Personalization with Unobserved Heterogeneity

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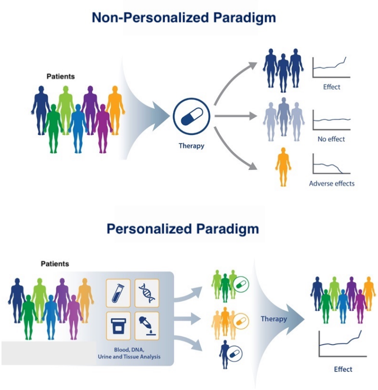
Motivation for Personalization

- Personalization is founded on the premise that individuals have heterogeneous responses to actions.
- Personalization algorithms aim to improve decision-making by identifying and exploiting this heterogeneity.



Unobserved and Heterogeneous Confounders

- Treatment effect (T) varies according to the value of unobserved confounders (U).

$$T := f(U) + N_T$$
$$Y := f(T, U, \boldsymbol{T \times U}) + N_Y$$

- Likely the norm in observational settings.

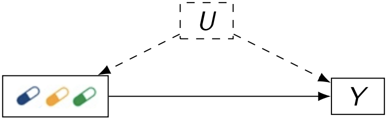


Figure: Observational setting.

Motivating Questions

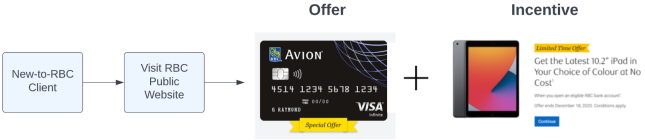
Given the high-level goal of personalization, and the context of unobserved heterogeneity:

- Alternatives to how I formulate this problem? For instance, what is a suitable causal estimand?
- What data do I need for identification?
- Is experimental data 'gold standard'?

Out-of-scope: Estimation (e.g., compare different estimators).

Motivating Example

- **Business objective:** Sell a credit card to new-to-RBC clients.
- **Current campaign**. All new-to-RBC clients who visited the RBC public site get a credit card offer + iPad incentive.



- **Business Goal: Personalize the incentive**. Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

Data Generating Process

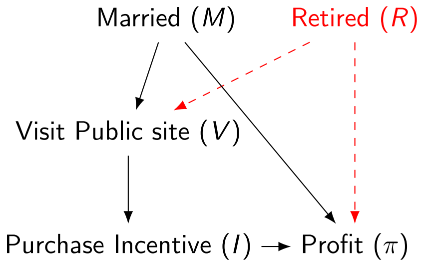


Figure: Observational setting.

Data Generating Process

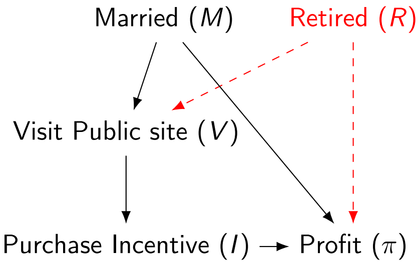


Figure: Observational setting.

$$P(R=1) = 0.5 \; , \; P(M=1) = 0.5$$

$$V := M \oplus R$$

$$I := V$$

	$R=0$		$R=1$	
	$M=1$	$M=0$	$M=1$	$M=0$
$I=1$	0.25	0.50	0.45	0.05
$I=0$	0.50	0.10	0.05	0.30

Table: $E[\pi|M,R,I]$. Highlighted cells reflect (new-to-RBC) client's 'natural' choice to visit the Public site or not.

Four Approaches to Personalizing the Incentive

Business Goal: Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

1. Associational Inference

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$$E[\pi | I = 0, M = 1] = 0.05$$
$$E[\pi | I = 1, M = 0] = 0.05$$
$$E[\pi | I = 0, M = 0] = 0.10$$

	$R = 0$		$R = 1$	
	$M = 1$	$M = 0$	$M = 1$	$M = 0$
$I = 1$	0.25	0.50	0.45	0.05
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Table: $E[\pi | M, R, I]$.

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	$M = 1$	$M = 0$	$M = 1$	$M = 0$
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Table: $E[\pi | M, R, I]$.

Decision Rule:

- If Visit Site $\wedge$ Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.25}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.30}$

Expected profit $= \boxed{0.275} = (0.25 + 0.30)/2$.

2. Interventional Inference + Post-Visit Randomization

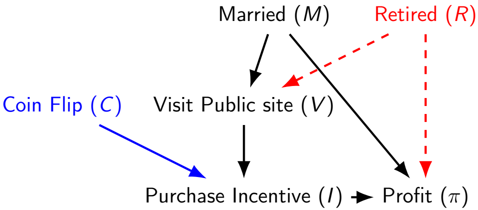

Figure: Causal DAG with post-visit randomization.

$$V := M \oplus \mathrm{R}$$
$$I := V \wedge C$$

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$$\mathcal{D}^*_{\mathsf{IPVR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}}\ E[\pi | do(I), M, V = 1]$$

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$$E[\pi | do(I = 1), M = 0, V = 1] = 0.05$$
$$E[\pi | do(I = 0), M = 0, V = 1] = 0.30$$

	$R = 0$		$R = 1$	
	$M = 1$	$M = 0$	$M = 1$	$M = 0$
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	$R=0$		$R=1$	
	$M=1$	$M=0$	$M=1$	$M=0$
$I=1$	0.25	0.50	0.45	0.05
$I=0$	0.50	0.10	0.05	0.30

Table: $E[\pi|M, R, I]$.

Decision Rule:

- If Visit Site $\wedge$ Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.50}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.30}$

Expected profit $= \boxed{0.40} = (0.50+0.30)/2$.

3. Interventional Inference + Full Randomization

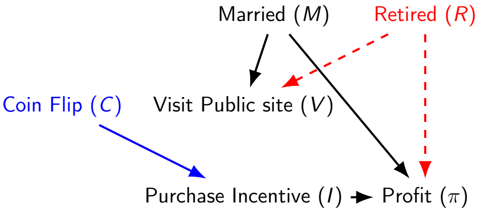

Figure: Causal DAG with A/B Test on all New-to-RBC clients.

$$V := M \oplus \mathrm{R}$$
$$I := C$$

3. Interventional Inference + Full Randomization

$$\mathcal{D}^*_{\mathsf{IFR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \; E[\pi | do(I), M]$$

$E[\pi|do(I = 1), M = 1] = 0.350 = (0.25 + 0.45)/2$
$E[\pi|do(I = 0), M = 1] = 0.275 = (0.50 + 0.05)/2$
$E[\pi|do(I = 1), M = 0] = 0.275 = (0.50 + 0.05)/2$
$E[\pi|do(I = 0), M = 0] = 0.200 = (0.10 + 0.30)/2$

	$R = 0$		$R = 1$	
	$M = 1$	$M = 0$	$M = 1$	$M = 0$
$I = 1$	0.25	0.50	0.45	0.05
$I = 0$	0.50	0.10	0.05	0.30

Table: $E[\pi|M, R, I]$.

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	$R=0$		$R=1$	
	$M=1$	$M=0$	$M=1$	$M=0$
$I=1$	0.25	0.50	0.45	0.05
$I=0$	0.50	0.10	0.05	0.30

Table: $E[\pi|M, R, I]$.

Decision Rule:

- If Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.35}$
- If Not Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.275}$

Expected profit $= \boxed{0.315} = (0.35+0.275)/2$.

4. Counterfactual Inference

$$\mathcal{D}^*_{\mathsf{CI}}(M, I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'} | I = a, M]$$

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$$\mathcal{D}^*_{\mathrm{CI}}(M, I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'} | I = a, M]$$

1. Do I need to assume a parametric model to identify this causal estimand?
2. Do I need to assume that the conditioning set $\{M\}$ satisfies the *backdoor criterion* to identify this causal estimand?

4. Counterfactual Inference

$$\mathcal{D}^{*}_{\mathsf{CI}}(M, I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'} | I = a, M]$$

$$\begin{aligned} P(\pi_{a'}, M) &= P(\pi_{a'}, M, a') + P(\pi_{a'}, M, a) \\ &= P(\pi_{a'} | M, a') P(M, a') + P(\pi_{a'} | M, a) P(M, a) \end{aligned}$$

$$\begin{aligned} P(\pi_{a'} | M) &= P(\pi_{a'} | M, a') P(a' | M) + P(\pi_{a'} | M, a) P(a | M) \\ &= P(\pi | M, a') P(a' | M) + P(\pi_{a'} | M, a) P(a | M) \ \text{(Consistency)} \end{aligned}$$

$$\begin{aligned} P(\pi_{a'} | M, a) &= \frac{1}{P(a|M)} \left[P(\pi_{a'} | M) - P(\pi | M, a') P(a' | M) \right] \\ &= \boxed{\underbrace{\frac{1}{P(a|M)}}_{\textbf{observational}} \overbrace{\left[P\left(\pi | M, do(a') \right)\right.}^{\textbf{experimental}} - \underbrace{\left. P(\pi | M, a') P(a' | M) \right]}_{\textbf{observational}}} \end{aligned}$$

4. Counterfactual Inference

$\boxed{E(\pi_{I=\mathbf{1}}|M=1, I=0)} =$

$$\frac{1}{P(I=0|M=1)}\Big[E\Big(\pi|M=1, do(I=1)\Big) -$$
$$E(\pi|M=1, I=1)P(I=1|M=1)\Big].$$
$$= \frac{1}{1/2}(0.350 - 0.25 \times 1/2) = \mathbf{0.45}$$
$$> 0.05 = E(\pi_{I=\mathbf{0}}|M=1, I=0).$$

$\boxed{E(\pi_{I=\mathbf{1}}|M=0, I=0)} = \mathbf{0.50}$

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Table: $E[\pi|M, R, I]$.

Decision Rule:

- If Not Visit Site $\wedge$ Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.45}$
- If Not Visit Site $\wedge$ Not Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.50}$
- If Visit Site $\wedge$ Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.50}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{0.30}$

Expected profit $= \boxed{0.4375} = (0.45 + 0.50 + 0.50 + 0.30)/4.$

Summary of Methods

Criterion	Decision Rule	$E[\pi]$
$\mathcal{D}_{AI}$	• If Visit Site $\wedge$ Married $\rightarrow$ **Purchase Incentive** • If Visit Site $\wedge$ Not Married $\rightarrow$ **No Purchase Incentive**	.2750
$\mathcal{D}_{IPVR}$	**Never Purchase Incentive**	.4000
$\mathcal{D}_{IFR}$	**Always Purchase Incentive**	.3150
$\mathcal{D}_{CI}$	• If Visit Site $\wedge$ Married $\rightarrow$ **No Purchase Incentive** • If Visit Site $\wedge$ Not Married $\rightarrow$ **No Purchase Incentive** • If Not Visit Site $\wedge$ Married $\rightarrow$ **Purchase Incentive** • If Not Visit Site $\wedge$ Not Married $\rightarrow$ **Purchase Incentive**	**.4375**
$\mathcal{D}_{\textbf{Oracle}}$		**.4375**

Remarks

- Experimental data are 'gold standard' in the non-personalized paradigm because they remove the influence of unobserved confounders.
- In the personalization paradigm, experimental data alone is not 'gold standard' for estimating heterogeneous treatment effects in the presence of unobserved and heterogenous confounders.
- Experiments 'destroy' information that can be valuable to identify the values of unobserved confounders.
- Counterfactual-based decision making, which leads to a fusion of experimental and observational data, might be optimal for personalization.

Further Reading

- The expression derived from RDC works only in the binary treatment case. RDC-type randomization (Forney et al., 2017) was proposed to estimate counterfactual expressions empirically from an arbitrary number of treatments.
- This presentation is fundamentally inspired by this paper:
  - Elias Bareinboim, Andrew Forney, and Judea Pearl. 2015. Bandits with unobserved confounders: a causal approach. In Proceedings of the 28th International Conference on Neural Information Processing Systems - Volume 1 (NIPS'15).
  - Implementation: `https://github.com/leoguelman/mabuc`