Introduction to Self-stabilization

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Proof Techniques

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Conclusion 000 Reference

Outline

Self-stabilization

Hypothesis

Atomicity Scheduling

Composition

Fair Composition Crossover Composition

Proof Techniques

Transfer Function Convergence stairs

Conclusion

References

- $ightharpoonup U_0 = a$
- $ightharpoonup U_{n+1} = \frac{U_n}{2}$ if U_n is even
- $U_{n+1} = \frac{3U_n + 1}{2} \text{ if } U_n \text{ is odd}$

- $ightharpoonup U_0 = a$
- ▶ $U_{n+1} = \frac{U_n}{2}$ if U_n is even
- $U_{n+1} = \frac{3U_n + 1}{2} \text{ if } U_n \text{ is odd}$

n	0				1	5				l .		
U_n	7	11	17	26	13	20	10	5	8	4	2	1

$$ightharpoonup U_0 = a$$

►
$$U_{n+1} = \frac{U_n}{2}$$
 if U_n is even

$$U_{n+1} = \frac{3U_n+1}{2} \text{ if } U_n \text{ is odd}$$

27	41	62	31	47	71	107	161	242
121	182	91	137	206	103	155	233	350
175	263	395	593	890	445	668	334	167
251	377	566	283	425	638	319	479	719
1079	1619	2429	3644	1822	911	1367	2051	3077
4616	2308	1154	577	866	433	650	325	488
244	122	61	92	46	23	35	53	80
40	20	10	5	8	4	2	1	

- $ightharpoonup U_0 = a$
- ▶ $U_{n+1} = \frac{U_n}{2}$ if U_n is even
- $V_{n+1} = \frac{3U_n+1}{2}$ if U_n is odd
- Converges towards a "correct" behavior
 - ► 12121212121212121212121212...
 - ► Independent from the initial value

Enumerator of Even Numbers

Self-stabilization

00000000000000

```
unsigned char x = 0;
for (;;)
   printf ("%d ",x);
   x = x + 2;
   . . .
```

Self-stabilization 0000000000000

► Self-Stabilizing Enumerator of Even Numbers (Overflow Control)

```
unsigned char x;
for (;;)
   printf ("%d ",x);
   x = ((x = x + 2) > 254) ? 0: x + 2;
   . . .
```

Self-stabilization

Example

► Self-Stabilizing Enumerator of Even Numbers (Parity Check)

```
unsigned char x;
for (;;)
   printf ("%d ",x);
   x = (x \% 2) ? x + 1 : x + 2;
   . . .
```

► Self-Stabilizing Enumerator of Even Numbers (Parity Check—Reset)

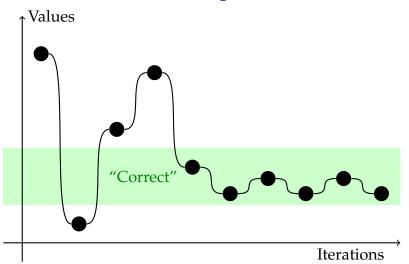
```
unsigned char x;
for (;;)
   printf ("%d ",x);
   x = (x \% 2) ? 0 : x + 2;
   . . .
```

Self-stabilization

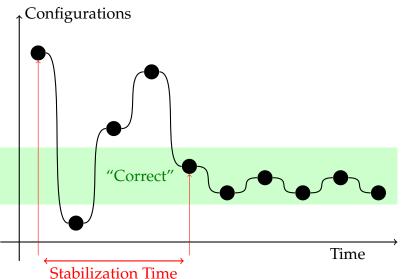
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Self-Stabilizing Enumerator of Even Numbers (Left Shift)

```
unsigned char x;
for (;;)
   printf ("%d ",x<<1);</pre>
   x++;
```



Self-stabilization



Self-stabilization 000000000000000

Memory Corruption

Example of a sequential program:

```
int x = 0;
if(x == 0)
  // code assuming x equals 0
else {
  // code assuming x does not equal 0
```

► Locality of information

- Locality of information
- Locality of time

- ► Locality of information
- Locality of time
- ► ⇒ non-determinism

- ► Locality of information
- Locality of time
- ▶ ⇒ non-determinism

Definition (Configuration)

Product of the local states of the system components.

Definition (Execution)

Interleaving of the local executions of the system components.

Definition (Classical System, a.k.a. Non stabilizing)

Starting from a particular initial configuration, the system immediately exhibits correct behavior.

Definition (Self-stabilizing System)

Starting from any initial configuration, the system eventually reaches a configuration from with its behavior is correct.

Self-stabilization

Definition (Self-stabilizing System)

Starting from any initial configuration, the system eventually reaches a configuration from with its behavior is correct.

Defined by Dijkstra in 1974

Self-stabilization

Definition (Self-stabilizing System)

Starting from any initial configuration, the system eventually reaches a configuration from with its behavior is correct.

- Defined by Dijkstra in 1974
- Advocated by Lamport in 1984 to addesss fault-tolerant issues

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Fault Tolerance

Definition ((Distributed) Task)

A task is **specified** in terms of:

Definition ((Distributed) Task)

A task is **specified** in terms of:

- ► **Safety**: *Bad actions*, which should not happen
 - At the intersection, traffic lights are green on two different axes.
 - Processes enter the critical section simultaneously.
 - Windows crashes.

Definition ((Distributed) Task)

A task is **specified** in terms of:

- ▶ **Safety**: *Bad actions*, which should not happen
- ► **Liveness**: *Good actions*, which should (eventually) happen
 - ▶ At the intersection, if one of the traffic lights is red then, it eventually becomes green.
 - ▶ Every process eventually enter the critical section.
 - ▶ *Windows* eventually reboots.

Definition (Fault)

A fault is an action that corrupts the task specification by changing the correct functioning of a system component.

Definition (Fault)

A fault is an action that corrupts the task specification by changing the correct functioning of a system component.

- ► At the intersection, traffic lights are off.
- ▶ A process requesting the critical section is stuck.
- Windows boot loops on a blue screen with white markings.

Definition (Fault)

A fault is an action that corrupts the task specification by changing the correct functioning of a system component.

- ▶ Type \rightarrow fail-stop, crash, omission, Byzantine, . . .
- Duration
- ▶ Detection Rate
- Correction Rate
- Frequency

Definition (Fault)

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Fault-tolerant algorithm ⇒ Tolerates a given class of faults

▶ **Masking FT**: Both *Safety* and *Liveness* must be guaranteed.

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Fault Tolerance

▶ Masking FT: Both *Safety* and *Liveness* must be guaranteed. Unfortunately, [FLP]!

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- ► **Masking FT**: Both *Safety* and *Liveness* must be guaranteed. Unfortunately, [FLP]!
- ► **Fail-Safe FT**: *Safety* guaranteed but not *Liveness*.
 - ► Traffic lights are red.
 - Transactions in databases.

- ► **Masking FT**: Both *Safety* and *Liveness* must be guaranteed. Unfortunately, [FLP]!
- ► **Fail-Safe FT**: *Safety* guaranteed but not *Liveness*.
- Non-Masking FT: Liveness guaranteed but not Safety.

- Masking FT: Both Safety and Liveness must be guaranteed. Unfortunately, [FLP]!
- ► **Fail-Safe FT**: *Safety* guaranteed but not *Liveness*.
- Non-Masking FT: Liveness guaranteed but not Safety.
 - ► Traffic lights are flashing orange.
 - Optimistic algorithm: Data replication mechanisms.

- ▶ Masking FT: Both *Safety* and *Liveness* must be guaranteed. Unfortunately, [FLP]!
- ► **Fail-Safe FT**: *Safety* guaranteed but not *Liveness*.
- ► Non-Masking FT: *Liveness* guaranteed but not *Safety*. Self-Stabilization: *Safety* eventually guaranteed.

Dijkstra' self-stabilizing algorithms

- Token-passing on a ring
- ► Token-passing on a chain with 4 states/process

Self-stabilization

Hypothesis Atomicity Scheduling

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Fair Composition
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Hypothesis

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Atomicity

An example of a "stabilizing" sequential program int x = 0;

```
while( x == x ) {
    x = 0;
    // code assuming x equals 0
}
```

Atomicity

► An example of a "stabilizing" sequential program

```
int x = 0;
...
while( x == x ) {
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}
```

Hypothesis

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```
0 iconst_0
1 istore_1
2 goto 7
...
5 iconst_0
6 istore_1
7 iload_1
8 iload_1
9 if icmpeq 5
```

Atomicity

► An example of a "stabilizing" sequential program

```
int x = 0;
...
while( x == x ) {
    x = 0;
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```

Hypothesis

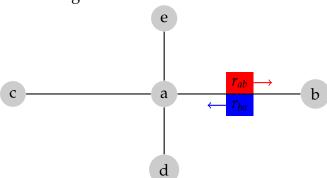
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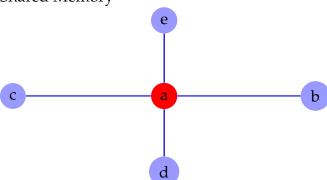
```
0 iconst_0
1 istore_1
2 goto 7
...
5 iconst_0
6 istore_1
7 iload_1
8 iload_1
9 if_icmpeq 5
```

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Shared Registers



Shared Memory



Message Passing

Shared Registers

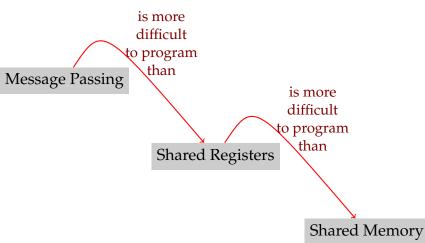
Shared Memory

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Communications





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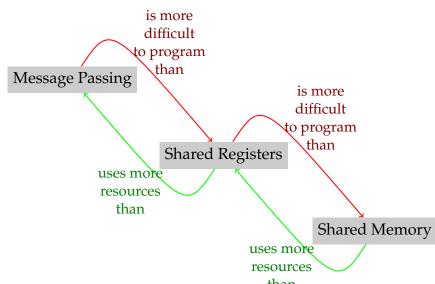
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Communications



Example

Definition (Shared Memory)

In one atomic step, read the states of all neighbors and write own state

Definition (Guarded command)

ightharpoonup Guard ightharpoonup Action

Example

Definition (Shared Memory)

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Definition (Guarded command)

- ightharpoonup Guard ightharpoonup Action
- Guard: predicate on the states of the neighborhood

Example

Definition (Shared Memory)

In one atomic step, read the states of all neighbors and write own state

Definition (Guarded command)

- ► Guard → Action
- Guard: predicate on the states of the neighborhood
- Action: executed if Guard evaluates to true

Hypothesis

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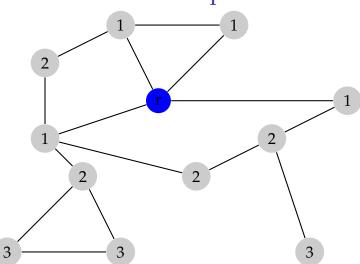
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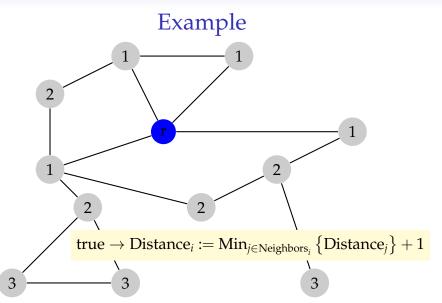
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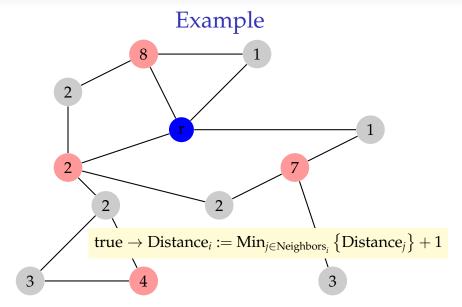
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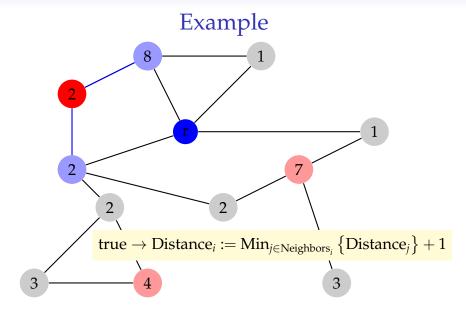
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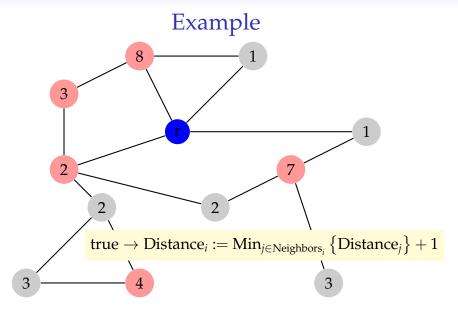














Hypothesis

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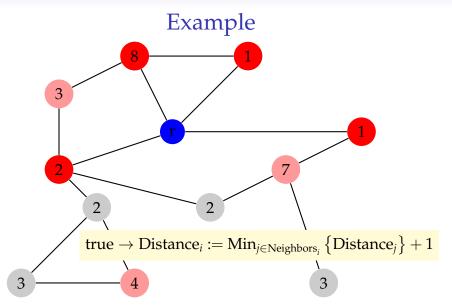
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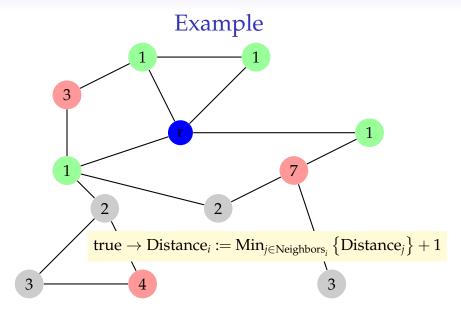
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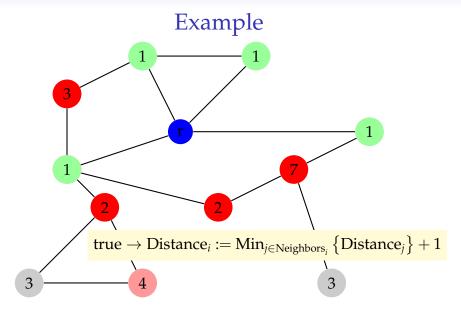
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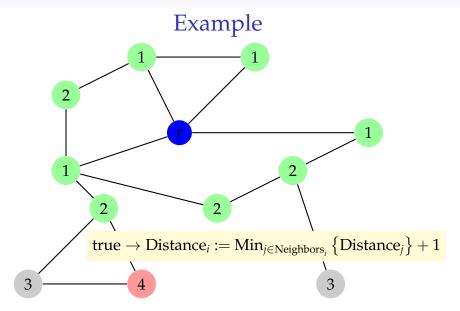




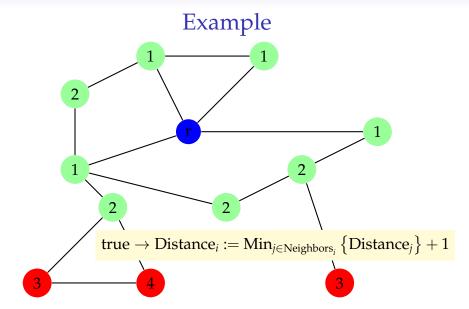














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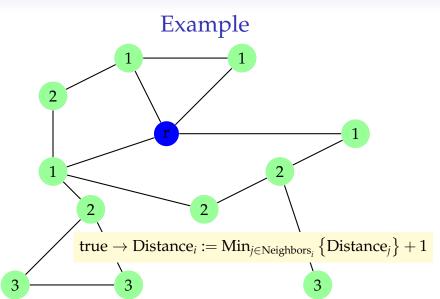
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Scheduling

Definition (Scheduler a.k.a. Daemon)

The daemon chooses among activable processors those that will execute their actions.

► The daemon can be seen as an adversary whose role is to prevent stabilization

 $\mathsf{true} \to \mathsf{color}_i := \mathsf{Min} \left\{ \Delta \setminus \{\mathsf{color}_j | j \in \mathsf{Neighbors}_i \} \right\}$

$$\Delta = \{ 0 \cdot 1 \}$$

a

b

 $\mathsf{true} \to \mathsf{color}_i := \mathsf{Min} \left\{ \Delta \setminus \{\mathsf{color}_j | j \in \mathsf{Neighbors}_i \} \right\}$

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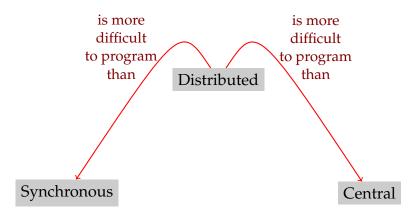
a —

b

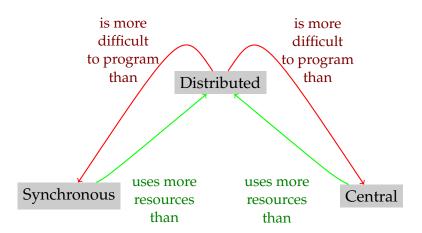
Distributed

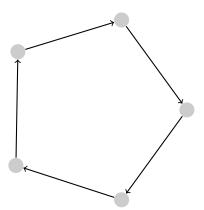
Synchronous

Central

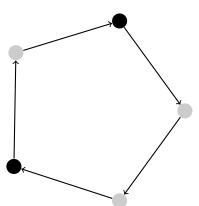


Spatial Scheduling

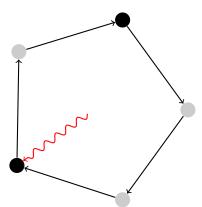




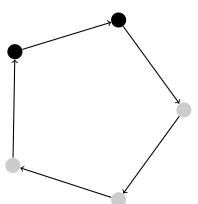




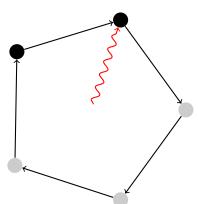




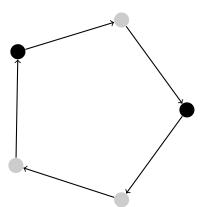




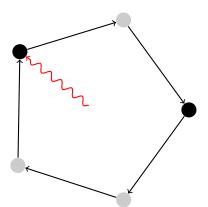




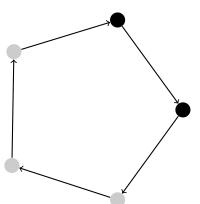




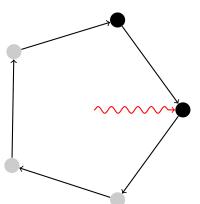




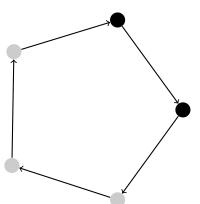




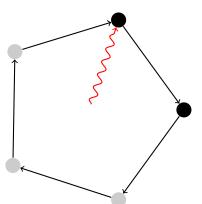




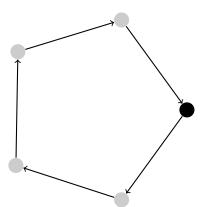














Unfair

Fair

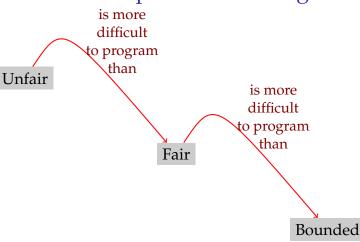
Bounded

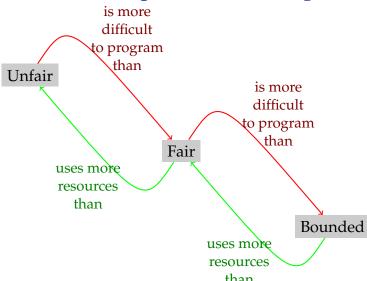
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Temporal Scheduling





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Fair Composition

Basic idea

- Compose several self-stabilizing algorithms $Al_1, Al_2, ... Al_k$ such that the results of algorithms $Al_1, Al_2, ... Al_i$ can be reused by Al_{i+1}
- ▶ Al_{i+1} can not detect whether algorithms $Al_1, Al_2, ... Al_i$ have stabilized, but behaves as if

Fair Composition

Basic idea

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Example with k = 2

- Two simple algorithms server and client are combined to obtain a more complex algorithm
- ► The server algorithm ensures that some properties (used by the client) will be eventually verified

Assume the server algorithm Al_1 solves a task defined by a set of legal executions T_1 , and the client algorithm Al_2 solves T_2

 A_i

Let A_i be the set of states of process P_i for Al_1 , and let $S_i = A_i \times B_i$ be the set of states of process P_i for Al_2 , where anytime P_i executes Al_2 , it modifies the B_i part of $A_i \times B_i$



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Fair Composition

Definition (Fair composition)

Al is a fair composition of Al_1 and Al_2 if, in Al, every process alternatively executes actions of Al_1 and Al_2

Theorem

If Al_2 is self-stabilizing for T_2 given T_1 , and if Al_1 is self-stabilizing for T_1 , then the fair composition of Al_1 and Al_2 is self-stabilizing for T_2



 Al_1 Al_2

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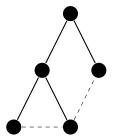
Theorem

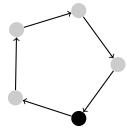
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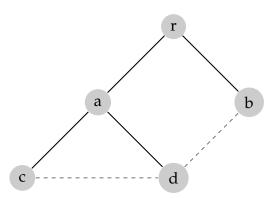
$$A_i \rightarrow B_i$$

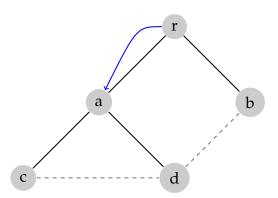
$$Al_1$$
 Al_2

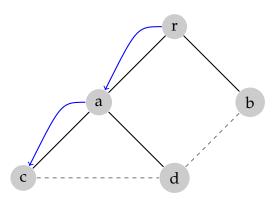
- We are given two self-stabilizing algorithms, one for constructing a tree, one for mutual exclusion on a ring
- We wish to construct a self-stabilizing mutual exclusion algorithm on general graphs

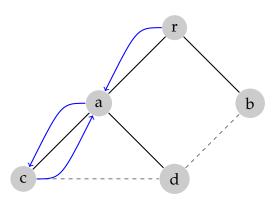


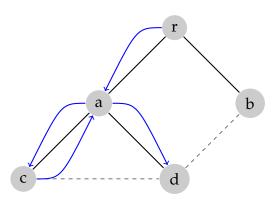


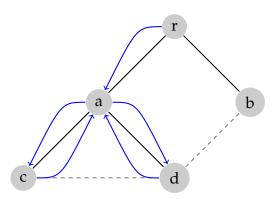


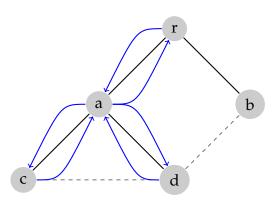


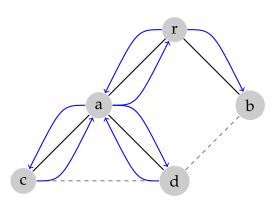


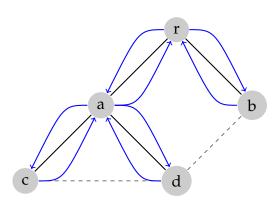












Hypothesis 0 00000 00000 Composition

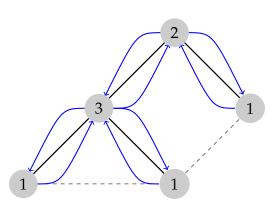
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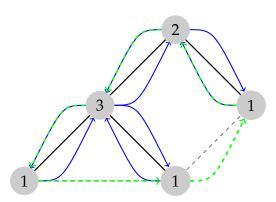
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Crossover Composition

Basic Idea

- ▶ We are given two algorithms Al_1 and Al_2
- ▶ Al_1 is correct with hypothesis H_1 and Al_2 is correct with hypothesis H_2
- \blacktriangleright H_2 is more restrictive than H_1

Definition (Crossover Composition)

The crossover composition is such that if Al_2 is conditionnally executed (only when Al_1 is executed), Al_2 is then correct with hypothesis H_1

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Conclusion 000 Reference

Self-stabilization

Hypothesis Atomicity Scheduling

Composition
Fair Composition
Crossover Composition

Proof Techniques
Transfer Function
Convergence stairs

Conclusion

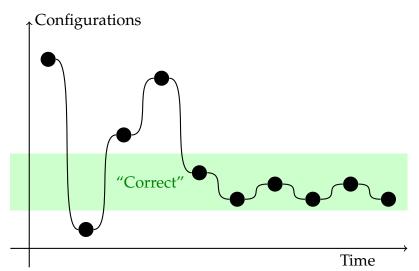
References

Transfer Function

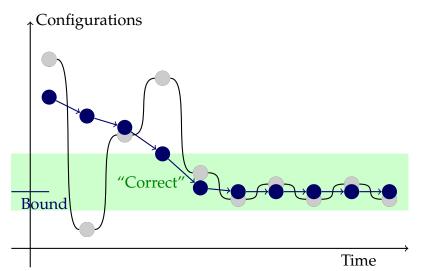
Basic Idea

- $ightharpoonup c_1
 ightharpoonup c_2
 ightharpoonup c_3
 ightharpoonup c_4
 ightharpoonup \cdots
 ightharpoonup c_i$
- ► $FP(c_1) > FP(c_2) > FP(c_3) > ... > FP(c_i) = bound$
- Used to prove convergence
- Can be used to compute the number of steps to reach a legitimate configuration

Transfer Function

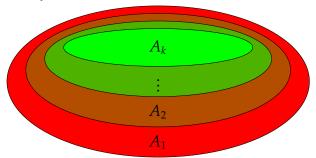


Transfer Function



Convergence stairs

- $ightharpoonup A_i$ is a predicate
- $ightharpoonup A_k$ is legitimate
- For any *i* between 1 and k, A_{i+1} is a refinement of A_i



Self-stabilization

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Fair Composition Crossover Composition

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Pros

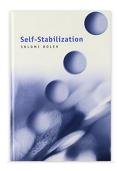
- ▶ The network does not need to be initialized
- When a fault is diagnosed, it is sufficient to identify, then remove or restart the faulty components
- ► The self-stabilization property does not depend on the nature of the fault
- ► The self-stabilization property does not depend on the extent of the fault

Self-stabilization

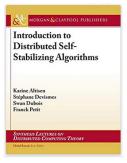
Cons

- ► *A priori,* "eventually" does not give any bound on the stabilization time
- ► *A priori*, nodes never know whether the system is stabilized or not
- ► A single failure may trigger a correcting action at every node in the network
- ► Faults must be sufficiently rare that they can be considered are transient

References



Self-Stabilization. S. Dolev The MIT Press, 2000.



Introduction to Distributed Self-Stabilizing Algorithms.

K. Altisen, S. Devismes, S. Dubois, and F. Petit Morgan & Claypool Publishers, 2019.