Q1: ✱ Note: Let TO = Tom has changed oil, TS = Tom speeding, TC = Tom crashes

a) Step 1: Condition to get distribution $P(TS, TC \mid TO = F)$

TS	TC	$P(TS, TC \mid TO = F)$
F	F	$0.18 / 0.6 = 0.3$
F	T	$0.12 / 0.6 = 0.2$
T	F	$0.06 / 0.6 = 0.1$
T	T	$0.24 / 0.6 = 0.4$

Step 2: Marginalize out TS to get $P(TC \mid TO = F)$

TC	$P(TC \mid TO = F)$
T	$0.2 + 0.4 = 0.6$
F	$0.3 + 0.1 = 0.4$

$\Rightarrow$ Answer: P(Tom crashes = T | Tom has changed car's oil = F)

$= \underline{0.6}$

b.)

1.b) Let A = Changing oil
B = Tom speeding
C = Tom crashing

If for any pair X, Y: $P(X \mid Y) = P(X)$ then X and Y are independent.

$p(A=T) = 0.12 + 0.08 + 0.04 + 0.16 = 0.4$
$p(A=T, B=T) = 0.04 + 0.16 = 0.2$
$p(B=T) = 0.06 + 0.24 + 0.04 + 0.16 = 0.5$

$$p(A \mid B) = \frac{p(A,B)}{p(B)} = \frac{0.2}{0.5} = 0.4 = p(A)$$

$\therefore$ A and B are independent

$p(B=T) = 0.5$
$p(B=T, C=T) = 0.24 + 0.16 = 0.4$
$p(C=T) = 0.12 + 0.24 + 0.08 + 0.16 = 0.6$

$$p(B \mid C) = \frac{p(B,C)}{p(C)} = \frac{0.4}{0.6} = 0.6\bar{6} \neq p(B)$$

$\therefore$ B and C are NOT independent

$p(A=T) = 0.4$
$p(A=T, C=T) = 0.08 + 0.16 = 0.24$
$p(C=T) = 0.6$

$$p(A \mid C) = \frac{p(A,C)}{p(C)} = \frac{0.24}{0.6} = 0.4 = p(A)$$

$\therefore$ A and C are independent

1.c) Let a = Tom has changed car's oil = T
b = Tom speeding = T
c = Tom crashes = T

$p(a,b,c) = 0.16$ according to the table

Now by chain rule:

$$p(a,b,c) = p(a|b,c) \times p(b|c) \times p(c)$$

$$= \frac{p(a,b,c)}{p(b,c)} \times \frac{0.4}{\cancel{0.6}} \times \cancel{0.6}$$

$$= \frac{0.16}{\cancel{0.4}} \times \cancel{0.4}$$

$$= 0.16$$

2.a) $0.46 \times 0.2 + 0.16 \times 0.3 + 0.18 \times 0.4 + 0.2 \times 0.5$
$= 0.312$

b) Let P = proportion of pokemons
S = successful catch at first try

Need $p(P \text{ is normal type} \mid S)$

$$= \frac{p(P \text{ is normal type}, S)}{p(S)}$$

$$= \frac{(0.18 \times 0.4 + 0.2 \times 0.5)}{0.312}$$

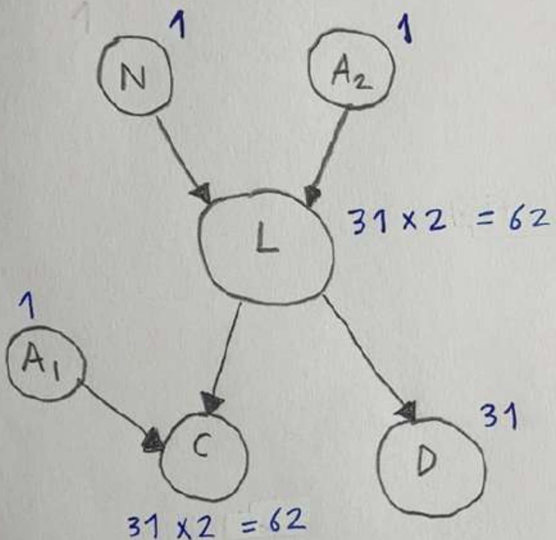
$$= \frac{0.172}{0.312}$$

$$= 0.55$$

Q3: JDP Representational Cost:

$$= 2^3 \times 31^3 - 1 = 8 \times 29791 - 1 = 238{,}327$$

Bayesian Network:



Cost $= 1 + 1 + 1 + 62 + 62 + 31 = 158$ probabilities need to be specified

Representational Saving = Cost JDP − Cost Bayesian

$$= 238327 - 158$$

$$= 238169$$