assignment_2_handout

November 25, 2024

1 Assignment 2 - Classification

Submitted by: - Jonas Henker, 7054995 - Leo Forster, 7055800

In this assigment you will be coding for a Classification task hands-on. (10 Points) Fill the part with #TODO

This notebook will guide you through the process of using logistic regression, LDA and QDA for classification tasks. You will start with a binary classification problem and attempt to solve it using logistic regression. You will then extend it to a more complicated dataset to classify it using LDA and QDA

Objectives:

- Implement logistic regression for binary classification.
- Understand differences between LDA and QDA.
- Implement LDA and QDA for multivariate data.

Imports and plotting function

[1]:
```
## Import Necessary Libraries
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification, make_blobs
from sklearn.linear_model import LinearRegression, LogisticRegression
from sklearn.metrics import accuracy_score, confusion_matrix
from sklearn.model_selection import train_test_split
```

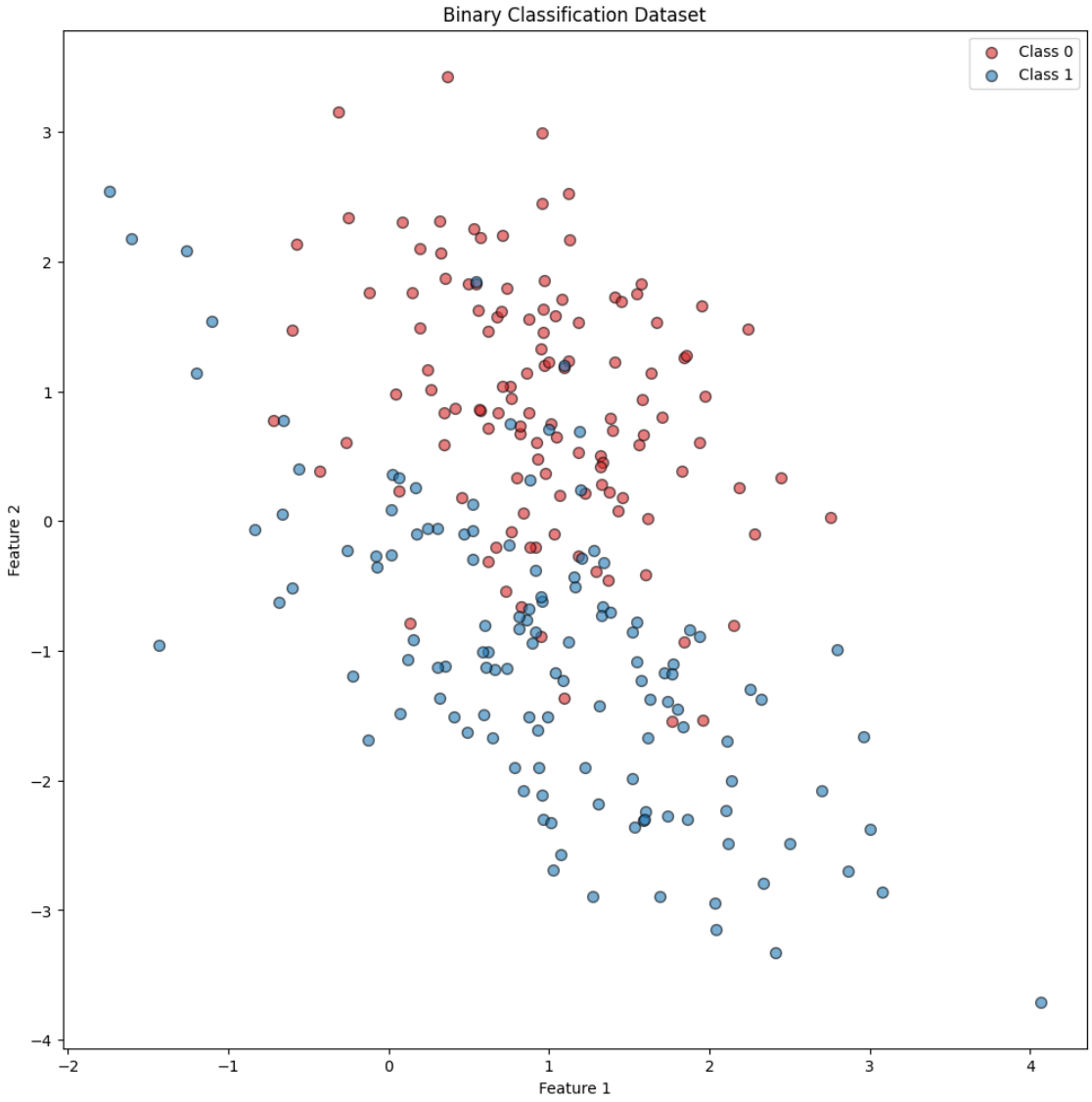
[2]:
```
import matplotlib as mpl
from matplotlib import colors

from sklearn.inspection import DecisionBoundaryDisplay

cmap = colors.ListedColormap(["tab:red", "tab:blue"])

def plot_ellipse(mean, cov, color, ax):
    v, w = np.linalg.eigh(cov)
    u = w[0] / np.linalg.norm(w[0])
```

```
    angle = np.arctan(u[1] / u[0])
    angle = 180 * angle / np.pi  # convert to degrees
    # filled Gaussian at 2 standard deviation
    ell = mpl.patches.Ellipse(
        mean,
        2 * v[0] ** 0.5,
        2 * v[1] ** 0.5,
        angle=180 + angle,
        facecolor=color,
        edgecolor="black",
        linewidth=2,
    )
    ell.set_clip_box(ax.bbox)
    ell.set_alpha(0.4)
    ax.add_artist(ell)


def plot_result(estimator, X, y, ax):
    cmap = colors.ListedColormap(["tab:red", "tab:blue"])
    DecisionBoundaryDisplay.from_estimator(
        estimator,
        X,
        response_method="predict_proba",
        plot_method="pcolormesh",
        ax=ax,
        cmap="RdBu",
        alpha=0.3,
    )
    DecisionBoundaryDisplay.from_estimator(
        estimator,
        X,
        response_method="predict_proba",
        plot_method="contour",
        ax=ax,
        alpha=1.0,
        levels=[0.5],
    )
    y_pred = estimator.predict(X)
    X_right, y_right = X[y == y_pred], y[y == y_pred]
    X_wrong, y_wrong = X[y != y_pred], y[y != y_pred]
    ax.scatter(X_right[:, 0], X_right[:, 1], c=y_right, s=20, cmap=cmap,␣
 ↪alpha=0.5)
    ax.scatter(
        X_wrong[:, 0],
        X_wrong[:, 1],
        c=y_wrong,
        s=30,
```

```
        cmap=cmap,
        alpha=0.9,
        marker="x",
    )
    ax.scatter(
        estimator.means_[:, 0],
        estimator.means_[:, 1],
        c="yellow",
        s=200,
        marker="*",
        edgecolor="black",
    )

    if isinstance(estimator, LinearDiscriminantAnalysis):
        covariance = [estimator.covariance_] * 2
    else:
        covariance = estimator.covariance_
    plot_ellipse(estimator.means_[0], covariance[0], "tab:red", ax)
    plot_ellipse(estimator.means_[1], covariance[1], "tab:blue", ax)

    ax.set_box_aspect(1)
    ax.spines["top"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.set(xticks=[], yticks=[])
```

Logistic Regression for Classification

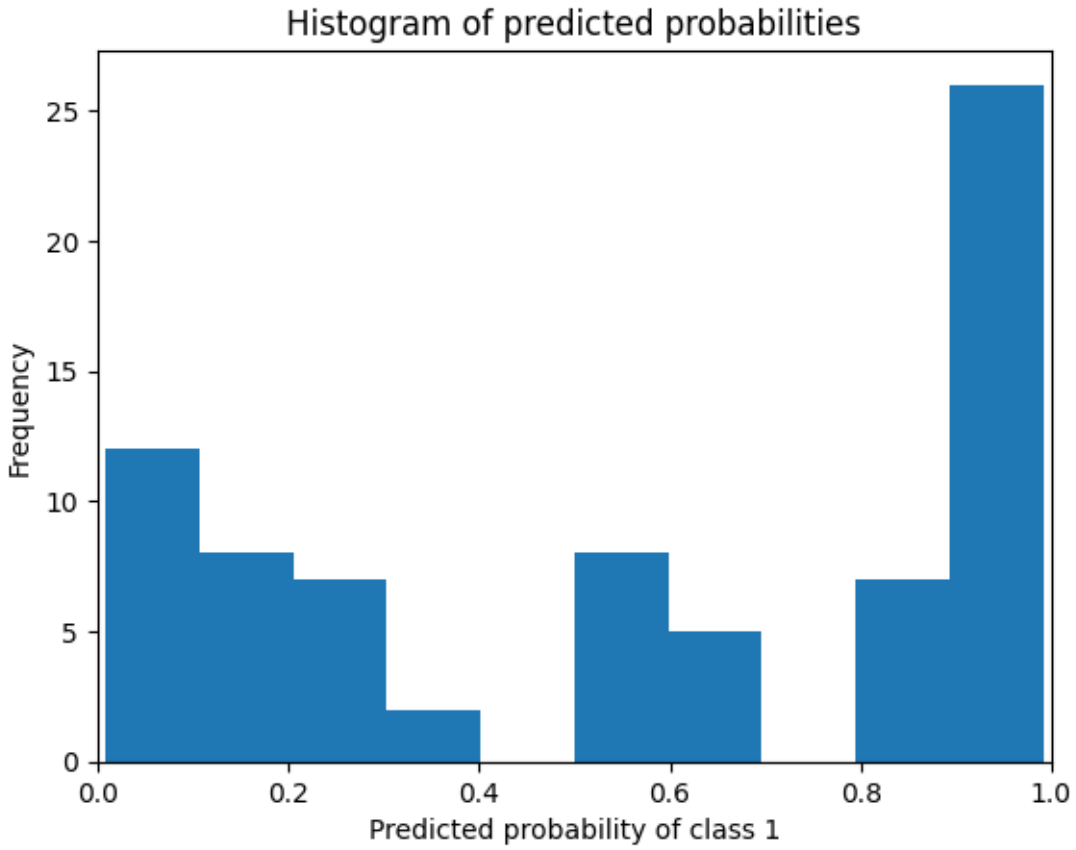
[3]:
```
# Generate a synthetic dataset
X, y = make_classification(n_samples=250, n_features=2, n_redundant=0,
                           n_informative=2, n_clusters_per_class=1,␣
 ↪random_state=1)

# Visualize the dataset
plt.figure(figsize=(12,12))
# Plot points for Class 0
plt.scatter(X[y == 0, 0], X[y == 0, 1], color=cmap(0.1), s=50, label='Class 0',␣
 ↪alpha=0.6, edgecolors='k')
# Plot points for Class 1
plt.scatter(X[y == 1, 0], X[y == 1, 1], color=cmap(0.9), s=50, label='Class 1',␣
 ↪alpha=0.6, edgecolors='k')
plt.title('Binary Classification Dataset')
plt.xlabel('Feature 1')
plt.ylabel('Feature 2')
plt.legend(['Class 0', 'Class 1'])
plt.show()
```



Task 1: Check range of outputs by plotting the histogram of them comment on the reason of the interval. (2 Points) Answer: The peaks at 0 & 1 show that the model ist mostly confident about its classifications. But the values around 0.6 could suggest, that more training or a more complex model is needed, to classify these values.

[4]:
```
# Split the dataset into training and test sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3,␣
 ↪random_state=42)

# Fit a linear regression model
logistic_regressor = LogisticRegression()
logistic_regressor.fit(X_train, y_train)
```

```
# Predict on the test set
y_pred_prob_all = logistic_regressor.predict_proba(X_test)

# Select only the probabilities for class 1
y_pred_prob = y_pred_prob_all[:, 1]

# TODO - Print histogram of predicted probabilities
plt.hist(y_pred_prob, bins=10)
plt.xlim(0, 1)
plt.title('Histogram of predicted probabilities')
plt.xlabel('Predicted probability of class 1')
plt.ylabel('Frequency')
plt.show()
```

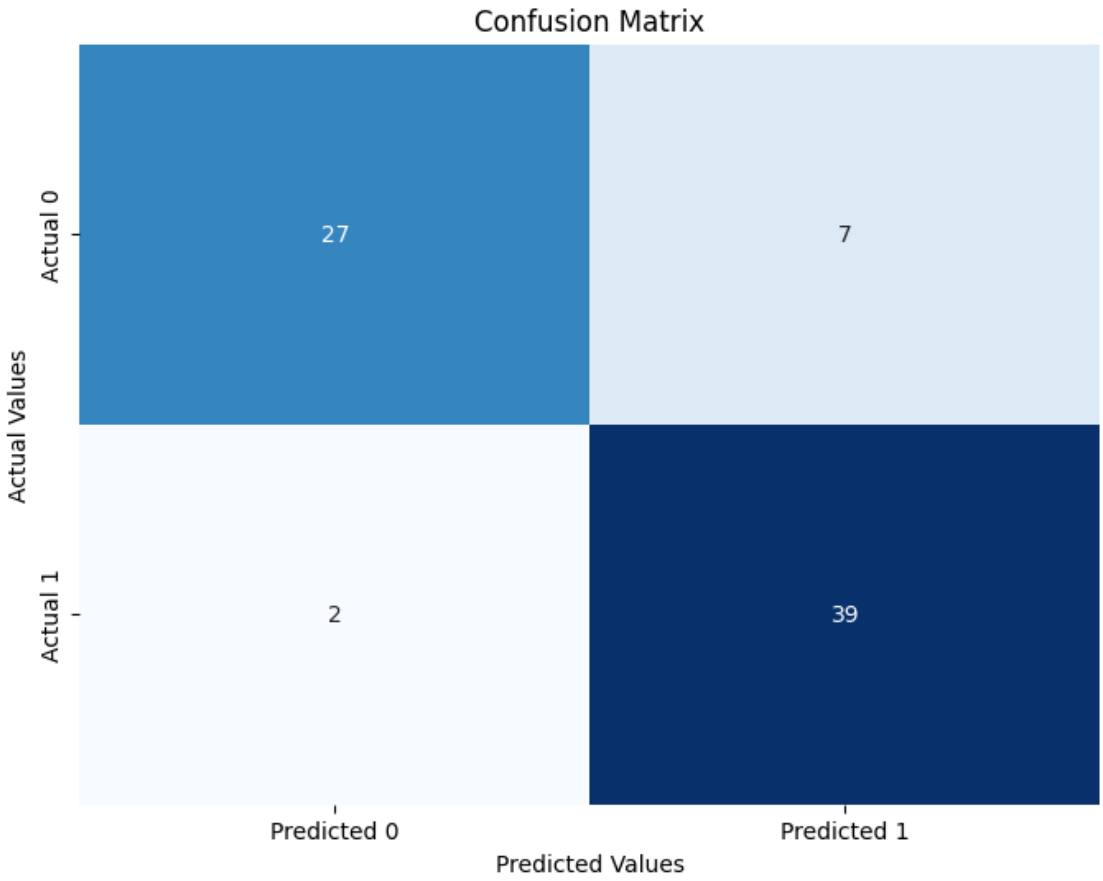


Task 2: Convert the probability predictions to binary class labels using 0.5 as threshold and compute Type-1 and Type-2 error. (2 Points)

[5]:
```
# TODO - Your code here
y_pred_class = (y_pred_prob > 0.5).astype(int)
```

```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn.metrics import accuracy_score, confusion_matrix


# Evaluate the model
accuracy = accuracy_score(y_test, y_pred_class)
print(f'Accuracy of Linear Regression Classifier: {accuracy:.2f}')

# Confusion Matrix
cm = confusion_matrix(y_test, y_pred_class)

FP = cm[0, 1]  # False Positive
FN = cm[1, 0]  # False Negative
TN = cm[0, 0]  # True Negative
TP = cm[1, 1]  # True Positive

# TODO - Your code here # Calculate Type I and Type II error rates
type1_error_rate = FP / (FP + TN)
type2_error_rate = FN / (FN + TP)

print(f'Type I Error Rate (False Positive Rate): {type1_error_rate:.2f}')
print(f'Type II Error Rate (False Negative Rate): {type2_error_rate:.2f}')

# Visualize the confusion matrix with heatmap
cm_df = pd.DataFrame(cm, index=['Actual 0', 'Actual 1'], columns=['Predicted␣
 ↪0', 'Predicted 1'])

plt.figure(figsize=(8, 6))
sns.heatmap(cm_df, annot=True, fmt='g', cmap='Blues', cbar=False)
plt.title('Confusion Matrix')
plt.ylabel('Actual Values')
plt.xlabel('Predicted Values')
plt.show()
```

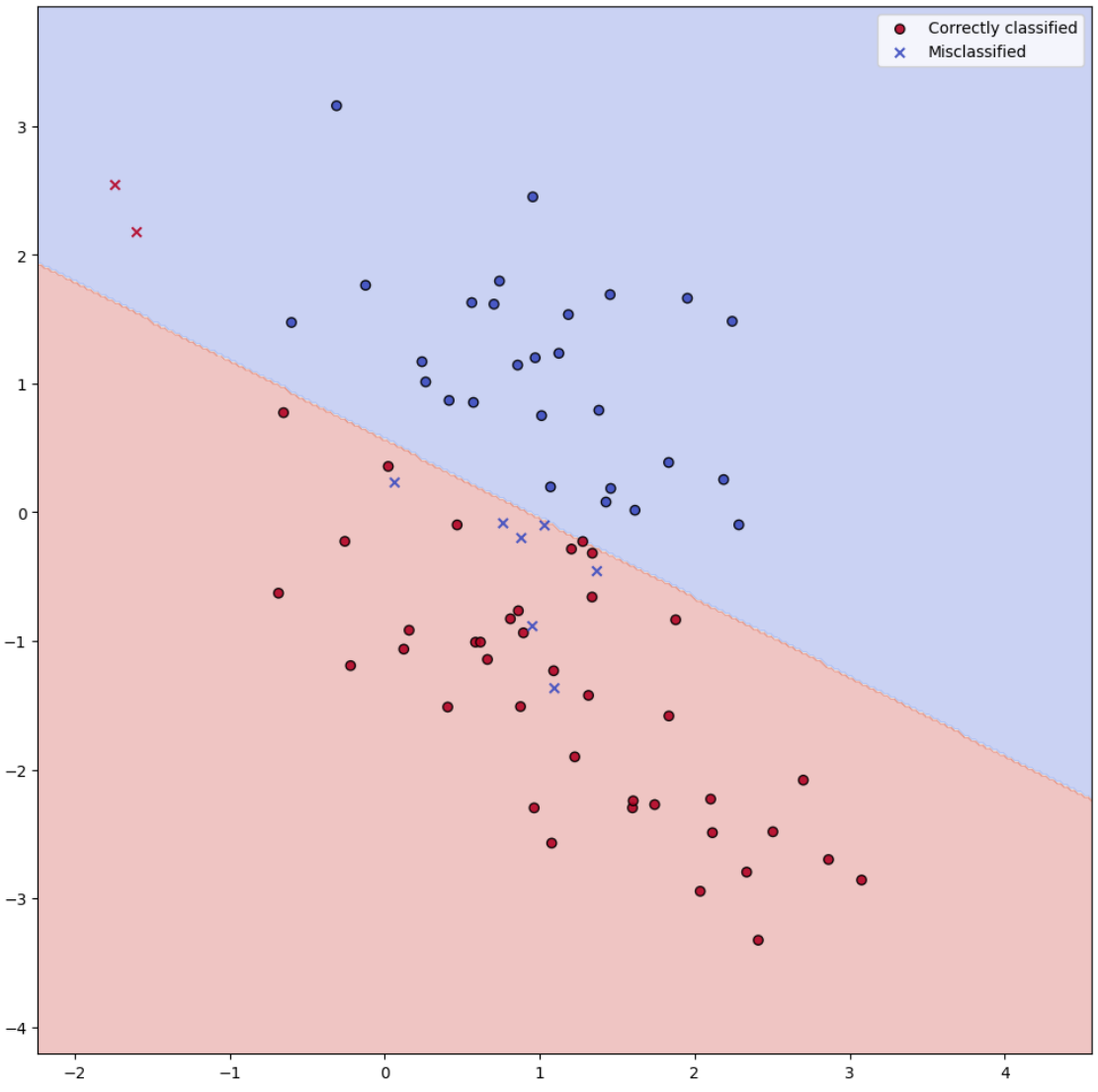
```
Accuracy of Linear Regression Classifier: 0.88
Type I Error Rate (False Positive Rate): 0.21
Type II Error Rate (False Negative Rate): 0.05
```



Task 3: Determine correct predictions. (1 Point)

[6]:
```
# TODO - Your code here
X_right = X_test[y_test == y_pred_class]
y_right = y_test[y_test == y_pred_class]
X_wrong = X_test[y_test != y_pred_class]
y_wrong = y_test[y_test != y_pred_class]


# Visualize the decision boundaries
# Define the mesh grid
x_min, x_max = X[:, 0].min() - 0.5, X[:, 0].max() + 0.5
y_min, y_max = X[:, 1].min() - 0.5, X[:, 1].max() + 0.5
xx, yy = np.meshgrid(np.linspace(x_min, x_max, 300),
                     np.linspace(y_min, y_max, 300))

# Predict using logistic regression on the mesh grid
Z = logistic_regressor.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
```

```
plt.figure(figsize=(12, 12))
plt.contourf(xx, yy, Z, alpha=0.3, cmap='coolwarm')
plt.scatter(X_right[:, 0], X_right[:, 1], c=y_right, cmap='coolwarm', alpha=0.
 ↪9, label='Correctly classified', edgecolors='k')
plt.scatter(
        X_wrong[:, 0],
        X_wrong[:, 1],
        c=y_wrong,
        cmap='coolwarm',
        alpha=0.9,
        marker="x",
        label='Misclassified'
    )
plt.legend()
plt.show()
```



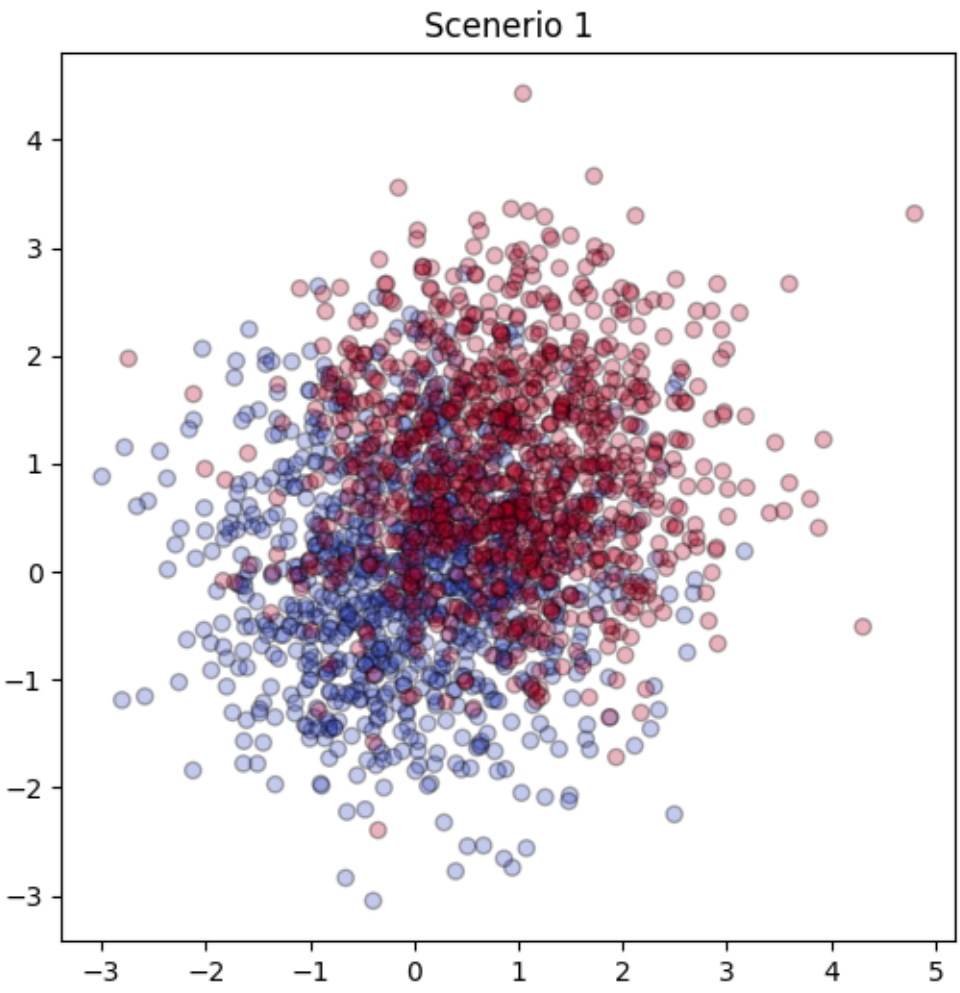
Exploring Gaussian Distributed Data Below we generate three datasets with different Gaussian distributions.

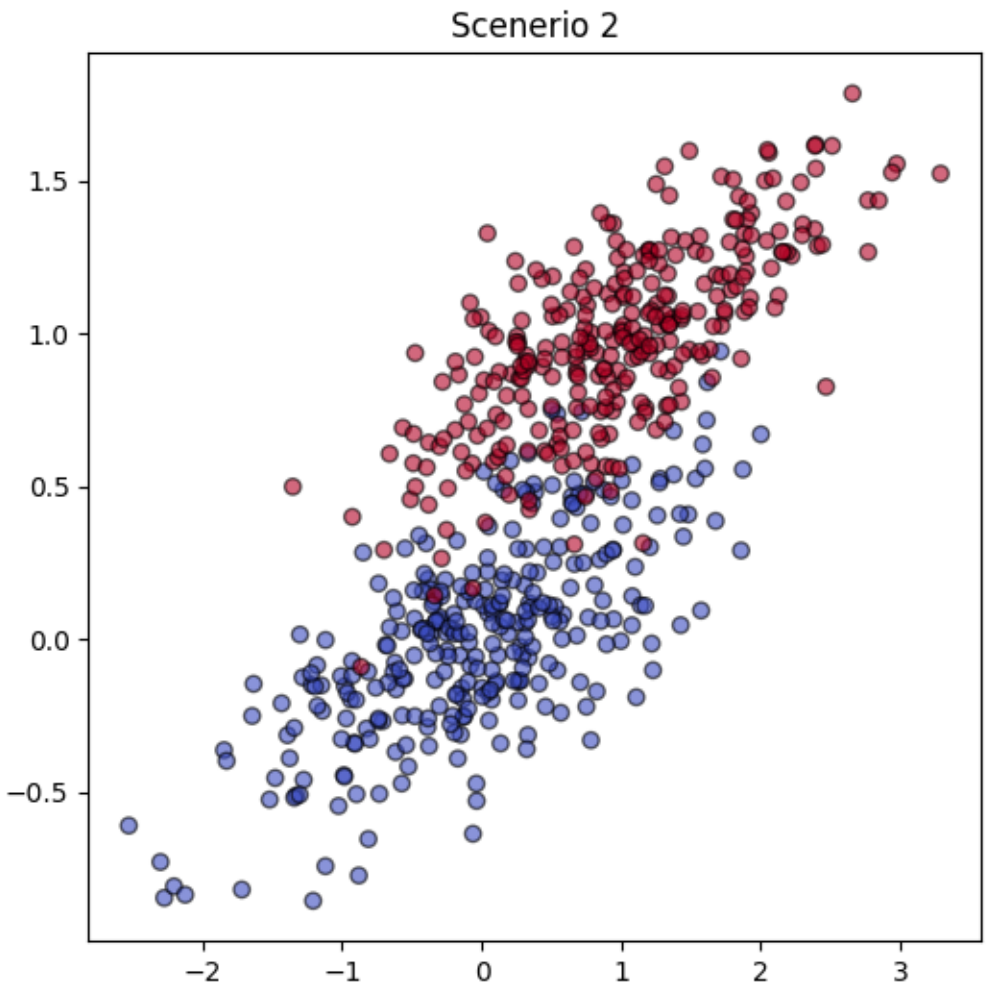
First, we define a function to generate synthetic data. It creates two blobs centered at `(0, 0)` and `(1, 1)`. Each blob is assigned a specific class. The dispersion of the blob is controlled by the parameters `cov_class_1` and `cov_class_2`, that are the covariance matrices used when generating the samples from the Gaussian distributions.

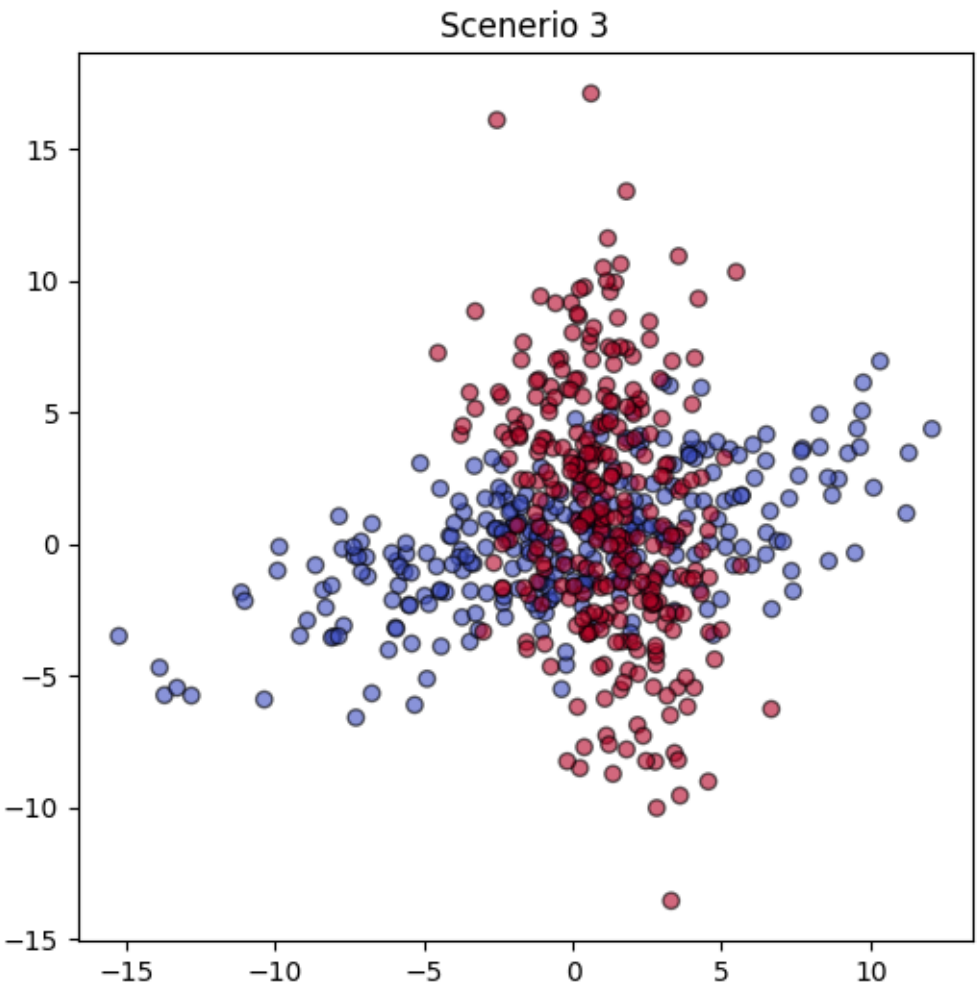
[7]:
```
def make_data(n_samples, n_features, cov_class_1, cov_class_2, seed=0):
    rng = np.random.RandomState(seed)
    X = np.concatenate(
        [
            rng.randn(n_samples, n_features) @ cov_class_1,
            rng.randn(n_samples, n_features) @ cov_class_2 + np.array([1, 1]),
        ]
    )
    y = np.concatenate([np.zeros(n_samples), np.ones(n_samples)])
    return X, y


covariance = np.array([[1, 0], [0, 1]])
X_1, y_1 = make_data(
    n_samples=1_000,
    n_features=2,
    cov_class_1=covariance,
    cov_class_2=covariance,
    seed=0,
)
covariance = np.array([[0.0, -0.23], [0.83, 0.23]])
X_2, y_2 = make_data(
    n_samples=300,
    n_features=2,
    cov_class_1=covariance,
    cov_class_2=covariance,
    seed=0,
)
cov_class_1 = np.array([[0.0, -1.0], [2.5, 0.7]]) * 2.0
cov_class_2 = cov_class_1.T
X_3, y_3 = make_data(
    n_samples=300,
    n_features=2,
    cov_class_1=cov_class_1,
    cov_class_2=cov_class_2,
    seed=0,
)
```

```
#PLOT DATASETS
plt.figure(figsize=(6, 6))
print(X_1.shape, y_1.shape)
import matplotlib.pyplot as plt
plt.scatter(X_1[:, 0], X_1[:, 1], c=y_1, cmap='coolwarm', edgecolors='k',␣
 ↪alpha=0.3)
plt.title('Scenerio 1')#('Scenerio Covariance')
plt.show()

plt.figure(figsize=(6, 6))
plt.scatter(X_2[:, 0], X_2[:, 1], c=y_2, cmap='coolwarm', edgecolors='k',␣
 ↪alpha=0.6)
plt.title('Scenerio 2')#('Shared Covariance')
plt.show()

plt.figure(figsize=(6, 6))
plt.scatter(X_3[:, 0], X_3[:, 1], c=y_3, cmap='coolwarm',␣
 ↪edgecolors='k',alpha=0.6)
plt.title('Scenerio 3')#('Different Covariance')
plt.show()
```

```
(2000, 2) (2000,)
```


Scenerio 1
4
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
4
5


Scenerio 2
1.5
1.0
0.5
0.0
−0.5
−2
−1
0
1
2
3



Task 4: Match the plotted scenerios with the following cases (1 Points) - Shared Covariance: Scenario 2 - Different Covariance: Scenario 3 - Isotropic Covariance: Scenario 1

Remember LDA and QDA:

- **Multivariate LDA**:

$$P(y = k \mid \mathbf{x}) \propto \exp\left(-\frac{1}{2}(\mathbf{x} - \mu_k)^\top \Sigma^{-1}(\mathbf{x} - \mu_k)\right)$$

- **Multivariate LDA Decision Function**:

$$\delta_k(\mathbf{x}) = \mathbf{x}^\top \Sigma^{-1}\mu_k - \frac{1}{2}\mu_k^\top \Sigma^{-1}\mu_k + \log P(y = k)$$

  - $ _k $ is the mean vector for class $ k $,
  - $ $ is the shared matrix ,
  - $ | | $ is the determinant of $ $,
  - $ P(y = k) $ is the prior probability for class $ k $.

- Assumes $ _1 = _2 = ...= _k $.

- **Multivariate QDA**:

$$P(y = k \mid \mathbf{x}) \propto \frac{1}{|\Sigma_k|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu_k)^\top \Sigma_k^{-1}(\mathbf{x} - \mu_k)\right)$$

- **QDA Decision Function**:

$$\delta_k(\mathbf{x}) = -\frac{1}{2}\log|\Sigma_k| - \frac{1}{2}(\mathbf{x} - \mu_k)^\top \Sigma_k^{-1}(\mathbf{x} - \mu_k) + \log P(y = k)$$

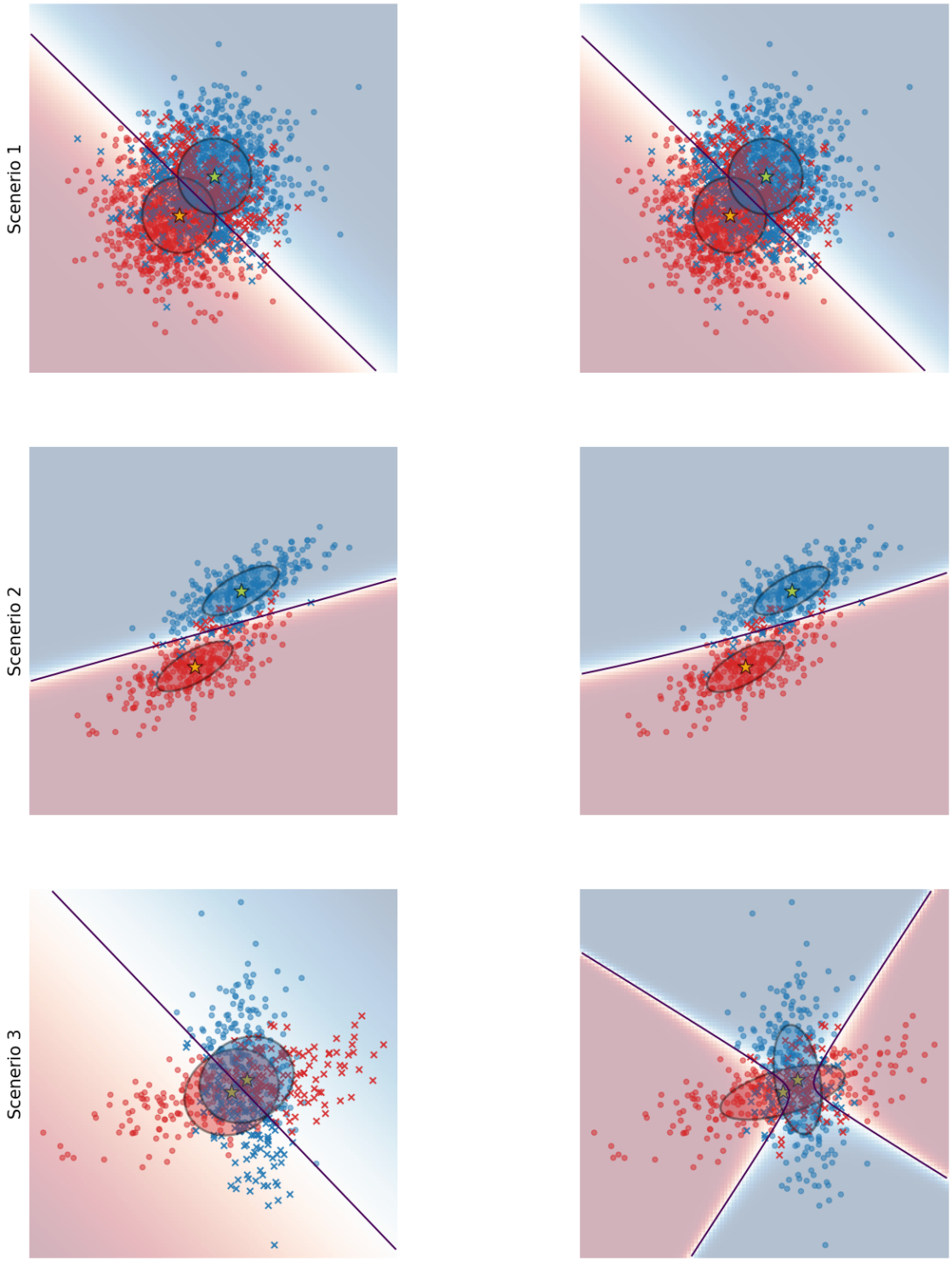
  - $ _k $ is the mean vector for class $ k $,
  - $ _k $ is the covariance matrix for class $ k $,
  - $ | _k| $ is the determinant of $ _k $,
  - $ P(y = k) $ is the prior probability for class $ k $.
  - Assumes $ _1 _2 ... _k $.

Task 5: Assume that we need to perform binary classification for plotted Scenarios 1, 2, and 3. Choose one classification method (QDA or LDA) to use in each scenario. If you believe both methods are suitable, always select the simpler one. (2 Points) - Scenerio 1: LDA - Scenerio 2: LDA - Scenerio 3: QDA

See the performance of LDA and QDA for each case

[8]:
```
import matplotlib.pyplot as plt
plt.rcParams['figure.facecolor'] = 'white'
from sklearn.discriminant_analysis import (
    LinearDiscriminantAnalysis,
    QuadraticDiscriminantAnalysis,
)

fig, axs = plt.subplots(nrows=3, ncols=2, sharex="row", sharey="row",␣
 ↪figsize=(16,20))
lda = LinearDiscriminantAnalysis(solver="svd", store_covariance=True)
qda = QuadraticDiscriminantAnalysis(store_covariance=True)

for ax_row, X, y in zip(
    axs,
    (X_1, X_2, X_3),
    (y_1, y_2, y_3),
):
    lda.fit(X, y)
    plot_result(lda, X, y, ax_row[0])
    qda.fit(X, y)
    plot_result(qda, X, y, ax_row[1])

axs[0, 0].set_ylabel("Scenerio 1", fontsize=15)
axs[1, 0].set_ylabel("Scenerio 2", fontsize=15)
axs[2, 0].set_ylabel("Scenerio 3", fontsize=15)
fig.suptitle(
```

```
    "Linear Discriminant Analysis (Left) vs Quadratic Discriminant Analysis␣
 ↪(Right)",
    y=0.94,
    fontsize=15,
)
plt.show()
```


Linear Discriminant Analysis (Left) vs Quadratic Discriminant Analysis (Right)
Scenerio 1
Scenerio 2
Scenerio 3

Task 6: For each of the scenerios plotted above, compare behaviour of decision boundaries with your words for LDA (left column) and QDA (right column). (2 Points) - Scenerio 1: In both cases, the decision boundaries are linear, thus using LDA is more fitting, given the lower complexity - Scenerio 2: In both cases, the decision boundaries are linear or near linear, thus using LDA is more fitting, given the lower complexity - Scenerio 3: In this scenario, the quadratic boundaries of QDA fit the data better, thus QDA is the better choice.