Equivalent Propositions 동치 명제

Two propositions are *equivalent* if they always have the same truth value 같은 진리값을 가지면

Example: Show using a truth table that the conditional is equivalent 동치 to the contrapositive 대우.

Solution:

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Using a Truth Table to Show Non-Equivalence
진리표 사용하여 동치가 아님을 보이기

Example: Show using truth tables that neither the converse 역 nor inverse 이 of an implication are not equivalent to the implication.

Solution:

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$q \rightarrow p$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Problem

How many rows are there in a truth table with *n* propositional variables? N개의 명제변수가 있을 때 진리표의 행의 개수는?

Solution: 2^n

Precedence of Logical Operators
명제 연산자의 우선순위

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$p \vee q \rightarrow \neg r$ is equivalent to $(p \vee q) \rightarrow \neg r$

If the intended meaning is $p \vee (q \rightarrow \neg r)$

then parentheses must be used.

Applications 응용 of Propositional Logic

Section 1.2

Applications of Propositional Logic: Summary

Translating English to Propositional Logic

영어를 명제논리로 표현

System Specifications

Boolean Searching

Logic Puzzles

Logic Circuits

AI Diagnosis Method (Optional)

Translating English Sentences

Steps to convert an English sentence to a statement in propositional logic 변환 단계

- Identify atomic propositions 단순 명제 찾기and represent using propositional variables 명제변수로 표현.
- Determine appropriate logical connectives 적절한 논리 연산자 결정

"If I go to Harry's or to the country, I will not go shopping."

- *p*: I go to Harry's
- *q*: I go to the country.
- *r*: I will go shopping.

If *p* or *q* then not *r*.

$$\left(p \vee q\right) \rightarrow \neg r$$

Example

Problem: Translate the following sentence into propositional logic:

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

One Solution: Let *a*, *c*, and *f* represent respectively "You can access the internet from campus," "You are a computer science major," and "You are a freshman."

$$a \to (c \vee \neg f)$$

Propositional Equivalences
명제 동치

Section 1.3

Section Summary 2

Tautologies 항진, Contradictions 모순, and Contingencies 불확정명제.

Logical Equivalence

- Important Logical Equivalences 중요한 논리 동치들
- Showing Logical Equivalence 논리적 동치 증명방법

Propositional Satisfiability 만족가능성

Tautologies, Contradictions, and Contingencies

A *tautology 항진* is a proposition which is always true 항상 참인 명제.

- Example: $p \vee \neg p$

A *contradiction 모순* is a proposition which is always false 항상 거짓인 명제.

- Example: $p \wedge \neg p$

A *contingency 불확정명제* is a proposition which is neither a tautology nor a contradiction, such as p *항진이나 모순이 아닌 명제*

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Logically Equivalent 논리적 동치

Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology 항진.

We write this as $p \Leftrightarrow q$ or as $p \equiv q$ where p and q are compound propositions.

Two compound propositions p and q are equivalent if and only if the columns in a truth table giving their truth values agree.

This truth table shows that $\neg p \vee q$ is equivalent to $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Augustus De Morgan
1806-1871

This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Key Logical Equivalences 1

Identity Laws: $p \wedge T \equiv p, \qquad p \vee F \equiv p$

Domination Laws: $p \vee T \equiv T, \qquad p \wedge F \equiv F$

Idempotent laws: $p \vee p \equiv p, \qquad p \wedge p \equiv p$

Double Negation Law: $\neg(\neg p) \equiv p$

Negation Laws: $p \vee \neg p \equiv T, \qquad p \wedge \neg p \equiv F$

Key Logical Equivalences 2

Commutative Laws: $p \vee q \equiv q \vee p, \quad p \wedge q \equiv q \wedge p$

Associative Laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributive Laws: $(p \vee (q \wedge r)) \equiv (p \vee q) \wedge (p \vee r)$

$(p \wedge (q \vee r)) \equiv (p \wedge q) \vee (p \wedge r)$

Absorption Laws: $p \vee (p \wedge q) \equiv p \quad p \wedge (p \vee q) \equiv p$

More Logical Equivalences

TABLE 7 Logical Equivalences Involving Conditional Statements.

$$p \to q \equiv \neg p \lor q$$
$$p \to q \equiv \neg q \to \neg p$$
$$p \lor q \equiv \neg p \to q$$
$$p \land q \equiv \neg(p \to \neg q)$$
$$\neg(p \to q) \equiv p \land \neg q$$
$$(p \to q) \land (p \to r) \equiv p \to (q \land r)$$
$$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$$
$$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$$
$$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$
$$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Constructing New Logical Equivalences

We can show that two expressions are logically equivalent by developing a series of logically equivalent statements.

To prove that $A \equiv B$ we produce a series of equivalences beginning with A and ending with B.

$$
\begin{gathered}
A \equiv A_1 \\
\vdots \\
A_n \equiv B
\end{gathered}
$$

Keep in mind that whenever a proposition (represented by a propositional variable) occurs in the equivalences listed earlier, it may be replaced by an arbitrarily complex compound proposition.

Equivalence Proofs 1

Example: Show that $\neg\left(p \vee (\neg p \wedge q)\right)$

is logically equivalent to $\neg p \wedge \neg q$

Solution:

$\neg\left(p \vee (\neg p \wedge q)\right) \equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
$\equiv \neg p \wedge \left[\neg(\neg p) \vee \neg q\right]$	by the first De Morgan law
$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
$\equiv F \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv F$
$\equiv (\neg p \wedge \neg q) \vee F$	by the commutative law for disjunction
$\equiv (\neg p \wedge \neg q)$	By the identity law for **F**

Equivalence Proofs [2]

Example: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Solution:

$$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$$ by truth table for →

$$\equiv (\neg p \vee \neg q) \vee (p \vee q)$$ by the first De Morgan law

$$\equiv (\neg p \vee p) \vee (\neg q \vee q)$$ by associative and commutative laws laws for disjunction

$$\equiv T \vee T$$ by truth tables

$$\equiv T$$ by the domination law

Propositional Satisfiability 만족가능성

A compound proposition is *satisfiable* if there is an assignment of truth values to its variables that make it true 진리표에서 참이 하나라도 있으면 만족가능함 . When no such assignments exist, the compound proposition is *unsatisfiable*.

A compound proposition is unsatisfiable if and only if its negation is a tautology .

Questions on Propositional Satisfiability

Example: Determine the satisfiability of the following compound propositions:

$$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$$

Solution: Satisfiable. Assign **T** to *p, q,* and *r*.

$$(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$$

Solution: Satisfiable. Assign **T** to *p* and ***F*** to *q*.

$$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$$

Solution: Not satisfiable. Check each possible assignment of truth values to the propositional variables and none will make the proposition true.