

Q1

$$(a) \min_{y, z} \delta_C(y) + \delta_D(z) \\ \text{s.t. } y - z = 0$$

(b) subproblem - y:

$$\begin{aligned} & \arg\min_y \left\{ \frac{1}{\sigma} \delta_C(y) + \frac{1}{2} \|y - z + \sigma^{-1} x\|^2 \right\} \\ \Rightarrow & \arg\min_y \frac{1}{2} \|y - z + \sigma^{-1} x\|^2 \quad \text{s.t. } y \in C \\ \Rightarrow & \Pi_C (z - \sigma^{-1} x) \\ \therefore & y^{k+1} = \Pi_C (z^k - \sigma^{-1} x^k) \end{aligned}$$

subproblem - z:

$$\begin{aligned} & \arg\min_z \left\{ \frac{1}{\sigma} \delta_D(z) + \frac{1}{2} \|y - z + \sigma^{-1} x\|^2 \right\} \\ \Rightarrow & \arg\min_z \frac{1}{2} \|y - z + \sigma^{-1} x\|^2 \quad \text{s.t. } z \in D \\ \Rightarrow & \Pi_D (y + \sigma^{-1} x) \\ \therefore & z^{k+1} = \Pi_D (y^{k+1} + \sigma^{-1} x^k) \end{aligned}$$

$$x^{k+1} = x^k + \tau \cdot \sigma \cdot (y^{k+1} - z^{k+1})$$

$$(c) y^0 = z^0 = x^0 = 0, \tau = 1, \sigma = 1$$

$$\textcircled{1} y^1 = \Pi_C (0 - 0) = 1$$

$$z^1 = \Pi_D (1 + 0) = 1.5$$

$$x^1 = 0 + 1 \cdot 1 \cdot (1 - 1.5) = -0.5$$

$$\textcircled{2} y^2 = \Pi_C (1.5 + 0.5) = 2$$

$$z^2 = \Pi_D (1 - 0.5) = 1.5$$

$$x^2 = -0.5 + 1 \cdot 1 \cdot (2 - 1.5) = 0.$$

Q2

(a)

- The difference between L and L_0 is 0.0178.
- The difference between S and S_0 is 0.0942.
- The number of iterations used to converge is 63.
- The running time for algorithm to converge is 5.84 seconds.

(b)

With reduced SVD and sigma iteration:

- The difference between L and L_0 is 0.0020.
- The difference between S and S_0 is 0.0039.
- The number of iterations used to converge is 21.
- The running time for algorithm to converge is 1.47 seconds.

(c)

- The number of iterations used to converge is 25.
- The running time used is 368.09 seconds.
- The term rk defined in question is $7.082e-5$.
- The rank of estimated matrix L is 56.
- The number of nonzero entries in estimated matrix S is 133194638.