Mc Graw Hill Education

The Foundations: Logic and Proofs 논리와 증명

Chapter 1, Part I: Propositional Logic
명제 논리

With Question/Answer Animations

Chapter Summary

Propositional Logic 명제

- The Language of Propositions
- Applications
- Logical Equivalences

Predicate Logic

- The Language of Quantifiers
- Logical Equivalences
- Nested Quantifiers

Proofs 증명

- Rules of Inference
- Proof Methods
- Proof Strategy

Propositional Logic Summary

The Language of Propositions

- Connectives 연결자
- Truth Values 진리값
- Truth Tables 진리표

Applications 응용

- Translating English Sentences 영어문장
- System Specifications
- Logic Puzzles
- Logic Circuits

Logical Equivalences 논리적 동치

- Important Equivalences
- Showing Equivalence
- Satisfiability

Propositional Logic

Section 1.1

Section Summary [1]

Propositions

Connectives

- Negation 부정
- Conjunction 논리곱
- Disjunction 논리합
- Implication 함축; contrapositive 대우, inverse 이, converse 역
- Biconditional 쌍조건

Truth Tables

Propositions

A *proposition* is a declarative sentence that is either true or false.
명제는 참 또는 거짓인 평서문

Examples of propositions:예제

a) The Moon is made of green cheese.

b) Trenton is the capital of New Jersey.

c) Toronto is the capital of Canada.

d) $1 + 0 = 1$

e) $0 + 0 = 2$

Examples that are not propositions.

a) Sit down!

b) What time is it?

c) $x + 1 = 2$

d) $x + y = z$

Propositional Logic

Constructing Propositions 명제 만들기

- Propositional Variables 명제 변수: *p*, *q*, *r*, *s*, …
- The proposition that is always true is denoted by **T** 항상 참인 명제 and the proposition that is always false is denoted by **F** **항상 거짓인 명제**

- Compound Propositions 복합 명제; constructed from logical connectives and other propositions 논리 연결자와 명제들로 구성
  - Negation ¬
  - Conjunction ∧
  - Disjunction ∨
  - Implication →
  - Biconditional ↔

Compound Propositions: Negation

The *negation* of a proposition p is denoted by $\neg p$ and has this truth table:

p	$\neg p$
T	F
F	T

Example: If p denotes "The earth is round.", then $\neg p$ denotes "It is not the case that the earth is round," or more simply "The earth is not round."

Conjunction 논리곱

The *conjunction* of propositions p and q is denoted by $p \wedge q$ and has this truth table:

p	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example: If p denotes "I am at home." and q denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining."

Disjunction 논리합

The *disjunction* of propositions p and q is denoted by $p \vee q$ and has this truth table:

p	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example: If p denotes "I am at home." and q denotes "It is raining." then $p \vee q$ denotes "I am at home or it is raining."

The Connective Or in English

In English "or" has two distinct meanings.

- "Inclusive Or" - In the sentence "Students who have taken CS202 or Math120 may take this class," we assume that students need to have taken one of the prerequisites, but may have taken both. This is the meaning of disjunction. For $p \vee q$ to be true, either one or both of p and q must be true.
- "Exclusive Or" - When reading the sentence "Soup or salad comes with this entrée," we do not expect to be able to get both soup and salad. This is the meaning of Exclusive Or (Xor). In $p \oplus q$, one of p and q must be true, but not both. The truth table for $\oplus$ is:

p	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication

If p and q are propositions, then $p \rightarrow q$ is a *conditional statement 조건문* or *implication 함축* which is read as "if p, then q" and has this truth table :

p	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: If p denotes "I am at home." and q denotes "It is raining." then $p \rightarrow q$ denotes "If I am at home then it is raining."

In $p \rightarrow q$, p is the *hypothesis 가정* (*antecedent* or *premise*) and q is the *conclusion* 결론(or *consequence*).

Understanding Implication 2

One way to view the logical conditional is to think of an obligation or contract.

- "If I am elected, then I will lower taxes."
- "If you get 100% on the final, then you will get an A."

If the politician is elected and does not lower taxes, then the voters can say that he or she has broken the campaign pledge. Something similar holds for the professor. This corresponds to the case where p is true and q is false.

Different Ways of Expressing $p \to q$

if p**, then** q

if p, q

q **unless** $\neg p$

q **if** p

q **whenever** p

q **follows from** p

p **implies** q

p **only if** q

q **when** p

p **is sufficient for** q

q **is necessary for** p

a necessary condition 필요조건 for p **is** q

a sufficient condition 충분조건 for q **is** p

Converse, Contrapositive, and Inverse

From $p \to q$ we can form new conditional statements .

- $q \to p$ is the **converse** of $p \to q$
- $\neg q \to \neg p$ is the **contrapositive** of $p \to q$
- $\neg p \to \neg q$ is the **inverse** of $p \to q$

Example: Find the converse, inverse, and contrapositive of "It raining is a sufficient condition for my not going to town."

Solution:

converse: If I do not go to town, then it is raining.

inverse: If it is not raining, then I will go to town.

contrapositive: If I go to town, then it is not raining.

Biconditional 쌍조건문

If p and q are propositions, then we can form the *biconditional* proposition $p \leftrightarrow q$, read as "p if and only if q." The biconditional $p \leftrightarrow q$ denotes the proposition with this truth table:

p	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

If p denotes "I am at home." and q denotes "It is raining." then $p \leftrightarrow q$ denotes "I am at home if and only if it is raining."

Expressing the Biconditional

Some alternative ways "p if and only if q" is expressed in English:

- p **is necessary and sufficient for** q
- **if** p **then** q, **and conversely**
- p **iff** q

Truth Tables For Compound Propositions 복합명제의 진리표

Construction of a truth table 진리표 만드는 법:

Rows 행

- Need a row for every possible combination of values for the atomic propositions 단순명제의 진리값의 모든 가능한 조합

Columns 열

- Need a column for the compound proposition (usually at far right) 복합명제 하나에 하나의 열
- Need a column for the truth value of each expression that occurs in the compound proposition as it is built up.
  - This includes the atomic propositions

Example Truth Table

Construct a truth table for $p \vee q \rightarrow \neg r$

p	q	r	$\neg r$	$p \vee q$	$p \vee q \rightarrow \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T