Personalization with Unobserved Heterogeneity

Leo Guelman

Head Statistician, DNA RBC Royal Bank

Motivation for Personalization

- Personalization is founded on the premise that individuals have heterogeneous responses to actions.
- Personalization algorithms aim to improve decision-making by identifying and exploiting this heterogeneity.



Personalized Paradigm



Unobserved and Heterogeneous Confounder (UHC)

 Treatment effect (T) varies according to the value of an unobserved confounder (U).

$$T := f(U) + N_T$$

$$Y := f(T, U, T \times U) + N_Y$$

- Existence of UHCs is the most sensible assumption in practice.
- UHCs introduce challenges to personalization.

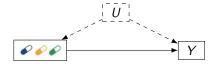


Figure: Observational setting.

Motivating Questions

Given the high-level goal of personalization, and the context of UHCs:

- What alternatives do I have to formulate this problem? For instance, what is a suitable causal estimand?
- What data do I need for identification?
- Is experimental data 'gold standard'?

Out-of-scope: Estimation (e.g., compare different estimators for personalization).

Business Setting

- Business objective: Sell a credit card to new-to-RBC clients.
- **Current campaign**. All new-to-RBC clients who visited the RBC public site get a credit card offer + iPad incentive.



 Business Goal: Personalize the incentive. Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.

Data Generating Process

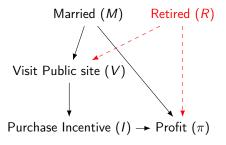


Figure: Observational setting.

Data Generating Process

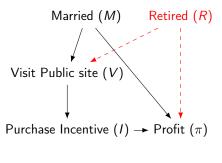


Figure: Observational setting.

$$P(R = 1) = 0.5$$
, $P(M = 1) = 0.5$
 $V := M \oplus R$
 $I := V$

	R = 0		R = 1	
	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
I = 0	50	10	5	30

Table: $E[\pi|M,R,I]$. Highlighted cells reflect (new-to-RBC) client's 'natural' choice to visit the Public site or not.

Four Approaches to Personalizing the Incentive

Business Goal: Identify which new-to-RBC clients should receive an iPad incentive in the future to maximize the expected profitability of the campaign.



1. Associational Inference



$$\mathcal{D}^*_{\mathsf{AI}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi|I,M]$$

1. Associational Inference



$$\mathcal{D}_{\mathsf{AI}}^*(M) = \underset{I \in \mathsf{0},1}{\operatorname{argmax}} \ E[\pi|I,M]$$

$$E[\pi|I=1, M=1] = 25$$

 $E[\pi|I=0, M=1] = 5$
 $E[\pi|I=1, M=0] = 5$
 $E[\pi|I=0, M=0] = 10$

	R = 0		R = 1	
	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
I = 0	50	10	5	30

Table: $E[\pi|M, R, I]$.

1. Associational Inference



$$\mathcal{D}_{\mathsf{AI}}^*(M) = \underset{I \in \mathsf{0},1}{\operatorname{argmax}} \ E[\pi|I,M]$$

$$E[\pi|I=1, M=1] = 25$$

 $E[\pi|I=0, M=1] = 5$
 $E[\pi|I=1, M=0] = 5$
 $E[\pi|I=0, M=0] = 10$

	R = 0		R = 1	
	M = 1	M = 0	M = 1	M = 0
I = 1	25	50	45	5
I = 0	50	10	5	30

Table: $E[\pi|M,R,I]$.

Decision Rule:

- If Visit Site \wedge Married \rightarrow Purchase Incentive \rightarrow $E[\pi] = 25$
- If Visit Site \land Not Married \rightarrow No Purchase Incentive \rightarrow $E[\pi] = 30$

Expected profit =
$$27.5$$
 = $(25+30)/2$.



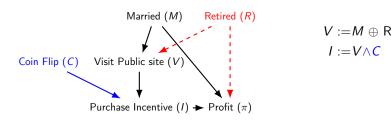


Figure: Causal DAG with post-visit randomization.



$$\mathcal{D}^*_{\mathsf{IPVR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi|do(I), M, V = 1]$$



$$\mathcal{D}^*_{\mathsf{IPVR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi | do(I), M, V = 1]$$

$$E[\pi|do(I=1), M=1, V=1] = 25$$

 $E[\pi|do(I=0), M=1, V=1] = 50$
 $E[\pi|do(I=1), M=0, V=1] = 5$
 $E[\pi|do(I=0), M=0, V=1] = 30$

	R = 0		R =	= 1
	M = 1	M = 0	M = 1	M = 0
I = 1	25	50	45	5
I = 0	50	10	5	30

Table: $E[\pi|M,R,I]$.



$$\mathcal{D}^*_{\mathsf{IPVR}}(M) = \underset{I \in [0,1]}{\operatorname{argmax}} \ E[\pi|do(I), M, V = 1]$$

$$E[\pi|do(I=1), M=1, V=1] = 25$$

 $E[\pi|do(I=0), M=1, V=1] = 50$
 $E[\pi|do(I=1), M=0, V=1] = 5$
 $E[\pi|do(I=0), M=0, V=1] = 30$

	R = 0			= 1
	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
<i>I</i> = 0	50	10	5	30

Table: $E[\pi|M, R, I]$.

Decision Rule:

- If Visit Site \land Married \rightarrow No Purchase Incentive \rightarrow $E[\pi] = 50$
- If Visit Site \wedge Not Married \rightarrow No Purchase Incentive \rightarrow $E[\pi] = 30$

Expected profit =
$$\boxed{40}$$
 = $(50+30)/2$.

3. Interventional Inference + Full Randomization



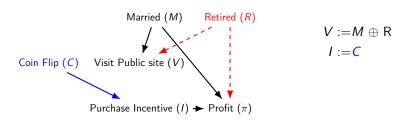


Figure: Causal DAG with Full Randomization.

3. Interventional Inference + Full Randomization



$$\mathcal{D}^*_{\mathsf{IFR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi|do(I), M]$$

$$E[\pi|do(I=1), M=1] = 35.0 = (25+45)/2$$

 $E[\pi|do(I=0), M=1] = 27.5 = (50+5)/2$
 $E[\pi|do(I=1), M=0] = 27.5 = (50+5)/2$
 $E[\pi|do(I=0), M=0] = 20.0 = (10+30)/2$

	R = 0		R = 1	
	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
I = 0	50	10	5	30

Table: $E[\pi|M,R,I]$.

3. Interventional Inference + Full Randomization



$$\mathcal{D}^*_{\mathsf{IFR}}(M) = \underset{I \in 0,1}{\operatorname{argmax}} \ E[\pi|do(I), M]$$

$$E[\pi|do(I=1), M=1] = 35.0 = (25 + 45)/2$$

 $E[\pi|do(I=0), M=1] = 27.5 = (50 + 5)/2$
 $E[\pi|do(I=1), M=0] = 27.5 = (50 + 5)/2$
 $E[\pi|do(I=0), M=0] = 20.0 = (10 + 30)/2$

	R = 0		R = 1	
-	M = 1	M = 0	M = 1	M = 0
/ = 1	25	50	45	5
I = 0	50	10	5	30

Table: $E[\pi|M,R,I]$.

Decision Rule:

- If Married \rightarrow Purchase Incentive \rightarrow $E[\pi] = 35$
- If Not Married \rightarrow Purchase Incentive \rightarrow $E[\pi] = 27.5$

Expected profit =
$$\boxed{31.5}$$
 = $(35+27.5)/2$.



$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'}|I=a,M]$$



$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'}|I=a,M]$$

• Do we need to assume a parametric model to identify this causal estimand?



$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'}|I = a,M]$$

- Do we need to assume a parametric model to identify this causal estimand?
 - No. Only unit-level counterfactuals require a parametric model for identification.
 - ▶ There is nothing personal about personalization!



$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'}|I = a,M]$$

- Oo we need to assume a parametric model to identify this causal estimand?
 - No. Only unit-level counterfactuals require a parametric model for identification.
 - There is nothing personal about personalization!
- ② Do we need to assume that the conditioning set $\{M\}$ satisfies the *backdoor criterion* to identify this causal estimand?



$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in \ 0,1}{\operatorname{argmax}} \ E[\pi_{a'}|I = a,M]$$

- Oo we need to assume a parametric model to identify this causal estimand?
 - No. Only unit-level counterfactuals require a parametric model for identification.
 - There is nothing personal about personalization!
- ② Do we need to assume that the conditioning set $\{M\}$ satisfies the *backdoor criterion* to identify this causal estimand?
 - In general, population-level counterfactuals can be identified non-parametrically using just the causal graph − i.e., assuming the conditioning set Z = {M} satisfies the backdoor criterion.
 - An exception is when the treatment is binary and both experimental and observational data are available.



$$\mathcal{D}^*_{\mathsf{RDC}}(M,I) = \underset{a' \in [0,1]}{\operatorname{argmax}} \ E[\pi_{a'}|I = a,M]$$

$$P(\pi_{a'},M) = P(\pi_{a'},M,a') + P(\pi_{a'},M,a)$$

$$= P(\pi_{a'}|M,a')P(M,a') + P(\pi_{a'}|M,a)P(M,a)$$

$$P(\pi_{a'}|M) = P(\pi_{a'}|M,a')P(a'|M) + P(\pi_{a'}|M,a)P(a|M)$$

$$= P(\pi|M,a')P(a'|M) + P(\pi_{a'}|M,a)P(a|M) \text{ (Consistency)}$$

$$P(\pi_{a'}|M,a) = \frac{1}{P(a|M)} \Big[P(\pi_{a'}|M) - P(\pi|M,a')P(a'|M) \Big]$$

$$= \underbrace{\frac{1}{P(a|M)} \Big[P(\pi_{a'}|M,a) - P(\pi|M,a')P(a'|M) \Big]}_{\text{observational}}$$



$$E(\pi_{I=1}|M=1,I=0) =$$

$$\begin{split} \frac{1}{P(I=0|M=1)} \Big[E\Big(\pi|M=1, do(I=1)\Big) - \\ E(\pi|M=1, I=1) P(I=1|M=1) \Big]. \\ &= \frac{1}{1/2} (35-25\times 1/2) = 45 \\ &> 5 = E(\pi_{I=0}|M=1, I=0). \end{split}$$

$$E(\pi_{I=1}|M=0,I=0)$$
 = 50

$$E(\pi_{I=0}|M=1,I=1) = 50$$

$$E(\pi_{I=0}|M=0,I=1)$$
 = 30

	R = 0		R = 1	
	M = 1	<i>M</i> = 0	M = 1	M = 0
/ = 1	25	50	45	5
<i>I</i> = 0	50	10	5	30

Table: $E[\pi|M,R,I]$.



$$E(\pi_{I=1}|M=1,I=0) =$$

$$\frac{1}{P(I=0|M=1)} \left[E\left(\pi|M=1, do(I=1)\right) - \\ E\left(\pi|M=1, I=1\right) P(I=1|M=1) \right].$$

$$= \frac{1}{1/2} (35 - 25 \times 1/2) = 45$$

$$> 5 = E\left(\pi_{I=0} \mid M=1, I=0\right).$$

	R =	= 0	R =	= 1
	M = 1	M = 0	M = 1	M = 0
/ = 1 / = 0	25 50	50 10	45 5	5 30

Table: $E[\pi|M,R,I]$.

$$E(\pi_{I=1}|M=0, I=0) = 50$$

$$E(\pi_{I=0}|M=1, I=1) = 50$$

$$E(\pi_{I=0}|M=0, I=1) = 30$$

Decision Rule:

- If Not Visit Site \land Married \rightarrow Purchase Incentive \rightarrow $E[\pi] = 45$
- ullet If Not Visit Site \wedge Not Married o Purchase Incentive o $E[\pi] = {f 50}$
- If Visit Site \land Married \rightarrow No Purchase Incentive \rightarrow $E[\pi] = 50$
- If Visit Site \wedge Not Married \rightarrow No Purchase Incentive \rightarrow $E[\pi] = 30$

Expected profit =
$$\boxed{43.75}$$
 = $(45 + 50 + 50 + 30)/4$.

Summary of Methods

Criterion	Decision Rule	$E[\pi]$
\mathcal{D}_{AI}	 If Visit Site ∧ Married → Purchase Incentive If Visit Site ∧ Not Married → No 	27.50
$\mathcal{D}_{\mathit{IPVR}}$	Purchase Incentive Never Purchase Incentive	40.0
$\mathcal{D}_{\mathit{IFR}}$	Always Purchase Incentive	31.50
\mathcal{D}_{RDC}	• If Visit Site \wedge Married \rightarrow No Purchase Incentive	43.75
	• If Visit Site \wedge Not Married \rightarrow No Purchase Incentive	
	• If Not Visit Site \land Married \rightarrow Purchase Incentive	
	$ \begin{tabular}{ll} \bullet & \mbox{If Not Visit Site} \land \mbox{Not Married} \rightarrow \\ \mbox{Purchase Incentive} \\ \end{tabular} $	
\mathcal{D}_{Oracle}		43.75

Remarks

- Experimental data are 'gold standard' in the non-personalized paradigm because they remove the influence of unobserved confounders.
- In the personalization paradigm, experimental data alone is not 'gold standard' for estimating heterogeneous treatment effects in the presence of (UHCs).
- Experiments 'destroy' information that can be valuable to 'recover' these confounders.
- Counterfactual-based decision-making leads to a fusion of experimental and observational data for personalization.

Further Reading

- This presentation is fundamentally inspired by this paper:
 - ▶ Elias Bareinboim, Andrew Forney, and Judea Pearl. 2015. Bandits with unobserved confounders: a causal approach. In Proceedings of the 28th International Conference on Neural Information Processing Systems Volume 1 (NIPS'15).
 - ► Implementation: https://github.com/leoguelman/mabuc.
- The expression derived from RDC works only in the binary treatment case.
 - ▶ RDC-type randomization (Forney et al., 2017) was proposed to estimate counterfactual expressions empirically from an arbitrary number of treatments.