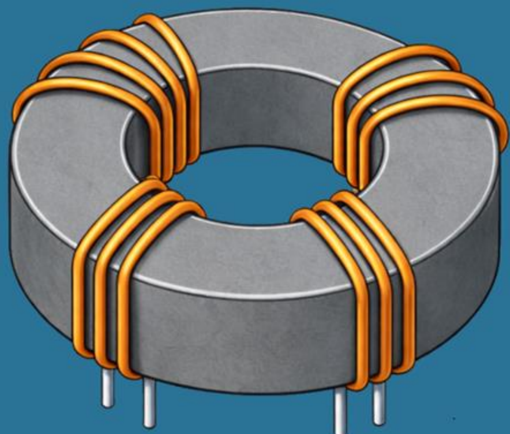
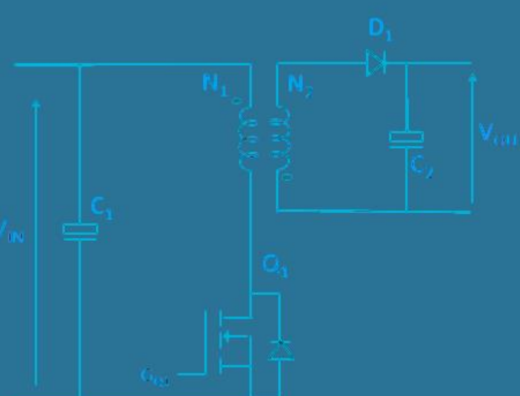
Electromagnetic Fundamentals and Design of High-Frequency Transformers for Switching Converters





Leonardo Chieco

Electromagnetic Fundamentals and Design of High-Frequency Transformers for Switching Converters

In this tutorial we will address the design of high-frequency transformers starting from the physical and electrical fundamentals that govern their operation. After defining what is meant by an HF transformer, we will analyze the fundamental phenomena underlying electromagnetic conversion: the relationship between voltage and flux, the role of magnetizing current, core saturation, magnetic losses, and the physical meaning of quantities such as B, H, and magnetizing inductance. On these theoretical foundations we will then move to practice, applying the developed concepts to the design of two widely used conversion topologies: the flyback converter, as an example of energy storage and transfer through a coupled inductor, and the push–pull converter, as a case of transformer use in the more classical sense.

Enjoy the reading!

The Author

Leonardo Chieco is an electronic engineer with over 20 years of experience in the design and development of automation control software (PC/PLC), electronic boards for industrial applications, firmware, robotics, and mechatronics.

LinkedIn: https://www.linkedin.com/in/leonardo-chieco-53550b129/

1. Basic Principles

A high-frequency transformer is a device in which electrical energy is transferred through a time-varying magnetic field. Unlike low-frequency transformers, where many phenomena can be neglected or “hidden” by the large iron mass and long variation times, in HF transformers every physical aspect clearly emerges: the nonlinearity of magnetic materials, core losses, parasitic currents, and the dynamics of the magnetic field become central to correct system operation. For this reason, it is essential to start from the fundamental concepts of magnetism, in particular the quantities B and H and their relationship with the current and voltage applied to the windings.

The quantity H, called the magnetizing field, is directly related to the current flowing in the windings. When a current flows through a coil, it generates a magnetic field that exists even if there is only air inside it. If a magnetic core is inserted, the field H is still determined by the current and the number of turns, while the material decides how to respond to that excitation. In simplified terms, increasing the current in the primary increases H, regardless of whether the core is ferrite, iron, or absent.

The response of the magnetic material to this excitation is described by the quantity B, the magnetic flux density. B represents the physical result of magnetization: how many magnetic flux lines pass through a given area of the core. In ferromagnetic materials, such as the ferrites used in HF transformers, B increases much more rapidly than in air because the material provides a low-reluctance path for the magnetic flux. However, this growth is not unlimited. As H increases, B initially grows almost linearly, then more and more slowly, until it reaches material saturation. In saturation the core can no longer “accept” additional flux, and any further increase in current produces only an increase in H, without a corresponding increase in B. In other words, the cause (H) increases, but not the effect (B).

To truly understand how HF transformers are designed, it is important to clarify how B and H are actually calculated and, above all, what those calculations represent from a physical point of view.

Let us start with the magnetizing field H.
H is the quantity most directly connected to the current flowing in the windings. In a real transformer, when a current flows in the primary, it generates a magnetic field that tends to “push” the flux through the core. From an electromagnetic point of view, H is proportional to the magnetomotive force applied along the magnetic circuit. For a closed core, such as an E-core or a toroid, H is calculated as:

$$H = N \cdot \frac{I}{l_m}$$

where N is the number of turns, I is the current flowing through them, and l_m is the mean length of the magnetic path in the core.

This formula has a very clear physical meaning: more turns or more current mean a greater magnetizing effort, while a longer magnetic path makes it harder to magnetize the core. H therefore depends exclusively on the geometry of the magnetic circuit and on the current, not on the material. Even if the core were replaced by air, for the same N, I, and l_m, the value of H would be the same.

The magnetic flux density B, on the other hand, is the result of the material’s magnetization. As a first approximation, in the linear region of the B–H characteristic, B is related to H by:

$$B = \mu \cdot H$$

where $\mu = \mu_0 \cdot \mu_r$ is the permeability of the material.
Recall that the permeability of free space is:

$$\mu_0 = 4 \cdot \pi \cdot 10^{-7} \frac{Henry}{m}$$

while μ_r depends on the material.

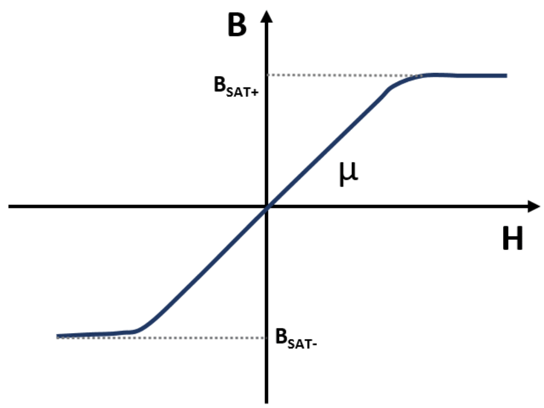


Figure 1: B–H characteristic

In ferromagnetic materials μ_r is much greater than 1, so even a relatively small H produces a high B. However, this relationship holds only as long as the material does not approach saturation. When the core enters the nonlinear region of the B–H curve, μ_r decreases and B grows less and less despite the increase in H. This is why HF transformers are never designed to operate near saturation.

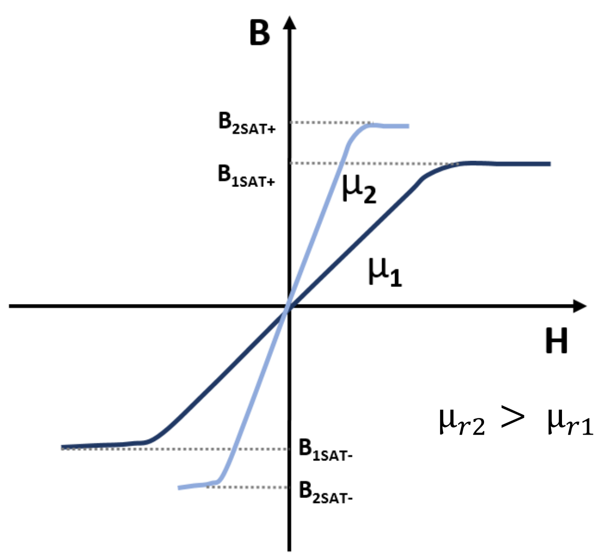


Figure 2: B(H) curves for different μr

In ferrites, the value of μ_r typically lies in the range 2000 – 4000 $\frac{Henry}{m}$.

In high-frequency transformer design, B is almost never calculated starting from H. What the designer actually specifies is the voltage applied to the primary and the time for which it is applied. For this reason, B is almost always calculated using Faraday’s law.

Applying Faraday’s law to a winding with N turns, the applied voltage is related to the time variation of the magnetic flux. If a constant voltage is applied for a given time interval, the variation in flux density can be written as:

$$\mathbf{\Delta B} = \frac{\boldsymbol{V \cdot t}}{\boldsymbol{N \cdot A_e}}$$

where V is the applied voltage, t is the application time, N is the number of turns, and A_e is the effective cross-sectional area of the core.

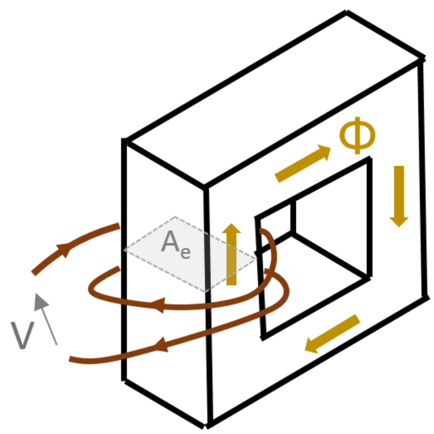


Figure 3: Application of Faraday’s law

This formula is one of the most important in HF transformer design because it clearly shows that the variation of B depends on the voltage–time product, not on the current. For the same voltage, increasing the application time increases B; for the same time, increasing the voltage increases B. Increasing the number of turns or the core area, instead, reduces the excursion of B.

For example, if a transformer is driven by a 0–V_{DC} square wave, the voltage is applied to the primary only for a fraction of the period. In this case, the time t coincides with the conduction time, which is linked to frequency and duty cycle. It follows that, at higher frequency, the same voltage produces a smaller variation of B. This is the physical reason why increasing frequency allows the core size to be reduced.

Once B has been calculated using Faraday's law, it is possible to indirectly obtain H. If the core operates in the linear region, the relation B = μ·H can be used.

Knowing H, the current required to generate it can be derived using Ampère's law.

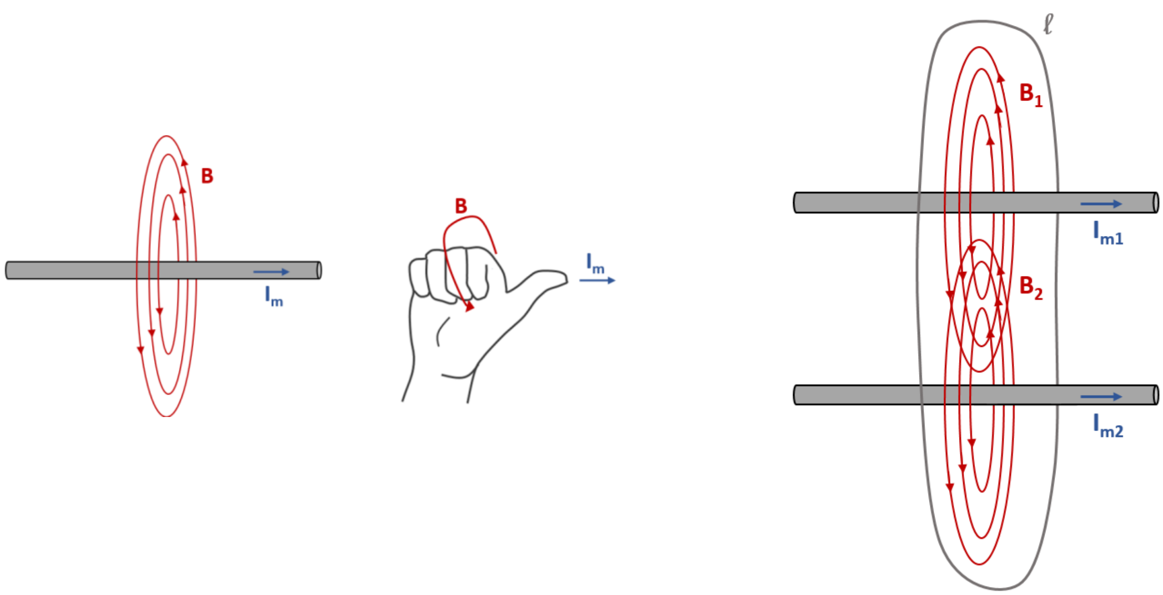


Figure 4: Ampère's Law

Ampère's law states that the line integral (circulation) of the magnetic field along a closed path is equal to the sum of the enclosed electric currents multiplied by the permeability of free space μ_0.

$$\oint \vec{B}\, d\vec{l} = \mu_0\, I_m \quad \rightarrow \quad \oint \vec{B}\, d\vec{l} = \mu_0 \sum I_{m_i} \ \textit{in the case of multiple currents}$$

In ferromagnetic materials, Ampère's law is modified to include material magnetization. Instead of using only the permeability of free space (μ_0), the magnetic permeability of the material ($\mu = \mu_0 \mu_r$) is used, which is much higher. The integral law becomes:

$$\oint \vec{B}\, d\vec{l} = \mu\, I_{m,\text{Enclosed}}$$

If all enclosed currents are equal, as in the case of a winding (solenoid), and recalling that $\mathrm{B} = \mu\,\mathrm{H}$, we obtain:

$$\oint \vec{H}\, d\vec{l} = N\, I_m = \mathcal{F}$$

where N is the number of turns. The product NI is called the magnetomotive force $\mathcal{F}$.

Now consider a transformer or inductor with N turns, magnetizing current I_m and a closed magnetic path in the core. Assume that H is approximately uniform and that the flux follows the mean path of the core.

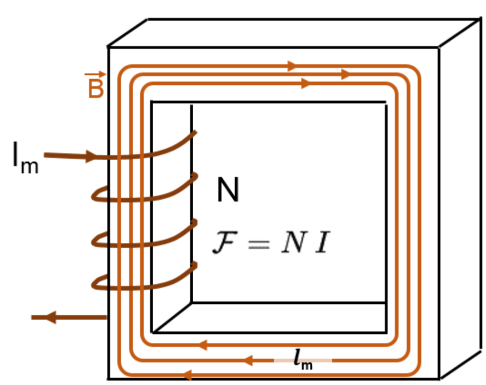


Figure 5: Flux generated by a current I_m and confined in a ferromagnetic material

Then the integral in Ampère's law becomes:

$$\oint \vec{H}\, d\vec{l} = H \oint dl = H\, l_m$$

Where l_m is the mean magnetic path length of the magnetic circuit.
Putting everything together, we obtain:

$$H\, l_m = N I_m \;\; \rightarrow \;\; \boldsymbol{I_m} = \frac{\boldsymbol{H\, l_m}}{\boldsymbol{N}}$$

This is the current required to sustain the flux in the core and it is present even at no load, that is, with no load on the secondary. As long as B remains within safe limits (linear region), the magnetizing current is relatively small. When B approaches saturation, however, the effective permeability of the material decreases, H must increase greatly to obtain a small increase in B, and the magnetizing current rises rapidly. This behavior is often the first warning sign of a poorly sized transformer.

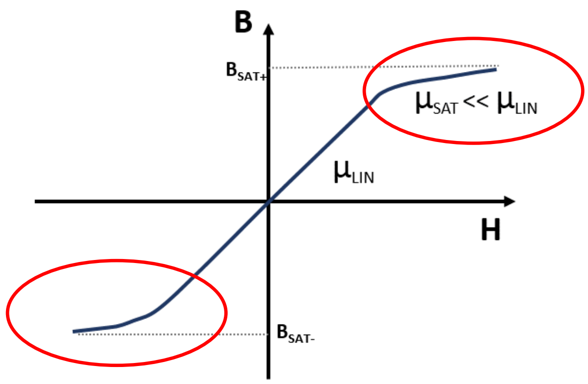


Figura 6: Behavior of μ in the saturation region

Magnetic flux is denoted by the letter Φ (phi) nd represents the "total number" of magnetic field lines passing through a given surface. It is measured in Webers (Wb) and is a global quantity related to the entire core cross-section. Magnetic flux density, denoted by B, is instead the magnetic flux per unit area, measured in $\frac{weber}{m^2} = Tesla$ and is a local quantity describing how "dense" the magnetic field is at a point in the material.

The relationship between the two quantities is simple and fundamental:

$$\Phi = \mathrm{B} \cdot \mathrm{A}$$

where A is the area crossed by the flux.
If the flux density varies by an amount ΔB, then the flux varies by an amount ΔΦ, and the two variations are related by:

$$\Delta\Phi = \Delta\mathrm{B} \cdot \mathrm{A}$$

if the area crossed by the flux can be considered constant, as is the case for most real cores.

Faraday's law, in its most general form, relates the induced voltage to the variation of magnetic flux ΔΦ. For a winding with N turns, it states that the voltage (electromotive force) is proportional to the time derivative of the linked flux:

$$\mathrm{v(t)} = \mathrm{N} \cdot \frac{d\ \Phi(\mathrm{t})}{dt}$$

In design calculations, B is often introduced because the flux Φ is expressed as the product of flux density (B) and core area (A_e).
Substituting $\Phi = \mathrm{B}\ \mathrm{A}_e$ into Faraday's law yields:

$$\mathrm{v(t)} = \mathrm{N} \cdot \mathrm{A}_e \cdot \frac{d\ \mathrm{B(t)}}{dt}$$

This clearly shows that the voltage controls the time variation of B, but only because it controls the variation of the flux Φ through the core area.

Integrating this relationship over time, assuming a constant voltage applied for a time interval t, we obtain:

$$\Delta\Phi = \frac{1}{N} \cdot \int v(t)\ dt \quad \rightarrow \quad \boldsymbol{\Delta\mathrm{B}} = \frac{\mathbf{1}}{\boldsymbol{N} \cdot \mathbf{A}_{\boldsymbol{e}}} \cdot \int \boldsymbol{v}(\boldsymbol{t})\ \boldsymbol{dt}$$

This form highlights the key concept of transformer design: the voltage–time product (volt-seconds) determines the flux variation and therefore the variation of flux density in the core. Referring to ΔB is simply a practical way to normalize the flux with respect to the core area and directly compare it with the material limits.

From a physical point of view, therefore, the process is as follows: the voltage applied to the primary imposes a variation of magnetic flux in the core; this flux variation, divided by the core area, appears as a variation in flux density B. When B reaches the maximum value allowed by the material, the core enters saturation, regardless of how much current is flowing at that moment.

Clarifying this distinction is essential because it avoids a very common mistake: thinking that the flux or its variation is “inside” a turn or a current. In reality, flux is a quantity of the magnetic field in the core, while B is the most convenient way to describe it locally and compare it with material characteristics. In HF transformers, design is always carried out in terms of allowable ΔB, but it must be remembered that what the voltage physically imposes is $\Delta \Phi$, and that ΔB is only its normalized form with respect to core geometry.

We have seen that the relationship between B and H is represented by the B(H) characteristic, which encompasses much of the physical behavior of the core. We must note, however, that over a complete magnetization and demagnetization cycle, B does not follow the same path in the forward and return directions, giving rise to the hysteresis loop, as shown in the following figure.

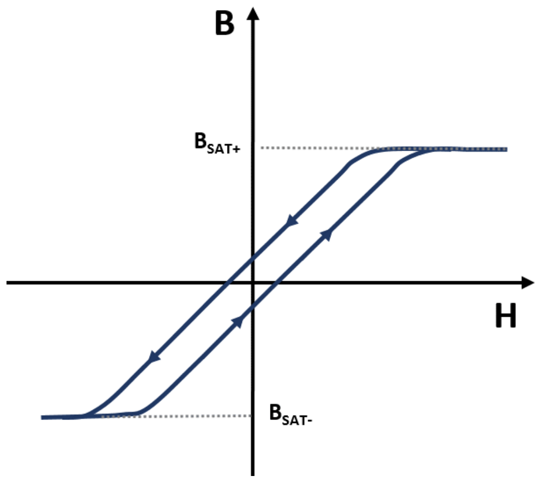


Figure 7: B hysteresis loop

The area of this loop represents energy dissipated as heat in the material during each cycle. In high-frequency transformers, where this cycle is repeated tens or hundreds of thousands of times per second, hysteresis losses quickly become dominant if the material is not chosen correctly.

The Transformer

We can now describe the operation of the transformer starting from these basic physical concepts and following a logical path that naturally leads to the fundamental relationship between voltages and number of turns.

Let us imagine an ideal transformer, consisting of two electrically isolated windings that are magnetically coupled through a core. When a time-varying voltage is applied to the primary, nothing happens immediately at the secondary from an electrical point of view, since there is no direct connection. What does appear immediately, however, is an electric field along the primary wire, which sets the charges in motion and generates a current. This current, even if initially very small, produces a magnetizing field H in the core. In turn, this magnetizing field generates a magnetic flux density B that is distributed in the magnetic circuit, following the minimum-reluctance path provided by the core.

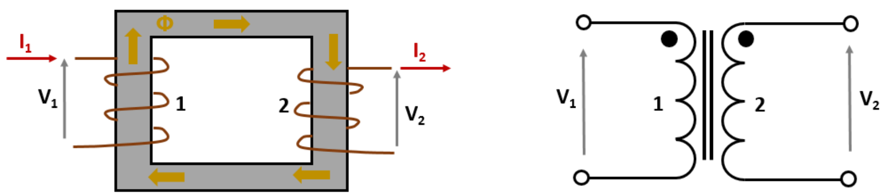


Figure 8: Ideal Transformer

The presence of the core causes the magnetic flux to be largely confined and guided within the material. From a physical point of view, the magnetic flux Φ that arises in the core is common to all windings that link it. This is the key point: the primary and secondary "see" the same magnetic flux, because they both wind around the same magnetic circuit. It does not matter what load is connected to the secondary or what current flows through it; as long as the core is not saturated and the flux remains well confined, the linked flux is the same for all windings.

Let us now return to Faraday's law. For each winding, the induced voltage is proportional to the number of turns and to the rate of change of the linked magnetic flux. For the primary we can state that the applied voltage is equal to the voltage induced by the varying flux, while for the secondary the terminal voltage is entirely induced by that same flux. Conceptually, for the primary the applied voltage serves to create a variation of flux in the core, whereas for the secondary the voltage arises as a consequence of that flux variation.

For the secondary we must remember that the induced voltage always has a polarity such as to oppose the variation of flux that generated it. Therefore:

$$\mathrm{v}_{secondary}(\mathrm{t}) = -\,\mathrm{N}\ \cdot \frac{d\ \Phi(\mathrm{t})}{dt}$$

That is, if the flux increases in one direction, the induced voltage generates a current that tends to create an opposing flux.

Writing Faraday's law explicitly for the two windings, we find that the primary voltage is proportional to the number of primary turns and to the time derivative of the flux, while the secondary voltage is proportional to the number of secondary turns and to the same identical time derivative of the flux. From a mathematical point of view, this means that both voltages contain the same term related to the time variation of the flux, but multiplied by a different number of turns.

At this point the step is natural. Since the magnetic flux is the same and varies in the same way for both windings, the ratio between the two voltages does not depend on the flux itself, but only on the ratio between the numbers of turns. In mathematical terms:

$$v_1(t) = N_1 \frac{d\,\Phi(t)}{dt}$$

$$v_2(t) = N_2 \frac{d\,\Phi(t)}{dt}$$

Dividing the expression for the primary voltage by that of the secondary, the term related to the flux variation cancels out, leaving an extremely simple relationship: the ratio of the voltages is equal to the ratio of the turns.

$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2} \quad \rightarrow \quad \boldsymbol{\frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2}}$$

In other words, if one winding has twice as many turns as the other, the voltage appearing at its terminals will be twice as high, for the same magnetic flux in the core. This relationship does not depend on the waveform, the frequency, or the core material, as long as the transformer operates in the linear regime and the flux is not distorted by saturation. It is a direct consequence of Faraday's law and of the physical fact that all windings link the same magnetic flux.

It is important to emphasize that this derivation makes no reference whatsoever to load currents. The voltage ratio is determined exclusively by the number of turns and by the flux variation.

In the previous figure we notice the presence of a small dot near the symbol of the inductor or winding. This indicates the dot convention and is used when two or more inductors are magnetically coupled, as in transformers. The dot indicates the instantaneous polarity of the induced voltages: if the current enters the dotted terminal of one coil, then the voltage induced on the other coil will be positive at its dotted terminal. In practice, the dotted terminals have the same magnetic phase.

Let us now complete the picture by introducing the reasoning about currents, maintaining the same physical approach and showing how current behavior is a direct consequence of magnetic flux and energy conservation.

Once again, let us start from the ideal transformer, with a linear core, perfect coupling, and no losses. We have already seen that the voltage applied to the primary imposes a variation of magnetic flux in the core and that this flux is common to all windings. As long as the secondary is open, only the magnetizing current flows in the primary, that is, the current required to generate the magnetic field that sustains the flux. This current is determined by the magnetizing inductance of the primary and is generally relatively small. Its only role is to create and maintain the magnetic flux required by the applied voltage.

The magnetizing inductance represents the ability of a winding to draw current in order to create only the magnetic flux in the core, without transferring power to the load; in other words, it is the inductance "seen" by the winding when it serves only to magnetize the magnetic circuit. The magnetizing inductance depends on:

- Geometry of the magnetic circuit
- Number of turns
- Core permeability
- Presence of an air gap

The current drawn by the magnetizing inductance L_m is the magnetizing current I_m (which serves only to create the flux). We will examine this concept in more detail later.

The situation changes when a load is connected to the secondary. As soon as a voltage appears at the secondary terminals, a current begins to flow in the load. This secondary current is not an isolated phenomenon: it in turn produces a magnetizing field. From a physical point of view, the current in the secondary tends to generate its own magnetic flux in the core. The direction of this flux, according to Lenz's law, is such as to oppose the change in flux that generated it. In other words, the secondary current attempts to reduce the magnetic flux in the core.

This point is crucial. If the flux in the core were to decrease, the induced voltage in both the primary and the secondary would immediately change, violating the balance imposed by the supply voltage. But this does not happen. The magnetic flux in the core remains practically unchanged, because it is the voltage applied to the primary that determines its time behavior. Consequently, the system reacts automatically: an additional current appears in the primary, in addition to the magnetizing current, whose purpose is to compensate for the magnetizing effect of the secondary current.

From the point of view of magnetomotive forces, this means that the magnetomotive force generated by the primary must balance that generated by the secondary, so that the total flux in the core remains the one imposed by the applied voltage. Under ideal conditions, the sum of the magnetomotive forces along the magnetic circuit must be equal to that required to sustain the flux. Since the magnetizing current is small, the main balance occurs between the "load" currents of the two windings.

This leads to a very simple but powerful relationship. If we denote by N_1 and I_1 the number of turns and current of the primary, and by N_2 and I_2 those of the secondary, magnetic equilibrium requires that the products $N_1 \cdot I_1$ and $N_2 \cdot I_2$ be equal in magnitude and opposite in direction. In other words, the current that appears in the primary due to the load is such as to cancel the magnetizing effect of the secondary current. It follows directly that the primary current is proportional to the secondary current and inversely proportional to the number of turns.

This relationship can also be interpreted from an energy point of view. In an ideal transformer there are no losses, so the power entering the primary must be equal to the power leaving the secondary. Since we have already shown that the voltage ratio is equal to the turns ratio, in order for power to be conserved the current ratio must be the inverse of the voltage ratio, and therefore the inverse of the turns ratio. Current behavior is therefore not an additional assumption, but a consequence of energy conservation and magnetic field equilibrium.

Mathematically, we have seen that the magnetizing field in the core is given by Ampère's law:

$$\oint \vec{H} \cdot \vec{dl} = \sum N_k \, I_k$$

That is, the circulation of the field $\vec{H}$ along a closed path is equal to the algebraic sum of the conduction currents linked with that path.

For a closed and uniform magnetic circuit (our transformer):

$$H \, l_m = N_1 \, I_1 - \, N_2 \, I_2$$

where the minus sign accounts for the opposite current directions according to Lenz's convention.

l_m denotes the mean length of the magnetic circuit, that is, the average distance traveled by the magnetic flux inside the core (measured in meters). Physically, it is the magnetic equivalent of the length of an electrical circuit:

- in an electrical circuit, current flows through a wire of length l;
- in a magnetic circuit, flux flows along a closed path of length l_m.

Let us write the primary current as the sum of two contributions:

$$I_1 = I_m + I_{1L}$$

where:

- I_m is the magnetizing current (the one present with no load on the secondary);
- I_{1L} is the current due to the load (reflected) on the secondary.

Substituting into Ampère's law:

$$H\, l_m = N_1(I_m + I_{1L}) - N_2 I_2$$

But, by definition of the magnetizing current:

$$H\, l_m = N_1\, I_m$$

Subtracting the last two equations we obtain:

$$0 = N_1\, I_{1L} - N_2 I_2 \quad \rightarrow \quad \boldsymbol{\frac{I_2}{I_{1L}} = \frac{N_1}{N_2}}$$

This is the fundamental relationship between currents in the ideal transformer.
Note that if we multiply, term by term, the voltage and current relationships, we obtain:

$$V_1\, I_{1L} = V_2 I_2 \;\rightarrow\; P_1 = P_2 \;\rightarrow\; \boldsymbol{P_{\mathrm{IN}} = P_{\mathrm{OUT}}}$$

This means that, under ideal conditions (no losses), power is conserved.

Technical Curiosity

Physically, the transformer behaves like an “impedance adapter.” A load connected to the secondary is reflected to the primary as an equivalent load whose value depends on the square of the turns ratio. If the secondary has fewer turns than the primary, the voltage is reduced and the current increased; consequently, from the primary side the load appears higher in voltage and lower in current. This mechanism occurs without any direct electrical contact, but exclusively through the shared magnetic field.

$$Z_1 = \frac{V_1}{I_{1L}} \qquad\qquad Z_2 = \frac{V_2}{I_2}$$

Therefore:

$$Z_1 = \frac{N_1}{N_2}\ \frac{V_2}{\frac{N_2}{N_1}\ I_2} = (\frac{N_1}{N_2})^2\ Z_2\ \rightarrow\ \frac{Z_1}{Z_2} = (\frac{N_1}{N_2})^2$$

We have seen that the voltage ratio between input and output is determined by the turns ratio between primary and secondary according to the law:

$$\frac{\mathrm{v}_1(\mathrm{t})}{\mathrm{v}_2(\mathrm{t})} = \frac{\mathrm{N}_1}{\mathrm{N}_2}$$

Let us now look at the methodology used to calculate the number of turns of the primary.

2. Flux and Primary Winding

The time evolution of the magnetic flux depends on the way the primary winding is driven, according to Faraday's law. The purpose of this chapter is to define a study methodology applied to different use cases.

Bipolar Signal with 50% Duty Cycle

Let us consider a transformer powered by a bridge converter. The voltage applied to the primary is a bipolar square wave with a 50% duty cycle, as shown in the following figure.

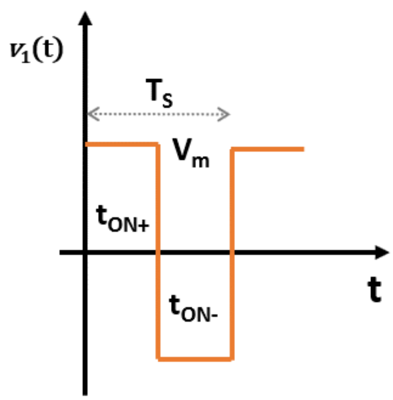


Figure 9: Bipolar PWM signal

Let us define:

- V_m: maximum primary supply voltage
- f_s: switching frequency
- $T_s = \frac{1}{f_s}$: period
- N_1: umber of primary turns
- A_e: effective core cross-sectional area

During each period, the voltage is:

$$v_1(t) = \begin{cases} +V_m \ for \ T_{ON+} = \dfrac{T_s}{2} \\ -V_m \text{ for } T_{ON-} = \dfrac{T_s}{2} \end{cases}$$

Since the duty cycle is 50%, we have: $T_{ON+} = T_{ON-} = \frac{T_s}{2}$.

Let us calculate the evolution of the magnetic flux and of B(t) using Faraday's law:

$$v_1(t) = N_1 \frac{d\Phi}{dt}$$

and since:

$$\Phi = B A_e$$

we obtain:

$$\frac{v_1(t)}{N_1} = \frac{d\Phi}{dt} = \frac{d(BA_e)}{dt} = A_e \frac{dB}{dt} \quad \rightarrow \quad \frac{d\mathrm{B}}{dt} = \frac{v_1(t)}{N_1 A_e}$$

Integrating to obtain B(t):

$$\boldsymbol{B(t) = B(0) + \frac{1}{N_1 A_e} \int v_1(t)\, dt}$$

We observe that over one period the integral of the voltage $v_1(t)$ is zero.

$$\int_0^{T_s} v_1(t)\, dt = 0$$

Therefore, there is no flux drift: after each period, B returns to its initial value B(0).

If the circuit does not impose B(0), then infinitely many valid solutions exist. However, the transformer core has no absolute memory or internal reference.

Thus, if $v(t)$ is antisymmetric, B(t) will also be antisymmetric and will naturally oscillate around zero, so we can assume B(0)=0.

Hence, in this case, B(t) varies in an antisymmetric manner. Let us examine its behavior.

During the positive pulse:

$$\frac{d\mathrm{B}}{dt} = \frac{v_1(t)}{N_1 A_e} = \frac{V_m}{N_1 A_e}$$

Integrating:

$$\Delta \mathrm{B}_+ = \frac{V_m\, T_{ON+}}{N_1 A_e}$$

This relation indicates that B(t) increases linearly during T_{ON+}.

During the negative pulse, in the following half-period:

$$\frac{d\mathrm{B}}{dt} = \frac{-V_m}{N_1 A_e}$$

Integrating:

$$\Delta \mathrm{B}_- = \frac{-V_m T_{ON-}}{N_1 A_e}$$

This relation indicates that B(t) decreases linearly during T_{ON-}.

We observe that: $|\Delta \mathrm{B}_-| = |\Delta \mathrm{B}_+|$.

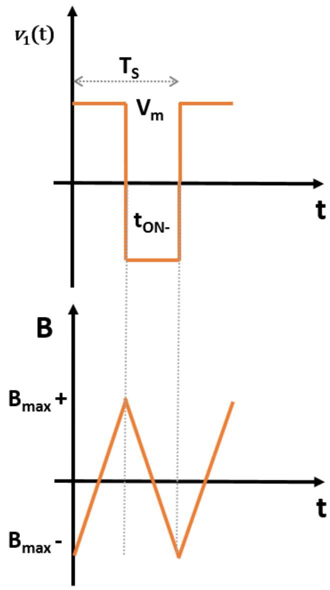


Figure 10: Evolution of B(t)

Since the peak-to-peak excursion of B is:

$$\Delta B_{pp} = \Delta B_{+} = -\Delta B_{-} = \frac{V_m\, T_{ON}}{N_1 A_e}$$

we obtain:

$$B_{MAX+} = -B_{MAX-} = \frac{V_m\, T_{ON}}{2\, N_1 A_e}$$

The maximum value of B (B_{MAX}) must remain below the saturation flux density (B_{SAT}), with a safety margin:

$$B_{MAX} \leq 0.6 \ldots 0.8\, B_{SAT} = B_{\text{Admissible}}$$

Imposing the condition:

$$\frac{V_m T_{ON}}{2\, N_1 A_e} \leq B_{\text{Admissible}}$$

we obtain:

$$N_1 \geq \frac{V_m T_{ON}}{2\, A_e\, B_{\text{Admissible}}}$$

Thus, we have computed the required number of primary turns.

From what we have seen so far, we have learned that:

- the voltage does not fix the value of B, but the slope of B(t);
- increasing the frequency (i.e., reducing T_s and T_{ON}) reduces the flux excursion;
- increasing the number of turns N_1 reduces the slope;
- saturation depends on applied volt-seconds, not on load current.

Note

What happens if the duty cycle is not 50%? Since the transformer, from a magnetic standpoint, behaves like an integrator of the voltage applied to the primary, a duty cycle different from 50% introduces a non-zero volt-second imbalance. Under these conditions, the variation of magnetic induction within each period does not cancel out but leaves a residual that accumulates over successive periods. The result is a progressive shift of the magnetic operating point: for duty cycles greater than 50%, the core drifts toward $+B_{max}$, while for duty cycles less than 50%, the drift occurs toward $-B_{max}$. This cycle-by-cycle accumulation increases the initial value B(0) of the following period and leads to core saturation.

Unipolar PWM Signal with 50% Duty Cycle

Let us assume that $v_1(t)$ is a PWM signal between V_m and zero with a 50% duty cycle.

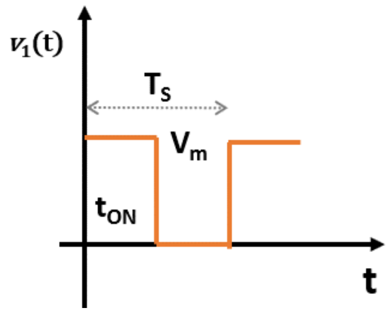


Figure 11: Unipolar PWM signal

$$v_1(t) = \begin{cases} V_m \, for \, T_{ON} = \dfrac{T_s}{2} \\ 0 \text{ for } \, T_s - T_{ON} = \dfrac{T_s}{2} \end{cases}$$

Starting again from T_{ON} phase:

$$\frac{d\mathrm{B}}{dt} = \frac{V_m}{N_1 A_e}$$

integrating:

$$\Delta\mathrm{B} = \frac{V_m \, T_{ON}}{N_1 A_e}$$

moreover:

$$B(t) = B(0) + \frac{1}{N_1 A_e} \int_0^{T_s} v_1(t) \, dt$$

Equation 1

$$\int_0^{T_s} v_1(t) \, dt = V_m T_{ON} = V_m \frac{1}{2 f_s}$$

From Equation 1:

$$\int_0^{T_s} v_1(t) \, dt = \Delta\mathrm{B} \, N_1 A_e$$

Combining last two equations:

$$\int_0^{T_s} v_1(t)\, dt = V_m \frac{1}{2f_s} = N_1 A_e\, \Delta B$$

which yields:

$$V_m \frac{1}{2f_s} = N_1 A_e\, \Delta B \;\; \rightarrow \;\; V_m = 2N_1 A_e\, \Delta B\, f_s$$

Since B varies between $+B_{MAX}$ and $-B_{MAX}$, we have

$$\Delta B = 2\, B_{MAX} \rightarrow B_{MAX} = \frac{\Delta B}{2}$$

so:

$$V_{\mathrm{m}} = N_1 A_e B_{\mathrm{max}} f_s$$

and therefore:

$$\boldsymbol{N_1} \geq \frac{\boldsymbol{V_{\mathrm{m}}}}{\boldsymbol{A_e B_{\mathrm{max}} f_s}}$$

Now comes the key conceptual point.

With this waveform, the flux increases until $\frac{T_s}{2}$, and then remains constant. It never returns back. This is because:

$$\int_0^{T_s} v_1(t)\, dt \neq 0$$

and this implies that B(t) drifts over time.

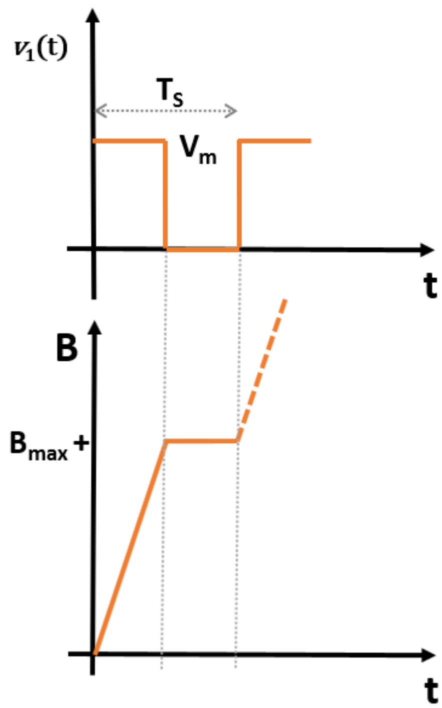


Figure 12: B(t) with unipolar waveform

If there is no flux reset mechanism, the flux increases cycle by cycle and the core saturates. We will later see how to address this problem.

Sinusoidal Signal with Angular Frequency ω

Starting again from Faraday's law:

$$v(t) = N_1 A_e \frac{dB(t)}{dt}$$

Assume the applied sinusoidal voltage is:

$$v(t) = V_m \sin(\omega t)\,, \qquad \omega = 2\pi f$$

Integrating:

$$B(t) = B(0) + \frac{1}{N_1 A_e} \int_0^t v(\tau) d\tau$$

$$B(t) = B(0) + \frac{1}{N_1 A_e} \int_0^t V_m \sin(\omega\tau)\, d\tau$$

Recalling $\int \sin(\,\omega t)\, dt = \,-\,\frac{1}{\omega}\cos(\omega t) + constant$

$$B(t) = B(0) - \frac{V_m}{N_1\, A_e\, \omega} \cos(\omega t) + \frac{V_m}{N_1\, A_e\, \omega} constant$$

We choose $B(0) = -\frac{V_m}{N_1 A_e \omega}\ constant$ such that the flux oscillates symmetrically around zero, since the voltage ($v(t)$) does as well:

$$B(t) = \frac{V_m}{N_1\, A_e\, \omega} \cos\,(\omega t)$$

A sinusoidal voltage produces a sinusoidal flux shifted by 90°.

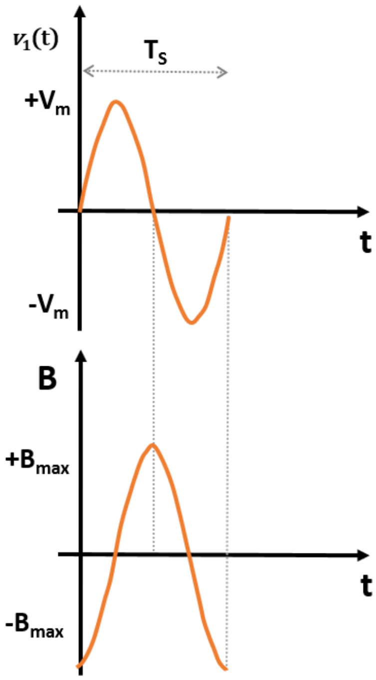


Figure 13: B(t) with sinusoidal waveform

The maximum flux $B_{\max}$ is:

$$B_{\max} = \frac{V_m}{N_1\, A_e\, \omega} \Rightarrow V_m = N_1\, A_e\, \omega\, B_{\max}$$

since $\omega = 2\pi f_s$:

$$V_m = 2\pi\, f_s\, N_1\, A_e\, B_{\max}$$

In RMS terms:

$$V_{RMS} = \frac{V_m}{\sqrt{2}} \approx 4.44\, f_s\, N_1\, A_e\, B_{\max}$$

The factor 4.44 is the classical coefficient for sinusoidal waveforms. This is why in traditional AC transformers we always find:

$$V_{RMS} = 4.44\, f_s\, N_1\, A_e\, B_{\max}$$

Imposing the non-saturation condition:

$$B_{\max} < B_{\text{Admissible}}$$

We can calculate N_1:

$$\boldsymbol{N_1 \geq \frac{V_m}{2\pi\, f_s\, A_e\, B_{\text{Admissible}}}}$$

This is the primary design formula.

3. Winding window and Area Product

The *winding window area* A_w is the area available in the core bobbin to accommodate conductors. This area cannot be completely filled with copper due to:

- insulation between turns and windings,
- enamel, tapes, safety margins,
- unavoidable geometric voids (due to circular conductors).

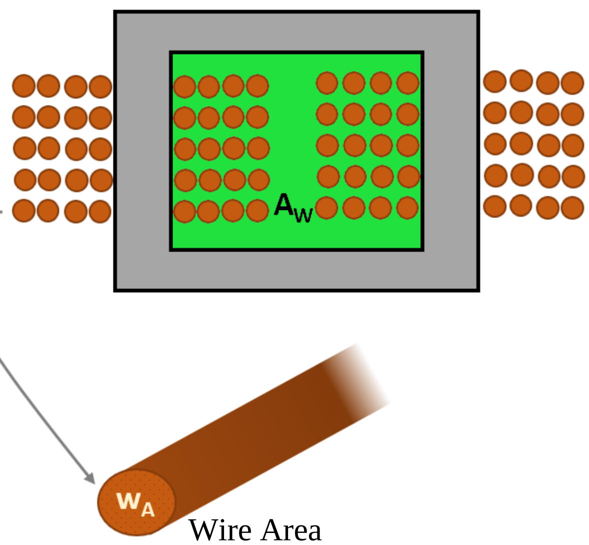


Figure 14: Winding window area concept

Typically, about 50% of A_w s physically occupied by copper, so each winding can occupy roughly 25% of the available space.
To account for this, a fill factor k_u typically between 0.4 and 0.6, is introduced:

$$A_{Cu,tot} = k_u A_w$$

where $A_{Cu,tot}$ is the area that can actually be occupied by copper.

The conductor cross-section is determined by the current density J. A typical value is 4.5 $\frac{A_{RMS}}{mm^2}$.
For example, for 3 A_{RMS} :

$$w_A \geq \frac{3\ A_{RMS}}{4.5\ \frac{A_{RMS}}{mm^2}} = 0.7\ mm^2\ (\sim 19\ AWG)$$

In general:

$$A_w = \frac{(\ w_{A1}\ N_1 + \ \ w_{A2}\ N_2 + \cdots + w_{Ai}\ N_i\ )}{k_u}$$

where w_{Ai} is the cross-section of the conductor of the i-th winding and N_i is the number of its turns.

The Area Product is defined as:

$$A_p = A_e A_w$$

where:

- A_e= = effective core area (magnetic constraint)
- A_w= winding window area (electrical constraint)

A_p represents how much power can be transferred while respecting core and winding constraints.

In the previous chapter, we saw that for a bipolar primary drive signal $v_1(t)$ with a 50% duty cycle, defined as:

$$v_1(t) = \begin{cases} +V_m\ for\ 0 \leq t \leq \frac{T_s}{2} \\ -V_m\ for\ \frac{T_s}{2} < t \leq T_s \end{cases}$$

We previously saw that, in this case:

$$N_1 \geq \frac{V_m}{4\ f_s\, A_e\, B_{Admissible}}$$

Therefore, the smaller A_e, the more turns are needed to avoid saturation.

The RMS current in the primary can be expressed as:

$$I_{1_{RMS}} \approx \frac{P_o}{V_1}$$

The copper section required is:

$$w_A = \frac{I_{1_{RMS}}}{J}$$

Where J is the permissible current density measured in $\frac{A_{RMS}}{m^2}$.

The area required for the primary winding is:

$$A_{cu,\text{prim}} = N_1 w_A = N_1\ \frac{I_{1_{RMS}}}{J}$$

Because:

- each turn carries the current $I_{1_{RMS}}$
- each turn has its own copper cross-section $w_A = \frac{I_{1_{RMS}}}{J}$.

The feasibility condition (the conductors must enter the core) is given by:

$$N_1 \frac{I_1}{J} + N_2 \frac{I_2}{J} \leq k_u A_w$$

However, the contribution of the secondary is often ignored because:

- In preliminary designs, it is assumed that the secondary winding has a cross-section similar to, or proportional, to the primary: the primary is calculated and the secondary is resized later.
- If the secondary voltage is much higher and the current lower, its space requirement is relatively small, so it can be temporarily ignored.
- This approach simplifies the initial estimate of N_1 and the area product A_p without having to delve into the details of secondary turns ratios and currents.

If we require more accurate sizing, especially in high-current flyback or push-pull converters with high turns ratios, both windings must be considered.

In this tutorial, we will only consider the primary winding. Therefore, the feasibility condition is given only by:

$$N_1 \frac{I_1}{J} \; \leq \; k_u A_w$$

Equation 2

Let's remember the value of $N_1$and I_1:

$$N_1 = \frac{V_m}{4 \; f_s \; A_e \; B_{Ammissibile}}, \qquad I_1 = \frac{P_o}{V_m}$$

Let's substitute these expressions into Equation 2:

$$\frac{V_m}{4 \, f_s \; A_e \; B_{Ammissibile}} \cdot \frac{P_o}{V_m \, J} \; \leq \; k_u A_w$$

so:

$$\frac{P_o}{4 \, A_e \; B_{Ammissibile} \; f_s \, J} \; \leq \; k_u A_w$$

We multiply by A_e:

$$A_p = A_e A_w \geq \frac{P_o}{4\, k_u\, J\, B_{Ammissibile}\, f_s}$$

This is the expression for A_p or this specific drive signal.

What happens if $v_1(t)$ is not a PWM between $+V_m$ e $-V_m$? The study methodology is the same. Let's look at it in detail.

In the sinusoidal case we found:

$$V_m = 2\pi\, f_s\, N_1\, A_e\, B_{\max} \qquad V_{RMS} = \frac{V_m}{\sqrt{2}} \approx 4.44\, f_s\, N_1\, A_e\, B_{\max}$$

$$N_1 \geq \frac{V_{RMS}}{4.44\, f_s\, A_e\, B_{Ammissibile}}$$

So, doing the calculations:

$$A_p \geq \frac{P_o}{4.44\, k_u\, J\, B_{Ammissibile}\, f_s}$$

Finally, let's look at the case of the unipolar PWM signal.

$$v_1(t) = \begin{cases} V_m\ for\ T_{ON} = \dfrac{T_s}{2} \\ 0 \text{ for } T_s - T_{ON} = \dfrac{T_s}{2} \end{cases}$$

We had found:

$$N_1 \geq \frac{V_\mathrm{m}}{A_e\, B_{\max}\, f_s}$$

So:

$$A_p \geq \frac{P_o}{k_u\, J\, B_{Ammissibile}\, f_s}$$

The general form (always true) is:

$$V_1 = k_f\, N_1\, A_e\, B_{\max}\, f_s \rightarrow N_1 = \frac{V_1}{k_f\, A_e\, B_{\max}\, f_s} \rightarrow \boldsymbol{A_p = A_e A_w \geq \frac{P_o}{k_f\, k_u\, J\, B_{\max}\, f_s}}$$

where:

- $k_f = 4 \rightarrow$ bipolar
- $k_f = 4.44 \rightarrow$ sinusoidal RMS
- $k_f = 1 \rightarrow$ unipolar

Drive voltage Pattern	B(t)	Range of B(t)	Number of primary turns N_1	Ap
Unipolar PWM	Sawtooth pattern: B increases linearly during ON and is reset during OFF	$\Delta B = \frac{V \cdot D}{N_1\, A_e\, f_s}$	$N_1 \geq \frac{V \cdot D}{B_{adm}\, A_e\, f_s}$	$\frac{P_o}{k_u\, J\, B_{\max}\, f_s}$
Symmetrical bipolar PWM (±V, DC = 50%)	Triangular pattern centered at zero	$\Delta B = \frac{V}{2\, N_1\, A_e\, f_s}$	$N_1 \geq \frac{V}{4\, B_{adm}\, A_e\, f_s}$	$\frac{P_o}{4\, k_u\, J\, B_{\max}\, f_s}$
Sinusoidal	B(t) sinusoidal, 90° out of phase with the voltage	$B(t) = B_{max} \cos(2\pi f_s t)$	$N_1 \geq \frac{\sqrt{2}\, V_{rms}}{2\pi f_s\, B_{adm}\, A_e}$	$\frac{P_o}{4.44\, k_u\, J\, B_{\max}\, f_s}$

4. Figure of Merit

In electrical engineering and in the design of transformers and converters, the figure of merit is a parameter that summarizes the efficiency of a design or the capability of a component to transfer energy while respecting physical limits.
The figure of merit is a number or a combination of physical quantities that allows different design solutions to be compared without having to simulate or build the entire circuit.
It is used to quickly assess whether a core, a transformer, a converter, or a material is suitable for a certain power or frequency level.

For a high-frequency transformer, the magnetic–electric figure of merit can be the Area Product, previously defined as:

$$A_p = A_e \cdot A_w$$

where:

- A_e= effective core area (magnetic part)
- A_w= winding window area (electrical part)

The larger A_p is, the more power the transformer can transfer without saturating the core or overheating the copper.

In the context of high-frequency transformer or inductor design, the product frequency × ΔB is a true magnetic figure of merit, because it measures how much “magnetic work” the core can perform without saturating.
Let us see why.

For a winding with N turns on a core with area A_e:

$$v(t) = NA_e \frac{dB}{dt}$$

Integrating over a time interval Δt:

$$\Delta B = \frac{1}{NA_e} \int v(t)dt$$

If the signal is periodic with frequency $f_s = \frac{1}{T_S}$ and assumes a constant average value over each half-period (PWM or square wave), then:

$$V_{\text{eff}} \sim N \, A_e \, \Delta B \, f_s$$

where:

- ΔB indicates how much the flux varies between positive and negative peaks (flux excursion)
- f_s indicates how many times per second the flux is "recharged"

Therefore, the product $f_s \Delta B$ represents the average rate of flux variation, which must be supported by the core without saturation.

So:

- For the same ΔB, increasing the frequency allows a higher voltage to be applied and more power to be transferred.
- For the same frequency, increasing ΔB allows more energy to be stored and more power to be transferred.

5. Magnetic Losses

Magnetic losses are the power dissipated inside the core of a transformer or inductor when the magnetic flux varies over time. They represent one of the main physical limitations in the design of magnetic devices, especially at high frequencies.
From a physical point of view, these losses arise because the magnetic material does not respond ideally to the magnetizing field: part of the supplied energy is used to realign magnetic domains along the hysteresis cycle and is lost as heat at every cycle, while another part is dissipated in the form of eddy currents induced in the material by the time variation of the flux.

Let us examine this in detail.

When a voltage is applied to the primary, a variation of flux $\Delta B(t)$ is imposed. The magnetic material is non-ideal and dissipates energy for two main reasons:

- Magnetic hysteresis: magnetic domains do not follow the magnetic field H instantaneously. Each magnetization cycle requires energy to realign the domains.
- Eddy currents: the variation of flux induces closed electric currents within the material, which dissipate power due to the Joule effect.

As a result, a certain amount of energy is lost at each cycle and converted into heat.

Hysteresis Losses

The energy lost per cycle per unit volume is equal to the area of the hysteresis loop:

$$W_h = \oint \vec{H} \cdot d\vec{B}$$

This energy does not depend on time, but only on the B–H trajectory.

Since the cycle is repeated $f_s$times per second:

$$P_h = f_s \cdot V_c \cdot \oint \vec{H} \cdot d\vec{B}$$

where V_c is the core volume.
Hysteresis losses are usually estimated using an empirical relationship:

$$P_h = k_h \, f_s \, \mathrm{B}_{\mathrm{max}}^{\alpha}$$

where:

- P_h = hysteresis losses (W/kg oppure W/m³, depending on the definition of k_h)
- f_s = frequency (Hz)
- B_{max} = maximum magnetic flux density (T)
- k_h, α = empirical constants of the material (provided by the manufacturer)

Typically:

- α≈ $1.6 \div 2.2$ for ferrites and magnetic steels
- k_h strongly depends on the material and temperature

Hysteresis losses arise because, at each magnetization cycle, magnetic domains must be continuously realigned, dissipating energy.
For this reason, they:

- increase linearly with frequency
- increase exponentially with B_{max}

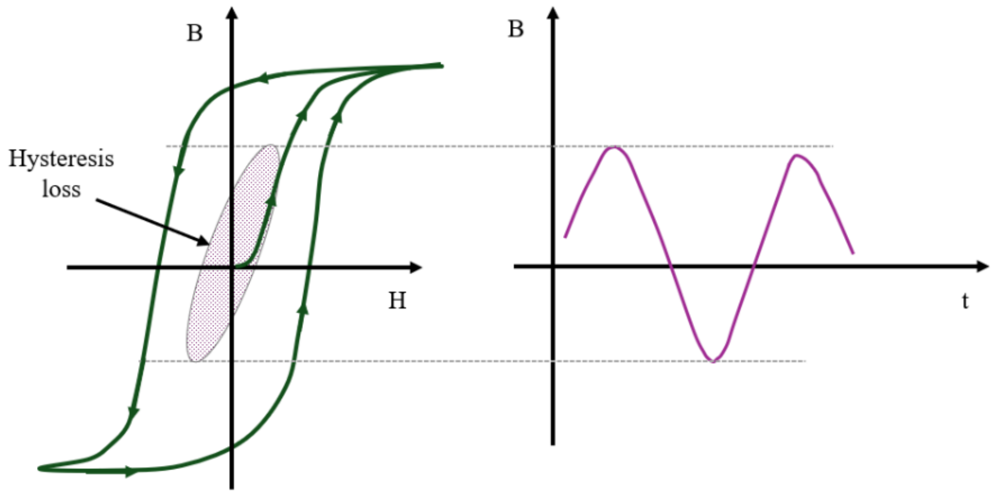


Figure 15: Hysteresis Losses

Eddy Current Losses

A variation of $B(t)$ induces an internal electric field:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This generates closed currents inside the material, leading to Joule losses proportional to:

$$P_e \propto f_s^2 B_{\max}$$

These losses strongly depend on:

- material resistivity
- lamination thickness (in laminated cores)
- microstructure (in ferrites)

For this reason, ferrites—with their high resistivity—are ideal for high-frequency applications.

Unified Model

The Steinmetz equation, which is an empirical formula, describes total core losses in magnetic circuits.
For sinusoidal signals, the equation is:

$$P_{core} = k\, f_s^{\alpha}\, B_{\max}^{\beta}\, V_c$$

where:

- k, α, β are provided in the core datasheet
- V_c is the core volume

Typical values for ferrites: $\alpha \approx 1.2 \div 1.5$, $\beta \approx 2.3 \div 2.8$

The fundamental concept to keep in mind is the following: losses depend on how fast and how often the flux is forced to vary.

Real Core

The following figures show the magnetic loss diagrams of a 3F3 ferrite as a function of frequency and temperature.
A good rule of thumb is to set a target loss of about 100 mW/cm^3 equivalent to 100 kW/m^3.

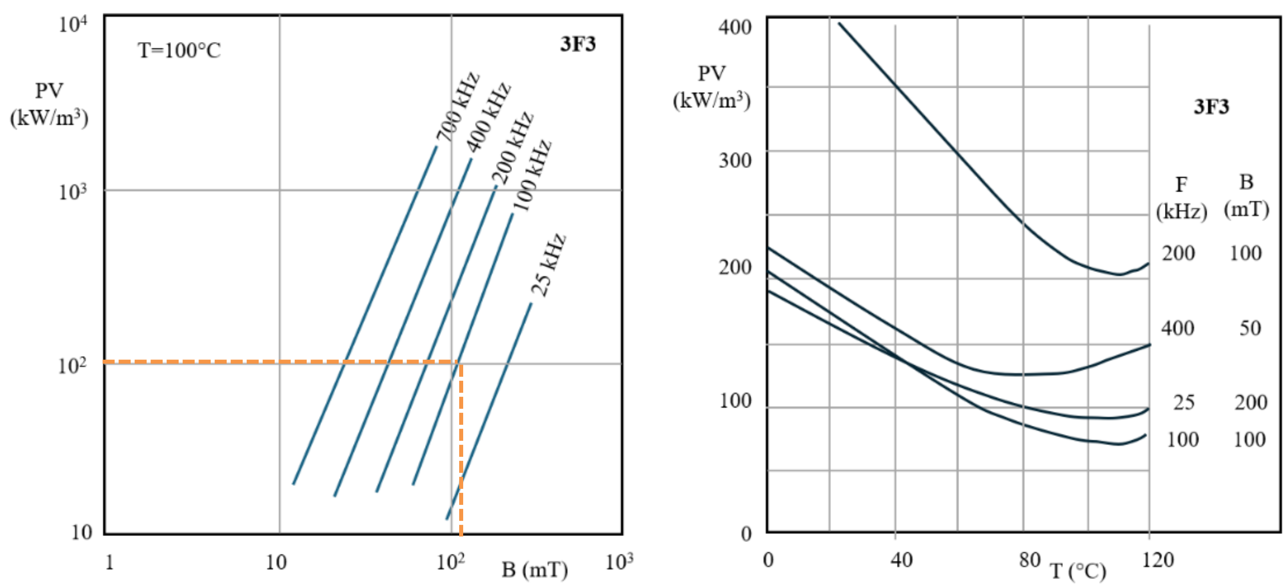


Figure 16: Magnetic losses of a 3F3 ferrite from the Ferroxcube datasheet

For example, if a transformer operates at 100 kHz, the flux density should be around 200 mT (millitesla).
In the figure on the right, losses at different frequencies and temperatures are shown. As can be seen, heating the core leads to a reduction in losses.
Therefore, if a 3F3 core is chosen, keeping it cold would be a mistake.

For further information, refer to the Ferroxcube datasheet “3F3 Material specification”.

It is also necessary to consider the figure of merit $f \cdot \Delta B$ which should be as high as possible because it appears in the denominator of the expression for A_P. The larger it is, the smaller the core dimensions will be.
Once the operating frequency is fixed, the core with the highest figure of merit should be selected.

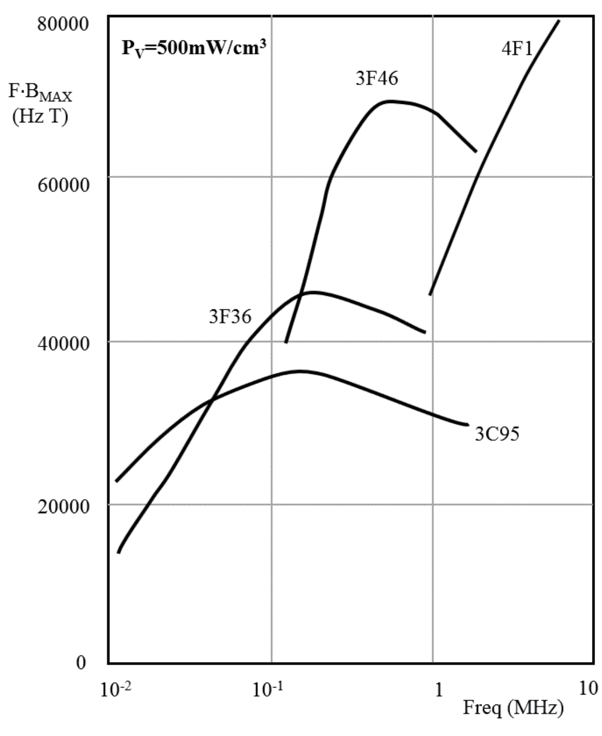


Figure 17: Example of $F \cdot B_{MAX}$ curves for different materials at constant loss, from Ferroxcube datasheet

For further information, consult:
https://www.ferroxcube.com/de-DE/download/download/228

6. Air Gap

The air gap is an intentional interruption of the magnetic circuit of the core, in which the ferromagnetic material is replaced by air (or by a material with very low permeability, close to μ_0).

Reluctance is the “opposing mechanism” that hinders the creation of magnetic flux in a magnetic circuit.
In other words, it is the magnetic equivalent of electrical resistance. In an electrical circuit, resistance opposes the flow of current, while in a magnetic circuit, reluctance opposes the flow of magnetic flux Φ.

Hopkinson’s law (or Ohm’s law for magnetic circuits) states that:

$$\boldsymbol{\mathcal{F}} = \boldsymbol{\Phi} \cdot \boldsymbol{\mathcal{R}}$$

where

- $\mathcal{F}$ is the magnetomotive force (MMF), equal to $N \cdot I$ (number of turns × current), analogous to electrical voltage;
- Φ is the magnetic flux, analogous to electrical current;
- $\mathcal{R}$ is the reluctance, analogous to electrical resistance.

From a physical point of view, just as electrical resistance depends on the length, cross-sectional area, and resistivity of a conductor, reluctance is defined as:

$$\mathcal{R} = \frac{l}{\mu A}$$

where:

- l is the average length of the path followed by the magnetic flux within the considered material. In the core, it is the average length of the magnetic circuit (often indicated as l_e n datasheets), while in the air gap it is the physical gap length g. It is measured in meters.

- The permeability μ indicates how easily the material magnetizes. It follows that: $\mu = \mu_0\,\mu_r$ where: $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the permeability of free space and μ_r is the relative permeability of the material.
  Typical values are:
  - air / gap: $\mu_r \approx 1$
  - ferrites: $\mu_r \sim 10^3$–10^4.

  Permeability is measured in Henries per meter [H/m]
- A is the cross−sectional area perpendicular to the magnetic flux lines. In datasheets it is often indicated as A_e. It is analogous to the conductor cross-section in electrical circuits and is measured in square meters.

The larger l is, the harder it is to create flux; the larger the product $\mu \cdot A$, the easier it is to create flux.
The unit of reluctance is ampere-turn per weber (A·turn/Wb), equivalent to H^{-1} (the inverse of inductance).

In practice, it is convenient to work with reluctance because it greatly simplifies the calculation of complex magnetic systems, allowing the use of analogies with electrical circuit laws (series/parallel combinations, Ohm's law, Kirchhoff's laws) to compute flux, MMF, and magnetization distribution.

The following table shows the main correspondence between magnetic and electrical quantities:

Magnetic circuit	**Quantity**	**Electrical analog**	**Quantity**
Magnetomotive force	$\mathcal{F} = \mathrm{N} \cdot \mathrm{I}$	Voltage	V
Magnetic flux	Φ	Electric current	I
Reluctance	$\mathcal{R} = \dfrac{l}{\mu \cdot A}$	Resistance	$\mathcal{R} = \dfrac{\rho \cdot \mathrm{l}}{A}$
Permeance	$\mathcal{P} = \dfrac{1}{\mathcal{R}}$	Conductance	$G = \dfrac{1}{R}$

Thus, a core with a winding carrying a current I can be represented by an equivalent electrical circuit, in which the winding acts as a magnetomotive force source N·I, while the magnetic path of the core is modelled by a reluctance $\mathcal{R}$, as shown in the following figure.

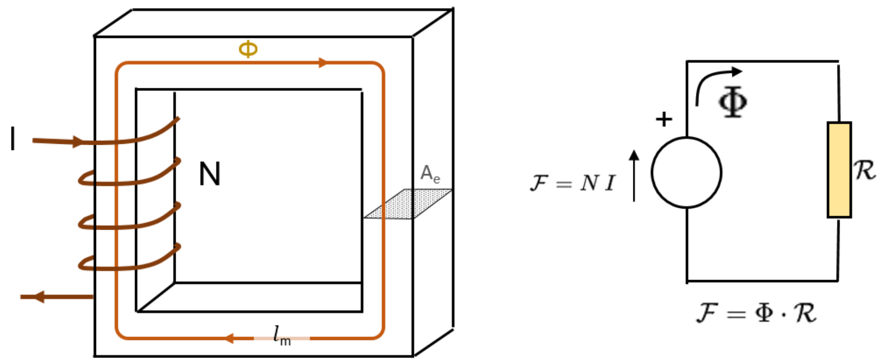


Figure 18: Equivalence between magnetic and electrical circuits

Core with air gap

Let us now consider a core with an air gap. The magnetic path is divided into two sections with different characteristics:

- Ferromagnetic section (core): low reluctance because μ is very high
- Air gap: very high reluctance because μ is low (air)

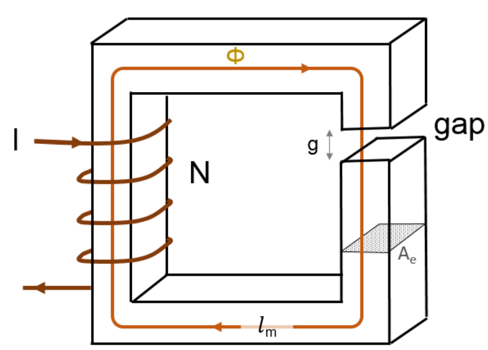


Figure 19: Core with gap

From a magnetic point of view, the air gap drastically increases the reluctance of the circuit, while from an electrical point of view it manifests itself as a reduction of the magnetizing inductance and an increase in the current required to generate flux.

The total reluctance is obtained, as in a series electrical circuit, by summing the two contributions (neglecting other effects):

$$\mathcal{R}_{tot} = \mathcal{R}_{core} + \mathcal{R}_{gap}$$

For the core:

$$\mathcal{R}_{core} = \frac{l_m}{\mu_0\,\mu_r\,A_e}$$

For the air gap:

$$\mathcal{R}_{gap} = \frac{g}{\mu_0\,A_e}$$

Since $\mu_r \gg 1$, even a very small gap dominates the series:

$$\mathcal{R}_{gap} \gg \mathcal{R}_{core}$$

therefore:

$$\mathcal{R}_{tot} \approx \mathcal{R}_{gap}$$

Thus, in the presence of a gap:

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_{tot}} = \frac{N\,I}{\mathcal{R}_{tot}} \approx \frac{N\,I\,\mu_0\,A_e}{g}$$

From which:

$$\boldsymbol{B} = \frac{\boldsymbol{\Phi}}{\boldsymbol{A_e}} = \frac{\boldsymbol{\mu_0\,N\,I}}{\boldsymbol{g}}$$

Therefore, B depends almost exclusively on the current I and the gap length g and not on the permeability of the core material.

Thus, with an air gap:

- the flux is mainly determined by the current
- the nonlinearity of the core has a reduced effect
- the equivalent B-H curve becomes almost linear
- the core no longer “decides” the flux: the electrical circuit does

Magnetizing Inductance

The magnetizing inductance (L_m) is the ability of an electrical circuit (a coil or solenoid) to store energy in a magnetic field generated by the flow of current and, above all, to oppose changes in that current by producing a counter-electromotive force, according to Lenz’s law. It is measured in Henries (H) and quantifies the ratio between linked magnetic flux and current:

$$L_m = \frac{\Phi_{TOT}}{I}$$

The higher the inductance, the greater its ability to oppose changes in current.

In detail:

$$L_m = \frac{\Phi_{TOT}}{I} = \frac{N\,\Phi}{I}$$

We also know that

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_{tot}} = \frac{N\,I}{\mathcal{R}_{tot}}$$

Substituting into the definition of L_m:

$$L_m = \frac{N\,\Phi}{I} = \frac{N}{I}\,\frac{N\,I}{\mathcal{R}_{tot}} = \frac{N^2}{\mathcal{R}_{tot}}$$

With a dominant gap we have seen that:

$$\mathcal{R}_{tot} \approx \mathcal{R}_{gap} = \frac{g}{\mu_0 A_e}$$

Thus:

$$L_m \approx \frac{\mu_0 \, N^2 A_e}{g}$$

Equation 3

Note that L_m does not depend on μ_r .

The magnetic energy that can be stored in L_m is:

$$E = \frac{1}{2} L_m \, I^2$$

But physically, the energy is stored in the field:

$$E = \frac{1}{2} \int \vec{B} \cdot \vec{H} \, dV$$

Due to the previous considerations on total reluctance (dominant gap), we can say that $\vec{B} = \mu_0 \overrightarrow{H}$. Therefore:

$$E = \frac{1}{2\mu_0} \int \vec{B} \cdot \vec{B} \, dV$$

Integrating over the volume of the gap ($V = A_e \, g$) we obtain:

$$E \approx \frac{B^2}{2 \, \mu_0} A_e \, g$$

The core does not store energy; it only guides it. The energy is instead stored in the air gap, which is why the core does not risk saturating.

How to Design the Air Gap

The sizing of the air gap starts from the required energy E and the peak current I_{pk}.

The required inductance is obtained as:

$$E = \frac{1}{2} L_m \, I^2 \quad \rightarrow \quad L_m = \frac{2E}{I_{pk}^2}$$

Then the gap is calculated as:

$$g = \frac{\mu_0 \, N^2 A_e}{L_m}$$

Finally, the non-saturation condition is verified:

$$B_{max} = \frac{\mu_0 \, N \, I_{pk}}{g} \leq B_{Admissible}$$

The air gap transforms a ferromagnetic core from a dominant element into a passive support of the magnetic field.
A practical sizing example will be shown later.

7. Skin Effect

The skin effect is an electrical phenomenon that appears in conductors carrying high-frequency alternating currents, such as those used in HF transformers and inductors. In practice, the current tends to concentrate near the surface of the conductor, reducing the effective area available for current flow and therefore increasing the apparent resistance of the wire.

The current penetration depth, called skin depth δ, depends on frequency f, permeability μ, and resistivity ρ of the material according to:

$$\delta = \sqrt{\frac{2\,\rho}{\omega\,\mu}}$$

with $\omega = 2\,\pi\,f$.
It is clear that the higher the frequency, the smaller δ, and the greater the resistance increase.

In HF transformer design, the skin effect requires reducing the conductor cross-section per single turn or using Litz wire, where many thin enameled wires are twisted together to increase the available surface area and keep current density within acceptable limits. Ignoring the skin effect can lead to losses higher than those estimated by low-frequency calculations and to transformer overheating.

At low frequency, the current distribution in a wire is almost uniform, but as frequency increases, we find that charge or current is effectively carried only by the outer part of the wire.

Assume a copper conductor with resistivity ρ$\approx 1.68 \cdot 10^{-8}\ \Omega \cdot m$. The permeability of pure copper is $\mu \approx \mu_0$.

It follows that:

$$\delta = \sqrt{\frac{2\,\rho}{\omega\,\mu_{cu}}} \approx \sqrt{\frac{2\ \cdot 1.678 \cdot 10^{-8}\,\Omega \cdot m}{2\pi f\ rad/s \cdot\ 4\pi \cdot 10^{-7}\ H/m}} \approx 2.063\ \sqrt{\frac{1}{f_{kHz}}}\ \ [mm]$$

For example, if current flows at 10 kHz in a copper wire, δ is about 0.66 mm, while at 1 MHz it is reduced to 66 μm.

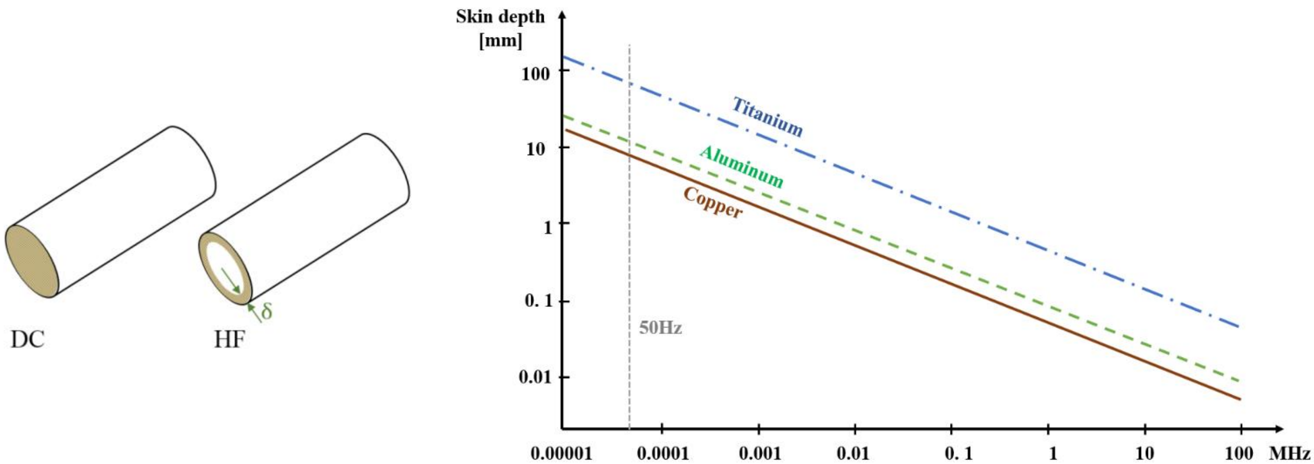


Figure 20: Skin effect

8. Design a flyback converter

The flyback converter does not integrate a transformer in the traditional sense, but a coupled inductor. It does not transfer energy directly from primary to secondary as an ideal transformer would but operates in two distinct phases.

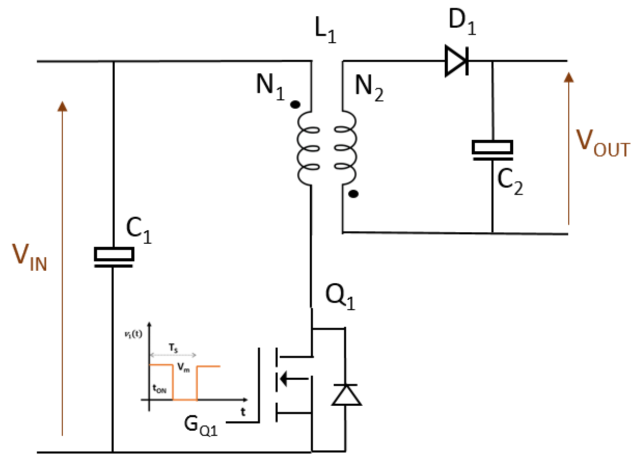


Figure 21: Schematic diagram of a flyback converter

During the switch-on phase of the power switch (MOSFET), energy is stored in the magnetic field of the core; during the switch-off phase, this energy is transferred to the load through the secondary winding.
From a magnetic point of view, therefore, the core does not operate around zero flux as in a classical transformer, but is intentionally brought to a certain level of magnetization to store energy.
Summarizing:

- During the ON phase:
    - the MOSFET applies v_{in} to the primary
    - energy is stored in the magnetic field
    - the secondary is off
- During the OFF phase:
    - the MOSFET is off
    - the magnetic field discharges
    - energy flows to the secondary

We observe that the primary and secondary never conduct simultaneously.

In a flyback converter, the primary of the transformer is driven by a unipolar PWM voltage (always positive), so from what we have seen, we expect the flux to always increase, as there is no mechanism capable of enforcing:

$$\int_0^{T_s} v_{in}(t)\ dt = 0$$

On the contrary, in this case, by Faraday's law:

$$\Delta B = \frac{1}{N_1\, A_e}\int_0^{T_s} v_{in}(t)\ dt > 0$$

So, cycle after cycle, the operating point shifts until saturation occurs.
From this point of view, using an air gap is of no use, since the flux would continue to grow. At most, it could delay saturation, but nothing more.
Therefore, it is necessary to introduce a mechanism capable of generating a reset voltage on the primary in order to zero ΔB during the cycle:

$$\int_0^{T_s} v_{in}(t)\ dt = \int_0^{T_{ON}} v_{in}(t)\ dt + \int_{T_{ON}}^{T_s} -v_{reset}(t)\ dt = 0$$

By doing so, the flux no longer grows indefinitely, and B(t) remains confined within a finite band.
But then, what is the purpose of the air gap? It serves to manage the flux level for a given current.
Indeed, with a dominant gap:

$$B_{max} = \frac{\mu_0\, N_1\, I_{pk}}{g}$$

Without gap:

$$B = \mu_{CORE}\, H \qquad\qquad H = \frac{N_1\, I}{l_m}$$

Thus, B grows significantly for small I. Even if ΔB is limited, the absolute value of B can still be too high.

Summarizing, the reset determines if the flux returns, while the gap determines how much flux corresponds to a given current.

In flyback converters, both aspects are used: reset voltage and air gap. In fact:

- the reset voltage prevents flux drift
- the gap allows energy storage, fixes B_{max} for a given I_{pk}, and makes the system tolerant to load variations

We note that without an air gap, even with perfect reset, the core would saturate at very low currents.
We have seen that a core saturates when the magnetic induction B reaches the maximum value the material can sustain, B_{sat}.
Beyond this point:

- μ_{CORE} collapses (the B(H) curve tends to become parallel to the H axis)
- H increases, but B does not (or very slowly)
- the inductance drops sharply ($L_m = \frac{{N_1}^2}{\mathcal{R}} = \mu_{CORE} \frac{{N_1}^2 A_e}{l_m}$)
- the current rises almost without limit ($v = L_m \frac{dI}{dt}$ if v is constant and L_m decreases, then $\frac{dI}{dt}$ must increase to keep the product constant)

In a core without a gap:

$$B = \mu_{CORE} H \qquad\qquad H = \frac{N_1 I}{l_m}$$

so:

$$B = \mu_{CORE}\, H = \frac{\mu_{CORE}\, N_1\, I}{l_m}$$

Given that:

- $\mu_{CORE} = \mu_0\, \mu_r$
- μ_r of ferrite is on the order of 2000–3000

It takes a very small current to reach B_{sat}.

Let's make a numerical example to better understand this concept.
Suppose:

$\mu_r = 2000$
$l_m = 5\ cm = 0.05\ m$
$N_1 = 50$
$B_{sat} = 0.3\ T$

The saturation current is:

$$I_{sat} = \frac{B_{sat}\, l_m}{\mu_0 \mu_r\, N_1} = \frac{0.3 \cdot 0.05}{4\pi \cdot 10^{-7} \cdot 2000 \cdot 50} \approx 0.12\ A$$

A decidedly low value for a power transformer.
In the flyback, instead, the primary current must rise significantly, and all the energy is stored as:

$$E = \frac{1}{2}\, L_m\, I_{pk}^2$$

With a dominant gap:

$$H = \frac{N_1\, I}{g} \rightarrow B = \mu_0\, \frac{N_1\, I}{g}$$

We note that μ_r disappears, and the B/I slope is lower.
With the same previous data, let's calculate the saturation current with an air gap.
Suppose we have a gap of 1 mm = 0.001 m.

$$I_{sat} = \frac{B_{sat}\ g}{\mu_0\ N_1} = \frac{0.3\ \cdot 0.001}{4\pi \cdot 10^{-7} \cdot 50} = 4.8\ A$$

Now the flyback can operate.

So, we must remember that:

- Without a gap:
  - the magnetic field "enters immediately" into the material
  - the core cannot accept energy

- With a gap:
  - the magnetic field "expands in the gap"
  - energy is stored in the gap, not in the material
  - the core becomes just a flux guide

Now, we introduce another fundamental concept, namely the mode of operation.

Mode of operation: DCM-BCM-CCM

Discussing CCM, DCM, and BCM in a flyback means describing how the magnetizing current (which is also the energy stored in the core) behaves from one switching period to the next.

During the MOSFET ON phase, the voltage $v_{in}(t)$ is applied to the primary and the magnetizing current increases linearly.

Recall that:

$$v_{in}(t) = L_m\ \frac{d\ I_m(t)}{dt} \rightarrow\ \ I_m(t) =\ \frac{v_{in}(t)}{L_m}\ \ t$$

So, in the ON phase, $I_m(t) =\ I_{primary}(t)$, it is precisely the primary current that generates the magnetizing current.

During the OFF phase, energy is transferred to the secondary, and the magnetizing current decreases.

When the MOSFET is off, $I_{primary}(t) = 0$ because the primary is open, but the flux cannot change instantaneously, so the magnetizing current must continue to flow, as a consequence of the stored energy, and transfers to the secondary.

Since the secondary is subject to the same flux (neglecting losses) generated by the primary, we expect a voltage $v_S(t)$ across the winding which, by Faraday's law, satisfies:

$$\frac{v_P}{N_1} = -\frac{v_S}{N_2} \rightarrow v_P = -v_S\frac{N_1}{N_2} = -\frac{N_1}{N_2}(v_{OUT} + V_D)$$

Where v_{OUT} is the output voltage of the converter (across the output capacitor) and V_D is the diode voltage (~0.7 V).

Note:
The voltage on the secondary is inverted with respect to the primary, a concept formalized by the dots near the inductances (see figure below).

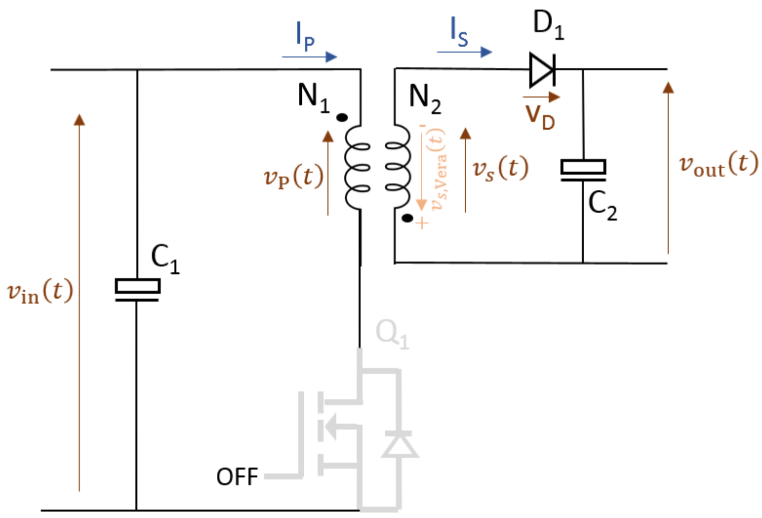


Figure 22: Mosfet OFF phase

Thus, the appearance of $v_S(t)$, combined with the mosfet being off, generates a reflected voltage on the primary:

$$v_P = -v_S\frac{N_1}{N_2}$$

We have previously seen that L_{m} is defined as:

$$L_{\mathrm{m}} = \frac{N_1^2}{\mathcal{R}}$$

So, it is referred to the primary winding.

The reference system for the magnetizing current I_{m} is therefore the primary, even though, at the beginning of the OFF phase, the effects appear as a (peak) current in the secondary equal to:

$$I_{\mathrm{s}} = I_{\mathrm{m}} \frac{N_1}{N_2}$$

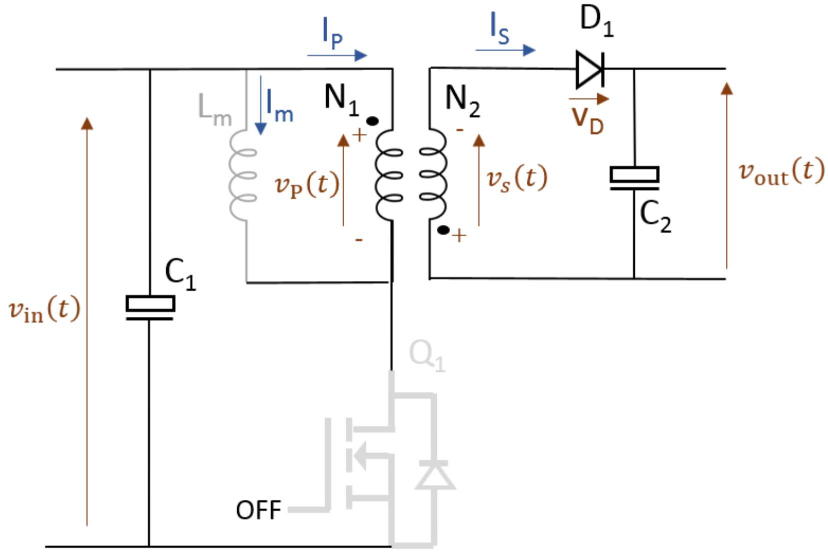


Figure 23: Magnetizing inductance and I_m in the OFF phase

Therefore, we can say that in the OFF phase:

$$v_P(t) = L_{\mathrm{m}} \frac{d\, I_{\mathrm{m}}}{dt} \quad \rightarrow \quad \frac{d\, I_{\mathrm{m}}}{dt} = \frac{v_P(t)}{L_{\mathrm{m}}} = -\frac{v_S(t)}{L_{\mathrm{m}}} \frac{N_1}{N_2}$$

$$\frac{d\, I_{\mathrm{m}}}{dt} = -\frac{v_S(t)}{L_{\mathrm{m}}} \frac{N_1}{N_2}$$

Considering $v_S(t)$ practically constant over the PWM period T_S thanks to the output capacitor, we can say that the magnetizing current decreases linearly, and so does the secondary current, properly scaled by the turns ratio.

From the previous relation, we can also derive:

$$\Delta I_{\mathrm{m}} = -\frac{v_S}{L_{\mathrm{m}}}\frac{N_1}{N_2}\Delta \mathrm{t}$$

So, the energy discharge time is given by:

$$t_{discharge} = \frac{L_{\mathrm{m}}\, I_{\mathrm{m,pk}}}{v_s}\frac{N_2}{N_1}$$

Now, depending on how quickly the magnetizing current falls to zero, the mode of operation changes. What distinguishes CCM, DCM, and BCM is whether and when this current reaches zero.

Assume: $t_{OFF} = (1 - \mathrm{D})\mathrm{T}_S$

- $t_{discharge} < t_{OFF}$ → DCM
- $t_{discharge} = t_{OFF}$ → BCM
- $t_{discharge} > t_{OFF}$ → CCM

In detail:

- **DCM**: At the end of the OFF phase, $I_{\mathrm{m}}(t) = 0$ before the period ends.
  All energy stored in the core is transferred to the load, and there is an interval in which no current flows.
  Magnetically, the flux always returns to zero.

- **CCM**: $I_{\mathrm{m}}(t) > 0$ for the entire period, so part of the energy remains in the core at the end of the cycle; the magnetizing current never nulls, and the flux oscillates around an average value.

- **BCM**: $I_{\mathrm{m}}(t) = 0$ exactly at the end of the period. It is the boundary between DCM and CCM. Many flyback controllers operate in BCM as it allows:

- natural zero-current switching
- EMI reduction
- simpler magnetic design
- no flux accumulation

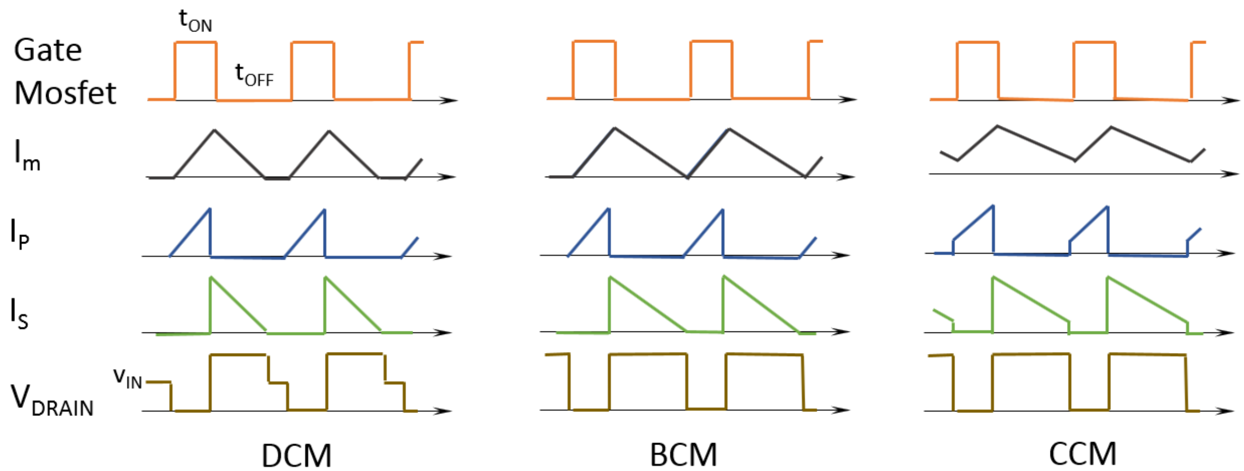


Figure 24: Current waveforms in the three operation modes

Having made these clarifications, we now return to the design of the converter.

Converter design

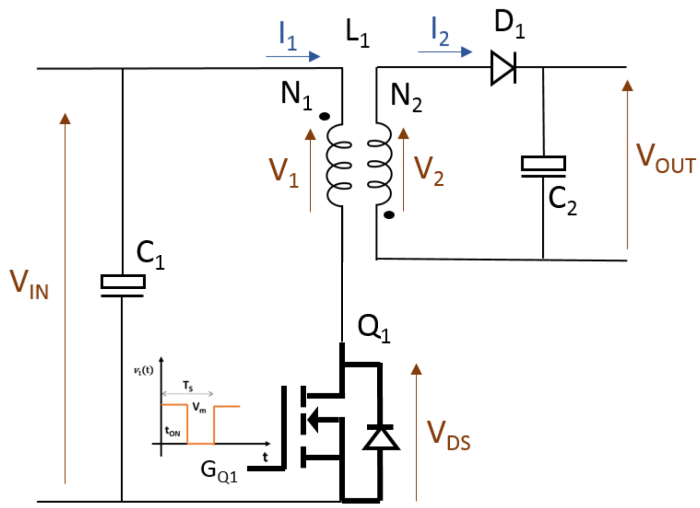


Figure 25: Schematic of a flyback power supply highlighting voltages and currents

Let's define the design specifications:

- Input voltage: 120 - 375 VDC (previously rectified)
- Output voltage: V_{OUT}=12 V, P_{OUT}=50 W
- Switching frequency: f_s = 100 kHz
- Operating mode: CCM / BCM
- Ferrite core: typical E-core, $A_e = 125\ mm^2 = 1.25\ 10^4\ m^2$
- Maximum allowed flux: $B_{max} = 0.22T$
- Maximum duty cycle (classic isolated flyback): $D_{max} = 0.45$
- Peak current $I_{PK} = 5A$ (depends on chosen mosfet)

Let's consider the two phases of the mosfet.

ON Phase

During the MOSFET on-time, the DC voltage $v_{in}(t)$ is applied to the primary. Faraday's law imposes:

$$v_{in}(t) = N_1 A_e \frac{dB}{dt}$$

Integrating:

$$\mathrm{B(t)} = B(0) + \frac{v_{in}(t)}{N_1\ A_e}\ t$$

In steady-state operation, $\mathrm{B}(0)$ does not change from cycle to cycle: $\mathrm{B}(0) = B(n\ T_s)$.
We reason in terms of $\Delta\mathrm{B}$.

$$\Delta\mathrm{B} = \frac{v_{in}(t)}{N_1\ A_e}\ T_{ON}$$

Equation 4

Since the magnetic circuit is dominated by the air gap, we have seen that the maximum B value is:

$$\mathrm{B}_{MAX} = \frac{\mu_0 \, N_1 \, I_{PK}}{g}$$

This equation is crucial because it shows that B_{MAX} does not depend on the ferrite but only on current and gap.
From Faraday's law (Equation 4):

$$N_1 = \frac{\mathrm{v}_{in,max} \, T_{ON,max}}{\mathrm{A}_e \mathrm{B}_{MAX}}$$

At maximum duty cycle D_{max}:

$$T_{ON,max} = \frac{D_{max}}{f_s}$$

Substituting the numeric values:

$$N_1 = \frac{\mathrm{v}_{in,max} \, D_{max}}{\mathrm{A}_e \mathrm{B}_{MAX} \, f_s} = \frac{375 \text{ V} \cdot 0.45}{1.25 \; 10^4 \; m^2 \; \cdot 0.22T \; \cdot 10^5} \approx 62 \; spire$$

The energy stored per cycle is:

$$E = \frac{P_0}{f_s} = \frac{50 \; W}{10^5 \; Hz} = 5 \cdot 10^{-4} \; J$$

But energy is also:

$$E = \frac{1}{2} \mathrm{L}_m \; \mathrm{I}_{PK}^2$$

so:

$$\mathrm{L}_m = \frac{2\,E}{\mathrm{I}_{PK}^2} = \frac{2 \cdot 5 \cdot 10^{-4}\,J}{5^2\,A^2} = 40\,\mu H$$

We have previously seen (Equation 3) that for a core with dominant gap:

$$L_m = \frac{\mu_0\,N_1^2\,\mathrm{A}_e}{g} \quad \rightarrow \quad g = \frac{\mu_0\,N_1^2\,\mathrm{A}_e}{L_m} = \frac{4\,\pi\,10^{-7} \cdot 62^2 \cdot 1.25 \cdot 10^{-4}}{40 \cdot 10^{-6}} \approx 1.5\ \mathrm{mm}$$

OFF Phase

When the OFF-time begins, the flux cannot instantaneously drop to zero and continues to circulate in the core, affecting both primary and secondary.
Since:

$$v_P(\mathrm{t}) = \mathrm{N}_1\,\frac{d\,\Phi(\mathrm{t})}{dt}, \qquad v_S(\mathrm{t}) = -\mathrm{N}_2\,\frac{d\,\Phi(\mathrm{t})}{dt}$$

We can write:

$$v_P(t) = -\frac{N_1}{N_2}\,v_s(t) = -\frac{N_1}{N_2}\,(v_{out}(t) + V_D) \;=\; v_{REF}(t)$$

Thus, on the primary, even without current flow, a “reflected” (negative) voltage appears, denoted as$v_{REF}(t)$. This equation can be used to calculate N_2:

$$N_2 = N_1\,\frac{(\,v_{out} + v_D)}{v_{REF}}$$

The mosfet drain voltage is:

$$V_{DRAIN} = V_{IN} + v_{REF} = V_{IN} + \frac{N_1}{N_2}(V_{OUT} + V_D)$$

The choice of v_{REF} is a compromise between MOSFET stress and complete flux reset.

Suppose we choose $v_{REF} = 100\mathrm{V}$ and $v_D = 0.7V$. Then:

$$N_2 = N_1 \frac{(v_0 + v_D)}{v_{REF}} = 62 \frac{12.7}{100} = 8 \; turns$$

$$V_{DRAIN} = V_{IN} + v_{REF} = 375 + 100 = 475V$$

At the end of the design, we always check that B is within acceptable ranges:

$$\mathrm{B}_{MAX} = \frac{\mu_0 N_1 I_{PK}}{g} < \mathrm{B}_{SATURAZIONE} \rightarrow \quad \mathrm{B}_{MAX} = \frac{4 \pi 10^{-7} \cdot 62 \cdot 5}{1.5 \cdot 10^{-3}} \approx 0.26 \, T$$

It is slightly high ($B_{max} = 0.22T$), so we can increase g or reduce I_{PK}.

The figure below shows the relevant voltages and currents to facilitate understanding of the physical and electronic phenomena.

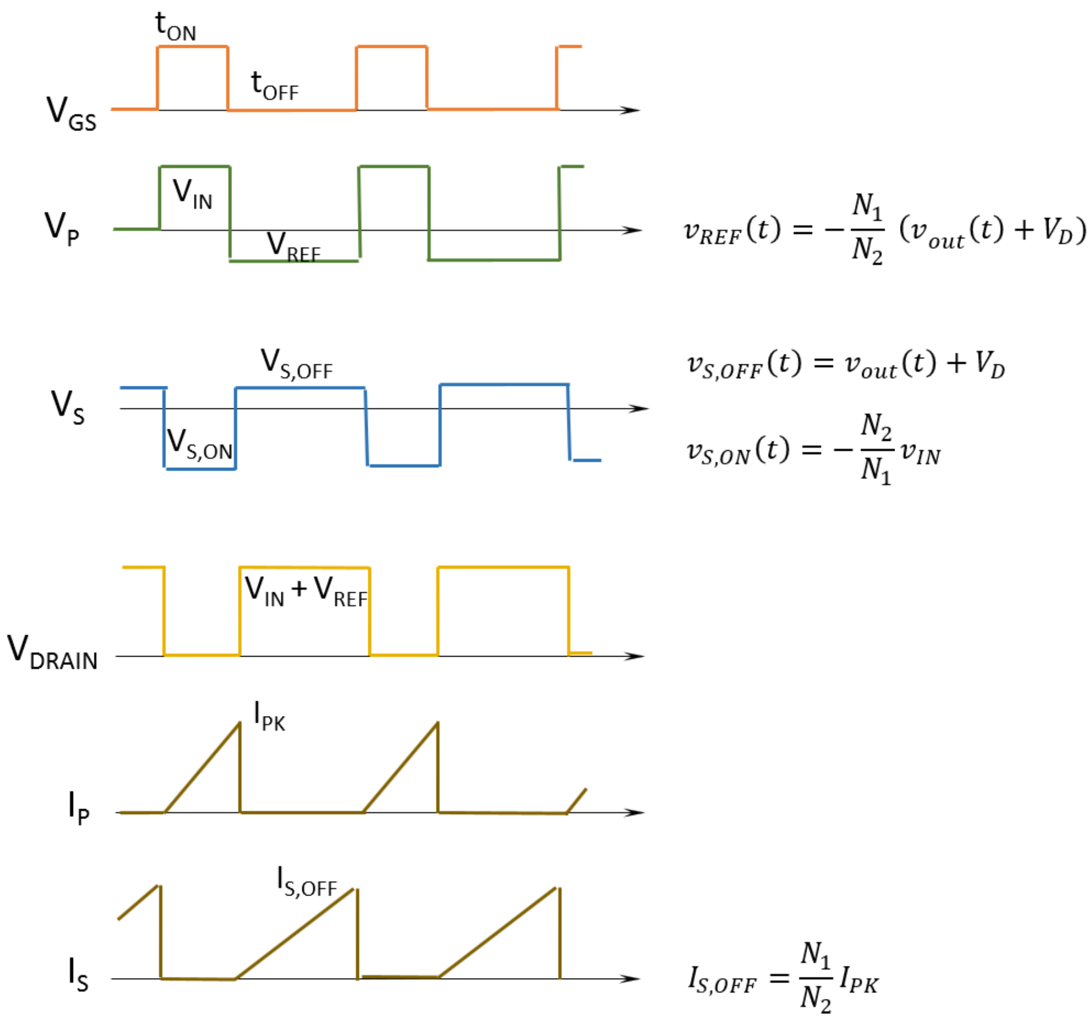


Figure 26: Relevant voltage and current waveforms

RCD Network (Resistor, Capacitor, Diode)

In the flyback converter, the RCD network on the primary acts as a dissipative snubber to limit the mosfet drain overvoltage caused by the energy stored in the transformer leakage inductance. When the mosfet turns off, the magnetizing current is transferred to the secondary, but the current associated with the leakage has no path and generates a rapid increase in drain voltage. The diode of the RCD network conducts when this voltage exceeds the clamp level, allowing the capacitor to absorb the leakage energy. This energy is subsequently dissipated through the resistor before the next switching cycle. The clamp voltage must be chosen as a compromise between mosfet stress and converter efficiency:

too low values increase snubber losses, while too high values may exceed the safety margin of the device's maximum drain voltage.

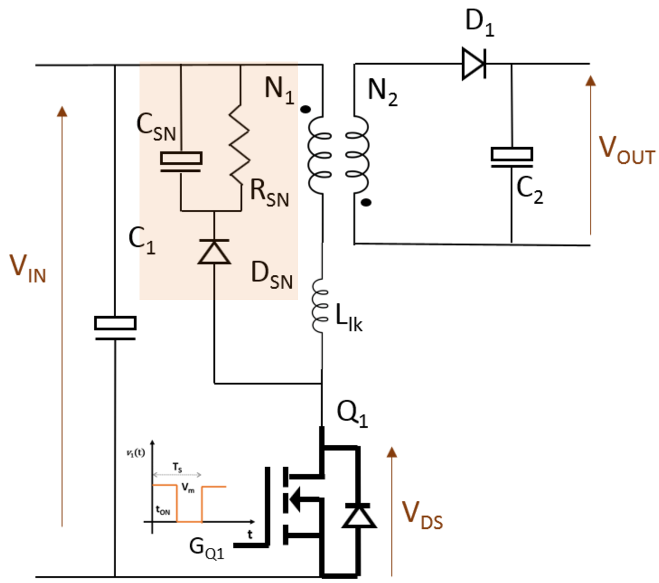


Figure 27: RCD network

We previously set:

$$v_{RF} = 100V$$

So that:

$$v_{DRAIN,IDEAL} = v_{in} + v_{REF} = 375V + 100V = 475V$$

Generally, the MOSFET is operated such that the drain voltage is at most 80% of the BVdss (drain-source breakdown voltage), which in this case we assume to be 650V.

$$v_{DS,PEAK} \leq 0.8 \cdot BV_{DSS} = 0.8 \cdot 650\,V = 520\,V$$

The maximum allowable overvoltage due to the leakage inductance L_{lk} , is:

$$v_{SN} = 520 - 475 = 45V \rightarrow v_{SN} = 50V\ (for\ safety)$$

The RCD network must be designed to meet this requirement.
First, we must estimate L_{lk}. Typically, it is set equal to 1%–3% of the primary inductance. A reference value for the power in question could be 2 μH.

The design of the RCD network is based on the energy of the leakage inductance:

$$E_{lk} = \frac{1}{2}\, L_{lk}\, I_{pk}^2$$

In this example:

$$E_{lk} = \frac{1}{2}\, L_{lk}\, I_{pk}^2 = \frac{1}{2} \cdot\ 2 \cdot\ 10^{-6}\ \cdot 5^2 = 25\ \mu\text{J}$$

This energy is dissipated every cycle, so the power involved is:

$$P_{SN} = E_{lk} \cdot f_S = 25\ \mu\text{J} \cdot 100\ kHz = 2.5\ W$$

The resistance R_{SN} must dissipate P_{SN} at voltage v_{SN} :

$$R = \frac{v_{SN}^2}{P_{SN}} = \frac{50^2}{2.5} = 1000\ \Omega$$

With nominal power greater than 5 W and, of course, non-inductive.

The capacitor must instantaneously absorb the leakage inductance energy when the mosfet turns off, limiting the rate and magnitude of the voltage spike on the drain, so it must absorb the energy E_{lk} without the voltage rising too much. In practice, at turn-off, the leakage energy is transferred to the capacitor, the capacitor voltage rises by a certain ΔV, and subsequently the resistor slowly discharges the capacitor before the next cycle.
We impose:

$$E_{lk} = \frac{1}{2}\, C\, \left(V_{C,max}^2 - V_{C,min}^2\right)$$

where $v_{C,min} = v_{SN}$ and $v_{C,max} = v_{SN} + \Delta v_{SN}$

Then:

$$E_{lk} = \frac{1}{2}\, C \left(V_{C,max}^{2} - V_{C,min}^{2}\right) = \frac{1}{2}\, C\, \left((v_{SN} + \Delta v_{SN})^{2} - v_{SN}^{2}\right) \simeq \mathrm{C}\, v_{SN}\, \Delta v_{SN}$$

We choose:

$$\Delta v_{SN} = 0.2 \cdot v_{SN} = 10V$$

Then:

$$\mathrm{C} \geq \frac{E_{lk}}{v_{SN} \cdot \Delta v_{SN}} = \frac{25\ \mu\mathrm{J}}{50V \cdot 10V} = 50nF$$

We can choose C=47nF with a working voltage greater than 100 V.
Regarding the diode, it must have a maximum repetitive reverse voltage $V_{RRM} \geq v_{SN}$ = 50V. It is preferable that it be "ultrafast," with low t_{rr}, and its maximum current greater than I_{pk}.
Finally, the time constant of R_{SN} and C_{SN} must be much smaller than the PWM period:

$$\tau = \mathrm{R}_{SN}\mathrm{C}_{SN} \ll T_S = \frac{1}{f_s}$$

Typically:

$$\tau \approx \frac{T}{5} \dots \frac{T}{10}$$

Note

he inductance L of a winding on a magnetic core is:

$$L = \frac{N^2 \, \mu_0 \, \mu_r \, \mathrm{A}_e}{l_m}$$

Where l_m is the mean magnetic path length in the core and A_e is the effective cross-section of the core.

Many ferrite datasheets give an inductance per square turn, called A_L, in nH/turn². Then:

$$L = N^2 \mathrm{A}_L$$

This is the most practical and quick formula.

9. Push-pull converter

Let us now consider a push-pull converter with transformer (or Half-Bridge / Full-Bridge), where the primary is driven by two mosfets that alternately apply the voltage between+V_{DC} e -V_{DC}, thus generating an alternating magnetic field in the core.

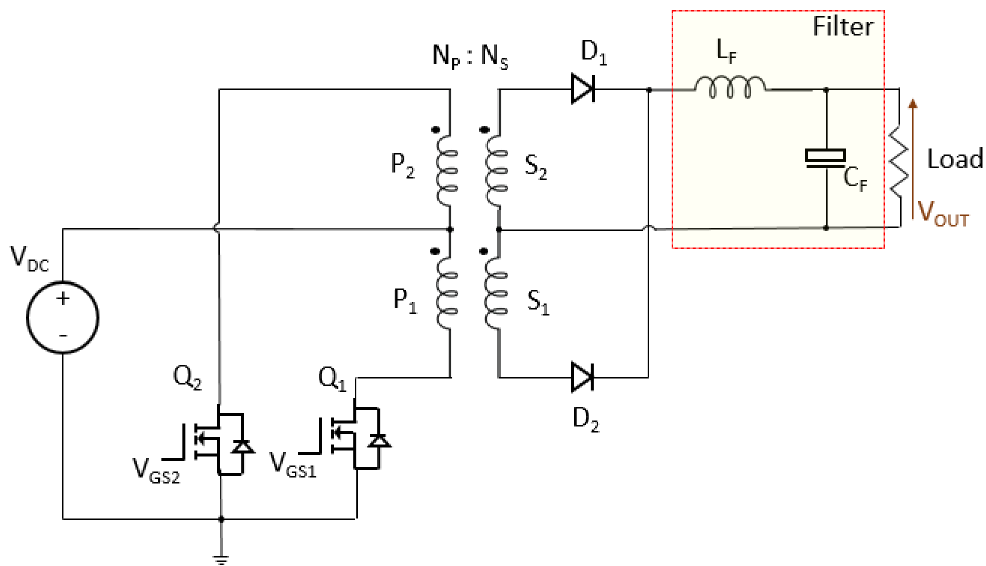


Figure 28: Principle schematic of a push-pull converter

We immediately notice that push-pull converters typically use a transformer equipped with a center tap at the midpoint of both the primary and secondary windings.

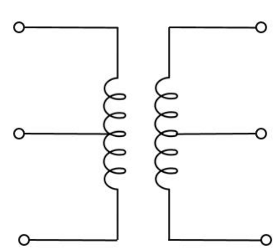


Figure 29: Transformer with center taps

Design specifications

- Input voltage: $V_{DC} = 375V$
- Output voltage: $V_{OUT} = 12V$
- Power: $P_{OUT} = 50W$
- Switching frequency: $f_S = 100\ kHz$
- Ferrite core: E-core, $A_e = 125\ mm^2, B_{max} = 0.22T$
- Operating mode: CCM/BCM

In a push-pull converter, the primary sees a symmetrical alternating voltage $\pm V_{DC}$. The half-period duration is:

$$t_{ON} = \frac{1}{2\,f_S} = \frac{1}{2 \cdot 100000} = 5\ \mu s$$

The two mosfets of the converter operate in a complementary manner: when one is on, the other is off, and vice versa. However, real mosfets do not switch instantaneously, but exhibit finite turn-on and turn-off times. Without adequate precautions, during transitions a conduction overlap may occur, with both devices simultaneously on, causing a direct short circuit of the supply.

To prevent this condition, the mosfets are driven by introducing a dead time, i.e., a time interval during which both mosfets are intentionally turned off, ensuring the absence of simultaneous conduction and protecting the converter. This time is extremely small compared to t_{ON}, so in the following calculations it can be neglected.

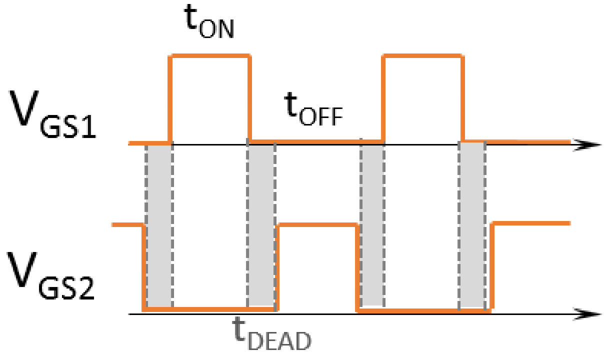


Figure 30: Insertion of the "dead time"

We can identify three operating states of the circuit.

Mosfet Q_1 ON, Mosfet Q_2 OFF, D_1 conducting, D_2 blocked

Mosfet Q_1 conducts, allowing current to flow in the primary winding P_1. In this case as well, the current magnetizes the transformer core, generating a flux with opposite sign with respect to the previous mode and inducing a voltage on the secondary windings.

With this new polarity:

- diode D_1 is forward-biased and allows current generated in secondary winding S_2 to flow;
- diode D_2 is reverse-biased and therefore blocked.

The current flowing through D_1 is delivered to the LC filter, which smooths it and supplies energy to the load. Also in this mode, power transfer occurs actively through the transformer.

The input voltage $+V_{DC}$ is applied to primary P_1 and remains approximately constant during the conduction interval.

As a consequence, $\frac{dB}{dt} = constant$, so B increases linearly over time starting from an initial value (ideally close to zero if the system is balanced).

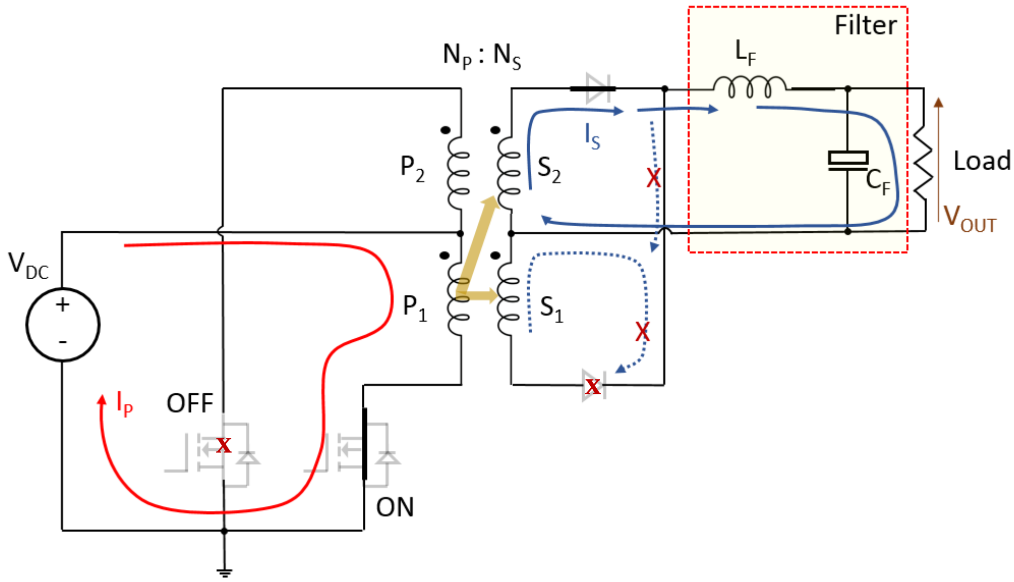


Figure 31: First phase - Q_1 ON, Q_2 OFF

Mosfet Q_1 OFF, Mosfet Q_2 ON, D_1 blocked, D_2 conducting

Mosfet Q_2 turns on, allowing current to flow in primary winding P_2 of the transformer. The current flowing in P_2 generates a magnetic flux in the core, which induces a voltage on the secondary windings.

Due to the polarity of the induced voltage:

- diode D_1 is reverse-biased and therefore blocked;
- diode D_2 is forward-biased and allows current generated in secondary winding S_1 to flow.

The current flowing through D_2 is filtered by the output LC filter and then delivered to the load. In this phase, energy is actively transferred from the primary side to the secondary side of the converter.

In this case, the voltage applied to primary P_2 has opposite polarity compared to the previous phase, but the absolute value of the voltage is similar.

Also in this phase, $\frac{dB}{dt} = constant$, but the constant is negative, therefore B decreases linearly over time.

The two half-waves produce a symmetric excursion of the flux density around zero.

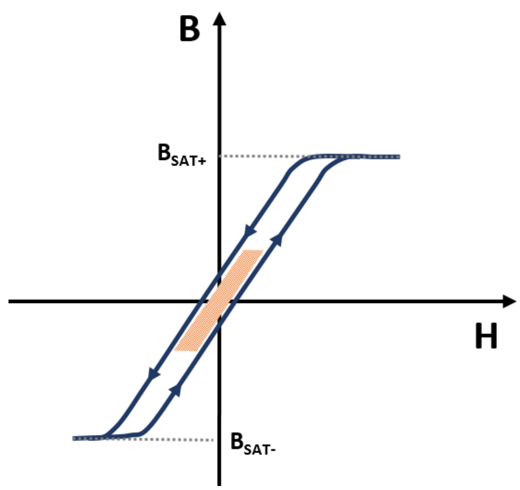


Figure 32: B–H characteristic

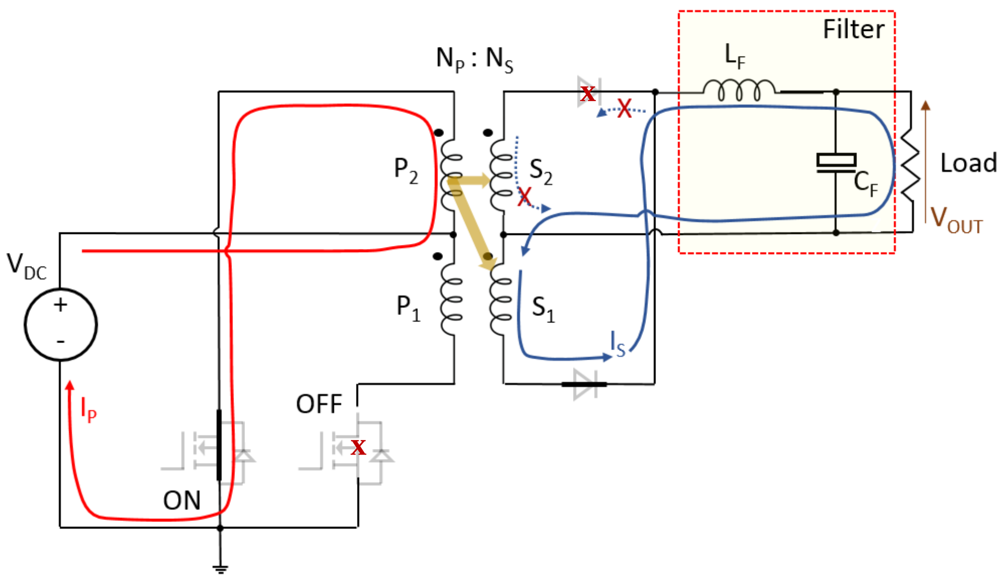


Figure 33: Second phase - Q_1 OFF, Q_2 ON

Mosfet Q_1 OFF, Mosfet Q_2 OFF, D_1 conducting, D_2 conducting

Between the end of one active mode and the beginning of the next, a short time interval is always introduced in which both transistors are off. This interval is known as dead time and is fundamental for the correct operation of the converter. During the dead time:

- no mosfet is conducting and no voltage is applied to the primary;
- no significant voltage is induced on the secondary windings;
- the energy supplied to the load comes exclusively from the output LC filter, which behaves as an energy reservoir.

The current provided by the LC filter inductor flows through the load and returns to the center tap of the secondary, from which it splits into the two secondary windings. In this condition, equal and opposite voltages are established across the two windings, canceling each other.
As a result, the voltage upstream of the LC filter is zero, while the load continues to be supplied continuously thanks to the energy previously stored in the filter.
The dead time is also essential to:

- avoid short circuits between the two mosfets (shoot-through);
- reduce stress on power devices;
- ensure reliable converter operation.

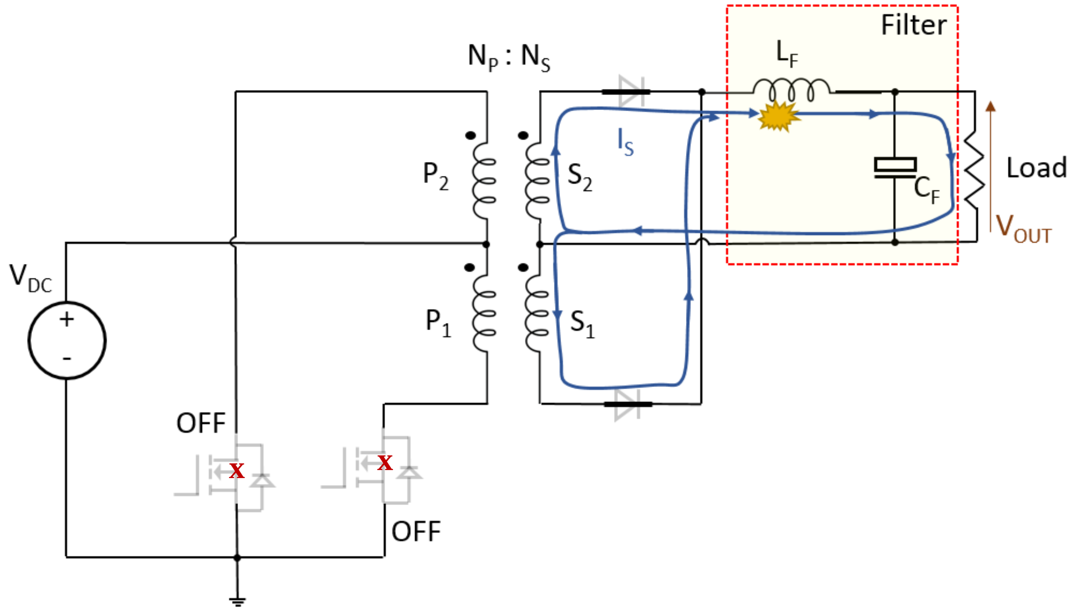


Figure 34: Third phase - Q_1 OFF, Q_2 OFF

The following figure shows the characteristic waveforms.

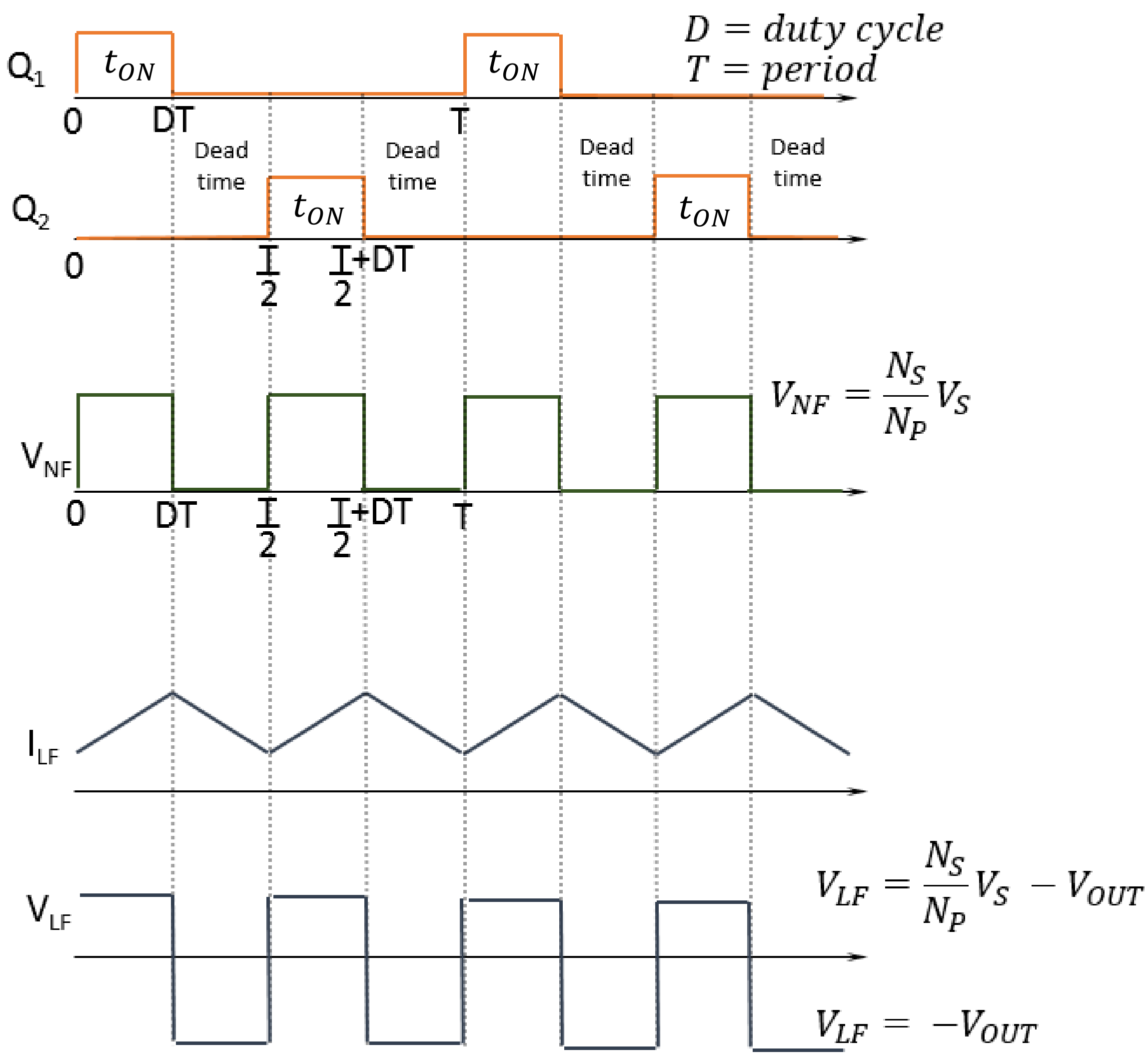


Figura 35: Relevant waveforms

Primary turns calculation

From Faraday's law for a winding on a magnetic core:

$$v_p(t) = N_p \frac{d\Phi}{dt} = N_p\, A_e \frac{dB}{dt}$$

Integrating over the half-period t_{ON}:

$$\Delta B_{max} = \frac{V_p \, t_{ON}}{N_p \, A_e} \quad \rightarrow \quad N_p = \frac{V_p \, t_{ON}}{\Delta B_{max} \, A_e}$$

where:

- $V_p = V_{DC}$ (amplitude over the half-period)
- $t_{ON} = \frac{1}{2\, f_S}$ (conduction time of one mosfet; 50% duty cycle)
- N_p = number of primary turns
- A_e = effective core cross-section

Substituting numerical values:

$$N_p = \frac{V_p \, t_{ON}}{\Delta B_{max} \, A_e} = \frac{375V \ \cdot \ 5\ \mu s}{0.22T \ \cdot \ 1.25 \cdot 10^{-4}\, m^2} = 68 \ turns$$

Regarding wire diameter and available window area, the same considerations discussed previously apply.

Knowing N_p, we can estimate the primary inductance L_p:

$$L_p = \frac{{N_p}^2 \ \mu_0 \ \mu_r \ A_e}{l_m} = \frac{68^2 \ \cdot 4\pi \cdot 10^{-7} \ \frac{H}{m} \cdot 2000 \cdot \ 1.25 \cdot 10^{-4}\, m^2}{0.05\, m} = 7.3\, mH$$

This is the total primary inductance and is useful for calculating peak current.

In CCM, the power transferred from primary to secondary (assuming unit efficiency) is:

$$P = V_{DC} \ \cdot \ I_{avg,primary}$$

Faraday's law for an inductor states:

$$V_L = L \ \frac{di}{dt}$$

Applying it to the primary, since $V_L = V_{DC} = constant$, the derivate $\frac{di}{dt}$ is constant, therefore the current $i(t)$ varies linearly with time. Since the two mosfets operate complementarily, the current has a triangular waveform over the period.

If the current rises linearly from 0 to I_{PK} in time T_{SP} (half-period of one mosfet):

$$I_{avg,primary} = \frac{1}{T_{SP}} \int_0^{T_{SP}} \frac{I_{PK}}{T_{SP}} \, t \, dt = \frac{I_{PK}}{T_{SP}^{\,2}} \int_0^{T_{SP}} t \, dt = \frac{I_{PK}}{T_{SP}^{\,2}} \frac{T_{SP}^{\,2}}{2} = \frac{I_{PK}}{2}$$

Thus:

$$I_{avg,primario} = \frac{1}{2} I_{PK} \quad \rightarrow \quad I_{PK} = 2\, I_{avg,primario} = 2\, \frac{P}{V_{DC}}$$

In RMS terms:

$$I_{RMS} = \frac{P_{OUT}}{V_{DC}} \sqrt{2} \approx 0.2 \text{ A}$$

Secondary turns calculation

From the transformer relationship:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

But $V_S = V_{OUT} + V_D$, therefore:

$$N_S = \frac{V_{OUT} + V_D}{V_{P,Average}} N_P$$

Where:

- $V_{P,Average}$ =V_{DC} (half-period voltage)
- $V_D \approx 0.7\,\mathrm{V}$ (diode voltage drops)

$$N_S = \frac{V_{OUT} + V_D}{V_{P,Average}} N_P = \frac{12 + 0.7}{375} \cdot 68 \approx 3\ turns$$

At the end of the design, core saturation must be verified:

$$\Phi_{MAX} = \mathrm{B}_{MAX}\,\mathrm{A}_e = 0.22 \cdot 1.25\,10^{-4} = 27.5\ \mu\mathrm{Wb}$$

If necessary, add an air gap in the core to avoid saturation.

IMPORTANT NOTE: the number of turns refers to a single half-winding. The total winding is never active simultaneously.

Output LC filter design

The output current is:

$$I_{OUT} = \frac{P_{OUT}}{V_{OUT}} = \frac{50}{12} \approx 4.2\mathrm{A}$$

Assume an acceptable output voltage ripple of 1%:

$$\Delta V_{OUT} = 12V \cdot 0.01 = 0.12V$$

and a current ripple of 20%:

$$\Delta I_{OUT} = 4.2A \cdot 0.20 = 0.84A$$

The voltage across the inductor is:

$$\mathrm{V}_L = L\,\frac{dI_L}{dt}$$

Discretizing:

$$\Delta t\ \mathrm{V}_L = L\ \Delta I_L\ \rightarrow\ L = \frac{\Delta t\ \mathrm{V}_L}{\Delta I_L}$$

We can assume $\mathrm{V}_L = \mathrm{V}_{OUT}$ during the ripple interval. In practice, the voltage upstream of the inductor is V_{OUT} except during dead time, where it tends to zero. Therefore, when one of the two diodes conduct, the voltage across the filter inductor is zero, while during dead time, assuming sufficient output capacitance:

$$\mathrm{V}_L = -\mathrm{V}_{OUT}$$

Assume a dead time of $0.5\ \mu\mathrm{s}$.
The output filter inductance is:

$$L_F = \frac{V_{OUT}\ t_{dead}}{\Delta I_L} = \frac{12\mathrm{V} \cdot\ 0.5\ \mu\mathrm{s}}{0{,}84\mathrm{A}} = \approx 7\mu\mathrm{H}$$

The filter capacitor is calculated assuming (conservatively) that during dead time it supplies the output current I_{OUT}.

$$I_C = C_F\ \frac{dv}{dt}\quad \rightarrow\quad \Delta V_{OUT} = \frac{1}{C_F}\int_0^{t_{dead}} I_C\ dt\quad \rightarrow\quad \Delta V_{OUT} = \frac{I_{OUT}\ \ t_{dead}}{C_F}$$

Thus:

$$C_F = \frac{I_{OUT}\ \ t_{dead}}{\Delta V_{OUT}} = \frac{4.2\mathrm{A}\ \cdot\ 0.5\ \mu\mathrm{s}}{0.12V} \approx 18\mu\mathrm{F}$$

This is the minimum value; in practice, a larger capacitance is usually selected (for safety), e.g. 100 µF, with low ESR. It is preferable to connect multiple capacitors in parallel.

Transformer core selection

The choice of the magnetic core in push-pull converters is a fundamental aspect, as it directly affects core saturation, magnetic losses, power density, and converter efficiency. Since in this circuit configuration the magnetic flux density varies bipolarly with an approximately triangular waveform, the core material must be capable of withstanding an adequate excursion of B without reaching saturation.

The saturation flux density B_{sat} of the material must be significantly higher than the maximum flux expected during operation, introducing an appropriate safety margin to account for imbalances between the push-pull branches, component tolerances, and thermal variations.

The effective core cross-section must be chosen to limit flux density excursion according to:

$$\Delta B_{max} = \frac{V_p\, t_{ON}}{N_p\, A_e}$$

A core with too small a cross-section would require a high number of turns or would rapidly saturate, while an oversized core would increase size and cost without real benefits.

Finally, core geometry must be chosen considering the trade-off between ease of winding, leakage inductance, coupling between primary and secondary, and thermal management. In push-pull converters, reducing leakage inductance is particularly important, as it can generate overvoltages on the mosfets during switching. For this reason, geometries with good magnetic coupling such as toroidal cores or EER / ETD shapes are often preferred.

10. Conclusions

In this tutorial, the fundamental principles of high-frequency transformers and their use in flyback and push–pull converters have been analyzed. Although both solutions make use of a magnetic component that appears similar at first glance, the analysis has shown that the role, operating mode, and design criteria of the transformer are profoundly different in the two cases.

Beyond the formulas and calculation procedures presented, it is essential to emphasize that the design of high-frequency transformers cannot be approached from a purely theoretical standpoint. Phenomena such as actual core saturation, leakage inductances, parasitic capacitances, and losses in the windings and magnetic material mainly become evident through practical experimentation.