4.4 - Recursive Algorithms

Monday, March 9, 2020 8:08 AM

- Example
  - $2^n = 2 \cdot 2^{n-1}$
  - $f(n) = 2f(n-1)$
- Example
  - $n! = n \cdot (n-1)!$
- Example: Fibonacci Numbers
  - $f_{n+1} = f_n + f_{n-1}$
- Example - 47
  - $\sum_{k=1}^{n} f_k = f_{n+2} - 1; n \geq 1$
  - Proof (using induction)
    - $n = 1$
      - $f_1 = 1$
      - $f_1 = f_3 - 1 = 2 - 1 = 1$
    - Assume valid up to n
      - $\sum_{k=1}^{n+1} f_k = \sum_{k=1}^{n} f_k + f_{n+1}$
      - By assumption $\sum_{k=1}^{n} f_k = f_{n+2} - 1$
      - $= f_{n+2} - 1 + f_{n+1} = f_{n+3} - 1$
- Example - 27
  - $f_n =$ fibonacci number
  - For $n \geq 6, f_n > \left(\frac{3}{2}\right)^{n-1}$
  - Induction
    - $n = 6$
      - $f_6 = 8 > \left(\frac{3}{2}\right)^5$
    - Assume true until n
      - $f_{n+1} > \left(\frac{3}{2}\right)^n$
      - $f_{n+1} = f_n + f_{n-1}$
      - $f_{n+1} > \left(\frac{3}{2}\right)^{n-1} + \left(\frac{3}{2}\right)^{n-2}$
      - $f_{n+1} > \left(\frac{3}{2}\right)^{n-2} \left(\frac{3}{2} + 1\right)$
      - $f_{n+1} > \left(\frac{3}{2}\right)^{n-2} \left(\frac{5}{2}\right)$
- Example - 28
  - $f_n \leq 2^{n-1}, n \geq 1$
  - Induction
    - $n = 1$
      - $f_1 = 1 = 2^{1-1}$
    - Assume true up to n
      - $f_{n+1} = f_n + f_{n-1}$
      - $f_{n+1} \leq 2^n?$
      - $f_{n+1} \leq 2^{n-1} + 2^{n-2} = 2^{n-2}(2+1) \leq 2^{n-2} \cdot 4 = 2^n$