The Foundations: Logic and Proofs

Chapter 1, Part III: Proofs 증명

Rules of Inference
추론 규칙

Section 1.6

Revisiting the Socrates Example

We have the two premises 전제:

- “All men are mortal.” 모든 사람은 죽는다.
- “Socrates is a man.” 소크라테스는 사람이다

And the conclusion 결론:

- “Socrates is mortal.” 소크라테스는 죽는다.

How do we get the conclusion from the premises? 전제로부터 결론을 어떻게 이끌어낼까?

The Argument 주장

We can express the premises 전제 (above the line) and the conclusion 결론 (below the line) in predicate logic 술어논리 as an argument 주장:

$$\begin{array}{l} \forall x\left(Man(x) \rightarrow Mortal(x)\right) \\ \underline{\qquad Man(Socrates) \qquad} \\ \therefore \quad Mortal(Socrates) \end{array}$$

이것이 valid argument 유효한 주장임을 보일 것임

Valid Arguments 1 유효한 주장

유효한 주장을 만드는 두 단계

추론규칙이 유효한 주장 만드는데 핵심 요소

1. Propositional Logic 명제 논리

   Inference Rules 추론 규칙들

2. Predicate Logic 술어 논리

   Inference rules for propositional logic plus additional inference rules to handle variables and quantifiers.

Arguments in Propositional Logic

A *argument 주장* in propositional logic 명제 논리 is a sequence of propositions 일련의 명제들. All but the final proposition 마지막 명제 외 모두 are called *premises 전제*. The last statement 마지막 명제 is the *conclusion 결론*.

The argument 명제 is valid 유효 if the premises 전제 imply the conclusion 결론. An *argument form 주장 서식* is an argument that is valid no matter what propositions are substituted into its propositional variables.

If the premises 전제 are $p_1, p_2, \ldots, p_n$ and the conclusion 결론 is q then $(p_1 \land p_2 \land \ldots \land p_n) \to q$ is a tautology.

Inference rules 추론규칙 are all argument simple argument forms that will be used to construct more complex argument forms.

Rules of Inference for Propositional Logic: Modus Ponens

$$\begin{array}{r} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:

$$(p \land (p \rightarrow q)) \rightarrow q$$

Example:

Let *p* be "It is snowing."

Let *q* be "I will study discrete math."

"If it is snowing, then I will study discrete math."

"It is snowing."

"Therefore , I will study discrete math."

Modus Tollens

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$$

Example:

Let *p* be "it is snowing."

Let *q* be "I will study discrete math."

"If it is snowing, then I will study discrete math."

"I will not study discrete math."

"Therefore , it is not snowing."

Hypothetical Syllogism

$$\begin{array}{r} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

Corresponding Tautology:

$$((p \to q) \land (q \to r)) \to (p \to r)$$

Example:

Let *p* be "it snows."

Let *q* be "I will study discrete math."

Let *r* be "I will get an A."

"If it snows, then I will study discrete math."

"If I study discrete math, I will get an A."

"Therefore , If it snows, I will get an A."

Disjunctive Syllogism

$$\begin{array}{l} \quad p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$$

Corresponding Tautology:

$$(\neg p \wedge (p \vee q)) \rightarrow q$$

Example:

Let *p* be "I will study discrete math."

Let *q* be "I will study English literature."

"I will study discrete math or I will study English literature."

"I will not study discrete math."

"Therefore , I will study English literature."

Addition

$$\frac{p}{\therefore\ p \vee q}$$

Corresponding Tautology:

$$p \rightarrow (p \vee q)$$

Example:

Let *p* be "I will study discrete math."

Let *q* be "I will visit Las Vegas."

"I will study discrete math."

"Therefore, I will study discrete math or I will visit

Las Vegas."

Simplification

$$\frac{p \wedge q}{\therefore p}$$

Corresponding Tautology:

$$(p \wedge q) \rightarrow p$$

Example:

Let *p* be "I will study discrete math."

Let *q* be "I will study English literature."

"I will study discrete math and English literature"

"Therefore, I will study discrete math."

Conjunction

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Corresponding Tautology:

$$((p) \wedge (q)) \rightarrow (p \wedge q)$$

Example:

Let *p* be "I will study discrete math."

Let *q* be "I will study English literature."

"I will study discrete math."

"I will study English literature."

"Therefore, I will study discrete math and I will study English literature."

Resolution

$$\begin{array}{l} \neg p \vee r \\ \underline{p \vee q} \\ \therefore q \wedge r \end{array}$$

Resolution plays an important role in AI and is used in Prolog.

Corresponding Tautology:

$$((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$$

Example:

Let *p* be "I will study discrete math."

Let *r* be "I will study English literature."

Let q be "I will study databases."

"I will not study discrete math or I will study English literature."

"I will study discrete math or I will study databases."

"Therefore, I will study databases or I will study English literature."