Open Quantum System

Lectures:

Last time:

- Daniel Manzano, A short introduction to the Lindblad master equation (*all*)
- Breuer and Petruccione, The Theory of Open Quantum Systems (*ch. 3 - 4.3*)
- Daniel A. Lidar, Notes on the Theory of Open Quantum Systems (*up to ch. 12*)

Today:

- Breuer and Petruccione, The Theory of Open Quantum Systems (*ch. 3 - 4.3*)
- Daniel A. Lidar, Notes on the Theory of Open Quantum Systems (*up to ch. 12*)
- B.Kraus, H.P.Buchler, S. Diehl, A. Kantian, A. Micheli, & P. Zoller, Preparation of Entangled States by Quantum Markov Processes
- Buča, B., Tindall, J. & Jaksch, D. Non-stationary coherent quantum many-body dynamics through dissipation
- Victor V. Albert & Liang Jiang, Symmetries and conserved quantities in Lindblad master equations
- Cameron Booker, Berislav Buča, Dieter Jaksch, Non-stationarity and Dissipative Time Crystals: Spectral Properties and Finite-Size Effects

November 5, 2023

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\begin{aligned}
\frac{d}{dt}\rho(t) = {} & [H + H_{\text{LS}}, \rho(t)] + \\
& + \sum_{k,l,\omega} \gamma_{k,l}(\omega) \left[S_l(\omega)\rho(t)S_k^\dagger(\omega) - \frac{1}{2}\left\{ S_k^\dagger(\omega)S_l(\omega), \rho(t) \right\} \right]
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  - With $\gamma_{k,l}$, $\pi_{k,l}$ - defined via $\Gamma_{k,l}(\omega)$'s real and imaginary parts.

$$\begin{aligned} H_{\text{LS}} = \sum_{\omega,k,l} \pi_{k,l}(\omega)S_k^\dagger(\omega)S_l(\omega) \\ \Gamma_{k,l}(\omega) = \int_0^\infty e^{i\omega s}\text{Tr}_B\left[B_k^\dagger(t)B_l(t-s)\rho_B(0)\right] \end{aligned} \tag{2}$$

- With the operators $S_k(\omega)$ defined via

$$S_k(\omega) = \sum_{\epsilon' - \epsilon = \omega} \Pi_\epsilon S_k \Pi_{\epsilon'} \equiv \sum_{\epsilon' - \epsilon = \omega} |\epsilon\rangle\langle\epsilon| \, S_k \, |\epsilon'\rangle\langle\epsilon'| \tag{3}$$

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  which we can diagonalize over (k, l), by defining $\gamma(\omega) = U\Sigma(\omega)U^\dagger$, with $\Sigma(\omega) = \text{Diag}(\varsigma_k(\omega))$ and the transformation of the jump operators - $L_k(\omega) = \sum_l U_{lk} S_l(\omega)$. Yielding

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Lindblad

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- Note: relaxation mechanisms are always present in practice.

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- For the most widely used cases, one supposes **ergodicity**, meaning the state explores the accessible state space, as illustrated in *figure (a)*, before reaching the supposed stationarity.

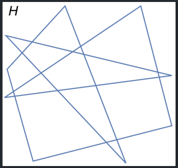

Figure 1: Illustration of ergodicity. [1]

[1] Buča, B., Tindall, J. & Jaksch, D. Non-stationary coherent quantum many-body dynamics through dissipation

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$$\exists \quad \left\{\{\varphi_m, |\varphi_m\rangle\} \mid \mathscr{L}\,|\varphi_m\rangle = \varphi_m\,|\varphi_m\rangle \text{ and } \{\varphi_m\} \in \sigma(\mathscr{L})\right\}$$

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- Dark Hamiltonian $\mathfrak{H}$ - driver of the *non-decaying* subspaces.

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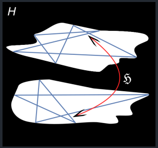

Figure 2: Decoherence-free subspaces, driven by $\mathfrak{H}$.[2]

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$$\dot{\rho} = \mathscr{L}\rho = \mathscr{E}(\rho) - Q\rho - \rho Q, \quad \mathscr{E} \equiv 2\sum_l g_l c_l \rho c_l^\dagger, \quad Q \equiv P - iH, \quad P \equiv \sum_l g_l c_l^\dagger c_l$$

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- For a steady-state ρ_∞, one clearly has $\mathscr{L}\rho_\infty = 0$
- Density ρ_n non stationary, if the corresponding eigenvalues ϱ_n are purely imaginary.

$$\begin{aligned} \mathscr{L}\rho_n &= \varrho_n \rho_n \\ &= -i\mathfrak{H}\rho_n = -i\lambda_n \rho_n \ , \quad \lambda \in \mathbb{R} \end{aligned} \quad (5)$$

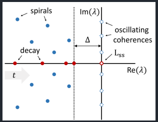

Figure 3: Meaning of eigenvalues of $\mathscr{L}$.

Non-dissipative subspaces

Let $|\Psi\rangle$ - a pure state, that is **not** affected by dissipation. Then,

- $|\Psi\rangle \in \mathrm{Ker}(P) \equiv \left\{|\Psi\rangle \;\;|\;\; \hat{c}_l\,|\Psi\rangle = 0\right\} = \mathrm{Ker}(P) \bigcap \mathrm{EigSp}(H) \equiv \mathscr{H}_{ND}$

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- Meaning that if for a density operator ρ, such that

$$\mathrm{Range}(\rho) = \mathrm{span}(\{|\varphi_i\rangle\}) \;\; \text{,with } \{\varphi_i\} \in \mathscr{H}_{ND} \tag{6}$$

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  - $\mathrm{EigSp}(\mathfrak{H}) = \{|\varphi_i\rangle\}$ such that $|\varphi_m\rangle \perp |\varphi_n\rangle$
  - $\mathfrak{H} |\varphi_m\rangle\langle\varphi_n| = (\omega_m - \omega_n) |\varphi_m\rangle\langle\varphi_n|$
  - For $\rho_{nm} = |\varphi_m\rangle\langle\varphi_n|, \;\; \rho^\dagger_{nm}\rho_{n'm'} = \delta_{nm}\delta_{n'm'}$
  - Hermicity - $\mathfrak{H} = \mathfrak{H}^\dagger$

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Theorem: conditions stationarity

The set $\{\varphi_m\} \in \mathrm{EigSp}(\mathfrak{H})$ with $|\varphi_m\rangle \perp |\varphi_n\rangle$ IFF

- $Q^\dagger |\varphi_n\rangle = \lambda |\varphi_n\rangle \quad , \quad \forall n$
- $c_k |\varphi_n\rangle = \lambda_{kn} |\varphi_n\rangle \quad$, with $\sum_l \gamma_l |\lambda_{ln}|^2 = \mathrm{Re}(\lambda_n)$
- Relations between λ_{kn} and γ_k:

$$\mathrm{Re}\left[\sum_k \gamma_k \left(2\lambda_{kn}\lambda_{km}^* - |\lambda_{kn}|^2 - |\lambda_{km}|^2\right)\right] = 0 \qquad \forall n, m$$

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- Corollary for eigenvalues ω_i for the dark Hamiltonian $\mathfrak{H}$:

$$\omega_n - \omega_m = \mathrm{Im}\left[\sum_k 2\gamma_k \lambda_{kn}\lambda_{km}^* - \langle\varphi_m| H |\varphi_n\rangle\right] \tag{7}$$

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If there exists no subspace $\mathscr{S} \subseteq \mathscr{H}$ with $\mathscr{S} \perp \mathscr{H}_{ND}$ such that $\hat{c}_l(\mathscr{S}) \subseteq \mathscr{S}, \quad \forall \hat{c}_l$, then the only oscillating coherences are the dark states, in the form $|\varphi_n\rangle\langle\varphi_m|$, such that $\{|\varphi_n\rangle\} \in \mathscr{H}_{DS}$ defined above.

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- Example of change of notation: $\sum_\mu \mapsto \sum_k \sum_\omega$.

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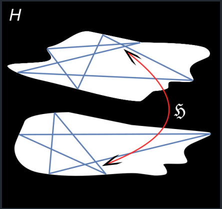
Corollary:

As a consequence, $\exists \rho_{mn} \equiv A^n \rho_\infty (A^m)^\dagger \quad m, n \in \mathbb{Z}$ and $\mathscr{L}\rho_{mn} = i(m-n)\lambda\rho_{mn}$



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Remark

It is important to distinguish the **mixed coherences** and **decoherence free subspace of dark space**.
Namely, $\hat{c}_l |\varphi_n\rangle = 0$ for dark states and $\hat{c}_l \rho_{nm} \hat{c}_l^\dagger \neq 0$

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- Steady-state $\rightleftarrows$ ergodicity & ETH
- **Dark states** $c_k |\varphi_n\rangle = 0 \ \forall n$ - invisible for decoherence.
- $\rho_{mn} = |\varphi_m\rangle\langle\varphi_n| \in \mathrm{EigSp}(\mathscr{L})$ with eigenval. $\lambda_{mn} = -i(E_n - E_m)$
- Mixed coherences $\rho_{nm} = (A^\dagger)^n \rho_\infty A^m$ also in $\mathrm{EigSp}(\mathscr{L})$ with eigenvalues $\lambda^j_{nm} = i(n-m)\omega_j$, with $\omega_j A_j = [H, A_j]$

Remark

It is important to distinguish the **mixed coherences** and **decoherence free subspace of dark space**. Namely, $\hat{c}_l |\varphi_n\rangle = 0$ for dark states and $\hat{c}_l \rho_{nm} \hat{c}_l^\dagger \neq 0$

Corollary

The mixed coherence states' evolution depends on dissipative processes.