Statistics and Discrete Structures

Exercise sheet 2: Functions

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SS26

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Exercise 1 (Injectivity, surjectivity, and bijectivity | 5 points)
Consider functions f, g, and h, and analyse their injectivity, surjectivity, and bijectivity.

$$\begin{aligned} f:&\quad [0,1] \to \mathbb{R}, && x \mapsto x^2;\\ g:&\quad [0,1] \to [0,1], && x \mapsto x^2;\\ h:&\ [-1,1] \to [0,1], && x \mapsto x^2. \end{aligned}$$

Exercise 2 (Cardinality | 5 points)
Let $A = (0,1)$ and $B = (0,1] \cup \{2\}$ be two subsets of $\mathbb{R}$. Prove or disprove $|A| = |B|$ without constructing any bijective function.

Exercise 3 (Left and right inverses | 5 points)
Consider the sets $A := \{2,3\}$ and $B := \{4,5,6\}$.

(a) *Compute the number of injective functions from A to B and from B to A.*
(b) *Compute the number of surjective functions from A to B and from B to A.*

Now, consider the functions

$$f := (\{(2,4),(3,6)\}, A, B) \quad \text{and} \quad g := (\{(4,3),(5,2),(6,3)\}, B, A).$$

Or, expressed differently,

$$\begin{aligned} &f : A \to B,\ n \mapsto 2n; \quad \text{and}\\ &g : B \to A,\ n \mapsto 2 + (n-5)^2. \end{aligned}$$

(c) *If there exists a left inverse of f, provide it. Analogously for g.*
(d) *If there exists a right inverse of f, provide it. Analogously for g.*
(e) *Is f a left or right inverse of g? Is g a left or right inverse of f?*

Exercise 4 (Image and preimage | 5 points)
Let $u, v : E \to F$ be functions, such that u is non-injective and v is non-surjective. Show that

(a) $\forall A \in \mathcal{P}(E) : A \subset u^{-1}(u(A))$,
(b) $\exists A \in \mathcal{P}(E) : u^{-1}(u(A)) \not\subset A$,
(c) $\exists B \in \mathcal{P}(F) : B \not\subset v(v^{-1}(B))$,
(d) $\forall B \in \mathcal{P}(F) : v(v^{-1}(B)) \subset B$.