Personalization with Unobserved Heterogeneity

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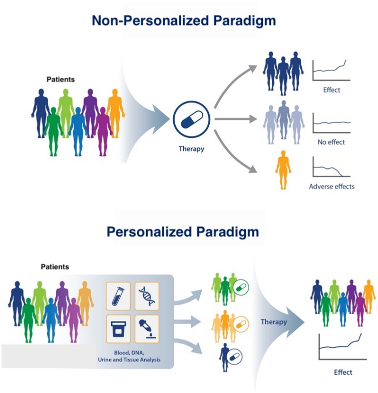
Motivation for Personalization

- Personalization is founded on the premise that individuals have heterogeneous responses to actions.
- Personalization algorithms aim to improve decision-making by identifying and exploiting this heterogeneity.



Unobserved and Heterogeneous Confounder (UHC)

- Treatment effect (T) varies according to the value of unobserved confounder/s (U).

$$T := f(U) + N_T$$
$$Y := f(T, U, \boldsymbol{T} \times \boldsymbol{U}) + N_Y$$

- Existence of UHCs is the most sensible assumption in practice.
- UHCs introduce challenges to personalization.

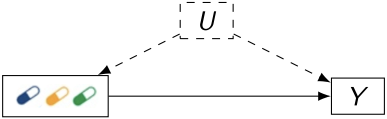


Figure: Observational setting.

Motivating Questions

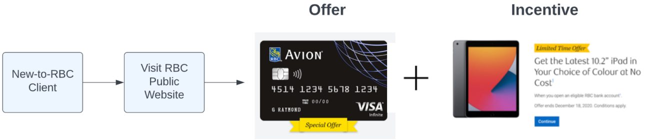
Given the high-level goal of personalization, and the context of UHCs:

- Alternatives to how I formulate this problem? For instance, what is a suitable causal estimand?
- What data do I need for identification?
- Is experimental data 'gold standard'?

Out-of-scope: Estimation (e.g., compare different estimators).

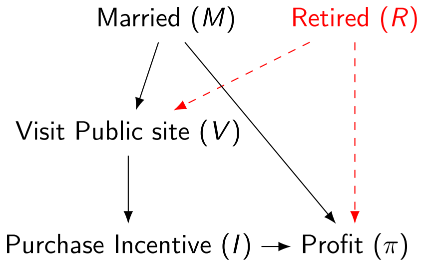
Motivating Example

- **Business objective:** Sell a credit card to new-to-RBC clients.
- **Current campaign**. All new-to-RBC clients who visited the RBC public site get a credit card offer + iPad incentive.



- **Business Goal: Personalize the incentive**. Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

Data Generating Process



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Data Generating Process

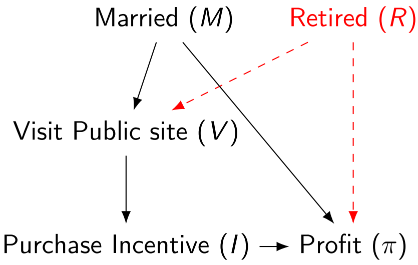

Figure: Observational setting.

$$P(R=1)=0.5\ ,\ P(M=1)=0.5$$
$$V := M \oplus \text{R}$$
$$I := V$$

	$R=0$		$R=1$	
	$M=1$	$M=0$	$M=1$	$M=0$
$I=1$	25	50	45	5
$I=0$	50	10	5	30

Table: $E[\pi|M,R,I]$. Highlighted cells reflect (new-to-RBC) client's 'natural' choice to visit the Public site or not.

Four Approaches to Personalizing the Incentive

Business Goal: Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

1. Associational Inference

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$$E[\pi | I = 1, M = 1] = 25$$
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$$E[\pi | I = 1, M = 0] = 5$$
$$E[\pi | I = 0, M = 0] = 10$$

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Decision Rule:

- If Visit Site $\wedge$ Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{25}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{30}$

Expected profit $= \boxed{27.5} = (25+30)/2.$

2. Interventional Inference + Post-Visit Randomization

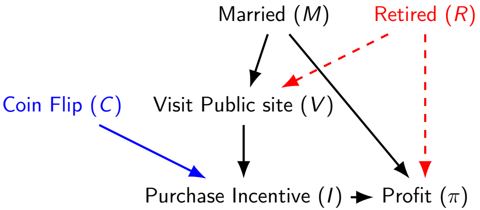

Figure: Causal DAG with post-visit randomization.

$$V := M \oplus \mathrm{R}$$
$$I := V \wedge C$$

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$$\mathcal{D}^*_{\mathsf{IPVR}}(M) = \operatorname*{argmax}_{I \in 0,1} E[\pi | do(I), M, V = 1]$$

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Decision Rule:

- If Visit Site $\wedge$ Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{50}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{30}$

Expected profit = $\boxed{40}$ = (50+30)/2.

3. Interventional Inference + Full Randomization

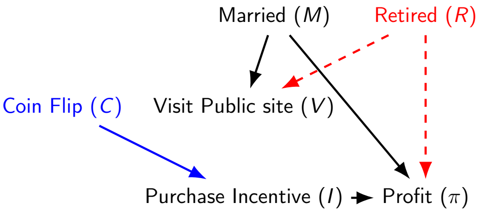

Figure: Causal DAG with Full Randomization.

$$V := M \oplus \mathrm{R}$$
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3. Interventional Inference + Full Randomization

$$\mathcal{D}^*_{\mathrm{IFR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \; E[\pi | do(I), M]$$

$E[\pi|do(I=1), M=1] = 35.0 = (25+45)/2$
$E[\pi|do(I=0), M=1] = 27.5 = (50+5)/2$
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Decision Rule:

- If Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] =$ **35**
- If Not Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] =$ **27.5**

Expected profit $= \boxed{31.5} = (35+27.5)/2$.

4. Counterfactual Inference

$$\mathcal{D}^*_{\mathsf{CI}}(M, I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'} | I = a, M]$$

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   - No. Only unit-level counterfactuals require a parametric model for identification.
   - There is nothing personal about personalization!

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2. Do we need to assume that the conditioning set $\{M\}$ satisfies the *backdoor criterion* to identify this causal estimand?

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1. Do we need to assume a parametric model to identify this causal estimand?
   - No. Only unit-level counterfactuals require a parametric model for identification.
   - There is nothing personal about personalization!

2. Do we need to assume that the conditioning set $\{M\}$ satisfies the *backdoor criterion* to identify this causal estimand?
   - In general, we do need to assume the conditioning set $Z = \{M\}$ satisfies the backdoor criterion.
   - An exception is when I is binary and both experimental and observational data are available.

4. Counterfactual Inference

$$\mathcal{D}^*_{\mathsf{CI}}(M, I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'} | I = a, M]$$

$$\begin{aligned} P(\pi_{a'}, M) &= P(\pi_{a'}, M, a') + P(\pi_{a'}, M, a) \\ &= P(\pi_{a'} | M, a')P(M, a') + P(\pi_{a'} | M, a)P(M, a) \end{aligned}$$

$$\begin{aligned} P(\pi_{a'} | M) &= P(\pi_{a'} | M, a')P(a' | M) + P(\pi_{a'} | M, a)P(a | M) \\ &= P(\pi | M, a')P(a' | M) + P(\pi_{a'} | M, a)P(a | M) \ \text{(Consistency)} \end{aligned}$$

$$\begin{aligned} P(\pi_{a'} | M, a) &= \frac{1}{P(a|M)} \Big[P(\pi_{a'} | M) - P(\pi | M, a')P(a' | M) \Big] \\ &= \boxed{ \underbrace{\frac{1}{P(a|M)}}_{\textbf{observational}} \Big[\overbrace{P\Big(\pi | M, do(a')\Big)}^{\textbf{experimental}} - \underbrace{P(\pi | M, a')P(a' | M)}_{\textbf{observational}} \Big] } \end{aligned}$$

4. Counterfactual Inference

$\boxed{E(\pi_{I=\mathbf{1}}|M=1, I=0)} =$

$$\frac{1}{P(I=0|M=1)}\Big[E\Big(\pi|M=1, do(I=1)\Big) - E(\pi|M=1, I=1)P(I=1|M=1)\Big].$$

$$= \frac{1}{1/2}(35 - 25 \times 1/2) = \mathbf{45}$$

$$> 5 = E(\pi_{I=\mathbf{0}}|M=1, I=0).$$

$\boxed{E(\pi_{I=\mathbf{1}}|M=0, I=0)} = \mathbf{50}$

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Table: $E[\pi|M,R,I]$.

Decision Rule:

- If Not Visit Site ∧ Married → Purchase Incentive → $E[\pi] = \mathbf{45}$
- If Not Visit Site ∧ Not Married → Purchase Incentive → $E[\pi] = \mathbf{50}$
- If Visit Site ∧ Married → No Purchase Incentive → $E[\pi] = \mathbf{50}$
- If Visit Site ∧ Not Married → No Purchase Incentive → $E[\pi] = \mathbf{30}$

Expected profit = $\boxed{43.75}$ = (45 + 50+ 50 +30)/4.

Summary of Methods

Criterion	Decision Rule	$E[\pi]$
$\mathcal{D}_{AI}$	• If Visit Site $\wedge$ Married $\rightarrow$ **Purchase Incentive** • If Visit Site $\wedge$ Not Married $\rightarrow$ **No Purchase Incentive**	27.50
$\mathcal{D}_{IPVR}$	**Never Purchase Incentive**	40.0
$\mathcal{D}_{IFR}$	**Always Purchase Incentive**	31.50
$\mathcal{D}_{CI}$	• If Visit Site $\wedge$ Married $\rightarrow$ **No Purchase Incentive** • If Visit Site $\wedge$ Not Married $\rightarrow$ **No Purchase Incentive** • If Not Visit Site $\wedge$ Married $\rightarrow$ **Purchase Incentive** • If Not Visit Site $\wedge$ Not Married $\rightarrow$ **Purchase Incentive**	**43.75**
$\mathcal{D}_{\mathbf{Oracle}}$		**43.75**

Remarks

- Experimental data are 'gold standard' in the non-personalized paradigm because they remove the influence of unobserved confounders.
- In the personalization paradigm, experimental data alone is not 'gold standard' for estimating heterogeneous treatment effects in the presence of (UHCs).
- Experiments 'destroy' information that can be valuable to recover these confounders.
- Counterfactual-based decision-making for personalization leads to a fusion of experimental and observational data.

Further Reading

- The expression derived from RDC works only in the binary treatment case.
  - RDC-type randomization (Forney et al., 2017) was proposed to estimate counterfactual expressions empirically from an arbitrary number of treatments.
- This presentation is fundamentally inspired by this paper:
  - Elias Bareinboim, Andrew Forney, and Judea Pearl. 2015. Bandits with unobserved confounders: a causal approach. In Proceedings of the 28th International Conference on Neural Information Processing Systems - Volume 1 (NIPS'15).
  - Implementation: `https://github.com/leoguelman/mabuc`.