Counting Problems

① Consider the word unusual. How many unique subsets of 5 letters of 7 exist? How many diff strings could be made from 5 of those 7 letters?

unique subsets: $\binom{4}{4}\cdot\binom{3}{3}\cdot\binom{4}{2}$

different strings from 5 letters: $5!\cdot\left(\frac{5!}{2!\,1!\,1!}\right)\cdot\left(\frac{5!}{3!\,1!\,1!}\right)$

3 U's, 3 places
2 N's, 2 places

② standard deck of playing cards, how many ways to form 5-card hand w/ 2 pairs?

2 values, pairs $\binom{13}{2}$

2 suits out of 4, 1st pair $\binom{4}{2}$
2 suits out of 4, 2nd pair $\binom{4}{2}$

5th card, there are 11 values after 2 are pick $\binom{11}{1}$ and 1 suit chosen out of 4 $\binom{4}{1}$

$= \binom{13}{2}\cdot\binom{4}{2}\cdot\binom{4}{2}\binom{11}{1}\binom{4}{1} = 78\cdot 6\cdot 6\cdot 11\cdot 4 = 123{,}552$

③ Vocabulary Serenade Problem

$\binom{16}{1}\cdot\binom{15}{6} =$

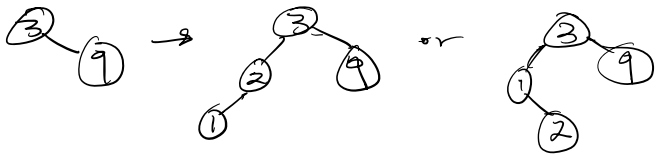
$\frac{16!}{1!(16-1)!} \cdot \frac{15!}{5!\times(15-6)!}$

$\rightarrow 16 \cdot 5005 = 80{,}080$ # of ways

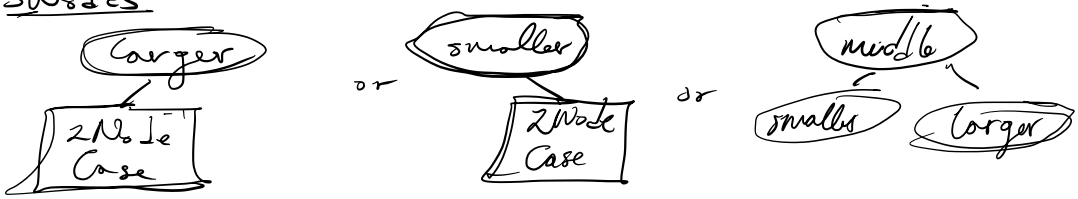
⑧ BST Problem

12 nodes, 1-12, Root = 3, RC = 9

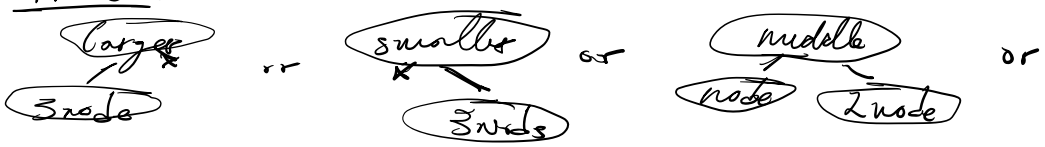
2 Nodes

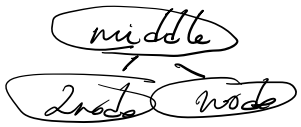


3 Nodes



4 Nodes





5 Nodes

= 2 × 5 × 42 ways

⑤ Covid Vaccine Problem

different combinations are there for the numbers of patients served by nurses served by nurses.?

$$\frac{10!}{(10-4)4!} = \frac{10!}{6!4!} = \frac{10\times9\times8\times7\times\cancel{6\times5\times4\times3\times2}}{\cancel{6\times5\times4\times3\times2}\cdot 4\times3\times2}$$

$$= 10\times3\times7$$

$$= 210 \text{ minutes}$$

Nurses

$$10P_4 = \frac{10!}{(10-4)!} = \frac{10\times9\times8\times7\times\cancel{6\times5\times4\times3\times2}}{\cancel{6\times5\times4\times3\times2}}$$

$$= 10\times8\times7 = 5040 \text{ Combinations}$$