

Análisis de Algoritmos

Actividad 7

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1. Cálculo de π usando el *método de Monte Carlo*.
 - Generar los puntos aleatorios usando *convergencia lineal*.
 - Generar los puntos aleatorios usando *secuencia de Halton*.
 - Generar los puntos aleatorios usando el generador valores aleatorios de su lenguaje de programación de preferencia.
2. Para cada caso graficar las curvas de convergencia, hacer al menos 10^6 iteraciones.
3. Analizar y determinar qué método fue más preciso.
4. Pensar cómo aproximar π con un número de cifras significativas dadas, por ejemplo, con un número de 4 cifras significativas el valor correspondiente sería 3.14159.

Solución:

La implementación del algoritmo del *generador lineal congruencial* usada en esta actividad está basada en el trabajo hecho por Schlegel [1] la cual a su vez está basada en una implementación dentro del estándar del lenguaje de programación `C` [2], siendo esta última referencia de la que se obtienen los parámetros adecuados para este generador.

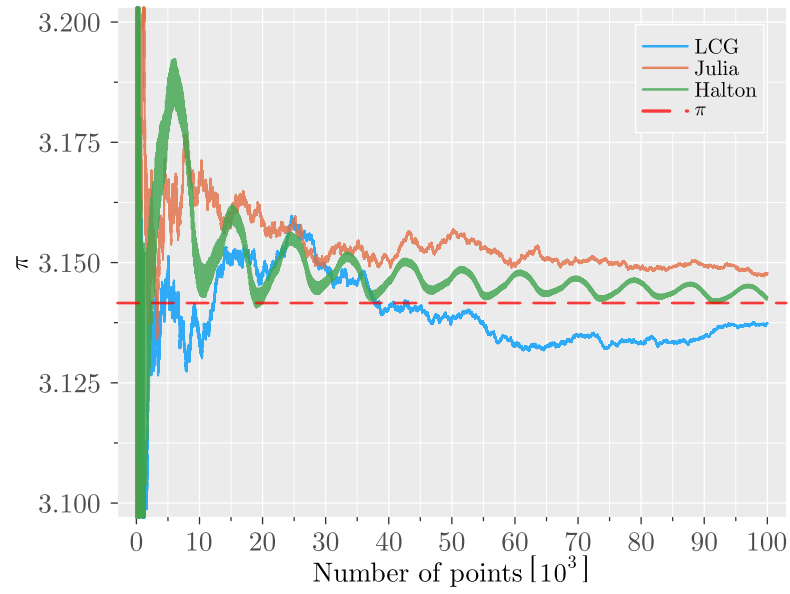


Figura 1: xxx

Apéndice

```

1  module ApproxPie
2
3  using Random
4  using FLoops
5
6  # Distance from the origin to a point
7  dist_from_origin(point) = point[1]^2 + point[2]^2
8  dist_from_origin(x, y) = x^2 + y^2
9
10 # Linear congruential generator
11 # https://aaronSchlegel.me/linear-congruential-generator-r.html
12 """
13     rngLcg(nvals)
14
15 Returns a list of pseudo-random numbers using a linear congruential generator. This
16 implementation uses the default parameters as those from the ANSI C implementation from
17 the year 2000 by Saucier.
18 """
19 function rngLcg(nvals)
20     # Parameters
21     m = 2^32
22     a = 1103515245
23     c = 12345
24     # The power of the distributed computing
25     seed = time() * 1000 > BigFloat # current system's time in micro seconds times 1000
26
27     numbers = zeros(nvals)
28     for i in eachindex(numbers)
29         seed = (a * seed + c) % m
30         numbers[i] = seed / m
31     end
32
33     return numbers
34 end
35
36 function rngLcg_floop(nvals)
37     # Parameters
38     m = 2^32
39     a = 1103515245
40     c = 12345
41     # The power of the distributed computing
42     seed = time() * 1000 # current system's time in micro seconds times 1000
43
44     numbers = zeros(nvals)
45     @floop for i in 1:nvals
46         # s = seed
47         # s = (a * s + c) % m
48         seed = (a * seed + c) % m
49         numbers[i] = seed / m
50     end
51
52     return numbers

```

```
53 end
54
55
56 # Halton sequence generator
57 """
58     rngHalton(nvals; base=2)
59
60 Returns a list of pseudo-random numbers generated using the Halton sequence. The
61 default 'base' is 2.
62 """
63 function rngHalton(nvals)
64     base = rand(2:150)
65     numbers = zeros(nvals)
66
67     for i in eachindex(numbers)
68         f = 1
69         r = 0
70
71         indx = i
72         while indx > 0
73             f = f / base
74             r = r + f * (indx % base)
75             indx = floor(indx / base)
76         end
77
78         numbers[i] = r
79     end
80
81     return numbers
82 end
83
84 # Julia's default random number generator
85 rngJulia(nvals) = rand(Float64, nvals)
86 # function rngJulia(nvals)
87 #     seed = rand(2:1500)
88
89 #     return rand(MersenneTwister(seed), Float64, nvals)
90 # end
91
92 function classifyPoints(points)
93     inside_circle = filter(x → dist_from_origin(x) <= 1, points)
94     outside_circle = filter(x → dist_from_origin(x) > 1, points)
95
96     return (in = inside_circle, out = outside_circle)
97 end
98
99 function classifyPoints(xvals, yvals)
100     inside_circle = []
101     outside_circle = []
102
103     for (x, y) in zip(xvals, yvals)
104         if dist_from_origin(x, y) > 1
105             push!(outside_circle, (x, y))
106         else
```

```

107         push!(inside_circle, (x, y))
108     end
109 end
110
111     return (in = inside_circle, out = outside_circle)
112 end
113
114 function pieApprox(classification)
115     ps = classification
116     count_circle = length(ps.in)
117     count_square = count_circle + length(ps.out)
118
119     return 4 * count_circle / count_square
120 end
121
122 # function piesApprox(npoints; rng="julia")
123 #     methods = Dict(
124 #         "julia" => rngJulia,
125 #         "halton" => rngHalton,
126 #         "lcg" => rngLcg)
127 #
128 #     method = methods[rng]
129 #     xvalues = method(npoints)
130 #     yvalues = method(npoints)
131 #
132 #     pies = zeros(npoints)
133 #     for i in eachindex(pies)
134 #         classify = classifyPoints(xvalues[begin:i], yvalues[begin:i])
135 #         pie = pieApprox(classify)
136 #         pies[i] = pie
137 #     end
138 #
139 #     return (pies=pies, xvalues=xvalues, yvalues=yvalues)
140 # end
141
142 function piesApprox(npoints; rng="julia")
143     methods = Dict(
144         "julia" => rngJulia,
145         "halton" => rngHalton,
146         "lcg" => rngLcg)
147     method = methods[rng]
148
149     xvalues = method(npoints)
150     yvalues = method(npoints)
151     pies = zeros(npoints)
152
153     for npoints_ in eachindex(pies)
154         xwindow = xvalues[begin:npoints_]
155         ywindow = yvalues[begin:npoints_]
156         points_inside_circle = count(x -> x[1]^2 + x[2]^2 <= 1, zip(xwindow, ywindow))
157         pies[npoints_] = 4 * points_inside_circle / npoints_
158     end
159
160     return (pies=pies, xvalues=xvalues, yvalues=yvalues)

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161 end
162
163 function plotFig(classification)
164     ps = classification
165     dx = 0.01    # extra space to not cut circles at the limits
166
167     fig = plot(size=(600,600),
168         xlims=(-dx,1+dx), ylims=(-dx,1+dx),
169         xticks=nothing, yticks=nothing)
170
171     scatter!(ps.out, label="", ms=4, ma=0.5, msw=0.5)
172     scatter!(ps.in, label="", ms=4, ma=0.5, msw=0.5)
173
174     xaxis!(fig, bordercolor="white")
175     yaxis!(fig, bordercolor="white")
176
177     return fig
178 end
179
180 function estimate_pi_floop(nMC)
181     radius = 1.
182     diameter = 2. * radius
183
184     @floop for i in 1:nMC
185         x = (rand() - 0.5) * diameter
186         y = (rand() - 0.5) * diameter
187         r = sqrt(x^2 + y^2)
188         if r <= radius
189             @reduce(n_circle += 1)
190         end
191     end
192
193     return (n_circle / nMC) * 4.
194 end
195
196
197 function rngLcg_floop(nvals)
198     # Parameters
199     m = 2^32
200     a = 1103515245
201     c = 12345
202     # The power of the distributed computing
203     seed = time() * 1000 > BigFloat    # current system's time in micro seconds times 1000
204
205     numbers = zeros(nvals)
206     @floop for i in eachindex(numbers)
207         seed = (a * seed + c) % m
208         numbers[i] = seed / m
209     end
210
211     return numbers
212 end
213
214 # julia> n = 5000; approximations = ap.piesApprox(n);

```

```
215 # julia> npoints = map(x → x[1], approximations); pies = map(x → x[2], approximations);  
216 # julia> scatter(npoints, pies, label="", ms=2, ma=0.8); hline!([π], label="", lw=2.5)  
217  
218 end # module ApproxPie
```

Referencias

- [1] Aaron Schlegel. Linear congruential generator for pseudo-random number generation with r. <https://aaronshlegel.me/linear-congruential-generator-r.html>, 2008. Visitado: 2022-11-20.
- [2] Richard Saucier. Computer generation of statistical distributions. Technical report, Army Research Lab Aberdeen Proving Ground MD, 2000.
- [3] Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B. Shah. Julia: A fresh approach to numerical computing. *SIAM Review*, 59(1):65–98, 9 2017.