3.54 Suppose that Y is a binomial random variable based on n trials with success probability p and consider $Y^\star = n - Y$.

a Argue that for $y^\star = 0, 1, \ldots, n$

$$P(Y^\star = y^\star) = P(n - Y = y^\star) = P(Y = n - y^\star).$$

b Use the result from part (a) to show that

$$P(Y^\star = y^\star) = \binom{n}{n - y^\star} p^{n-y^\star} q^{y^\star} = \binom{n}{y^\star} q^{y^\star} p^{n-y^\star}.$$

c The result in part (b) implies that $Y^\star$ has a binomial distribution based on n trials and "success" probability $p^\star = q = 1 - p$. Why is this result "obvious"?

3.55 Suppose that Y is a binomial random variable with $n > 2$ trials and success probability p. Use the technique presented in Theorem 3.7 and the fact that $E\{Y(Y-1)(Y-2)\} = E(Y^3) - 3E(Y^2) + 2E(Y)$ to derive $E(Y^3)$.

3.56 An oil exploration firm is formed with enough capital to finance ten explorations. The probability of a particular exploration being successful is .1. Assume the explorations are independent. Find the mean and variance of the number of successful explorations.

3.57 Refer to Exercise 3.56. Suppose the firm has a fixed cost of \$20,000 in preparing equipment prior to doing its first exploration. If each successful exploration costs \$30,000 and each unsuccessful exploration costs \$15,000, find the expected total cost to the firm for its ten explorations.

3.58 A particular sale involves four items randomly selected from a large lot that is known to contain 10% defectives. Let Y denote the number of defectives among the four sold. The purchaser of the items will return the defectives for repair, and the repair cost is given by $C = 3Y^2 + Y + 2$. Find the expected repair cost. [*Hint:* The result of Theorem 3.6 implies that, for any random variable Y, $E(Y^2) = \sigma^2 + \mu^2$.]

3.59 Ten motors are packaged for sale in a certain warehouse. The motors sell for \$100 each, but a double-your-money-back guarantee is in effect for any defectives the purchaser may receive. Find the expected net gain for the seller if the probability of any one motor being defective is .08. (Assume that the quality of any one motor is independent of that of the others.)

3.60 A particular concentration of a chemical found in polluted water has been found to be lethal to 20% of the fish that are exposed to the concentration for 24 hours. Twenty fish are placed in a tank containing this concentration of chemical in water.

a Find the probability that exactly 14 survive.

b Find the probability that at least 10 survive.

c Find the probability that at most 16 survive.

d Find the mean and variance of the number that survive.

3.61 Of the volunteers donating blood in a clinic, 80% have the Rhesus (Rh) factor present in their blood.

a If five volunteers are randomly selected, what is the probability that at least one does not have the Rh factor?

b If five volunteers are randomly selected, what is the probability that at most four have the Rh factor?

...allest number of volunteers who must be selected if we want to be at least

d Use Definition 3.5 and the result in part (b) to show that

$$V(X) = E\{[(X - E(X)]^2\} = E[(Y - \mu)^2] = \sigma^2;$$

that is, $X = Y + 1$ and Y have *equal* variances.

3.31 Suppose that Y is a discrete random variable with mean μ and variance σ^2 and let $W = 2Y$.

a Do you expect the mean of W to be larger than, smaller than, or equal to $\mu = E(Y)$? Why?

b Use Theorem 3.4 to express $E(W) = E(2Y)$ in terms of $\mu = E(Y)$. Does this result agree with your answer to part (a)?

c Recalling that the variance is a measure of spread or dispersion, do you expect the variance of W to be larger than, smaller than, or equal to $\sigma^2 = V(Y)$? Why?

d Use Definition 3.5 and the result in part (b) to show that

$$V(W) = E\{[W - E(W)]^2\} = E[4(Y - \mu)^2] = 4\sigma^2;$$

that is, $W = 2Y$ has variance four times that of Y.

3.32 Suppose that Y is a discrete random variable with mean μ and variance σ^2 and let $U = Y/10$.

a Do you expect the mean of U to be larger than, smaller than, or equal to $\mu = E(Y)$? Why?

b Use Theorem 3.4 to express $E(U) = E(Y/10)$ in terms of $\mu = E(Y)$. Does this result agree with your answer to part (a)?

c Recalling that the variance is a measure of spread or dispersion, do you expect the variance of U to be larger than, smaller than, or equal to $\sigma^2 = V(Y)$? Why?

d Use Definition 3.5 and the result in part (b) to show that

$$V(U) = E\{[U - E(U)]^2\} = E[.01(Y - \mu)^2] = .01\sigma^2;$$

that is, $U = Y/10$ has variance .01 times that of Y.

3.33 Let Y be a discrete random variable with mean μ and variance σ^2. If a and b are constants, use Theorems 3.3 through 3.6 to prove that

a $E(aY + b) = aE(Y) + b = a\mu + b.$

b $V(aY + b) = a^2V(Y) = a^2\sigma^2.$

3.34 The manager of a stockroom in a factory has constructed the following probability distribution for the daily demand (number of times used) for a particular tool.

y	0	1	2
$p(y)$	.1	.5	.4

It costs the factory \$10 each time the tool is used. Find the mean and variance of the daily cost for use of the tool.

3.4 The Binomial Probability Distribution

Some experiments consist of the observation of a sequence of identical and independent trials, each of which can result in one of two outcomes. Each item leaving a manufacturing production line is either defective or nondefective. Each shot in a sequence of firings at a target can result in a hit or a miss, and each of n persons