Universidad Veracruzna

Artificial Intelligence

Automata and Formal Languages

Activity 2

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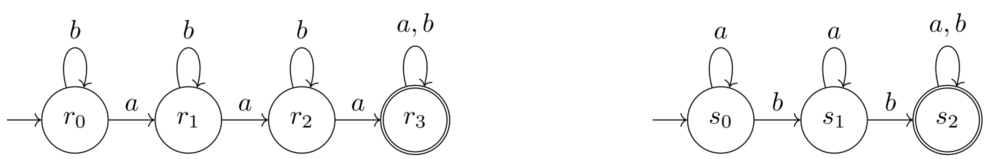
September 1, 2022

Build a DFA M_C that recognizes the regular language $C = \{w \mid w \text{ has at least 3 } a\text{'s and at least 2 } b\text{'s}\}$ and has an alphabet $\Sigma = \{a, b\}$.

Solution:

To move forward with the solution of the problem, it is important to mention the following theorem:

Theorem 1 *The class of regular languages is closed under the intersection operation.*



(a) Automaton M_A that recognizes A.

(b) Automaton M_B that recognizes B.

Figure 1: Automata in which the automaton M_C is divided into.

Lets us assume that the language C of the automaton M_C is a regular language. In fact, *a language is called a regular language if some finite automaton recognizes it*[1]. The finite automaton M_C and the langauge it recognizes, C, can be regarded as the intersection of two regular languages A and B corresponding to the automata M_A and M_B, respectively. The regular language C was divided to take advantage of the intersection theorem between regular languages and to ease its construction. The regular languages A and B can be expressed as,

$$
\begin{aligned}
A &= \{w \mid w \text{ has at least 3 } a\text{'s}\}, \\
B &= \{w \mid w \text{ has at least 2 } b\text{'s}\}.
\end{aligned}
$$

The diagram of the automaton M_A is shown in fig. 1a, and it is described by the 5-tuple $(Q_A, \Sigma, \delta_A, r_0, F_A)$ where

$$
\begin{aligned}
Q_A &= \{r_0, r_1, r_2, r_3\}, \\
F_A &= \{r_3\}.
\end{aligned}
$$

The alphabet Σ is the same for both M_A and M_B. Additionally, the transition function has the same form, i.e., $\delta_I : Q_I \times \Sigma \to Q_I$ where $Q_I = Q_A, Q_B$. Similarly, the diagram of M_B is shown in fig. 1b and it is described by the 5-tuple $(Q_B, \Sigma, \delta_B, s_0, F_B)$ where,

$$
\begin{aligned}
Q_B &= \{s_0, s_1, s_2\}, \\
F_B &= \{s_2\}.
\end{aligned}
$$

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[1]Introduction to the Theory of Computation - Michael Sipser.

The transition tables that describe $\delta_A : Q_A \times \Sigma \to Q_A$ and $\delta_B : Q_B \times \Sigma \to Q_B$ are:

$Q_A \backslash \Sigma$	a	b
r_0	r_1	r_0
r_1	r_2	r_1
r_2	r_3	r_2
r_3	r_3	r_3

$Q_B \backslash \Sigma$	a	b
s_0	s_0	s_1
s_1	s_1	s_2
s_2	s_2	s_2

In the same way, the 5-tuple that describes M_C is $(Q_C, \Sigma, \delta_C, t_0, F_C)$ has a set of states such that

$$\begin{aligned} Q_C &= \{t_{i,j} = (r_i, s_j) \mid r_i \in Q_A \wedge s_j \in Q_B\}\,, \\ &= Q_A \times Q_B\ . \end{aligned}$$

The symbol $t_{i,j}$ was used to denote $t_{i,j} = (r_i, s_j)$ for convenince of use throughout this assignment.

Also, the alphabet is the same as the one originally defined, and the initial state is now a 2-tuple $t_{0,0} = (r_0, s_0)$. On the other hand, the accept states, for the intersection, are given by

$$\begin{aligned} F_C &= \{t_{i,j} = (r_i, s_j) \mid r_i \in F_A \wedge s_j \in F_B\} \\ &= F_A \times F_B\ , \\ &= \{r_3\} \times \{s_2\}\ , \\ &= \{(r_3, s_2)\}\ . \end{aligned}$$

In contrast with the accept states coming from the union of two regular languages, the accept states in the intersection are only those whose both elements in the 2-tuple also belong to accept states.

Now, taking our attention to the transition function, it is defined as

$$\delta_C\left((r_i, s_j),\ a\right) = \left(\delta_A(r_i, a),\ \delta_B(s_j, a)\right)\ ,$$

where $(r_i, s_i) \in Q_C$. Thus, it is possible to contruct the transition table based on the individual transitions from δ_A and δ_B, as shown below:

$Q_C \backslash \Sigma$	a	b
(r_0, s_0)	(r_1, s_0)	(r_0, s_1)
(r_0, s_1)	(r_1, s_1)	(r_0, s_2)
(r_0, s_2)	(r_1, s_2)	(r_0, s_2)
(r_1, s_0)	(r_2, s_0)	(r_1, s_1)
(r_1, s_1)	(r_2, s_1)	(r_1, s_2)
(r_1, s_2)	(r_2, s_2)	(r_1, s_2)
(r_2, s_0)	(r_3, s_0)	(r_2, s_1)
(r_2, s_1)	(r_3, s_1)	(r_2, s_2)
(r_2, s_2)	(r_3, s_2)	(r_2, s_2)
(r_3, s_0)	(r_3, s_0)	(r_3, s_1)
(r_3, s_1)	(r_3, s_1)	(r_3, s_2)
(r_3, s_2)	(r_3, s_2)	(r_3, s_2)

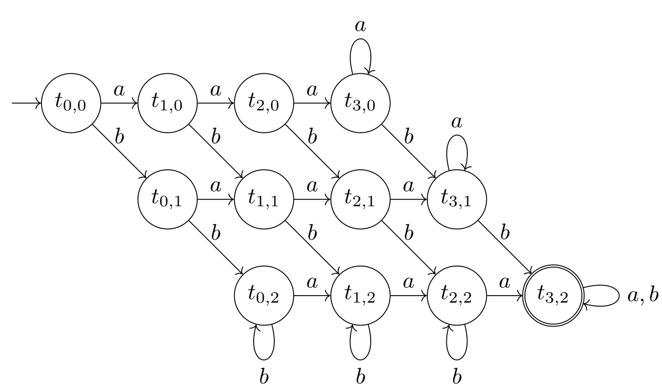

Figure 2: Automaton M_C that recognizes the regular language which is an intersection of the regular languages A and B.

$\therefore$ The resulting DFA M_C is shown in fig. 2 and it recognizes the regular language C which was constructed as an intersection of the regular languages A and B from M_A and M_B, respectively.