Using the Rules of Inference to Build Valid Arguments 추론규칙을 사용하여 유효한 주장 만들기

A *valid argument 유효한 주장* is a sequence of statements 일련의 문장들.

Each statement 각 문장 is either a premise 전제 or follows from previous statements by rules of inference 추론규칙으로 유도한 문장.

마지막 문장이 결론 The last statement is called conclusion.

A valid argument takes the following form 다음과 같은 형태:

$$S_1$$

$$S_2$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$S_n$$

$$\therefore C$$

Valid Arguments 2

Example 1: From the single proposition

$$p \wedge (p \rightarrow q)$$

Show that *q* is a conclusion.

Solution:

Step	**Reason**
1. $p \wedge (p \rightarrow q)$	Premise
2. p	Simplification using (1)
3. $p \rightarrow q$	Simplification using (1)
4. q	Modus Ponens using (2) and (3)

Valid Arguments 3

Example 2:

With these hypotheses:

"It is not sunny this afternoon and it is colder than yesterday."

"We will go swimming only if it is sunny."

"If we do not go swimming, then we will take a canoe trip."

"If we take a canoe trip, then we will be home by sunset."

Using the inference rules, construct a valid argument for the conclusion:

"We will be home by sunset."

Solution:

1. Choose propositional variables:

p : "It is sunny this afternoon." *r* : "We will go swimming." *t :* "We will be home by sunset."

q : "It is colder than yesterday." *s :* "We will take a canoe trip."

2. Translation into propositional logic:

$$\text{Hypotheses: } \neg p \wedge q, r \rightarrow p, \neg r \rightarrow s, s \rightarrow t$$

$$\text{Conclusion: } t$$

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Valid Arguments 4

3. Construct the Valid Argument

Step	**Reason**
1. $\neg p \wedge q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \rightarrow s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \rightarrow t$	Premise
8. t	Modus ponens using (6) and (7)

Handling Quantified Statements
한정기호가 들어간 문장 다루기

Valid arguments for quantified statements are a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference which include:

- Rules of Inference for Propositional Logic
- Rules of Inference for Quantified Statements

The rules of inference for quantified statements are introduced in the next several slides.

Universal Instantiation (UI)

$$\frac{\forall x P(x)}{\therefore P(c)}$$

Example:

Our domain consists of all dogs and Fido is a dog.

"All dogs are cuddly."

"Therefore, Fido is cuddly."

Universal Generalization (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

Used often implicitly in Mathematical Proofs.

Existential Instantiation (EI)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

Example:

"There is someone who got an A in the course."

"Let's call her *a* and say that *a* got an A"

Existential Generalization (EG)

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$

Example:

"Michelle got an A in the class."

"Therefore, someone got an A in the class."

Using Rules of Inference [1]

Example 1: Using the rules of inference, construct a valid argument to show that

"John Smith has two legs"

is a consequence of the premises:

"Every man has two legs." "John Smith is a man."

Solution: Let $M(x)$ denote "x is a man" and $L(x)$ " x has two legs" and let John Smith be a member of the domain.

Valid Argument:

Step	**Reason**
1. $\forall x\left(M(x) \rightarrow L(x)\right)$	Premise
2. $M(J) \rightarrow \mathrm{L}(J)$	UI from (1)
3. $M(J)$	Premise
4. $L(J)$	Modus Ponens using (2) and (3)

Using Rules of Inference 2

Example 2: Use the rules of inference to construct a valid argument showing that the conclusion

"Someone who passed the first exam has not read the book."

follows from the premises

"A student in this class has not read the book."

"Everyone in this class passed the first exam."

Solution: Let $C(x)$ denote "x is in this class," $B(x)$ denote "x has read the book," and $P(x)$ denote "x passed the first exam."

First we translate the premises and conclusion into symbolic form.

$$\begin{array}{l} \exists x(C(x) \wedge \neg B(x)) \\ \forall x(C(x) \rightarrow P(x)) \\ \hline \therefore \exists x(P(x) \wedge \neg B(x)) \end{array}$$

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Using Rules of Inference [3]

Valid Argument:

Step	Reason
1. $\exists x\left(C(x) \wedge \neg B(x)\right)$	Premise
2. $C(a) \wedge \neg \mathrm{B}(a)$	EI from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x\left(C(x) \rightarrow P(x)\right)$	Premise
5. $C(a) \rightarrow P(a)$	UI from (4)
6. $P(a)$	MP from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \wedge \neg B(a)$	Conj from (6) and (7)
9. $\exists x\left(P(x) \wedge \neg B(x)\right)$	EG from (8)

Returning to the Socrates Example

$$\begin{array}{l}\forall x\left(Man\left(x\right) \rightarrow Mortal\left(x\right)\right) \\ \quad Man\left(Socrates\right) \\ \hline \therefore \quad Mortal\left(Socrates\right)\end{array}$$

Solution for Socrates Example

Valid Argument

Step	**Reason**
1. $\forall x\left(Man\left(x\right) \to Mortal\left(x\right)\right)$	Premise
2. $Mam\left(Socrates\right) \to Mortal\left(Socrates\right)$	UI from $\left(1\right)$
3. $Mam\left(Socrates\right)$	Premise
4. $Mortal\left(Socrates\right)$	MP from $\left(2\right)$ and $\left(3\right)$

Universal Modus Ponens

Universal Modus Ponens combines universal instantiation and modus ponens into one rule.

$$\begin{array}{l} \forall x\big(P(x) \to Q(x)\big) \\ P(a),\ \text{where } a \text{ is a particular} \\ \underline{\text{element in the domain}} \\ \qquad \therefore Q(a) \end{array}$$

This rule could be used in the Socrates example.

Introduction to Proofs

Section 1.7