Personalization with Unobserved Heterogeneity

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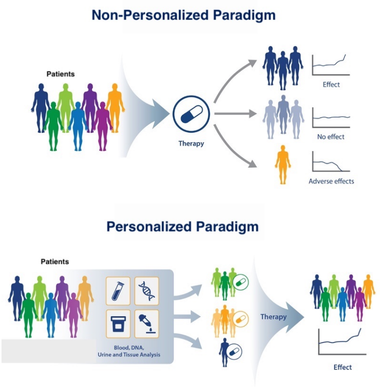
Motivation for Personalization

- Personalization is founded on the premise that individuals have heterogeneous responses to actions.
- Personalization algorithms aim to improve decision-making by identifying and exploiting this heterogeneity.



Unobserved and Heterogeneous Confounder (UHC)

- Treatment effect (T) varies according to the value of an unobserved confounder (U).

$$T := f(U) + N_T$$
$$Y := f(T, U, \boldsymbol{T} \times \boldsymbol{U}) + N_Y$$

- Presence of UHCs is arguably the most sensible assumption in practice.
- UHCs introduce challenges to personalization.

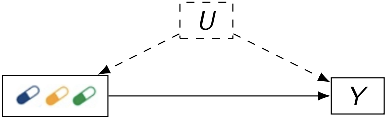


Figure: Observational setting.

Motivating Questions

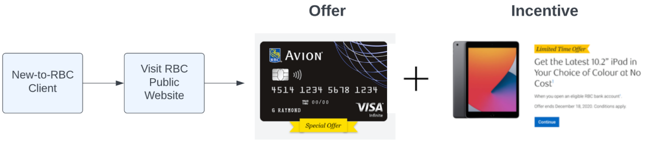
Given the goal of assigning the best treatment to each individual, and the context of UHCs:

- What alternatives do I have to express my objective optim. function?
- What is the corresponding data I need in each case?
- Which of these alternatives can theoretically achieve the oracle treatment assignment?

Out-of-scope: Estimation (e.g., compare different estimators for personalization).

Business Setting

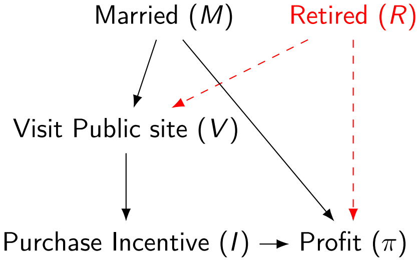
- **Business objective:** Sell a credit card to new-to-RBC clients.
- **Current campaign**. All new-to-RBC clients who visit the RBC public site get a credit card offer + iPad incentive.



- **Business Goal: Personalize the incentive**. Identify which new-to-RBC clients should receive an iPad incentive to maximize the expected profitability of the campaign.

$$\text{Profit} = \text{Revenue - Cost of iPad}.$$

Data Generating Process



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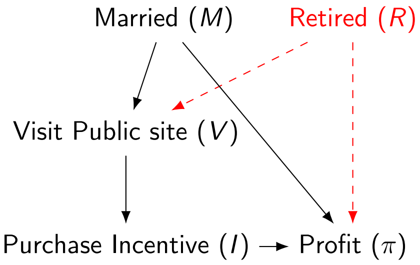

Figure: Observational setting.

$$P(R=1) = 0.5\ ,\ \ P(M=1) = 0.5$$
$$V := M \oplus \mathrm{R}$$
$$I := V$$

	$R=0$		$R=1$	
	$M=1$	$M=0$	$M=1$	$M=0$
$I=1$	25	50	45	5
$I=0$	50	10	5	30

Table: $E[\pi|M,R,I]$. Highlighted cells reflect (new-to-RBC) client's 'natural' choice to visit the Public site or not.

Four Approaches to Personalizing the Incentive

Business Goal: Identify which new-to-RBC clients should receive an iPad incentive to maximize the expected profitability of the campaign.

1. Associational Inference

$$\mathcal{D}^*_{\mathsf{AI}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \; E[\pi | I, M]$$

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Decision Rule:

- If Visit Site $\wedge$ Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{25}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{30}$

Expected profit $= \boxed{27.5} = (25+30)/2$.

2. Interventional Inference + Post-Visit Randomization

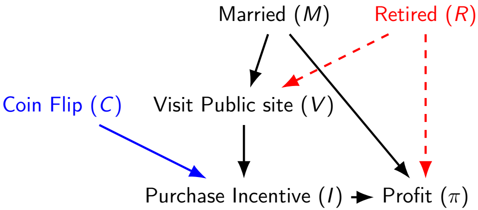

Figure: Causal DAG with post-visit randomization.

$$V := M \oplus \mathrm{R}$$
$$I := V \wedge C$$

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Table: $E[\pi|M, R, I]$.

Decision Rule:

- If Visit Site $\wedge$ Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{50}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{30}$

Expected profit = $\boxed{40}$ = (50+30)/2.

3. Interventional Inference + Full Randomization

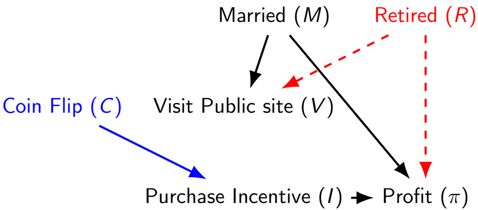

Figure: Causal DAG with Full Randomization.

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$E[\pi | do(I = 1), M = 1] = 35.0 = (25 + 45)/2$
$E[\pi | do(I = 0), M = 1] = 27.5 = (50 + 5)/2$
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Table: $E[\pi|M, R, I]$.

Decision Rule:

- If Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] =$ **35**
- If Not Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] =$ **27.5**

Expected profit $= \boxed{31.5} = (35+27.5)/2.$

4. Regret Decision Criterion

Puzzled by $E[\pi|I, M] \neq E[\pi|do(I), M]$.

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Eq.1 is a population-level counterfactual known as the *Conditional Average Treatment Effect on the Treated* (CATT).

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In general, identifiability of the CATT requires the causal graph (so I can condition on a set of covariates that satisfy the backdoor criterion).

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Is this causal estimand identifiable?

In general, identifiability of the CATT requires the causal graph (so I can condition on a set of covariates that satisfy the backdoor criterion).

An exception where we don't need the graph is when the treatment is binary and both experimental and observational data are available.

4. Regret Decision Criterion

$$\mathcal{D}^*_{\mathrm{RDC}}(M, I) = \underset{a' \in \ 0,1}{\mathrm{argmax}} \ E[\pi_{a'} | I = a, M]$$

$$\begin{aligned} P(\pi_{a'}, M) &= P(\pi_{a'}, M, a') + P(\pi_{a'}, M, a) \\ &= P(\pi_{a'} | M, a')P(M, a') + P(\pi_{a'} | M, a)P(M, a) \end{aligned}$$

$$\begin{aligned} P(\pi_{a'} | M) &= P(\pi_{a'} | M, a')P(a' | M) + P(\pi_{a'} | M, a)P(a | M) \\ &= P(\pi | M, a')P(a' | M) + P(\pi_{a'} | M, a)P(a | M) \ \text{(Consistency)} \end{aligned}$$

$$\begin{aligned} P(\pi_{a'} | M, a) &= \frac{1}{P(a|M)} \left[P(\pi_{a'} | M) - P(\pi | M, a')P(a' | M) \right] \\ &= \boxed{\underbrace{\frac{1}{P(a|M)}}_{\textbf{observational}} \left[\overbrace{P\Big(\pi | M, do(a')\Big)}^{\textbf{experimental}} - \underbrace{P(\pi | M, a')P(a' | M)}_{\textbf{observational}} \right]} \end{aligned}$$

4. Regret Decision Criterion

$$\boxed{E(\pi_{I=\mathbf{1}}|M=1, I=0)} =$$

$$\frac{1}{P(I=0|M=1)}\Big[E\Big(\pi|M=1, do(I=1)\Big) - E(\pi|M=1, I=1)P(I=1|M=1)\Big].$$

$$= \frac{1}{1/2}(35 - 25 \times 1/2) = \mathbf{45}$$

$$> 5 = E(\pi_{I=\mathbf{0}}|M=1, I=0).$$

$$\boxed{E(\pi_{I=\mathbf{1}}|M=0, I=0)} = \mathbf{50}$$

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$$= \frac{1}{1/2}(35 - 25 \times 1/2) = \mathbf{45}$$

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Table: $E[\pi|M, R, I]$.

Decision Rule:

- If Not Visit Site $\wedge$ Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{45}$
- If Not Visit Site $\wedge$ Not Married $\rightarrow$ Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{50}$
- If Visit Site $\wedge$ Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{50}$
- If Visit Site $\wedge$ Not Married $\rightarrow$ No Purchase Incentive $\rightarrow$ $E[\pi] = \mathbf{30}$

Expected profit $= \boxed{43.75} = (45 + 50 + 50 + 30)/4$.

Summary of Methods

Criterion	Decision Rule	$E[\pi]$
$\mathcal{D}_{AI}$	• If Visit Site ∧ Married → **Purchase Incentive** • If Visit Site ∧ Not Married → **No Purchase Incentive**	27.50
$\mathcal{D}_{IPVR}$	**Never Purchase Incentive**	40.0
$\mathcal{D}_{IFR}$	**Always Purchase Incentive**	31.50
$\mathcal{D}_{RDC}$	• If Visit Site ∧ Married → **No Purchase Incentive** • If Visit Site ∧ Not Married → **No Purchase Incentive** • If Not Visit Site ∧ Married → **Purchase Incentive** • If Not Visit Site ∧ Not Married → **Purchase Incentive**	**43.75**
$\mathcal{D}_{\textbf{Oracle}}$		**43.75**

Remarks

- All approaches used population/sub-population level estimands. There's nothing personal about personalization!
- Experimental data are 'gold standard' in the non-personalized paradigm because they remove the influence of unobserved confounders.
- In the personalization paradigm with UHCs, experimental data are not necessarily 'gold standard' for identifying heterogeneous treatment effects:
    - Experiments 'destroy' uncoded knowledge about these confounders.
    - Observational data (usually discarded when experimental data are available) might capture additional information about UHCs required to maximize payout.

Further Reading

- This presentation is fundamentally inspired by this paper:
  - Elias Bareinboim, Andrew Forney, and Judea Pearl. 2015. **Bandits with unobserved confounders: a causal approach**. In Proceedings of the 28th International Conference on Neural Information Processing Systems - Volume 1 (NIPS'15).
  - Implementation: `https://github.com/leoguelman/mabuc`.
- The expression derived from RDC works only in the binary treatment case.
  - RDC-type randomization (Forney et al., 2017) was proposed to estimate counterfactual expressions empirically from an arbitrary number of treatments.