

# Personalization with Unobserved Heterogeneity

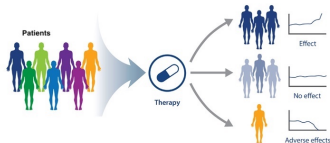
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RBC Royal Bank

# Motivation for Personalization

- Personalization is founded on the premise that individuals have heterogeneous responses to actions.
- Personalization algorithms aim to improve decision-making by identifying and exploiting this heterogeneity.

**Non-Personalized Paradigm**



**Personalized Paradigm**



# Unobserved and Heterogeneous Confounder (UHC)

- Treatment effect ( $T$ ) varies according to the value of unobserved confounder/s ( $U$ ).

$$T := f(U) + N_T$$

$$Y := f(T, U, \mathbf{T} \times \mathbf{U}) + N_Y$$

- Existence of UHCs is the most sensible assumption in practice.
- UHCs introduce challenges to personalization.

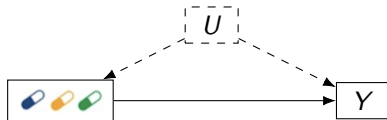


Figure: Observational setting.

# There's Nothing Personal About Personalization

# Motivating Questions

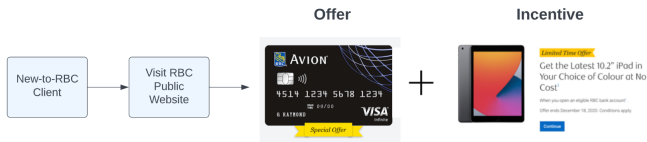
Given the high-level goal of personalization, and the context of UHCs:

- Alternatives to how I formulate this problem? For instance, what is a suitable causal estimand?
- What data do I need for identification?
- Is experimental data 'gold standard'?

**Out-of-scope:** Estimation (e.g., compare different estimators).

# Motivating Example

- **Business objective:** Sell a credit card to new-to-RBC clients.
- **Current campaign.** All new-to-RBC clients who visited the RBC public site get a credit card offer + iPad incentive.



- **Business Goal: Personalize the incentive.** Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

# Data Generating Process

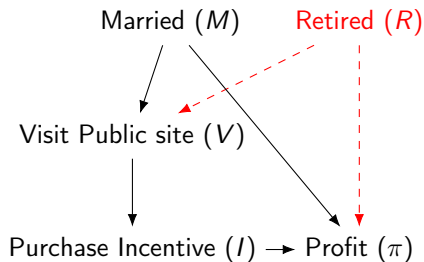


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# Data Generating Process

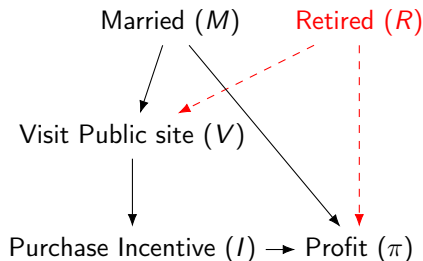


Figure: Observational setting.

$$P(R = 1) = 0.5, \quad P(M = 1) = 0.5$$

$$V := M \oplus R$$

$$I := V$$

	$R = 0$		$R = 1$	
	$M = 1$	$M = 0$	$M = 1$	$M = 0$
$I = 1$	25	50	45	5
$I = 0$	50	10	5	30

Table:  $E[\pi|M, R, I]$ . Highlighted cells reflect (new-to-RBC) client's 'natural' choice to visit the Public site or not.



# Four Approaches to Personalizing the Incentive

**Business Goal:** Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

Data Scientist 1



Data Scientist 2



Data Scientist 3



Data Scientist 4



# 1. Associational Inference

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$$E[\pi|I = 0, M = 1] = 5$$

$$E[\pi|I = 1, M = 0] = 5$$

$$E[\pi|I = 0, M = 0] = 10$$

	$R = 0$		$R = 1$	
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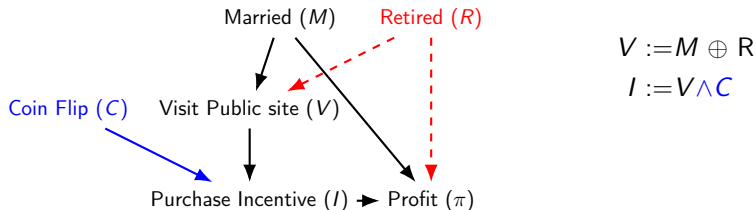
Table:  $E[\pi|M, R, I]$ .

## Decision Rule:

- If Visit Site  $\wedge$  Married  $\rightarrow$  Purchase Incentive  $\rightarrow E[\pi] = 25$
- If Visit Site  $\wedge$  Not Married  $\rightarrow$  No Purchase Incentive  $\rightarrow E[\pi] = 30$

Expected profit =  $\boxed{27.5} = (25+30)/2$ .

## 2. Interventional Inference + Post-Visit Randomization



**Figure:** Causal DAG with post-visit randomization.

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$$E[\pi | do(I = 0), M = 1, V = 1] = 50$$

$$E[\pi | do(I = 1), M = 0, V = 1] = 5$$

$$E[\pi | do(I = 0), M = 0, V = 1] = 30$$

	$R = 0$		$R = 1$	
	$M = 1$	$M = 0$	$M = 1$	$M = 0$
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	$R = 0$		$R = 1$	
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$I = 1$	25	50	45	5
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Table:  $E[\pi | M, R, I]$ .

### Decision Rule:

- If Visit Site  $\wedge$  Married  $\rightarrow$  No Purchase Incentive  $\rightarrow E[\pi] = 50$
- If Visit Site  $\wedge$  Not Married  $\rightarrow$  No Purchase Incentive  $\rightarrow E[\pi] = 30$

Expected profit =  $\boxed{40} = (50+30)/2$ .



### 3. Interventional Inference + Full Randomization

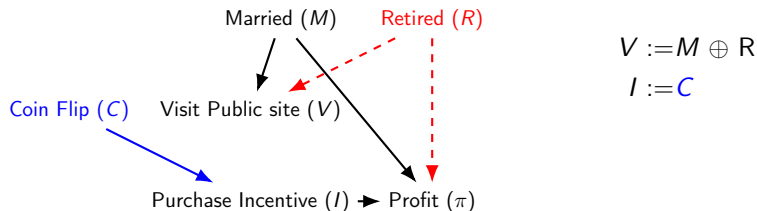


Figure: Causal DAG with Full Randomization.

### 3. Interventional Inference + Full Randomization

$$\mathcal{D}_{\text{IFR}}^*(M) = \operatorname{argmax}_{I \in \{0,1\}} E[\pi | do(I), M]$$

$$E[\pi | do(I = 1), M = 1] = 35.0 = (25 + 45)/2$$

$$E[\pi | do(I = 0), M = 1] = 27.5 = (50 + 5)/2$$

$$E[\pi | do(I = 1), M = 0] = 27.5 = (50 + 5)/2$$

$$E[\pi | do(I = 0), M = 0] = 20.0 = (10 + 30)/2$$

	$R = 0$		$R = 1$	
	$M = 1$	$M = 0$	$M = 1$	$M = 0$
$I = 1$	25	50	45	5
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	$R = 0$		$R = 1$	
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$I = 1$	25	50	45	5
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Table:  $E[\pi | M, R, I]$ .

#### Decision Rule:

- If Married  $\rightarrow$  Purchase Incentive  $\rightarrow E[\pi] = 35$
- If Not Married  $\rightarrow$  Purchase Incentive  $\rightarrow E[\pi] = 27.5$

$$\text{Expected profit} = \boxed{31.5} = (35 + 27.5)/2.$$

## 4. Counterfactual Inference

$$\mathcal{D}_{\text{CI}}^*(M, I) = \operatorname{argmax}_{a' \in \{0,1\}} E[\pi_{a'} | I = a, M]$$

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$$\mathcal{D}_{CI}^*(M, I) = \operatorname{argmax}_{a' \in \{0,1\}} E[\pi_{a'} | I = a, M]$$

- 1 Do I need to assume a parametric model to identify this causal estimand?
- 2 Do I need to assume that the conditioning set  $\{M\}$  satisfies the *backdoor criterion* to identify this causal estimand?

## 4. Counterfactual Inference

$$\mathcal{D}_{CI}^*(M, I) = \operatorname{argmax}_{a' \in \{0,1\}} E[\pi_{a'} | I = a, M]$$

$$\begin{aligned} P(\pi_{a'}, M) &= P(\pi_{a'}, M, a') + P(\pi_{a'}, M, a) \\ &= P(\pi_{a'} | M, a') P(M, a') + P(\pi_{a'} | M, a) P(M, a) \end{aligned}$$

$$\begin{aligned} P(\pi_{a'} | M) &= P(\pi_{a'} | M, a') P(a' | M) + P(\pi_{a'} | M, a) P(a | M) \\ &= P(\pi | M, a') P(a' | M) + P(\pi_{a'} | M, a) P(a | M) \quad (\text{Consistency}) \end{aligned}$$

$$P(\pi_{a'} | M, a) = \frac{1}{P(a | M)} \left[ P(\pi_{a'} | M) - P(\pi | M, a') P(a' | M) \right]$$

$$= \underbrace{\frac{1}{P(a | M)}}_{\text{observational}} \left[ \overbrace{P(\pi | M, do(a'))}^{\text{experimental}} - \underbrace{P(\pi | M, a') P(a' | M)}_{\text{observational}} \right]$$

## 4. Counterfactual Inference

$$E(\pi_{I=1}|M=1, I=0) =$$

$$\begin{aligned} & \frac{1}{P(I=0|M=1)} \left[ E(\pi|M=1, do(I=1)) - \right. \\ & \quad \left. E(\pi|M=1, I=1)P(I=1|M=1) \right]. \\ & = \frac{1}{1/2} (35 - 25 \times 1/2) = 45 \\ & > 5 = E(\pi_{I=0}|M=1, I=0). \end{aligned}$$

$$E(\pi_{I=1}|M=0, I=0) = 50$$

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$$E(\pi_{I=0}|M=0, I=1) = 30$$

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**Decision Rule:**

- If Not Visit Site  $\wedge$  Married  $\rightarrow$  Purchase Incentive  $\rightarrow E[\pi] = 45$
- If Not Visit Site  $\wedge$  Not Married  $\rightarrow$  Purchase Incentive  $\rightarrow E[\pi] = 50$
- If Visit Site  $\wedge$  Married  $\rightarrow$  No Purchase Incentive  $\rightarrow E[\pi] = 50$
- If Visit Site  $\wedge$  Not Married  $\rightarrow$  No Purchase Incentive  $\rightarrow E[\pi] = 30$

$$\text{Expected profit} = \boxed{43.75} = (45 + 50 + 50 + 30)/4.$$

	R = 0		R = 1	
	M = 1	M = 0	M = 1	M = 0
I = 1	25	50	45	5
I = 0	50	10	5	30

Table:  $E[\pi|M, R, I]$ .



# Summary of Methods

Criterion	Decision Rule	$E[\pi]$
$\mathcal{D}_{AI}$	<ul style="list-style-type: none"> <li>• If Visit Site <math>\wedge</math> Married <math>\rightarrow</math> <b>Purchase Incentive</b></li> <li>• If Visit Site <math>\wedge</math> Not Married <math>\rightarrow</math> <b>No Purchase Incentive</b></li> </ul>	27.50
$\mathcal{D}_{IPVR}$	<b>Never Purchase Incentive</b>	40.0
$\mathcal{D}_{IFR}$	<b>Always Purchase Incentive</b>	31.50
$\mathcal{D}_{CI}$	<ul style="list-style-type: none"> <li>• If Visit Site <math>\wedge</math> Married <math>\rightarrow</math> <b>No Purchase Incentive</b></li> <li>• If Visit Site <math>\wedge</math> Not Married <math>\rightarrow</math> <b>No Purchase Incentive</b></li> <li>• If Not Visit Site <math>\wedge</math> Married <math>\rightarrow</math> <b>Purchase Incentive</b></li> <li>• If Not Visit Site <math>\wedge</math> Not Married <math>\rightarrow</math> <b>Purchase Incentive</b></li> </ul>	43.75
$\mathcal{D}_{Oracle}$		43.75

## Remarks

- Experimental data are 'gold standard' in the non-personalized paradigm because they remove the influence of unobserved confounders.
- In the personalization paradigm, experimental data alone is not 'gold standard' for estimating heterogeneous treatment effects in the presence of unobserved and heterogeneous confounders.
- Experiments 'destroy' information that can be valuable to identify the values of unobserved confounders.
- Counterfactual-based decision making, which leads to a fusion of experimental and observational data, might be optimal for personalization.

## Further Reading

- The expression derived from RDC works only in the binary treatment case. RDC-type randomization (Forney et al., 2017) was proposed to estimate counterfactual expressions empirically from an arbitrary number of treatments.
- This presentation is fundamentally inspired by this paper:
  - ▶ Elias Bareinboim, Andrew Forney, and Judea Pearl. 2015. Bandits with unobserved confounders: a causal approach. In Proceedings of the 28th International Conference on Neural Information Processing Systems - Volume 1 (NIPS'15).
  - ▶ Implementation: <https://github.com/leoguelman/mabuc>