assignment_1_handout

November 8, 2024

1 Assignment 1 - Linear Regression

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In this assigment you will be coding for a Linear Regression task hands-on. (10 Points)

The notebook uses some popular libraries. If your environment is missing any of these libraries, you can install them using the following `pip` commands:

"'bash !pip install matplotlib seaborn scikit-learn

[1]:
```
import math

from sklearn.datasets import fetch_california_housing
import pandas as pd
from pandas.plotting import scatter_matrix
from scipy import stats
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import StandardScaler
```

[2]:
```
#make sizes bigger for readability
import matplotlib.pyplot as plt
plt.rcParams.update({'font.size': 17})
plt.rcParams["figure.figsize"] = (12,12)
```

1.1 Load and Explore Data

[3]:
```
# Load the California Housing dataset
housing = fetch_california_housing()
# Convert the dataset into a DataFrame
df = pd.DataFrame(housing.data, columns=housing.feature_names)
df['MedHouseVal'] = housing.target  # Add the target (median house value)
```

Number of Instances:

20640

Number of Attributes:

8 numeric, predictive attributes and the target

Attribute Information:

    MedInc median income in block group

    HouseAge median house age in block group

    AveRooms average number of rooms per household

    AveBedrms average number of bedrooms per household

    Population block group population

    AveOccup average number of household members

    Latitude block group latitude

    Longitude block group longitude

[4]:
```
display(df)
```

```
       MedInc  HouseAge  AveRooms  AveBedrms  Population  AveOccup  Latitude  \
0      8.3252      41.0  6.984127   1.023810       322.0  2.555556     37.88
1      8.3014      21.0  6.238137   0.971880      2401.0  2.109842     37.86
2      7.2574      52.0  8.288136   1.073446       496.0  2.802260     37.85
3      5.6431      52.0  5.817352   1.073059       558.0  2.547945     37.85
4      3.8462      52.0  6.281853   1.081081       565.0  2.181467     37.85
…       …       …       …       …         …       …       …
20635  1.5603      25.0  5.045455   1.133333       845.0  2.560606     39.48
20636  2.5568      18.0  6.114035   1.315789       356.0  3.122807     39.49
20637  1.7000      17.0  5.205543   1.120092      1007.0  2.325635     39.43
20638  1.8672      18.0  5.329513   1.171920       741.0  2.123209     39.43
20639  2.3886      16.0  5.254717   1.162264      1387.0  2.616981     39.37

       Longitude  MedHouseVal
0        -122.23        4.526
1        -122.22        3.585
2        -122.24        3.521
3        -122.25        3.413
4        -122.25        3.422
…           …           …
20635    -121.09        0.781
20636    -121.21        0.771
20637    -121.22        0.923
20638    -121.32        0.847
20639    -121.24        0.894

[20640 rows x 9 columns]
```

[5]:
```
#Explore data for missingness
print(df.info())
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 20640 entries, 0 to 20639
Data columns (total 9 columns):
 #   Column       Non-Null Count  Dtype
---  ------       --------------  -----
 0   MedInc       20640 non-null  float64
 1   HouseAge     20640 non-null  float64
 2   AveRooms     20640 non-null  float64
 3   AveBedrms    20640 non-null  float64
 4   Population   20640 non-null  float64
 5   AveOccup     20640 non-null  float64
 6   Latitude     20640 non-null  float64
 7   Longitude    20640 non-null  float64
 8   MedHouseVal  20640 non-null  float64
dtypes: float64(9)
memory usage: 1.4 MB
None
```

[6]:
```
#Check statistics of the data
print(df.describe())
```

```
             MedInc      HouseAge      AveRooms     AveBedrms    Population  \
count  20640.000000  20640.000000  20640.000000  20640.000000  20640.000000
mean       3.870671     28.639486      5.429000      1.096675   1425.476744
std        1.899822     12.585558      2.474173      0.473911   1132.462122
min        0.499900      1.000000      0.846154      0.333333      3.000000
25%        2.563400     18.000000      4.440716      1.006079    787.000000
50%        3.534800     29.000000      5.229129      1.048780   1166.000000
75%        4.743250     37.000000      6.052381      1.099526   1725.000000
max       15.000100     52.000000    141.909091     34.066667  35682.000000

           AveOccup      Latitude     Longitude   MedHouseVal
count  20640.000000  20640.000000  20640.000000  20640.000000
mean       3.070655     35.631861   -119.569704      2.068558
std       10.386050      2.135952      2.003532      1.153956
min        0.692308     32.540000   -124.350000      0.149990
25%        2.429741     33.930000   -121.800000      1.196000
50%        2.818116     34.260000   -118.490000      1.797000
75%        3.282261     37.710000   -118.010000      2.647250
max     1243.333333     41.950000   -114.310000      5.000010
```

[7]:
```
# Display the first few rows
print(df.head())
```

```
   MedInc  HouseAge  AveRooms  AveBedrms  Population  AveOccup  Latitude  \
0  8.3252      41.0  6.984127   1.023810       322.0  2.555556     37.88
```

```
1  8.3014      21.0  6.238137  0.971880      2401.0  2.109842     37.86
2  7.2574      52.0  8.288136  1.073446       496.0  2.802260     37.85
3  5.6431      52.0  5.817352  1.073059       558.0  2.547945     37.85
4  3.8462      52.0  6.281853  1.081081       565.0  2.181467     37.85

   Longitude  MedHouseVal
0    -122.23        4.526
1    -122.22        3.585
2    -122.24        3.521
3    -122.25        3.413
4    -122.25        3.422
```

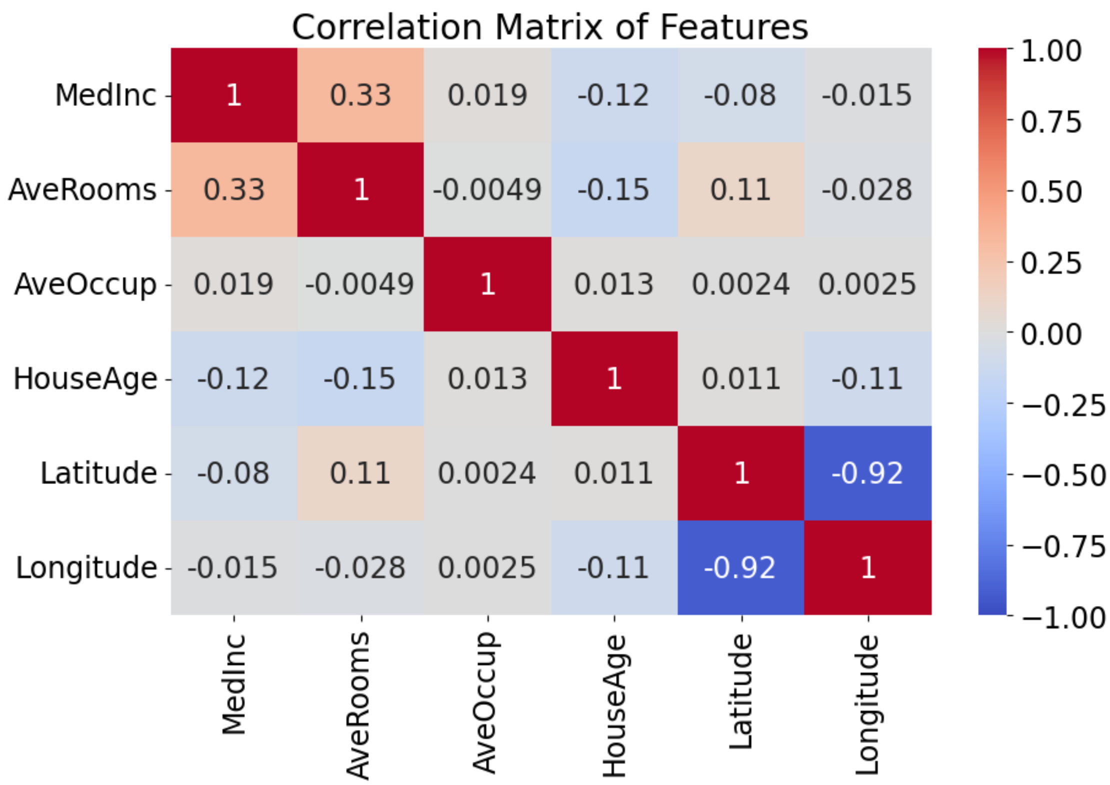
[8]:
```python
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from statsmodels.stats.outliers_influence import variance_inflation_factor

# Select multiple features for the correlation check
X_all = df[['MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude',␣
 ↪'Longitude']]

# Calculate correlation matrix
corr_matrix = X_all.corr()

# Visualize the correlation matrix
plt.figure(figsize=(10, 6))
sns.heatmap(corr_matrix, annot=True, cmap='coolwarm', vmin=-1, vmax=1)
plt.title('Correlation Matrix of Features')
plt.show()

# Note that correlation between Latitude and Longitude is coming from␣
 ↪geographical location of California
```



[9]:
```
#display scatter_matrix also
fig = plt.figure()
scatter_matrix(df,figsize =(25,25),alpha=0.9,diagonal="kde",marker="o")
```

[9]:
```
array([[<Axes: xlabel='MedInc', ylabel='MedInc'>,
        <Axes: xlabel='HouseAge', ylabel='MedInc'>,
        <Axes: xlabel='AveRooms', ylabel='MedInc'>,
        <Axes: xlabel='AveBedrms', ylabel='MedInc'>,
        <Axes: xlabel='Population', ylabel='MedInc'>,
        <Axes: xlabel='AveOccup', ylabel='MedInc'>,
        <Axes: xlabel='Latitude', ylabel='MedInc'>,
        <Axes: xlabel='Longitude', ylabel='MedInc'>,
        <Axes: xlabel='MedHouseVal', ylabel='MedInc'>],
       [<Axes: xlabel='MedInc', ylabel='HouseAge'>,
        <Axes: xlabel='HouseAge', ylabel='HouseAge'>,
        <Axes: xlabel='AveRooms', ylabel='HouseAge'>,
        <Axes: xlabel='AveBedrms', ylabel='HouseAge'>,
        <Axes: xlabel='Population', ylabel='HouseAge'>,
        <Axes: xlabel='AveOccup', ylabel='HouseAge'>,
        <Axes: xlabel='Latitude', ylabel='HouseAge'>,
        <Axes: xlabel='Longitude', ylabel='HouseAge'>,
        <Axes: xlabel='MedHouseVal', ylabel='HouseAge'>],
```

```
[<Axes: xlabel='MedInc', ylabel='AveRooms'>,
 <Axes: xlabel='HouseAge', ylabel='AveRooms'>,
 <Axes: xlabel='AveRooms', ylabel='AveRooms'>,
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 <Axes: xlabel='Latitude', ylabel='AveRooms'>,
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 <Axes: xlabel='MedHouseVal', ylabel='AveRooms'>],
[<Axes: xlabel='MedInc', ylabel='AveBedrms'>,
 <Axes: xlabel='HouseAge', ylabel='AveBedrms'>,
 <Axes: xlabel='AveRooms', ylabel='AveBedrms'>,
 <Axes: xlabel='AveBedrms', ylabel='AveBedrms'>,
 <Axes: xlabel='Population', ylabel='AveBedrms'>,
 <Axes: xlabel='AveOccup', ylabel='AveBedrms'>,
 <Axes: xlabel='Latitude', ylabel='AveBedrms'>,
 <Axes: xlabel='Longitude', ylabel='AveBedrms'>,
 <Axes: xlabel='MedHouseVal', ylabel='AveBedrms'>],
[<Axes: xlabel='MedInc', ylabel='Population'>,
 <Axes: xlabel='HouseAge', ylabel='Population'>,
 <Axes: xlabel='AveRooms', ylabel='Population'>,
 <Axes: xlabel='AveBedrms', ylabel='Population'>,
 <Axes: xlabel='Population', ylabel='Population'>,
 <Axes: xlabel='AveOccup', ylabel='Population'>,
 <Axes: xlabel='Latitude', ylabel='Population'>,
 <Axes: xlabel='Longitude', ylabel='Population'>,
 <Axes: xlabel='MedHouseVal', ylabel='Population'>],
[<Axes: xlabel='MedInc', ylabel='AveOccup'>,
 <Axes: xlabel='HouseAge', ylabel='AveOccup'>,
 <Axes: xlabel='AveRooms', ylabel='AveOccup'>,
 <Axes: xlabel='AveBedrms', ylabel='AveOccup'>,
 <Axes: xlabel='Population', ylabel='AveOccup'>,
 <Axes: xlabel='AveOccup', ylabel='AveOccup'>,
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[<Axes: xlabel='MedInc', ylabel='Latitude'>,
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 <Axes: xlabel='AveBedrms', ylabel='Latitude'>,
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 <Axes: xlabel='Latitude', ylabel='Latitude'>,
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 <Axes: xlabel='MedHouseVal', ylabel='Latitude'>],
[<Axes: xlabel='MedInc', ylabel='Longitude'>,
 <Axes: xlabel='HouseAge', ylabel='Longitude'>,
```
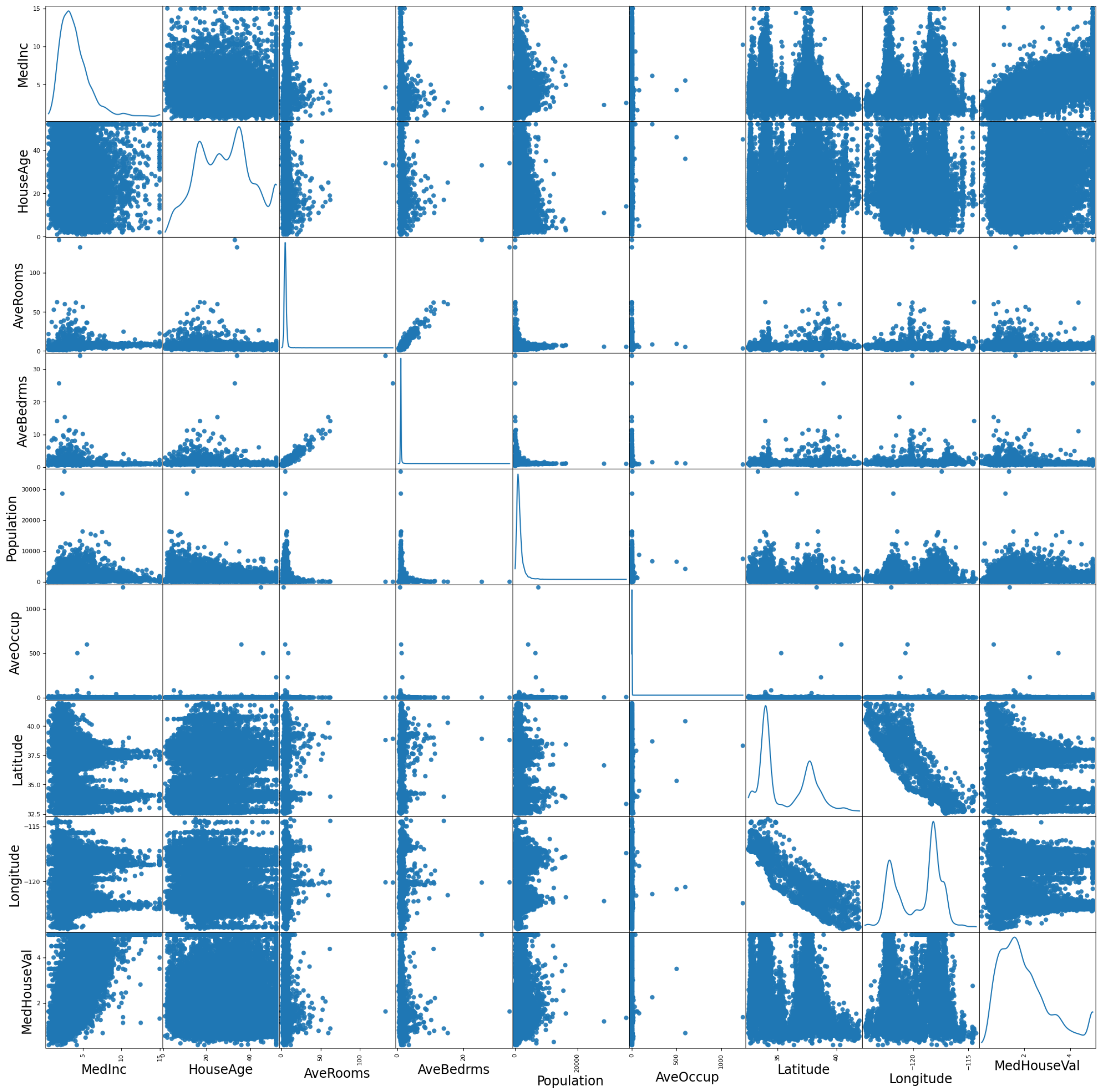

```
        <Axes: xlabel='AveRooms', ylabel='Longitude'>,
        <Axes: xlabel='AveBedrms', ylabel='Longitude'>,
        <Axes: xlabel='Population', ylabel='Longitude'>,
        <Axes: xlabel='AveOccup', ylabel='Longitude'>,
        <Axes: xlabel='Latitude', ylabel='Longitude'>,
        <Axes: xlabel='Longitude', ylabel='Longitude'>,
        <Axes: xlabel='MedHouseVal', ylabel='Longitude'>],
       [<Axes: xlabel='MedInc', ylabel='MedHouseVal'>,
        <Axes: xlabel='HouseAge', ylabel='MedHouseVal'>,
        <Axes: xlabel='AveRooms', ylabel='MedHouseVal'>,
        <Axes: xlabel='AveBedrms', ylabel='MedHouseVal'>,
        <Axes: xlabel='Population', ylabel='MedHouseVal'>,
        <Axes: xlabel='AveOccup', ylabel='MedHouseVal'>,
        <Axes: xlabel='Latitude', ylabel='MedHouseVal'>,
        <Axes: xlabel='Longitude', ylabel='MedHouseVal'>,
        <Axes: xlabel='MedHouseVal', ylabel='MedHouseVal'>]], dtype=object)
```

```
<Figure size 1200x1200 with 0 Axes>
```



1.1.1 1. Residual Sum of Squares (RSS)

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Where:

$$y_i \text{ is the actual value}$$

$$\hat{y}_i \text{ is the predicted value}$$

$$n \text{ is the number of observations}$$

1.1.2 2. Residual Standard Error (RSE)

$$RSE = \sqrt{\frac{RSS}{n - p - 1}}$$

Where:

$$RSS \text{ is the Residual Sum of Squares}$$

$$n \text{ is the number of observations}$$

$$p \text{ is the number of predictors (excluding the intercept)}$$

1.1.3 3. t-statistic

$$t = \frac{\hat{\beta}_j}{SE(\hat{\beta}_j)}$$

Where:

$$\hat{\beta}_j \text{ is the estimated coefficient for predictor } j$$

$$SE(\hat{\beta}_j) \text{ is the standard error of the estimated coefficient for predictor } j$$

1.1.4 4. p-value

$$p = 2 \cdot (1 - T(|t|, df))$$

Where:

$$t \text{ is the t-statistic}$$

$$df \text{ is the degrees of freedom, calculated as } n - p - 1$$

$$T \text{ is the CDF of the t-distribution}$$

1.2 Relevant Metrics

Task 1: Fill the missing parts (#TODO) of metric computations (1 Point Each, 3 Points)

[10]:
```
def compute_rss(y, y_pred):
    """
    Compute Residual Sum of Squares (RSS)
    y: array of true target values
    y_pred: array of predicted target values
    """
    rss = 0
    for y_elt, y_pred_elt in zip(y, y_pred):
        rss += math.pow(y_elt - y_pred_elt, 2)

    return rss


def compute_rse(y, y_pred, n, p):
    """
    Compute Residual Standard Error (RSE)
    y: array of true target values
    y_pred: array of predicted target values
    n: number of observations
    p: number of predictors
    """
    rss = compute_rss(y, y_pred)

    return math.sqrt(rss/(n-p-1))

def compute_pvalue(X, y, y_pred):
    '''
    Compute p-values for the coefficients of a linear regression model.

    X: array of features
    y: array of true target values
    y_pred: array of predicted target values
    return: p-values for each feature
    '''
    n, p = X.shape  # Number of observations (n) and number of predictors (p)

    # Compute RSS and RSE
    rss = compute_rss(y, y_pred)
    rse = compute_rse(y, y_pred, n, p)

    # # Add intercept (constant term) to the design matrix X
    X = np.c_[np.ones(n), X]

    # Calculate (X^T X)^-1
```

```
    XTX_inv = np.linalg.inv(np.dot(X.T, X))

    # Compute standard error (SE) for each coefficient
    se = np.sqrt(np.diagonal(rse ** 2 * XTX_inv))

    # Fit the model to compute the coefficients (betas)
    beta_hat = np.linalg.lstsq(X, y, rcond=None)[0]

    # Compute t-statistics for each coefficient
    t_stats = beta_hat / se

    degrees_of_freedom = n - p - 1

    # Compute p-values
    p_values = 2 * (1 - stats.t.cdf(np.abs(t_stats), df=degrees_of_freedom))

    return p_values
```

1.3 Linear Regression with single predictor

[11]:
```
# Select features and target
X = df[['AveRooms']]
#z-normalize the data for each column
X = (X - X.mean()) / X.std()
y = df['MedHouseVal']

# Split the data into training and testing sets (80% training, 20% testing)␣
 ↪with a fixing seed that ensures same split every time
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
 ↪random_state=42)

independent_scaler = StandardScaler()
X_train = independent_scaler.fit_transform(X_train)
X_test = independent_scaler.transform(X_test)

# Create a linear regression model
model_1 = LinearRegression()

# Train the model
model_1.fit(X_train, y_train)

# Get the coefficients
print(f"Intercept ( 0): {model_1.intercept_}")
print(f"Coefficients ( 1,  2): {model_1.coef_}")

#Compute RSS for training data
y_pred = model_1.predict(X_train)
```

```
# Compute RSS
rss = compute_rss(y_train, y_pred)

# Calculate R-squared
r_squared_all = model_1.score(X_train, y_train)

# Compute the p-value
p_value = compute_pvalue(X_train, y_train, y_pred)

# Display the coefficients and p-values in a DataFrame
coefficients = np.concatenate([[model_1.intercept_], model_1.coef_])
p_values = np.concatenate([ p_value])

display(pd.DataFrame(pd.DataFrame({'features': ['intercept'] + list(X.columns),␣
↪'coefficients': coefficients, 'p-values': p_values})))
print(f"RSS (test data): {rss}")
print(f"R-squared (test data): {r_squared_all}")
```

```
Intercept ( 0): 2.071946937378876
Coefficients ( 1,  2): [0.18323882]
```

	features	coefficients	p-values
0	intercept	2.071947	0.0
1	AveRooms	0.183239	0.0

```
RSS (test data): 21518.4672577652
R-squared (test data): 0.025117453148833846
```

Task 2: Use 'MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude', 'Longitude' as predictors. (2 Points)

[12]:
```
X_all = df[['MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude',␣
↪'Longitude']]
y = df['MedHouseVal']

# Split the data into training and testing sets (80% training, 20% testing)␣
↪with a fixing seed that ensures same split every time
X_train_all, X_test_all, y_train_all, y_test_all = train_test_split(X_all, y,␣
↪test_size=0.2, random_state=42)
independent_scaler = StandardScaler()
X_train_all = independent_scaler.fit_transform(X_train_all)
X_test_all = independent_scaler.transform(X_test_all)

# Fit the linear regression model
model_2 = LinearRegression()
model_2.fit(X_train_all, y_train_all)

# Predictions on the test set
```

```
y_pred_all = model_2.predict(X_test_all)

#Code this part
rss = compute_rss(y_test_all, y_pred_all)

# Calculate R-squared
r_squared_all = model_2.score(X_test_all, y_test_all)

# Compute the p-value
p_value = compute_pvalue(X_test_all, y_test_all, y_pred_all)

# Display the coefficients and p-values in a DataFrame
coefficients = np.concatenate([[model_2.intercept_], model_2.coef_])
p_values = np.concatenate([ p_value])

# pd.DataFrame({'features': ['intercept'] + list(X_all.columns), 'coefficients':
 ↪ coefficients, 'p-values': p_values})
display(pd.DataFrame({'features': ['intercept'] + list(X_all.columns),␣
 ↪'coefficients': coefficients, 'p-values': p_values}))
print(f"RSS (test data): {rss}")
print(f"R-squared (test data): {r_squared_all}")
```

```
    features  coefficients  p-values
0  intercept      2.071947  0.000000
1     MedInc      0.708366  0.000000
2   AveRooms      0.045937  0.010799
3   AveOccup     -0.037746  0.000000
4   HouseAge      0.124500  0.000000
5   Latitude     -0.977368  0.000000
6  Longitude     -0.931079  0.000000

RSS (test data): 2259.4509562626686
R-squared (test data): 0.5823077951522642
```

Task 3: Try model performance on different K values by using the code below, observe the effect of very large K values which one would you pick? (3 Points)

Answer: The optimal value is k=12, since it produces the lowest RSS & highest r-squared output.

[13]:
```
from sklearn.neighbors import KNeighborsRegressor
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error, r2_score

X_all = df[['MedInc', 'AveRooms', 'AveOccup', 'HouseAge', 'Latitude',␣
 ↪'Longitude']]
X_all = (X_all - X_all.mean()) / X_all.std()
y = df['MedHouseVal']
```

```
# Split the data into training and testing sets (80% training, 20% testing)␣
↪with a fixing seed that ensures same split every time
X_train_all, X_test_all, y_train_all, y_test_all = train_test_split(X_all, y,␣
↪test_size=0.2, random_state=42)
independent_scaler = StandardScaler()
X_train_all = independent_scaler.fit_transform(X_train_all)
X_test_all = independent_scaler.transform(X_test_all)

#Fit the KNN model (you can tune 'n_neighbors' for optimal performance)
knn_model = KNeighborsRegressor(n_neighbors=12)
knn_model.fit(X_train_all, y_train_all)

#Make predictions on the test set
y_pred_knn = knn_model.predict(X_test_all)

#Compute RSS and R-squared
rss_knn = compute_rss(y_test, y_pred_knn)
r2_knn = r2_score(y_test_all, y_pred_knn)
print(f"KNN Model RSS: {rss_knn}")
print(f"KNN Model R-squared: {r2_knn}")
```

```
KNN Model RSS: 1588.7586109921456
KNN Model R-squared: 0.7062949804877341
```

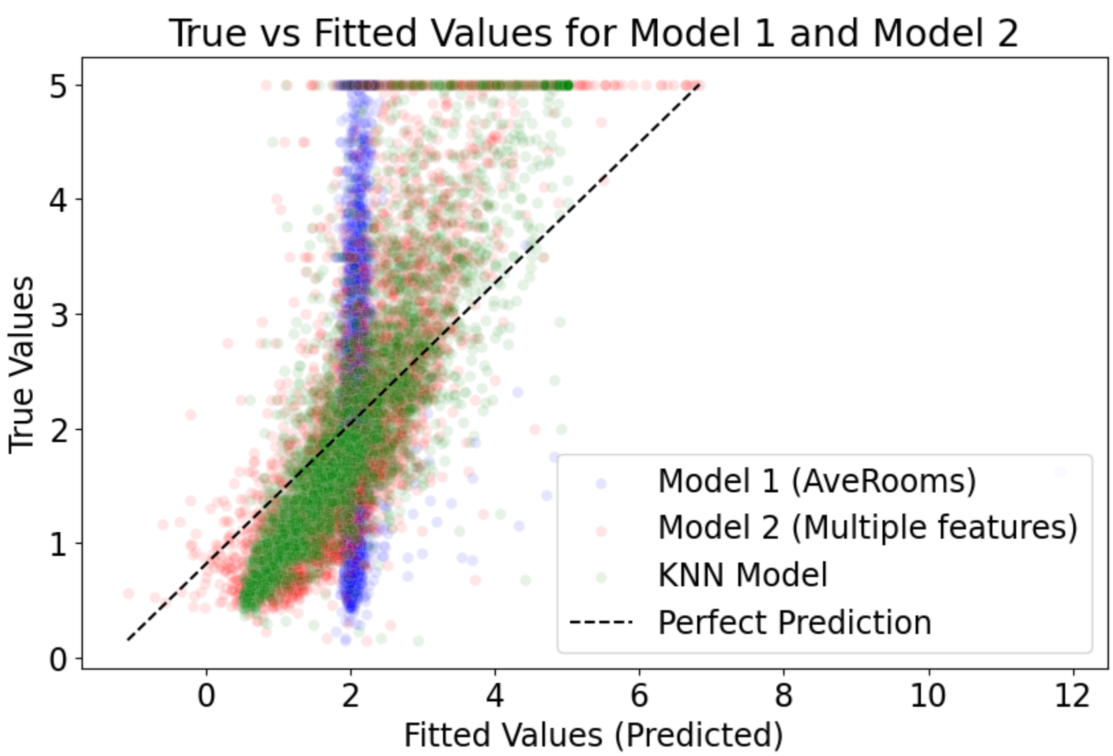
Task 4: Comment on R-squared and RSS values (1 Point)

R-Squared: This measures which percentage of the total variance in the data is explained by the model, the value lies between 1 & 0, the higher the better. RSS: The residual sum of squares measures how apart the predictions and the true values are, thus a good performing model has a low RSS.

1.4 Visualize results

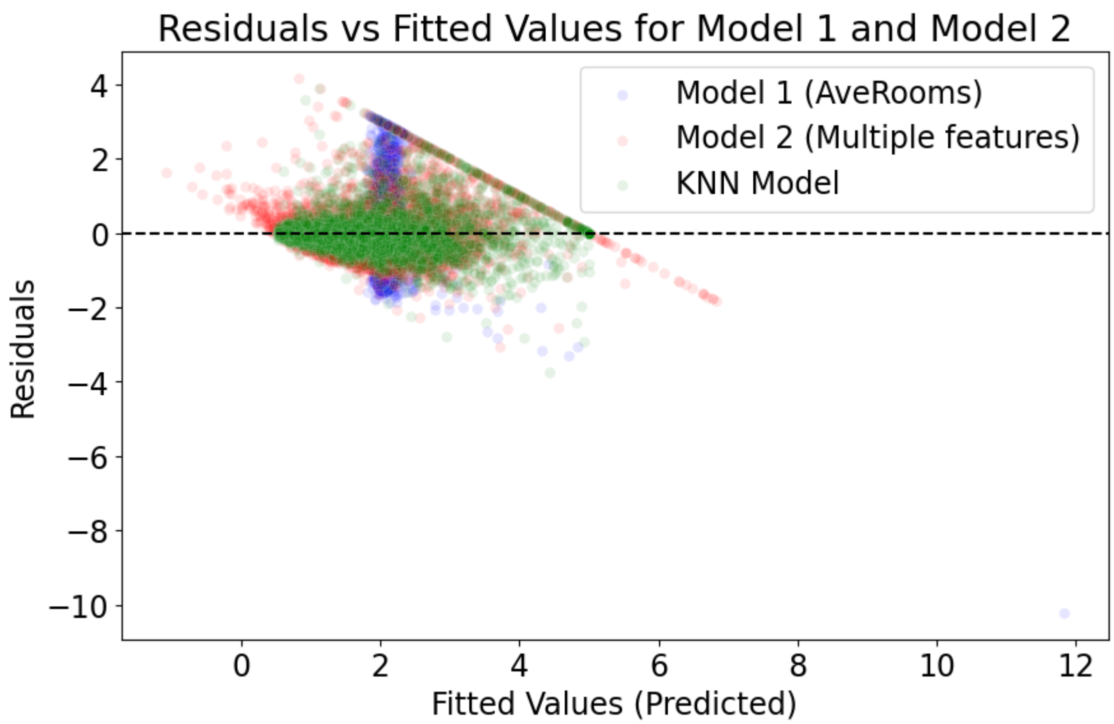
[14]:
```
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.metrics import mean_squared_error



# Make predictions on the test set for the first model (only AveRooms)
y_pred = model_1.predict(X_test)

# Make predictions on the test set for the second model (multiple features)
y_pred_all = model_2.predict(X_test_all)

# Make predictions using the KNN model
y_pred_knn = knn_model.predict(X_test_all)  # Use scaled features for KNN
```

```
plt.figure(figsize=(10, 6))

# Model 1: True vs Fitted (only AveRooms)
sns.scatterplot(x=y_pred, y=y_test, color='blue', label='Model 1 (AveRooms)',␣
↪alpha=0.1)

# Model 2: True vs Fitted (multiple features)
sns.scatterplot(x=y_pred_all, y=y_test_all, color='red', label='Model 2␣
↪(Multiple features)', alpha=0.1)

# KNN Model: True vs Fitted
sns.scatterplot(x=y_pred_knn, y=y_test_all, color='green', label='KNN Model',␣
↪alpha=0.1)

# Add perfect prediction line
plt.plot([min(y_pred_all), max(y_pred_all)], [min(y_test_all),␣
↪max(y_test_all)], color='black', linestyle='--', label='Perfect Prediction')

# Labels and title
plt.xlabel('Fitted Values (Predicted)')
plt.ylabel('True Values')
plt.title('True vs Fitted Values for Model 1 and Model 2')
plt.legend()
plt.show()
```



Task 5: Compute residuals (1 Point)

[15]:
```
### 2. Residuals vs Fitted

# Compute residuals
residuals_model_1 = y_test - y_pred
residuals_model_2 = y_test_all - y_pred_all
residuals_knn = y_test_all - y_pred_knn

plt.figure(figsize=(10, 6))

# Residuals vs Fitted for Model 1 (only AveRooms)
sns.scatterplot(x=y_pred, y=residuals_model_1, color='blue', label='Model 1␣
↪(AveRooms)', alpha=0.1)

# Residuals vs Fitted for Model 2 (multiple features)
sns.scatterplot(x=y_pred_all, y=residuals_model_2, color='red', label='Model 2␣
↪(Multiple features)', alpha=0.1)

# Residuals vs Fitted for KNN Model
sns.scatterplot(x=y_pred_knn, y=residuals_knn, color='green', label='KNN␣
↪Model', alpha=0.1)

# Add horizontal line at 0 (perfect prediction residual)
plt.axhline(0, color='black', linestyle='--')

# Labels and title
plt.xlabel('Fitted Values (Predicted)')
plt.ylabel('Residuals')
plt.title('Residuals vs Fitted Values for Model 1 and Model 2')
plt.legend()
plt.show()
```


Residuals vs Fitted Values for Model 1 and Model 2
Model 1 (AveRooms)
Model 2 (Multiple features)
KNN Model
Residuals
Fitted Values (Predicted)
4
2
0
−2
−4
−6
−8
−10
0
2
4
6
8
10
12