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Statistics and Discrete Structures

Exercise sheet 1: Logic and sets

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SS26

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Note #1: Sketches (e.g., Venn diagrams) are useful for intuition, but not acceptable as formal proofs or counterexamples.

Note #2: Use notation from the lecture notes.

Exercise 1 (Operations with propositions | 5 points)
Let A, B, and C be propositions. Prove

$$\big(A \wedge B \implies C\big) \iff \big(A \implies (B \implies C)\big)$$

(a) ...using truth tables.
(b) ...using operational calculus.

Exercise 2 (Equality of sets | 5 points)
Let E be an arbitrary set and $A, B, C \in \mathcal{P}(E)$.

(a) Show that if $A \cap B = A \cup B$ then $A = B$.
(b) Show that if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$. Is one of the two conditions sufficient?

Exercise 3 (Symmetric difference | 5 points)
Let E be an arbitrary set, and $A, B, C \in \mathcal{P}(E)$ (i.e., A, B, and C are subsets of E). Let us define the symmetric difference Δ as

$$A \,\Delta\, B := A \backslash B \cup B \backslash A.$$

Consider the propositions

$$P_1 := \big(x \in A \,\Delta\, B \implies x \in (A \cup C) \,\Delta\, (B \cup C)\big),$$
$$P_2 := \big(x \in A \,\Delta\, B \implies x \in (A \cap C) \,\Delta\, (B \cap C)\big).$$

(a) What property must the sets A, B, and C satisfy in order for P_1 to hold?
(b) What property must the sets A, B, and C satisfy in order for P_2 to hold?
(c) Provide the set of elements $x \in E$ which do not satisfy at least one of the propositions P_1 and P_2.

Exercise 4 (Convexity | 5 points)
Convexity is a fundamental concept in mathematics and is widely used in areas such as calculus, functional analysis, or optimization. It appears that the first more rigorous definition of convexity was already given by Archimedes (Archimedes of Syracuse, ca 287 - 212 B.C.). In this exercise, we will study some simple properties of convex sets, restricting ourselves to sets in $\mathbb{R}^2$.

We say that $A \subset \mathbb{R}^2$ *is convex if*

$$(\star) \qquad \forall t \in [0,1] : \forall x, y \in A : tx + (1-t)y \in A,$$

i.e., the line segment connecting the elements x *and* y *is completely included in the set* A.

Let $\mathcal{C} \subset \mathcal{P}(\mathbb{R}^2)$ *denote the set of all convex subsets of* $\mathbb{R}^2$. *Prove or disprove the following statements.*

(a) $\forall A_1, A_2 \in \mathcal{P}(\mathbb{R}^2) : A_1, A_2 \in \mathcal{C} \Rightarrow A_1 \cap A_2 \in \mathcal{C}$.
Hint: *Sketch convex sets and consider their intersections. Express your observations using the definition* ($\star$).

(b) $\forall A_1, A_2 \in \mathcal{P}(\mathbb{R}^2) : A_1 \cap A_2 \in \mathcal{C} \Rightarrow A_1, A_2 \in \mathcal{C}$.

(c) $\forall A_1, A_2, \dots, A_n \in \mathcal{P}(\mathbb{R}^2) : A_1, A_2, \dots, A_n \in \mathcal{C} \Rightarrow \bigcap_{i=1}^{n} A_i \in \mathcal{C}$.
Hint: *Use the associativity of the intersection of sets.*