# Homework 5

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***Assignment:*** Grammar for $L = \{1^k / k \text{ is composite}\}$
Any compound number is given by one of the following equations:

$c_1 = 2(n+2)$
$c_2 = 3(n+2)$
$c_3 = (1+6n)(1+6m) = 1+6n+6m+36mn = 1+6(m+n)+36mn \quad n, m \neq 0$
$c_4 = (5+6n)(5+6m) = 25+30m+30n+36mn = 25+30(m+n)+36mn$
$c_5 = (1+6n)(5+6m) = 5+30n+6m+36mn \quad n \neq 0$

A grammar for each equation

$$\begin{aligned} S_1 &\to S_1' II \\ S_1' &\to S_1' I \mid M_2 \\ M_2 I &\to 11 M_2 \\ M_2 &\to \varepsilon \end{aligned}$$

$$\begin{aligned} S_2 &\to S_2' II \\ S_2' &\to S_2' I \mid M_3 \\ M_3 I &\to 111 M_3 \\ M_3 &\to \varepsilon \end{aligned}$$

$$
\begin{aligned}
S_3 &\to 1S_3' \\
S_3' &\to S_3'' III_{mn} \\
S_3'' &\to S_3'' II_{mn} \mid M_{1x1} \\
M_{1x1}I &\to 1^6 M_{1x1} \\
M_{1x1}I_{mn} &\to 1^{36} M_{1x1} \\
M_{1x1} &\to \varepsilon
\end{aligned}
$$

$$
\begin{aligned}
S_4 &\to 1^{25} S_4' \\
S_4' &\to S_4' II_{mn} \mid M_{5x5} \\
M_{5x5}I &\to 1^{30} M_{5x5} \\
M_{5x5}I_{mn} &\to 1^{36} M_{5x5} \\
M_{5x5} &\to \varepsilon
\end{aligned}
$$

$$
\begin{aligned}
S_5 &\to 1^5 S_5' I_n I_{mn} \\
S_5' &\to S_5' I_n I_{mn} \mid S_5' I_m I_{mn} \mid M_{1x5} \\
M_{1x5}I_n &\to 1^{30} M_{1x5} \\
M_{1x5}I_m &\to 1^6 M_{1x5} \\
M_{1x5}I_{mn} &\to 1^{36} M_{1x5} \\
M_{1x5} &\to \varepsilon
\end{aligned}
$$

The grammar with starting rule

$$A \to S_1 \mid S_2 \mid S_3 \mid S_4 \mid S_5$$

generates all strings $1^k$ where $k$ is composite