

Deep Learning and Applications

DSA 5204 • Lecture 2
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Homework 1

Homework 1 has been uploaded on Canvas

Submission through Canvas submission folder

Due: 4th Feb 2023

Take note of the late submission policies



Project Instructions

A decorative network diagram in the top right corner, featuring a series of interconnected nodes and lines, resembling a web or a complex graph structure.

Please read Canvas for project instructions

TLDR version

- Form groups of 5-7 by the end of Week 3
- Three main components
 - **Proposal (Homework 2)**
 - **Presentation with Q&A (live, during last 2 classes)**
 - **Report**
- Any questions, please post on Canvas forums

Last Time

- **T**ask, **E**xperience, **P**erformance
Machine Learning = Improve P on T with E
- Linear regression models as a simplest example
$$f(\mathbf{x}) = \mathbf{w}^T \mathbf{x}$$
- Capture nonlinearity using linear basis model
$$f(\mathbf{x}) = \mathbf{w}^T \boldsymbol{\phi}(\mathbf{x})$$
- Today, we will look at a further extension of these ideas to neural networks



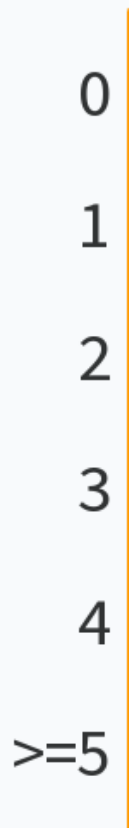
Motivation for Representation Learning and Deep Learning

Let's play a game – Round 1

1. Choose 3 different numbers out of $\{0, 1, \dots, 9\}$
2. Write them down on a piece of paper. Don't change them!
3. Now, for the game:
 1. I will give you a "magic" number
 2. You add 0, 1, 2 or all 3 of your chosen numbers, to get as close to this number as possible. Record the absolute difference
 3. Example: You chose $\{3, 5, 9\}$. I give 15. The closest you can get is $5 + 9 = 14$. Difference is 1. This is your score
 4. Example: You chose $\{3, 5, 9\}$. I give 1. The closest you can get is 0. Difference is 1. This is your score

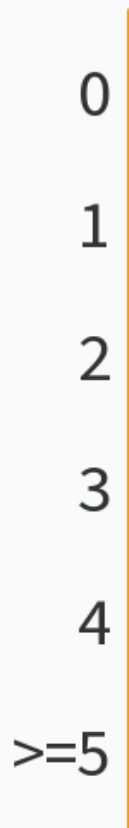


Round 1.1: The magic number is 12. What is your score?



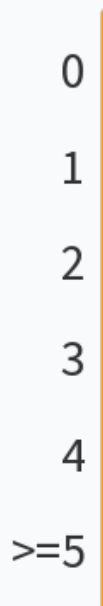


Round 1.2: The magic number is 2. What is your score?





Round 1.3: The magic number is 20. What is your score?



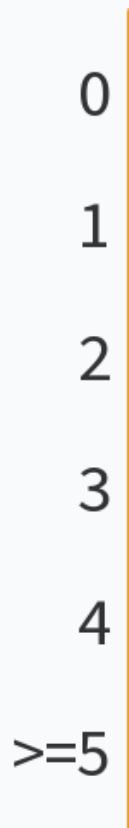
Round 2

For the next round, implement the following changes

- instead of picking 3 numbers out of $\{0,1, \dots, 9\}$, pick 6 instead
- Now, you can pick either 0, 1, 2 or 3 of the 6 numbers you picked to get close to the answer
- Example: you picked $\{0,1,2,3,4,8\}$
 - The given number is 5, you can pick 2, 3, and your score is 0
 - The given number is 18, you can pick 3, 4, 8, and your score is 3 (note that you cannot pick more than 3)

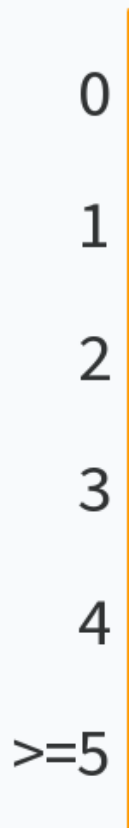


Round 2.1: The magic number is 4. What is your score?



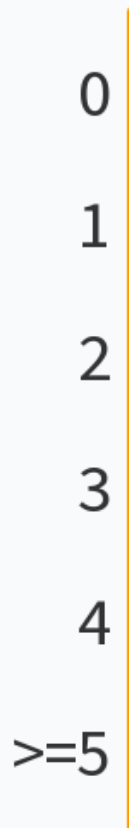


Round 2.2: The magic number is 10. What is your score?





Round 2.3: The magic number is 19. What is your score?



Recap



Round 1:

$$\text{Score} = \left| f - \sum_{i=1}^3 w_i \phi_i \right| \quad w_i \in \{0,1\}$$

Given number Your numbers

Round 2:

$$\text{Score} = \left| f - \sum_{i=1}^3 w_i \phi_i(f) \right| \quad w_i \in \{0,1\}$$

$\phi(f) = \{3 \text{ choices based on } f\}$

Why did we do better in round 2?

Back to linear basis models



Recall that linear basis models can be chosen with **universality**

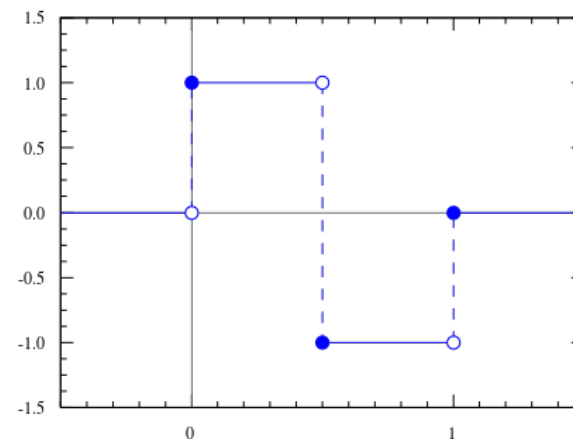
Examples (1D):

$$\phi(x) = (1, x, x^2, x^3, \dots)$$

$$\phi(x) = (1, \cos(x), \sin(x), \cos(2x), \sin(2x), \dots)$$

$$\phi(x) = \left(2^{\frac{n}{2}} \psi(2^n x - k) \right)_{n, k=1,2,\dots}$$

Haar Wavelet: $\psi(x) =$



A decorative network graph in the top right corner, consisting of numerous small blue circular nodes connected by thin, light blue lines, forming a complex web-like structure.

**Why don't we just use linear basis models
with a large m ?**

Approximation Efficiency

Goal: approximate some function f^*

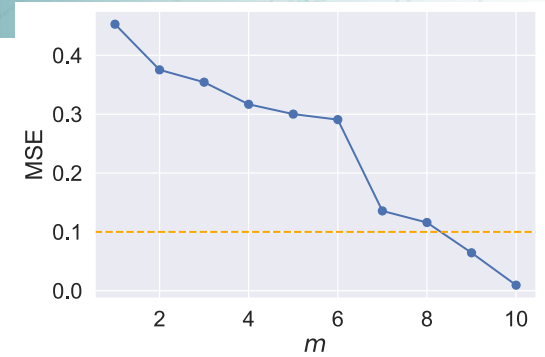
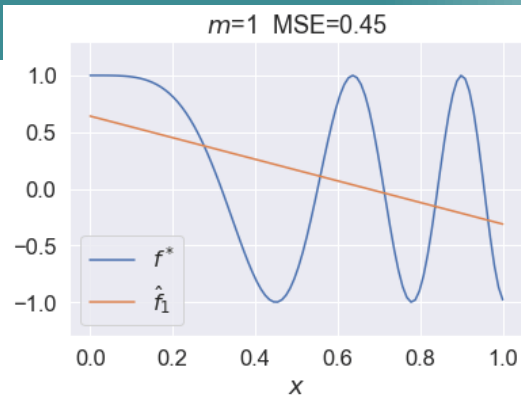
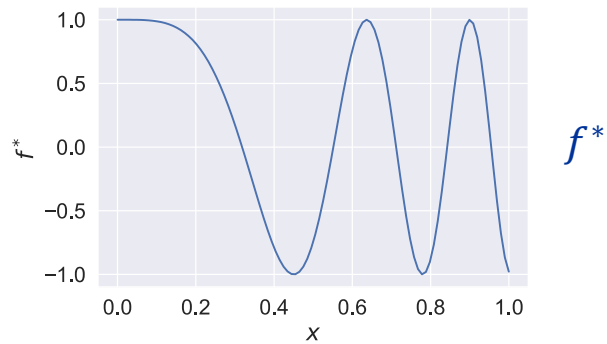
Consider two basis sets:

$$\phi_m(x) = (1, x, x^2, x^3, \dots, x^m)$$

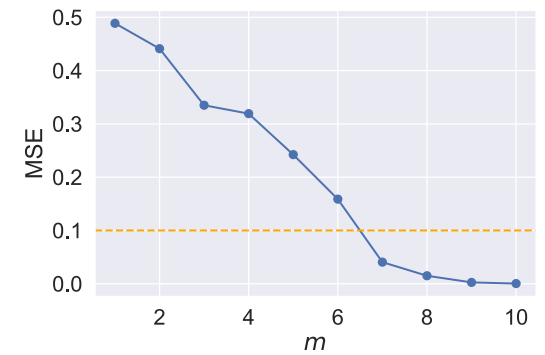
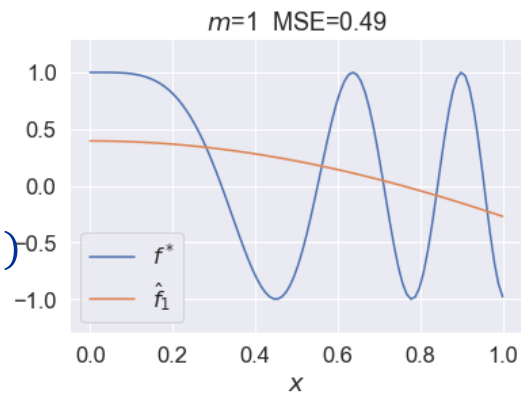
$$\phi_m(x) = (1, \cos(x), \cos(2x), \dots, \cos(mx))$$

Let us consider fitting some function f^* with varying basis and varying m

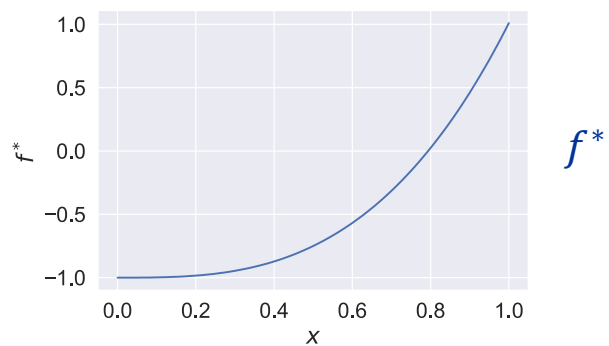
$$\phi_m(x) = (1, x, x^2, x^3, \dots, x^m)$$



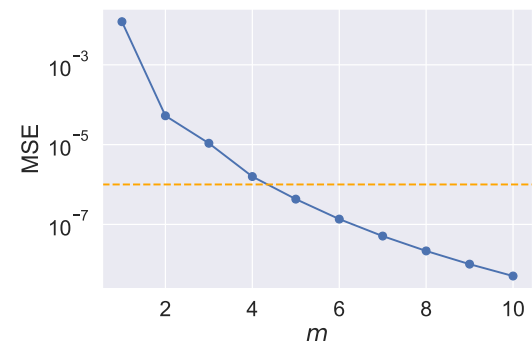
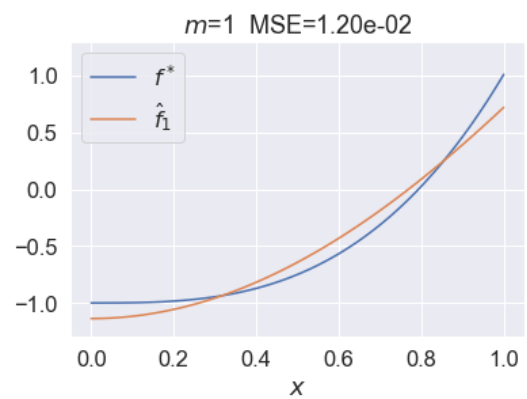
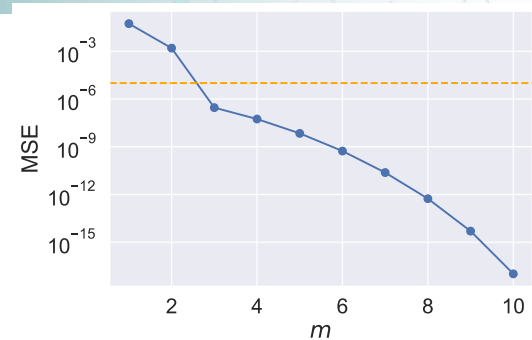
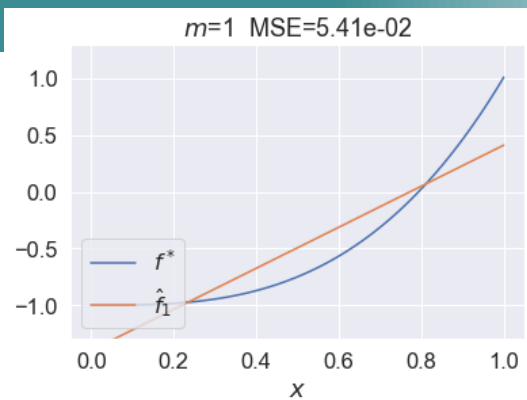
$$\phi_m(x) = (1, \cos(x), \cos(2x), \dots, \cos(mx))$$



$$\phi_m(x) = (1, x, x^2, x^3, \dots, x^m)$$



$$\phi_m(x) = (1, \cos(x), \cos(2x), \dots, \cos(mx))$$



Adaptive Approach



Previous experiments show

- How well we can learn with a given computational budget (m) depends **both** on the target (f^*) and the choice of basis (ϕ)
- No ϕ works universally well for **all** f^*
- Adapt the choice of ϕ to data!



Neural Networks as Adaptive Basis Models

Adaptive Basis Models



Usual linear basis models have the form

$$f(\mathbf{x}) = \mathbf{w}^T \boldsymbol{\phi}(\mathbf{x}) \quad \mathbf{x} \in \mathbb{R}^d, \mathbf{w} \in \mathbb{R}^m$$

Adaptive models

$$f(\mathbf{x}) = \mathbf{w}^T \boldsymbol{\phi}(\mathbf{x}; \boldsymbol{\theta}) \quad \mathbf{x} \in \mathbb{R}^d, \mathbf{w} \in \mathbb{R}^m, \boldsymbol{\theta} \in \mathbb{R}^p$$

The parameters $\boldsymbol{\theta}$ are learned from data to find the best possible basis.

Example (Nonlinear Approximation)

Consider

$$f^* = 1 + x^{10}$$

Polynomial basis

$$\boldsymbol{\phi}_m(x) = (1, x, x^2, \dots, x^{m-1})$$

Adaptive polynomial basis

$$\begin{aligned} \boldsymbol{\phi}_m(x; \boldsymbol{\theta}) &= (x^{\theta_0}, x^{\theta_1}, \dots, x^{\theta_{m-1}}) \\ \boldsymbol{\theta} &\in \mathbb{N}^m \end{aligned}$$

Learned Basis as Feature Maps



The basis $\{\phi(x; \theta): \theta \in \mathbb{R}^p\}$ is adapted to data by adjusting θ .

For this reason, we also call

$$\phi_i(\cdot; \theta): \mathbb{R}^d \rightarrow \mathbb{R}$$

feature maps, as they extract information from dataset.

Equivalently, they give good **representation** of the data

Example (Learning XOR)

The **exclusive or (XOR)** function on two binary variables

$$f^*(x_1, x_2) = \begin{cases} 1 & \text{exactly one of } x_i \text{ is 1} \\ 0 & \text{otherwise} \end{cases}$$

Let us build a simple linear model to represent this function



Data Matrix:

$$X = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \quad y = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Empirical Risk:

$$R(\boldsymbol{\theta}) = \frac{1}{4} \sum_{i=1}^4 (f^*(\mathbf{x}_i) - f(\mathbf{x}_i; \boldsymbol{\theta}))^2$$



Linear Model:

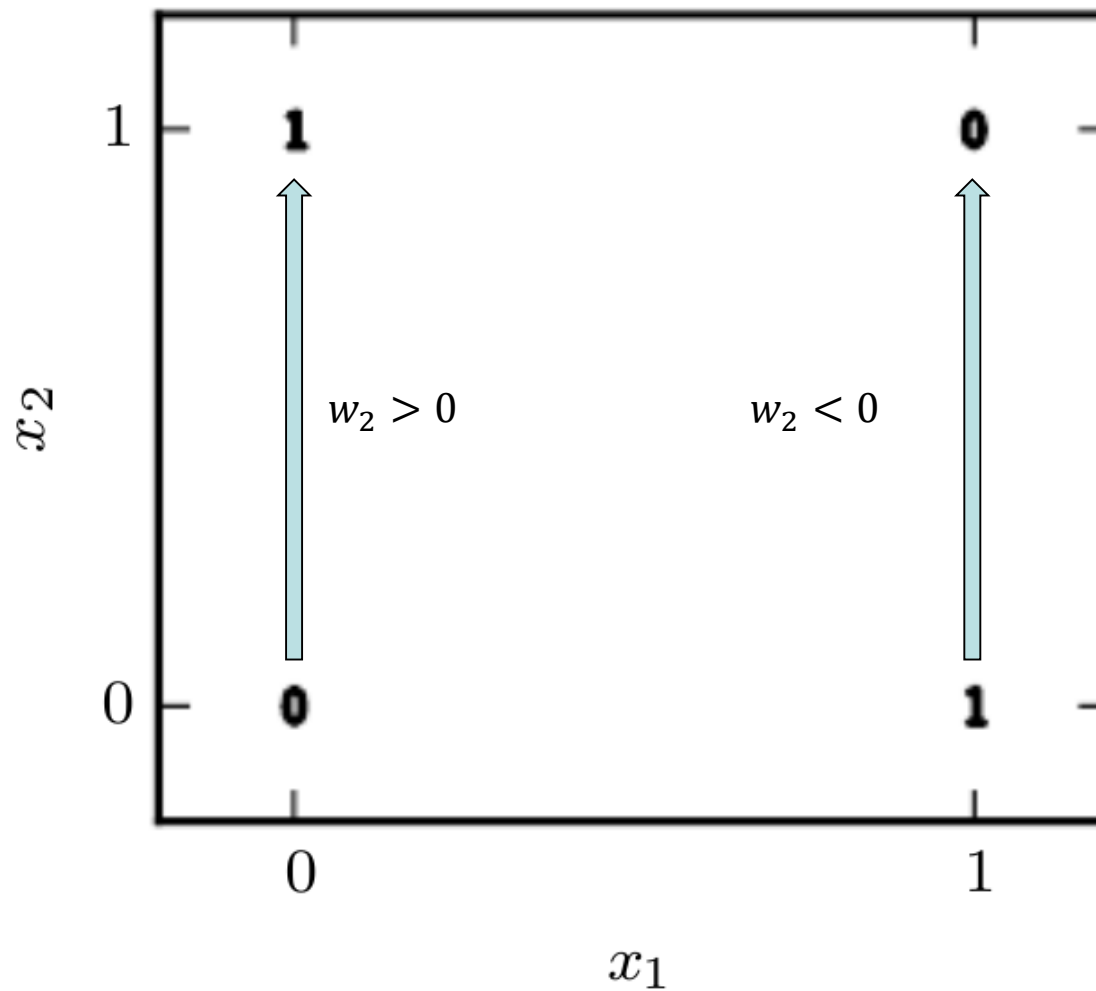
$$f(\mathbf{x}; \boldsymbol{\theta}) = f(\mathbf{x}; \mathbf{w}, b) = \mathbf{w}^T \mathbf{x} + b$$

Solving the regression problem gives

$$\mathbf{w} = (0, 0) \quad b = \frac{1}{2}$$

Hence, $f(\mathbf{x}; \mathbf{w}, b) \equiv \frac{1}{2}$

Why is linear model not enough?



Neural Network for the XOR Function



We are going to write $f = f^{(2)} \circ f^{(1)}$:

$$\begin{aligned} \mathbf{h} &= f^{(1)}(\mathbf{x}; W, \mathbf{c}) \\ y &= f^{(2)}(\mathbf{h}; \mathbf{w}, b) \end{aligned}$$

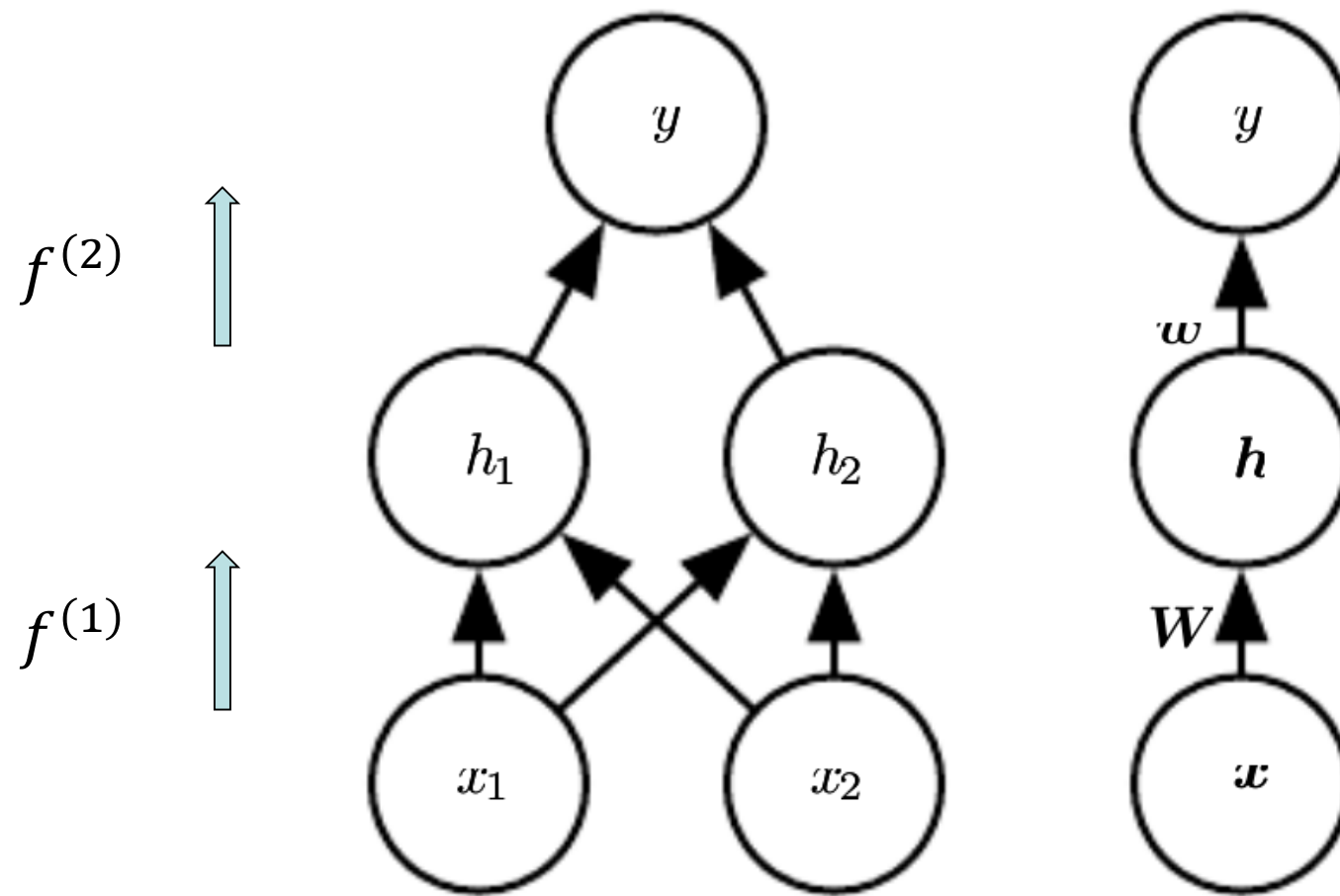
The vector \mathbf{h} is called a vector of **hidden units**

Its dimension is m , also known as the *width* of the hidden layer

The combined model is

$$f(\mathbf{x}; \boldsymbol{\theta}) \equiv f(\mathbf{x}; W, \mathbf{c}, \mathbf{w}, b) = f^{(2)}(f^{(1)}(\mathbf{x}; W, \mathbf{c}); \mathbf{w}, b)$$

Graph Representation of Neural Networks



Activation Functions



We have not specified how to choose $f^{(1)}$ and $f^{(2)}$

Simplest choice: linear functions

$$\begin{aligned}f^{(1)}(\mathbf{x}; W, \mathbf{c}) &= W\mathbf{x} + \mathbf{c} \\f^{(2)}(\mathbf{h}; \mathbf{w}, b) &= \mathbf{w}^T \mathbf{h} + b\end{aligned}$$

$$\begin{aligned}f^{(2)}\left(f^{(1)}(\mathbf{x})\right) &= \mathbf{w}^T (W\mathbf{x} + \mathbf{c}) + b \\&= \underbrace{(W^T \mathbf{w})^T}_{\mathbf{w}'} \mathbf{x} + \underbrace{(\mathbf{w}^T \mathbf{c} + b)}_{b'}\end{aligned}$$

We need some form of **nonlinearity**!

We pick the simplest type of nonlinearity

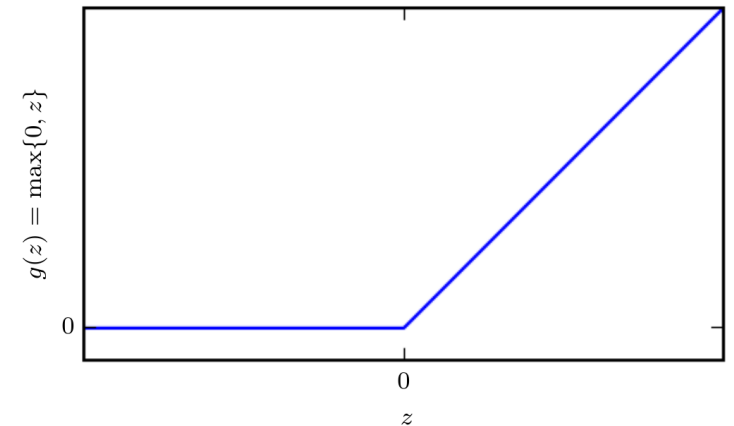
$$\mathbf{h} = f^{(1)}(\mathbf{x}; W, \mathbf{c}) = g(W\mathbf{x} + \mathbf{c})$$
$$y = f^{(2)}(\mathbf{h}; \mathbf{w}, b) = \mathbf{w}^T \mathbf{h} + b$$

The function $g: \mathbb{R} \rightarrow \mathbb{R}$ is called an **activation function**, and is applied **element-wise** to a vector

$$g(\mathbf{z})_i = g(z_i)$$

Simplest choice of activation:

$$g(z) = \max(0, z)$$



A Solution to the XOR Problem



Neural network model:

$$f(\mathbf{x}; W, \mathbf{c}, \mathbf{w}, b) = \mathbf{w}^T \max(0, W\mathbf{x} + \mathbf{c}) + b$$

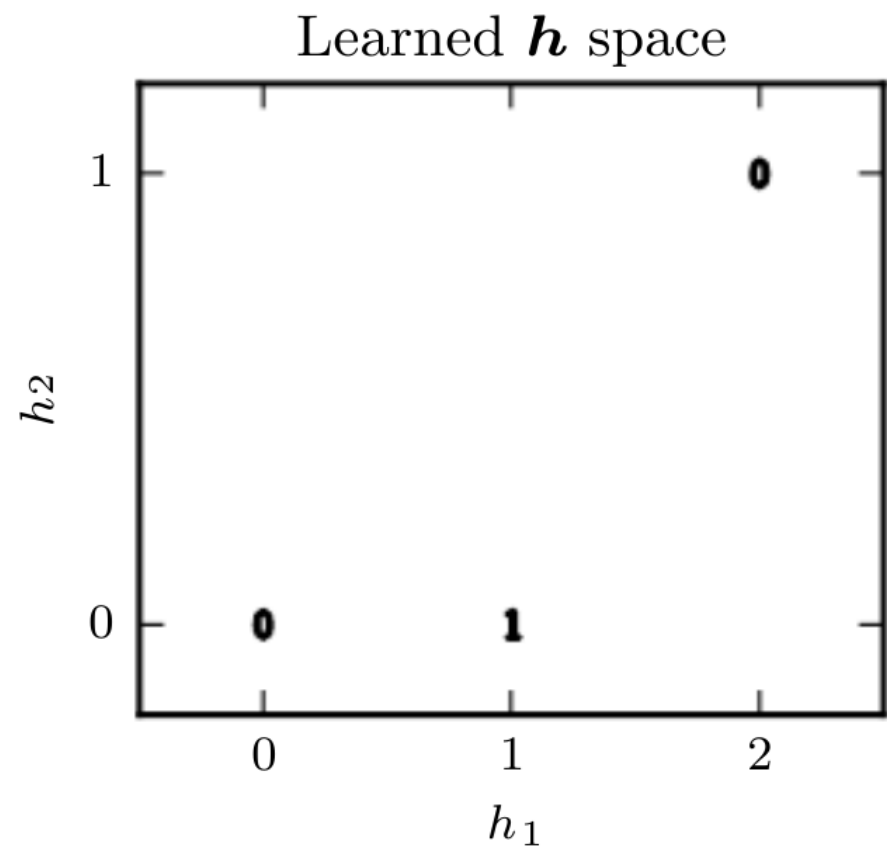
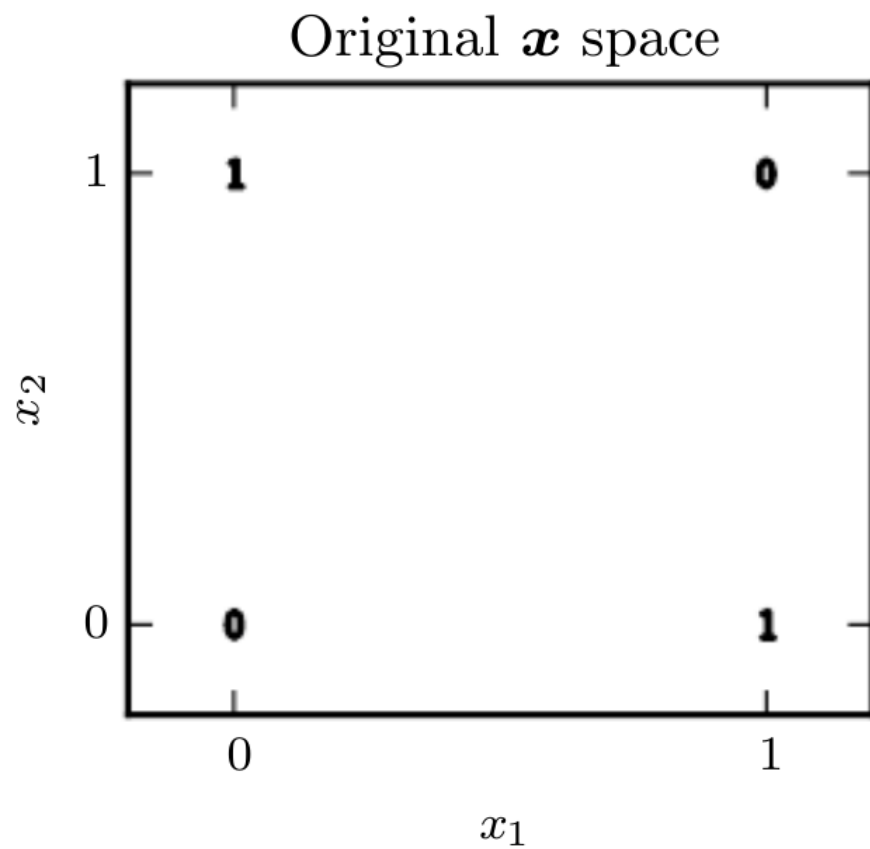
Let

$$W = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad b = 0$$

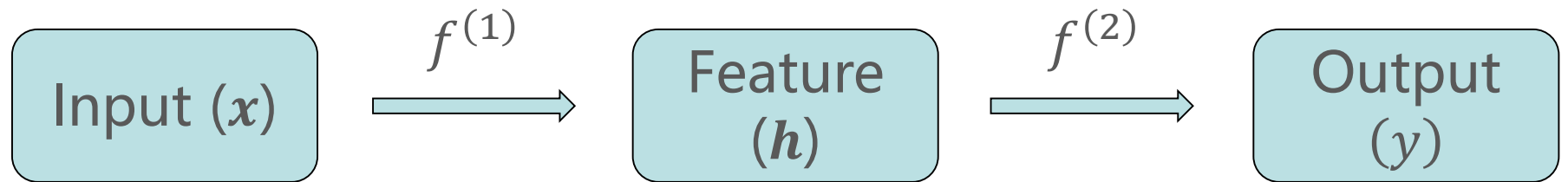
Then

$$f(X; W, \mathbf{c}, \mathbf{w}, b) = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = f^*(X)$$

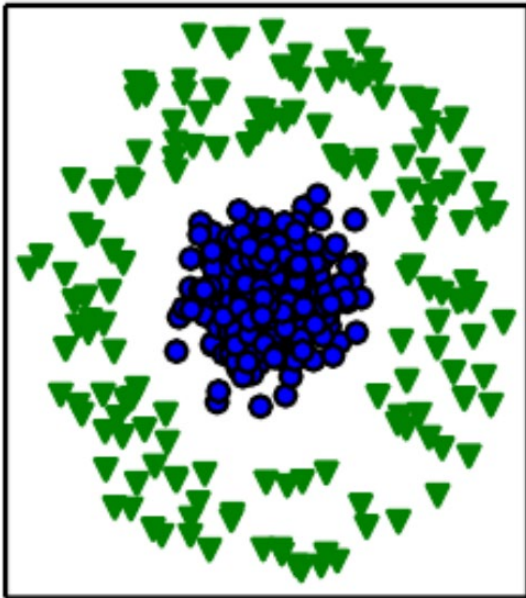
What is the role of the hidden units?



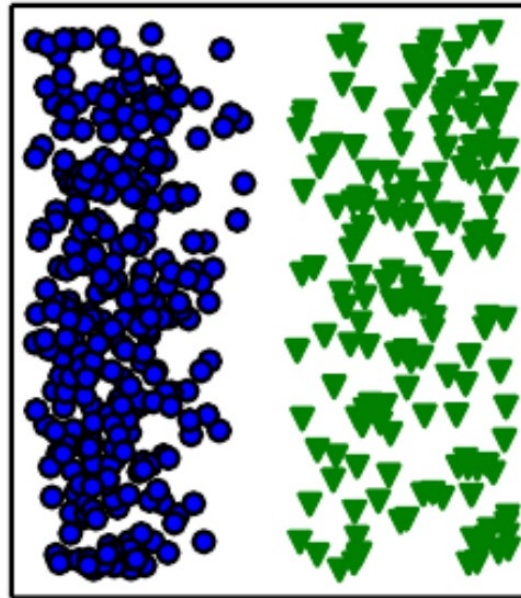
Feature Space



Cartesian coordinates



Polar coordinates



General Shallow (1-hidden-layer) Neural Networks



General neural networks for regression

$$f(\mathbf{x}; W, \mathbf{c}, \mathbf{w}, b) = w^T \underbrace{g(W\mathbf{x} + \mathbf{c})}_{\phi(\mathbf{x}; \theta)} + b$$

Variables: $W \in \mathbb{R}^{m \times d}$, $\mathbf{c} \in \mathbb{R}^m$, $\mathbf{w} \in \mathbb{R}^m$, $b \in \mathbb{R}$

- W and \mathbf{w} are called **weights**
- \mathbf{c} and b are called **biases**
- g is the activation function

Activation Functions



- Rectified Linear Unit (ReLU)

$$g(z) = \max(0, z)$$

- Sigmoid

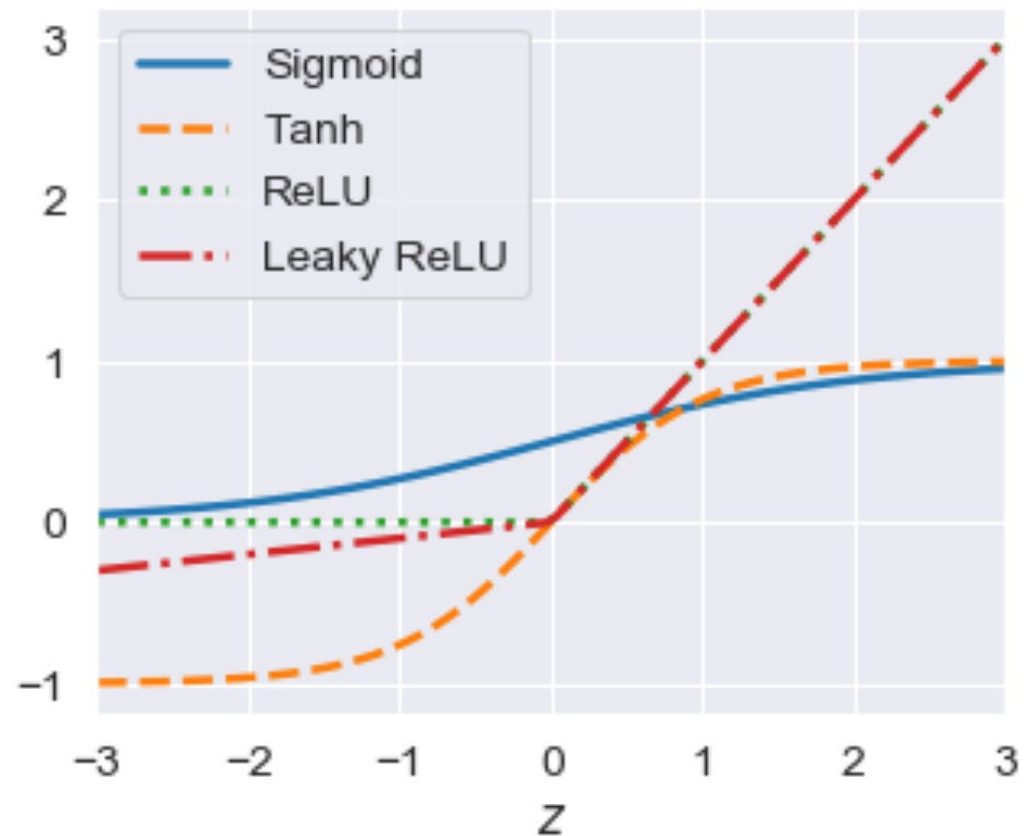
$$g(z) = \frac{1}{1 + e^{-z}}$$

- Tanh

$$g(z) = \tanh(z)$$

- Leaky-ReLU

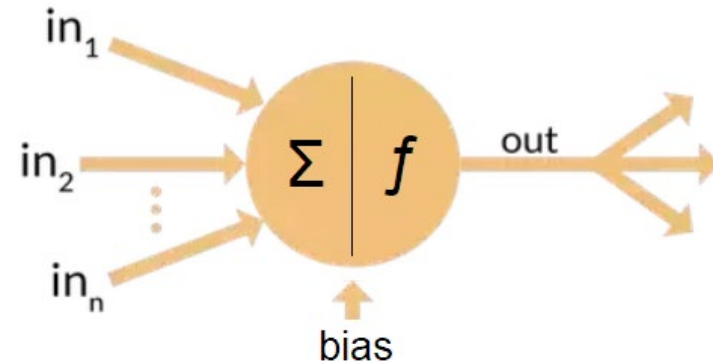
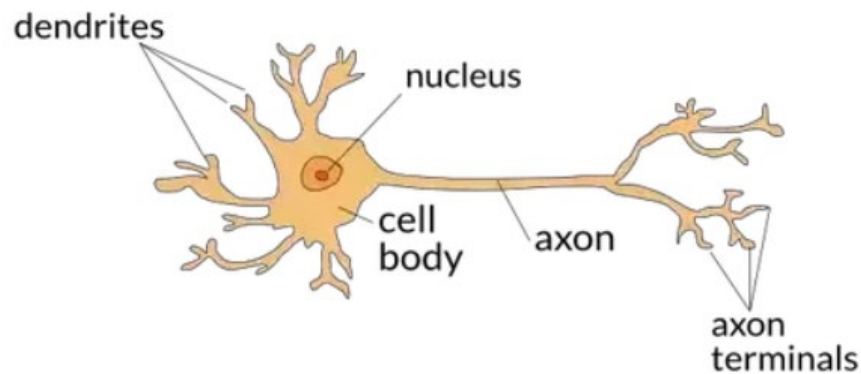
$$g(z) = \begin{cases} z & \text{if } z \geq 0 \\ \delta z & \text{if } z < 0 \end{cases}$$



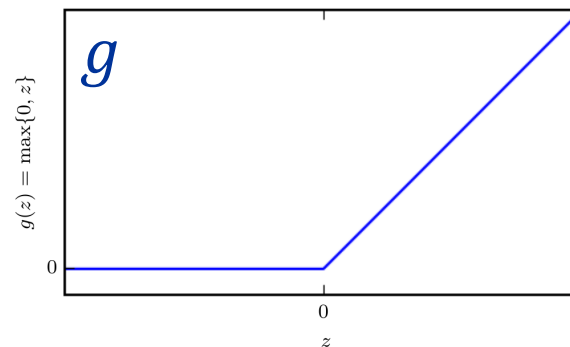
Historical Motivation: Modelling Neurons



Neural networks originated from an attempt to model collective interaction of neurons



Neuron: $g(W_i \cdot \mathbf{x} + c_i)$



Universal Approximation Theorem

A decorative network diagram in the top right corner, consisting of numerous light blue nodes connected by thin, light blue lines, forming a complex web-like structure.

One of the foundational results for neural networks is the **universal approximation theorem**.

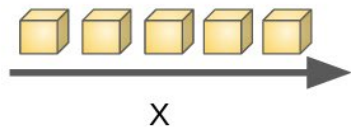
In words, it says the following:

Any continuous function f^ on a compact domain can be approximated by neural networks to **arbitrary precision**, provided there are enough neurons (m large enough).*

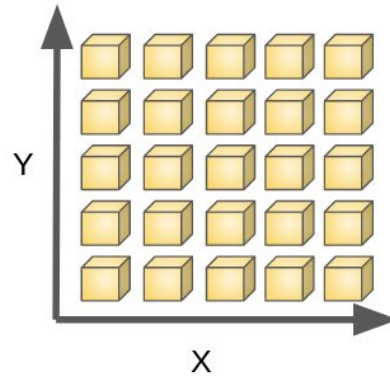
Why NNs over linear basis models?



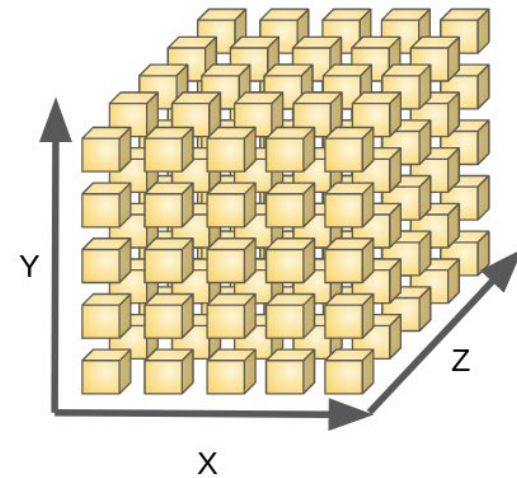
Curse of Dimensionality



$$\{x^j : j = 0, 1, \dots\}$$



$$\{x_1^{j_1} x_2^{j_2} : j_1, j_2 = 0, 1, \dots\}$$



$$\{x_1^{j_1} x_2^{j_2} x_3^{j_3} : j_1, j_2, j_3 = 0, 1, \dots\}$$



Under some technical assumptions, for any continuous (+ other conditions) function $f^*: [0,1]^d \rightarrow \mathbb{R}$, there exists a width- m neural network f_m such that

$$\|f^* - f_m\|^2 \leq \mathcal{O}(m^{-1})$$

This result is first proved in [Baron, 1993]

This is a **tremendous** improvement over linear basis models, where we usually have [Jackson, 1912]

$$\|f^* - f_m\|^2 \leq \mathcal{O}(m^{-\frac{2\alpha}{d}})$$

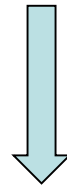
The constant α measures the smoothness of f^*

Neural Networks for Classification



Neural networks can be modified to handle classification problems, exactly in the same way as linear models

$$f(\mathbf{x}; W, \mathbf{c}, \mathbf{w}, b) = \mathbf{w}^T g(W\mathbf{x} + \mathbf{c}) + b$$



$$f(\mathbf{x}; W, \mathbf{c}, \mathbf{w}, b) = \sigma(V g(W\mathbf{x} + \mathbf{c}) + \mathbf{b})$$

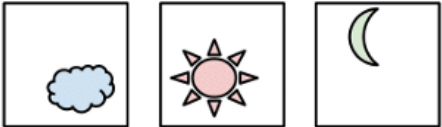

$$V \in \mathbb{R}^{K \times m}, \mathbf{b} \in \mathbb{R}^K, \sigma = \text{softmax}$$

Choice of Output Units



The soft-max output unit is appropriate for multi-class classification problems

There are also multi-label classification problems...

	Multi-Class	Multi-Label
C = 3	Samples 	Samples 
	Labels (t) [0 0 1] [1 0 0] [0 1 0]	Labels (t) [1 0 1] [0 1 0] [1 1 1]



In multilabel classification, a sigmoid output unit is more appropriate

$$\text{Softmax}(\mathbf{z})_i = \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

$$\text{Sigmoid}(\mathbf{z})_i = \frac{1}{1 + \exp(-z_i)}$$

Loss Functions

Empirical risk minimization requires the definition of loss functions

$$R_{\text{emp}}(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^N L(f^*(\mathbf{x}_i), f(\mathbf{x}_i, \boldsymbol{\theta}))$$

For regression problems, we usually pick the square loss (why?)

$$L(y, y') = \frac{1}{2} (y - y')^2$$

Loss Functions for Classification



Multiclass classification problem

- Here, $\mathbf{y} = f(\mathbf{x}; \boldsymbol{\theta})$ and $\mathbf{y}' = f^*(\mathbf{x})$ are probability vectors
- Usually pick the **cross-entropy loss**

$$L(\mathbf{y}, \mathbf{y}') = - \sum_j y'_j \log y_j$$

Multilabel classification problem

- Here, each \mathbf{y}, \mathbf{y}' is a vector whose coordinates are in $[0,1]$
- Usually pick the **binary cross-entropy loss**

$$L(\mathbf{y}, \mathbf{y}') = - \sum_j (y'_j \log y_j + (1 - y'_j) \log(1 - y_j))$$

Why cross-entropy over square?



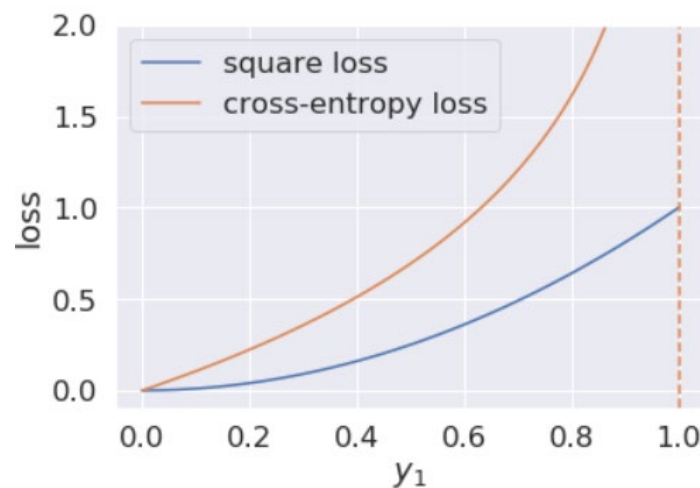
There are some good reasons to choose cross-entropy over the square loss for classification problems

- Better statistical interpretation
- Better numerical stability

2-Class case:

MSE: $L(y, y') = (y_1 - y'_1)^2$

CE: $L(y, y') = -y'_1 \log y_1 - (1 - y'_1) \log(1 - y_1)$



Surrogate Losses

A decorative network diagram in the top right corner, consisting of a series of interconnected nodes (small circles) and edges (thin lines), forming a complex web-like structure.

Importantly, for classification problems, whatever loss you choose, they are often **surrogates** for the true loss, which is the **accuracy**:

$$L(y, y') = \mathbb{I}_{y \neq y'}$$

This is known as the **zero-one loss**.

Why is this usually not used for optimization?



Gradient Based Learning

Empirical Risk Minimization

A decorative network graph in the top right corner, consisting of numerous light blue nodes connected by thin, light blue lines, forming a complex web-like structure.

Essentially, empirical risk minimization is an optimization problem

$$\min_{\boldsymbol{\theta}} R(\boldsymbol{\theta})$$

Recall that in linear regression, we solved the problem by setting

$$\nabla R(\boldsymbol{\theta}) = 0$$

However, very often there are no easy way to solve this equation

Iterative Methods

A decorative network graph is located in the top right corner of the slide. It consists of numerous small blue circular nodes connected by thin, light blue lines, forming a complex, interconnected web-like structure.

In this case, we will resort to iterative methods, where we make the loss smaller successively

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + F(\boldsymbol{\theta}_k)$$

And hopefully we achieve

$$R(\boldsymbol{\theta}_{k+1}) \leq R(\boldsymbol{\theta}_k)$$

Gradient Descent as an Iterative Method

Let us consider the optimization problem

$$\min_{\boldsymbol{\theta}} R(\boldsymbol{\theta}) \quad R: \mathbb{R}^p \rightarrow \mathbb{R}$$

Given $\boldsymbol{\theta}$, we want to make a small change to $\boldsymbol{\theta}$ in some appropriate direction $\boldsymbol{\phi}$, so that

$$R(\boldsymbol{\theta} + \epsilon \cdot \boldsymbol{\phi}) \leq R(\boldsymbol{\theta})$$

Here, ϵ is a small positive number



Taylor Expansion:

$$R(\boldsymbol{\theta} + \epsilon \boldsymbol{\phi}) = R(\boldsymbol{\theta}) + \epsilon \boldsymbol{\phi}^T \nabla R(\boldsymbol{\theta}) + \mathcal{O}(\epsilon^2)$$

We want the term $\epsilon \boldsymbol{\phi}^T \nabla R(\boldsymbol{\theta})$ to be as negative as possible, so we should take

$$\boldsymbol{\phi} \propto -\nabla R(\boldsymbol{\theta})$$

In other words, we want to push $\boldsymbol{\theta}$ in the negative gradient direction

$$\boldsymbol{\theta} \rightarrow \boldsymbol{\theta} - \epsilon \nabla R(\boldsymbol{\theta})$$

The Gradient Descent Algorithm

The direction $-\nabla R$ is called the **steepest descent direction**

This gives rise to a simple iterative algorithm for minimization

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \epsilon \nabla R(\boldsymbol{\theta}_k)$$

This is called **Gradient Descent** (GD)

The number ϵ is called the **step size** or the **learning rate**

Can we take ϵ as large as we like?

Example

Consider minimizing a quadratic function

$$R(\boldsymbol{\theta}) = \frac{1}{2} \|\boldsymbol{\theta}\|^2$$

Gradient descent iterates

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \epsilon \boldsymbol{\theta}_k$$

$$\boldsymbol{\theta}_k = (1 - \epsilon)^k \boldsymbol{\theta}_0$$

Example (Iterative Solution of Linear Regression)

Recall linear regression problem

$$\min_{\mathbf{w}} R(\mathbf{w}) = \frac{1}{2} \|\mathbf{X}\mathbf{w} - \mathbf{y}\|^2$$

Gradient descent iterates

$$\begin{aligned}\mathbf{w}_{k+1} &= \mathbf{w}_k - \epsilon \mathbf{X}^T (\mathbf{X}\mathbf{w}_k - \mathbf{y}) \\ &= (\mathbf{I} - \epsilon \mathbf{X}^T \mathbf{X}) \mathbf{w}_k + \epsilon \mathbf{X}^T \mathbf{y}\end{aligned}$$

Suppose $\mathbf{w}_k \rightarrow \mathbf{w}_\infty$ for some \mathbf{w}_∞ , what is \mathbf{w}_∞ ?

$$\mathbf{w}_\infty = (\mathbf{I} - \epsilon \mathbf{X}^T \mathbf{X}) \mathbf{w}_\infty + \epsilon \mathbf{X}^T \mathbf{y} \Rightarrow \mathbf{w}_\infty = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$

Convergence Analysis (Optional)



When does $\mathbf{w}_k \rightarrow \mathbf{w}_\infty$?

To see this, we can rewrite the GD iterations as

$$\mathbf{w}_{k+1} - \mathbf{w}_\infty = A(\mathbf{w}_k - \mathbf{w}_\infty) \quad \text{where} \quad A = (I - \epsilon X^T X)$$

Denote by $\mathbf{e}_k = \mathbf{w}_k - \mathbf{w}_\infty$ the error vector, then

$$\mathbf{e}_k = A^k \mathbf{e}_0$$

Let $\{\lambda_1(A), \dots, \lambda_d(A)\}$ be the real eigenvalues of A (why real?)

Then, if $\lambda_i(A) < 1$ for all i ,

$$\mathbf{e}_k = A^k \mathbf{e}_0 \rightarrow 0$$

This is ensured by $\epsilon \leq \frac{1}{\max_i \lambda_i(X^T X)}$ (Is this necessary?)



Suppose we take

$$\epsilon = \frac{1}{\max_i \lambda_i(X^T X)}$$

Then,

$$\|\mathbf{e}_k\| \leq \left(1 - \frac{1}{\kappa(X^T X)}\right)^k \|\mathbf{e}_0\|$$

The quantity

$$\kappa(X^T X) := \frac{\max_i \lambda_i(X^T X)}{\min_i \lambda_i(X^T X)} \geq 1$$

is called the **condition number** of $X^T X$. The bigger the condition number, the slower the convergence.

Convergence for General Losses



In general, one can prove that if ∇R is Lipschitz and $\nabla^2 R$ is uniformly bounded, then for small enough ϵ we have

$$\|\nabla R(\boldsymbol{\theta}_k)\| \rightarrow 0 \quad \text{as} \quad k \rightarrow \infty$$

Does this mean that $\boldsymbol{\theta}_k \rightarrow \boldsymbol{\theta}_\infty$ for some $\boldsymbol{\theta}_\infty$?

Steepest Descent to Optimize Neural Networks



Recall that the empirical risk minimization problem for NNs can be cast as an optimization problem

$$\min_{W, \mathbf{c}, \mathbf{w}, b} R(W, \mathbf{c}, \mathbf{w}, b) = \sum_{i=1}^N L(f^*(\mathbf{x}_i), f(\mathbf{x}_i; W, \mathbf{c}, \mathbf{w}, b))$$

And hence can be solved* iteratively by gradient descent

$$\begin{aligned} W_{k+1} &= W_k - \nabla_W R(W_k, \mathbf{c}_k, \mathbf{w}_k, b_k) \\ &\vdots \end{aligned}$$



Demo: Building Neural Networks in Keras

Tensorflow

Tensorflow is a open source deep learning framework developed by Google.

Main features

- Automatic differentiation
- Same code can be run on GPU (CUDA)
- Deployable to all sorts of devices (e.g. mobile phones)
- Eager execution starting from Tensorflow v2.0

Further Information and Tutorials

<https://www.tensorflow.org/>



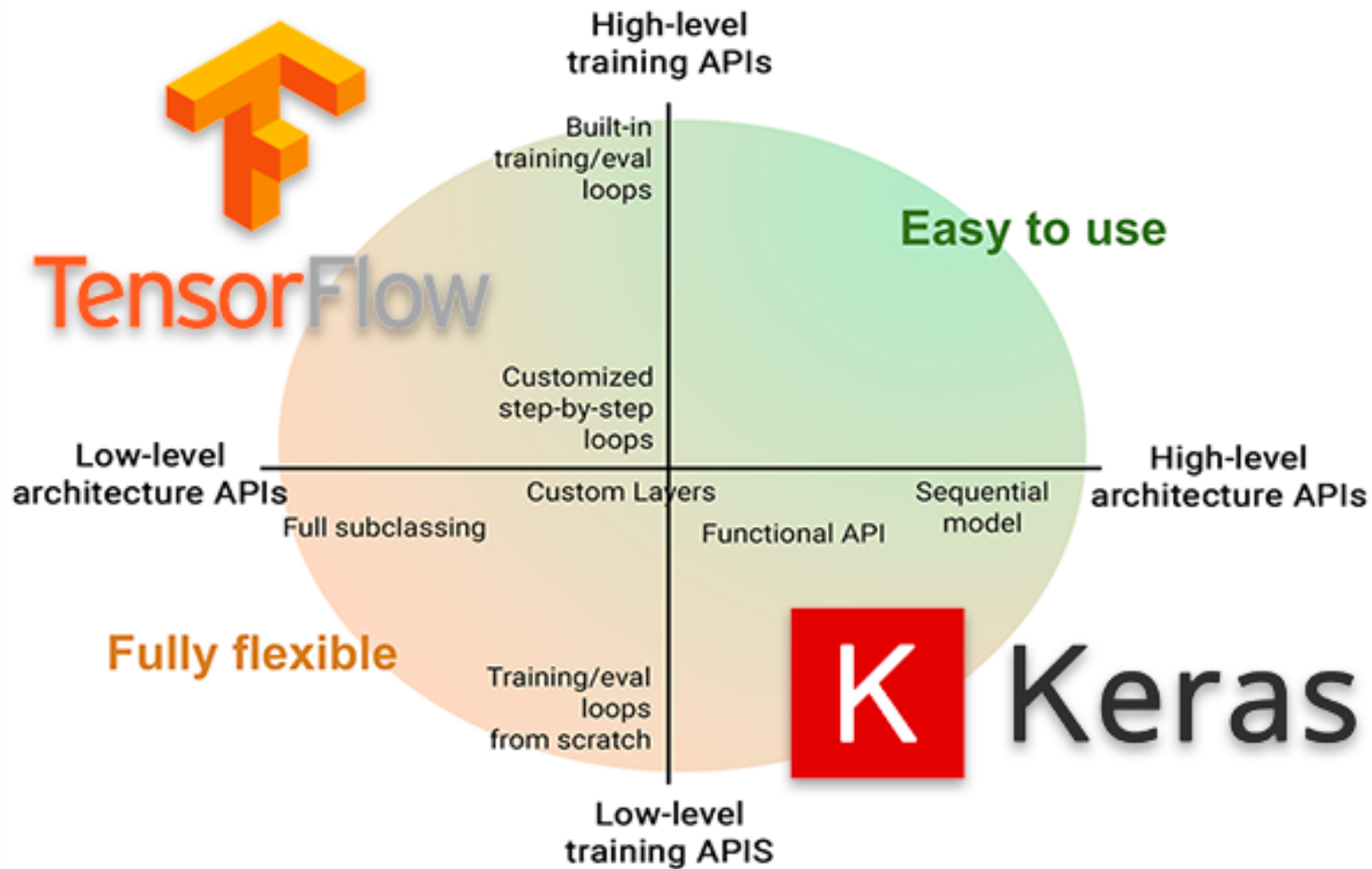
Keras

Keras is a high-level neural network API that makes model building and prototyping easier

- Uses Tensorflow (or CNTK, Theano) backend
- High-level abstraction (layers, models)
- “Disadvantage”: may need some lower level (tensorflow) knowledge to do some meaningful tinkering



Keras vs Tensorflow



Alternatives

The main alternative to the tensorflow/keras framework is **pytorch**

- Similar API and behavior to numpy
- Flexible for model and algorithm development
- See: <https://pytorch.org/>

The PyTorch logo, featuring the word "PYTORCH" in a bold, black, sans-serif font. The letter "O" is replaced by a stylized orange flame icon with a small purple dot at its base.

Computational Resources



For your project, you may require GPU computing resources for application-heavy type of problems

There are two sources for computational resources available to you:

- University HPC cluster (<https://nusit.nus.edu.sg/hpc/>)
- National Supercomputing Center (NSCC) (<https://www.nscg.sg>)