Universidad Veracruzna

Artificial Intelligence

Automata and Formal Languages

Assignment 3

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Convert the NFA shown in fig. 1, corresponding to the machine N, to a DFA:

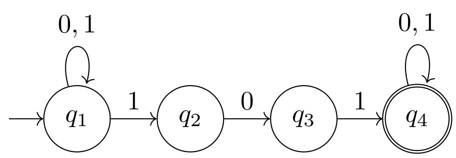

Figure 1: The NFA N.

Solution:

The automaton N is described by the 5-tuple $(Q, \Sigma, \delta, q_1, F)$. Its set of states $Q = \{q_1, q_2, q_3, q_4\}$, its alphabet $\Sigma = \{0, 1\}$, the start state is q_1, and the set of accept states is $F = \{q_4\}$. Additionally, N's transition table is shown below:

$Q\backslash\Sigma$	0	1
q_1	$\{q_1\}$	$\{q_1, q_2\}$
q_2	$\{q_3\}$	$\emptyset$
q_3	$\emptyset$	$\{q_4\}$
q_4	$\{q_4\}$	$\{q_4\}$

Now, to find a deterministic equivalent M of N it is necessary to compute the 5-tuple given by $(Q', \Sigma, \delta', q_1', F')$. The set of states of the deterministic automaton Q' correspond to all the possible states product of the power set of Q, given by $\mathbb{P}(Q)$,

$$\begin{aligned}
Q' = \{&\emptyset, \{q_1\}, \{q_2\}, \{q_3\}, \{q_4\}, \\
&\{q_1, q_2\}, \{q_1, q_3\}, \{q_1, q_4\}, \{q_2, q_3\}, \{q_2, q_4\}, \{q_3, q_4\}, \\
&\{q_1, q_2, q_3\}, \{q_1, q_2, q_4\}, \{q_1, q_3, q_4\}, \{q_2, q_3, q_4\}, \\
&\{q_1, q_2, q_3, q_4\}\} \ , \\
= \{&\emptyset, R_1, R_2, R_3, R_4, \\
&R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, R_{34}, \\
&R_{123}, R_{124}, R_{134}, R_{234}, \\
&R_{1234}\} \ .
\end{aligned}$$

The notation $R_{i,j,k,l}$ is used here for simplicity.

It can be observed that Q' is a set of all possible subsets of Q. But, as it will be shown later, not all subsets are *reachables* The transition function for the DFA equivalence is $\delta'(R,a) = \{q \in Q | q \in \delta(r,a) \text{ for some } r \in R\}$ which can also be interpreted as

$$\delta'(R,a) = \bigcup_{r \in R} \delta(r,a) \ .$$

The transitions $\delta'(R,a)$ of M are shown below:

$$\delta'(R_1,0) = R_1$$
$$\delta'(R_1,1) = R_{12}$$
$$\delta'(R_2,0) = R_3$$
$$\delta'(R_2,1) = \emptyset$$
$$\delta'(R_3,0) = \emptyset$$
$$\delta'(R_3,1) = R_4$$
$$\delta'(R_4,0) = R_4$$
$$\delta'(R_4,1) = R_4$$
$$\delta'(R_{12},0) = R_{13}$$
$$\delta'(R_{12},1) = R_{12} \cup R_{12} = R_{12}$$
$$\delta'(R_{13},0) = R_1 \cup \emptyset = R_1$$
$$\delta'(R_{13},1) = R_{12} \cup R_4 = R_{124}$$
$$\delta'(R_{14},0) = R_1 \cup R_4 = R_{14}$$
$$\delta'(R_{14},1) = R_{12} \cup R_4 = R_{124}$$
$$\delta'(R_{23},0) = R_3 \cup \emptyset = R_3$$
$$\delta'(R_{23},1) = \emptyset \cup R_4 = R_4$$
$$\delta'(R_{24},0) = R_3 \cup R_4 = R_{34}$$
$$\delta'(R_{24},1) = \emptyset \cup R_4 = R_4$$
$$\delta'(R_{34},0) = \emptyset \cup R_4 = R_4$$
$$\delta'(R_{34},1) = R_4 \cup R_4 = R_4$$
$$\delta'(R_{123},0) = R_1 \cup R_3 \cup \emptyset = R_{13}$$
$$\delta'(R_{123},1) = R_{12} \cup \emptyset \cup R_4 = R_{124}$$
$$\delta'(R_{124},0) = R_1 \cup R_3 \cup R_4 = R_{134}$$
$$\delta'(R_{124},1) = R_{12} \cup \emptyset \cup R_4 = R_{124}$$
$$\delta'(R_{134},0) = R_1 \cup \emptyset \cup R_4 = R_{14}$$
$$\delta'(R_{134},1) = R_{12} \cup R_4 \cup R_4 = R_{124}$$
$$\delta'(R_{234},0) = R_3 \cup \emptyset \cup R_4 = R_{34}$$
$$\delta'(R_{234},1) = \emptyset \cup R_4 \cup R_4 = R_4$$
$$\delta'(R_{1234},0) = R_1 \cup R_3 \cup \emptyset \cup R_4 = R_{134}$$
$$\delta'(R_{1234},1) = R_{12} \cup \emptyset \cup R_4 \cup R_4 = R_{124}$$
$$\delta'(\emptyset,0) = \emptyset$$
$$\delta'(\emptyset,1) = \emptyset$$

The above transitions were included only to shown how the union operation from the definition of δ' is supposed to operate but it was done directly with the R subsets instead of their elements $r \in R$. Additionally, the transitions above can be written in a transition table for better readability:

$Q' \backslash \Sigma$	0	1
$\emptyset$	$\emptyset$	$\emptyset$
R_1	R_1	R_{12}
R_2	R_3	$\emptyset$
R_3	$\emptyset$	R_4
R_4	R_4	R_4
R_{12}	R_{13}	R_{12}
R_{13}	R_1	R_{124}
R_{14}	R_{14}	R_{124}
R_{23}	R_3	R_4
R_{24}	R_{34}	R_4
R_{34}	R_4	R_4
R_{123}	R_{13}	R_{124}
R_{124}	R_{134}	R_{124}
R_{134}	R_{14}	R_{124}
R_{234}	R_{34}	R_4
R_{1234}	R_{134}	R_{124}

It is important to notice that in the definition of the equivalence of a NFA to a DFA, the start state needs to be the same for both cases. Thus, the execution path for M shall commence in R_1 and it will follow from there. This means that any other state in Q' that is not in anyway attached to the main path commencing in R_1 will not be reachable includding those conected to the main path but with no transition towards them.

To end the computation of M's 5-tuple we shall find the accept states F'. By definition, the accept states are $F' = \{R \in Q' | R \text{ contains an accpet state of } N\}$. Thus,

$$F' = \{R_4, R_{14}, R_{24}, R_{34}, R_{124}, R_{134}, R_{234}, R_{1234}\} \ .$$

The resulting DFA M from the transition table is shown in fig. 2. Clearly, the states R_{1234} and R_{123} are both connected to the main path but have no connection towards them, these two states are unreachable. Furthermore, there is a set of unreachable states which do not have any connection to the main path nor a start state.

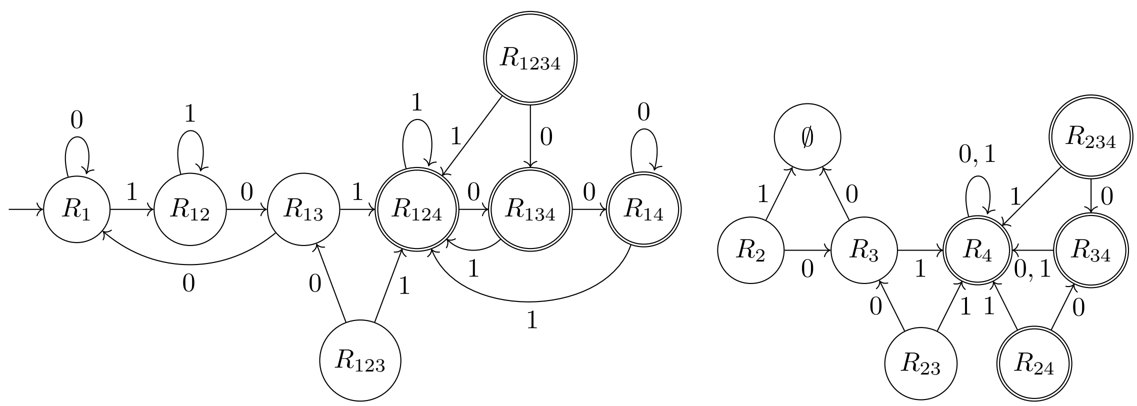

Figure 2: State diagram of the DFA M. Main path shown in the left, and unreachable set of states shown in the right.

Therefore, after purging the states that cannot be reached, the automaton M is left as the form shown in fig. 3, and it is the DFA equivalent to the NFA N.

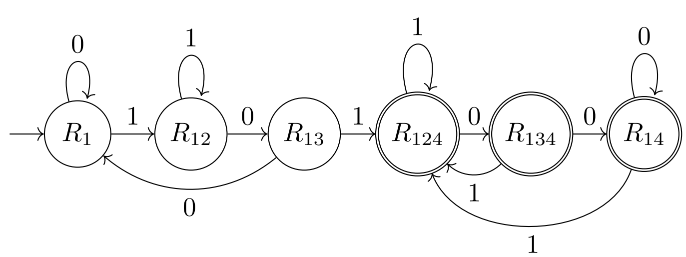

Figure 3: Final state diagram of M.