SCALER

Bit Manipulation

Hello Everyone

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Notes

Decimal Number System [0 - 9] base 10

$324 = 3*10^2 + 2*10^1 + 4*10^0$

$2563 = 2*10^3 + 5*10^2 + 6*10^1 + 3*10^0$

Binary Number System [0-1] base 2

1 1 0 $= 1*2^2 + 1*2^1 + 0*2^0$

$= 4 + 2 + 0$

$= 6$

3 2 1 0

1 1 0 1 $= 1*2^3 + 1*2^2 + 0*2^1 + 1*2^0$

$= 8 + 4 + 0 + 1$

$= 13$

1. Binary to Decimal Conversion

- $(1101)_2 = (\quad)_{10}$

$= 1*2^3 + 1*2^2 + 0*2^1 + 1*2^0$

$= 8 + 4 + 0 + 1$

$= 13$

6 5 4 3 2 1 0

1 0 1 1 0 1 0

2^6 2^4 2^3 2^1

$= 1*2^6 + 1*2^4 + 1*2^3 + 1*2^1$

$= 64 + 16 + 8 + 2$

$= 90$

4 3 2 1 0

1 0 0 1 0

$= 1*2^4 + 1*2^1$

$= 16 + 2$

$= 18$

2. Decimal to Binary

- $(20)_{10} = (\quad)_2$

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

remainder

4 3 2 1 0
1 0 1 0 0

$= 1*2^4 + 1*2^2$

$= 16 + 4$

$= 20$

- $(45)_{10} = (\quad)_2$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

5 4 3 2 1 0
$= 1\ 0\ 1\ 1\ 0\ 1$

$= 1*2^5 + 1*2^3 + 1*2^2 + 1*2^0$

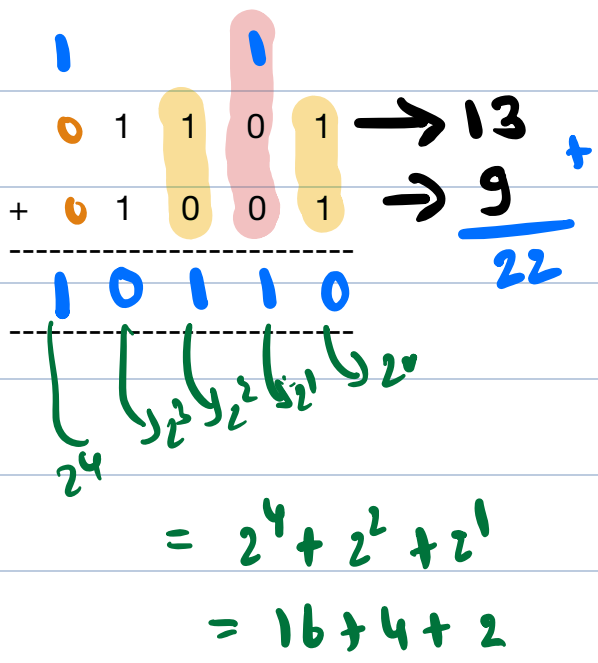
$= 32 + 8 + 4 + 1$

$= 45$

Addition of two decimal numbers -

```
  1 1
  3 6 8
+ 1 4 5
-------
  5 1 3
-------
```

Addition of two binary numbers -



Decimal	Binary
0	0
1	1
2	10
3	11

$= 2^4 + 2^2 + 2^1$

$= 16 + 4 + 2 = 22$

```
    1 1 0 1 0 1
+   1 0 0 1 1 0
-------------------
-------------------
```

```
      1  1
      1  0  1  1  0   → 22
+     0  0  1  1  1   → 7
-------------------
   1  1  1  0  1      → 29
-------------------
```

0 → false / unset bit
1 → True / set bit

Bitwise Operators

AND ↑ XOR ↗

! , & , | , ^ , << , >> - Advanced BM

↙ NOT ↓ OR

a	b	a&b	a|b	a^b	~a/!a
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

AND: Result would be 1 (set bit) if all bits = 1

OR: Result would be 1 if any bit = 1

XOR: 1) Addition without carry.

2) result = 1 if odd # 1's

3) " same same puppy shame",

NOT: Flip the bit

5 & 6 = ?

5 → 1 0 1

&

6 → 1 1 0

---

1 0 0

4

20 & 45 = ?

20 → 0 1 0 1 0 0

45 → 1 0 1 1 0 1

---

0 0 0 1 0 0 → 4

---

20 | 45 = ?

20 → 0 1 0 1 0 0

45 → 1 0 1 1 0 1

---

1 1 1 1 0 1

---

5 4 3 2 1 0

$= 2^5 + 2^4 + 2^3 + 2^2 + 2^0$

$= 32 + 16 + 8 + 4 + 1$

$= 61$

20 ^ 45 = ?

20 → 0 1 0 1 0 0

45 → 1 0 1 1 0 1

---

1 1 1 0 0 1

---

5 4 3 2 1 0

$= 2^5 + 2^4 + 2^3 + 2^0$

$= 32 + 16 + 8 + 1 = 57$

1 1 1 1

20 → 0 0 1 0 1 0 0

\+

45 → 0 1 0 1 1 0 1

---

1 0 0 0 0 0 1

6 5 4 3 2 1 0

$= 2^6 + 2^0 = 64 + 1 = 65$

Break : 10 - 10 pm

Negative Numbers

int → 32 bits long → 64 bits

⇒ for our understanding → 8 bit system

MSB ← LSB

	7	6	5	4	3	2	1	0
45 →	0	0	1	0	1	1	0	1
Flip bits →	1	1	0	1	0	0	1	0

[1's complement]

Add 1 →+ 1

	1	1	0	1	0	0	1	1
	7	6	5	4	3	2	1	0

$= -2^7 + 2^6 + 2^4 + 2^1 + 2^0$ [2's complement]

$= -128 + 64 + 16 + 2 + 1$

$= 211$

$= -128 + 83 = -45$

MSB

0

↓

No. is positive ✓

1

↓

No. is negative ✓

Binary representation of -3

	7	6	5	4	3	2	1	0
3 →	0	0	0	0	0	0	1	1
Flip →	1	1	1	1	1	1	0	0

$$
\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
+ & & & & & & & 1 \\
\hline
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
7 & 6 & 5 & 4 & 3 & 2 & 1 & 0
\end{array}
$$

$$
\begin{aligned}
&= -2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 \\
&= -128 + 64 + 32 + 16 + 8 + 4 + 1 \\
&= -128 + 125 \\
&= -3
\end{aligned}
$$

$10 \rightarrow \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}$

Flip $\rightarrow$ $\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}$

1

$$\begin{array}{r} 1\ 1\ 1\ 1\ 0\ 1\ 0\ 1 \\ +\qquad\qquad\qquad 1 \\ \hline 1\ 1\ 1\ 1\ 0\ 1\ 1\ 0 \end{array}$$

↳ −10

8 bit system :

$$\underset{-2^7}{1}\ \underset{2^6}{0}\ \underset{2^5}{0}\ \underset{2^4}{0}\ \underset{2^3}{0}\ \underset{2^2}{0}\ \underset{2^1}{0}\ \underset{2^0}{0}$$

$= -128$

Min → 1 0 0 0 0 0 0 0

max:

$$\underset{2^6}{0}\ \underset{2^5}{1}\ \underset{2^4}{1}\ \underset{2^3}{1}\ \underset{2^2}{1}\ \underset{2^1}{1}\ \underset{2^0}{1}\ \underset{}{1}$$

$= 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$

$= 127$

$= 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$

$a = 1 \quad , r = 2 \quad , n = 7$

$$= \frac{a(r^n - 1)}{(r-1)}$$

$$= \frac{1(2^7 - 1)}{(2-1)} \qquad = \frac{2^7 - 1}{1}$$

$$= 2^7 - 1$$

$[-128, 127]$

$[-2^7, 2^7 - 1]$.

Integer (32 bit system)

31 30 . . . . . 2 1 0

Min → 1 0 0 0 0

$= -2^{31} \simeq -10^9$

Max → 0 1 1 . . . . 1 1 1

$= 2^{30} + 2^{29} \dots + 2^1 + 2^0$

$= 2^{31} - 1 \simeq 10^9$

$= 2147483647$

$\simeq 2 * 10^9$

$[-2^{31}, 2^{31} - 1]$

Long (64 bit system)

63 62 61 ... 1 0

Min → 1 0 0 ..... 0 0

$= -2^{63} \simeq -9 * 10^{18}$

63 62 61 ... 1 0

Max → 0 1 1 ..... 1 1

$= 2^{63} - 1 \simeq 9 * 10^{18}$

Importance of constraints.
(also useful to predict TLE)

Q→ Given 2 integers a and b find their product.

$a = 10^5$ $b = 10^5$

`int c = a * b` ✗

$a * b = 10^5 * 10^5 = 10^{10}$

↓

(outside int range)

`long c = a * b;` ✗ right side calc → overflow

`long c = long(a * b)` ✗ → overflow

`long c = long(a) * b` ✓

long c = long(a) * long(b) ✓

long c = a * long(b) ✓

int * int → int
long * int → long

Q → Given an integer array find sum of an element.

long
~~int~~ sum = 0

for(i=0; i<n; i++)
{
    sum = sum + A[i];
}
return sum;

$1 <= n <= 10^5$

$1 <= A[i] <= 10^6$

Max sum = $10^5 * 10^6$

$= 10^{11}$

Doubt session

0 1 1 1 1 1 1 1

45.786