Long-Run Consumption and Inflation Risks in Stock and Bond Returns

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Abstract

We derive a long-run risk model with time-varying inflation non-neutrality and show that it matches a challenging set of moments describing the joint dynamics of stock returns, term structure of nominal bond yields and returns, as well as macroeconomic fundamentals. Furthermore, we match not only more moments than other long-run risk models, but also moments that remained unaddressed in the literature so far, i.e., the volatility of the risk-free rate and of the dividend-price ratio, and the dividend-price ratio ability to predict stock market returns, consumption and dividend growth rates. More importantly, we match this challenging set of moments, while simultaneously holding the risk aversion and elasticity of intertemporal substitution parameters low.

JEL Classification Codes: G11, G12, G13

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We derive a long-run risk model (LRR) with time-varying inflation non-neutrality that allows us to match joint dynamics of stock returns, term structure of nominal bond yields and returns, as well as macroeconomic fundamentals. Modeling these joint dynamics is challenging and, so far, has not been fully addressed in the literature. Even for the long-run risk model, despite the fact that it has been shown to be relatively successful in addressing many asset pricing puzzles,[1] there is only a partial evidence on whether this model can match the stock and bond markets dynamics.[2] Our goal is to fill this gap.

To price assets in the stock and bond markets, we introduce inflation effects into the otherwise standard long-run risk model of Bansal and Yaron (2004). There are two key new ingredients that we introduce to the LRR model. First, we allow for both neutral and non-neutral effects of inflation on the real economy. The neutral effects lead to the contemporaneous relation between expected consumption growth and expected inflation, and the non-neutral effects allow inflation to also predict consumption growth. Both are important in matching the risk premiums across the two markets. Non-neutral effects, additionally, allow inflation shock to affect marginal utility and hence inflation risk is priced in our economy.

Second, we allow inflation shock to predict future real consumption growth in a time-varying way. Unconditionally, inflation shock predicts consumption growth in a negative way in our model, as in, for example, Piazzesi and Schneider (2006). We, then, define a nominal-real covariance (NRC) process that determines the time-variation in this predictive regression, following Boons, Duarte, de Roon, and Szymanowska (2020). They show that the NRC defined this way, is similar to the NRC defined in Campbell, Sunderam, and Viceira (2017) as the negative of the stock market beta of a long-term bond, hence it can also capture the time-variation in the stock-bond comovement. In our model, when the NRC is negative, a positive inflation shock is a bad signal for future consumption growth, while it predicts consumption growth positively when the NRC is positive. The time-varying signaling role of

[1]For example, Bansal and Yaron (2004), and Bansal, Kiku, and Yaron (2012) match the joint dynamics of consumption, the equity risk premium and its time-variation, and the risk-free rate and volatility puzzles. Bansal and Shaliastovich (2012) focus on the joint dynamics of bond and foreign currency markets, and are able to account for the bond return predictability and the concomitant violations of the uncovered interest rate parity.

[2]See, for example, Eraker (2008), and Hasseltoft (2012).

inflation features in, for example, David and Veronesi (2013) and Boons, Duarte, de Roon, and Szymanowska (2020). The NRC is important in matching the volatility as well as predictive moments in both markets.

The nominal-real covariance introduces, thus, a new source of uncertainty in our economy. Despite the fact that, as in Bansal and Yaron (2004), we only have one stochastic volatility in the model, namely the stochastic volatility of the latent component of expected consumption growth, the expected consumption growth in our setup will also depend on the nominal-real covariance. Hence, the nominal-real covariance plays a dual role in our model, determining non only the time-variation in the predictive relation between inflation and consumption growth, but also driving the stochastic volatility of expected consumption.

We calibrate our model using US stock and bond markets data. We use annual real Personal Consumption Expenditure of nondurables and services to construct inflation and consumption variables. For the stock and bond market returns and yields, we use monthly data from the CRSP and Fama-Bliss databases, and convert them to annual frequency. We calibrate the model to target 52 moments in the generalized method of moments estimation. The moments aim to capture (joint) dynamics of consumption, inflation, stock and bond market returns and yields. Thus, we are able not only to match more moments than many of the previous studies but also address some of the criticism raised against the long-run risk models, most notably by Beeler and Campbell (2012). More important, we match this challenging set of moments, while simultaneously holding the risk aversion and elasticity of intertemporal substitution parameters low.

The model matches the key descriptive statistics of our exogenous processes, namely inflation, consumption and dividend growth rates, quite precisely. We are also able to match key descriptive statistics of stock market returns and the risk-free rate. Most importantly, we are able to match the volatility of the risk-free rate and the price-dividend ratio. These moments have been challenging for the other long-run risk models in the literature, see, for example, Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012), Beeler and Campbell (2012).

We are also able to address one of the main criticisms raised by Beeler and Campbell

(2012) against the long-run models, namely their ability to capture the evidence of predictability from the price-dividend ratio, that remained unaddressed in the literature thus far. Our model, consistent with the data, generates small and decreasing with the horizon predictability for the consumption and dividend growth rates, simultaneously implying high and increasing over the horizon predictability for the stock market returns.

Finally, our model, consistent with the data, implies an upward-sloping term structure of nominal bond yields and returns, a downward-sloping term structure of the yield volatility, and an upward-sloping one for the volatility of bond returns. To the best of our knowledge, we are the first to match key summary statistics for both yields and returns in the bond markets, on top of the joint dynamics of the macro variables and the stock market returns and predictability.

We contribute to the small but growing literature that aims to jointly model stock and bond returns. Research on modeling these two markets has traditionally treated them separately, with recent contributions attempting to propose unifying frameworks. To jointly model stock and bond markets, for example, Campbell and Viceira (2001) use the model of optimal consumption and portfolio choice; d'Addona and Kind (2006) the affine term structure model; Bekaert, Engstrom, and Xing (2009) the habit model; Koijen, Lustig, and Van Nieuwerburgh (2017) the dynamic no-arbitrage model; and Campbell, Sunderam, and Viceira (2017) a linear-quadratic model that account for the changing covariance of bonds with stocks. These contributions mainly focused on examining the relation between the aggregate stock and bond markets. In the context of the long-run risk model, Hasseltoft (2012) in a standard LRR model, and Eraker (2008) in a non-Gaussian model, attempt to match the dynamics of the two markets and the macroeconomy. We contribute to this literature by introducing time-varying non-neutral inflation effects, which allow us to address some unresolved empirical challenges (i.e., volatility of the price-dividend ratio, predictability). We are also the first ones to model both bond yields and returns, next to the stock markets and macro fundamentals.

We also contribute to the literature on the long-run risk models in general that originates from Bansal and Yaron (2004). We are not the first ones to augment the LRR model with some inflation processes (e.g., Koijen, Lustig, Van Nieuwerburgh, and Verdelhan (2010), Hasseltoft (2012), and Bansal and Shaliastovich (2012)). The key novelty of our extension is the addition of the nominal-real covariance. Boons, Duarte, de Roon, and Szymanowska (2020) also study the nominal-real covariance in the context of the long-run risk model, but they replace the latent long-run component of expected consumption growth with inflation and focus only on the stock market.

We also contribute to the literature that identifies and study the nominal-real covariance. The bond market literature has identified the nominal-real covariance as an important driver of the time-variation in bond prices and in the comovement of stocks and bonds. Campbell, Sunderam, and Viceira (2017) finds that the variation in the nominal-real covariance accounts for the changing correlation between stock and bond markets. Kang and Pflueger (2015) find that nominal-real covariance affects corporate bond yield spreads. The nominal-real covariance also features prominently in recent term structure literature that analyzes the implications of the zero lower bound (see, e.g., Gourio and Ngo (2016), Bilal (2017), Bretscher, Hsu, and Tamoni (2017)). We contribute to this literature by showing that the nominal-real covariance is also important for capturing the time-variation in stock and bond markets.

1 Model

Our model augments the long-run risk model of Bansal and Yaron (2004) with exogenous processes for inflation, expected inflation and the nominal-real covariance. Several papers in the literature have also augmented the LRR model with some inflation processes (e.g., Koijen, Lustig, Van Nieuwerburgh, and Verdelhan (2010), Hasseltoft (2012), and Bansal and Shaliastovich (2012)). The key novelty of our extension is the addition of the nominal-real covariance, an important driver of the time-variation in bond prices and in the comovement of stocks and bonds (e.g., Kang and Pflueger (2015), Campbell, Sunderam, and Viceira (2017), Boons, Duarte, de Roon, and Szymanowska (2020)).[3]

[3]Detailed derivations for all the results presented in this section can be found in the Online Appendix.

1.1 Exogenous Processes and Stochastic Discount Factor

The real side of the economy is given by Equations (1) through (4) that describe the exogenous processes for real consumption growth (Δc_{t+1}), long-run component of expected consumption growth (x_{t+1}), stochastic volatility of long run component of expected consumption growth ($\sigma_{x,t+1}$), and real dividend growth (Δd_{t+1}). The noimnal side of the economy is given by Equations (5) through (7) that describe the exogenous processes for inflation (π_{t+1}), expected inflation ($\bar{\pi}_{t+1}$), and the nominal-real covariance ($\sigma_{g,t+1}$).

$$\Delta c_{t+1} = \mu_c + x_t + \xi_c \sigma_{g,t-1} \varepsilon_{\pi,t} + \sigma_{x,t} \varepsilon_{c,t+1}, \tag{1}$$

$$x_{t+1} = \rho_x x_t + \phi_x \sigma_{x,t} \varepsilon_{x,t+1}, \tag{2}$$

$$\sigma^2_{x,t+1} = \sigma^2_x + v_x(\sigma^2_{x,t} - \sigma^2_x) + \phi_{s,xs} \varepsilon_{s,t+1} \tag{3}$$

$$\Delta d_{t+1} = \mu_d + \rho_{d,x} x_t + \xi_d \sigma_{g,t-1} \varepsilon_{\pi,t} + \phi_{d,xc} \sigma_{x,t} \varepsilon_{c,t+1}, \tag{4}$$

$$\pi_{t+1} = \mu_\pi + \bar{\pi}_t + \xi_\pi \varepsilon_{\pi,t} + \phi_\pi \varepsilon_{\pi,t+1}, \tag{5}$$

$$\overline{\pi}_{t+1} = \rho_{\overline{\pi}} \overline{\pi}_t + \phi_{\overline{\pi},x} \sigma_{x,t} \varepsilon_{x,t+1} + \phi_{\overline{\pi},x,p} \sigma_{x,t} \varepsilon_{p,t+1}, \tag{6}$$

$$\sigma_{g,t+1} = \sigma_g + \rho_g \left(\sigma_{g,t} - \sigma_g\right) + \phi_g \varepsilon_{g,t+1} \tag{7}$$

$$\varepsilon_{c,t+1}, \varepsilon_{x,t+1}, \varepsilon_{s,t+1}, \varepsilon_{\pi,t+1}, \varepsilon_{p,t+1}, \varepsilon_{g,t+1} \sim iid\, \mathcal{N}\left(0,1\right). \tag{8}$$

Real consumption growth (Δc_{t+1}) in Equation (1) is determined by its unconditional mean μ_c, two components of its conditional mean, x_t and $\xi_c \sigma_{g,t-1} \varepsilon_{\pi,t}$, and a real consumption shock $\varepsilon_{c,t+1}$. The first component of the conditional expected consumption growth, given in Equation (2), is a latent, predictable, long-run component of Bansal and Yaron (2004) that is persistent (ρ_x), and subject to $\varepsilon_{x,t+1}$ shock. Second component of the conditional expected consumption growth is new and arises because inflation shock $\varepsilon_{\pi,t}$, predicts consumption growth in a time-varying manner, where the sign and magnitude of this predictability are given by $\xi_c \sigma_{g,t-1}$. The time-varying volatility of (expected) consumption growth is given in

Equation (3), which is subject to $\varepsilon_{s,t+1}$ shock. Dividends in Equation (4) are subject to the same shocks as consumption, but with potentially different exposures.

The inflation in Equation (5) is an ARMA(1,1)-process determined by its unconditional mean μ_π, expected inflation $\bar{\pi}_t$ and inflation shock ε_π. Expected inflation given in Equation (6) is persistent ($\rho_{\overline{\pi}}$) and it is affected by its own shock $\varepsilon_{p,t+1}$ and a shock to long-run component of expected consumption growth $\varepsilon_{x,t+1}$. Both shocks are heteroscedastic and subject to the same volatility process $\sigma_{x,t}$. The nominal-real covariance, $\sigma_{g,t+1}$, follows the mean-reverting process given in Equation (7) and it is subject to shock $\varepsilon_{g,t+1}$. All the shocks in our economy, real and nominal, are standard normal and uncorrelated.

We assume a representative agent with Epstein-Zin utility. The (log) nominal stochastic discount factor (SDF) takes then the following form

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - (1-\theta)\, r_{c,t+1}, \tag{9}$$

where $r_{c,t+1}$ is the (log) return on the wealth portfolio, that is, the claim on aggregate consumption; $\delta \in (0,1)$ is the discount rate; $\psi > 0$ is the elasticity of intertemporal substitution (EIS); and $\theta = (1-\gamma)\,/\,(1-1/\psi)$, with $\gamma > 0$ the coefficient of relative risk aversion.

1.2 Inflation Effects in the Model

Our model implies two channels that lead to the relation between inflation and the real economy, (i) via $\varepsilon_{x,t+1}$ that affects expected inflation contemporaneously, and (ii) non-neutral, time-varying effects of inflation on consumption growth via $\varepsilon_{\pi,t}$.

First, a shock to the long-run component of expected consumption growth $\varepsilon_{x,t+1}$, affects expected inflation and leads to the contemporaneous relation between expected inflation and expected consumption growth, and to a predictive relation between expected inflation and consumption growth next period, i.e., $Cov_t(x_{t+1}, \overline{\pi}_{t+1}) = Cov_t(\Delta c_{t+2}, \overline{\pi}_{t+1}) = \phi_x \phi_{\overline{\pi},x} \sigma_{x,t}$. Furthermore, volatility of expected inflation and consumption are governed by the same process, hence these contemporaneous and predictive relations are time-varying and their sign and magnitude are determined by ϕ_x, $\phi_{\overline{\pi},x}$ and $\sigma_{x,t}$.

Second, a shock to inflation $\varepsilon_{\pi,t}$, predicts consumption growth and leads to a predictive relation between consumption growth and inflation $Cov_t(\Delta c_{t+2}, \pi_{t+1}) = \phi_\pi \xi_c \sigma_{g,t}$. This predictive relation is also time-varying, but its sign and magnitude are determined by ϕ_π, ξ_c and $\sigma_{g,t}$. This expression illustrates why we call $\sigma_{g,t}$ the nominal-real covariance.

Hence, there are two sources of uncertainty about the relation between inflation and the real economy in our model: the stochastic volatility of the long-run component of expected consumption growth $\sigma_{x,t+1}$, and the nominal-real covariance $\sigma_{g,t+1}$. Moreover, both of them determine the stochastic volatility of expected consumption growth, despite the fact that stochastic volatility of consumption growth is driven by $\sigma_{x,t+1}$ alone. Thus, the nominal-real covariance plays a dual role in our model, determining not only the time variation in the predictability of consumption with inflation but also the stochastic volatility of expected consumption growth.

1.3 Wealth-consumption ratio

To price the assets we conjecture (and later verify) that the log wealth-consumption ratio is linear quadratic in the state variables x_t, $\sigma^2_{x,t}$, $\overline{\pi}_t$, $\sigma_{g,t}$, ε_π, and has the following form

$$wc_t = A_0 + A_1 x_t + A_2\left(\sigma^2_{x,t} - \sigma^2_x\right) + A_3 \overline{\pi}_t + A_4 \sigma_{g,t-1}\varepsilon_{\pi,t} + A_5 \varepsilon_{\pi,t} + A_6\left(\sigma_{g,t} - \sigma_g\right) + A_7\left(\sigma^2_{g,t} - E\left[\sigma^2_{g,t}\right]\right), \tag{10}$$

where the loadings on the state variables are

$$A_1 = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho_x}, \qquad A_2 = \theta\left(1-\frac{1}{\psi}\right)^2 \frac{(1-\kappa_1\rho_x)^2 + \kappa_1^2\phi_x^2}{2(1-\kappa_1\rho_x)^2(1-\kappa_1\nu_x)} \tag{11}$$

$$A_3 = 0, \qquad A_4 = \left(1-\frac{1}{\psi}\right)\xi_c, \qquad A_5 = 0, \tag{12}$$

$$A_6 = \frac{2\kappa_1 A_7 \sigma_g \nu_g(\nu_g - 1)[1 + 2\theta\kappa_1 A_7 \phi_g^2]}{(\kappa_1\nu_g - 1) + 2\theta\kappa_1^2 A_7 \phi_g^2 \nu_g}, \tag{13}$$

$$A_7 = \theta^{-1}\frac{\psi(\nu_g^2\kappa_1 - 1) + \sqrt{\psi^2(1-\nu_g^2\kappa_1)^2 - 4(\psi-1)^2\phi_g^2\nu_g^2\theta^2\xi_c^2\kappa_1^4}}{4\nu_g^2\psi\phi_g^2\kappa_1^2}, \tag{14}$$

where $\kappa_1 \in (0,1)$ is a linearization constant.

The intuition behind A_1 and A_2 in Equation (11) is identical to the one of the standard long-run risk model. When $\psi > 1$ the intertemporal substitution effect dominates the wealth effects, hence agents invest more in response to higher expected growth, which leads to the increase in wealth-consumption ratio and a positive A_1. A_2 is negative when $\gamma > 1$ and $\psi > 1$, thus $\theta < 0$, leading to the negative relation between volatility and the wealth consumption ratio. The strength of this relation depends on how persistent the volatility process ν_x and long-run component of expected consumption ρ_x are.

A_3 through A_5 in Equation (12) capture the inflation effects in our model. Because we do not allow for non-neutral effects of (expected) inflation on consumption growth, the A_3 and A_5 are zero. Hence, the expected inflation and inflation shock do not affect the wealth-consumption ratio but they will become important in pricing the nominal bonds. A_4 captures our non-neutral effects of inflation on consumption growth, namely the time-varying predictability of consumption growth from inflation shock. In our calibrations we set $\xi_c < 0$, which implies $A_4 < 0$, therefore higher $\sigma_{g,t-1}\varepsilon_{\pi,t}$ decreases the wealth-consumption ratio. The time-varying $\sigma_{g,t-1}$ determines whether inflation shock itself has a positive or negative effect for expected consumption growth. When $\sigma_{g,t-1}$ is positive, a positive inflation shock is good news for expected consumption growth, but it is a bad signal of future growth when the nominal-real covariance is negative.

A_6 captures the direct effect of time-variation in the nominal-real covariance. In our calibrations $A_6 > 0$, hence a positive nominal-real covariance leads to higher wealth-consumption ratio because it signals that a positive inflation shock leads to lower expected future consumption growth (consistent with negative A_4).

A_7 captures the second-role of the nominal real-covariance in our model, namely that of stochastic volatility of expected consumption. This is a second source of stochastic volatility in our model, next to the stochastic volatility effects of $\sigma_{x,t}$ that are captured by A_2. The intuition is similar, though; higher the stochastic volatility of expected consumption growth is detrimental for asset prices and leads to a lower wealth-consumption ratio. Hence, when $\gamma > 1$ and $\psi > 1$, thus $\theta < 0$, the A_7 is negative, similar to A_2. The fact that nominal-real covariance can change sign is unimportant for this channel.

1.4 Prices of risk in the economy

Using the solution for the wealth-consumption ratio, allows us to write the equilibrium stochastic discount factor in terms of the fundamental shocks in our economy in the following way

$$\begin{aligned} m_{t+1} - E_t[m_{t+1}] &= -\lambda_c \sigma_{x,t} \varepsilon_{c,t+1} - \lambda_x \sigma_{x,t} \varepsilon_{x,t+1} - \lambda_s \varepsilon_{s,t+1} \\ &- \lambda_p \sigma_{x,t} \varepsilon_{p,t+1} - \lambda_\pi \sigma_{g,t} \varepsilon_{\pi,t+1} - \lambda_{g,t} \varepsilon_{g,t+1}, \end{aligned} \tag{15}$$

where the prices of risk in the economy are

$$\lambda_c = \gamma, \qquad \lambda_x = (\gamma - \frac{1}{\psi}) \frac{\kappa_1 \phi_x}{1 - \kappa_1 \rho_x}, \qquad \lambda_s = (1-\theta)\kappa_1 A_2 \phi_{s,xs}, \tag{16}$$

$$\lambda_p = 0, \qquad \lambda_\pi = (\gamma - \frac{1}{\psi}) \kappa_1 \xi_c, \tag{17}$$

$$\lambda_{g,t} = (1-\theta)\kappa_1 \phi_g (A_6 + 2A_7 \sigma_g + 2A_7 \nu_g (\sigma_{g,t} - \sigma_g)). \tag{18}$$

As in standard long-run risk model, the price of short-run consumption risk λ_c, is equal to the risk aversion coefficient, the prices of the long run risk λ_x is positive, and the price of its volatility risk λ_s negative, assuming early resolution of uncertainty.

Given that the contemporaneous innovations to inflation do not impact the real economy, the price of expected inflation shock λ_p is zero. However, a shock to inflation, because it predicts future consumption growth, is priced in our model. In our calibration ξ_c is negative hence λ_π is also negative assuming early resolution of uncertainty.

Finally, the shock to the nominal-real covariance ε_g is priced, because of the dual role the nominal-real covariance plays in our model: it controls the time-variation of the predictability of consumption growth with inflation shock and drives the stochastic volatility of expected consumption growth.

1.5 Model Implications for the Stock Market

We use the innovation in the stochastic discount factor in Equation (15), the prices of risk defined in Equations (16) through (18), and the equilibrium aggregate market price-dividend ratio to derive the equity risk premium. The price-dividend ratio has the same form as the wealth consumption ratio defined in Equation (10), given that dividends are exposed to the same risks as consumption

$$pd_t = D_0+D_1x_t+D_2\left(\sigma_{x,t}^2-\sigma_x^2\right)+D_3\overline{\pi}_t+D_4\sigma_{g,t-1}\varepsilon_{\pi,t}+D_5\varepsilon_{\pi,t}+D_6\left(\sigma_{g,t}-\sigma_g\right)+D_7\left(\sigma_{g,t}^2-E\left[\sigma_{g,t}^2\right]\right), \tag{19}$$

with the definitions of the loadings reported in the Appendix. The intuition behind the loadings on the state variables is also similar, except that they depend on how expected inflation and inflation shock affect dividend growth relative to how they affect consumption growth. Importantly, D_3 and D_5 are also zero for the same reasons as their counterparts in the wealth consumption ratio.

We can, thus, derive the equity premium on the aggregate market portfolio

$$E_t[r_{m,t+1}-r_{f,t}] = -Cov_t(m_{t+1},r_{m,t+1}) \tag{20}$$

$$= \lambda_c\beta_c\sigma_{x,t}^2+\lambda_x\beta_x\sigma_{x,t}^2+\lambda_s\beta_s+\lambda_\pi\beta_\pi\sigma_{g,t}^2+\lambda_{g,t}\beta_{g,t} \tag{21}$$

where

$$\beta_c = -\phi_{d,xc}, \qquad \beta_x=-\kappa_{1,m}D_1\phi_x, \qquad \beta_s=-\kappa_{1,m}D_2\phi_{s,xs}, \tag{22}$$

$$\beta_\pi = -\kappa_{1,m}D_4, \tag{23}$$

$$\beta_{g,t} = -\kappa_{1,m}D_6\phi_g-2\kappa_{1,m}D_7\sigma_g\phi_g-2\kappa_{1,m}D_7\phi_g\nu_g(\sigma_{g,t}-\sigma_g). \tag{24}$$

All the components of the risk premiums for the market portfolio are time-varying, except the risk premium for the stochastic volatility of long-run component of expected consumption growth, because the volatility of the volatility process is constant in our model. The short-run and long-run consumption risks vary with the volatility of the long-run component of consumption growth $\sigma^2_{x,t}$. The variation in the inflation and the nominal-real covariance risk premiums is driven by the nominal-real covariance, which again reflects the dual role of the nominal-real covariance in our model.

1.6 Model Implications for the Bond Market

By no-arbitrage, Gallmeyer, Hollifield, Palomino, and Zin (2007) show that the price of an n-period bond at time t is the expected discounted value of an $(n-1)$-period bond at time $t+1$:

$$P^b_t(n) = E_t[m_{t+1}P^b_{t+1}(n-1)]. \tag{25}$$

Log linear approximation of this equation gives:

$$p^b_t(n) = E_t[m_{t+1}]+E_t[p^b_{t+1}(n-1)]+\frac{1}{2}Var_t[m_{t+1}]+\frac{1}{2}Var_t[p^b_{t+1}(n-1)]+Cov_t[p^b_{t+1}(n-1), m_{t+1}], \tag{26}$$

where small letters denote logs of capital letter, i.e., $p^b_t(n) = log(P^b_t(n))$.[4]

We assume that the log bond prices $p^b_t(n)$, hence yields $y^b_t(n)$, are linear quadratic function of our state variables:

$$\begin{aligned} p^b_t(n) &= -B_0(n) - B_1(n)x_t - B_2(n)\left(\sigma^2_{x,t} - \sigma^2_x\right) - B_3(n)\overline{\pi}_t - B_4(n)\sigma_{g,t-1}\varepsilon_{\pi,t} \\ &- B_5(n)\varepsilon_{\pi,t} - B_6(n)\left(\sigma_{g,t} - \sigma_g\right) - B_7(n)\left(\sigma^2_{g,t} - E\left[\sigma^2_{g,t}\right]\right), \qquad (27) \\ y^b_t(n) &= -\frac{1}{n}p^b_t(n), \qquad (28) \end{aligned}$$

with the definitions of the loadings reported in the Appendix. Note that the main difference between stocks and bonds is that for stocks the price-dividend ratio is linear quadratic in the state variables, whereas for bonds it is the prices themselves that are linear quadratic in the state variables. The loadings, thus, represent the response of bond prices to movements

[4] Another way of seeing that this equation is true is by defining the log return of real bonds as $r^b_{t+1} = p^b_{t+1}(n-1) - p^b_t(n)$. Then the Euler equation gives $0 = E_t[m_{t+1}] + E_t[p^b_{t+1}(n-1) - p^b_t(n)] + \frac{1}{2}Var_t[m_{t+1}] + \frac{1}{2}Var_t[p^b_{t+1}(n-1) - p^b_t(n)] + Cov_t[p^b_{t+1}(n-1) - p^b_t(n), m_{t+1}]$, which is the same equation, since $p^b_t(n)$ is know at t.

in the state variables, and the intuition behind them is similar to the ones for the wealth-consumption and price-dividend ratios. The main difference is that expected inflation and inflation shocks are important in pricing the nominal bonds, hence $B_3(n)$ and $B_5(n)$ loadings are positive. For the real bonds these ratings are zero, for the same reason as their counterparts in the wealth-consumption and price-dividend ratios.

2 Data and Calibration

2.1 Data

To construct inflation and real consumption growth, we use annual data for the real Personal Consumption Expenditures (PCE) of nondurables and services available from the Bureau of Economic Analysis (BEA). We follow Piazzesi and Schneider (2006) and use the quantity indexes from Table 2.3.3, price indexes from Table 2.3.4, and PCE from Table 2.3.5. The data at annual frequency is available from 1929. Consumption is real PCE of nondurables and services, and inflation is the PCE inflation of non-durables and services. We use log-growth rates for both.

For the aggregate stock market, we use monthly CRSP's value-weighted return including distributions (VWRETD) available from 1925 in the CRSP Stock Market Indexes database. We construct annual returns by summing monthly log returns across the twelve months of the year. The nominal risk-free rate is the three-month yield from the Fama Risk Free Rates database within CRSP Monthly Treasury, also available from 1925. For annual frequency, we take the sum of log quarterly nominal risk-free rates for the quarters within the year.

To compute dividends and dividend-price ratio, we use monthly CRSP's value-weighted return including distributions (VWRETD) and excluding dividends (VWRETX), both from the CRSP Stock Market Indexes database. We then smooth monthly dividends by summing over the previous twelve months and measure annual dividends at the end of each year. The log dividend growth is the log difference of annual dividends, and the annual price-dividend ratio is the monthly price-dividend ratio at the end of the year.

Nominal bond prices and yields are from the CRSP Fixed Term Indices and Fama-Bliss Discount Bond databases and are available from 1952. For annual prices and yields, we use end-of-the year monthly log prices and log yields of bonds with maturity of 1, 2, 3, 4 and 5 years, and use them to construct log holding period returns and excess returns.

2.2 Calibration

Table 1 shows the calibrated parameters of the model. The model has 23 parameters: three preference parameters; seven parameters for the consumption, six for the inflation, and three for the nominal-real covariance processes; and four parameters for the aggregate stock market. All parameters are estimated in the generalized method of moments (GMM) system, except means of consumption growth rate, inflation and dividend growth rates that are set to match the sample evidence. The GMM estimation targets 52 moments that describe the (joint) dynamics of consumption, inflation, stock and bond markets.[5] The model is calibrated on a monthly decision interval. We, then, time-aggregate the moments from the model to annual frequency following Bansal, Kiku, and Yaron (2016).

For preference parameters, we estimate the time-discount factor $\delta = 0.9995$, and the risk aversion $\gamma = 10$ that are close to commonly calibrated values in other long-run risk studies. The elasticity of the intertemporal substitution ψ is estimated to be 1.1, which is considerably lower than the values typically used in other studies that vary between 1.5 up to 2.5. These preference parameters imply that agents in our model prefer early resolution of uncertainty given that $\gamma > \frac{1}{\psi}$.

Expected consumption growth in our model has two components. First, the latent, long-run expected consumption growth component is more persistent but has a smaller volatility multiplier than, for example, Bansal and Yaron (2004), Bansal and Shaliastovich (2012), but it is similar to Hasseltoft (2012). The second component of expected consumption growth rate is driven by the predictability of consumption growth rate from inflation shock. The exposure of consumption growth rate to the time-varying inflation shock ξ_c, is estimated

[5]The target moments are described in more detail in the Online Appendix.

to be negative. Moreover, in our calibration inflation predicts consumption growth with a negative sign unconditionally, but whether inflation shock itself has a positive or negative impact on consumption growth depends on the nominal-real covariance process $\sigma_{g,t}$.

Inflation follows and ARMA(1,1) process, with expected inflation that is also highly persistent but less so than the long-run component of expected consumption growth. The two components of the expected inflation volatility are estimated to have comparable multiples.

Compared with the long-run risk model without the inflation processes, the dividend growth rate is similarly sensitive to the long-run component of expected consumption growth as the consumption growth itself, as ρ_{dx} is close to 1. However, the dividend growth rate is more volatile than consumption growth rate, because it is more exposed to the two shocks that also affect consumption, namely the inflation shock and the real consumption shock. The exposure of dividend growth rate to the time-varying inflation shock ξ_d, is estimated to be negative, similar as for the consumption process, but whether inflation shock itself has a positive or negative impact depends, again, on the nominal-real covariance process $\sigma_{g,t}$.

3 Model Implications

3.1 Implications for the Macrovariables

Table 2 shows the descriptive statistics for the exogenous processes of the model: consumption growth rate, inflation and the dividend growth rate. We report their means, volatility and autocorrelation coefficients, as well as pair-wise correlation between them.

The results show that we can match the dynamics of the observable exogenous processes in our model, namely, the processes for the inflation, consumption and dividend growth rates. By construction, we can match the unconditional means of these processes. For the consumption growth and inflation we find that our model implies similar estimates of their annual volatilities, 2.2% for consumption growth and 2.4% for inflation in the data versus 2.4% for both in the model. The dividend growth rates are more volatile than consumption growth rates both in the data and in our model. Our model estimates a dividend growth rate volatility at 10%, which is comparable but slightly higher than the one observed in the data at 7%.

We can also match the autocorrelation coefficients of these processes, with quite similar estimates for the inflation and dividend growth rate, and a comparable though slightly smaller model estimate for the consumption growth, 48% in the data versus 34% in the model. Furthermore, we also match small but negative correlations between inflation and consumption growth rate, as well as between inflation and dividend growth rate. The former one is slightly higher in the model and the latter one is slightly higher in the data, but for both we can conclude that these processes are weakly, negatively correlated, both, in the data and in the model.

3.2 Implications for the Stock Market

Table 3 shows the model implications for the stock market. Panel A shows the descriptive statistics for the aggregate stock market index, real and nominal risk-free rate and for the dividend price ratio of the aggregate stock market. Panel B reports the R^2 statistics from the predictive regressions of price-dividend ratio on consumption growth rate, dividend growth rates, and stock market excess returns.

The results in Panel A show that we can match key characteristics, namely means, volatility and autocorrelation coefficients, of the stock market index, risk-free rate and the price-dividend ratio. Although the mean of the nominal risk-free rate is slightly overestimated in the model, it is important to notice that we are able to match its volatility but also mean and volatility of the real risk-free rate. The model underestimates slightly the volatility of the stock market returns, but more importantly, we can match very precisely the volatility of the price-dividend ratio. Matching, especially the volatility of the risk-free rate and the price-dividend ratio has been challenging for other long-run risk models (see, for example, Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012), Beeler and Campbell (2012)).

Because the long-run risk models introduce predictable and persistent component in consumption growth, which leads to the variation in both consumption and dividend growth rates, Beeler and Campbell (2012) call for assessing the long-run risk models by their ability to match the evidence for the price-dividend ratio's predictability for the stock market returns, consumption and dividend growth rates. Panel B aims at addressing this call by reporting the R^2-statistics from such predictive regressions. We consider a one-year horizon regressions, as well as long-horizon regressions when the left hand side variables, namely the stock market returns, consumption and dividend growth rates, are measured over three and five years.

In the data there is very little predictability from the price-dividend ratio for the consumption and dividend growth rates. There is some predictability at the one-year horizon but it dies out rather quickly. However, there is a strong predictability from the price-dividend ratio for the stock market returns, and it is increasing with the horizon. Our model, although unable to match the point estimates for the R^2 statistics exactly, generates the conclusions that are consistent with the data. The model implies small and decreasing with the horizon predictability for the consumption and dividend growth rates, simultaneously implying high and increasing over the horizon predictability for the stock market returns. Inability to match these moments has been one of the main criticisms raised by Beeler and Campbell (2012) that was still unaddressed in the literature.

3.3 Implications for the Bond Market

Table 4 shows the model implications for the bond market. Panel A shows means and volatility for the term structure of nominal bond yields, while Panel B shows the same statistics for the term structure of nominal bond returns.

In Panel A, the results show that the model is able to match the slope of the term structure of nominal yields quite precisely. The difference between the five- and one-year yields is 62 basis points in the data, while it is 67 basis points in our model, although, the average yields are slightly overstated in the model. Our model can also match a downward-sloping term structure of volatility, although it underestimates the point estimates by around 1 percentage point.

In Panel B, we show that our model can also match the upward-sloping term structure of nominal bond returns and their volatility. The slope of the term structure of the returns is a bit steeper in the model, with a two-year nominal bond return being slightly underestimated in the model, and the five-year return slightly overestimated. In contrast, the term structure of bond return volatility is more flat in the model, though it consistently increase in the data and in the model.

4 Conclusion

We show that the long-run risk model with time-varying inflation non-neutrality matches joint dynamics of stock returns, term structure of nominal bond yields and returns, as well as macroeconomic fundamentals. We introduce two key new elements into the otherwise standard long-run risk model of Bansal and Yaron (2004). First, we introduce neutral and non-neutral effects of inflation on the real economy that are important in matching the risk premiums in stock and bond markets. Second, we introduce the nominal-real covariance process that determines the variation in the predictive relation between inflation and consumption growth, which helps matching volatility as well as predictive moments in the two markets.

We calibrate our model using US stock and bond markets data and calibrate it to target 52 moments that capture (joint) dynamics of consumption, inflation, stock and bond market returns and yields. Thus, we are able not only to match more moments than many of the previous studies but also address some of the criticism raised against the long-run risk models, most notably by Beeler and Campbell (2012), i.e., the inability of the LRR models to match volatility of the risk-free rate and of the dividend-price ratio, and the dividend-price ratio ability to predict stock market returns, consumption and dividend growth rates. More importantly, we match this challenging set of moments, while simultaneously holding the risk aversion and elasticity of intertemporal substitution parameters low.

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Table 1: Calibrated Parameters of the Model

This table reports the configuration of the parameters used in the calibration of the model. The model is calibrated on a monthly decision interval, hence the parameters reported in the table refer to monthly frequency. Means of consumption growth, inflation and dividend growth rates are set to match the sample evidence.

Preferences		
Discount factor	δ	0.9995
Elasticity of intertemporal substitution	ψ	1.1
Risk aversion coefficient	γ	10
(Expected) Consumption		
Mean of real consumption growth	μ_c	0.0015
Consumption exposure to inflation shock	ξ_c	-0.0334
Persistence of long-run component	ρ_x	0.9980
Volatility multiple of long-run consumption shock	ϕ_x	0.0265
Mean of volatility	σ_x	0.0017
Persistence of volatility	ν_x	0.9961
Volatility of volatility	$\phi_{s,xs}$	3.56E-07
(Expected) Inflation		
Mean of inflation	μ_π	0.0027
Inflation MA(1) coefficients	ξ_π	1.81E-04
	ϕ_π	-0.0011
Persistence of expected inflation	$\rho_{\overline{\pi}}$	0.9874
Volatility multiple of long-run consumption shock	$\phi_{\overline{\pi}x}$	-0.1269
Volatility multiple of expected inflation shock	$\phi_{\overline{\pi},xp}$	-0.1438
Nominal-real covariance		
Mean of nominal-real covariance	σ_g	-0.2138
Persistence of nominal-real covariance	ρ_g	0.9993
Volatility of nominal-real covariance	ϕ_g	-0.0035
Aggregate market		
Mean of dividend growth	μ_d	0.0021
Exposure of dividend growth to long-run component	ρ_{dx}	1.0802
Dividend growth exposure to inflation shock	ξ_d	-0.1470
Volatility multiple of real consumption shock	$\phi_{d,xc}$	2.5496

Table 2: Calibrated Moments for the Exogenous Processes
This table shows the descriptive statistics for the exogenous processes of the model: consumption growth rate, inflation and the dividend growth rate. All the moments are expressed on an annual basis and in percentage points.

	Data	Model
Consumption growth rate		
Mean	1.83	1.83
Std	2.19	2.42
AR(1)	48.42	33.89
Inflation		
Mean	3.21	3.21
Std	2.36	2.44
AR(1)	81.27	88.71
Dividend growth rate		
Mean	2.46	2.46
Std	6.60	9.86
AR(1)	22.56	25.38
Correlations		
(Inflation, Consumption growth rate)	-9.52	-23.69
(Inflation, Dividend growth rate)	-26.10	-12.53

Table 3: Model Implications for the Stock Market
This table shows the model implications for the stock market. Panel A shows the descriptive statistics for the aggregate stock market index, real and nominal risk-free rate and for the dividend price ratio of the aggregate stock market index. Panel B shows the results of running predictive regressions using the price-dividend ratio as a predictor for consumption growth, dividend growth rates and stock market returns. All the moments are expressed on an annual basis and in percentage points.

	Data	Model
Panel A: Descriptive statistics		
Mean stock market return	6.28	6.32
Std stock market return	16.39	13.62
AR(1) stock market return	-1.30	3.21
Mean nominal risk-free rate	3.52	4.92
Std nominal risk-free rate	3.18	3.34
Mean price-dividend ratio	3.38	3.28
Std price-dividend ratio	0.45	0.50
AR(1) price-dividend ratio	87.12	96.67
Mean real risk-free rate	1.55	1.62
Std real risk-free rate	2.05	2.59
Panel B: Predictability from price-dividend ratio		
$\sum_{j=1}^{J}(\Delta c_{t+j}) = \alpha + \beta pd_t + \varepsilon_{t+j}$		
R^2, $j = 1$	6.00	1.66
R^2, $j = 3$	1.30	0.61
R^2, $j = 5$	0.00	0.46
$\sum_{j=1}^{J}(\Delta d_{t+j}) = \alpha + \beta pd_t + \varepsilon_{t+j}$		
R^2, $j = 1$	9.20	1.48
R^2, $j = 3$	5.90	0.43
R^2, $j = 5$	3.90	0.28
$\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t+j}) = \alpha + \beta pd_t + \varepsilon_{t+j}$		
R^2, $j = 1$	4.40	3.92
R^2, $j = 3$	17.00	11.05
R^2, $j = 5$	26.90	17.27

Table 4: Model Implications for the Bond Market
This table shows the model implications for the bond market. Panel A shows the descriptive statistics for the term structure of nominal yields, while Panel B shows the same statistics for the term structure of nominal bond returns. All the moments are expressed on an annual basis and in percentage points.

Maturity	Data		Model	
	Mean	Std	Mean	Std
Panel A: Nominal bond yields				
1 year	4.20	3.20	4.47	2.26
2 years	4.39	3.17	4.61	2.04
5 years	4.81	3.04	5.14	1.76
Panel B: Nominal bond returns				
2 years	5.13	3.67	4.91	4.18
3 years	5.50	4.44	5.28	4.68
4 years	5.69	5.30	5.68	5.19
5 years	5.85	6.10	6.08	5.69