Status hierarchy and group cooperation: A generalized model

Hsuan-Wei "Wayne" Lee

June 30, 2021

中央研究院
社会学研究所
Institute of Sociology, Academia Sinica

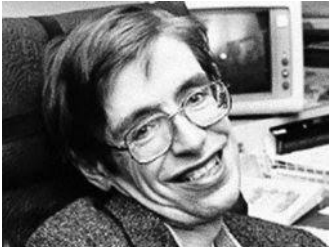
Outline

- Research overview: from complex systems to social networks
- Status organizes cooperation
- Status hierarchy and group cooperation: A generalized model
- Conclusion and future work
- Code sharing

Part 1: From complex systems to social networks

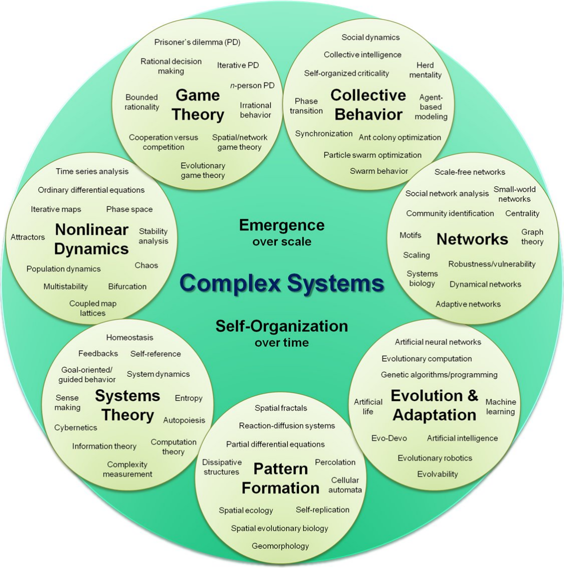
Complexity

- Complexity characterizes the behavior of a **system** or model whose components interact in multiple ways and follow **local rules**, meaning there is no reasonable higher instruction to define the various possible interactions.
- Stephen Hawking "I think the next century will be the century of complexity."



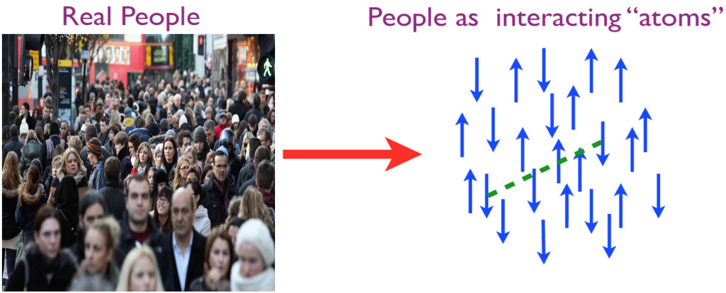
[1] https://en.wikipedia.org/wiki/Stephen_Hawking

Related research fileds



[1] https://en.wikipedia.org/wiki/Complex_system

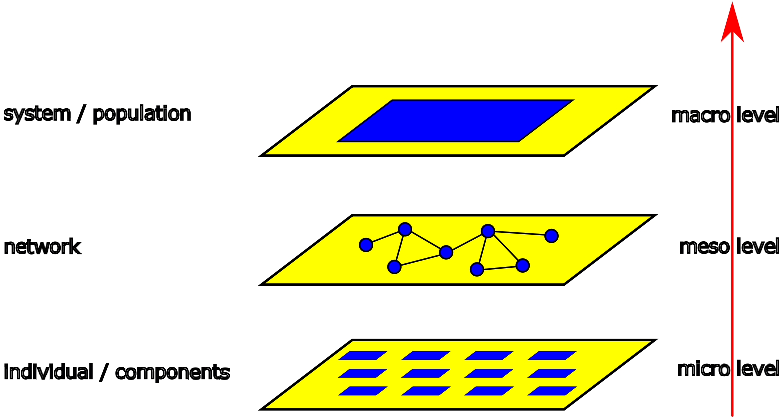
Social dynamics modeling



- **States of nodes**:
  binary, multistate, discrete/continuous, multidimensional
- **Structure of system**:
  systematic, random, heterogeneous, time dependent

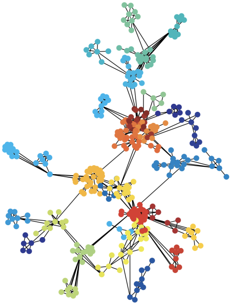
A model should be as simple as possible, but no simpler.
– A. Einstein

What are networks?



- a representation / an approach
- structure for complexity
- structure above individuals / components
- structure below system / population

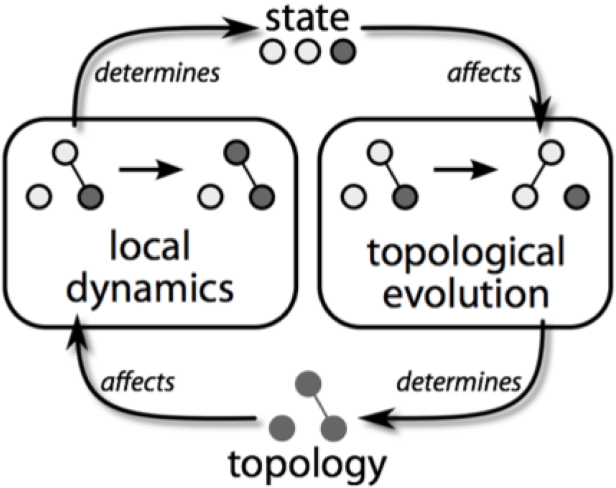
Are social networks different from other networks?



- homophily (birds of a feather flock together)
- high clustering
- small world

[1]Figure from (Porter, Onnela, and Mucha, 2009)

Coevolving networks

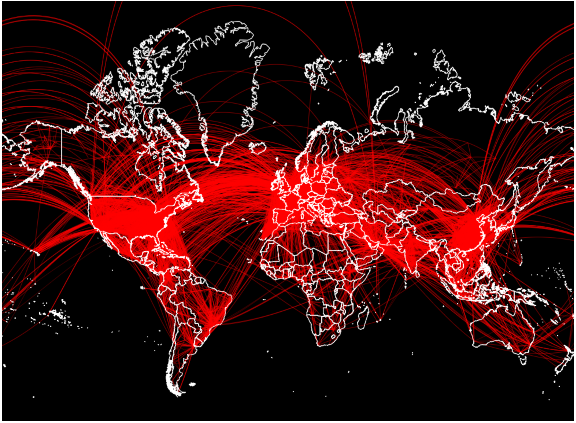


[1]Figure from (Gross and Blasius, 2008)

How do beliefs and behavior spread?

[1]Nishant Malik, Feng Shi, **Hsuan-Wei Lee** and Peter J. Mucha, "Transitivity Reinforcement in the Coevolving Voter Model", Chaos: An Interdisciplinary Journal of Nonlinear Science (2016).

How does a pandemic happen?



[1] **Hsuan-Wei Lee**, Nishant Malik, Feng Shi and Peter J. Mucha, "Social Clustering in Epidemic Spread on Co-evolving Networks", Physical Review E (2019).

Why we are this cooperative?

[1]**Hsuan-Wei Lee**, Nishant Malik and Peter J. Mucha, " Evolutionary Prisoner's Dilemma Games Coevolving on Adaptive Networks", Journal of Complex Networks (2018).

[2]**Hsuan-Wei Lee**, Colin Cleveland, and Attila Szolnoki, " Small Fraction of Selective Cooperators Can Elevate General Wellbeing Significantly", Physica A: Statistical Mechanics and its Applications, forthcoming.

Respondent-driven capture-recapture with anonymity

[1]https://www.accidentallygay.com
[2]https://www.directivecommunication.net
[3]https://www.youtube.com/watch?v=gt_IoIWE8Q0

---

[1]Bilal Khan, **Hsuan-Wei Lee**, Ian Fellows and Kirk Dombrowski, "One-step Estimation of Networked Population Size: Respondent-Driven Capture-Recapture with Anonymity", PLoS ONE (2018).

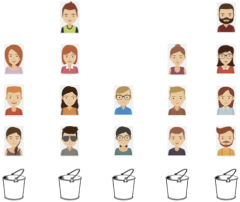
[2]Bilal Khan, **Hsuan-Wei Lee**, Courtney R. Thrash, and Kirk Dombrowski, "Agency and Social Constraint among Victims of Domestic Minor Sex Trafficking: A Method for Measuring Free Will", Social Science Research (2018).

Research in Alaska

[1] https://carolnorrisphotography.com/product/alaskan-village
[2] https://newsela.com/read/native-american-heritage-photos

Social Network Analysis through Perceptual Tomography (SNAPT)

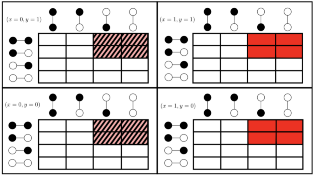
- We introduce a new method for acquiring and interpreting data on **cognitive (or perceptual)** networks.
- We refer to the method as perceptual tomography, it aggregates **multiple 3rd-party data** on the perceived presence or absence of individual properties and pairwise relationships.



[1]**Hsuan-Wei Lee**, Miranda Melson, Jerreed Ivanich, Patrick Habecke, G. Robin Gauthier, Lisa Wexler, Bilal Khan, and Kirk Dombrowski "Mapping the Structure of Perceptions in Helping Networks of Alaska Natives", PLoS ONE (2018).

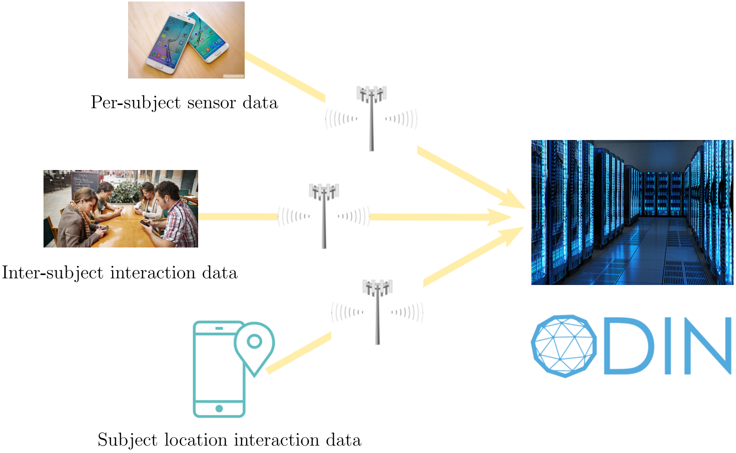
Network interventions and diffusion

- We introduce a formal method for the evaluation of behavioral interventions within **community-based participatory research**. Pre-/post- responses for a range of suicide prevention activities were collected from **intervention participants** and **non-participants** living in the communities.
- Comparisons across these four dyad classes allowed the researchers to assess the **social effects** on intervention outcomes related to behavior change and prevention.



[1] **Hsuan-Wei Lee**, G. Robin Gauthier, Jerreed D. Ivanich, Lisa Wexler, Bilal Khan, and Kirk Dombrowski, "A method for assessing the success and failure of community-level interventions in the presence of network diffusion, social reinforcement, and related social effects" (under review).

Diffusion and selection processes in dynamic networks



[1] In preparation, with Zeki Bilgin, Kirk Dombrowski and Bilal Khan.

Part 2: Status organizes cooperation

Status organizes cooperation

Status Organizes Cooperation: An Evolutionary Theory of Status and Social Order[1]

Noah P. Mark
University of North Carolina at Charlotte

Status behaviors are verbal and nonverbal behaviors that elevate or lower the status of the individuals who perform them and that can combine to create status hierarchies in task groups. This article identifies how status behavior could promote contributions to public goods in social dilemma situations through its ability to organize a group—that is, through its ability to generate a status hierarchy and mobilize contributions from all members of a group—and offers an explanation for the evolution of cooperative status behavior. The evolutionary viability of cooperative status behavior derives from its orientation toward hierarchy: status behavior tends to produce hierarchy in groups, and status behavior discriminates between the presence and absence of hierarchy. Cooperative status actors achieve mutual cooperation when they encounter each other but are protected from exploitation when a collectively recognized hierarchy is not present. This article shows that a strategy of cooperative status behavior can proliferate under conditions that otherwise make cooperation impossible.

INTRODUCTION

Identifying and understanding sources of social order is an essential task for sociology (Durkheim [1893] 1984; Axelrod 1984; Hechter 1987; Kollock 1998; Bowles and Gintis 2011). A growing literature argues that status be-

[1] I thank Cecilia Ridgeway, Jack Thomas, Lisa Walker, Murray Webster, and Joseph Whitmeyer for helpful comments and discussion. Direct correspondence to Noah P. Mark, University of North Carolina at Charlotte, Department of Sociology, Fretwell 476, 9201 University City Boulevard, Charlotte, North Carolina 28223. E-mail: nmark@uncc.edu

0002-9602/2018/12306-0002$10.00

AJS Volume 123 Number 6 (May 2018): 1601–34 1601

[1]Mark, N. P. (2018). Status organizes cooperation: an evolutionary theory of status and social order. American Journal of Sociology, 123(6), 1601-1634.

Status behavior and status behavior's orientation toward hierarchy

- Social psychological research on status shows that when a group of individuals are working together to achieve a common task goal.
- The members of the group engage in a set of behaviors–called **status behavior**–that tend to elevate or lower the status of the person who performs the behavior.
- A critical feature of status behavior is its **orientation toward hierarchy**: when people are in a collectively oriented, task-oriented group, they seek to establish a status hierarchy, and they attend to whether a status hierarchy has emerged.

Earlier work on the functions of status

Status functions by **compensating** cooperation:

- Davis-Moore theory of stratification
- Exchange theory
- Status as a selective incentive

Status functions by **organizing** cooperation:

- Status organizes social interaction
- How organization by a status hierarchy benefits groups

The evolution of status behavior

In one iteration of the model, four events occur:

1. individuals are assigned to groups
2. preplay communication occurs
3. each group plays a one-shot, *n*-person prisoner's dilemma
4. fitness levels of status cooperators and defectors of the present generation

Preplay signaling and the potential formation of status hierarchies

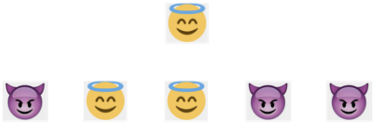
- In a group composed of status cooperators and defectors, preplay signaling may produce an apparent status hierarchy: a signaling pattern identical to that shown by status cooperators organized by a status hierarchy but in which one or more of the actors in low-status positions are defectors who will not accept the influence of the high-status actor.
- In a group composed of defectors, preplay signaling produces no status hierarchy.
- In a group composed of status cooperators and defectors, preplay signaling may also produce no apparent status hierarchy.

Preplay signaling and the potential formation of status hierarchies

Two possible outcomes:

no leader

one leader



Population dynamics

- To model change in the composition of the population across successive generations, Mark adopt the linear replicator dynamic suggested by Maynard Smith (1982).

  $$pSC_{t+1} = pSC_t \times W(SC)/(pSC_t \times W(SC) + pD_t \times W(D)),$$

  $$pD_{t+1} = pD_t \times W(D)/(pSC_t \times W(SC) + pD_t \times W(D)),$$

  where
  pSC_t: the proportion of status cooperators at time t
  pD_t: the proportion of defectors at time t
  $W(SC)$: the expected payoff to a status cooperator
  $W(D)$: the expected payoff to a defector.

Evolutionarily stable strategies

We have some basic definitions as follows.

- We say the **fitness (payoff)** of an organism in a population is the expected payoff it receives from an interaction with a random member of the population.
- We say that a strategy T **invades** a strategy S at level x, for some small positive number x, if an x fraction of the underlying population uses T and a $1 - x$ fraction of the underlying population uses S.
- Finally, we say that a strategy S is **evolutionarily stable** if there is a (small) positive number y such that when any other strategy T invades S at any level $x < y$, the fitness of an organism playing S is strictly greater than the fitness of an organism playing T.

General symmetric game

		Organism 2 S	Organism 2 T
Organism 1	S	a, a	b, c
	T	c, b	d, d

- The expected payoff to an organism playing S in a random interaction in this population:

$$a(1-x) + bx$$

- The expected payoff to an organism playing T in a random interaction in this population:

$$c(1-x) + dx$$

- Therefore, S is evolutionarily stable if for all sufficiently small values of $x > 0$, the inequality

$$a(1-x) + bx > c(1-x) + dx$$

holds.

The range of status cooperation is a stable strategy in social dilemmas

TABLE 1
RANGE OF c/b ACROSS WHICH STATUS COOPERATION IS A STABLE STRATEGY IN SOCIAL DILEMMAS

N	Lower Bound[a] $= 1/n$	Upper Bound[b] $= (n^{(n-1)})/(n^n - (n-1)^n)$
2.	1/2	2/3
3.	1/3	9/19
4.	1/4	64/175
5.	1/5	625/2,101
10.	1/10	.1535

[a] If $c/b \leq 1/n$, the game in which actors meet is not a social dilemma.
[b] If $c/b \geq (n^{(n-1)})/(n^n - (n-1)^n)$, status cooperation is not stable against invasion by defection.

Assortative mixing

- A bias parameter τ to manipulate the strength of the assortative bias in the formation of groups
- The parameter τ takes values on the interval $[0, 1]$: $\tau = 0$ yields unbiased random mixing, and $\tau = 1$ yields perfect assortativity; that is, when $\tau = 1$, every group will be homogeneous.
- Want to determine whether assortative mixing lets status cooperation invade a population of defectors and, if so, to determine the level of assortativity required for status cooperation to invade.

Assortative mixing

$$W(SC) = ((\tau + (1-\tau)\times pSC)^2)\times((1-((2/3)^3))\times b + ((2/3)^3)\times c)$$
$$+ ((4/3)\times(\tau + (1-\tau)\times pSC)\times(1-\tau)\times pD + (2/3)$$
$$\times(1-\tau)\times pD\times(1-\tau)\times pSC)$$
$$\times((1-((2/3)^2))\times(2/3)\times b + ((2/3)^2)\times c) \tag{6a}$$
$$+ ((1/3)\times((1-\tau)\times pD)^2 + (2/3)\times(1-\tau)\times pD$$
$$\times(\tau + (1-\tau)\times pD))$$
$$\times(((1/3)^2)\times b + (2/3)\times c),$$

$$W(D) = ((1/3)\times((1-\tau)\times pSC)^2 + (2/3)\times(1-\tau)$$
$$\times pSC\times(\tau + (1-\tau)\times pSC))$$
$$\times(1-((2/3)^2))\times(2/3)\times b$$
$$+ ((4/3)\times(\tau + (1-\tau)\times pD)\times(1-\tau)\times pSC \tag{6b}$$
$$+ (2/3)\times(1-\tau)\times pSC\times(1-\tau)\times pD)$$

Part 3: Status hierarchy and group cooperation –A generalized model

Status hierarchy and group cooperation–A generalized model

Cornell University

We gratefully acknowledge support from the Simons Foundation and member institutions.

arXiv.org > econ > arXiv:2004.00944

Search... All fields Search

Help | Advanced Search

Economics > General Economics

[Submitted on 2 Apr 2020 (v1), last revised 17 Aug 2020 (this version, v3)]

Status hierarchy and group cooperation: A generalized model

Hsuan-Wei Lee, Yen-Ping Chang, Yen-Sheng Chiang

In a refreshing mathematical investigation, Mark (2018) shows that status hierarchy may facilitate the emergence of cooperation in groups. Despite the contribution, the present paper notes that there are limitations in Mark's model that makes it less realistic than it could in explaining real-world experiences. Consequently, we present a more generalized modified framework in which his model is a special case, by developing and introducing a new hierarchy measure into the model to estimate the cooperation level in a set of hierarchical structures omitted in Mark's work yet common in everyday life-- those with multiple leaders. We derived the conditions under which cooperation can emerge in these groups, and verified our analytical predictions in agent-based computer simulations. In so doing, not only does our model elaborate on its predecessor and support Mark's general prediction. For theory, our work further reveals two novel phenomena of group cooperation: Both the relative number of cooperators to defectors in groups and the assortativity among these different roles can backfire; they are not always the higher, the better for cooperation to thrive. For methodology, the hierarchy measure developed and our model using the measure may also be applied in future research on a wide range of related topics.

Comments: 42 pages, 11 figures
Subjects: **General Economics (econ.GN)**; Physics and Society (physics.soc-ph)
Cite as: arXiv:2004.00944 **[econ.GN]**
(or arXiv:2004.00944v3 **[econ.GN]** for this version)

Submission history

From: Hsuan-Wei Lee [view email]
[v1] Thu, 2 Apr 2020 11:31:02 UTC (1,256 KB)
[v2] Thu, 11 Jun 2020 15:44:59 UTC (2,595 KB)
[v3] Mon, 17 Aug 2020 10:00:29 UTC (3,683 KB)

Download:

- PDF
- Other formats

(license)

Current browse context:
econ.GN
< prev | next >
new | recent | 2004

Change to browse by:
econ
physics
physics.soc-ph
q-fin
q-fin.EC

References & Citations

- NASA ADS
- Google Scholar
- Semantic Scholar

Export Bibtex Citation

Bookmark

[1] Lee, H. W., Chang, Y. P., & Chiang, Y. S. (2020). Status hierarchy and group cooperation: A generalized model of Mark (2018). arXiv preprint arXiv:2004.00944.

Why we want to extend the model? –The existence of multi-leadership

[1]https://nbafamily.fandom.com/
[2]https://fadeawayworld.net/

Why we want to extend the model? –The cost of repeated structure formation

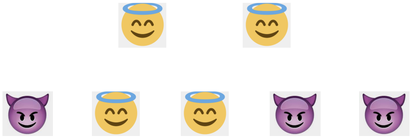
- We do not want to ignore the cost of repeated structure formation and leadership negotiation.
- This cost was implicitly assumed to be zero when multi-leader hierarchies are dropped without a cost in Mark's model yet is common and influential in the real world.
- If the signaling process generates multiple leaders in a round in our model, rather than dropping the round and starting all over as Mark's model would, we estimate the hierarchicalness of this–following his terminology–unapparent structure.

The evolution of status behavior (our version)

In one iteration of the model, four events occur:

1. individuals are assigned to groups
2. preplay communication occurs
3. each group plays a one-shot, *n*-person prisoner's dilemma
4. fitness levels of status cooperators and defectors of the present generation

We generalize this model by allowing multi-leader hierarchies to pass phase (2) and, therefore, forcing cooperators to make decisions on whether to contribute in such "unapparent" structures in phase (3).



Hierarchical score and the cooperation probability $H_n(x)$

OPEN ACCESS Freely available online PLoS one

Hierarchy Measure for Complex Networks

Enys Mones[1], Lilla Vicsek[2], Tamás Vicsek[1,3]*

1 Department of Biological Physics, Eötvös Loránd University, Budapest, Hungary, **2** Institute of Sociology and Social Policy, Corvinus University of Budapest, Budapest, Hungary, **3** Biological Physics Research Group of Hungarian Academy of Sciences, Budapest, Hungary

Abstract

Nature, technology and society are full of complexity arising from the intricate web of the interactions among the units of the related systems (e.g., proteins, computers, people). Consequently, one of the most successful recent approaches to capturing the fundamental features of the structure and dynamics of complex systems has been the investigation of the networks associated with the above units (nodes) together with their relations (edges). Most complex systems have an inherently hierarchical organization and, correspondingly, the networks behind them also exhibit hierarchical features. Indeed, several papers have been devoted to describing this essential aspect of networks, however, without resulting in a widely accepted, converging concept concerning the quantitative characterization of the level of their hierarchy. Here we develop an approach and propose a quantity (measure) which is simple enough to be widely applicable, reveals a number of universal features of the organization of real-world networks and, as we demonstrate, is capable of capturing the essential features of the structure and the degree of hierarchy in a complex network. The measure we introduce is based on a generalization of the m-reach centrality, which we first extend to directed/partially directed graphs. Then, we define the global reaching centrality (GRC), which is the difference between the maximum and the average value of the generalized reach centralities over the network. We investigate the behavior of the GRC considering both a synthetic model with an adjustable level of hierarchy and real networks. Results for real networks show that our hierarchy measure is related to the controllability of the given system. We also propose a visualization procedure for large complex networks that can be used to obtain an overall qualitative picture about the nature of their hierarchical structure.

Citation: Mones E, Vicsek L, Vicsek T (2012) Hierarchy Measure for Complex Networks. PLoS ONE 7(3): e33799. doi:10.1371/journal.pone.0033799

Editor: Stefano Boccaletti, Technical University of Madrid, Italy

Received January 2, 2012; **Accepted** February 17, 2012; **Published** March 28, 2012

Funding: This work was supported by the EU FP7 COLLMOT Grant No: 227878. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

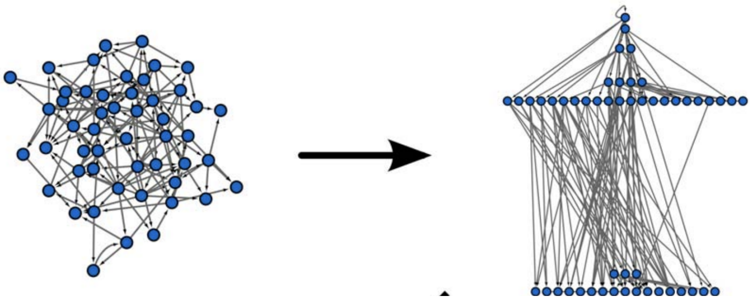
Competing Interests: The authors have declared that no competing interests exist.

* E-mail: vicsek@hal.elte.hu

---

[1] Mones, E., Vicsek, L., & Vicsek, T. (2012). Hierarchy measure for complex networks. PloS one, 7(3), e33799.

Hierarchical score and the cooperation probability $H_n(x)$



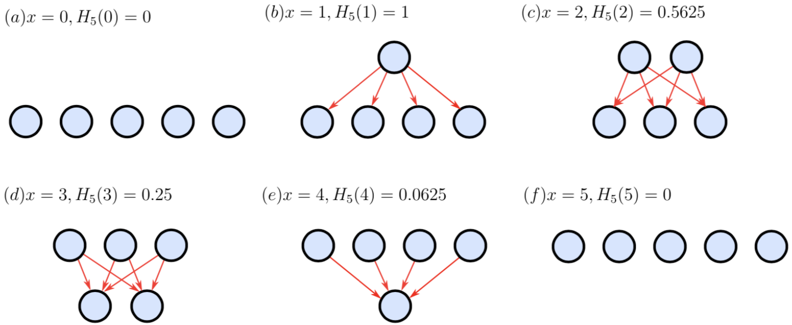
[1] Mones, E., Vicsek, L., & Vicsek, T. (2012). Hierarchy measure for complex networks. PloS one, 7(3), e33799.

Hierarchical score and the cooperation probability $H_n(x)$

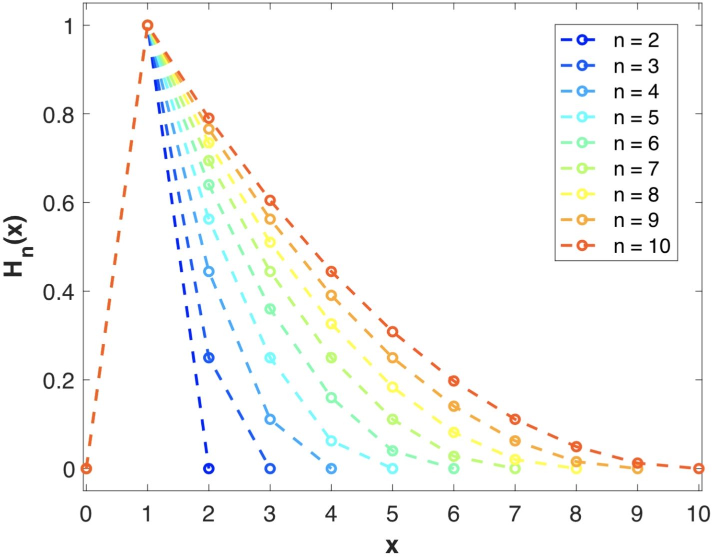
- We define the hierarchicalness $H_n(x)$ of this kind of two-level cooperation structures with group size n and x players at the top level using the general reaching centrality of network G.
- That is,

$$H_n(x) = \begin{cases} \frac{(n-x)\cdot\frac{n-x}{n-1}}{n-1} = \left(\frac{n-x}{n-1}\right)^2 & \text{if } 0 < x \leqslant n \\ 0 & \text{if } x = 0. \end{cases}$$

Illustration of $H_5(x), x = 0, 1, \cdots, 5$



Values of $H_n(x)$, for $n = 0, 1, \cdots, 10$



Some calculation (I)

The expected payoffs for the status cooperators and defectors for $n = 2$:

$$W(C) = f_c \cdot \left\{ \binom{2}{2} \left(\frac{1}{2}\right)^2 \left(\frac{2-1}{2}\right)^0 \cdot c + \binom{2}{0} \left(\frac{1}{2}\right)^0 \left(\frac{2-1}{2}\right)^2 \cdot c + \binom{2}{1} \left(\frac{1}{2}\right)^1 \left(\frac{2-1}{2}\right)^1 \cdot b \right\}$$
$$+ (1 - f_c) \cdot \left\{ \binom{1}{0} \left(\frac{1}{2}\right)^0 \left(\frac{2-1}{2}\right)^1 \cdot c + \binom{1}{1} \left(\frac{1}{2}\right)^1 \left(\frac{2-1}{2}\right)^0 \cdot \frac{b}{2} \right\}$$

$$W(D) = c + f_c \cdot \left\{ \binom{1}{1} \left(\frac{1}{2}\right)^1 \left(\frac{2-1}{2}\right)^0 \cdot \frac{b}{2} \right\}$$

The expected payoffs for the status cooperators and defectors for $n = 3$:

$$
\begin{aligned}
W(C) &= (1-f_c)^2 \cdot \left(\frac{2}{3}\right) \cdot c \\
&+ \binom{2}{1} f_c(1-f_c) \cdot \left\{ \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 (1 - H_3(2)) \right\} \cdot c \\
&+ (f_c)^2 \cdot \left\{ \left(\frac{2}{3}\right)^3 + \binom{3}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 (1 - H_3(2)) + \left(\frac{1}{3}\right)^3 \right\} \cdot c \\
&+ (1-f_c)^2 \cdot \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) \cdot b \\
&+ \binom{2}{1} f_c(1-f_c) \cdot \left\{ \binom{2}{1} \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) \right. \\
&+ \left. \binom{2}{2} \left(\frac{1}{3}\right)^2 \cdot \left[\binom{2}{1} (H_3(2))(1 - H_3(2)) \left(\frac{1}{3}\right) + \binom{2}{2} (H_3(2))^2 \left(\frac{2}{3}\right) \right] \right\} \cdot b \\
&+ (f_c)^2 \cdot \left\{ \binom{3}{1} \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \binom{3}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 \right. \\
&\left. \cdot \left[\binom{3}{1} (H_3(2))(1 - H_3(2))^2 \left(\frac{1}{3}\right) + \binom{3}{2} (H_3(2))^2 (1 - H_3(2))^1 \left(\frac{2}{3}\right) + (H_3(2))^3 \right] \right\} \cdot b
\end{aligned}
$$

$$
\begin{aligned}
W(D) &= c + 2 f_c(1-f_c) \cdot \left\{ \left(\frac{1}{3}\right) (H_3(1)) \left(\frac{1}{3}\right) \right\} \cdot b \\
&+ f_c^2 \left\{ \binom{2}{1} \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^2 \left[\binom{2}{1} (H_3(2))(1 - H_3(2)) \left(\frac{1}{3}\right) + (H_3(2))^2 \left(\frac{2}{3}\right) \right] \right\} \cdot b
\end{aligned}
$$

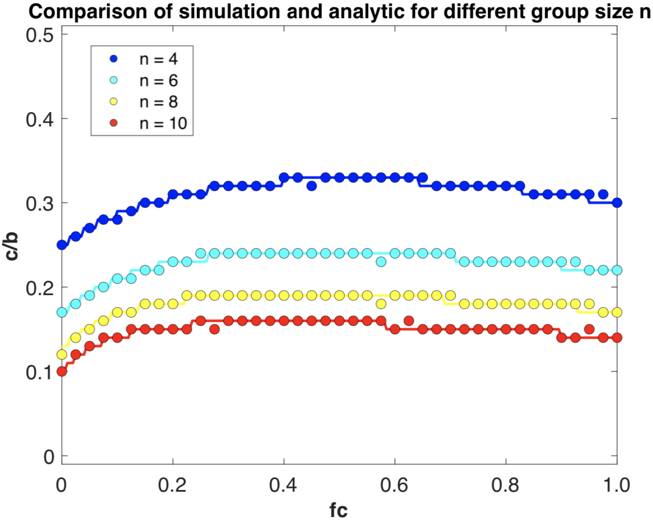
Some calculation (III)

The expected payoffs for the status cooperators and defectors for general *n*:

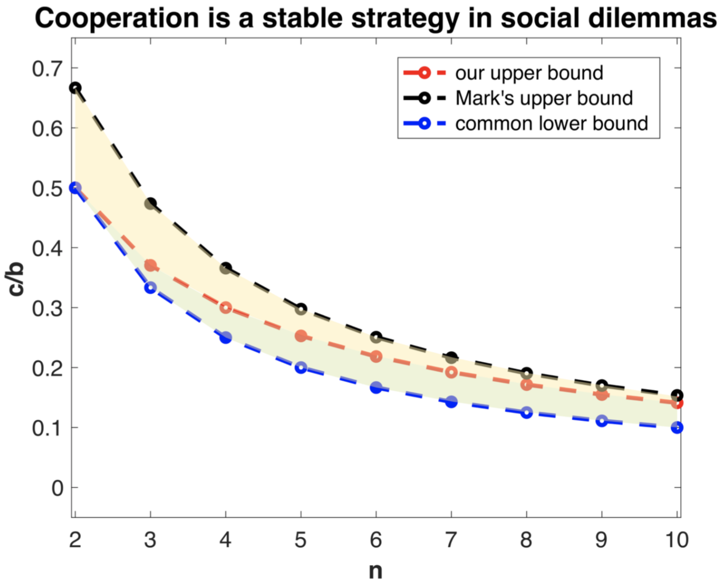
$$W(C) = \sum_{i=0}^{n-1} \binom{n-1}{i} (f_c)^i (1-f_c)^{n-1-i} \cdot \left[\sum_{j=0}^{i+1} \binom{i+1}{j} \left(\frac{1}{n}\right)^j \left(\frac{n-1}{n}\right)^{i+1-j} (1-H_n(j)) \right] \cdot c$$
$$+ \sum_{i=0}^{n-1} \binom{n-1}{i} (f_c)^i (1-f_c)^{n-1-i} \cdot \left\{ \sum_{j=0}^{i+1} \binom{i+1}{j} \left(\frac{1}{n}\right)^j \left(\frac{n-1}{n}\right)^{i+1-j} \right.$$
$$\left. \cdot \left[\sum_{k=0}^{i+1} \binom{i+1}{k} (H_n(j))^k (1-H_n(j))^{i+1-k} \left(\frac{k}{n}\right) \right] \right\} \cdot b$$

$$W(D) = c + \sum_{i=0}^{n-1} \binom{n-1}{i} (f_c)^i (1-f_c)^{n-1-i}$$
$$\cdot \left\{ \sum_{j=0}^{i} \binom{i}{j} \left(\frac{1}{n}\right)^j \left(\frac{n-1}{n}\right)^{i-j} \cdot \left[\sum_{k=0}^{i} \binom{i}{k} (H_n(j))^k (1-H_n(j))^{i-k} \left(\frac{k}{n}\right) \right] \right\} \cdot b$$

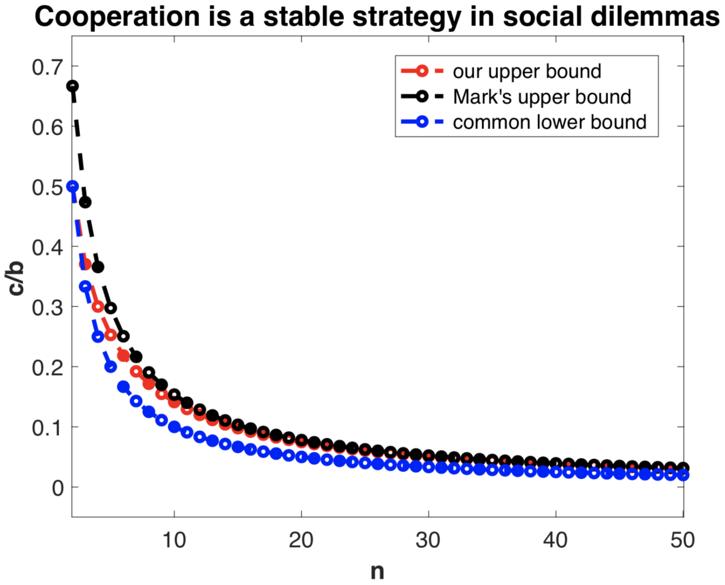
Comparison of simulations and analytics for different group size *n*



Cooperation in social dilemmas



Cooperation in social dilemmas for large n



The expected payoffs for a status cooperator

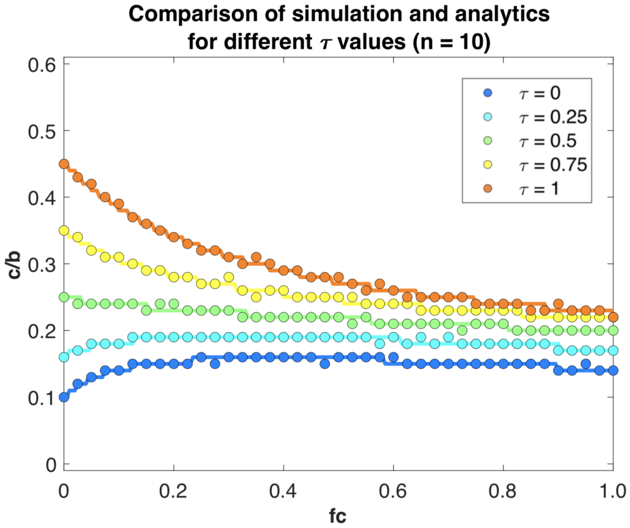
$$
\begin{aligned}
W(C) = \sum_{i=0}^{n-1} \Bigg\{ \left(\frac{i+1}{n}\right) \Bigg\{ & \left(\frac{1}{i+1}\right) \binom{n-1}{i} \big[\tau + (1-\tau) f_c\big]^{i} \big[(1-\tau)(1-f_c)\big]^{n-i-1} \\
& + \left(\frac{i}{i+1}\right) f_c \binom{n-1}{i} \big[\tau + (1-\tau) f_c\big]^{i-1} \big[(1-\tau)(1-f_c)\big]^{n-i-1} \Bigg\} \\
& + \left(\frac{n-i-1}{n}\right) \Bigg\{ (1-f_c) \binom{n-1}{i} \big[(1-\tau) f_c\big]^{i} \big[\tau + (1-\tau)(1-f_c)\big]^{n-i-2} \Bigg\} \Bigg\} \\
& \cdot \left[\sum_{j=0}^{i+1} \binom{i+1}{j} \left(\frac{1}{n}\right)^{j} \left(\frac{n-1}{n}\right)^{i+1-j} (1 - H_n(j)) \right] \cdot c \\
& + \sum_{i=0}^{n-1} \Bigg\{ \left(\frac{i+1}{n}\right) \Bigg\{ \left(\frac{1}{i+1}\right) \binom{n-1}{i} \big[\tau + (1-\tau) f_c\big]^{i} \big[(1-\tau)(1-f_c)\big]^{n-i-1} \\
& + \left(\frac{i}{i+1}\right) f_c \binom{n-1}{i} \big[\tau + (1-\tau) f_c\big]^{i-1} \big[(1-\tau)(1-f_c)\big]^{n-i-1} \Bigg\} \\
& + \left(\frac{n-i-1}{n}\right) \Bigg\{ (1-f_c) \binom{n-1}{i} \big[(1-\tau) f_c\big]^{i} \big[\tau + (1-\tau)(1-f_c)\big]^{n-i-2} \Bigg\} \Bigg\} \\
& \cdot \Bigg\{ \sum_{j=0}^{i+1} \binom{i+1}{j} \left(\frac{1}{n}\right)^{j} \left(\frac{n-1}{n}\right)^{i+1-j} \\
& \cdot \left[\sum_{k=0}^{i+1} \binom{i+1}{k} (H_n(j))^{k} (1 - H_n(j))^{i+1-k} \left(\frac{k}{n}\right) \right] \Bigg\} \cdot b
\end{aligned}
$$

A multi-leader model with assortative mixing (II)

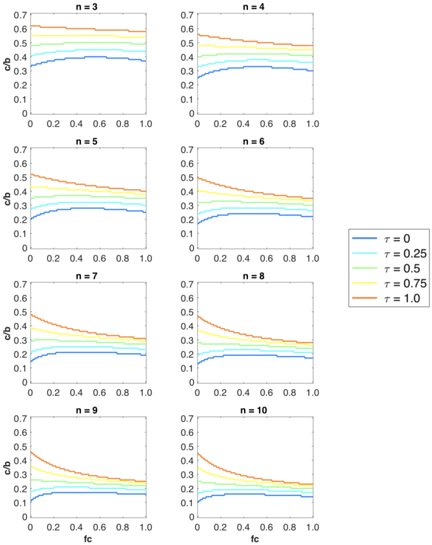
The expected payoffs for a defector

$$
\begin{aligned}
W(D) = c + \sum_{i=0}^{n-1} \Bigg\{ \left(\frac{i}{n}\right) \Bigg\{ f_c \binom{n-1}{i} \big[\tau + (1-\tau) f_c\big]^{i-1} \big[(1-\tau)(1-f_c)\big]^{n-i-1} \Bigg\} \\
+ \left(\frac{n-i}{n}\right) \Bigg\{ \left(\frac{1}{n-i}\right) \binom{n-1}{i} \big[(1-\tau) f_c\big]^{i} \big[\tau + (1-\tau)(1-f_c)\big]^{n-i-1} \\
+ \left(\frac{n-i-1}{n-i}\right) (1-f_c) \binom{n-1}{i} \big[(1-\tau) f_c\big]^{i} \big[\tau + (1-\tau)(1-f_c)\big]^{n-i-2} \Bigg\} \Bigg\} \\
\cdot \Bigg\{ \sum_{j=0}^{i} \binom{i}{j} \left(\frac{1}{n}\right)^{j} \left(\frac{n-1}{n}\right)^{i-j} \cdot \Bigg[\sum_{k=0}^{i} \binom{i}{k} (H_n(j))^{k} (1 - H_n(j))^{i-k} \left(\frac{k}{n}\right) \Bigg] \Bigg\} \cdot b
\end{aligned}
$$

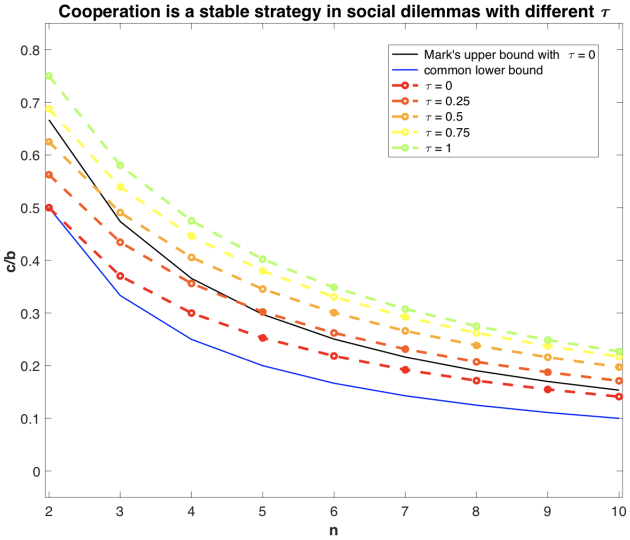
Comparison of $W(C) = W(D)$ with different mixing ratio τ



$W(C) = W(D)$ lines in different settings of group size n and assortative level τ



Cooperation is a stable strategy in social dilemmas with different assortative level τ



Conclusion

- We avoid the cost of repeated structure formation resulting from the Mark's model.
- With both analytical and numerical examination, we present an extension to a single-leadership model by gauging the hierarchicalness of these multiple-leader cases.
- We also estimate the level of cooperation mobilized by the leader-"s."
- We show that when these multiple-leader scenarios are taken into account and properly handled, the emergence of cooperation against the invasion of defection converges to the level Mark proposed.

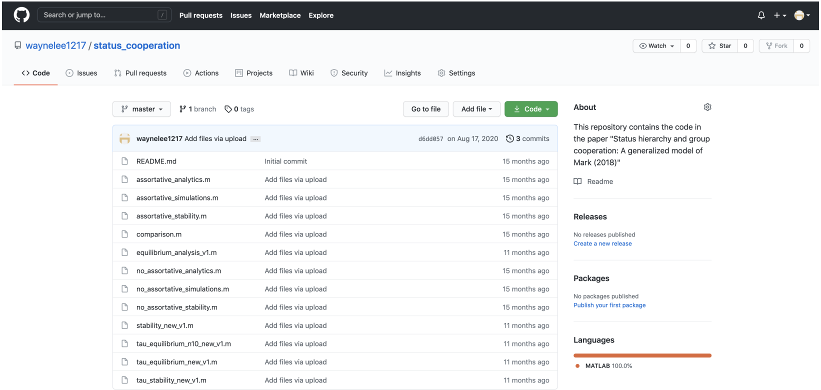
Current work

- Deal with the problem of "memory"
- Extend the model to an arbitrary level of hierarchies
- Make the distribution of benefit uneven by introducing the Gini coefficient
- Investigate the evolutionary interplay of status hierarchy and group cooperation

Questions?

Part 4: Code sharing

Code sharing



[1] https://github.com/waynelee1217/status_cooperation

Simulation

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Code description: This code is used to plot the status cooperation with
% hierarchy pursing with no assortative mixing. Here we plot the
% averaged simulation results over 100,000 trials. The sizes of groups are
% set to be 4, 6, 8, and 10.

% Author: Hsuan-Wei Wayne Lee
% Contact information: hwwaynelee@sinica.edu.tw
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% parameter setting
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

n = 10;          % group size (temporary)
fc = (n-1)/n;    % fraction of C (temporary), put this here is dangerous since we will change n later
tau = 0;         % tau
c_prob = 1/n;    % prob. for a C going to top level, put this here is dangerous since we will change n later

b = 1;
c = 0.1;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% simulation
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

trial = 100000;
WC_WD_all = zeros(2000,trial);
colors = jet(4);

for n = 4:2:10

    n

    fc = (n-1)/n;
    c_prob = 1/n;

for kk = 1:trial
```

Simulation

```
WC_WD_vec = [];

for c = 0:0.01:0.50

    for fc = 0:0.025:1

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        % WC
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        node_matrix1 = zeros(n, 4);

        for ii = 1:n

            node_matrix1(ii,1) = ii; % node id

            if ii == 1
               node_matrix1(ii,2) = 1; % being a cooperator
            end

            if rand < fc && ii > 1
               node_matrix1(ii,2) = 1; % being a cooperator
            end

            if node_matrix1(ii,2) == 1 && rand < c_prob
               node_matrix1(ii,3) = 1; % going to the top level
            end

        end

        x = sum(node_matrix1(:,3)); % number of nodes on the top level

        H_value1 = H_n(n,x);

        for ii = 1:n

            if node_matrix1(ii,2) == 1 && rand < H_value1
```

Simulation

```
       node_matrix1(ii,4) = 1; % contributing
    end

end

count_C_not_contribute = 0;

for ii = 1:n

    if node_matrix1(ii,2) == 1 && node_matrix1(ii,4) == 0
       count_C_not_contribute = count_C_not_contribute + 1;
    end

end


%WC = c*(count_C_not_contribute/n)+sum(node_matrix1(:,4))*b/n; % payoff
WC = c*(1-H_value1)+sum(node_matrix1(:,4))*b/n;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% WD
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

node_matrix2 = zeros(n, 4);

for ii = 1:n

    node_matrix2(ii,1) = ii; % node id

    if  ii == 1
       node_matrix2(ii,2) = 0; % being a D
    end

    if rand < fc && ii > 1
       node_matrix2(ii,2) = 1; % being a cooperator
    end

    if node_matrix2(ii,2) == 1 && rand < c_prob
       node_matrix2(ii,3) = 1; % going to the top level
```

Simulation

```
                end

            end

            x = sum(node_matrix2(:,3)); % number of nodes on the top level

            H_value2 = H_n(n,x);

            for ii = 1:n

                if node_matrix2(ii,2) == 1 && rand < H_value2
                   node_matrix2(ii,4) = 1; % contributing
                end

            end

            WD = c + sum(node_matrix2(:,4))*b/n; % payoff

            WC_WD = WC - WD;

            WC_WD_vec = [WC_WD_vec WC_WD];


        end

    end

    WC_WD_all(:,kk) = WC_WD_vec;
    kk;


end

WC_WD_vec_ave = mean(WC_WD_all,2);
WC_WD_matrix = reshape(WC_WD_vec_ave,[41,51])';
```

Simulation

```matlab
set(gca,'Xtick',1:8:41,'XTickLabel',{'0','0.2','0.4','0.6','0.8','1.0'},'FontSize',16)
set(gca,'Ytick',[1 11 21 31 41 51],'YTickLabel',{'0','0.1','0.2','0.3','0.4','0.5'},'FontSize',16)


% xlabel('fc','FontSize',16,'FontWeight','bold')
% ylabel('c/b','FontSize',16,'FontWeight','bold')
% title('n = 3','FontSize',16,'FontWeight','bold')
% hcb=colorbar
% title(hcb,'WC - WD')


value = [];
for ii = 1:41
    [M,I] = min(abs(WC_WD_matrix(:,ii)));
    value_new = I;
    value = [value value_new];
end

xx = 1:5:201;
yyy = value;
%plot(xx, yyy,'LineWidth',2,'Color',colors((n-2)/2, :))

sz = 60;
scatter(xx,yyy,sz,'MarkerEdgeColor',[0 0 0],...
              'MarkerFaceColor',colors((n-2)/2, :),...
              'LineWidth',0.3)

hold on


end

set(gca,'Xtick',1:40:201,'XTickLabel',{'0','0.2','0.4','0.6','0.8','1.0'},'FontSize',16)
set(gca,'Ytick',[1 11 21 31 41 51],'YTickLabel',{'0','0.1','0.2','0.3','0.4','0.5'},'FontSize',16)
axis([1 201 0 52])
xlabel('fc','FontSize',16,'FontWeight','bold')
ylabel('c/b','FontSize',16,'FontWeight','bold')
title('MATLAB simulation','FontSize',16,'FontWeight','bold')
title('Comparison of simulation and analytic for different group size n','FontSize',16,'FontWeight','bold')
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Code description: This code is used to plot the status cooperation with
% hierarchy pursing with no assortative mixing. Here we plot the
% analytical approximation to the simulations. The sizes of groups are
% set to be 4, 6, 8, and 10.

% Author: Hsuan-Wei Wayne Lee
% Contact information: hwwaynelee@sinica.edu.tw
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;
clc;
close all;

% hold on

n = 5; % you probably want to fix n

fc = 0.8;
b = 1; % fix b = 1 here, since we only care about c/b
c = 1;

color = jet(4);

for n = 4:2:10

    n

% average payoff for D

diff_WC_WD = zeros(51,201);

count_col = 1;

for fc = 0:0.005:1

    count_row = 1;

    for c = 0:0.01:0.50
```

Analytics

```
        % average payoff for D

        sum_i = 0;

        for ii = 1:(n-1)

            sum_j = 0;

            for jj = 2:ii

               sum_k = 0;

               for kk = 1:ii

                   sum_k = sum_k + nchoosek(ii,kk)*H_n(n,jj)^kk*(1-H_n(n,jj))^(ii-kk)*(kk*b/n);

               end

               sum_j = sum_j + nchoosek(ii,jj)*(1/n)^jj*((n-1)/n)^(ii-jj)*sum_k;

            end


            sum_i = sum_i + nchoosek(n-1,ii)*fc^ii*(1-fc)^(n-1-ii)*(ii*(1/n)*((n-1)/n)^(ii-1)*(ii*b/n)+sum_j);
        end

        triple_sum_D = sum_i;

        W_D = c + triple_sum_D ;

        % average payoff for C

        first_term = 0;

        sum_i = 0;

        for ii = 0:(n-1)
```

Analytics

```
            sum_j = 0;

            for jj = 2:(ii+1)

                sum_j = sum_j + nchoosek((ii+1),jj)*(1/n)^jj*((n-1)/n)^(ii+1-jj)*(1-H_n(n,jj));

            end

            sum_i = sum_i + nchoosek(n-1,ii)*fc^ii*(1-fc)^(n-1-ii)*(nchoosek((ii+1),0)*(1/n)^0*((n-1)/n)^(ii+1)+sum_j);
        end

        first_term = (fc^(n-1)*(1/n)^n + sum_i)*c;

        sum_ii = 0;

        for ii = 0:(n-1)

            sum_j = 0;

            for jj = 2:(ii+1)

               sum_k = 0;

               for kk = 0:(ii+1)

                   sum_k = sum_k + nchoosek(ii+1,kk)*H_n(n,jj)^kk*(1-H_n(n,jj))^(ii+1-kk)*((kk)*b/n);

               end

               %sum_j = sum_j + nchoosek((ii+1),jj)*(1/n)^jj*((n-1)/n)^(ii+1-jj)*H_n(n,jj)*sum_k;
               %sum_j = sum_j + nchoosek((ii+1),jj)*(1/n)^jj*((n-1)/n)^(ii+1-jj)*sum_k;
               sum_j = sum_j + nchoosek((ii+1),jj)*(1/n)^jj*((n-1)/n)^(ii+1-jj)*sum_k;

            end

            sum_ii = sum_ii + nchoosek(n-1,ii)*fc^ii*(1-fc)^(n-1-ii)*(nchoosek((ii+1),1)*(1/n)*((n-1)/n)^(ii)*((ii+1)*b/n)+sum_j);

        end
```

Analytics

```
        second_term = sum_ii;

        W_C = first_term + second_term;

        W_diff = W_C - W_D;

        diff_WC_WD(count_row,count_col) = W_diff;

        count_row = count_row + 1;

    end

    count_col = count_col + 1;

end


% imagesc(diff_WC_WD)
% set(gca,'YDir','normal')

diff_zero = zeros(1,0);

for ii = 1:201

  [M, I] = min(abs(diff_WC_WD(:,ii)));
  diff_zero = [diff_zero,I];

end


hold on
value_b = diff_zero;
xx = 1:201;
yy = value_b;
yyy_n = value_b;
plot(xx, yyy_n,'LineWidth',2,'Color',color((n-2)/2,:))

end

set(gca,'Xtick',1:40:201,'XTickLabel',{'0','0.2','0.4','0.6','0.8','1.0'},'FontSize',16)
```

Analytics

```
set(gca,'Xtick',1:40:201,'XTickLabel',{'0','0.2','0.4','0.6','0.8','1.0'},'FontSize',16)
set(gca,'Ytick',[1 11 21 31 41 51],'YTickLabel',{'0','0.1','0.2','0.3','0.4','0.5'},'FontSize',16)

axis([1 201 0 52])

xlabel('fc','FontSize',16,'FontWeight','bold')
ylabel('c/b','FontSize',16,'FontWeight','bold')
title('MATLAB analysis','FontSize',16,'FontWeight','bold')
box on

% diff_WC_WD_matrix = reshape(diff_WC_WD,[21,26]);
% diff_WC_WD_matrix = diff_WC_WD_matrix';
% imagesc(diff_WC_WD_matrix)
% set(gca,'YDir','normal')

% set(gca,'Xtick',1:20:101,'XTickLabel',{'0','0.2','0.4','0.6','0.8','1.0'},'FontSize',16)
% set(gca,'Ytick',[1 301 601 901 1201 1501],'YTickLabel',{'0','0.1','0.2','0.3','0.4','0.5'},'FontSize',16)
%
% xlabel('','FontSize',16,'FontWeight','bold')
% ylabel('c/b','FontSize',16,'FontWeight','bold')
% title('n = 3','FontSize',16,'FontWeight','bold')
% hcb=colorbar
% title(hcb,'WC - WD')
%
% hold on
% value = [];
% for mm = 1:101
%     [M,I] = min(abs(diff_WC_WD_matrix(:,mm)));
%     value_new = 0.02 * I;
%     value = [value value_new];
% end
%
% value_b = value./30;
% xx = 1:101;
% yy = value_b;
% yyy_n3 = value_b * 1500;
% plot(xx, yyy_n3,'LineWidth',2,'Color','r')
```