_i)^2$. External conflict ($ec$) measures how different are the opinions of the neighbours with each other. $ec = \sum_{(i,j)\epsilon E} w_{ij}(z_i - z_j)^2$. Controversy ($c$) measures the variation of the opinions in the network and is independent of the social graph structure. $c = \sum_i {z_i}^2$. These measures are not independent. Reducing one of them results in the increase of another. This leads to the conservation law of conflict. $S^T S = ic + 2ec + c$. There are two methods of minimising the conflict of the network for each of the ACR and WCR problems. One is a gradient method that considers deleting and adding edges simultaneously and the other is a descent method that suggests deleting or adding a single edge. Chen, Lijffijt and De Bie used small world random networks and random networks with binomial and power law degree distribution to find out what types of networks have the highest risks for every conflict measure they defined. A small world network is a type of graph in which most nodes are not neighbours of one another, but the neighbours of any given node are likely to be neighbours of each other and most nodes can be reached from every other node by a small number of hops or steps. They found that the small world networks are the most high-risk for the ic metric. For c and r the most high-risk network depends on the density.[5]

2.4 Reducing Controversy by connecting Opposing Views

Chapter 3

Premilinaries and Problem Definition

3.1 The Friedkin and Johnsen Model

The model will use the information about the opinion of the user, internal and external, but also the constant update of the external opinions of the neighbourhood of the user e.g. the friend list or the accounts the user follows to compute an opinion vector. This vector is a metric for the whole social graph that can give us insight about its current situation. The vector values range from [-1,1]. Values closer to the range limits indicate bigger polarization. Polarized graphs create groups of nodes that are strongly connected with each other and feedback to one another the same extreme opinion over a topic. These groups can be seen clearly in the illustration of filter bubbles and often associated with politics and controversial issues of our society. Using a certain number of users we can achieve a reduction on the polarization of the network.

We can educate a group of users with the opposite view, and in terms of our model that means that we can modify the social graph by adding a connection between users of different opinions.

Let $G = (V, E)$ be a connected undirected graph representing a network. Let z be the vector of expressed opinions for the whole network. Each value of the vector represents a node and can be computed with the opinion-formation model of Friedkin and Johnsen as follows.

$$z_i = \frac{w_{ii} * si + \sum_{j \epsilon N(i)} w_{ij} * z_j}{w_{ii} + \sum_{j \epsilon N(i)} w_{ij}} \tag{3.1}$$

Where s_i denotes the internal and z_i the expressed opinion of a user. The internal opinion of a user corresponds to the views that inherently has for a controversial topic while the expressed one is the views that the user shares on a social network with his neighbours. The length of the opinion vector $||z||^2$ measures the polarization of the network. To make the polarization independent of its network we can normalize it by dividing it with the length of the vector z. An equivalent way of obtaining the vector z from a graph is the following: if L is the laplacian matrix of a graph $G = (V, E)$, and I is the identity matrix, then $z = (L + I)^{-1}S$ [6].

3.2 Measuring the polarization

We measure the polarization by its distance from a neutral opinion. We can quantify this with the length of the vector of the second norm L_2^2 [3].

$$\pi(z) = ||z||_2^2 \tag{3.2}$$

This value can be independent of the network if we normalize it by dividing with the size of the graph.

3.3 A small example of the Friedkin and Johnsen Model

We will now present a small example so we can build a basic understanding of the Friedkin and Johnsen Model. Consider a small graph that consists of two nodes, u and v with internal opinions of 1 and -1 and $w_{uu} = w_{vu} = w_{vv} = 1$.

$$z_u = \frac{1*1+1*1+1*(-1)}{1+2} = \frac{1}{3} \quad , \quad z_v = \frac{1*(-1)+1*(-1)+1*1}{1+2} = -\frac{1}{3} \tag{3.3}$$

$$\pi(z) = ||z||_2^2 = \sqrt{(\frac{1}{3})^2 + (-\frac{1}{3})^2}^{\,2} = \frac{2}{9} \tag{3.4}$$

We did not normalize the polarization index here by dividing with the size of the graph as we have a simple example.

3.4 Problem Definition

Real world events such as Brexit and the 2016 U.S. presidential elections gives us a clear hint about the polarization our society is witnessing. Social media polarization has a strong effect on politics, opinion formation and how people interact with each other in a society. Users of social media are now receiving biased information that amplify their own viewpoints. Enclosed in their filter bubble, they will ignore everyone else and only acknowledge opinions that fit their own reality. In combination with fake news a malicious entity could use social media as a tool to polarize certain groups of people for their own interest. Problem 3 and 4 examine this case. Reducing online polarization is crucial, Problem 1 and 2 can help combat this phenomenon.

Problem 1. Let $C \subseteq V \times V$ a set of edges that are not in the graph. We want to find a subset of $S \subseteq C$ of k edges whose addition to a graph G leads to the greatest reduction of $\pi(z)$.

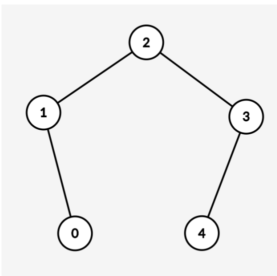
Problem 2. Let S a set of edges that are proposed for addition in the graph G from Problem 1. We want to find the probabilities of these changes being accepted.

Problem 3. Let $V \times V$ the set of edges of graph G. We want to find a subset of edges $S = k, k \in V \times V$ whose removal from the graph G leads to the greatest increase of $\pi(z)$.

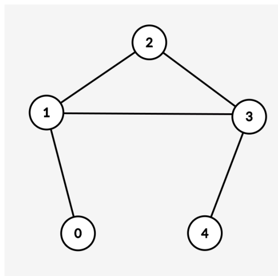
Problem 4. Let S the set of edges that are proposed for removal in the graph G from Problem 3. We want to find the probabilities of these changes being accepted.

3.5 Monotonicity of the Problem

We observe that $\pi(z)$ is not monotone with respect to the edge additions. This means that adding an edge will not necessarily decrease the polarization index. We will show that this is true with a counter example. In the network 3.1 nodes 0, 2 and 3 have a value of $s_i = -1$, and nodes 2 and 4 have a value of $s_i = +1$. For both examples we assume that $w_{ii} = w_{ij} = w_{ji} = 1$ and n the number of nodes. We will now compute the polarization index of the original graph



(a)



(b)

Figure 3.1: Edge addition between opposed opinions.

$$L+I = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}, \qquad (L+I)^{-1} = \begin{pmatrix} \frac{34}{55} & \frac{13}{55} & \frac{1}{11} & \frac{2}{55} & \frac{1}{55} \\ \frac{13}{55} & \frac{26}{55} & \frac{2}{11} & \frac{4}{55} & \frac{2}{55} \\ \frac{1}{11} & \frac{2}{11} & \frac{5}{11} & \frac{2}{11} & \frac{1}{11} \\ \frac{2}{55} & \frac{4}{55} & \frac{2}{11} & \frac{26}{55} & \frac{13}{55} \\ \frac{1}{55} & \frac{2}{55} & \frac{1}{11} & \frac{13}{55} & \frac{34}{55} \end{pmatrix},$$

$$s = \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}, \qquad (L+I)^{-1}s = \begin{pmatrix} \frac{34}{55} & \frac{13}{55} & \frac{1}{11} & \frac{2}{55} & \frac{1}{55} \\ \frac{13}{55} & \frac{26}{55} & \frac{2}{11} & \frac{4}{55} & \frac{2}{55} \\ \frac{1}{11} & \frac{2}{11} & \frac{5}{11} & \frac{2}{11} & \frac{1}{11} \\ \frac{2}{55} & \frac{4}{55} & \frac{2}{11} & \frac{26}{55} & \frac{13}{55} \\ \frac{1}{55} & \frac{2}{55} & \frac{1}{11} & \frac{13}{55} & \frac{34}{55} \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{-27}{55} \\ \frac{1}{55} \\ \frac{-5}{11} \\ \frac{-21}{55} \\ \frac{17}{55} \end{pmatrix}$$

$$\pi(z) = \frac{||z||_2^2}{n} = \frac{\sqrt{(\frac{-27}{55})^2 + (\frac{1}{55})^2 + (\frac{-5}{11})^2 + (\frac{-21}{55})^2 + (\frac{17}{55})^2}^2}{5} = 0.13785123966 \tag{3.5}$$

We will now compute the polarization index after the addition of the edge $1 \to 3$.

$$L + I = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 \\ 0 & -1 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}, \qquad (L+I)^{-1} = \begin{pmatrix} \frac{59}{99} & \frac{19}{99} & \frac{1}{11} & \frac{8}{99} & \frac{4}{99} \\ \frac{19}{99} & \frac{38}{99} & \frac{2}{11} & \frac{16}{99} & \frac{8}{99} \\ \frac{1}{11} & \frac{2}{11} & \frac{5}{11} & \frac{2}{11} & \frac{1}{11} \\ \frac{8}{99} & \frac{16}{99} & \frac{2}{11} & \frac{38}{99} & \frac{19}{99} \\ \frac{4}{99} & \frac{8}{99} & \frac{1}{11} & \frac{19}{99} & \frac{59}{99} \end{pmatrix},$$

$$s = \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}, \qquad (L+I)^{-1}s = \begin{pmatrix} \frac{59}{99} & \frac{19}{99} & \frac{1}{11} & \frac{8}{99} & \frac{4}{99} \\ \frac{19}{99} & \frac{38}{99} & \frac{2}{11} & \frac{16}{99} & \frac{8}{99} \\ \frac{1}{11} & \frac{2}{11} & \frac{5}{11} & \frac{2}{11} & \frac{1}{11} \\ \frac{8}{99} & \frac{16}{99} & \frac{2}{11} & \frac{38}{99} & \frac{19}{99} \\ \frac{4}{99} & \frac{8}{99} & \frac{1}{11} & \frac{19}{99} & \frac{59}{99} \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{-53}{99} \\ \frac{-7}{99} \\ \frac{-5}{11} \\ \frac{-29}{99} \\ \frac{35}{99} \end{pmatrix}$$

$$\pi(z) = \frac{||z||_2^2}{n} = \frac{\sqrt{(\frac{-53}{99})^2 + (\frac{-7}{99})^2 + (\frac{-5}{11})^2 + (\frac{-29}{99})^2 + (\frac{35}{99})^2}^2}{5} = 0.14180185695 \tag{3.6}$$

We can see an increase of the polarization index after adding this particular edge. This example was discovered after brute-forcing different graph topologies with different combinations of opinion values.

Lemma 3.1. The polarization index does not necessarily decrease after an edge addition between opposing views.

3.6 Polarization in a complete graph

Given a polarized graph G we will compute the polarization index $\pi(z)$ before and after converting the graph G to a full graph.

Table 3.1: Polarization Before and after converting to a full graph

Dataset	Number of Nodes	Number of edges	Average Degree	$\pi(z)$
Karate Before	34	78	4.5882	0.35857
Karate After	34	561	33	0.00081
Books Before	105	441	8.4000	0.44046
Books After	105	5460	104.0000	0.00453
Blogs Before	1490	16718	22.4403	0.27909
Blogs After	1490	1109308	1489.0040	0.00030

We can see the results from the karate, books and blogs datasets at table 3.1 The results leads us to the following lemma.

Lemma 3.2. The polarization index does not drop to zero in a fully connected graph.

Chapter 4

Algorithms

4.1 Intuition

To solve this problem we have to evaluate all possible edge combinations. Even for greedy heuristics we need to limit the edge candidates. The algorithm considers nodes with a high expressed value. According to our model the smallest decrease is happening when we connect a value near zero and a relatively high value.

We will now see why this statement holds by examining how the expressed opinion changes with an addition in the Friedkin and Johnsen model. Consider an arbitrary example with two nodes inside a network. Node a has $z_a = -0.02$ and node b has $z_b = 0.5$. Also for this example we assume that $w_{ii} = w_{ij} = w_{ji} = 1$.

If we connect these two nodes with an edge and re calculate the expressed opinions both of the z_i denominators will be increased by one. This emerges from the fact that both nodes will have an additional neighbour and that all weights equal with one. The numerator of the one node a will be increased by a lot and the numerator of the node b will be decreased by a small value.

The new z_a will not change a lot because the big addition in the numerator will approach the +1 addition of the denominator. On the other hand the new z_b will see a big change as the numerator had a small decrease thus creating a big decrease overall for this node. We can clearly see that only one of the two nodes will amount to a big decrease.

Now consider a second example of two nodes node c has $z_c = -0.8$ and node d has $z_d = 0.9$. After the addition node d will see a big decrease because we add two conflicting values that almost neutralise each other on the numerator but the addition of the +1 on the denominator stands still. On the other hand node c will also see a big decrease for the same reason. With this type of connection both of the nodes have a significant decrease.

4.2 Experiment by removing edges

Bellow we examine the removal of edges from a social graph and their result in polarization. We also use the edge betweenness centrality. The edge betweenness centrality is defined as the number of the shortest paths that go through an edge in a graph or network.(add cite Girvan and Newman 2002).

In the tables following Sign and Addition refer to the multiplication and the addition of the opinions of the nodes that are attached to the specific edge examined.

We can clearly see that there is not a direct association between the edge betweenness centrality and the decrease in polarization. For example in the karate dataset edge (20,34) has almost the same betweenness centrality with edge (32,34). The first is among the edges that their removal contributes in one of the biggest polarization decreases while the other is among the ones with the smallest. A second thing that we can see in all three datasets is that the biggest decrease is coming from the removal of edges that connect opposing opinions.

Table 4.1: Edges with the 5 largest decrease (Karate Dataset)

Edge	Betweeness Centrality	Polarization Decrease	Sign	Addition
(1, 32)	0.12725	0.04669	-	0
(20, 34)	0.059384	0.03470	-	0
(14, 34)	0.06782	0.02924	-	0
(2, 31)	0.03228	0.02505	-	0
(3, 28)	0.04119	0.02068	-	0

Table 4.2: Edges with the 5 smallest decrease (Karate Dataset)

Edge	Betweeness Centrality	Polarization Decrease	Sign	Addition
(6, 7)	0.00297	0.0	+	-2
(5, 11)	0.00297	$5.55111 * 10^{-17}$	+	-2
(4, 8)	0.00336	$3.04869 * 10^{-7}$	+	-2
(1, 4)	0.02049	$1.38023 * 10^{-5}$	+	-2
(32, 34)	0.05339	$1.61826 * 10^{-5}$	-	+2

Table 4.3: Edges with the 5 largest decrease (Blogs Dataset)

Edge	Betweenness Centrality	Polarization Decrease	Sign	Addition
(213, 793)	0.00219	0.00091	-	0
(600, 1183)	0.00439	0.00074	-	0
(523, 1375)	0.00110	0.00070	-	0
(325, 1159)	0.00110	0.00069	-	0
(632, 1000)	0.00110	0.00069	-	0

Table 4.4: Edges with the 5 smallest decrease (Blogs Dataset)

Edge	Betweenness Centrality	Polarization Decrease	Sign	Addition
(384, 385)	$9.01465 * 10^{-7}$	0.0	+	-2
(301, 644)	$2.11840 * 10^{-5}$	$4.64017 * 10^{-13}$	+	-2
(775, 1369)	$4.23796 * 10^{-5}$	$8.28614 * 10^{-13}$	+	-2
(233, 736)	$1.37432 * 10^{-5}$	$1.26054 * 10^{-12}$	+	-2
(1330, 1410)	$4.08651 * 10^{-5}$	$2.12324 * 10^{-12}$	+	+2

Table 4.5: Edges with the 5 largest decrease (Books Dataset)

Edge	Betweeness Centrality	Polarization Decrease	Sign	Addition
(53, 76)	0.06290	0.01985	-	0
(46, 102)	0.04914	0.01541	+	-2
(19, 77)	0.04367	0.01458	+	+2
(9, 51)	0.02812	0.01000	-	0
(49, 72)	0.06809	0.00952	-	0

Table 4.6: Edges with the 5 smallest decrease (Books Dataset)

Edge	Betweeness Centrality	Polarization Decrease	Sign	Addition
(13, 40)	0.00305	$3.89807 * 10^{-9}$	+	+2
(35, 37)	0.00078	$8.65544 * 10^{-9}$	+	+2
(88, 89)	0.00036	$1.00835 * 10^{-8}$	+	-2
(65, 69)	0.00072	$1.51261 * 10^{-8}$	+	-2
(35, 36)	0.00146	$2.92727 * 10^{-8}$	+	+2

4.3 Heuristics

In this section we consider some heuristic algorithms for minimising $\pi(z)$. All the heuristics use the intuition that connecting the most extreme opinions of each community draw both of them into neutrality. The algorithms use two lists. One for each viewpoint sorted according to their opinion value. The naive Algorithm 4.1 stops when the first k edges are found.

Algorithm 4.1 Naive minimization of $\pi(z)$

INPUTS: Graph G; k number of edges to add; X, Y, the set of vertices of each viewpoint ϵ [-1,0] and [0,1] respectively.

OUTPUT: List of k edges that minimize the polarization index $\pi(z)$

$EdgesToAdd \leftarrow EmptyList$;
for $i = 1 : n$ do
    $Vertex\ u = X[i]$
    for $j = 1 : n$ do
        $Vertex\ v = Y[i]$
        if edge(u, v) does not exist in the graph then
            Compute $\pi(z)$, the decrease if the edge (u, v) is added;
            Append edge (u, v) to EdgesToAdd;
        end if
    end for
end for
$Sorted \leftarrow sort(EdgesToAdd)$ by the decrease of $\pi(z)$ by decreasing order;
Return top k from $Sorted$

An improvement can be done by using the $\pi(z)$ as a condition to continue. A list of $\pi(z)$ will be created with a traversal. This list will contain the polarization of each node of one of the viewpoints after adding an edge between him and the most extreme of the other. This list can be used to break the second loop and reduce the number of computations and can be seen in 4.2. Even though the worst complexity is the same with 4.1 it will not reach them in an average case.

Algorithm 4.2 Greedy minimization of $\pi(z)$

INPUTS: Graph G; k number of edges to add; Number of nodes n; X, Y, the sorted set of vertices according to polarization index of each viewpoint ϵ [-1,0] and [0,1] respectively.

OUTPUT: List of k edges that minimize the polarization index $\pi(z)$

$EdgesToAdd \leftarrow EmptyList$;
$PolarizationFirstPass \leftarrow EmptyList$;
$Vertex\ u = X[0]$
for $i = 1 : n$ **do**
    $Vertex\ v = Y[i]$
    Add edge (u, v) and compute $\pi(z)$;
    Append the result to $PolarizationFirstPass$;
    Remove the edge (u, v) from the graph to stay as previous;
end for
for $i = 1 : n$ **do**
    $Vertex\ u = X[i]$
    for $j = 1 : n$ **do**
        $Vertex\ v = Y[i]$
        Compute $\pi(z)$, the decrease if the edge (u, v) is added;
        if $i \neq k$ **then**
            if $\pi(z) > PolarizationFirstPass[i + 1]$ **then**
                break;
            end if
        end if
        Append edge (u, v) to EdgesToAdd;
        if $SizeOf(EdgesToAdd) = k$ **then**
            return EdgesToAdd;
        end if
    end for
end for

We can improve further Algorithm 4.2 by removing the need for the first pass. Instead the $\pi(z)$ a function of distance can be used as condition to continue. The same holds for the complexity with 4.2.

Algorithm 4.3 Distance minimization of $\pi(z)$

INPUTS: Graph G; k number of edges to add; Number of nodes n; X, Y, the sorted set of vertices according to polarization index of each viewpoint ϵ [-1,0] and [0,1] respectively.

OUTPUT: List of k edges that minimize the polarization index $\pi(z)$

```
EdgesToAdd ← EmptyList;
for i = 1 : n do
    Vertex u = X[i]
    for j = 1 : n do
        Vertex v = Y[i]
        if i ≠ k then
            if Abs(v) + Abs(u) ⩽ Abs(X[i + 1]) + Abs(Y[0]) then
                continue;
            end if
        end if
        Compute π(z), the decrease if the edge (u, v) is added;
        Append edge (u, v) to EdgesToAdd;
        if SizeOf(EdgesToAdd) = k then
            return EdgesToAdd;
        end if
    end for
end for
```

Bellow we can see the results of these algorithms in two datasets.

Table 4.7: Heuristics algorithm comparison

Algorithm	Dataset	Before	After	time
Naive	Karate	0.35857	0.23116	0.93457
Merge	Karate	0.35857	0.15205	0.02955
Distance	Karate	0.35857	0.13423	0.00262
Naive	Books	0.44046	0.35138	30.53548
Merge	Books	0.44046	0.34692	0.29411
Distance	Books	0.44046	0.34719	0.01033

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