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M01 Algorithmics

Solutions Exercise Sheet 01

Dr. Sofia Botti and Andrea Angino
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Exercises discussed in class

The exercises will be discussed in class on Tuesday, February 10, 2026 at 7:00pm in the Moodle course.

Exercise 1 *In each of the following, prove whether f is bijective, injective, surjective, or none:*

i. $f : \mathbb{R} \longrightarrow \mathbb{R}$ *defined by* $f(x) = x^2$
ii. $f : \mathbb{N} \longrightarrow \mathbb{R}$ *defined by* $f(x) = x^2$
iii. $f : \mathbb{R} \longrightarrow \mathbb{R}$ *defined by* $f(x) = x^3$

Homework

The solutions of the homework are due on February 17, 2026 at 7:00pm on the Moodle page `https://moodle.fernuni.ch/course/view.php?id=XXXX`.

Homework 1 (15 points) *Recall the definition of the absolute value of $x \in \mathbb{R}$, i.e.*

$$|x| = \begin{cases} x, & \text{if } x \geq 0, \\ -x, & \text{if } x < 0. \end{cases}$$

i. *(5 points) Let $a \in \mathbb{R}$. Show that $\max\{a, 0\} = \frac{1}{2}(a + |a|)$ and $\min\{a, 0\} = \frac{1}{2}(a - |a|)$.*
   Hint: For both equalities, consider the two cases, i.e. $a \geq 0$ and $a < 0$.
ii. *(5 points) Let $a, x \in \mathbb{R}$, $a \geq 0$. Prove the following:*
$$|x| \leq a \iff -a \leq x \leq a.$$
   Hint: Remember that in proving a statement $A \iff B$, you need to prove both $A \implies B$ and $B \implies A$.
iii. *(5 points) Prove the generalized triangle inequality: Let $x, y \in \mathbb{R}$. Then, we have:*
$$|x - y| \geq |x| - |y| \text{ and } |x + y| \geq |x| - |y|.$$
   Hint: You are allowed to use the triangle inequality in your proof.

Homework 2 (15 points) *A function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is called strictly decreasing if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$.*

i. *(10 points) Show that if f is strictly decreasing, then f is injective.*
ii. *(5 points) In Python, plot a graph of a strictly decreasing function of your choosing. Remember to add proper labels to your axes and a title to your plot. Use* `matplotlib` *for plotting.*

Homework 3 (20 points)
i. *(10 points) In each of the following, prove whether f is bijective, injective, surjective, or none (see Hints: Injections/surjections - video).*

a) $f : \mathbb{R} \longrightarrow \mathbb{R}$*, defined by* $f(x) = (x+2)^3$

b) $f : \mathbb{R} \longrightarrow \mathbb{R}$*, defined by* $f(x) = \frac{1}{4^x}$

c) $f : \mathbb{R} \longrightarrow [-1, 1]$*, defined by* $f(x) = \cos(x)$

d) $f : \mathbb{R} \longrightarrow \mathbb{R}$*, defined by* $f(x) = \cos(x)$

ii. (10 points)

a) Suppose that X *and* Y *are finite and non-empty sets and that the function* $f : X \longrightarrow Y$ *is injective. Describe and prove which conclusion is possible regarding the number of elements in* X *in relation to the number of elements in* Y*.*

b) Suppose that X *and* Y *are finite and non-empty sets and that the function* $f : X \longrightarrow Y$ *is surjective. Describe and prove which conclusion is possible regarding the number of elements in* X *in relation to the number of elements in* Y*.*