### Patterns in Language Usage

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Language Quantitative Linguistics Structuralism Language Structure

### Language

#### language: system of communication

Language is a system that uses signs to encode and decode information to convey communication.

Language must be statable in a finite form (Davidson, 2005; von Humboldt, 1836; Chomsky, 1969; Apostel et al., 1957).



Language is a human capacity for acquiring and using complex systems of communication. A language in this sense is a system of signs for encoding and decoding information. Language is a social process of human iteration, an then it is made possible by our biological capabilities (and so limited by its biological restrictions) and shaped by our psychological aspects.

Language has a fundamentally social function. Processes of human interaction along with domain-general cognitive processes shape the structure and knowledge of language.

Recent research across a variety of disciplines in the cognitive sciences has demonstrated that patterns of use strongly affect how language is acquired, is structured, is organized in cognition, and changes over time. However, there is mounting evidence that processes of language acquisition, use and change are not independent from one another but are facets of the same system.

Is there Thought without Language?

"Thoughts are forms conceived in the mind, rather than the forms perceived through the five senses. Thought and thinking are the processes by which these concepts are perceived and manipulated. Thinking allows beings to model the world and to represent it according to their objectives, plans, ends and desires."

In 1967 Donald Davidson (philosopher) published "Truth and Meaning", in which he argued that any learnable language must be statable in a finite form, even if it is capable of a theoretically infinite number of expressions — as we may assume that natural human languages are, at least in principle. If it could not be stated in a finite way then it could not be learned through a finite, empirical method such as the way humans learn their languages.

We are finite beings whose mastery of the indefinitely many expressions of our language must somehow arise out of our mastery of finite resources. Otherwise, there would be an unbounded number of distinct things to learn in learning a language, which would make language learning impossible for finite beings like ourselves. The linguistic competence of a finite being of our sort must be the result of the interaction of a finite number of basic competencies.

### Language

- language unpredictable nature
- recurring patterns, language 'universal'
- structures and organization
- the principle of least effort (Zipf, 1949) : fast and robust communication



Patterns in Language Usage 57 60 Language Language Language

Language

- language unpredictable
   nature
- structures and organization
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  (Zipf, 1949): fast and
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Languages have unpredictable nature and without it there would be no communication at all. If it was a certain deterministic event, it would have no information associated.

The linguistic analysis of a language is the observation of certain recurring patterns. Some patterns that are said to be frequently found in the languages of the world are called 'universal'.

To study and understand how languages works, it is important to find/define its structures and the way they are organized to build communication.

The idea proposed by Zipf (1949) is that language works seeking the principle of least effort. The process of communication is better when it is possible to transmit information in a fast and robust way.

# Language

Language is a complex adaptive system.

- (1) multiple agents
- (2) adaptive
- (3) perception and motivation
- (4) emergence of patterns

We argue that this system is best construed as a complex adaptive system (CAS). This system is radically different from the static system of grammatical principles characteristic of the widely held generativist approach. Instead, language as a complex adaptive system of dynamic usage and its experience involves the following key features: (1) The system consists of multiple agents (the speakers in the speech community) interacting with one another. (2) The system is adaptive, that is, speakers' behavior is based on their past interactions, and current and past interactions together feed forward into future behavior. (3) A speaker's behavior is the consequence of competing factors ranging from perceptual mechanics to social motivations. (4) The structures of language emerge from interrelated patterns of experience, social interaction, and cognitive processes.

### Language



Figure: Dragon fractal.

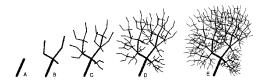
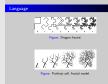


Figure: Purkinje cell, fractal model.



Our assumption is that the complex patterns and rules observed in language communication are emergent from the multiple interactions of speech agents over time. We believe there are simple fundamental laws that rule a multitude of microscopic interactions which create the observed macroscopic properties apparently complex. We believe language presents a fractal nature.

# Quantitative Approach

- systematic empirical investigation of phenomena via statistical, mathematical or computational techniques
- develop and employ mathematical models, theories and/or hypotheses pertaining to phenomena
- measurement: empirical observation

An immense number of of properties and processes in language which can be detected and analysed only with quantitative methods on the basis of quantitative concepts: features and interrelations which can be expressed only by numbers or rankings

It can be shown that these properties of linguistic elements and their interreations abide by universal laws, which can be formulated in a strict mathematical way - in analogy to the laws of the well-known natural sciences. Emphasis has to be put on the fact that these laws are stochastic; they do not capture single cases (this would neither be expected nor possible), they rather predict the probabilities of certain events or certain conditions in a whole.

A law can be said to be a statement representing universal patterns in the world (the phenomenological type of law) or universal mechanisms (the representational or mechanistic type).

types of laws: 1) probability distributions, i.e. it makes predictions about the number of units of a given property - Zipf-Mandelbrot Law 2) functional type, because these laws link two (or more) variables, i.e. properties - Menzerath's Law, which relates the size of linguistic constituents to the size of the corresponding construct 3) developmental type - a property is related to time - Piotrowski Law, which represents the development (increase and/or decrease) of the portion of new units or forms over time.

# Quantitative Linguistics - History I

- date back in the ancient Greek applications of combinatorics
- 718-791, philologist and lexicographer Al-Khalil ibn Ahmad permutations and combinations to list all possible Arabic words with and without vowels
- ▶ 1564-1614, William Bathe *Janua Linguarum*, the world's first language teaching texts, where he had compiled a list with 5.300 essential words
- ▶ the first scientific counts of units of language or text were published already in the 19th century as a means of linguistic description in Germany, Förstemann (1846, 1852) and Drobisch (1866), in Russia, Bunjakovskij (1847), in France Bourdon (1892), in Italy, Mariotti (1880) and in the USA, probably Sherman (1888)

# Quantitative Linguistics - History II

- ▶ the Russian mathematician Andrey Andreyevich Markov who created the base of the theory of Markov chains in 1913
- George Kingley Zipf was the first to set up a theoretical model in order to explain the observations and to find a mathematical formula for the corresponding function - the famous "Zipf's Law" (1935, 1949)
- Benoît Mandelbrot (1953, 1959, 1961a, 1961b)
- Shannon and Weaver (1949) Information Theory
- Gustav Herdan (1954, 1956, 1960, 1962, 1964 1966, 1969),
   Rajmund.G. Piotrowski (1959, 1968, 1979) and Walter
   Meyer-Eppler (1959)
- today, Quantitative Linguistics is a well-developed scientific discipline with a broad applicational impact

# Laws in Quantitative Linguistics

- Zipf-Mandelbrot frequency and rank are inversely related
- Menzerath the size of linguistic constituents decreases as the size of the corresponding construct increases
- Heaps (Herdan) relation between lexical size and text size
- ▶ Piotrowski development of new units or forms over time

Laws in Quantitative Linguistics (Universität Trier) http://lql.uni-trier.de Glottopedia http://www.glottopedia.org/

spreading process.

aws in Quantitative Linguistics

Glottopedia http://www.elottopedia.org/

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   Laws in Quantitative Linguistics (Universität Trier) http://lql.uni-trier.de

The Piotrowski's law states over the language change, proposing a mathematical model to it and delimiting which path could changes produce. The basic idea is that the change comes from the realization of one pearson and that may produces a gradual spread. The greater the number of people that takes on this change, the faster will be the

#### Structuralism

#### Structural linguistics

- ▶ Ferdinand de Saussure Course in General Linguistics in 1916.
- theoretical paradigm: elements must be understood in terms of their relationship to a larger, overarching system or structure
- linguistic signs were composed of two parts: signifier and signified
- linguistic levels: the phonemes, morphemes, lexical categories, noun phrases, verb phrases, and sentence types

"En matière de langue on s'est toujours contenté d'opérer sur des unités mal définies" 1 (de Saussure, 1916).

 $<sup>^{1}\</sup>mbox{In language's matter it has always been sufficient to operate on ill-defined units.$ 

Structuralism

"En matière de langue on s'est toujours contenté d'opérer sur des <u>unités mal définies" <sup>1</sup> (de Saussure, 1916).</u>

<sup>1</sup>In language's matter it has always been sufficient to operate on il-defined units.

The structuralism brings a new paradigma where we understand language as system where elements stablish relations one to another and to a largerr structure creating the whole.

It creates a dissociation of the signifier and signified and brings the linguistc analysis to multiple levels.

# Language Structure - Categorization

The categorization process in each language is different, although some common aspects are observed in all/most of them  $\rightarrow$  different sounds and rules.

```
Vowels systems.
```

```
English ο ο w ə i ι e ε æ a u ʊ ∧ α

Swedish ο i: y: ι: y e: Ø: ε ε: œ a u: ʊ ο: α: θ ʉ

Japanese ο w i ε a w

Portuguese ο ο i e ε a u
```

The sound of languages are also not equal across the multitude of languages in the World. Not only the sounds differ from language to language, but also the way they might be combined.

One big difficulty in learning a foreign language lays on the fact that different languages make different categorizations.

# Language Structure - Combinations

```
Phonemic Rules
```

```
The consonantal cluster [ts] in
```

German is allowed.

examples: 'Konferenz' ([kɔnfe'rɛnts]), 'Zeit' ([tsaɪt])

English is allowed only word initially.

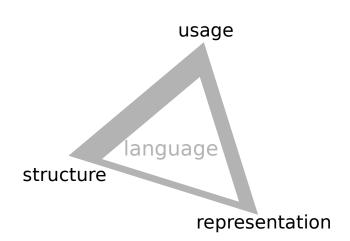
examples: 'cats' ([kæts]), 'splits' ([splits])

Classical Arabic no multiconsonant onsets are allowed at all.

We observe that speech inventory among languages is quite diverse, but there seems to exist some cornerstones and some patterns that are prefered over others. Phonemic rules are also different among different languages, creating different patterns, but some of them are more frequent.

It is important to investigate and understand what the reasons, what are the ubiquontous patterns and from this knowledge formulate/construct an explanation for how language works.

### Language Structure, Usage and Representation





"In particular, the frequency with which individual words or sequences of words are used and the frequency with which certain patterns recur in a language affects the nature of mental representation and in some cases the actual phonetic shape of words" (Bybee, 2003).

"It is certainly possible that the way language is used affects the way it is represented cognitively, and thus the way it is structured" (Bybee, 2003).

"The proposal that frequency of use affects representation suggests a very different view of lexical storage and its interaction with other aspects of the grammar or phonology than that assumed in most current theories. Structuralist and generative theories assume that the lexicon is a static list, and that neither the rules nor the lexical forms of a language are changed at all by instances of use" (Bybee, 2003).

# Frequency and Complexity

Frequency of occurrence is inversely proportional to complexity (Zipf, 1949).

- words
- phones, clusters
- within a language or among various languages of the world

Complexity and occurrence are intrinsically related to the way languages change.

Zipf (1949) proposes that the frequency a phoneme in a language occurs is inversely proportional to its complexity. The frequency of clusters should also be inversely proportional to the complexity of the cluster and now, the complexity of a cluster needs to be defined by the relation created by its parts.

In the history of any language, the phonemic system undergoes constant changes that may affect the complexity and occurrence frequency of phonemes.

# **UCLA Phonological Segment Inventory Database**

The UCLA Phonological Segment Inventory Database (or UPSID) is a statistical survey of the phoneme inventories in 451 of the world's languages. The database was created by American phonetician Ian Maddieson for the University of California, Los Angeles (UCLA) in 1984 and has been updated several times.

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http://www.linguistics.ucla.edu/faciliti/sales/software.htm
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Book: Patterns of sounds (Maddieson, 1984).

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JCLA Phonological Segment Inventory Database

http://www.linguistics.ucla.edu/faciliti/sales/ software.htm Book: Patterns of sounds (Maddisson, 1984).

Patterns of Sounds describes the frequency and distributional patterns of the phonemic sounds in a large and representative sample of the world's languages. Questions of the frequency and co-occurrence of the particular segment types are discussed in detail and possible explanations for the patterns observed are evaluated.

# **UPSID** - Number of Segments

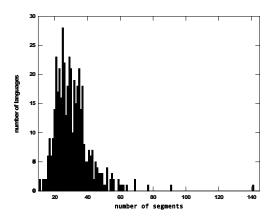
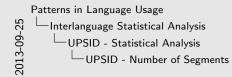
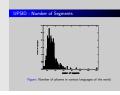


Figure: Number of phones in various languages of the world.





Among the 451 languages in the UPSID databese, the minimum number of segments used by a language is 11, in only two languages (Pirahã, spoken in Brazil by around 300 speakers; and Rotokas, spoken in Papua New Guinea by approximately 4,300 speakers). The language with the highest number of segments found in the UPSID database has 141 segments (the !Xu language, also called !Kung, is spoken by fifteen thousand speakers in Namibia and Angola). On average, the languages are built on 31 segments. Figure 3 bellow shows the histogram of languages regarding the number of segments used in each of them. 66.3% of the languages have a repertoire from 20 to 37 speech sounds. The languages use between 1.2% and 15.3% of all available speech sounds (919).

# **UPSID** - Segments in Languages

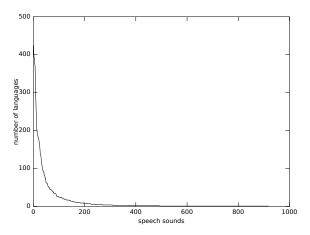
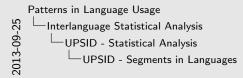


Figure: Number of languages where a give speech sounds occurs.





46.5% of the sounds appear in only one language. 80% of the sounds appear in 10 or fewer languages.

### **UPSID** - Segments in Languages

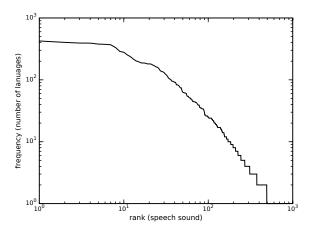


Figure: Number of languages for a given speech sound.

#### **UPSID** - Number of Vowels

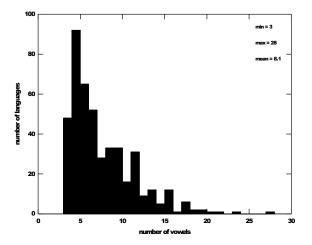
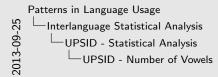
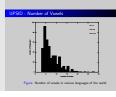


Figure: Number of vowels in various languages of the world.





Considering that the database with 451 languages has in its inventory 652 consonants (71%) and 269 vowels (29%), we see that across the languages, the number of vowels used corresponds from 0.9% to 17.9% (averaging 3.0%) of the possible vowels.

#### **UPSID** - Number of Consonants

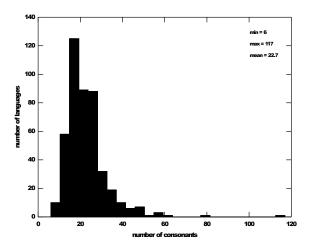
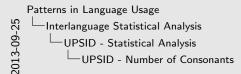
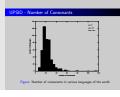


Figure: Number of consonants in various languages of the world.





The number of consonants used goes from 0.9% to 14.6% (averaging 3.5%) of the consonants in the database (which has an inventory of 652 consonants).

#### **UPSID** - Consonant-Vowels Ratio

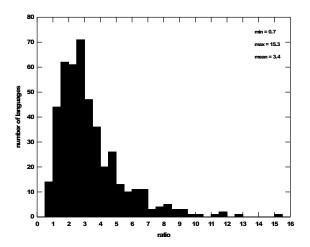
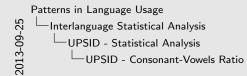
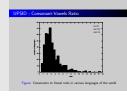


Figure: Consonants to Vowel ratio in various languages of the world.





klao	0.6875	yagua	0.91667	()		rutul	11.8
andoke	0.73333	cubeo	0.91667	acoma	9.2		
vanimo	0.75	panare	0.92308	shilha	9.3333	jaqaru	12
apinaye	0.76471	kaingang	0.92857	coola	9.3333		
dan	0.85	kashmiri	0.96429	yanyuwa	9.6667	tsimshian	12.667
bruu	0.90909	japreria	1	dahalo	10.2		
barasano	0.91667	maxakali	1	hadza	11.4	haida	15.333

#### **UPSID** - Consonant-Vowels Ratio CDF

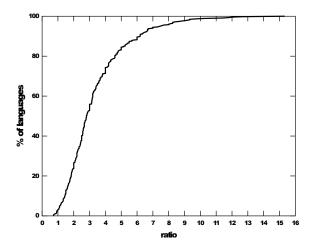


Figure: Cumulative distribution of the Consonant-Vowel ratio.

#### **UPSID** - Consonants

Table: List of the 20 most frequent consonants in UPSID.

consonant	m	k	j	р	W	b	h
n. of languages	425	403	378	375	332	287	279
frequency	94.2	89.4	83.8	83.2	73.6	63.6	61.9
consonant	g	ŋ	7	n	S	t∫	ſ
n. of languages	253	237	216	202	196	188	187
frequency	56.1	52.6	47.9	44.8	43.5	41.7	41.5
consonant	t	f	I	ņ	ţ	ŋ	
n. of languages	181	180	174	160	152	141	
frequency	40.1	39.9	38.6	35.5	33.7	31.3	

#### **UPSID** - Vowels

Table: List of the 10 most frequent vowels in UPSID.

consonant	i	а	u	3	о/э
n. of languages	393	392	369	186	181
frequency	87.1	86.9	81.8	41.2	40.1
consonant	e/ε	Э	0	е	а
n. of languages	169	162	131	124	83
frequency	37.5	35.9	29.0	27.5	18.4

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The most frequent vowels are not so frequent across languages in comparison to the most frequent consonants.

#### **UPSID** - number of phones vs. frequency index

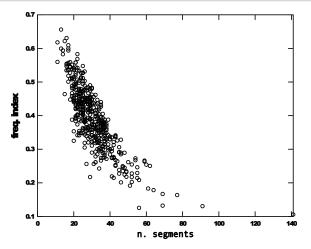
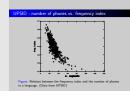
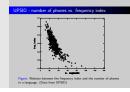


Figure: Relation between the frequency index and the number of phones in a language. (Data from UPSID)



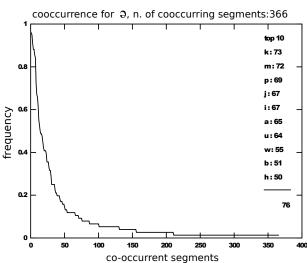
Frequency index is the arithmetic average of the segment frequencies of a language. A language with mostly rare segments will have a low frequency index, whereas a language with mostly common sounds will have a high frequency index. A frequency index of 0.1 means that a language has many very rare segments; 0.7 means it has many common segments; the average frequency index of all languages is 0.39.

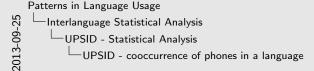


Note that there is a relation between frequency index and number of segments in a language. That is, if a language has only few segments, it is likely that these are rather common in the languages in UPSID. On the other hand, a language with many segments will also have many segments that are uncommon in the UPSID database. This does not necessarily mean that certain sounds are more natural but it is a probabilistic effect: if you make a pot with many red marbles, few green marbles, and other marbles with different colors and you draw a small random sample (i.e. 10 marbles) you will have mostly red marbles. If you draw a large random sample (e.g. 100 marbles) you will have many single colored ones.

Frequency index: mean=0.39, max=0.66, min=0.11.

#### Figure: Co-occurrence frequency for phones in relation to [ə]. [ə] occurs in 76 languages (16.8%) and has 366 cooccurring phones (39.8%). Cooccurring phones: [k] (96.0%), [m] (94.7%), [p] (90.7%), etc.



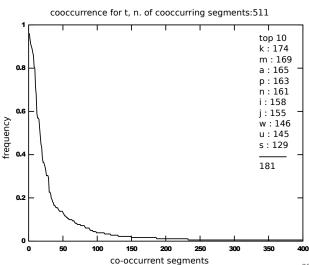




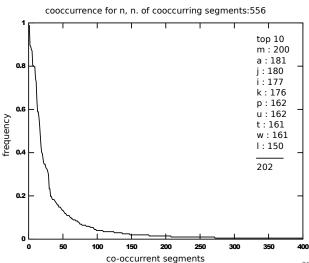
This graph presents the co-occurrence frequency for phones in relation to [a]. The phone [a] occurs in 76 languages (16.8%). It has 366 cooccurring phones in UPSID, that means 39.8% of 919 phones. Among them, the phone [k] has a frequency of 96.0% (73/76). [m] follows with 94.7% (72/76) and [a] with 90.7% (69/76).

#### Figure: Co-occurrence frequency for phones in relation to [t]. [t] occurs in 181 languages (40.1%) and has 511 cooccurring phones (55.6%). Cooccurring phones: [k] (96.1%), [m] (93.3%),

[a] (91.2%), etc.

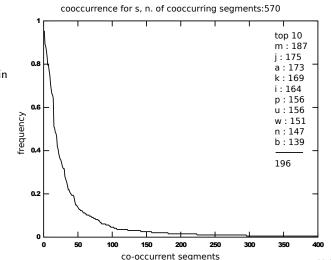


#### Figure: Co-occurrence frequency for phones in relation to [n]. [n] occurs in 202 languages (44.7%) and has 556 cooccurring phones (60.5%). Cooccurring phones: [m] (99.0%), [a] (89.6%), [j] (89.1%), etc.

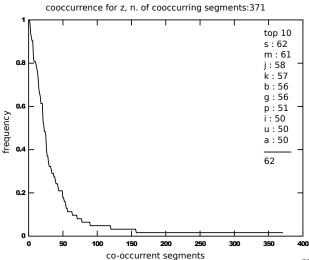


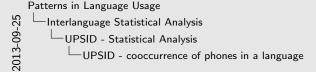
# Figure:

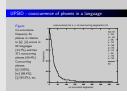
Co-occurrence frequency for phones in relation to [s]. [s] occurs in 196 languages (43.5%) and has 570 cooccurring phones (62.0%). Cooccurring phones: [m] (95.4%), [j] (89.3%), [a] (88.3%), etc.



#### Figure: Co-occurrence frequency for phones in relation to [z]. [z] occurs in 62 languages (13.7%) and has 371 cooccurring phones (40.4%). Cooccurring phones: [s] (100%), [m] (98.4%), [j] (93.5%), etc.







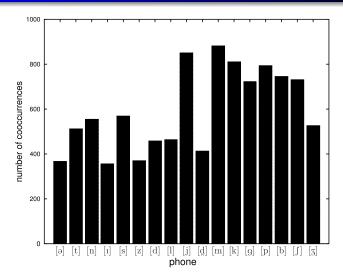
Always when the phone [z] (voiced alveolar fricatives) is present, the phone [s] (voiceless alveolar fricatives) is also present.

Table: List of phones and their top 8 co-occurring pairs with their relative frequency of occurrence (data from UPSID).

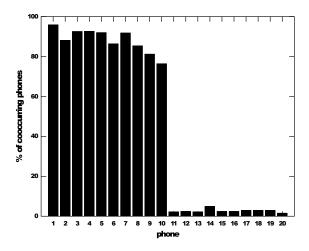
phone		co-occurring phone with their respective relative frequency (%)									
Э	k: 96.1	m: 94.7	p: 90.8	j: 88.2	i : 88.2	a: 85.5	u: 84.2	w : 72.4			
t	k: 96.1	m: 93.4	a: 91.2	p: 90.1	n: 89.0	i : 87.3	j: 85.6	w: 80.7			
n	m: 99.0	a: 89.6	j: 89.1	i : 87.6	k : 87.1	p: 80.2	u: 80.2	t: 79.7			
I I	m: 97.3	j: 91.9	k : 86.5	a: 83.8	p: 83.8	ช : 74.3	w : 71.6	b : 66.2			
s	m: 95.4	j: 89.3	a: 88.3	k : 86.2	i : 83.7	p: 79.6	u: 79.6	w : 77.0			
z	s: 100.0	m: 98.4	j: 93.5	k: 91.9	b: 90.3	g: 90.3	p: 82.3	i : 80.6			
d	b: 96.7	m: 94.2	i: 91.7	a: 90.8	j: 90.0	n: 89.2	g: 87.5	u: 86.7			
I	m: 98.9	j: 89.1	k : 86.8	n: 86.2	a: 86.2	i : 85.6	p: 81.0	w: 79.9			
i	m: 93.9	u: 91.6	k : 89.6	a: 89.1	p: 82.7	j: 82.7	w : 73.8	b: 65.1			
ď	b : 97.5	m: 96.2	g: 93.8	j: 88.8	k : 85.0	t : 83.8	i : 80.0	p: 76.2			
m	k : 89.2	i : 86.8	a: 86.6	j: 85.2	p: 82.6	u: 81.6	w: 74.1	b : 64.2			
n	m: 99.0	a: 89.6	j: 89.1	i : 87.6	k : 87.1	p: 80.2	u: 80.2	t: 79.7			
k	m: 94.0	p: 91.3	i : 87.3	a: 86.6	j: 83.4	u: 82.1	w : 73.2	b : 62.8			
g	b: 96.4	m: 95.3	i : 89.3	j: 87.4	k : 86.2	u: 84.2	a: 83.0	p: 76.7			
р	k: 98.1	m: 93.6	a: 87.2	i : 86.7	j: 82.9	u: 81.3	w: 71.7	b : 60.5			
b	m: 95.1	i: 89.2	k : 88.2	j: 86.8	g: 85.0	u: 84.3	a: 84.3	p: 79.1			
ſ	m: 94.7	j: 91.4	k : 88.8	i : 84.0	a: 82.9	p: 80.2	u: 78.1	w : 74.3			
3	∫: 95.1	m: 95.1	j: 90.2	k: 90.2	b: 82.0	g: 80.3	i : 78.7	p: 78.7			

As we take one bilabial consonants as reference, the others appear in the top-10 list, showing a strong adhesion of one bilabial consonant to another. Observing now the voiced alveolar sibilant fricative [z], its voiceless counterpart always appear, but doing the other way around analysis, taking [s] as a reference, its voiced counterpart [z] has a relative frequency of co-occurrence of 31.6%, figuring as the 29th in the list. Analyzing other pairs like [t]-[d] (alveolar plosive), [k]-[g] (velar plosive), [p]-[b] (bilabial plosive) and [ʒ]-[ʃ] (palato-alveolar fricative), it seems that the existence of the voiced counterpart subjects the existence of the voiceless much more emphatically than the other way around.

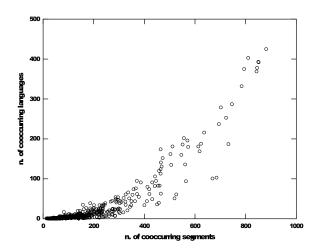
# Cooccurrence for a given phone



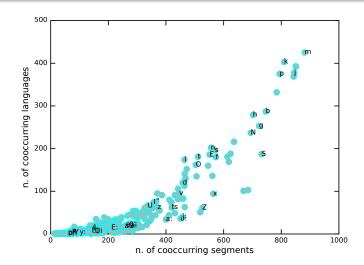
### Cooccurrence: most frequent vs least frequent



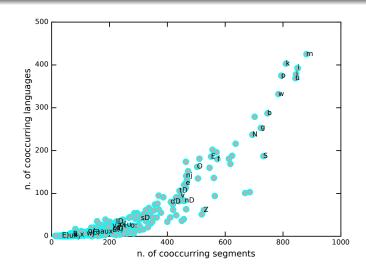
#### Cooccurrence: languages and segments



#### Cooccurrence: languages and segments - German



#### Cooccurrence: languages and segments - French



# Approach

- synchrony
- written corpus
- pronouncing dictionary
- statistical analysis



Although "languages are simultaneously products of history and entities existing at particular times" (Good, 2008) and both diachrony and synchrony aspects are important to determine what languages are, we focus here only on the synchrony aspects.

To carry out the analysis of it, we use large text databases, because it is much easier to be acquired, stored and handled.

A pronouncing dictionary is used to convert text into phones.

It is possible then to derive statistical analysis on this new database which statistically resembles a speech database.

#### Text Database

Project Gutenberg is a volunteer effort to provide free dgital access to cultural works. It was founded by Michael S. Hart in 1971 and is the oldest digital library. Most of its items are public domain books. Project Gutenberg claimed over 42,000 items in its collection (March 2013) (Project Gutenberg, 2013).

Google Ngram is a large database of n-grams (words combinations) based originally on 5.2 million books, published between 1500 and 2008, containing 500 billion words in American English, British English, French, German, Spanish, Russian, or Chinese (Michel et al., 2011b).

Project Custobleg is a volunteer effort to provide free digital access to cultural works. It was founded by McKealat S. Mart in 1071 and in the oblest digital blave, Most Gustellow (See 1) and in the oblest digital blave, Most Gustellow (March 2013) (Project Gustellow, 2013) (Project Gustellow, 2013) (Project Gustellow, 2013) (Project Gustellow, 2013) (Congle Nyami is a large database of pagma (contilionation) based originally on 3.2 million books, combination) based originally on 3.2 million books, combination (See 1) and originally on 3.2 million books, consideration (See 1) and originally on 3.2 million books, combination (See 1) and originally on 3.2 million books, combination (See 1) and originally on 3.2 million books, combination (See 1) and original distribution (See 1)

French, German, Spanish, Russian, or Chinese (Michel et al., 2011b).

Project Gutenberg is a volunteer effort to digitize and archive cultural works, to encourage the creation and distribution of eBooks. Founded in 1971 by Michael S. Hart, it is the oldest digital library. Most of the items in its collection are the full texts of public domain books. The project tries to make these as free as possible, in long-lasting, open formats that can be used on almost any computer. As of December 2009, Project Gutenberg claimed over 33,000 items in its collection.

Today it is viable to have large databases and it is relatively easy do complile such databases and manage them. That could not be dreamt a few years ago. It would be extremily good to have large speech databases in order to study language as a spoken phenomena, investigate its usage and change, but it is a much harder task to acchieve this goal.

reCAPTCHA is a user-dialogue system originally developed by Luis von Ahn, Ben Maurer, Colin McMillen, David Abraham and Manuel Blum at Carnegie Mellon University's main Pittsburgh campus, and acquired by Google in September 2009.

# **Pronouncing Dictionary**

Dictionary: Carnegie Mellon University Pronouncing Dictionary

- is a machine-readable pronunciation dictionary for North American English that contains over 125,000 words and their transcriptions (Weide, 2008).

Phoneme set: 39 phonemes.

Symbols : ARPAbet.

Phoneme	IPA	Example	Translation	IPA Translation
AE	[æ]	at	AE T	[æt]
В	[b]	be	B IY	[bi]
S	[s]	sea	S IY	[si]

ronouncin	g Dic	tionary		
Dictionary	- is a Nort	a machine- h America	readable prons n English that	Pronouncing Dictiona inciation dictionary for contains over 125,000 s (Weide, 2008).
Phoneme se Symbol				
	: AF	PAbet.	Translation	IPA Translation
Symbol	: AF	PAbet.		IPA Translation
Symbol	: AF	Example	Translation	

To carry out the statistical analysis of a spoken language, we assume that a written language presents a very similar characteristics, and then the statistical analysis using a written text database and a text-to-phone transcription would be quite reasonable.

Arpabet is a phonetic transcription code developed by Advanced Research Projects Agency (ARPA) as a part of their Speech Understanding Project (1971-1976). It represents each phoneme of General American English with a distinct sequence of ASCII characters. Arpabet has been used in several speech synthesizers, including Computalker for the S-100 (Altair) system, SAM for the Commodore 64, SAY for the Amiga and TextAssist for the PC and Speakeasy from Intelligent Artefacts (see ST\_Robotics) which used the Votrax SC01 speech synthesiser IC. It is also used in the CMU Pronouncing Dictionary.

# Words Frequency

(1)	the: 775911	(16)	for: 107245	(31)	by: 63944	(46)	if: 38421
(2)	and: 471916	(17)	as: 102009	(32)	which: 63051	(47)	there : 38209
(3)	of: 414499	(18)	not : 96636	(33)	she: 57839	(48)	we: 37944
(4)	to: 350613	(19)	be: 86896	(34)	they: 57770	(49)	when: 37385
(5)	a: 277321	(20)	but: 81643	(35)	from: 56128	(50)	their : 36721
(6)	in: 226505	(21)	had: 80327	(36)	or: 52089	(51)	who: 36109
(7)	i: 200689	(22)	at: 76688	(37)	so : 51617	(52)	an: 35485
(8)	that : 173083	(23)	her: 75761	(38)	said: 50040	(53)	your: 33401
(9)	he: 162183	(24)	on: 75493	(39)	no: 48930	(54)	would : 32582
(10)	it: 145364	(25)	my: 73879	(40)	are: 45831	(55)	do: 31225
(11)	was: 130804	(26)	him: 72258	(41)	one: 43822	(56)	out: 30165
(12)	his: 129300	(27)	have : 68463	(42)	what : 41575	(57)	then: 29682
(13)	you: 118473	(28)	this: 67572	(43)	them: 41320	(58)	been: 29502
(14)	with: 114122	(29)	all: 65960	(44)	were : 40475	(59)	up: 28860
(15)	is: 112640	(30)	me: 64560	(45)	will: 39733		

# Words Frequency and Zipf Law

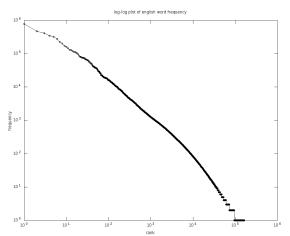


Figure: Log-log plot of words rank versus frequency of occurrence.

### Letters Frequency

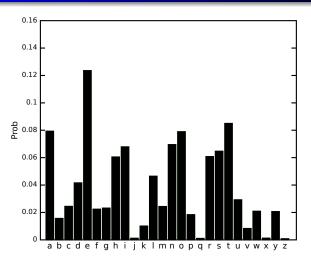


Figure: Relative frequency of letters in text.

#### Letters Frequency



Figure: Frequency distributions of the 26 most common Latin letters across some languages (source: Wikipedia).

#### Words Frequency - Transcribed

- DH AH: 775911 (1)
- AH N D: 471916 (2) AH V: 414499
- T UW: 350613 (4)

(3)

- (5) AH: 277321
- IH N: 226505 (6)
- AY: 200689 (7)
- DH AE T: (8) 173083
- HH IY: 162183 (9)
- IH T: 145364 (10)
- (11)W AA Z: 130804
- HH IH Z: (12) 129300
- Y UW: 118473 (13)
- (14) WIH DH: 114122

- IH Z: 112640 (15)
- F AO R: 107245 (16)
- AE Z: 102009 (17)(18)N AA T: 96636
- (19)B IY: 86896
- (20)B AH T: 81643
- (21)HH AE D: 80327 (22)AE T: 76688
- (23)HH ER: 75761
- (24)AA N: 75493
- (25)M AY: 73879
- (26)HH IH M: 72258
- (27)HH AE V: 68463
- (28)DH IH S: 67572
- (29)AO L: 65960
- (30)M IY: 64560

- (31)B AY: 63944
- W IH CH: 63051 (32)
- SH IY: 57839 (33)DH EY: 57770 (34)
- FRAHM: (35)
- 56128 AO R: 52089 (36)
- S OW: 51617 (37)
- S EH D: 50040 (38)
- N OW: 48930 (39)
- AA R: 45831 (40)
- (41)W AH N: 43822
- (42)W AH T: 41575 DH EH M: (43)
- 41320
- W ER: 40475 (44)
- W IH L: 39733 (45)

- IH F: 38421 (46)
- DH EH R: 38209 (47)
- (48)W IY: 37944
- (49)W EH N: 37385
- DH EH R: 36721 (50)HH UW: 36109
- AE N: 35485 (52)
- Y AO R: 33401 (53)
- (54)W UH D: 32582
- D UW: 31225 (55)AO L AW T: (56)
- 30165
- DH EH N: 29682 (57)
- B IH N: 29502 (58)
- (59)AH P: 28860

## Phones Frequency

```
(1) a: 44539
                (11) m: 13072 (21) æ: 8635
                                                 (31) q: 3351
 (2) t: 33131
                (12) 3<sup>4</sup>: 12640 (22) b: 8390
                                                 (32) t[: 2501
 (3) n: 31928
                (13) k : 12308
                                 (23) u: 7972
                                                 (33) j : 2462
 (4) i: 28845
                (14) w: 11107
                                 (24) p: 7501
                                                 (34) \theta : 2309
 (5) s: 21928
                                 (25) o: 7429
                (15) z : 10744
                                                 (35) v: 2276
 (6) d: 20032
                (16) ð: 10720
                                 (26) ei: 6196
                                                 (36) au: 2242
 (7) r: 18563
                (17) v: 10407
                                 (27) ai: 6148
                                                 (37) dz: 2100
 (8) i: 16482
                (18) h: 10009
                                 (28) ou : 5283
                                                 (38) or: 326
 (9) I: 15816
                (19) f: 9391
                                 (29) [: 4915]
(10) ε : 13896
                                 (30) n: 4861
                                                 (39) 3:314
                (20) a: 8744
```

### Phones Frequency - Log-log plot

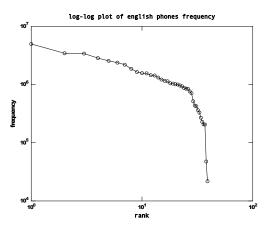


Figure: Log-log plot of phones rank versus frequency of occurrence.



This is not an example of distribution with large number of rare events ... where "rare events are common" We might say that there are no rare events. Zipf's law is found only on distributions with large number of rare events.

The simple "maximum likelihood" method for predicting estimate the probability of an event-type that has occurred r times in N trials as r/N. This generally works well if r is fairly large (and if the world doesn't change too much). But as r gets smaller, the maximum likelihood estimate gets worse. And if r is zero, it may still be quite unwise to bet that the event-type in question will never occur in the future. Even more important, the fraction of future events whose past counts are zero may be substantial.

One of the present time (2000) characteristics of the study of Zipf's law is the increase of interest in the more general concept of "distributions with a large number of rare events" (LNRE distributions), which was introduced and first systematically studied by (Khmaladze, 1987).

### Probability of occurrence of [a] across words rank

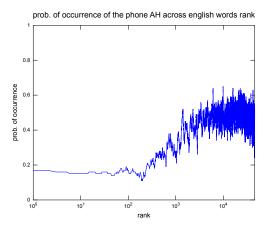
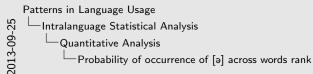
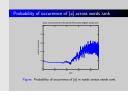


Figure: Probability of occurrence of [ə] in words versus words rank.





We might wonder whether some phones are more frequent then others because they are present in most frequent words, what would lead them along to a higher frequency of occurrence. The present graphic and the following ones shows this is a false hypothesis.

### Probability of occurrence of [t] across words rank

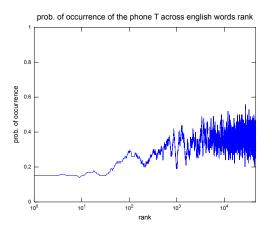


Figure: Probability of occurrence of [t] in words versus words rank.

### Probability of occurrence of [n] across words rank

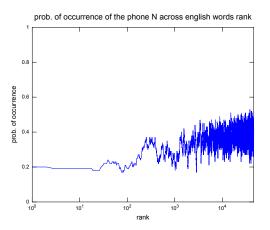


Figure: Probability of occurrence of [n] in words versus words rank.

### Probability of occurrence of [v] across words rank

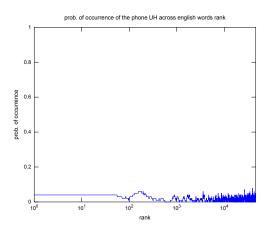


Figure: Probability of occurrence of [v] in words versus words rank.

### Probability of occurrence of [ɔi] across words rank

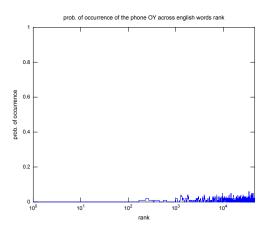


Figure: Probability of occurrence of [ɔi] in words versus words rank.

### Probability of occurrence of [3] across words rank

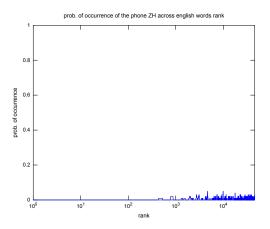


Figure: Probability of occurrence of [3] in words versus words rank.

# **Diphones**: Frequency of occurrence

```
ən: 1296408
                                əl: 372847
                          (12)
                                                           hæ: 237724
                                                                                (34) ar: 193525
     ðə: 785354
                          (13)
                                εn: 349905
                                                           hı: 237490
                                                     (24)
     nd: 784028
                                nt: 337193
                                                           əm: 233793
                          (14)
                                                     (25)
     st: 651129
                          (15)
                                æt: 284460
                                                     (26)
                                                           ri: 219997
 (4)
                                                                              (1120) uai: 1
     əv: 489267
                                wı: 282526
                                                           li: 219652
 (5)
                          (16)
                                                     (27)
                                                                              (1121)
                                ət: 265396
                                                                                      эıa : 1
 (6)
     or: 472069
                          (17)
                                                     (28)
                                                           an: 213755
     ın: 470069
                           (18)
                                er: 264293
                                                     (29)
                                                           æn: 213169
                                                                              (1122) b[:1
     tu: 425544
                                rə: 261447
                                                           sə: 210385
 (8)
                          (19)
                                                     (30)
                                                                              (1123) pv: 1
     tə: 420825
                           (20)
                                ıt: 260544
                                                     (31)
                                                           ıs: 208619
 (9)
                                                                              (1124) iv: 1
(10)
     ın: 387096
                           (21)
                                əs: 246996
                                                     (32)
                                                           ðæ: 194602
(11) iz: 380357
                           (22)
                               ju: 245715
                                                     (33)
                                                           əf: 193570
                                                                              (1125) εου: 1
```

# Diphones Frequency - Log-log plot

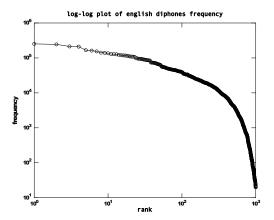


Figure: Log-log plot of the diphones frequency of occurrence versus their rank.

## Diphones: phone-normalized frequency of occurrence

```
(1) ʒə
            (5) dʒə
                         (9) ju
                                     (13) \text{ ət}  (17) \text{ əp } (1122) \text{ aih }
(2) ðə
            (6) bə
                        (10) əm
                                     (14) nd
                                                            (1123) εου
(3) <sub>a</sub>v
                        (11) or
                                     (15) kə (1120) zs (1124) ddʒ
(4) (a
                                     (16) əq (1121) uai (1125) tv
            (8) əf
                        (12) əb
```

### Diphones Normalized Frequency - Log-log plot

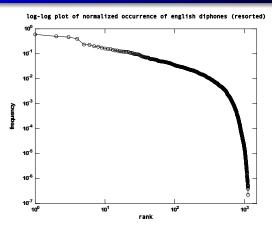
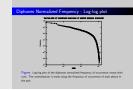


Figure: Log-log plot of the diphones normalized frequency of occurrence versus their rank. The normalization is made using the frequency of occurrence of each phone in the pair.



the normalization is total number of diphones divided by the occurrence of either one of the phones in the diphone pair

### **Diphones Conditional Probability**

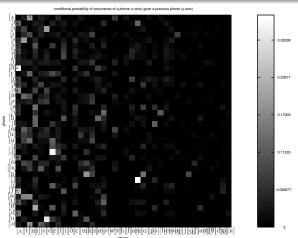
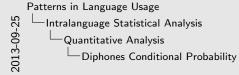


Figure: Probability of occurrence of a phone given another previous phone.





We observe in the figure that the most frequent phones (in the left part of the figure) also have an average higher conditional probability of occurrence regardless which the prior phone is.

$$(30,22) = [j u]$$

$$(25,7) = [5 r]$$

$$(37,6) = [v d]$$

$$(10,1)=[\delta \ ext{ə}]$$

#### Triphone Probabilities

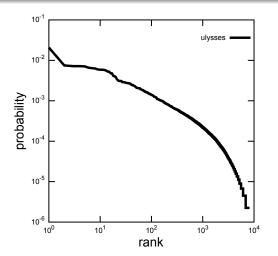


Figure: Triphone Probability (from Ulysses).

### Syllable Probabilities

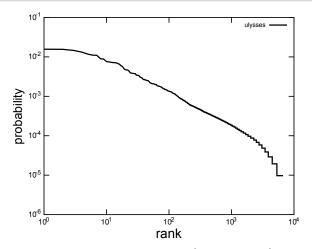


Figure: Syllable Probability (from Ulysses).

#### Words Length

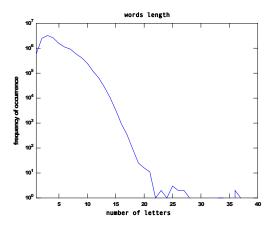


Figure: Frequency of occurrence of words of a given length (letters).

### Words Phones Length

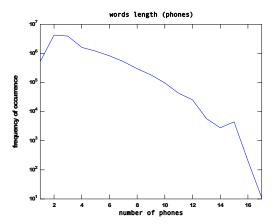


Figure: Frequency of occurrence of words of a given length (phones).

### Average word length (letters)

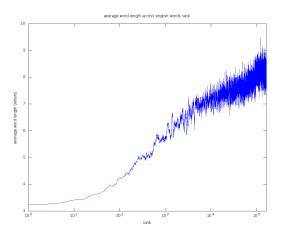


Figure: Average word length (letters) across word rank.

## Average word length (phones)

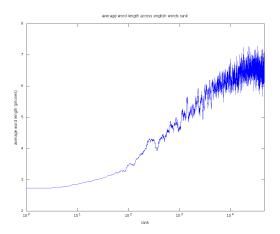


Figure: Average word length (phones) across word rank.

### Frequency of occurrence and frequency index

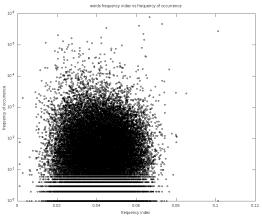


Figure: Each spot represents a word with a given frequency of occurrence and a frequency index. For a better visualization the frequency of occurrence is displayed in a logarithm scale.

### Frequency of occurrence vs frequency index (density plot)

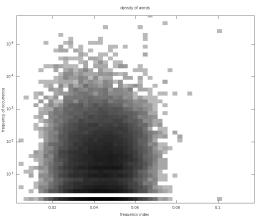


Figure: This picture is the density of words in each partition on the frequency of occurrence vs. frequency index space. The largest number is displayed in black and it refers to 578 words. White represents no word found in a spot.

### Words Frequency and Zipf Law

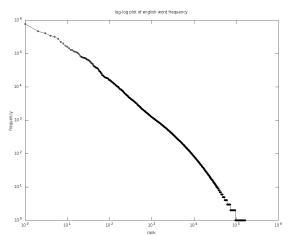


Figure: Log-log plot of words rank versus frequency of occurrence.

### Zipf's Law

Zipf law states that there is a relationship between the word's frequency of appearance in texts and its rank, the product of them is roughly a constant.

Power law relation:

$$f(k; s, N) = Ck^{-s} = \frac{k^{-s}}{\sum_{n=1}^{N} n^{-s}}$$
 (1)

f frequency

k rank

N number of elements in the set

s characterizing exponent

N number of elements in the set s characterizing exponent

As already pointed out by Li (1992), "Zipf's law is not a deep law in natural language as one might first have thought. It is very much related the to particular representation one chooses, i.e., rank as the independent variable." Li (1992) showed that random texts also exhibit Zipf's law-like curves.

### Zipf's Law

The normalizing constant C in Zipf's law might also be written as

$$C = \frac{1}{H_{N,s}} \tag{2}$$

where  $H_{N,s}$  is known as the generalized harmonic number

$$H_{N,s} = \sum_{n=1}^{N} \frac{1}{n^s} \ . \tag{3}$$

(converges for s > 1)

# Zipf's exponent

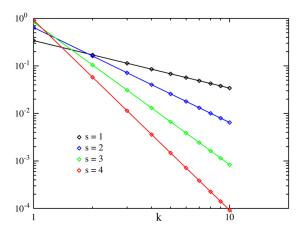


Figure: Probability mass function with N = 10. (Wikipedia)

# The ubiquity of Zipf's law

This seemingly universal law is present in various phenomena:

- (1) earthquakes
- (2) avalanches
- (3) populations
- (4) firms
- (5) stocks
- (6) game of life

## Gutenberg-Richter law for earthquakes

Relation between magnitude m of a earthquake and the number N of earthquakes with magnitude greater of equal to m:

$$\log(N) = -bm + a . \quad (4)$$

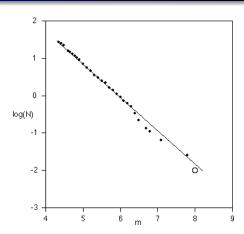


Figure: Data from the South of California from 1932 to 1972 (Turcotte, 1989).

### Sandpile

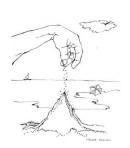


Figure: Sandpile avalanches (Bak, 1999).

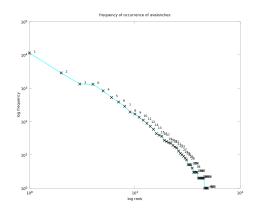


Figure: Frequency of avalanches.

### **Population**



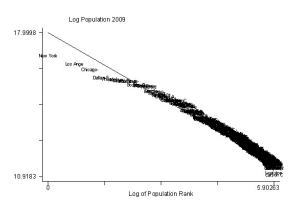
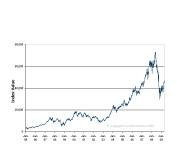


Figure: Population of American metropolitan areas (Edward L. Glaeser).

#### **Stocks**



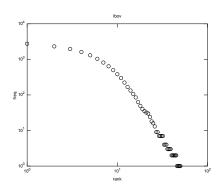


Figure: Variations on the Ibov index, using the idea presented by Mandelbrot (1963).

## Game of Life

 cellular automata (Stanisław Ulam, John von Neumann, 1940)

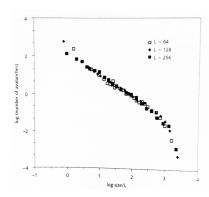


Figure: Avalanches (number of births and deaths until a static configuration is reached) in the game of life (Maslov et al., 1994).

A cellular automaton (pl. cellular automata) is a discrete model studied in computability theory, mathematics, physics, complexity science, theoretical biology and microstructure modeling. A cellular automaton consists of a regular grid of cells, each in one of a finite number of states, such as on and off. The grid can be in any finite number of dimensions. For each cell, a set of cells called its neighborhood is defined relative to the specified cell. An initial state (time t=0) is selected by assigning a state for each cell. A new generation is created (advancing t by 1), according to some fixed rule (generally, a mathematical function) that determines the new state of each cell in terms of the current state of the cell and the states of the cells in its neighborhood. Typically, the rule for updating the state of cells is the same for each cell and does not change over time, and is applied to the whole grid simultaneously.

The concept was originally discovered in the 1940s by Stanislaw Ulam and John von Neumann while they were contemporaries at Los Alamos National Laboratory. While studied by some throughout the 1950s and 1960s, it was not until the 1970s and Conway's Game of Life, a two-dimensional cellular automaton, that interest in the subject expanded beyond academia. In the 1980s, Stephen Wolfram engaged in a systematic study of one-dimensional cellular automata, or what he calls elementary cellular automata; his research assistant Matthew Cook showed that one of these rules is Turing-complete. Wolfram published A New Kind of Science in 2002, claiming that cellular automata have applications in many fields of science. These include computer processors and cryptography.

#### Random Texts

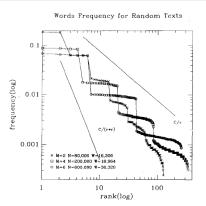


Figure: Word frequency vs rank for random generated symbols equally distributed. Symbol set size: M=2, 4, and 6. Reference Zipf's law for s=1 and 2 are displayed. (Li, 1992; Miller, 1957)

# Random Text vs Ulysses

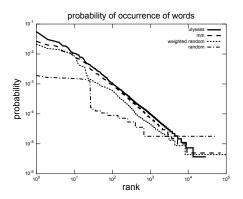


Figure: Random text and *Ulysses* compared.

# Random Texts vs Ulysses: word length

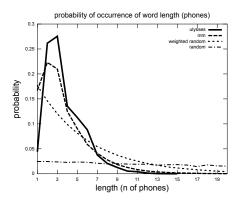


Figure: Word probability for a given length.

# Markov Processes and Zipf's law

Kanter and Kessler (1995) shows that a 2-parameter random Markov process constructed with *N* states and biased random transitions gives rise to a stationary distribution where the probabilities of occurrence of the states exhibit a rank-ordered frequencies of occurrence of words given by Zipf's law.

## Random Text Model - Biemann

Biemann (2007) proposes a model of Random Text generation that takes the properties of neighboring co-occurrence into account and introduces the notion of sentences in random text.

It is is basically composed of: A word generator that produces random words composed of letters and a sentence generator that composes random sentences of words.

## Random Text Model - Biemann

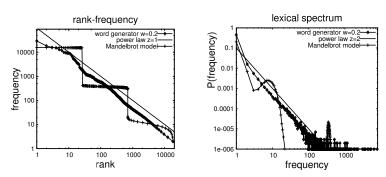


Figure: Rank-frequency distribution and lexical spectrum for the word generator in comparison to the Mandelbrot model (Biemann, 2007).

# **Analysis of Smaller Units**

Word is considered the smallest free form that can be uttered or written and carries a meaning (Bloomfield, 1926).

Basic units of speech perception:

- phones (Pisoni, 1982)
- diphones (Klatt, 1979)
- triphones
- syllables (Studdert-Kennedy, 1976)
- demisyllables (Fujimura and Lovins, 1978)

## Distinctive Features



Figure: English Distinctive Features Table.



The theory proposes the existence of a small finite set of features that may be used to analyze speech sounds, in such way that the description of each speech sound in terms of these features is unique. The analysis into distinctive features is a unambiguous process to associate speech sounds into arrays of features that describe these sounds. The classical statement of distinctive features was brought by Chomsky and Halle (1968).

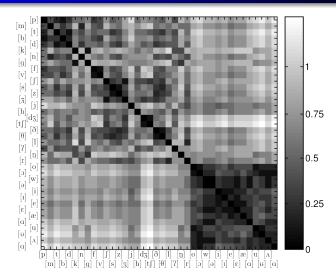
The most important contribution to phonology from the distinctive feature theory is that a set of segments may be analyzed into some features and it is possible to identify classes of segments in rules, creating the notion of natural class, as a set of sounds that has certain phonetic features in common and is affected in the same way in the same environment.

#### Distinctive Feature Distance

#### definition: distance measure between two segments

we define a distance measure of two segments as the number of features not shared between them (that dissimilarity definition resembles the natural class definition of Flemming (2005))

# Dissimilarity Matrix



## MDS - Vowels

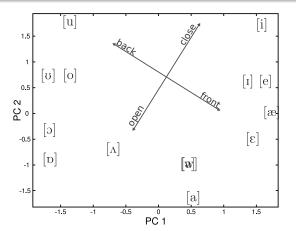
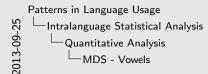
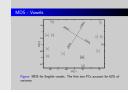


Figure: MDS for English vowels. The first two PCs account for 63% of variance.





Multidimensional scaling (MDS) is a means of visualizing the level of similarity of individual cases of a dataset. An MDS algorithm aims to place each object in N-dimensional space such that the between-object distances are preserved as well as possible. Each object is then assigned coordinates in each of the N dimensions. The number of dimensions of an MDS plot N can exceed 2 and are specified a priori. Choosing N=2 optimizes the object locations for a two-dimensional scatterplot.

## MDS - Consonants

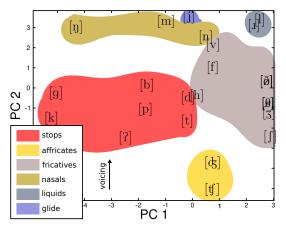


Figure: MDS for English consonants. The first two PCs account for 46% of variance.

## Intradistances - triphones

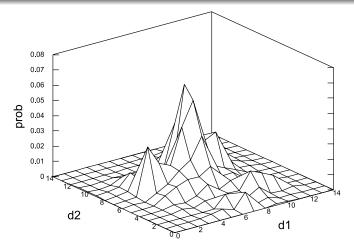


Figure: Probability of triphones given its intradistances.

# Frequency of occurrence of words in logarithmic scale

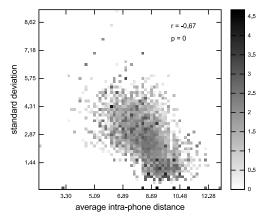


Figure: Relation in words between frequency of occurrence, average intra-phone distances and standard deviation of these distances.

# Zipf Exponent

The Zipf's exponent characterizes the source's distribution.

- ▶ natural languages  $s \approx 1$  (Piotrovskii et al., 1994)
- children's speech  $s \approx 1.66$  (Piotrovskii et al., 1994)
- ▶ military combat text  $s \approx 1.42$  (Kolguškin, 1960)
- ▶ adult and infant dolphins  $s \approx 1.1$  and  $s \approx 0.87$ , respectively (McCowan et al., 1999)
- authorship attribution (Havlin, 1995)
- ▶ noncoding DNA sequence  $s \approx 0.36$  and coding DNA sequence  $s \approx 0.20$  (Mantegna et al., 1994)

Probability density function for a Zipf distribution:

$$f(k|s,N) = \frac{1/k^s}{\sum_{n=1}^{N} n^{-s}} = \frac{1/k^s}{H_{N,s}},$$
 (5)

Maximum likelihood estimation (MLE).

$$L(s|k_{1}, \dots, k_{M}, N) = f(k = (k_{1}, k_{2}, \dots, k_{M})|s, N)$$

$$= \prod_{m=1}^{M} f(k_{m}|s, N)$$

$$= \left(\frac{1}{H_{N,s}}\right)^{M} \prod_{1}^{M} \frac{1}{k_{m}^{s}}.$$
 (6)

The logarithm of the likelihood function:

$$\ln L(s|k_1, \dots, k_M, N) = -M \ln H_{N,s} - s \sum_{m=1}^{M} \ln k_m.$$
 (7)

Solution: the parameter s such that

$$\frac{\partial \ln L(s|k,N)}{\partial s} = 0 , \qquad (8)$$

if

$$\frac{\partial^2 \ln L(s|k,N)}{\partial s^2} < 0. (9)$$

We need to solve

$$\frac{\partial \ln L(s|k_1,\cdots,k_M,N)}{\partial s} = -M \frac{G_{N,s}}{H_{N,s}} - \sum_{m=1}^{M} \ln k_m = 0 , \qquad (10)$$

given that

$$\frac{d^2}{ds^2} \ln L(s|k_1, \cdots, k_M, N) = M \frac{G_{N,s}^2 - I_{N,s} H_{N,s}}{H_{N,s}^2} < 0.$$
 (11)

where

$$G_{N,s} = -\sum_{s=1}^{N} n^{-s} \ln n , \qquad (12)$$

$$I_{N,s} = \sum_{n=0}^{N} n^{-s} \ln^2 n . {13}$$

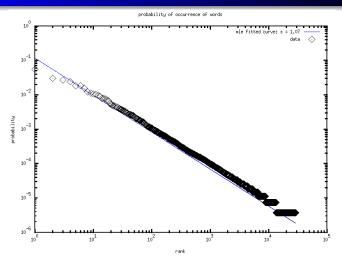


Figure: Zipf fit to *Ulysses* words ( $s_{mle} = 1.0738$ ).

# Zipf-Mandelbrot Fit

The Zipf-Mandelbrot distribution is given by

$$f(k|s,q,N) = \frac{1/(k+q)^s}{\sum_{n=1}^{N} (n+q)^{-s}} = \frac{1/(k+q)^s}{H_{N,s,q}} .$$
 (14)

The logarithm of the likelihood is

$$\ln L(s|k_1,\cdots,k_M,N) = -M \ln H_{N,s,q} - s \sum_{m=1}^M \ln(k_m+q)$$
 (15)

# Zipf-Mandelbrot Fit

Now we need to solve:

$$\frac{\partial \ln L(s|k_1,\cdots,k_M,N)}{\partial s} = -M \frac{G_{N,s,q}}{H_{N,s,q}} - \sum_{m=1}^M \ln(k_m+q) = 0 , (16)$$

$$\frac{\partial^2 \ln L(s|k_1, \cdots, k_M, N)}{\partial s^2} = M \frac{G_{N,s,q}^2 - I_{N,s,q} H_{N,s,q}}{H_{N,s,q}^2} < 0.$$
 (17)

$$\frac{\partial \ln L(s|k_1,\cdots,k_M,N)}{\partial q} = sM \frac{H_{N,s+1,q}}{H_{N,s,q}} - sH_{N,1,q} = 0 , \qquad (18)$$

$$\frac{\partial^{2} \ln L(s|k_{1}, \cdots, k_{M}, N)}{\partial q^{2}} = s^{2} M \frac{H_{N, s+1, q}^{2}}{H_{N, s, q}^{2}} - s(s+1) M \frac{H_{N, s+2, q}}{H_{N, s, q}} + sH_{N, 2, q} < 0.$$
(19)

# Zipf-Mandelbrot Fit

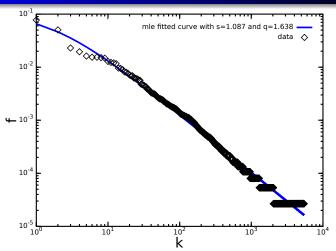


Figure: Zipf-Mandelbrot fit to Hamlet words ( $s_{mle} = 1.087 \ q_{mle} = 1.638$ ).

# Lexical Spectrum or Inverse Zipf

"Understanding the origins and evolution of language requires an appropriate identification of its universal features. One of the most obvious is the statistical distribution of word abundances. The second form is called the lexical spectrum or the inverse Zipf's distribution" (Ferrer-i-Cancho and Solé, 2002).

## **Inverse Zipf**

The number of words for a given frequency of occurrence

$$N(f) = af^{-\beta} \tag{20}$$

where

N(f) number of words with a given frequency f

f frequency of occurrence

a and  $\beta$  parameters

The exponents are related by

$$\beta = \frac{1}{s} + 1 \ . \tag{21}$$

# Inverse Zipf

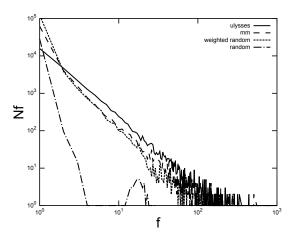


Figure: Inverse Zipf. Natural and Artificial Texts compared.

# Smoothing

The maximum likelihood estimator predicts that the probability of a word not seen in the corpus is zero.

Smoothing is used to overcome this problem.

- (1) Add-one estimator (Laplace, 1902)
- (2) Good-Turing estimator (Good, 1953)
- (3) Simple Good-Turing estimator (Gale and Sampson, 1995)

## Good-Turing

The Good-Turing method states that  $N_0=N_1$  (the total probability of all unseen events is equal to the sum of probabilities of all events that occur only once) and the number of observations are adjusted by

$$f^* = (f+1)\frac{E[N_{f+1}]}{E[N_f]}$$
 (22)

 $E[\cdot]$  expectation of a random variable

f\* adjusted number of observations

f observed frequency of occurrence

 $N_f$  number of types observed f times

# Simple Good-Turing

Gale and Sampson (1995) proposes a simple way to do a Good-Turing estimation by choosing  $E[\cdot]$  so that

$$E[N_{f+1}] = E[N_f] \left(\frac{f}{f+1}\right) \left(1 - \frac{E[N_1]}{N}\right)$$
 (23)

leading to

$$p_f^* = p_f \left( 1 - \frac{E[N_1]}{N} \right) . \tag{24}$$

# Simple Good-Turing

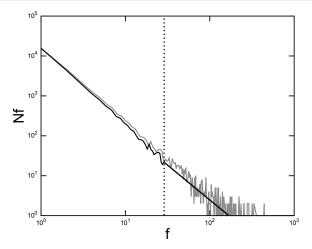


Figure: Simple Good-Turing.

# How Zipf's law emerges

According to Ferrer-i-Cancho and Solé (2003); Ferrer-i-Cancho (2005a), Zipf's law is a manifestation of a complex system operating between order and disorder.

A communication system should minimize  $\Omega$ 

$$\Omega(\lambda) = -\lambda I(S, R) + (1 - \lambda)H(S) , \qquad (25)$$

where

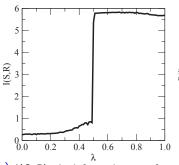
S set of signs (words)

R set of estimula

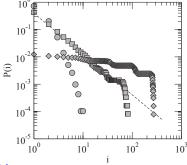
I(S,R) transmitted information

H(S) cost

# How Zipf's law emerges



(a) I(S,R), the information transfer between words and meanings, versus  $\lambda$ , the parameter regulating the balance between maximizing I(S,R) and minimizing the entropy of words.



(b) P(i), the probability of the i-th most likely word in the system for  $\lambda=0.49$  (circles),  $\lambda=0.498$  (squares) and  $\lambda=0.5$  (diamonds). The dashed line contains the theoretical curve for  $\lambda=0.498$ .

Figure: Some computational results on the model where meaning probabilities are governed by the internal structure of the communication system. The size of the system is n = m = 400 (i.e. 400 words and meanings). Figure reproduced from Ferrer-i-Cancho (2005b).

Entropy of a Zipfian distributed source:

$$\overline{H} = -\frac{1}{\ln 2} \sum_{k=1}^{N} Ck^{-s} \ln(Ck^{-s})$$

$$= \frac{sC}{\ln 2} \sum_{k=1}^{N} \frac{\ln k}{k^{s}} - \frac{\ln C}{\ln 2}.$$
(26)

We propose lower and upper bounds to estimate the entropy

$$B_I \le \sum_{k=1}^N k^{-s} \ln k \le B_u. \tag{27}$$

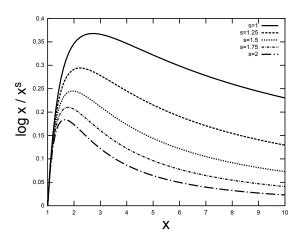


Figure: Function  $f(x) = \ln x/x^s$  for different values of s.

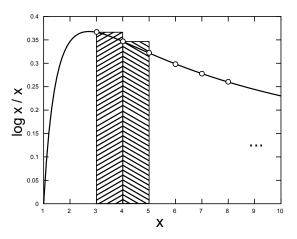


Figure: Left Riemann sum approximation of the integral.

The bounds are given by

$$B_{I} = \int_{3}^{N} \frac{\ln x}{x^{s}} dx + \frac{\ln 2}{2^{s}} + \frac{\ln N}{N^{s}}$$

$$\leq \sum_{n=1}^{N} \frac{\ln n}{n^{s}}$$

$$\leq \int_{3}^{N-1} \frac{\ln x}{x^{s}} dx + \frac{\ln 3}{3^{s}} + \frac{\ln 2}{2^{s}} + \frac{\ln N}{N^{s}} = B_{u}, \quad (28)$$

and the estimated Entropy is bounded by

$$\frac{sC}{\ln 2}B_I - \frac{\ln C}{\ln 2} = H_I \le \overline{H} \le H_u = \frac{sC}{\ln 2}B_u - \frac{\ln C}{\ln 2} \ . \tag{29}$$

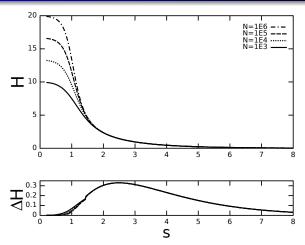


Figure: Entropy H (in bits) as a function of the Zipf exponent s and the number of types N. The upper plot presents the average Entropy estimated and the lower plot presents the difference between the upper and lower bounds of the entropy estimated.

Table: Entropy of real texts (bits), with and without SGT smoothing, compared with the estimated entropy (bits) using the parameter N (number of types) found in the text, parameter s (Zipf exponent) found by a Maximum Likelihood Estimation (MLE) and the flattening parameter g, also found by MLE.

source	N	estimated parameters			entropy		estimated entropy	
		Zipf	Zipf-Mandelbrot		normal	sgt	Zipf	Zipf-
		S	S	q	IIOIIIIai	- sgt	Zibi	Mandelbrot
Alice	3016	0.992	1.172	3.27	8.49	8.79	8.55	8.73
Hamlet	5447	0.991	1.087	1.64	9.04	9.08	9.09	9.13
Macbeth	4017	0.969	1.009	0.56	9.00	9.00	9.02	9.04
Shakespeare	29847	1.060	1.172	2.33	9.52	9.57	9.60	9.69
Ulysses	34391	1.025	1.085	1.18	10.19	10.25	10.22	10.25

## Heaps' law (Herdan's law)

There is a relation between

T text size

V<sub>T</sub> vocabulary length

given by

$$V_T \propto T^{\alpha}$$
 ,  $\alpha < 1$  . (30)

Leijenhorst et al. (2005) shows that a Zipfian distributed source presents this relation between text size and vocabulary length.

## Heaps' law (Herdan's law)

The expected number of types is monotonically increasing with the sample size. It converges to the length of the underlying lexicon.

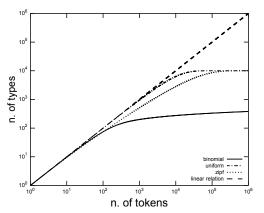


Figure: The recurrence equation is used to estimate the expected number of types for a sample with a certain number of tokens.  $_{115/121}$ 

# Heaps' law (Herdan's law)

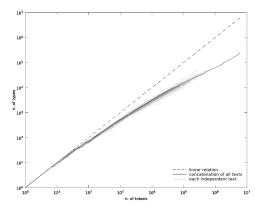


Figure: The relation between the number of tokens and types in 35 books from Gutenberg Database is presented in gray.

#### Menzerath's law

The longer the whole, the smaller the parts.

Menzerath (1928) observed the decreasing relation between syllable's length and length of a word (number of syllables).

The change relative rate on the length of the constituent is inversely proportional to the length of the construct.

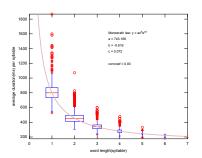
$$\frac{y'}{y} = \frac{b}{x}. (31)$$

y constituent length

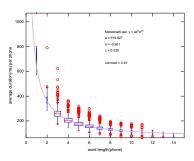
x construct length

Solution:  $y = ax^b$ .

#### Menzerath's law



(a) Average syllable duration as words get longer (number of syllables).



(b) Average phone duration as words get longer (number of phones).

#### Menzerath's law

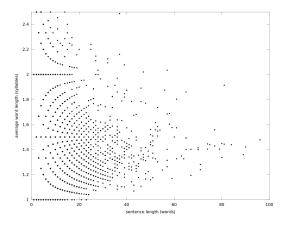


Figure: Relation between sentence length (number of words) and average words length (number of syllables) in a sentence.

#### Conclusions I

- usage-based driven language (Bybee, 2001, 2003, 2006; Ellis, 2002)
- humans track co-occurrence patterns and statistical regularities (Saffran et al., 1996a, 1999; Saffran and Wilson, 2003)
- regularities among several languages
  - some speech segments are very common (example: [m] appear in 94.2% of the languages)
  - ▶ 427 out of 919 speech sounds appear just once among all 425 languages
  - ▶ 90% of the languages use at most 6 times more consonants than vowels

#### Conclusions II

- a few phones have many co-occurring segments in their languages and are also found in many different languages (wildcards)
- ubiquity of scaling laws in nature
- a compared study of 'intermittent silence', Markov model and natural text
- compare different sources using the Zipf-Maldelbrot model
- mle method to fit the best model
- entropy estimation estimation of a Zipf-Maldelbrot source
- Menzerath's law and Heap's law
- distance measure between phones using Feature Theory

We adopt here a usage-based theory of language in which its cognitive organization is based directly on one's experience with it, rather than being an abstract set of rules or structures that are only indirectly related to experience with language. We see grammar as a network built up from the categorized instances of language use.

Since the grammar is based on usage, it contains many details of co-occurrence as well as a record of the probabilities of occurrence and co-occurrence. The evidence for the impact of usage on cognitive organization includes the fact that language users are aware of specific instances of constructions that are conventionalized and the multiple ways in which frequency of use has an impact on structure.

The latter include speed of access related to token frequency, priming, morphological and phonetic properties of high and low frequency words, as well as the importance of usage to the process of grammaticalization. (In linguistics, grammaticalization is a process by which words representing objects and actions (i.e. nouns and verbs) transform through sound change and language migration to become grammatical markers (affixes, prepositions, etc.).)

A number of recent experimental studies (Saffran et al., 1996b, 1999; Saffran and Wilson, 2003) show that both infants and adults track co-occurrence patterns and statistical regularities in artificial grammars. Such studies indicate that subjects learn patterns even when the grammar corresponds to no meaning or communicative intentions. It is thus not surprising that in actual communicative settings, the co-occurrence of words has an impact on cognitive representation. There is indeed evidence from multiple sources that such cognitive changes occur, and contribute to the shape of grammar.

The cognitive representations underlying language use are built up by the categorization of utterances into exemplars and exemplar clusters based on their linguistic form as well as their meaning and the context in which they have been experienced (Pierrehumbert, 2001).

The fact that proposed 'universals of grammar' appear to be highly general can be seen as a by-product of this process: grammars are emergent phenomena dependent on idiosyncratic localized interactions.

The ubiquity of scaling laws in nature suggests that complex systems organize them- selves, they are not just a collection of random independent realizations, but exhibit similar patterns and relations that are observed in different scales. Complex systems are more than mere collections of independent random variables, they have interdependent effects and behaviours.

Only the Markov model was able to result in similar patterns to those observed in natural texts. Frequency of 1-gram, 2-grams, 3-grams, 'word' length, occurrence of hapax legomenon, Zipf and inverse Zipf (lexical spectrum) relations.

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The exponent parameter in the Zipf distribution is important to determine the statistical properties of that distribution. We observe that different type of sources present different exponent values and we argue that this exponent is related to the balance between order and disorder in a Complex System, the amount of information produced by the source and the underlying lexicon size (and the possibility of being virtually infinite). In order to compare data that follow a power law relation, we need to find the best fit to the data. We have presented the usage of the maximum likelihood approach to determine the exponent parameters that gives the best fit. We have derived the equations which lead to a root finding problem, for which the solution is the maximum likelihood estimated parameter. The same procedure is used to find the exponent and flattening parameters of a Zipf-Mandelbrot model. This procedure was tested using real text data and the results show a good performance.

Menzerath's law states a relation on the usage of tiles to create a whole. According to this law, the length of the whole is inversely proportional to the length of the parts.

Heap's law: sublinear relation between the text length and the vocabulary size. Zipf's law implies Heap's law, but any distribution would lead to a convergent sublinear lexical growth in relation to the text length.

The Feature Theory might be providential since it give us a direct and formal way to quantify a phone characteristics making it possible to create a distance measure between two phones. We propose to measure the distance between two phones by the number of distinctive features not shared by them. We believe this measure might be a meaningful choice. It could be used alongside frequency of occurrence to draw better explanations to phonological process and cognitive representation.

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