Problem 3:

a) $x \leq \xi$:

since $x \leq \xi$

$$f_1(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 \cdot 0$$
$$= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

$a_1 = \beta_0$
$b_1 = \beta_1$
$c_1 = \beta_2$
$d_1 = \beta_3$

b) $x > \xi$:

$$f_2(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)^3$$
$$= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^3 - \beta_4 3\xi x^2 + \beta_4 3\xi^2 x - \beta_4 \xi^3$$

$a_2 = \beta_0 - \beta_4 \xi^3$
$b_2 = \beta_1 + 3\beta_4 \xi^2$
$c_2 = \beta_2 - 3\beta_4 \xi$
$d_2 = \beta_3 + \beta_4$

c) $f_1(\xi) = f_2(\xi)$

$$f_1(\xi) = \beta_0 + \beta_1 \xi + \beta_2 \xi^2 + \beta_3 \xi^3$$

$$f_2(\xi) = \beta_0 - \beta_4 \xi^3 + (\beta_1 + 3\beta_4 \xi^2)\xi + (\beta_2 - 3\beta_4 \xi)\xi^2 + (\beta_3 + \beta_4)\xi^3$$
$$= \beta_0 - \beta_4 \xi^3 + \beta_1 \xi + 3\beta_4 \xi^3 + \beta_2 \xi^2 - 3\beta_4 \xi^3 + \beta_3 \xi^3 + \beta_4 \xi^3$$
$$= \beta_0 - \cancel{\beta_4 \xi^3} + \beta_1 \xi + \cancel{3\beta_4 \xi^3} + \beta_2 \xi^2 - \cancel{3\beta_4 \xi^3} + \beta_3 \xi^3 + \cancel{\beta_4 \xi^3}$$

$$\beta_0 + \beta_1 \xi + \beta_2 \xi^2 + \beta_3 \xi^3 = \beta_0 + \beta_1 \xi + \beta_2 \xi^2 + \beta_3 \xi^3 \quad \checkmark$$

d) $f_1(x)' = \beta_1 + 2\beta_2 x + 3\beta_3 x^2$

$$f_2(x)' = \beta_1 + 3\beta_4 \xi^2 + 2\beta_2 x - 6\beta_4 \xi x + 3\beta_3 x^2 + 3\beta_4 x^2$$

$$f_1(\xi)' = \beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2$$

$$f_2(\xi)' = \beta_1 + 3\beta_4 \xi + 2\beta_2 \xi - 6\beta_4 \xi^2 + 3\beta_3 \xi^2 + 3\beta_4 \xi^2$$
$$= \beta_1 + \cancel{3\beta_4 \xi} + 2\beta_2 \xi - \cancel{6\beta_4 \xi^2} + 3\beta_3 \xi^2 + \cancel{3\beta_4 \xi^2}$$
$$= \beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2$$

$$\beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2 = \beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2 \quad \checkmark$$

e) $f_1(x)'' = 2\beta_2 + 6\beta_3 x$

$$f_2(x)'' = 2\beta_2 - 6\beta_4 \xi + 6\beta_3 x + 6\beta_4 x$$

$$f_1(\xi)'' = 2\beta_2 + 6\beta_3 \xi$$
$$f_2(\xi)'' = 2\beta_2 - 6\beta_4 \xi + 6\beta_3 \xi + 6\beta_4 \xi$$
$$= 2\beta_2 - \cancel{6\beta_4 \xi} + 6\beta_3 \xi + \cancel{6\beta_4 \xi}$$
$$= 2\beta_2 + 6\beta_3 \xi$$

$$2\beta_2 + 6\beta_3 \xi = 2\beta_2 + 6\beta_3 \xi \quad \checkmark$$

2)

a) With $\lambda \to \infty$, the right part of the function dominates, and since $g^{(3)}$ is less restrictive than $g^{(4)}$, g_1 will fit the training data better, because it is more flexible. Thus g_1 will have the smaller training RSS.

b) Since g_1 is more flexible, it generalizes worse than g_2. Thus g_2 will have the smaller test RSS.

c) For $\lambda = 0$, the terms become equal, thus both curves will have the same training/test RSS.