CSC384h: Intro to Artificial Intelligence

Probabilistic Reasoning Temporal Models

- This material is covered in Chapter 15 (we cover a subset of this chapter)

Fahiem Bacchus, University of Toronto

Uncertainty

- In many practical problems we want to reason about a sequence of observations
  - Speech recognition
  - Robot localization
  - User attention
  - Medical monitoring

- Need to introduce time (or space) into our models

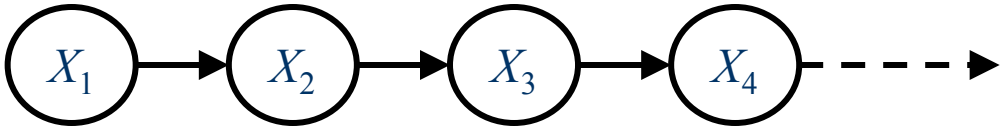
Markov Models

- Have one feature X (perhaps with a very large number of possible states). We want to track the probability of different values of X (the probability distribution over X) as it changes over time
- We make multiple copies of X, one for each time point i (we use a discrete model of time): X_1, X_2, X_3, …, X_n …,
- A Markov Model is specified by the two following assumptions

1. The current state Xt is conditionally independent of the earlier states given the previous state.
$$\Pr(X_t|X_1, \cdots X_{t-1}) = \Pr(X_t|X_{t-1})$$
2. The transitions between Xt_{-1} and Xt are determined by probabilities that do not change over time (stationary probabilities).
$\Pr(X_t|X_{t-1})$ is the same for all time points t

Markov Models

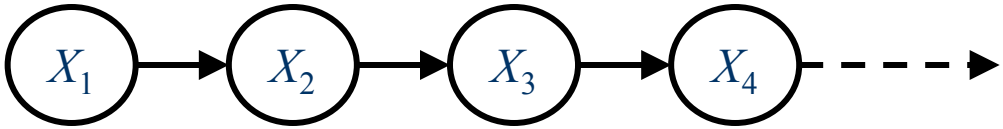
- These assumptions give rise to the following Bayesian Network



- $Pr(X_1,X_2,X_3,....) = Pr(X_1)Pr(X_2|X_1)Pr(X_3|X_2)$ … (Assumption 1)

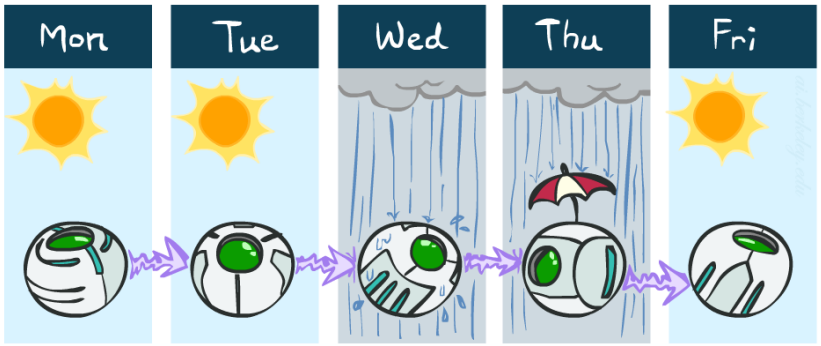
- And all the CPTs (except $Pr(X_1)$) are the same (Assumption 2)

Markov Models



- D-Separation also shows us that X_{t-1} is conditionally independent of X_{t+1}, X_{t+2}, … given X_t
  - The current state separates the past from the future.

Example Markov Chain Weather



- States: X = {rain, sun}

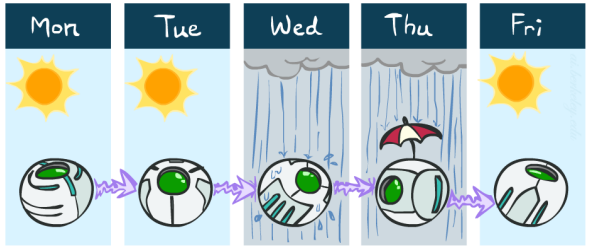
- Initial distribution:
  $Pr(X_1=sun) = 1.0$

CPT $Pr(X_t \mid X_{t-1})$:

X_{t-1}	X_t	$Pr(X_t \mid X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example Markov Chain Weather



- $Pr(X_1=sun) = 1.0$
- What is the probability distribution after one step, $Pr(X_2)$?
- Use summing out rule with X_1

$$P(X_2 = \text{sun}) = P(X_2 = \text{sun}|X_1 = \text{sun})P(X_1 = \text{sun}) + P(X_2 = \text{sun}|X_1 = \text{rain})P(X_1 = \text{rain})$$

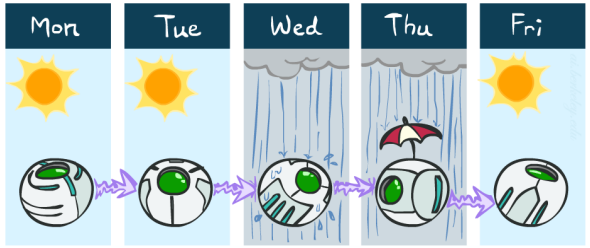
$$0.9 \cdot 1.0 + 0.3 \cdot 0.0 = 0.9$$

CPT Pr(Xt | Xt-1):

X_{t-1}	X_t	$Pr(X_t \mid X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example Markov Chain Weather



- What is the probability distribution on the day t, $Pr(X_t)$?
- Sum out over X_{t-1}

$$P(x_1) = \text{known}$$

$$P(x_t) = \sum_{x_{t-1}} P(x_{t-1}, x_t)$$
$$= \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$

Forward simulation
Compute $Pr(X_2)$ then $Pr(X_3)$ then $Pr(X_4)$ …

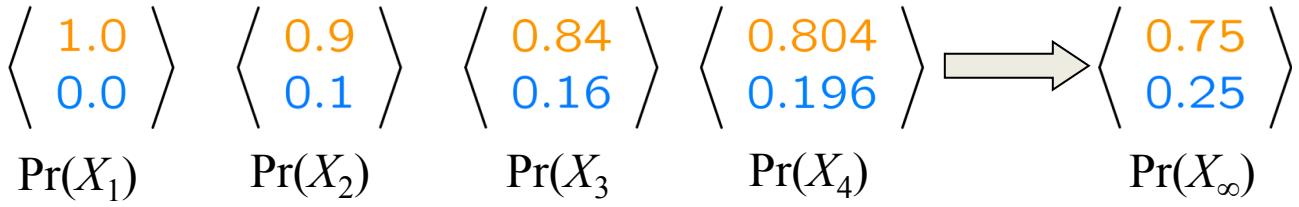
CPT Pr(Xt | Xt-1):

X_{t-1}	X_t	$Pr(X_t \mid X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

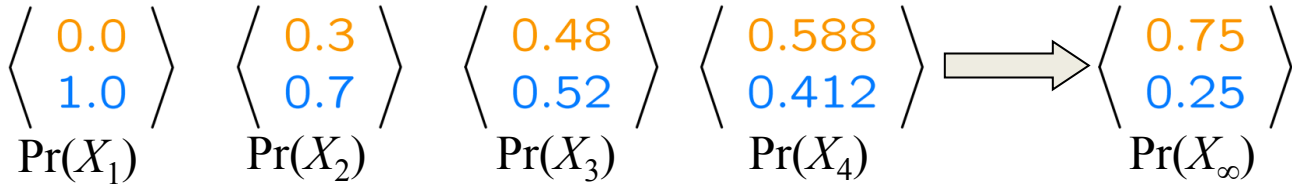
[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example Run of Forward Computation

- From initial observation of sun



- From initial observation of rain



- From yet another initial distribution $Pr(X_1)$:

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Stationary Distributions

- For most Markov chains:
  - Influence of the initial distribution gets less and less over time.
  - The distribution we end up in is independent of the initial distribution

- Stationary distribution:
  - The distribution we end up with is called the stationary distribution of the chain
  - It satisfies

$$\Pr_\infty(X = d') = \sum_{d \in Dom[X]} \Pr(X = d' | X = d) \Pr_\infty(X = d')$$

  - That is the stationary distribution does not change on an forward progression
  - We can compute it by solving these simultaneous equations (or by forward simulating the system many times; forward simulation is generally computationally more effective)

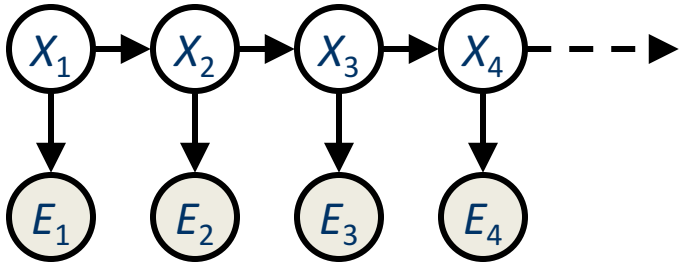
Web Link Analysis

- PageRank over a web graph
  - Each web page is a state
  - Initial distribution: uniform over pages
  - Transitions:
    - With prob. c, uniform jump to a random page
    - With prob. 1-c, follow a random outlink
- Stationary distribution
  - Will spend more time on highly reachable pages
  - E.g. many ways to get to the Acrobat Reader download page
  - Somewhat robust to link spam
  - Google 1.0 returned the set of pages containing all your keywords in decreasing rank, now all search engines use link analysis along with many other factors (rank actually getting less important over time)

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

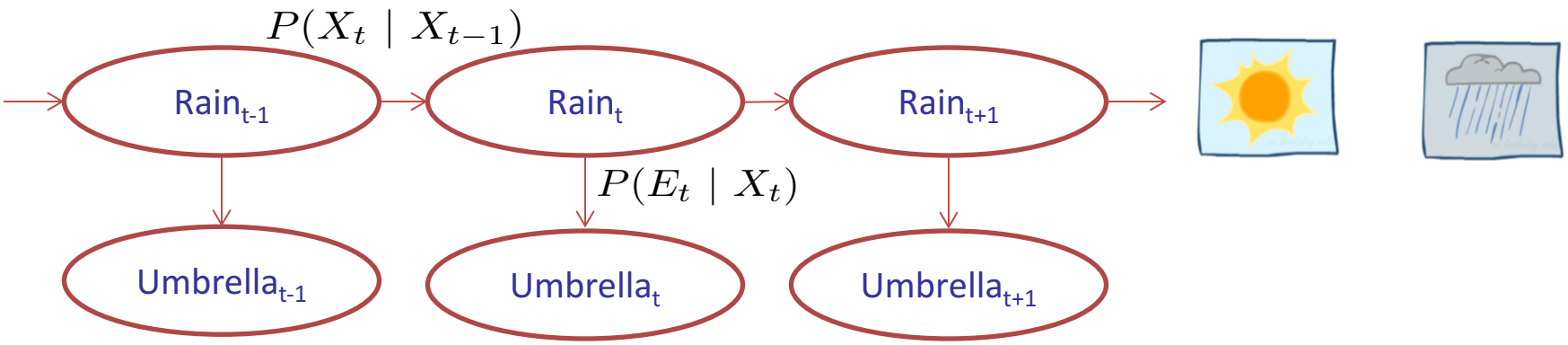
Hidden Markov Models

- Markov chains not so useful for most agents
  - Need observations to update your beliefs
- Hidden Markov models (HMMs)
  - Underlying Markov chain over states X
  - But you also observe outputs (effects) at each time step



[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Weather HMM



- An HMM is defined by:
  - Initial distribution: $Pr(X_1)$
  - Transitions: $Pr(X_t|X_{t-1})$
  - Emissions: $Pr(E_t|X_t)$

R_t	R_{t+1}	$P(R_{t+1}\|R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

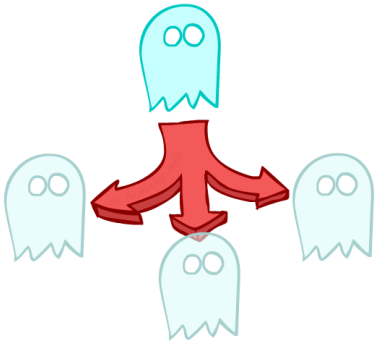
R_t	U_t	$P(U_t\|R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Ghostbusters HMM

- $Pr(X_1)$ = uniform

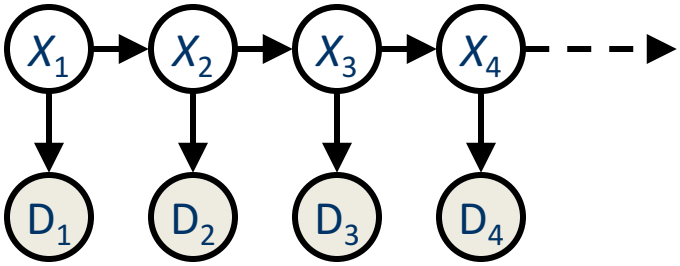
- $Pr(X|X')$ = usually move clockwise, but sometimes move in a random direction or stay in place



1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$P(X_1)$

- $Pr(D|X)$ = Observe distance to ghost (using a noisy sonar)

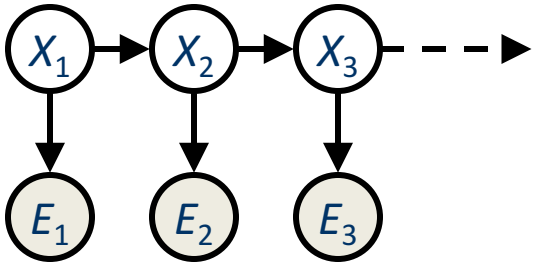


1/6	1/6	1/2
0	1/6	0
0	0	0

$P(X|X'=<1,2>)$

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

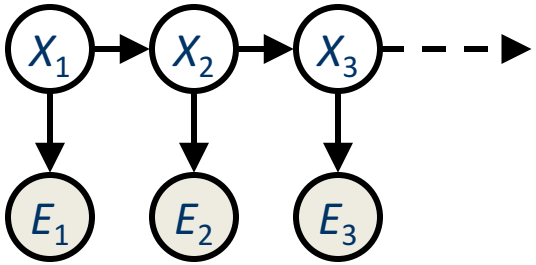
Joint Distribution of an HMM



Assumptions:

1. $\Pr(X_t|X_{t-1}, E_{t-1}, \cdots, E_1, X_1) = \Pr(X_t|X_{t-1})$
   The current state Xt is conditionally independent of the earlier states and evidence given the previous state.
2. $\Pr(X_t|X_{t-1})$ is the same for all time points t
   The transitions between X_{t-1} and X_t are determined by probabilities that do not change over time (stationary probabilities).
3. $\Pr(E_t|X_t, E_{t-1}, X_{t-1}, \cdots, E_1, X_1) = \Pr(E_t|X_t)$
   The current evidence is conditionally independent of all earlier states and evidence given the current state.

Joint Distribution of an HMM



Independencies

As with Markov Chains, the past is independent of the future (and vice versa) given the current state. (Easy to see by D-Separation)

But note that two evidence items are not independent, unless one of the intermediate states is known.

Real HMM Examples

- Speech recognition HMMs:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)
- Machine translation HMMs:
  - Observations are words (tens of thousands)
  - States are translation options
- Robot tracking:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Filtering / Monitoring

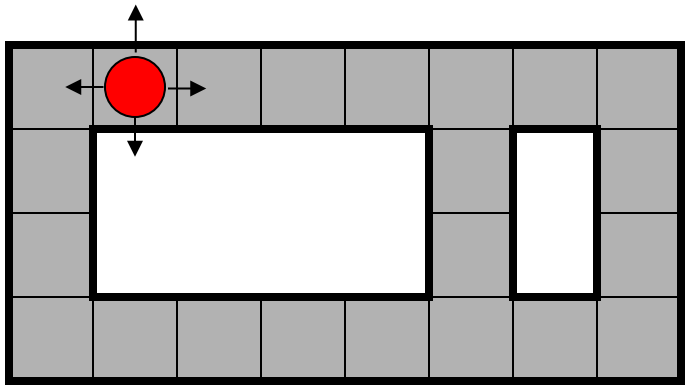
- Filtering, or monitoring, is the task of tracking $Pr(X_t \mid e_1, \ldots, e_t)$ over time: the probability distribution over feature X updated by all evidence accumulated so far.
- $P(X_1)$ is the initial distribution over feature X. (Usually we start with a uniform distribution, $Pr(X_1 = d)$ is the same for all d)
- As time passes, and we get observations, we update our distribution over X, i.e., we move from $\Pr(X_{t-1}|e_{t-1}, e_{t-2}, \cdots, e_1)$ to $\Pr(X_t|e_t, e_{t-1}, \cdots, e_1)$

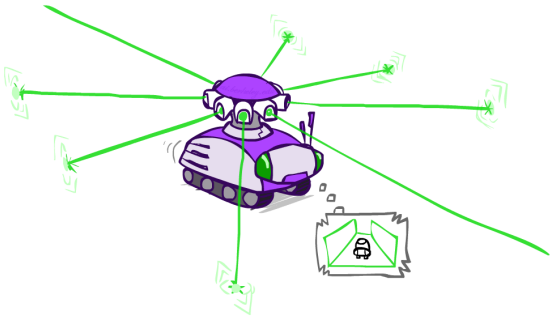
Filtering / Monitoring

- The equations for updating HMMs existed long before Bayes Nets. But the equations can be derived from Variable Elimination over the HMM Bayes Net.

Example: Robot Localization

Example from Michael Pfeiffer





t=0

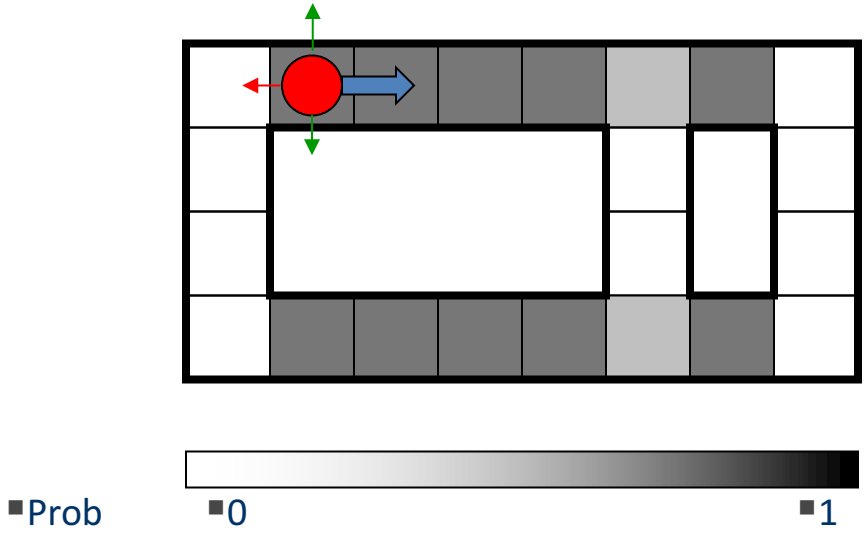
Sensor model: can read in which directions there is a wall, never more than 1 mistake

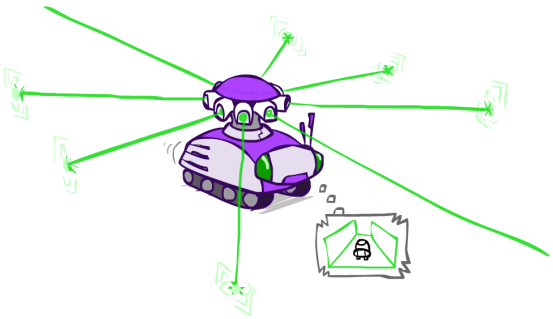
Motion model: Either executes the move, or the robot with low probabiity does not move at all. Cannot move in wrong direction

Initially uniform distribution over where robot is located—equally likely to be anywhere.

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Robot Localization





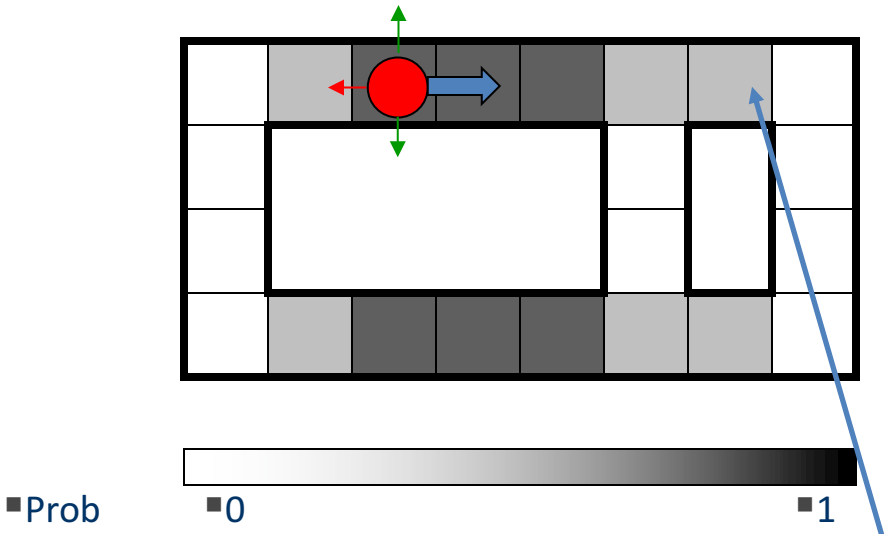
Initially don't know where you are. Observe a wall above and below, no wall to the left or right. Low probability of 1 mistake, 2 mistakes not possible

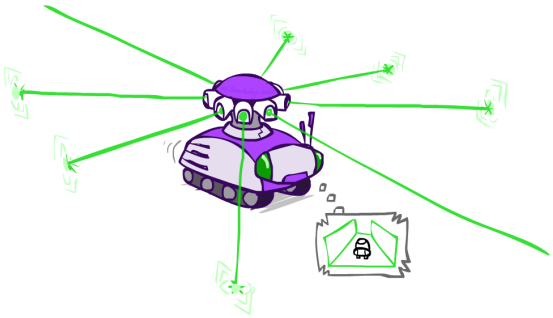
White: impossible to get this reading (more than one mistake)

Lighter grey: was possible to get the reading, but less likely because it required 1 mistake

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Robot Localization





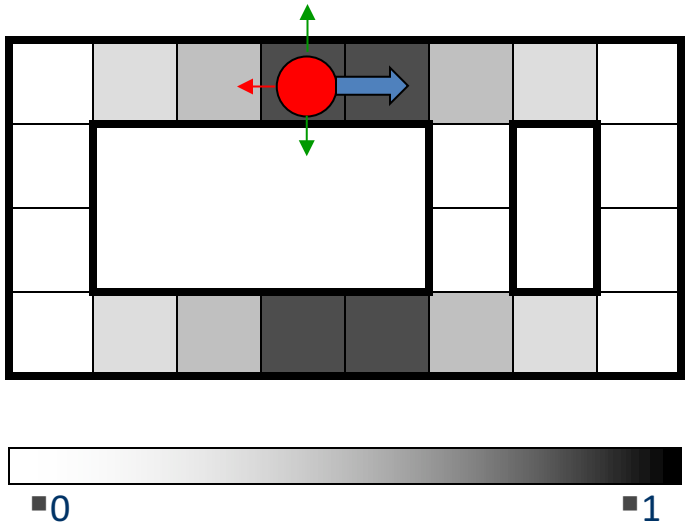
t=2

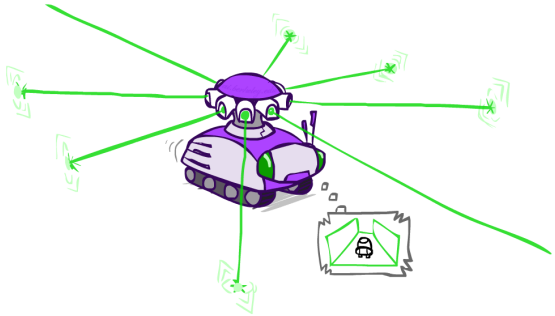
Move right. Low probability didn't move, else must have moved right.
Still observing wall above and below

can only be here if
(a) was at low probability square to the left
(b) was at this square and action didn't work.

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Robot Localization

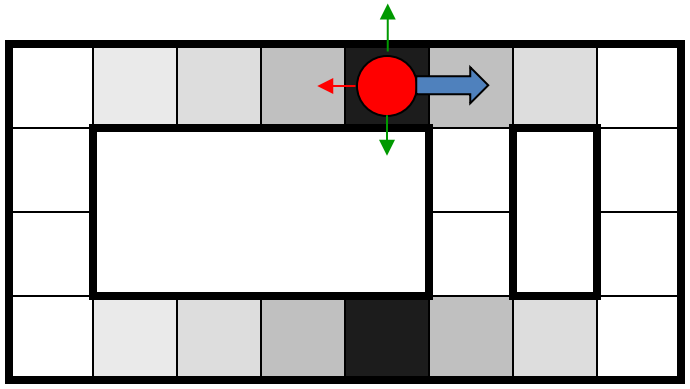


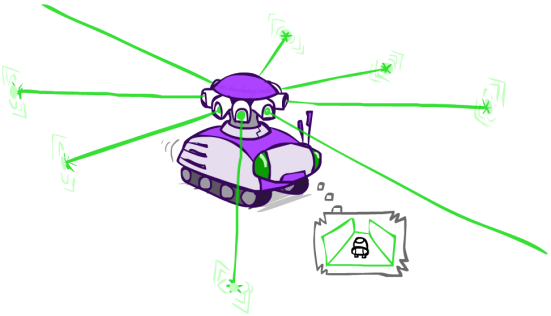


t=3

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Robot Localization



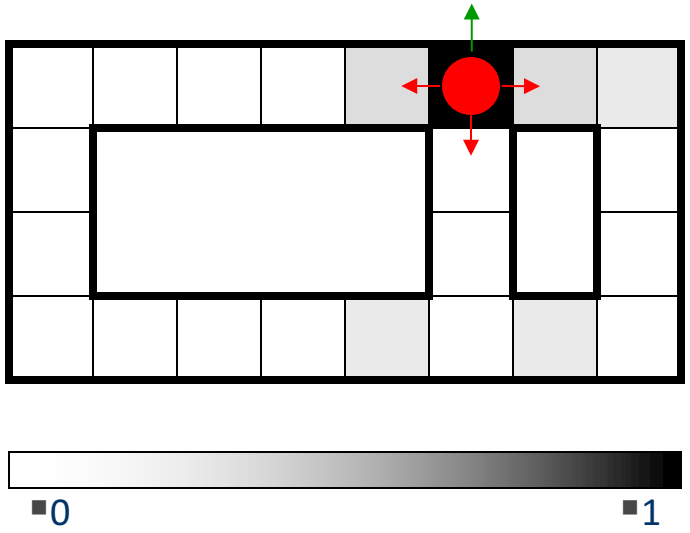


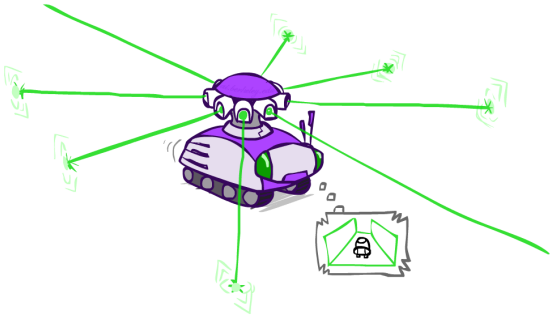
Prob 0 1

t=4

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Robot Localization





t=5

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Update rules for HMMs

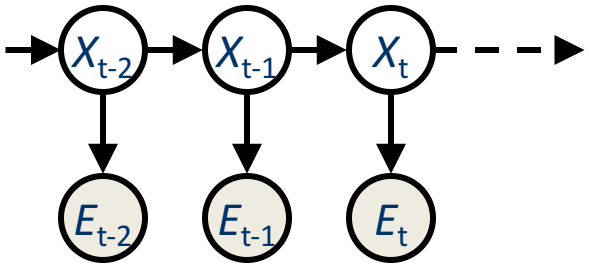
- HMMs are a type of Bayes Net so we can use Variable Elimination to compute $\Pr(X_t|e_t, e_{t-1}, \cdots, e_1)$ at all time points t.

- If we compute $\Pr(X_{t-1}|e_{t-1}, \cdots, e_1)$ using VE we will see that most of the information we need to compute the X_t has already been done.

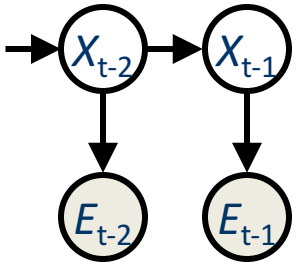
- Reusing this information allows us to derive simple update rules.

VE for $\Pr(X_{t-1}|e_{t-1},\cdots,e_1)$

Relevance reasoning shows that all future variables $X_t, E_t, X_{t+1}, E_{t+1}, \cdots$ are irrelevant. ~~X_{t-1} is the query, and e_{t-1} is evidence: Only ancestors of X_{t-1}~~



Relevance



VE for $\Pr(X_{t-1}|e_{t-1},\cdots,e_1)$

Order of Elimination X_1, …, X_{t-1} (all E_i variables have been instantiated with values e_i)

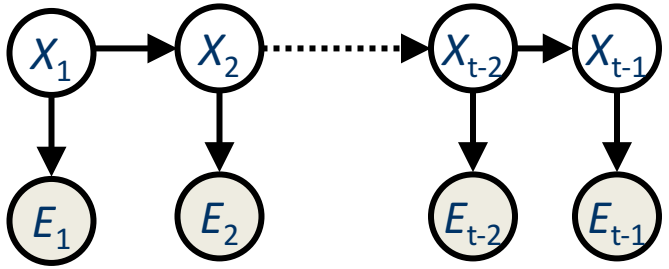
X_1: $\Pr(X_1)\Pr(e_1|X_1)\Pr(X_2|X_1)$

X_2: $\Pr(e_2|X_2)\Pr(X_3|X_2)$

…

X_{t-2}: $\Pr(e_{t-2}|X_{t-2})\Pr(X_{t-1}|X_{t-1})$

X_{t-1}: $\Pr(e_{t-1}|X_{t-1})$



VE for $\Pr(X_{t-1}|e_{t-1},\cdots,e_1)$

Summing out X_1 we get a factor over X_2, summing out over X_2 we get a factor over X_3 … summing out X_{t-2} we get a factor over X_{t-1}

X_1: $\Pr(X_1)\Pr(e_1|X_1)\Pr(X_2|X_1)$

X_2: $\Pr(e_2|X_2)\Pr(X_3|X_2)\ F_2(X_2)$

…

X_{t-2}: $\Pr(e_{t-2}|X_{t-2})\Pr(X_{t-1}|X_{t-1})\ F_{t-2}(X_{t-2})$

X_{t-1}: $\Pr(e_{t-1}|X_{t-1})\ F_{t-1}(X_{t-1})$

VE for $\Pr(X_{t-1}|e_{t-1},\cdots,e_1)$

X_1: $\Pr(X_1)\Pr(e_1|X_1)\Pr(X_2|X_1)$
X_2: $\Pr(e_2|X_2)\Pr(X_3|X_2)\,F_2(X_2)$
…
X_{t-2}: $\Pr(e_{t-2}|X_{t-2})\Pr(X_{t-1}|X_{t-1})\,F_{t-2}(X_{t-2})$
X_{t-1}: $\Pr(e_{t-1}|X_{t-1})\,F_{t-1}(X_{t-1})$

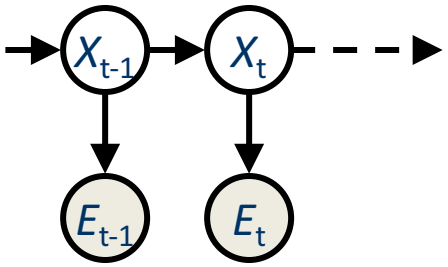
$\Pr(X_{t-1}|e_{t-1},\cdots,e_1) = normalize(\Pr(e_{t-1}|X_{t-1})\,F_{t-1}(X_{t-1}))$

At time step t we have already computed this vector of values for time step t-1. (One number for each value of X_{t-1})

Base case is $\Pr(X_1|e_1) = normalize(\Pr(e_1|X_1)\Pr(X_1))$

VE for $\Pr(X_t|e_{t-1},\cdots,e_1)$

Update Rule #1: Time has passed but new observation not yet made.



X_1: $\Pr(X_1)\Pr(e_1|X_1)\Pr(X_2|X_1)$

X_2: $\Pr(e_2|X_2)\Pr(X_3|X_2)$

…

X_{t-2}: $\Pr(e_{t-2}|X_{t-2})\Pr(X_{t-1}|X_{t-1})$

X_{t-1}: $\Pr(e_{t-1}|X_{t-1})\Pr(X_t|X_{t-1})$

X_t:

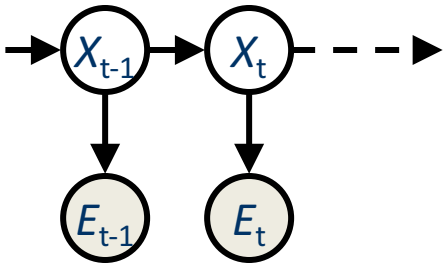
VE buckets are the same as before except

a) Add a bucket for X_t (this is initially empty)

b) Extra factor $\Pr(X_t|X_{t-1})$ in bucket for X_{t-1}

VE for $\Pr(X_t|e_{t-1},\cdots,e_1)$

Sum out variables as before obtaining exactly the same factors



X_1: $\Pr(X_1)\Pr(e_1|X_1)\Pr(X_2|X_1)$

X_2: $\Pr(e_2|X_2)\Pr(X_3|X_2)\,F_2(X_2)$

…

X_{t-2}: $\Pr(e_{t-2}|X_{t-2})\Pr(X_{t-1}|X_{t-1})\,F_{t-2}(X_{t-1})$

X_{t-1}: $\Pr(e_{t-1}|X_{t-1})\,F_{t-1}(X_{t-1})\Pr(X_t|X_{t-1})$

X_t: $F_t(X_t)$

We obtain a factor over X_t

$$F_t(X_t) = \sum_{d\in Dom[X_{t-1}]} \Pr(e_{t-1}|X_{t-1})\,F_{t-1}(X_{t-1})\Pr(X_t|X_{t-1})$$

VE for $\Pr(X_t|e_{t-1},\cdots,e_1)$

$$F_t(X_t) = \sum_{d\in Dom[X_{t-1}]} \Pr(e_{t-1}|X_{t-1})\, F_{t-1}(X_{t-1}) \Pr(X_t|X_{t-1})$$

$$= \sum_{d\in Dom[X_{t-1}]} \alpha \Pr(X_{t-1}|e_{t-1},\cdots,e_1) \Pr(X_t|X_{t-1})$$

Since we already computed that

$$\Pr(X_{t-1}|e_{t-1},\cdots,e_1) = \text{normalize}(\Pr(e_{t-1}|X_{t-1})\, F_{t-1}(X_{t-1}))$$
$$= \Pr(e_{t-1}|X_{t-1})\, F_{t-1}(X_{t-1}))\,/\alpha$$

(α was the normalization constant)

Update rule for $\Pr(X_t|e_{t-1},\cdots,e_1)$

Finally:

$$\Pr(X_t|e_{t-1},\cdots,e_1)$$
$$= \text{normalize}(F_t(X_t))$$
$$= \text{normalize}(\sum_{d\in Dom[X_{t-1}]} \alpha \Pr(X_{t-1}|e_{t-1},\cdots,e_1)\Pr(X_t|X_{t-1}))$$
$$= \text{normalize}(\sum_{d\in Dom[X_{t-1}]} \Pr(X_{t-1}|e_{t-1},\cdots,e_1)\Pr(X_t|X_{t-1}))$$

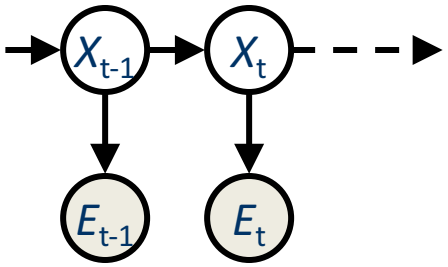
We can drop α because we are normalizing.

PROGRESS IN TIME

$$\Pr(X_t|e_{t-1},\cdots,e_1) = \text{normalize}(\sum_{d\in Dom[X_{t-1}]} \Pr(X_{t-1}|e_{t-1},\cdots,e_1)\Pr(X_t|X_{t-1}))$$

VE for $\Pr(X_t|e_t,\cdots,e_1)$

Now we deal with new evidence e_t
Variable Elimination will look similar except



X_1: $\Pr(X_1)\Pr(e_1|X_1)\Pr(X_2|X_1)$
X_2: $\Pr(e_2|X_2)\Pr(X_3|X_2)\,F_2(X_2)$
…
X_{t-2}: $\Pr(e_{t-2}|X_{t-2})\Pr(X_{t-1}|X_{t-1})\,F_{t-2}(X_{t-1})$
X_{t-1}: $\Pr(e_{t-1}|X_{t-1})\,F_{t-1}(X_{t-1})\Pr(X_t|X_{t-1})$
X_t: $F_t(X_t)\Pr(e_t|X_t)$

But we add a new factor $\Pr(e_t|X_t)$ to the X_t bucket then normalize

VE for $\Pr(X_t|e_t,\cdots,e_1)$

Hence $\Pr(X_t|e_{t-1},\cdots,e_1) = normalize(F_t(X_t)\Pr(e_t|X_t))$

We already saw that

$$\text{normalize}(F_t(X_t)) = \Pr(X_t|e_{t-1},\cdots,e_1)$$

So

$$F_t(X_t) = \alpha\Pr(X_t|e_{t-1},\cdots,e_1)$$

And

$$\Pr(X_t|e_{t-1},\cdots,e_1) = normalize(\alpha\Pr(X_t|e_{t-1},\cdots,e_1)\Pr(e_t|X_t))$$

We can remove α as we are normalizing

Update rule for $\Pr(X_t|e_t, \cdots, e_1)$

Finally:

Observe

$$\Pr(X_t|e_t, \cdots, e_1) = \text{normalize}(\Pr(X_t|e_{t-1}, \cdots, e_1)\Pr(e_t|X_t))$$

HMM update rules, recap.

Initial

$\Pr(X_1) = initial\ distribution$ (usually uniform)

Observe

$\Pr(X_t|e_t, \cdots, e_1)$ =

normalize($\Pr(X_t|e_{t-1}, \cdots, e_1)$ $\Pr(e_t|X_t)$)

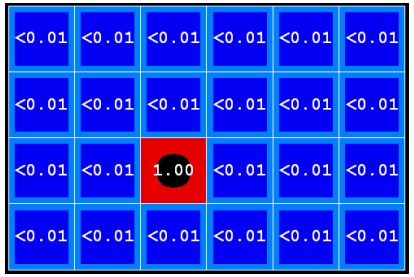
PROGRESS IN TIME

$\Pr(X_t|e_{t-1}, \cdots, e_1)$ =

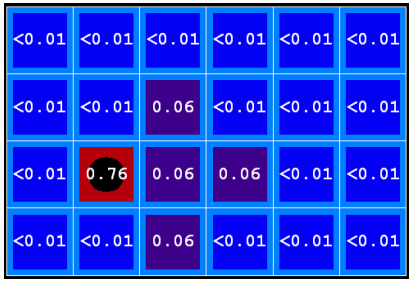
normalize($\sum_{d \in Dom[X_{t-1}]} \Pr(X_{t-1}|e_{t-1}, \cdots, e_1)$ $\Pr(X_t|X_{t-1})$)

Example: Passage of Time

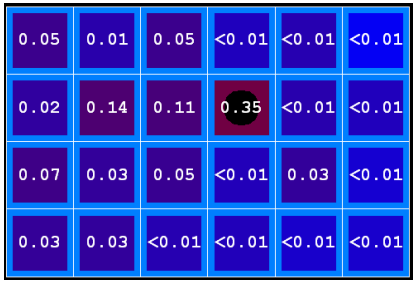
- As time passes, uncertainty "accumulates"
  - (Transition model: ghosts usually go clockwise)



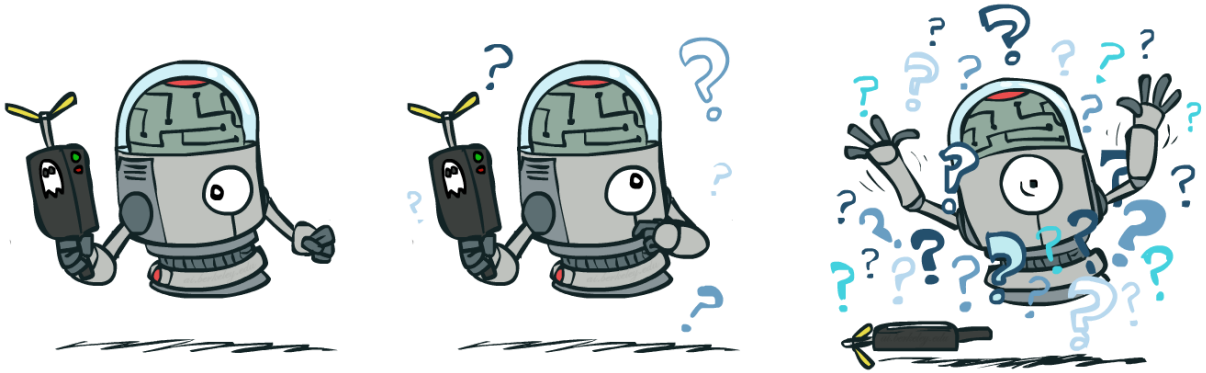
T = 1



T = 2



T = 5



[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Example: Observation

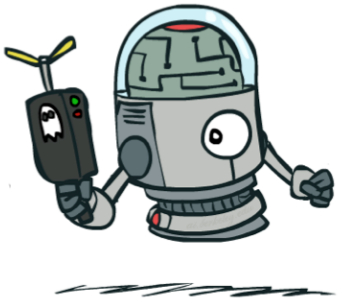
- As we get observations, beliefs get reweighted, uncertainty "decreases"

0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

Before observation

<0.01	<0.01	<0.01	<0.01	0.02	<0.01
<0.01	<0.01	<0.01	0.83	0.02	<0.01
<0.01	<0.01	0.11	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

After observation



$$\Pr(X_t|e_t, \dots, e_1) \propto \Pr(e_t|X_t) \Pr(X_{t-1}|e_{t-1}, \dots, e_1)$$

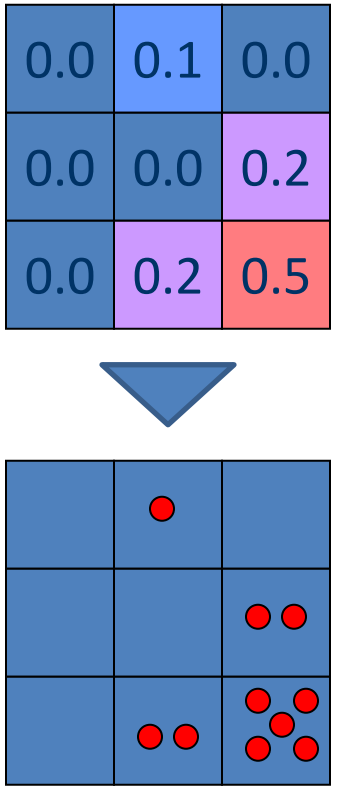
[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Approximate Inference

- Often the Bayes net is not solvable by Variable Elimination: under any ordering of the variables we end up with a factor that is too large to compute (or store).
- Since we are trying to compute a probability (which only predicts the likelihood of an event occurring) it is natural to consider approximating answer.
- Approximation can also be used in HMMs.

Particle Filtering

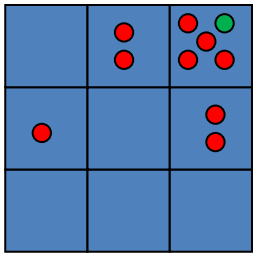
- Filtering: approximate solution
- Sometimes |X| is too big to use exact inference
  - |X| may be too big to even store B(X)
  - E.g. X is continuous
- Solution: approximate inference
  - Track samples of X, Pr(X)
  - Samples are called particles, each sample specifies one particular value that X might have.
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not distributions over X
- This is how robot localization works in practice



[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Representation: Particles

- We approximate Pr(X) by a list of N particles. Each particle corresponds to single value for X: (1,1), (1,2), (2,2), (2,3) etc. These particles serve as a representation for Pr(X).
  - Generally, N << |X|
  - So the list of particles have size more manageable than storing all possible values for feature X.



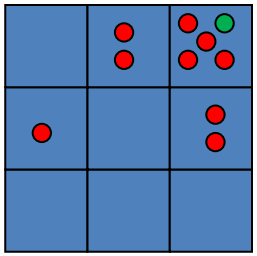
Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

- Pr(X=d) approximated by number of particles with value d (note than more than one particle can have the same value.
  - For many values X=d we might have Pr(X=d) = 0!
  - More particles, more accuracy
- Initially all particles have equal weight of 1
- In our examples the values for X are positions of the ghost on in the pacman grid.

Representation: Particles

Pr(X=d) ≈ (aproximately equal)

(#particles with value d)/
(Total # of particles)



Particles:

(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Filtering with Particles

- We want to approximate exact HMM inference with a collection of particles.

- This means that we must at every stage we must progress the particles to the next time step so that they approximate $\Pr(X_t|e_{t-1}, \cdots, e_1)$

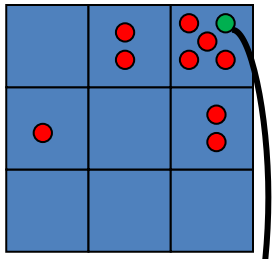
- And then update the particles to account for the next observation so that they approximate $\Pr(X_t|e_t, \cdots, e_1)$

Particle Filtering: Elapse Time

- Each particle is moved to the next step by sampling its next position from the transition model.
  - Each particle **p** is "asserting" that X_t**=p.** If X_t=**p** then it will transition to X_{t+1}=d with probability
    $$\Pr(X_{t+1} = d' | X_t = \boldsymbol{p})$$
  - Hence if we sample randomly from this distribution we will get a new particle
  - For each particle **p** we replace **p** with a new particle drawn randomly from the distribution
    $$\Pr(X_{t+1} | X_t = \boldsymbol{p})$$
    - Here, most samples move clockwise, but some move in another direction or stay in place
- This captures the passage of time
  - If enough samples, close to exact values before and after (consistent)

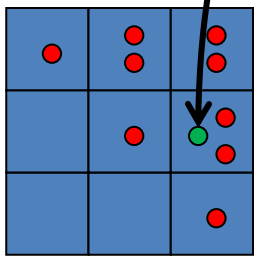
Particles :

(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)



Particles:

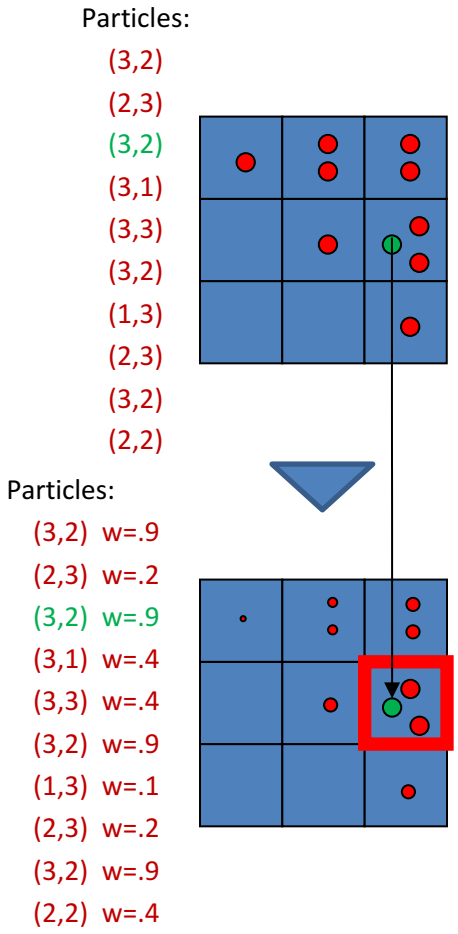
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)



Particle Filtering: Observe

- Observations are fixed, we don't sample these.
  - Instead we use the observation to reweight the particles.
  - Particles that are unlikely given the observation become less likely.
  - So for each particle **p** we set wt(**p**) = **Pr(e|X=p)**

    That is, **p** asserts that X has value **p**. If it did then the probability we would see observation **e** would be **Pr(e|X=p)**



Particle Filtering: Observe

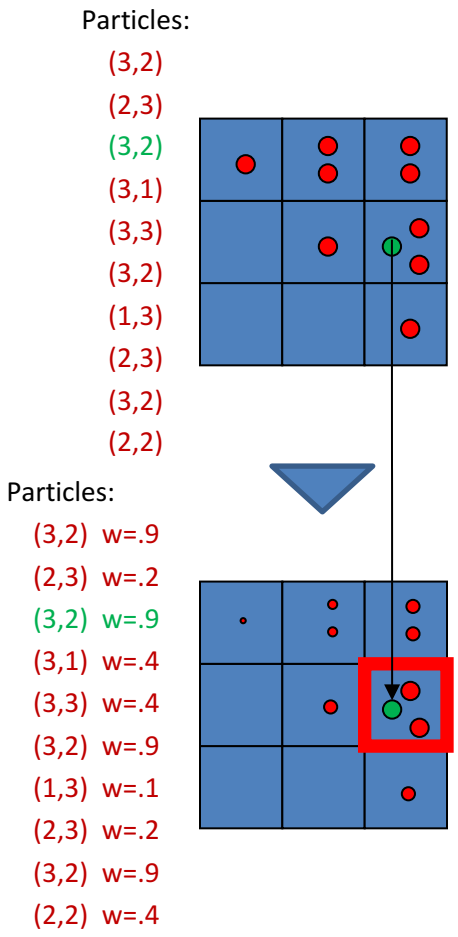
- But now with weighted particles our approximate probabilities
  Pr(X=d) = #particles with value d/
  Total number of particles

  No longer makes sense.

- We could instead use

$$\frac{\sum_{\boldsymbol{p}\ with\ value\ d} \boldsymbol{wt(p)}}{\sum_{all\ \boldsymbol{p}} wt(p)}$$

- As our approximate probability. But then we would also have to transfer the particle weights when time elapses. And we would accumulate many very low weight particles.



Particle Filtering: Back to unit weights

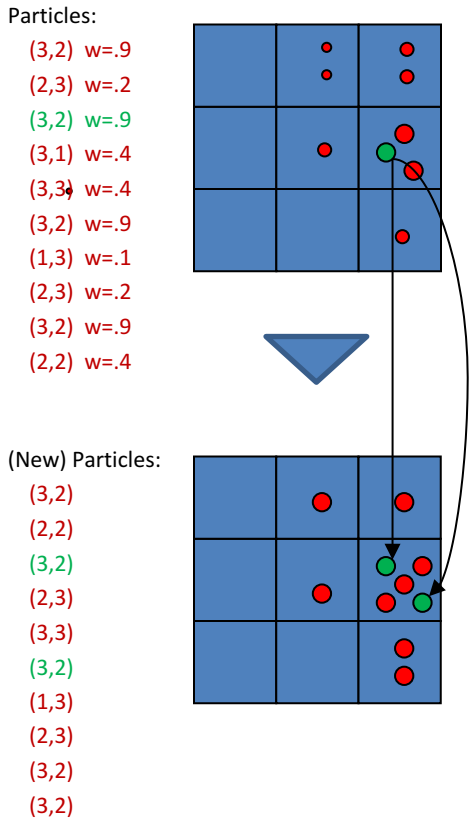
- Rather than do this we convert back to weight 1 particles by resampling.

- If we are using N particles, we sample N weighted particles from our set of weighted particles. These is newly chosen weighted particles are given unit weight and used in the next time step. This is called **resampling** the particles.

- Specifically, for N times we select any particle **p** from our weighted particles with probability

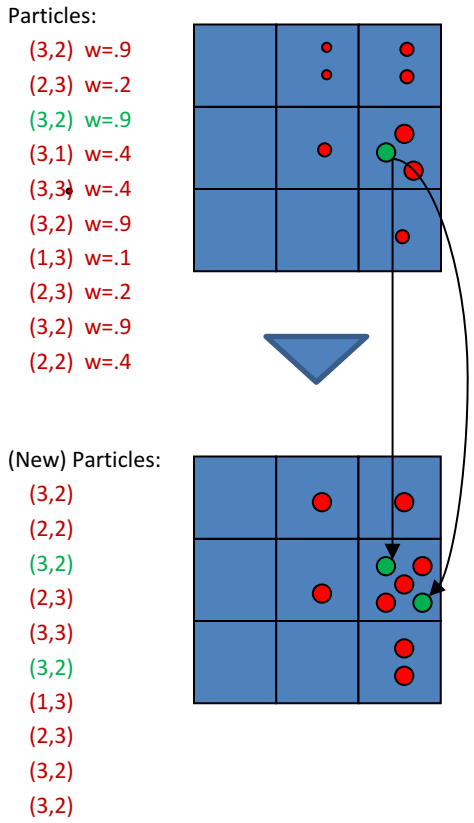
$$wt(\boldsymbol{p}) / \sum_{all\ p'} wt(p')$$

- E.g., we normalize the weights of all particles, and then select randomly from resulting distribution.



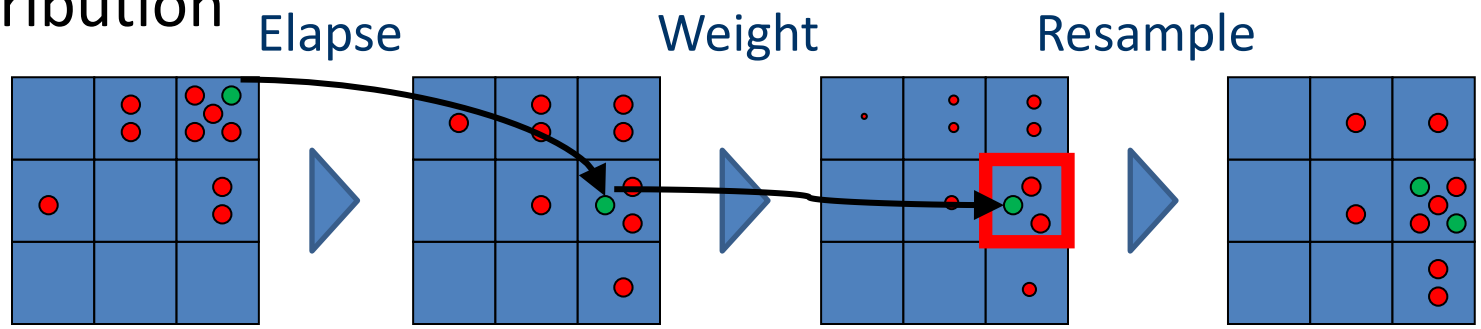
Particle Filtering: Back to unit weights

- E.g., particles = {p1, p2, p3, p4, p5}
- wt(p1) = 0.1, wt(p2) = 0, wt(p3) = .8, wt(p4) = .8, wt(p5) = .5
- Normalize the weights: wt(p1)=0.0455, wt(p2)=0, wt(p3)=0.36, wt(p4)=0.36, wtp5) = 0.23
- Now we sample this set of particles 5 times selecting p1 with probability 0.0455, p2 with probability 0, p3 with probability 0.36, p4 with probability 0.36, and p5 with probability 0.23
- Note that p2 drops out—it will never be sampled. This way we an get rid of samples with very low probability.



Recap: Particle Filtering

- Particles: track samples of states rather than an explicit distribution



- Particles:
  - (3,3)
  - (2,3)
  - (3,3)
  - (3,2)
  - (3,3)
  - (3,2)
  - (1,2)
  - (3,3)
  - (3,3)
  - (2,3)

- Particles:
  - (3,2)
  - (2,3)
  - (3,2)
  - (3,1)
  - (3,3)
  - (3,2)
  - (1,3)
  - (2,3)
  - (3,2)
  - (2,2)

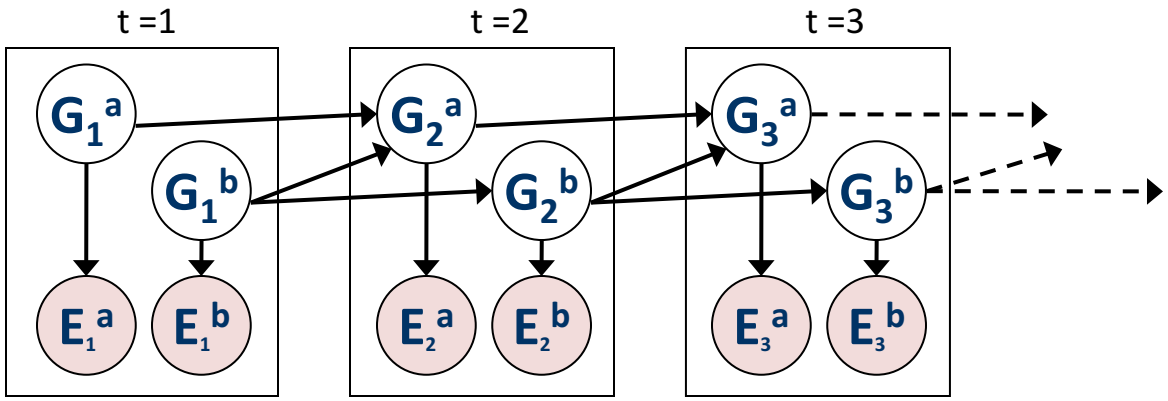
- Particles:
  - (3,2) w=.9
  - (2,3) w=.2
  - (3,2) w=.9
  - (3,1) w=.4
  - (3,3) w=.4
  - (3,2) w=.9
  - (1,3) w=.1
  - (2,3) w=.2
  - (3,2) w=.9
  - (2,2) w=.4

- (New) Particles:
  - (3,2)
  - (2,2)
  - (3,2)
  - (2,3)
  - (3,3)
  - (3,2)
  - (1,3)
  - (2,3)
  - (3,2)
  - (3,2)

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Dynamic Bayes Nets (DBNs)

- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables from time *t* can condition on those from *t-1*

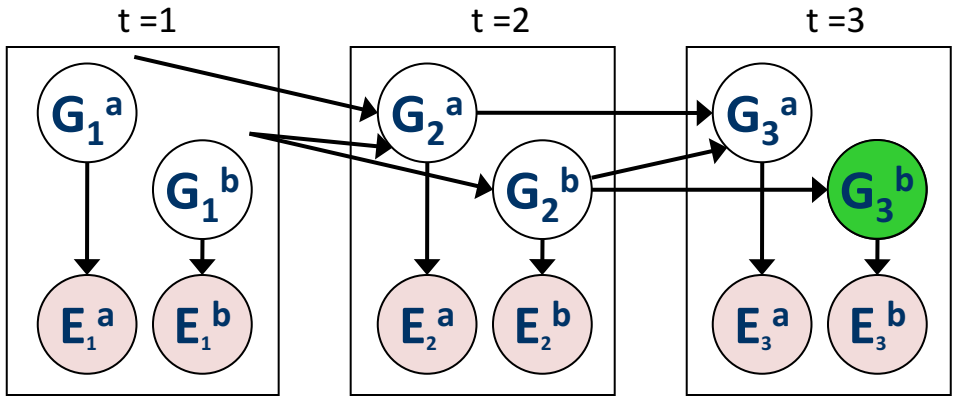


- Dynamic Bayes nets are a generalization of HMMs

[Slide created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.

Exact Inference in DBNs

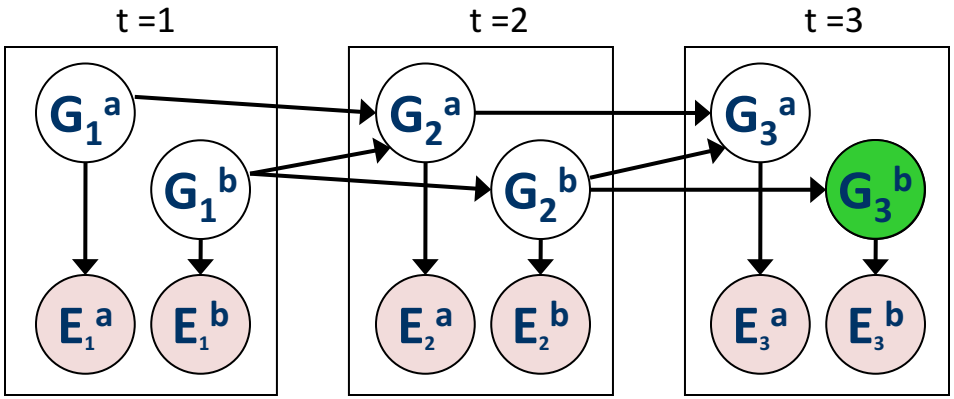
- Variable elimination applies to dynamic Bayes nets
- Procedure: "unroll" the network for T time steps, then eliminate variables until $P(X_T | e_{1:T})$ is computed



- Online belief updates: Eliminate all variables from the previous time step; store factors for current time only (just like HMMs but the updates are not as simple)

Particle Filtering in DBN

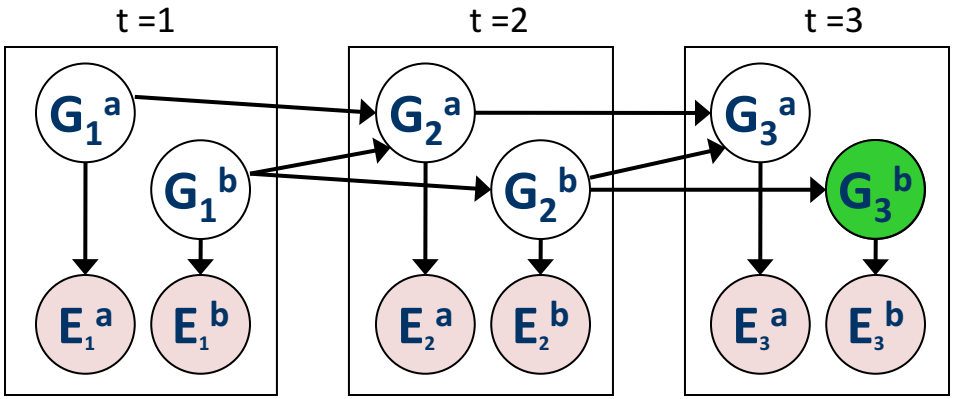
- As with HMMs we can have a set of particles. Now each particle must represent the state of all variables in the Bayes net at time step **t** except for the evidence variables (as these will become known)
- In this example, the particles will have a pair of values ($G_t^a = \mathbf{p}_a, G_t^b = \mathbf{p}_b$)



- To progress time for each particle we would sample a new value for G_{t+1}^a and G_{t+1}^b from the CPT given in the Bayes net: $Pr(G_{t+1}^a | G_t^a = \mathbf{p}_a)$ and $Pr(G_{t+1}^b | G_t^a = \mathbf{p}_a, G_t^b = \mathbf{p}_b)$. Note that we can sample each value $\mathbf{p'_a}$ **and** $\mathbf{p'_b}$ in the new particle **p'** separately since are G_{t+1}^a and G_{t+1}^b are independent in this Bayes net given $G_t^a = \mathbf{p}_a$ and $G_t^b = \mathbf{p}_b$

Particle Filtering in DBN

- Then to update by observation, we would weight each particle **p=(p**$_a$, **p**$_b$) by the product of Pr(e_{t+1}^a| G_{t+1}^a = **p**$_a$) * Pr(e_{t+1}^b| G_{t+1}^b = **p**$_b$)
- Again because E_{t+1}^a and E_{t+1}^b are independent of each other given G_{t+1}^a, G_{t+1}^b in this Bayes net.



- If the variables and evidence at each time step t are dependent on each other then the updates to the particles are a bit more complex.