Derive the posterior mean and variance for a prior given by $w \sim N(\mu_0, \Lambda_0^{-1})$:

$$P(w|D) \propto P(D|w) \cdot P(w)$$

let $\sigma^2 = a^{-1}$

$$\propto e^{-\frac{1}{2\sigma^2}(w^T\Phi - y)^T(w^T\Phi - y)} \cdot e^{-\frac{\Lambda_0}{2}(w-\mu_0)^T(w-\mu_0)}$$

$$= e^{-\frac{a}{2}\left[(w^T\Phi - y)^T(w^T\Phi - y) + \frac{\Lambda_0}{a}(w-\mu_0)^T(w-\mu_0)\right]}$$

$$= e^{-\frac{a}{2}\left[\Phi^T w w^T \Phi \underbrace{- \Phi^T w y - y^T w^T \Phi}_{-2w^T\Phi^T y} + y^T y + \frac{\Lambda_0}{a} w^T w \underbrace{- \frac{\Lambda_0}{a} w^T \mu_0 - \frac{\Lambda_0}{a} \mu_0^T w}_{-2\frac{\Lambda_0}{a} w^T \mu_0} + \frac{\Lambda_0}{a} \mu_0^T \mu_0\right]}$$

$$= e^{-\frac{1}{2}\left(w^T \Lambda w - 2w^T \Lambda \mu + \boxed{}\right)}$$

$$\therefore \Lambda = a\Phi^T\Phi + \Lambda_0$$

$$\cancel{-2w^T}(a\Phi^T y + \Lambda_0 \mu_0) = \cancel{-2w^T}\Lambda \mu \Rightarrow \mu = \Lambda^{-1}(a\Phi^T y + \Lambda_0 \mu_0)$$

$$\therefore P(w|D) \sim N(\mu, \Lambda^{-1}) = N(\Lambda^{-1}(a\Phi^T y + \Lambda_0 \mu_0), (a\Phi^T\Phi + \Lambda_0)^{-1})$$