Homework 8

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April 4, 2025

Remark: Recall that a sheaf $\mathcal{F}$ on a space X is flabby if for every $U \subseteq V$, the restriction $\mathcal{F}(V) \to \mathcal{F}(U)$ is surjective. (Or equivalently, every $\mathcal{F}(X) \to \mathcal{F}(U)$ is surjective)

Exercise 1: Suppose

$$0 \longrightarrow \mathcal{F} \longrightarrow \mathcal{G} \longrightarrow \mathcal{H} \longrightarrow 0$$

is an exact sequence of sheaves.

1. Show that if $\mathcal{F}$ is flabby, then

$$0 \longrightarrow \mathcal{F}(U) \longrightarrow \mathcal{G}(U) \longrightarrow \mathcal{H}(U) \longrightarrow 0$$

   is exact for every $U \subseteq X$ open.

2. Show that if $\mathcal{F}$ and $\mathcal{G}$ are flabby, then $\mathcal{H}$ is flabby.

Hint for 1: *For surjectivity, fix $s \in \mathcal{H}(U)$ and consider the set of pairs (t, V) where $V \subseteq U$ open and t is mapped to $s|_V$ along the morphism $\mathcal{G}(U) \to \mathcal{H}(U)$. Then use Zorn's Lemma.*