Composite quantum collision models

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A collision model (CM) is a framework to describe open quantum dynamics. In its memoryless version, it models the reservoir $\mathcal{R}$ as consisting of a large collection of elementary ancillas: the dynamics of the open system $\mathcal{S}$ results from successive collisions of $\mathcal{S}$ with the ancillas of $\mathcal{R}$. Here, we present a general formulation of memoryless composite CMs, where $\mathcal{S}$ is partitioned into the very open system under study S coupled to one or more auxiliary systems $\{S_i\}$. Their composite dynamics occurs through internal $S-\{S_i\}$ collisions interspersed with external ones involving $\{S_i\}$ and the reservoir $\mathcal{R}$. We show that important known instances of quantum non-Markovian dynamics of S—such as the emission of an atom into a reservoir featuring a Lorentzian, or multi-Lorentzian, spectral density or a qubit subject to random telegraph noise—can be mapped on to such memoryless composite CMs.

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I. INTRODUCTION

A longstanding problem in the field of open quantum system dynamics is the derivation of an effective description of the reduced dynamics of a system $\mathcal{S}$ in contact with the surrounding environment, i.e., of a master equation having the reduced density operator of $\mathcal{S}$ as the only unknown [1–3]. This is in general a highly nontrivial task for quantum non-Markovian dynamics. Note that even the very meaning non-Markovianity and its correct measure are currently the focus of intense investigations [4]. Sometimes the approximations made to describe non-Markovian dynamics can lead to master equations (MEs), which do not preserve trace and complete positivity.

Quantum collision models, first introduced in Ref. [5] and more recently studied in Refs. [6,7] have proved to be a promising tool to analyze quantum non-Markovian dynamics [8–13] as well as of quantum thermodynamical systems (see, e.g., Refs. [14–16]). In its standard, memoryless, version, a collision model describes the reservoir as a large collection of elementary constituents or ancillas and the joint dynamics as a sequence of pairwise system-ancilla unitary collisions. The resulting reduced nonunitary dynamics of $\mathcal{S}$, in the continuous-time limit, can be shown [7] to be described by a Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation [1–3]. A memoryless collision model thus entails a fully Markovian evolution for the open system as long as the ancillas are initially in a product state, they do not mutually interact and the system collides only once with each of the ancillas. To account for non-Markovian processes in collision models, one has to somehow relax such assumptions, e.g., by allowing the initial reservoir state to be correlated [8,11] or enabling interancillary interactions between next system-ancilla interactions [9,12].

Collision-model-based approaches are promising for the study of non-Markovian dynamics for at least two reasons: (i) they allow for the possibility to decompose a complicated open dynamics in terms of discrete elementary processes (each usually involving a pair of low-dimensional systems) and (ii) they make possible the derivation of well-behaved non-Markovian master equations [12,13]. Remarkably, (i) in particular can suggest schemes to perform experimental simulations of non-Markovian dynamics [17] or provide valuable theoretical tools in the analysis of very large Hilbert space problems [18] and time-delayed quantum feedback [19]. Concerning (ii), in particular, in 2013 two of us [12] showed that one can use a collision model to work out a non-Markovian master equation that is both capable to interpolate between a Markovian and a strongly non-Markovian regime and, additionally, is ensured to be completely positive and trace preserving, namely two requirements that are in general hard to meet at the same time. This master equation – recently generalized and reinterpreted in [20] – prompted a number of studies within a more general open-quantum-systems framework [21,22].

An open issue in quantum collision models is their descriptive power. While being an advantageous tool in many respects, a collision model is itself rather abstract. One thus naturally wonders whether (and how), given a non-Markovian open dynamics, this can be reproduced through a suitably built collision model. In the case of a qubit, Rybar *et al.* showed that any nonunitary dynamics can be described through a collision model provided that the initial reservoir state is chosen accordingly. Typically, however, this requires the preparation of a multipartite correlated state of all the reservoir ancillas, which may be an experimentally demanding task. Concerning collision models with initial uncorrelated reservoir states, instead, very few instances of non-Markovian dynamics were so far demonstrated to be reproducible through a collision model.

A recurrent situation in which a non-Markovian dynamics emerges (e.g., in quantum optics or condensed matter scenarios) is when the interaction between a small quantum system S and a large reservoir $\mathcal{R}$ is not direct but bridged by an auxiliary quantum system S_1 [23,24]. The prototypical tool for describing such dynamics is a GKSL master equation for the joint state of S and S_1, where the Hamiltonian term of the Lindbladian superoperator features in particular a direct coherent S-S_1 coupling while the non-Hamiltonian one depends on a set

of jump operators defined in the Hilbert space of S_1 only. While the resulting joint dynamics of S-S_1 is evidently Markovian, the one of S is in general non-Markovian. The question is now raised whether a collision model effectively describing the evolution of S in the continuous-time limit can be defined for such an important class of non-Markovian dynamics.

One is thereby intuitively led to consider a memoryless collision model (noninteracting and initially uncorrelated ancillas) where however the system $\mathcal{S}$ undergoing repeated collisions with the ancillas is now multipartite, being composed by S (the very open system under study) and an auxiliary system S_1. Natural requirements would be to let S be uncoupled from the reservoir, but allow for a direct $S-S_1$ coherent interaction to occur between collisions. The main aim of the present paper is to formulate in a rigorous way a theoretical framework showing that it is indeed possible to define a family of quantum collision models—which we call composite collision models—that are precisely based on this intuitive idea and reproduce the class of non-Markovian dynamics described above. The discrete dynamics of such models can be thought as consisting of internal collisions—enabling a crosstalk between S and S_1—interspersed with collisions between the auxiliary system S_1 only and the reservoir ancillas. The effectiveness of this framework is illustrated in the case of some specific instances of composite collision models, showing in particular that, e.g., the known non-Markovian decay of an atom in a lossy cavity or the dynamics of a qubit subject to random telegraph noise can emerge through a collision-model-based formulation. The collision models we introduce are naturally extended, as we show, to the case of a manifold of auxiliary systems $\{S_1, S_2, \ldots\}$.

The use of a bipartite collision model to describe a damped Jaynes-Cummings-model dynamics was introduced in Ref. [14] and then investigated in more detail in Ref. [25]. Here, this result emerges as a specific instance of our composite collision model framework. In particular, we present a thorough discussion of the conditions to match in order for such effective description to hold in the continuous-time limit.

On a rather general ground, any open system dynamics of a system S arises as the partial trace over the environmental degrees of freedom of the joint unitary dynamics entailed by the system-reservoir total Hamiltonian model [1–3]. Such environmental model has on the one hand a clear physical meaning while, on the other hand, allows for a joint dynamics where a large number of degrees of freedom are involved. In contrast, a collision model dynamics takes place through a succession of elementary interactions—each involving only a small reservoir subunit—but its connection to a realistic physical scenario is less straightforward. In the light of this, given a microscopic environmental model, it would be highly desirable to devise a general method to associate a collision model yielding the same open system dynamics in the continuous-time limit. Here, we take a first step towards this challenging goal by showing that such mapping is possible for some specific environmental models. This can be the case for a qubit that is coupled in a purely dissipative or dispersive fashion to a bosonic bath when the spectral density has a Lorentzian or multi-Lorentzian shape, as we show.

The outline of this paper is the following. In Sec. II, we review the standard quantum collision model leading to a GKSL master equation in its continuous-time limit. In Sec. III, we show how and under what conditions the collision model of Sec. II can be extended to include an internal system dynamics described by a corresponding free Hamiltonian. The theoretical framework so formulated is then used in Sec. IV as the basis to define a composite quantum collision model in the bipartite case. In Sec. V, we illustrate a prominent instance of such models, which in the continuous-time limit effectively reproduces the open dynamics of an atom decaying in a lossy cavity (damped Jaynes-Cummings model). In Sec. VI, we study another instance of composite bipartite collision model based on a dispersive $S-S_1$ coupling, either with respect to S_1 or S. Correspondingly, the resulting collision model can describe either a qubit subjected to random-telegraph noise or a qubit undergoing a purely dephasing dynamics. In Sec. VII, we show how to extend the composite bipartite collision model of Sec. IV to the multipartite case. An instance, based on a tripartite collision model, is then presented in Sec. VIII and shown to be able to reproduce the dynamics of an atom dissipatively coupled to a reservoir featuring a SD that is the sum of two Lorentzian distributions. Finally, in Sec. IX we draw our conclusions.

II. MEMORYLESS COLLISIONAL MODEL AND THE MARKOVIAN MASTER EQUATION

In this section, we will briefly review how the standard Markovian GKSL master equation (ME) is naturally derived by a collisional memoryless model of open dynamics. In such a model a quantum reservoir $\mathcal{R}$ consists of a large ensemble of identical noninteracting ancillas $\{R_n\}$ all in the same initial state. The system $\mathcal{S}$ interacts with the environment via a sequence of collisions, i.e., short interactions, with each of the ancillas. The initial joint state of $\mathcal{S}-\mathcal{R}$ is assumed to be the product state

$$\sigma_0 = \rho_0 \otimes (\eta \otimes \eta \otimes \ldots), \tag{1}$$

where ρ_0 is the initial state of $\mathcal{S}$ while η is the common initial state of all the ancillas. Both ρ_0 and η can in general be mixed. The state η can always be expressed in diagonal form in terms of its eigenstates $\{|m\rangle\}$ and associated probabilities $\{p_m\}$ as

$$\eta = \sum_m p_m \, |m\rangle\langle m|, \tag{2}$$

where $\{|m\rangle\}$ form an orthonormal basis of the ancilla Hilbert space. In the memoryless version of the model the reservoir is assumed to be so large that the system never collides twice with the same ancilla, therefore the open dynamics of $\mathcal{S}$ takes place through pairwise short interactions between $\mathcal{S}$ and each reservoir ancilla: $\mathcal{S}-R_1$, $\mathcal{S}-R_2$, $\mathcal{S}-R_3, \ldots$, in such a way that at each step $\mathcal{S}$ collides with a fresh ancilla that is still in state η. A schematic sketch of the model dynamics is given in Fig. 1(a).

It is assumed that all the collisions have the same duration τ, each being described by the unitary evolution operator $\hat{U}_{\mathcal{S}n}$ given by (we set $\hbar = 1$ throughout)

$$\hat{U}_{\mathcal{S}n} = e^{-i\hat{H}_{\mathcal{S}n}\tau}, \tag{3}$$

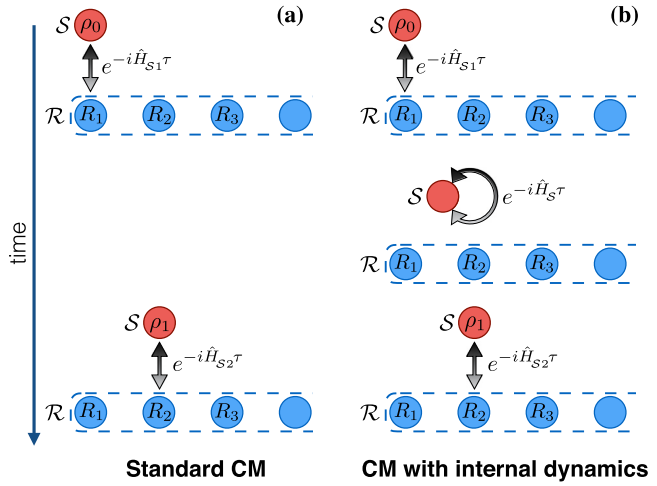

FIG. 1. (a) Standard memoryless collision model: the system $\mathcal{S}$ undergoes successive collisions with the reservoir ancillas $\{R_n\}$, each corresponding to an interaction Hamiltonian $\hat{H}_{\mathcal{S}n}$. (b) Collision model with internal dynamics: unlike (a), an intrasystem collision (corresponding to a free system Hamiltonian $\hat{H}_{\mathcal{S}}$) takes place between two next system-ancilla collisions. In either case of collision model, only the first steps are sketched, the next ones being obtained by simple iteration.

with

$$\hat{H}_{\mathcal{S}n} = g\,\hat{h}_{\mathcal{S}n}, \tag{4}$$

where g is a coupling rate and $\hat{h}_{\mathcal{S}n}$ is a dimensionless Hermitian interaction Hamiltonian acting on the joint $\mathcal{S}-R_n$ Hilbert space.

Let ρ_n be the state of $\mathcal{S}$ at the (generic) nth step, i.e., just after the collision with the nth ancilla:

$$\rho_n = \mathrm{Tr}_{\mathcal{R}}\{\hat{U}_{\mathcal{S}n}\rho_{n-1}\,\eta\,\hat{U}^{\dagger}_{\mathcal{S}n}\} \equiv \mathrm{Tr}_{R_n}\{\hat{U}_{\mathcal{S}n}\rho_{n-1}\,\eta\,\hat{U}^{\dagger}_{\mathcal{S}n}\}. \tag{5}$$

Continuous-time limit

As we assumed each collision to last for a short time τ, we approximate $\hat{U}_{\mathcal{S}n}$ [cf. Eq. (3)]up to the second order in τ as

$$\hat{U}_{\mathcal{S}n} \simeq \mathbb{1}_{\mathcal{S}n} - i\,\hat{H}_{\mathcal{S}n}\tau - \frac{\hat{H}^2_{\mathcal{S}n}}{2}\tau^2. \tag{6}$$

When this is substituted into Eq. (5) the variation of ρ_n due to a single collision, to second order in τ, is

$$\begin{aligned}\Delta\rho_n = {} & \mathrm{Tr}_{R_n}\{-i\,[\hat{h}_{\mathcal{S}n},\rho_n\eta]\}g\tau + \mathrm{Tr}_{R_n}\big\{\hat{h}_{\mathcal{S}n}(\rho_n\eta)\hat{h}_{\mathcal{S}n} \\ & - \tfrac{1}{2}\big[\hat{h}^2_{\mathcal{S}n},\rho_n\eta\big]_+\big\}(g\tau)^2\end{aligned} \tag{7}$$

with $\Delta\rho_n = \rho_n - \rho_{n-1}$, $[\hat{C},\hat{D}] = \hat{C}\hat{D} - \hat{D}\hat{C}$ and $[\hat{C},\hat{D}]_+ = \hat{C}\hat{D} + \hat{D}\hat{C}$. In line with standard procedures in open quantum system theory [1–3], it is also assumed [13] that

$$\mathrm{Tr}_{R_n}\{\hat{h}_{\mathcal{S}n}\eta\} = 0. \tag{8}$$

This assumption can be made with no loss of generality since, when the average (8) is nonzero, it amounts just to a renormalization of the $\mathcal{S}$ Hamiltonian and can thereby be incorporated in the free-system Hamiltonian [13,26].

Let now $t_n = n\tau$ (with $n = 0,1,\ldots$) be the discrete time variable up to the nth step. As one can equivalently regard the collision model as the interaction of $\mathcal{S}$ with only one ancilla, whose state is refreshed to η at times t_n, the collision time here plays the role of the usual environment self-correlation time in standard microscopic derivations of the GKSL master equation [1]. This time is, strictly speaking, finite. To pass from the discrete dynamics to the continuous-time one we must therefore realize that what we have in mind is a sort of coarse graining over a finite time. From a formal viewpoint, we carry out this by taking the limit $n \gg 1$ and $\tau \simeq 0$ in such a way that $t_n \to t$ with t being now a continuous-time variable. Accordingly, $\Delta\rho_n/\tau \to d\rho/dt$. At the same time we assume that the product $\gamma = g^2\tau$ remains finite. Note that in microscopic derivations γ is proportional to the self-correlation time. In the continuous-time limit just described, thereby, the finite-difference equation (7) takes the form of a continuous-time master equation

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) \tag{9}$$

with the superoperator $\mathcal{L}$ given by

$$\mathcal{L}(\rho) = \gamma\sum_{\mu\nu}\left(\hat{A}_{\mu\nu}\rho\hat{A}^{\dagger}_{\mu\nu} - \frac{1}{2}[\hat{A}^{\dagger}_{\mu\nu}\hat{A}_{\mu\nu},\rho]_+\right). \tag{10}$$

Here, $\{\hat{A}_{\mu\nu}\}$ are jump operators in the $\mathcal{S}$ Hilbert space defined by

$$\hat{A}_{\mu\nu} = \sqrt{p_\nu}\;\langle\mu|\hat{h}_{\mathcal{S}n}|\nu\rangle, \tag{11}$$

where [cf. Eq. (2)] $|\mu\rangle$ and $|\nu\rangle$ are two orthonormal eigenstates of η, i.e., elements of the basis $\{|m\rangle\}$, and p_ν the νth eigenvalue of η [owing to the collision model translational invariance, jump operators (11) and thus $\mathcal{L}$ are independent of ancilla R_n].

III. MEMORYLESS COLLISION MODEL WITH INTERNAL DYNAMICS

The standard collision model of the previous section can be modified to allow for an internal dynamics of $\mathcal{S}$ to take place as well. Specifically, we assume that, between two consecutive system-ancilla collisions, $\mathcal{S}$ undergoes a unitary dynamics governed by a free Hamiltonian $\hat{H}_{\mathcal{S}}$ as shown in Fig. 1(b). We will refer to this process as an intrasystem collision. A step is now defined so to incorporate one intrasystem collision, lasting a time τ_s followed by a system-ancilla one, lasting a time τ_n. The system evolution after the nth step is again described by Eq. (5), but $\hat{U}_{\mathcal{S}n}$ is now given by

$$\hat{U}_{\mathcal{S}n} = e^{-i\hat{H}_{\mathcal{S}n}\tau_n}\,e^{-i\hat{H}_{\mathcal{S}}\tau_s}, \tag{12}$$

where the $S-R_n$ interaction Hamiltonian $\hat{H}_{\mathcal{S}n}$ is the same as Eq. (4) while

$$\hat{H}_{\mathcal{S}} = J\,\hat{h}_{\mathcal{S}} \tag{13}$$

is the free Hamiltonian of $\mathcal{S}$ with characteristic frequency J and where $\hat{h}_S$ is a dimensionless operator defined in the $\mathcal{S}$ Hilbert space. In the following we want to reproduce a coherent dynamics, generated by $\hat{h}_S$, together with an incoherent dynamics, due to the system-ancilla collisions. To be consistent with this assumption, while we coarse grain on the incoherent dynamics, which means that τ_n will be assumed

to be small but finite, we will assume $\tau_s \ll \tau_n$ Consistently, $\hat{U}_{\mathcal{S}n}$ in Eq. (12) is approximated as

$$\hat{U}_{\mathcal{S}n} \simeq \mathbb{1}_{\mathcal{S}n} - i(\hat{H}_{\mathcal{S}}\tau_s + \hat{H}_{\mathcal{S}n}\tau_n) - \frac{\hat{H}^2_{\mathcal{S}n}}{2}\tau_n^2. \tag{14}$$

Equation (14) can be obtained by approximating in Eq. (12) $e^{-i\hat{H}_{\mathcal{S}}\tau}$ up to first in τ_s and $e^{-i\hat{H}_{\mathcal{S}n}\tau}$ to second order in τ_n, respectively, and then neglecting terms $\sim\tau_s\tau_n^2$ as well as terms $\tau_s\tau_n$. Note that, given the approximations made, in particular neglecting terms $\sim gJ$, the two unitaries in Eq. (12) commute: it is therefore irrelevant whether the system-ancilla collision occurs before or after the intrasystem one. Thereby, one can equivalently regard the two elementary collisions as if they occurred simultaneously and assume [cf. Eq. (12)], $\hat{U}_{\mathcal{S}n} \simeq e^{-i(\hat{H}_{\mathcal{S}}+\hat{H}_{\mathcal{S}n})\tau}$. In particular, this makes legitimate to set (see a few lines below) $\dot{\rho} \simeq \Delta\rho_n/\tau$ even if the time step consists of two subsequent collisions duration τ_n and τ_s.

Proceeding now in analogy with the previous section, we get the identity

$$\begin{aligned}\Delta\rho_n = &-i\,[\hat{H}_{\mathcal{S}},\rho_n]\tau + \mathrm{Tr}_{R_n}\{\hat{H}_{\mathcal{S}n}\rho_n\eta\hat{H}_{\mathcal{S}n}\}\tau^2 \\ &- \tfrac{1}{2}\mathrm{Tr}_{R_n}\left\{\left[\hat{H}^2_{\mathcal{S}n},\rho_n\eta\right]_+\right\}\tau^2.\end{aligned} \tag{15}$$

Correspondingly, in the continuous-time limit we end up the master equation

$$\frac{d\rho}{dt} = -i\,[\hat{\mathcal{H}}_{\mathcal{S}},\rho] + \mathcal{L}(\rho), \tag{16}$$

where the superoperator $\mathcal{L}$ has the same form as in Eq. (10) with the associated jump operators given by Eq. (11).

We point out that, as in the previous model, $\mathcal{S}$ undergoes a Lindbladian (hence Markovian) dynamics and that the internal dynamics of $\mathcal{S}$ only appears in the Hamiltonian term of the right-hand side of Eq. (16). This is a consequence of the fact that are treating the system-reservoir dynamics in a coarse-grained fashion, while the system's internal dynamics is taken into account in full detail.

Note that the collision model with no internal dynamics of Sec. II is effective even in the presence of a system free Hamiltonian provided that $[\hat{H}_{\mathcal{S}},\hat{H}_{\mathcal{S}n}] = 0$. In such a case, it indeed corresponds to the interaction picture. If $[\hat{H}_{\mathcal{S}},\hat{H}_{\mathcal{S}n}] \neq 0$, though, this is no longer true since the system-ancilla interaction Hamiltonian in Sec. II is assumed to be time independent. To avoid time dependencies regardless of such commutation relationship, the $\mathcal{S}$ internal dynamics thus must be explicitly involved in the collisional dynamics, as shown above.

IV. COMPOSITE COLLISION MODEL

We are now ready to discuss in detail the composite quantum collision model that is central to our study. This is in fact a specific instance of the collision model with internal dynamics analyzed in the previous section, where $\mathcal{S}$ is a bipartite system (in Sec. VII we will discuss the extension to the multipartite case). Specifically, $\mathcal{S}$ comprises subsystems S and S_1 (see Fig. 2) with S embodying the very open system under study, while S_1 plays the role of an auxiliary system (note that in the collision model with internal dynamics of the previous section $\mathcal{S} \equiv S$). By definition, the free Hamiltonian of $\mathcal{S}$ reads

$$\hat{H}_{\mathcal{S}} = \hat{H}_{S_1} + \hat{V}_{SS_1}, \tag{17}$$

where $\hat{H}_{S_1}$ is the free Hamiltonian of S_1 (the one of S is assumed to be zero) and $\hat{V}_{SS_1}$ is the interaction Hamiltonian of S and S_1. As for the $\mathcal{S}-R_n$ interaction [cf. Eq. (4)], this takes the form

$$\hat{H}_{\mathcal{S}n} = \hat{W}_{S_1n} = g\,\hat{w}_{S_1n} \tag{18}$$

with $\hat{w}_{S_1n}$ a dimensionless operator acting on the Hilbert space of subsystem S_1 and ancilla R_n (g is the associated coupling strength). System S is thus not subject to any direct interaction with R_n. A sketch of the collision model dynamics is given in Fig. 2.

The master equation in the continuous-time limit thus reads

$$\frac{d\rho}{dt} = -i\,[\hat{\mathcal{H}}_{\mathcal{S}},\rho] + \mathcal{L}_{S_1}(\rho) \tag{19}$$

with $\mathcal{L}_{S_1}(\rho)$ having a form analogous to Eq. (10) with

$$\hat{A}^{(1)}_{\mu\nu} = \sqrt{p_\nu}\;{}_{R_n}\langle\mu|\hat{w}_{S_1R_1}|\nu\rangle_{R_n}, \tag{20}$$

$$\gamma = g^2\tau. \tag{21}$$

The jump operators $\{\hat{A}^{(1)}_{\mu\nu}\}$ act in the Hilbert space of S_1. In the next two sections, we discuss two important instances of bipartite composite collision model and we show how they are related with known relevant classes of open quantum system dynamics.

V. ATOM IN A LOSSY CAVITY

Based on the definitions in Sec. IV, consider now the case where S and S_1 are, respectively, a qubit and a bosonic mode. Let $\{\hat{\sigma}_+,\hat{\sigma}_z\}$ be the usual Pauli spin operators associated with S, while $\hat{\alpha}$ ($\hat{\alpha}^\dagger$) is the annihilation (creation) bosonic operator for the auxiliary system S_1. The nth reservoir ancilla R_n is modeled as as a bosonic mode with associated annihilation (creation) operator $\hat{a}_n$ ($\hat{a}_n^\dagger$). By definition, [cf. Eqs. (17) and (18)]

$$\hat{H}_{S_1} = \Delta\hat{\alpha}^\dagger\hat{\alpha}, \quad \hat{V}_{SS_1} = G(\hat{\sigma}_-\hat{\alpha}^\dagger + \mathrm{H.c.}), \tag{22}$$

$$\hat{W}_{S_1n} = g\,(\hat{\alpha}\hat{a}_n^\dagger + \mathrm{H.c.}), \tag{23}$$

hence both the $S-S_1$ and S_1-R_n interaction take place under the rotating wave approximation (RWA).

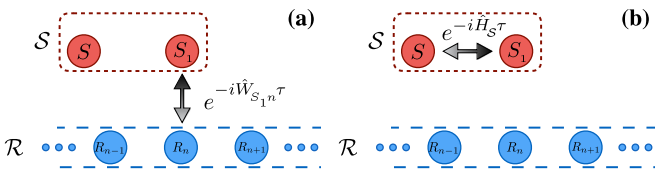

FIG. 2. (a) System-ancilla collision and (b) intrasystem collision in a composite bipartite collision model. System $\mathcal{S}$ comprises the very open system S under study and an auxiliary system S_1. In (a), note that only S_1 is involved in the collision with a reservoir ancilla. In (b), Hamiltonian $\hat{H}_{\mathcal{S}}$ in particular accounts for a direct $S-S_1$ interaction. Apart from these specifications, the general discrete dynamics takes place analogously to Fig. 1(b).

To illustrate the dynamics of the collision model defined this way, we consider the zero-temperature dynamics occurring when S is initially in its excited state, while both S_1 and the ancillas are in their vacuum states (hence, in particular, $\eta = |0\rangle\langle 0|$). The total number of excitations of $\mathcal{S}-\mathcal{R}$ is conserved at each collision, as follows from the form of Eqs. (22) and (23). Given the considered initial state, the process thus takes place within the single-excitation sector of the Hilbert space. Based on this, we use a compact notation according to which the overall initial state is denoted by $|10\mathbf{0}\rangle$, where the first two quantum numbers refer to S and S_1, respectively, while $\mathbf{0}$ refers to the reservoir ancillas (indicating that they are all in the vacuum states). With the same notation, $|01\mathbf{0}\rangle$ is the state with the single excitation localized in S_1 while $|001_i\rangle$ is the state with the single excitation localized on the ith reservoir ancilla S_i. At any step n, the joint state thus reads

$$|\Phi^{(n)}\rangle = \varepsilon^{(n)}|10\mathbf{0}\rangle + \beta^{(n)}|01\mathbf{0}\rangle + \sum_{i=1}^{n}\lambda_i^{(n)}|001_i\rangle. \tag{24}$$

Here, the superscript "n" labels the nth time step, while the subscript "i" on λ labels the ith ancilla. Note that the last sum in the equation above runs up to $i = n$ since at the end of the nth step ancillas labeled by index $i \geqslant n+1$ are still unexcited.

State $|\Phi^{(n)}\rangle$ is connected with $|\Phi^{(n-1)}\rangle$ as $|\Phi^{(n)}\rangle = e^{-i\hat{W}_{S_1 n}\tau}e^{-i(\hat{H}_{S_1}+\hat{V}_{SS_1})\tau}|\Phi^{(n-1)}\rangle$ [cf. Eqs. (12), (22), and (23)]. In Appendix A, we show that this allows to express coefficients $\{\varepsilon_n, \beta_n\}$ as linear functions of $\{\varepsilon_{n-1}, \beta_{n-1}\}$ through the 2×2 transformation matrix $\mathbf{M}$ given by

$$\mathbf{M} = e^{-i\frac{\Delta}{2}\tau}\begin{pmatrix} z & -i\frac{G}{\Omega}\sin(\Omega\tau) \\ -i\frac{G}{\Omega}\sin(\Omega\tau)\cos(g\tau) & z^*\cos(g\tau)\end{pmatrix}, \tag{25}$$

where

$$\Omega = \frac{1}{2}\sqrt{\Delta^2+4G^2}, \quad z = \cos(\Omega\tau) + i\frac{\Delta}{2\Omega}\sin(\Omega\tau). \tag{26}$$

Upon iteration,

$$\begin{pmatrix}\varepsilon^{(n)}\\ \beta^{(n)}\end{pmatrix} = \mathbf{M}^n\begin{pmatrix}\varepsilon^{(0)}\\ \beta^{(0)}\end{pmatrix}, \tag{27}$$

where in our case $\varepsilon^{(0)} = 1$ while $\beta^{(0)} = 0$. Eq. (27) in particular allows to compute step by step the evolution of the excitation amplitude of S up to any desired time $n\tau$.

In Fig. 3, we use Eq. (27) to illustrate how the discrete-step evolution of the S excited-state population depends on the collision time τ in the paradigmatic case of zero detuning ($\Delta = 0$) and $g = \sqrt{G/\tau}$ (we use G as the frequency unit and let g be τ dependent in a way that $g^2\tau$ is fixed to G). If τ is not short enough, a continuous-time approximation of the dynamics fails (see cases $\tau = 2G^{-1}$ and $\tau = G^{-1}$ in Fig. 3). Collision times of the order of $\tau \sim 0.1G^{-1}$ or shorter are already enough to determine a smooth evolution as a function of the step number n. For the considered parameters, the S dynamics in this limit exhibits damped oscillations. These originate from the $S-S_1$ coupling Hamiltonian term $\hat{V}_{SS_1}$ [cf. Eqs. (22)], which in absence of reservoir would induce a continuous excitation exchange between S and the auxiliary system S_1. The effect of the reservoir is to damp the amplitude of such energy exchange. These features can be explained by noting that for $\tau \ll G^{-1}$ the conditions required for master equation (19) to hold (see Sec. IV) are matched. Using that $\eta = |0\rangle\langle 0|$, the master equation takes the explicit form [cf. Eqs. (22) and (23)]

$$\begin{aligned}\dot{\rho} = &-i[\Delta\hat{\alpha}^\dagger\hat{\alpha} + G(\hat{\sigma}_-\hat{\alpha}^\dagger + \text{H.c.}), \rho] \\ &+ \gamma\left(\hat{\alpha}\rho\hat{\alpha}^\dagger - \tfrac{1}{2}[\hat{\alpha}^\dagger\hat{\alpha}, \rho]_+\right)\end{aligned} \tag{28}$$

with $\gamma = g^2\tau$ [in passing, note that condition (8) is fulfilled]. This is the well-known master equation (in the rotating frame) occurring in the damped Jaynes-Cummings (JC) model [23] describing the dynamics of a two-level atom of frequency ω_0 coupled with rate G to a single-mode cavity of frequency ω_c, where $\Delta = \omega_c - \omega_0$ is the detuning while γ represents the cavity dissipation rate.

For $\rho(0) = |e\rangle_S\langle e||0\rangle_{S1}\langle 0|$, the joint state of $\mathcal{S}$ and $\mathcal{R}$ at time t must have the same form as Eq. (24) with $\varepsilon^{(n)} \to \varepsilon(t)$, $\beta^{(n)} \to \beta(t)$ and $\lambda_i^{(n)} \to \lambda_i(t)$. This alongside master equation (28) then entail that $\varepsilon(t)$ obeys the integrodifferential equation (see Appendix B)

$$\dot{\varepsilon} = -G^2\int_0^t dt'\, e^{-i(\Delta - i\frac{\gamma}{2})(t-t')}\varepsilon(t'), \tag{29}$$

the solution of which reading

$$\varepsilon(t) = e^{-i\frac{\Delta}{2}t}e^{-\frac{\gamma}{4}t}\left[\cos\left(\frac{\delta t}{2}\right) + i\,\frac{\omega_1}{\delta}\sin\left(\frac{\delta t}{2}\right)\right] \tag{30}$$

with

$$\omega_1 = \Delta - i\frac{\gamma}{2}, \quad \delta = \sqrt{4G^2+\omega_1^2}.$$

For $\Delta = 0$ (zero detuning), $\delta = \sqrt{G^2 - \gamma^2/4}$. Hence, for $\gamma \leqslant 2G$ and $\gamma > 2G$ the excitation probability $|\varepsilon(t)|^2$, respectively, exhibits damped oscillations and a monotonic (in general nonexponential) decay [1]. In Fig. 3, we compare the time evolution of the excitation probability of S predicted by Eq. (30) corresponding to master equation (28) with the exact discrete dynamics of the collision model computed through Eq. (27). The agreement between these is excellent for collision times shorter than $\tau \sim 10^{-1}G^{-1}$.

A. Connection with a microscopic environmental model

Consider the microscopic environmental model defined by the Hamiltonian

$$\hat{H}_{AF} = \omega_0\,\hat{\sigma}_+\hat{\sigma}_- + \sum_k \omega_k\,\hat{a}_k^\dagger\hat{a}_k + \sum_k \mu_k(\hat{\sigma}_-\hat{a}_k^\dagger + \hat{\sigma}_+\hat{a}_k), \tag{31}$$

describing a two-level atom A of frequency ω_0 in dissipative contact (under RWA) with a bath of bosonic modes (field), labeled by index k, with frequency ω_k and bosonic annihilation (creation) operator $\hat{a}_k$ ($\hat{a}_k^\dagger$) and atom-mode coupling rate μ_k. In the continuous limit, $\omega_k \to \omega$, $\mu_k \to \mu(\omega)$ and $\sum_k \to \int d\omega\,\rho(\omega)$ with $\rho(\omega)$ the field density of states.

Consider the spontaneous emission process with the atom initially in its excited state and the bath modes initially in their vacuum state. Let $\varepsilon(t)$ be the probability amplitude to find the atom in its excited state at time t. Given Hamiltonian (31), it can be shown [1] that $\varepsilon(t)$ is governed by the general

integrodifferential equation

$$\dot{\varepsilon} = -\int_0^t ds \left[\int d\omega J(\omega) e^{i(\omega_0-\omega)(t-s)}\right]\varepsilon(s), \tag{32}$$

where $J(\omega) = [\mu(\omega)]^2\rho(\omega)$ is the spectral density.

Now, for a Lorentzian spectral density given by

$$J(\omega) = \frac{\Gamma_0}{2\pi}\frac{\kappa^2}{(\omega-\omega_0-\Delta)^2+\kappa^2}, \tag{33}$$

it can be shown that the *exact* solution for $\varepsilon(t)$ coincides with Eq. (30) provided that

$$\gamma = 2\kappa, \quad G = \sqrt{\frac{\Gamma_0\kappa}{4}}. \tag{34}$$

As long as the open dynamics of the two-level system is concerned, this in fact establishes an equivalence (first pointed out by Garraway [27]) between the environmental model (31) and the master equation (28) of the damped JC model. This is intuitively clear once in Eq. (33) $\omega_0+\Delta$ is interpreted as the resonance frequency of a cavity mode and κ as the bandwidth of a lossy cavity. Within our framework, given the previously shown correspondence between master equation (28) and the collision model in Eqs. (22) and (23), we can thus establish a correspondence between such composite collision model (in the continuous-time limit) and the microscopic environmental model in Eq. (31). Starting from the latter, we can thereby construct an associated composite (bipartite) collision model defined by Eqs. (22), (23), and the parameters: Δ,

$$g = \sqrt{\frac{2\kappa}{\tau}}, \quad G = \sqrt{\frac{\Gamma_0\kappa}{4}}, \tag{35}$$

where we used $\gamma = g^2\tau$ in combination with Eqs. (34).

To summarize, given the environmental microscopic model in Eq. (31), in the case of a Lorenztian spectral density [cf. Eq. (33)], one can construct a composite collision model through Eqs. (22), (23), and (35) which, in the continuous-time limit, reproduces the same open system dynamics.

VI. RANDOM TELEGRAPH NOISE AND PURE DEPHASING

In the next instance of composite bipartite collision model that we consider, S, S_1 and R_n are all qubits. By definition [cf. Eqs. (17) and (18)],

$$\hat{V}_{SS_1} = G\,\hat{K}_S\hat{K}_{S_1}, \quad \hat{W}_{S_1n} = g(\hat{\sigma}_{1-}\hat{\sigma}_{n+} + \hat{\sigma}_{1+}\hat{\sigma}_{n-}) \tag{36}$$

with $\hat{K}_S$ ($\hat{K}_{S_1}$) a Hermitian operator on S (S_1). We take as initial state of each ancilla, a thermal state $\eta = \frac{1}{2}(1-\xi)|0\rangle\langle 0| + \frac{1}{2}(1+\xi)|1\rangle\langle 1|$ with $\xi = \tanh(\beta)$ and β the $\mathcal{R}$'s inverse temperature.

In the continuous-time limit (see Sec. IV), the collision model defined this way gives rise to the master equation for the $S-S_1$ state

$$\dot{\rho} = -i\big[G\,\hat{K}_S\hat{K}_{S_1},\rho\big] + \Gamma_+\big(\hat{\sigma}_{1-}\rho\,\hat{\sigma}_{1+} - \tfrac{1}{2}[\hat{\sigma}_{1+}\hat{\sigma}_{1-},\rho]_+\big) + \Gamma_-\big(\hat{\sigma}_{1+}\rho\,\hat{\sigma}_{1-} - \tfrac{1}{2}[\hat{\sigma}_{1-}\hat{\sigma}_{1+},\rho]_+\big), \tag{37}$$

where $\Gamma_\pm = \gamma(1\pm\xi)/2$ with $\gamma = g^2\tau$. We will consider next the collision models arising from two different choices of operators $\hat{K}_S$ and $\hat{K}_{S_1}$.

A. Random telegraph noise

In this first instance, we set $\hat{K}_S = \hat{H}_S/G$, $\hat{K}_{S_1} = \hat{\sigma}_{1z}$, and $\xi = 0$ (hence the ancillary initial states are all maximally mixed). Operator $\hat{H}_S$ can be interpreted as a Hamiltonian operator on S. Let $|\pm\rangle_{S_1}$ be the state of S_1 such that $\hat{H}_S|\pm\rangle_{S_1} = \pm|\pm\rangle_{S_1}$. Tracing over S_1, the S reduced state is given by $\rho_S(t) = \rho_{S+}(t) + \rho_{S-}(t)$ with $\rho_{S\pm}(t) = {}_{S_1}\langle\pm|\rho(t)|\pm\rangle_{S_1}$. In the continuous-time limit, the master equation (37) gives rise to the following pair of coupled master equations

$$\dot{\rho}_{S\pm} = -i[\pm\hat{H}_S,\rho_{S\pm}] \pm \frac{1}{t_c}(\rho_{S-} - \rho_{S+}), \tag{38}$$

which describe the well-known dynamics of a quantum system S subjected to random telegraph noise [28] in the case of a single bistable fluctuator featuring a correlation time $t_c = 2/g^2\tau$. In such a case, $\pm\hat{H}_S$ is the Hamiltonian corresponding to the fluctuator's classical state labeled by "$\pm$", in turn defining one of the two possible trajectories along which S can evolve. The possibility to derive a random telegraph noise qubit dynamics from a bipartite master equation of the form Eq. (37) was pointed out in Ref. [29].

B. Pure dephasing

Let us now set $\hat{K}_S = \hat{\sigma}_z$ and $\hat{K}_{S_1} = \hat{\sigma}_{1x}$ with states $|0\rangle$ ($|1\rangle$) being the S state such that $\hat{\sigma}_z|0\rangle = |0\rangle$ ($\hat{\sigma}_z|1\rangle = -|1\rangle$) and ρ_0 the initial state of S. The populations $\langle m|\rho_0|m\rangle$ for $m = 0,1$ will be clearly unaffected by the collision process due to the dispersive nature of the $S-S_1$ coupling while, as shown in Appendix C, the coherences, at the nth step are

$$\langle 0|\rho_n|1\rangle = \langle 1|\rho_n|0\rangle^* = f_n\langle 0|\rho_0|1\rangle,$$

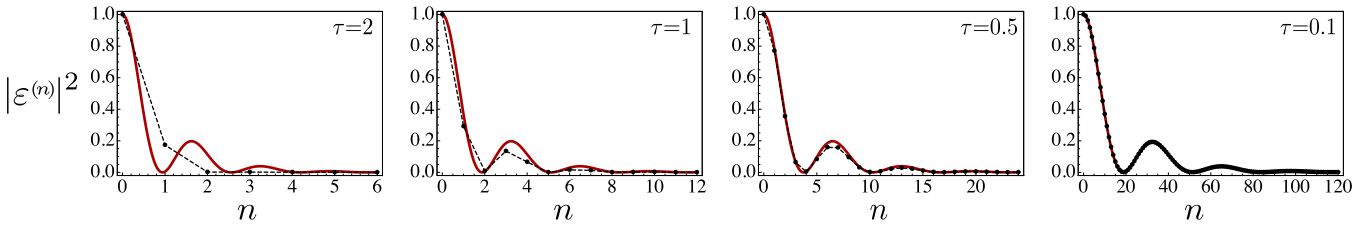

FIG. 3. Excited-state population $|\varepsilon^{(n)}|^2$ of S against the step number n in the case of the composite collision model specified by Eqs. (22) and (23) for different values of the collision time τ (in units of G^{-1}) and for $g = \sqrt{G/\tau}$, $\Delta = 0$. For each set value of τ, the solid curve shows the behavior of the excitation probability [cf. Eq. (30)] predicted by master equation (28) for $t = n\tau$ and $\gamma = g^2\tau$. For $\tau = 10^{-1}G^{-1}$ the solid curve is in fact indistinguishable from the exact discrete dynamics.

where the step-dependent dephasing factor f_n reads

$$f_n = \frac{1}{2}\left[\left(\frac{(c_g+1)c_G+\kappa}{2}\right)^n + \left(\frac{(c_g+1)c_G-\kappa}{2}\right)^n\right] + \frac{(1-c_g)c_G}{\kappa} \times \frac{1}{2}\left[\left(\frac{(c_g+1)c_G+\kappa}{2}\right)^n - \left(\frac{(c_g+1)c_G-\kappa}{2}\right)^n\right] \quad (39)$$

with

$$c_g = \cos(2g\tau), \quad c_G = \cos(2G\tau), \quad \kappa = \sqrt{(c_g-1)^2 c_G^2 - 4c_g s_G^2}.$$

To carry out the continuous-time limit, in line with Sec. III, we expand up to the second order in $g\tau$ and first order in $G\tau$. This way, neglecting terms proportional to $\sim G^2 g^2$ and taking $n\tau \to t$ and $g^2\tau \to \gamma$, the continuous-time limit of $(1-c_g)c_G/\kappa$ turns out to be $g^2\tau/\kappa_c$ while $(c_g+1)c_G \pm \kappa$ becomes $2(1 - g^2\tau \pm \kappa_c\tau)$, where

$$\kappa_c = \sqrt{\gamma^2 - 4G^2}. \quad (40)$$

With these approximations the step-dependent decoherence factor (39) takes the continuous-time form

$$f(t) = e^{-\gamma t}\left[\cosh(\kappa_c t) + \frac{\gamma \sinh(\kappa_c t)}{\kappa_c}\right]. \quad (41)$$

This result can also be derived through a direct solution of master equation (37).

Similarly to Sec. V A, also the present collision model can be associated with a corresponding microscopic environmental model yielding the same S open dynamics. To see this, consider a qubit S dispersively coupled to a bosonic reservoir according to the Hamiltonian

$$\hat{H} = \omega_0 \hat{\sigma}_+ \hat{\sigma}_- + \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_k \mu_k \hat{\sigma}_z (\hat{a}_k + \hat{a}_k^\dagger), \quad (42)$$

where the difference with respect to Eq. (31) is that the interaction is now dispersive. As in Sec. V A, the reservoir spectral density in the continuous limit is given by $J(\omega) = [\mu(\omega)]^2 \rho(\omega)$.

This model can be solved exactly [1,30,31], the corresponding master equation for the qubit (in the interaction picture) reading

$$\dot{\rho}_S = \gamma(t)(\sigma_z \rho_S \sigma_z - \rho_S). \quad (43)$$

If the environment is initially in the vacuum state, the time-dependent dephasing rate $\gamma(t)$ takes the form

$$\gamma(t) = \int_0^\infty d\omega \sin(\omega t)\frac{J(\omega)}{\omega}. \quad (44)$$

Accordingly, the coherences decay as $\langle 0|\rho(t)|1\rangle = e^{-\Lambda(t)}\langle 0|\rho(0)|1\rangle)$ with $\Lambda(t)$ related to $\gamma(t)$ as $\Lambda(t) = 2\int_0^t \gamma(t')dt'$. Equation (44) shows that $J(\omega)/\omega$ is the Fourier-Sine transform of $\gamma(t)$. Correspondingly,

$$J(\omega) = \omega \int_0^\infty dt \sin(\omega t)\gamma(t). \quad (45)$$

In the continuous-time limit of our collision model, we can identify $\Lambda(t) = -\log[f(t)]$ and thereby $\gamma(t) = 1/2\dot{\Lambda}(t) = -1/2\dot{f}(t)/f(t)$. Using next Eq. (45) with the help of Eq. (41), we thus find that for $\gamma > 2G$ the equivalent spectral density of our collision model is given by

$$J(\omega) = \sum_{j=0}^\infty \frac{(\gamma-\kappa_c)^j}{(\gamma+\kappa_c)^{j+2}} \frac{4G^2\kappa_c(j+1)^2}{(j+1)^2 + \omega^2/(4\kappa_c^2)}. \quad (46)$$

We thus find that our collision model yields the same reduced dynamics of S obtained from a microscopic environmental model where the reservoir spectral density consists of a series of Lorentzian-shaped distributions (with positive weights). Note that all of these are centered at the same frequency with a width that increases with index j. In the limit where the coupling rate G is much smaller than the decay rate γ, only the first term of the sum dominates in a way that the spectral density reduces to a single Lorentzian.

VII. EXTENSION TO THE MULTIPARTITE CASE

So far we have considered composite collision models where the system $\mathcal{S}$ comprises the very open system S and a single auxiliary system S_1. In this section, we present an extension of the collision model to account for multiple auxiliary systems. Furthermore, in this new scenario, we will in addition allow each ancilla to be multipartite. Such extension enables a collision model-based description of certain open dynamics that cannot be captured in the simple bipartite case, as we will show in the next section.

Both $\mathcal{S}$ and each reservoir ancilla are now assumed to be multipartite. Specifically, $\mathcal{S}$ comprises $N+1$ subsystems $S, S_1, S_2, \ldots, S_N$ with S embodying the very open system under study and where $\{S_i\}$ are auxiliary systems. Furthermore, the nth reservoir ancilla R_n is N-partite, its subsystems, referred to as subancillas in the following, being $\{R_{n1}, R_{n2}, \ldots, R_{nN}\}$. The free Hamiltonian of $\mathcal{S}$ is now defined by [cf. Eq. (17)]

$$\hat{H}_\mathcal{S} = \sum_{i=1}^N \left(\hat{H}_{S_i} + \hat{V}_{SS_i}\right) + \sum_{i<j} \hat{V}_{S_i S_j}, \quad (47)$$

where $\hat{H}_{S_i}$ is the free Hamiltonian of subsystem S_i, $\hat{V}_{SS_i}$ is the interaction Hamiltonian of S and S_i, while $\hat{V}_{S_i S_j}$ [not appearing in Eq. (17)] describes the S_i–S_j coupling between different auxiliary systems. The $\mathcal{S}$–R_n interaction [cf. Eq. (18)] is generalized as

$$\hat{H}_{\mathcal{S}n} = \sum_{i=1}^N \hat{W}_{S_i n_i} = g_i \hat{w}_{S_i n_i} \quad (48)$$

with $\hat{W}_{S_i n_i}$ the interaction Hamiltonian of subsystem S_i and subancilla R_{n_i}. Note that $\hat{V}$ operators describe interactions internal to the system $\mathcal{S}$, while the $\hat{W}$'s correspond to system-ancilla interactions. Also, note that the latter ones take place only between subsystems and subancillas labeled by corresponding indexes. The bipartite composite model of Sec. IV is retrieved in the special case $N = 1$. A sketch of a composite tripartite collision model, corresponding to $N = 2$, is given in Fig. 4.

We assume the initial state of each ancilla to be the product state $\eta = \otimes_{i=1}^N \xi_i$, with ξ_i the initial state of each subancilla, and that [cf. Eqs. (8) and (48)]

$$\mathrm{Tr}_{R_{n_i}}\left\{\hat{W}_{S_i n_i}\xi_i\right\} = 0. \quad (49)$$

This entails $\mathrm{Tr}_{R_n}\{\hat{H}_{\mathcal{S}n}\eta\} = 0$. Under the above conditions, an identity analogous to Eq. (15) holds.

Once $\hat{H}_{\mathcal{S}n}$ is expressed through Eq. (48), we note that the resulting cross terms in the $\hat{W}$ operators vanish because of Eq. (49). In the continuous-time limit, we thus end up with a master equation for ρ (state of $\mathcal{S}$) that reads

$$\frac{d\rho}{dt} = -i\,[\hat{\mathcal{H}}_{\mathcal{S}},\rho] + \sum_{i=1}^{N}\mathcal{L}_{S_i}(\rho) \tag{50}$$

with

$$\begin{aligned}\mathcal{L}_{S_i}(\rho) &= \gamma_i \sum_{\mu\nu}\left\{\hat{A}^{(i)}_{\mu\nu}\,\rho\big(\hat{A}^{(i)}_{\mu\nu}\big)^{\dagger} - \frac{1}{2}\big[\big(\hat{A}^{(i)}_{\mu\nu}\big)^{\dagger}\hat{A}^{(i)}_{\mu\nu},\rho\big]_{+}\right\},\\ \hat{A}^{(i)}_{\mu\nu} &= \sqrt{p_\nu}\langle\mu|\hat{w}_{S_i n_i}|\nu\rangle\\ \gamma_i &= g_i^2\tau,\end{aligned} \tag{51}$$

where the initial subancilla state is $\xi_i = \sum_m p_m|m\rangle_i\langle m|$ (here $|\mu\rangle$ and $|\nu\rangle$ are two generic elements of the orthonormal basis $\{|m\rangle\}$ in the Hilbert space of subancilla R_{n_i}). The jump operator $\hat{A}^{(i)}_{\mu\nu}$ and its H.c. act in the Hilbert space of the auxiliary system S_i with $i = 1,\ldots,N$. In the following section, we show how the composite model defined above provides a collision-model description of a quantum emitter subject to a multi-Lorentzian spectral density.

VIII. MULTI-LORENTZIAN SPECTRAL DENSITY

Here, we address a composite collision model, which can be regarded as an extension of the model described in Sec. V to the case $N = 2$ (hence $\mathcal{S}$ is now tripartite). The model features two auxiliary systems S_1 and S_2, each modeled as a bosonic mode with associated annihilation (creation) operator $\hat{\alpha}_i^{\dagger}$ ($\hat{\alpha}_i$) for $i = 1,2$. Correspondingly, each reservoir ancilla R_n is now bipartite, consisting of subancillas R_{n1} and R_{n2}, each modeled as a bosonic mode of annihilation (creation) operator $\hat{a}_{ni}$ ($\hat{a}_{ni}^{\dagger}$). The dynamics of this collision model is generated by the following Hamiltonian [cf. Eqs. (47) and (48)]

$$\hat{H}_{S_i} = \Delta_i\hat{\alpha}_i^{\dagger}\hat{\alpha}_i, \quad \hat{V}_{SS_i} = G_i(\hat{\sigma}_-\hat{\alpha}_i^{\dagger} + \text{H.c.}), \tag{52}$$

$$\hat{V}_{S_1S_2} = c\,(\hat{\alpha}_1\hat{\alpha}_2^{\dagger} + \text{H.c.}), \quad \hat{W}_{S_i n} = g_i(\hat{\alpha}_i\hat{a}_{ni}^{\dagger} + \text{H.c.}) \tag{53}$$

with $i = 1,2$. Note that, in general, the auxiliary systems S_1 and S_2 can be subjected to a mutual interaction (with associated coupling rate c).

We again restrict our analysis to the single-excitation subspace, with S initially in its excited state, while S_1, S_2 and all the ancillas are initially in their ground state (hence now $\eta = |0\rangle_{R_{n1}}\langle 0| \otimes |0\rangle_{R_{n2}}\langle 0|$). Introducing a compact notation analogous to that employed in Sec. V, the single-excitation initial state is thus denoted by $|100\mathbf{0}\rangle$, the first three quantum numbers now referring to S, S_1 and S_2, respectively. The total number of excitations of $\mathcal{S}-\mathcal{R}$ is again conserved. Similarly to Eq. (24), the joint state at an arbitrary step n is now of the form

$$\begin{aligned}|\Phi^{(n)}\rangle = \varepsilon^{(n)}|100\mathbf{0}\rangle + \beta_1^{(n)}|010\mathbf{0}\rangle + \beta_2^{(n)}|001\mathbf{0}\rangle\\ + \sum_{i=1}^{n}\sum_{j=1}^{2}\lambda_{ij}^{(n)}|001_{ij}\rangle\end{aligned}$$

with $\beta_j^{(n)}$ ($\lambda_{ij}^{(n)}$) the excitation probability amplitude of S_j (R_{nj}) at the nth step.

By carrying out an analysis similar to the one carried on in section V, one can show that in the continuous-time limit the present collision model yields for S an open dynamics equivalent to that of a microscopic environmental model of the form (31) where the spectral density is a sum of two Lorentzian functions.

In line with Ref. [27], we consider the two cases respectively specified by [see Eqs. (52) and (53)] (i) $c = 0$ and (ii) $G_2 = 0$, $\Delta_{1,2} = \Delta$. In the case (i), we obtain that the equivalent spectral density is the sum of two spectral densities with positive weights

$$J(\omega) = \sum_{i=1,2}\frac{4G_i^2/\gamma_i}{2\pi}\frac{(\gamma_i/2)^2}{(\omega-2\Delta_i)^2+(\gamma_i/2)^2}$$

with $\gamma_i = g_i^2\tau$. Hence, the rate $\gamma_i/2$ and ratio $4G_i^2/\gamma_i$ give respectively the width and maximum of each distribution. In the case (ii), instead, under the condition $\gamma_1 - \gamma_2 > 2c$, the equivalent spectral density turns out to be

$$J(\omega) = \left(\frac{\kappa_+}{2\pi}\frac{\lambda_+^2}{(\omega-2\Delta)^2+\lambda_+^2}\right) - \left(\frac{\kappa_-}{2\pi}\frac{\lambda_-^2}{(\omega-2\Delta)^2+\lambda_-^2}\right),$$

where

$$\begin{aligned}\lambda_\pm &= (\gamma_1+\gamma_2 \pm \chi)/4\\ \kappa_\pm &= \frac{2G^2\{8c^2(\chi \mp 2\gamma_2) \pm (\gamma_1-\gamma_2)\gamma_2[\gamma_1-(\gamma_2\pm\chi)]\}}{\chi^2(4c^2+\gamma_1\gamma_2)}\end{aligned}$$

with $\chi = \sqrt{(\gamma_1-\gamma_2)^2 - 16c^2}$. Such a combination of Lorentzian distributions (with weights of opposite signs) can be used as a simplified model of a reservoir featuring a band-gapped spectrum [1,27].

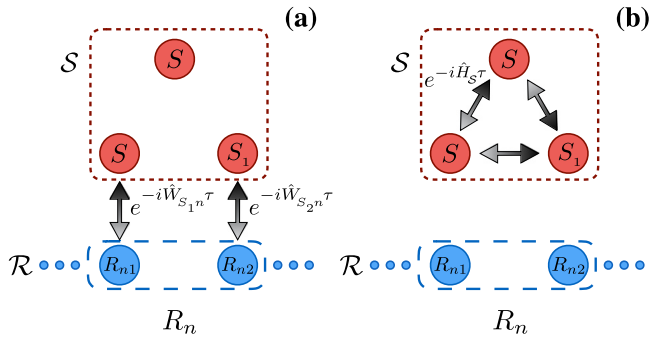

FIG. 4. (a) System-ancilla collision and (b) intrasystem collision in a composite tripartite collision model. System $\mathcal{S}$ is tripartite comprising the very open system S under study and the auxiliary systems $\{S_1, S_2\}$, while ancilla R_n is bipartite comprising subancillas $\{R_{n1}, R_{n2}\}$. Note that a system-ancilla collision (a) occurs through pairwise interactions between the auxiliary systems and the corresponding reservoir subancillas.

IX. CONCLUSIONS

In this paper, we have introduced a class of quantum collision models, which we called composite collision models. Their definition is inspired by multipartite Lindblad-type master equations to describe non-Markovian dynamics, where the open system under study is coherently coupled to one or more auxiliary systems, which are in turn in contact with Markovian baths.

In one such collision model, the very open system under study S (undergoing non-Markovian dynamics in general) is coupled to one or more auxiliary (in general mutually interacting) systems $\{S_i\}$, which in turn interact with the reservoir ancillas. We have presented a comprehensive discussion of the continuous-time limit in which the collision model is effectively described by a master equation, in particular the conditions on the collision time and Hamiltonian parameters to fulfill.

We have shown that this collision-model-based framework can accommodate some known relevant instances of non-Markovian dynamics, such as an atom decaying in a lossy cavity, a qubit subjected to random telegraph or purely dephasing noise and a quantum emitter in dissipative contact with a reservoir featuring a spectral density that is the sum of two Lorentzian distributions.

It was also illustrated that some specific microscopic environmental models can interestingly be associated with suitably built corresponding composite collision models yielding the same open system dynamics in the continuous-time limit. The theory presented here strengthens the role that collision models can play as an alternative, advantageous approach for tackling quantum non-Markovian dynamics.

An open question left is whether collision models can be constructed to describe certain classes of non-Markovian dynamics that cannot be captured by the framework developed here, such as the decay of an atom in a photonic-band-gap medium where the corresponding reservoir spectral density exhibits van Hove singularities, or, if not, whether such impossibility can be given an insightful physical meaning [32].

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APPENDIX A

To derive Eq. (27), which in particular yields the open dynamics of S in the collision model of Sec. V, we first define the unitaries $\hat{U}_{S_1 n} = e^{-i\hat{W}_{S_1 n}\tau}$ and $\hat{U}_{\mathcal{S}} = e^{-i(\hat{H}_{S_1}+\hat{V}_{SS_1})\tau}$. Next, we use that $\hat{U}_{S_1 n}$ acts on the nth ancilla and subsystem S_1 only, while $\hat{U}_{\mathcal{S}}$ acts only on S and S_1. Also, either of these unitary operators does not change the total number of excitations. Thereby, in the single-excitation sector of the total Hilbert space, the only three states to be affected by the application of $\hat{U}_{S_1 n}\hat{U}_{\mathcal{S}}$ are $\{|10\mathbf{0}\rangle, |01\mathbf{0}\rangle, |00\,1_n\rangle\}$, all the remaining ones being invariant. Thus, in virtue of Eq. (24) for $n \to n-1$,

$$|\Phi^{(n)}\rangle = \hat{U}_{S_1 R_n}\hat{U}_{\mathcal{S}}\Big[\varepsilon^{(n-1)}|10\mathbf{0}\rangle + \beta^{(n-1)}|01\mathbf{0}\rangle + \sum_{i=1}^{n-1}\gamma_i^{(n-1)}|001_i\rangle\Big]. \tag{A1}$$

Based on Eqs. (22) and (23), the effective matrix representation of $\hat{U}_{S_1 n}\hat{U}_{\mathcal{S}}$ in the subspace $\{|10\mathbf{0}\rangle, |01\mathbf{0}\rangle, |00\,1_n\rangle\}$ can be calculated as

$$\mathbf{U}_{S_1 n}\mathbf{U}_{\mathcal{S}} = \begin{pmatrix} e^{-i\frac{\Delta}{2}\tau}\big[\cos(\Omega\tau) + i\frac{\Delta}{2\Omega}\sin(\Omega\tau)\big] & \frac{G}{2\Omega}e^{-i(\frac{\Delta}{2}+\Omega)\tau}(1-e^{2i\Omega\tau}) & 0 \\ \frac{G}{4\Omega}(1+e^{2ig\tau})(1-e^{2i\Omega\tau})e^{-i(\Omega+\frac{\Delta}{2}+g)\tau} & e^{-i\frac{\Delta}{2}\tau}\cos(g\tau)\big[\cos(\Omega\tau) - i\frac{\Delta}{2\Omega}\sin(\Omega\tau)\big] & -i\sin(g\tau) \\ \frac{G}{4\Omega}(1-e^{2ig\tau})(1-e^{2i\Omega\tau})e^{-i(\Omega+\frac{\Delta}{2}+g)\tau} & -ie^{-i\frac{\Delta}{2}\tau}\sin(g\tau)\big[\cos(\Omega\tau) - i\frac{\Delta}{2\Omega}\sin(\Omega\tau)\big] & \cos(g\tau) \end{pmatrix},$$

where $\Omega = \sqrt{\Delta^2 + 4G^2}/2$ [see Eqs. (26)]. This alongside Eq. (24) thus yield

$$\varepsilon^{(n)} = e^{-i\frac{\Delta}{2}\tau}\Big[\cos(\Omega\tau) + i\frac{\Delta}{2\Omega}\sin(\Omega\tau)\Big]\varepsilon^{(n-1)} + \frac{G}{2\Omega}e^{-i(\frac{\Delta}{2}+\Omega)\tau}(1-e^{2i\Omega\tau})\beta^{(n-1)}, \tag{A2}$$

$$\beta^{(n)} = \frac{G}{4\Omega}(1+e^{2ig\tau})(1-e^{2i\Omega\tau})e^{-i(\Omega+\frac{\Delta}{2}+g)\tau}\varepsilon^{(n-1)} + e^{-i\frac{\Delta}{2}\tau}\cos(g\tau)\Big[\cos(\Omega\tau) - i\frac{\Delta}{2\Omega}\sin(\Omega\tau)\Big]\beta^{(n-1)}, \tag{A3}$$

$$\lambda_n^{(n)} = \frac{G}{4\Omega}(1-e^{2ig\tau})(1-e^{2i\Omega\tau})e^{-i(\Omega+\frac{\Delta}{2}+g)\tau}\varepsilon^{(n-1)} - ie^{-i\frac{\Delta}{2}\tau}\sin(g\tau)\Big[\cos(\Omega\tau) - i\frac{\Delta}{2\Omega}\sin(\Omega\tau)\Big]\beta^{(n-1)}. \tag{A4}$$

Equations (A2) and (A3) can be expressed in matrix form as

$$\begin{pmatrix}\varepsilon^{(n)}\\ \beta^{(n)}\end{pmatrix} = \mathbf{M}\begin{pmatrix}\varepsilon^{(n-1)}\\ \beta^{(n-1)}\end{pmatrix},$$

where the 2×2 matrix $\mathbf{M}$ [cf. Eq. (25)] is the upper-left 2×2 block of matrix $\mathbf{U}_{S_1 R_n}\mathbf{U}_{\mathcal{S}}$.

APPENDIX B: DERIVATION OF EQ. (30)

For $\eta = |0\rangle\langle 0|$ and $\rho(0) = |e\rangle_S\langle e||0\rangle_{S1}\langle 0|$, due to the conservation of the total number of excitations [cf. Eqs. (22) and (23)] the joint $\mathcal{S}-\mathcal{R}$ state at time t has the form

$$|\Phi(t)\rangle = \varepsilon(t)|10\mathbf{0}\rangle + \beta(t)|01\mathbf{0}\rangle + \sum_{i=1}^{n}\lambda_i(t)|001_i\rangle. \tag{B1}$$

Upon trace over $\mathcal{R}$ of $|\Phi(t)\rangle\langle\Phi(t)|$, the $\mathcal{S}$'s density matrix then reads

$$\rho(t) = |\varepsilon(t)|^2|10\rangle\langle 10| + |\beta(t)|^2|01\rangle\langle 01| + (\varepsilon(t)[\beta(t)]^*|10\rangle\langle 01| + \text{H.c.}) + (1 - |\varepsilon(t)|^2 - |\beta(t)|^2)|00\rangle\langle 00|. \tag{B2}$$

Plugging this into master equation (28) yields the following set of equations:

$$\begin{aligned}
\frac{d}{dt}|\varepsilon|^2 &= iG(\varepsilon\beta^* - \varepsilon^*\beta),\\
\frac{d}{dt}|\beta|^2 &= -iG(\varepsilon\beta^* - \varepsilon^*\beta) - \gamma|\beta|^2,\\
\frac{d}{dt}(\varepsilon\beta^*) &= -\frac{\gamma}{2}\varepsilon\beta^* + i[G(|\varepsilon|^2 - |\beta|^2) + \Delta\varepsilon\beta^*].
\end{aligned} \tag{B3}$$

It is easily checked that these are equivalent to the system of differential equations in the excitation amplitudes

$$\dot{\varepsilon} = -iG\beta, \tag{B4}$$

$$\dot{\beta} = -i\left(\Delta - i\frac{\gamma}{2}\right)\beta - iG\varepsilon. \tag{B5}$$

Solving Eq. (B5) as a function of $\varepsilon(t)$ under the initial condition $\beta(0) = 0$ yields

$$\beta(t) = -i\,G\int_0^t dt'\, e^{-i(\Delta - i\frac{\gamma}{2})(t-t')}\varepsilon(t'), \tag{B6}$$

which when replaced in Eq. (B4) gives rise to the integrodifferential equation (29). The solution (30) of Eq. (29) can be worked out by taking the Laplace transform of each equation side so as to end up with an algebraic equation in the Laplace transform of $\varepsilon(t)$. Once the inverse Laplace transform is computed, Eq. (30) is obtained.

APPENDIX C

At step n, the $S-S_1$ joint state (i.e., the $\mathcal{S}$'s one) reads

$$\rho_n = \mathrm{Tr}_{R_n}\{\hat{U}_{Sn}\rho_{n-1}\,\eta\,\hat{U}_{Sn}^\dagger\} = \mathcal{F}[\rho_{n-1}] = \mathcal{F}^n[\rho_0] \tag{C1}$$

with $\hat{U}_{Sn} = \exp(-i\hat{V}_{SS_1}\tau)\exp(-i\hat{W}_{S_1R_n}\tau)$ [cf. Eq. (36) and Sec. VI B], where we have defined the bipartite quantum map $\mathcal{F}$ acting on S and S_1.

Next, we take as local operator basis for S and S_1 the set of Hermitian operators

$$\hat{\mathcal{G}}_{\alpha 0} = \mathbb{I}_\alpha/\sqrt{2}, \quad \hat{\mathcal{G}}_{\alpha 1} = \sigma_{\alpha x}/\sqrt{2}, \quad \hat{\mathcal{G}}_{\alpha 2} = \sigma_{\alpha y}/\sqrt{2}, \quad \hat{\mathcal{G}}_{\alpha 3} = \sigma_{\alpha z}/\sqrt{2}$$

with $\alpha = S, S_1$. Accordingly, the bipartite operator basis for the joint $S-S_1$ system is given by $\hat{G}_{kj} = \hat{\mathcal{G}}_{Sk}\otimes\hat{\mathcal{G}}_{S_1 j}$.

In this representation, map $\mathcal{F}$ corresponds to a 16×16 matrix $\mathbf{F}$, whose entries are given by $\mathbf{F}_{(kj,k'j')} = \mathrm{Tr}\{\hat{G}_{kj}\mathcal{F}[\hat{G}_{k'j'}]\}$, while the bipartite state ρ_n is turned into the 16-dimensional column vector $\mathbf{r}_n$ defined by $r_{n,kj} = \mathrm{Tr}\{\hat{G}_{kj}\rho_n\}$. Matrix $\mathbf{F}$ and vector $\mathbf{r}_0$ are then computed as

$$\mathbf{F} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c_g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & c_G c_g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_g s_G\\
-s_g^2 & 0 & 0 & c_G c_g^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_g^2 s_G & 0\\
0 & 0 & 0 & 0 & c_G & 0 & 0 & 0 & 0 & -s_G & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & c_G c_g & 0 & 0 & -c_g s_G & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & c_g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -c_G s_g^2 & 0 & 0 & c_g^2 & 0 & s_G s_g^2 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & s_G & 0 & 0 & c_G & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & c_g s_G & 0 & 0 & 0 & 0 & c_G c_g & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_g & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -s_G s_g^2 & 0 & 0 & -c_G s_g^2 & 0 & 0 & c_g^2 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_g & 0 & 0\\
0 & 0 & 0 & -c_g s_G & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_G c_g & 0\\
0 & 0 & c_g^2 s_G & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -s_g^2 & 0 & 0 & c_G c_g^2
\end{pmatrix},$$

$$\mathbf{r}_0 = \tfrac{1}{2}\left\{1,0,0,-1,\rho_0^{(01)}+\rho_0^{(10)},0,0,-\left(\rho_0^{(01)}+\rho_0^{(10)}\right),i\left(\rho_0^{(10)}-\rho_0^{(01)}\right),0,0,-i\left(\rho_0^{(10)}-\rho_0^{(01)}\right),\rho_0^{(11)}-\rho_0^{(00)},0,0,\rho_0^{(00)}-\rho_0^{(11)}\right\}$$

with $s_X = \sin(2X\tau)$ and $c_X = \cos(2X\tau)$ for $X = g, G$ and $\rho_0^{(ij)} = \langle i|\rho|j\rangle$ for $i, j = 0, 1$.

Evaluating next matrix $\mathbf{F}^n$ and applying it on $\mathbf{r}_0$, one can calculate $\mathbf{r}_n$ and eventually return to the density-matrix description through $\rho_n = \sum_{kj} r_{kj}\hat{G}_{kj}$. This way, we end up with Eq. (39) where $f_n = \frac{1}{2}[(\mathbf{F}^n)_{(21,21)} + (\mathbf{F}^n)_{(31,31)}]$.

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