DSA5204 Lecture 2

Saturday, 21 January 2023 9:53 AM

Example: Linear Model for learning XOR

$$X = \begin{pmatrix} x_1 & x_2 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad y = f^*(X) = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Linear model

$$f(x) = w^T x + b = w_1 x_1 + w_2 x_2 + b.$$

$$X' = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad w' = \begin{pmatrix} w_1 \\ w_2 \\ b \end{pmatrix}$$

Empirical Risk Minimization

$$\min_{w'} \frac{1}{2} \| X'w' - y \|^2$$

Least Square Formula: $\hat{w} = (X'^T X')^{-1} X'^T y$

so, $\hat{w} = \begin{pmatrix} 0 \\ 0 \\ 1/2 \end{pmatrix}$

Best linear model is

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$$f(x) = 1/2 \quad , \quad \hat{y} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$