# Personalization with Unobserved Heterogeneity

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#### Motivation for Personalization

- Personalization is founded on the premise that individuals have heterogeneous responses to actions.
- Personalization algorithms aim to improve decision-making by identifying and exploiting this heterogeneity.



#### Personalized Paradigm



# Unobserved and Heterogeneous Confounder (UHC)

 Treatment effect (T) varies according to the value of an unobserved confounder (U).

$$T := f(U) + N_T$$
  
 $Y := f(T, U, T \times U) + N_Y$ 

- Presence of UHCs is arguably the most sensible assumption in practice.
- UHCs introduce challenges to personalization.

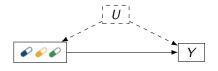


Figure: Observational setting.

### Motivating Questions

Given the high-level goal of personalization, and the context of UHCs:

- How should I express my objective function? What is a suitable causal estimand?
- What data do I need for identification?
- Is experimental data 'gold standard'?

Out-of-scope: Estimation (e.g., compare different estimators for personalization).

## **Business Setting**

- Business objective: Sell a credit card to new-to-RBC clients.
- **Current campaign**. All new-to-RBC clients who visited the RBC public site get a credit card offer + iPad incentive.



 Business Goal: Personalize the incentive. Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

# Data Generating Process

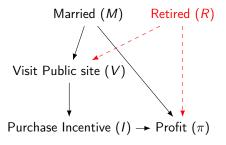


Figure: Observational setting.

# Data Generating Process

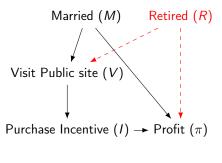


Figure: Observational setting.

$$P(R = 1) = 0.5$$
,  $P(M = 1) = 0.5$   
 $V := M \oplus R$   
 $I := V$ 

	R = 0		R = 1	
	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
I = 0	50	10	5	30

Table:  $E[\pi|M,R,I]$ . Highlighted cells reflect (new-to-RBC) client's 'natural' choice to visit the Public site or not.

# Four Approaches to Personalizing the Incentive

**Business Goal**: Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.



### 1. Associational Inference



$$\mathcal{D}_{\mathsf{AI}}^*(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi|I,M]$$

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$$E[\pi|I=1, M=1] = 25$$
  
 $E[\pi|I=0, M=1] = 5$   
 $E[\pi|I=1, M=0] = 5$   
 $E[\pi|I=0, M=0] = 10$ 

	R = 0		R = 1	
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Table:  $E[\pi|M,R,I]$ .

#### **Decision Rule:**

- If Visit Site  $\wedge$  Married  $\rightarrow$  Purchase Incentive  $\rightarrow$   $E[\pi] = 25$
- If Visit Site  $\land$  Not Married  $\rightarrow$  No Purchase Incentive  $\rightarrow$   $E[\pi] = 30$

Expected profit = 
$$27.5$$
 =  $(25+30)/2$ .



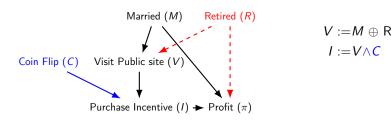


Figure: Causal DAG with post-visit randomization.



$$\mathcal{D}^*_{\mathsf{IPVR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi|do(I), M, V = 1]$$



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 $E[\pi|do(I=0), M=1, V=1] = 50$   
 $E[\pi|do(I=1), M=0, V=1] = 5$   
 $E[\pi|do(I=0), M=0, V=1] = 30$ 

	R = 0		R =	= 1
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 $E[\pi|do(I=0), M=0, V=1] = 30$ 

	R = 0			= 1
	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
<i>I</i> = 0	50	10	5	30

Table:  $E[\pi|M,R,I]$ .

#### **Decision Rule:**

- If Visit Site  $\land$  Married  $\rightarrow$  No Purchase Incentive  $\rightarrow$   $E[\pi] = 50$
- If Visit Site  $\wedge$  Not Married  $\rightarrow$  No Purchase Incentive  $\rightarrow$   $E[\pi] = 30$

Expected profit = 
$$\boxed{40}$$
 =  $(50+30)/2$ .

### 3. Interventional Inference + Full Randomization



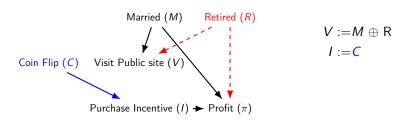


Figure: Causal DAG with Full Randomization.

### 3. Interventional Inference + Full Randomization



$$\mathcal{D}^*_{\mathsf{IFR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi|do(I), M]$$

$$E[\pi|do(I=1), M=1] = 35.0 = (25+45)/2$$
  
 $E[\pi|do(I=0), M=1] = 27.5 = (50+5)/2$   
 $E[\pi|do(I=1), M=0] = 27.5 = (50+5)/2$   
 $E[\pi|do(I=0), M=0] = 20.0 = (10+30)/2$ 

	R = 0		R = 1	
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/ = 1	25	50	45	5
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Table:  $E[\pi|M,R,I]$ .

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 $E[\pi|do(I=0), M=0] = 20.0 = (10 + 30)/2$ 

	R = 0		R = 1	
-	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
I = 0	50	10	5	30

Table:  $E[\pi|M,R,I]$ .

#### **Decision Rule:**

- If Married  $\rightarrow$  Purchase Incentive  $\rightarrow$   $E[\pi] = 35$
- If Not Married  $\rightarrow$  Purchase Incentive  $\rightarrow$   $E[\pi] = 27.5$

Expected profit = 
$$\boxed{31.5}$$
 =  $(35+27.5)/2$ .





$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{I=a'}|I=a,M]$$



Puzzled by  $E[\pi|I, M] \neq E[\pi|do(I), M]$ .

$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{I=a'}|I=a,M]$$

• Do we need to assume a parametric SCM to identify this causal estimand?



$$\mathcal{D}_{\mathsf{RDC}}^*(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{I=a'}|I=a,M]$$

- Oo we need to assume a parametric SCM to identify this causal estimand?
  - No. Only unit-level counterfactuals require a parametric model for identification.
  - ▶ There is nothing personal about personalization!



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  - There is nothing personal about personalization!
- ② Do we need the causal graph to non-parametrically identify this causal estimand?



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- Oo we need to assume a parametric SCM to identify this causal estimand?
  - No. Only unit-level counterfactuals require a parametric model for identification.
  - There is nothing personal about personalization!
- ② Do we need the causal graph to non-parametrically identify this causal estimand?
  - ▶ In general, yes. Population-level counterfactuals require the causal graph.
  - An exception is when the treatment is binary and both experimental and observational data are available.



$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in [0,1]}{\operatorname{argmax}} \ E[\pi_{a'}|I = a,M]$$

$$P(\pi_{a'},M) = P(\pi_{a'},M,a') + P(\pi_{a'},M,a)$$

$$= P(\pi_{a'}|M,a')P(M,a') + P(\pi_{a'}|M,a)P(M,a)$$

$$P(\pi_{a'}|M) = P(\pi_{a'}|M,a')P(a'|M) + P(\pi_{a'}|M,a)P(a|M)$$

$$= P(\pi|M,a')P(a'|M) + P(\pi_{a'}|M,a)P(a|M) \text{ (Consistency)}$$

$$P(\pi_{a'}|M,a) = \frac{1}{P(a|M)} \Big[ P(\pi_{a'}|M) - P(\pi|M,a')P(a'|M) \Big]$$

$$= \underbrace{\frac{1}{P(a|M)} \Big[ P(\pi_{a'}|M,a) - P(\pi|M,a')P(a'|M) \Big]}_{\text{observational}}$$



$$E(\pi_{I=1}|M=1,I=0) =$$

$$\begin{split} \frac{1}{P(I=0|M=1)} \Big[ E\Big(\pi|M=1, do(I=1)\Big) - \\ E(\pi|M=1, I=1) P(I=1|M=1) \Big]. \\ &= \frac{1}{1/2} (35-25\times 1/2) = 45 \\ &> 5 = E(\pi_{I=0}|M=1, I=0). \end{split}$$

$$E(\pi_{I=1}|M=0,I=0)$$
 = 50

$$E(\pi_{I=0}|M=1,I=1) = 50$$

$$E(\pi_{I=0}|M=0,I=1) = 30$$

	R = 0		R = 1	
	M = 1	<i>M</i> = 0	M = 1	M = 0
/ = 1	25	50	45	5
<i>I</i> = 0	50	10	5	30

Table:  $E[\pi|M,R,I]$ .



$$E(\pi_{I=1}|M=1,I=0) =$$

$$\frac{1}{P(I=0|M=1)} \left[ E\left(\pi|M=1, do(I=1)\right) - \\ E\left(\pi|M=1, I=1\right) P(I=1|M=1) \right].$$

$$= \frac{1}{1/2} (35 - 25 \times 1/2) = 45$$

$$> 5 = E\left(\pi_{I=0} \mid M=1, I=0\right).$$

	R =	= 0	R =	= 1
	M = 1	M = 0	M = 1	M = 0
/ = 1 / = 0	25 50	50 10	45 5	5 30

Table:  $E[\pi|M,R,I]$ .

$$E(\pi_{I=1}|M=0, I=0) = 50$$

$$E(\pi_{I=0}|M=1, I=1) = 50$$

$$E(\pi_{I=0}|M=0, I=1) = 30$$

#### **Decision Rule:**

- If Not Visit Site  $\land$  Married  $\rightarrow$  Purchase Incentive  $\rightarrow$   $E[\pi] = 45$
- ullet If Not Visit Site  $\wedge$  Not Married o Purchase Incentive o  $E[\pi] = {f 50}$
- If Visit Site  $\land$  Married  $\rightarrow$  No Purchase Incentive  $\rightarrow$   $E[\pi] = 50$
- If Visit Site  $\wedge$  Not Married  $\rightarrow$  No Purchase Incentive  $\rightarrow$   $E[\pi] = 30$

Expected profit = 
$$\boxed{43.75}$$
 =  $(45 + 50 + 50 + 30)/4$ .

# Summary of Methods

Criterion	Decision Rule	$E[\pi]$
$\mathcal{D}_{AI}$	<ul> <li>If Visit Site ∧ Married →         Purchase Incentive</li> <li>If Visit Site ∧ Not Married → No</li> </ul>	27.50
$\mathcal{D}_{\mathit{IPVR}}$	Purchase Incentive  Never Purchase Incentive	40.0
$\mathcal{D}_{\mathit{IFR}}$	Always Purchase Incentive	31.50
$\mathcal{D}_{RDC}$	• If Visit Site $\wedge$ Married $\rightarrow$ No Purchase Incentive	43.75
	• If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive	
	• If Not Visit Site $\land$ Married $\rightarrow$ Purchase Incentive	
	$ \begin{tabular}{ll} \bullet & \mbox{If Not Visit Site} \land \mbox{Not Married} \rightarrow \\ \mbox{Purchase Incentive} \\ \end{tabular} $	
$\mathcal{D}_{Oracle}$		43.75

#### Remarks

- Experimental data are 'gold standard' in the non-personalized paradigm because they remove the influence of unobserved confounders.
- In the personalization paradigm with UHCs, experimental data are not necessarily 'gold standard' for identifying heterogeneous treatment effects:
  - Experiments 'destroy' uncoded knowledge about these confounders.
  - Interventional inference does not capture the information required to maximize payout.
- Counterfactual-based decision-making leads to a fusion of experimental and observational data, which might capture additional information about UHCs required to maximize payout.

# Further Reading

- This presentation is fundamentally inspired by this paper:
  - ▶ Elias Bareinboim, Andrew Forney, and Judea Pearl. 2015. **Bandits with unobserved confounders: a causal approach**. In Proceedings of the 28th International Conference on Neural Information Processing Systems Volume 1 (NIPS'15).
  - ► Implementation: https://github.com/leoguelman/mabuc.
- The expression derived from RDC works only in the binary treatment case.
  - RDC-type randomization (Forney et al., 2017) was proposed to estimate counterfactual expressions empirically from an arbitrary number of treatments.