Patterns in Language Usage

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Outline

Language

language: system of communication

Language is a complex system that uses signs to encode and decode information to convey communication.

Language must be statable in a finite form (Davidson, 2005; von Humboldt, 1836; Chomsky, 1969; Apostel et al., 1957; Wang, 1991).

Language

- language unpredictable nature
- recurring patterns, language 'universal'
- structures and organization
- the principle of least effort (Zipf, 1949) : fast and robust communication
- maximize information transmission

Language

Language is a complex adaptive system.

(1) multiple agents

(2) adaptive

(3) perception and motivation

(4) emergence of patterns

Language

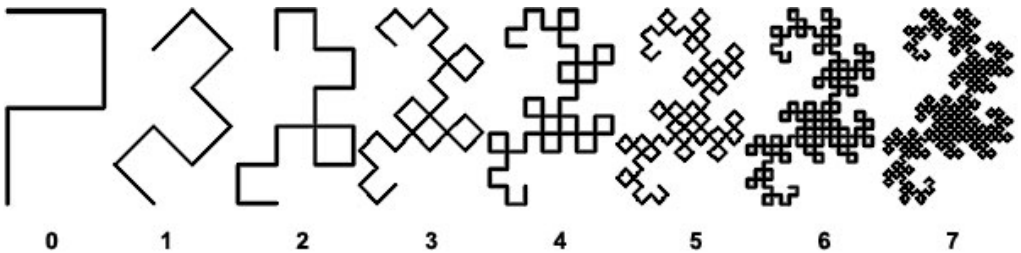


Figure: Dragon fractal.

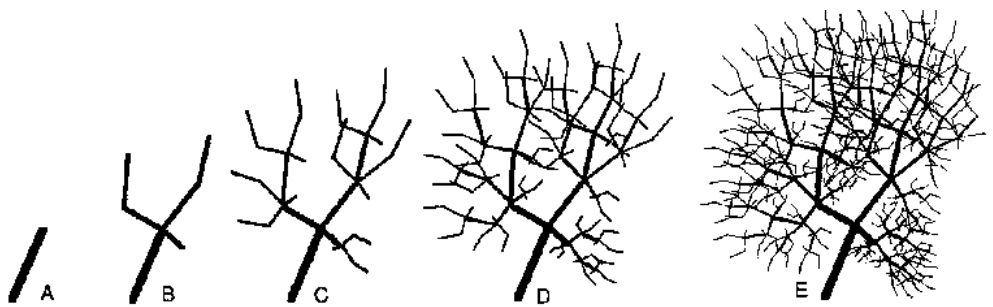


Figure: Purkinje cell, fractal model.

Quantitative Approach

- systematic empirical investigation of phenomena via statistical, mathematical or computational techniques
- develop and employ mathematical models, theories and/or hypotheses pertaining to the phenomena
- measurement: empirical observation

Quantitative Linguistics - History I

- date back in the ancient Greek - applications of combinatorics
- 718-791, philologist and lexicographer Al-Khalil ibn Ahmad - permutations and combinations to list all possible Arabic words with and without vowels
- 1564-1614, William Bathe - *Janua Linguarum*, the world's first language teaching texts, where he had compiled a list with 5.300 essential words

Quantitative Linguistics - History II

- the first scientific counts of units of language or text were published already in the 19th century as a means of linguistic description - in Germany, Förstemann (1846, 1852) and Drobisch (1866), in Russia, Bunjakovskij (1847), in France Bourdon (1892), in Italy, Mariotti (1880), in England, Augustus De Morgan (1851) and in the USA, probably Sherman (1888)
- the Russian mathematician Andrey Andreyevich Markov who created the base of the theory of Markov chains in 1913
- George Kingley Zipf was the first to set up a theoretical model in order to explain the observations and to find a mathematical formula for the corresponding function - the famous "Zipf's Law" (1935, 1949)
- Benoît Mandelbrot (1953, 1959, 1961a, 1961b)

Quantitative Linguistics - History III

- Shannon and Weaver (1949) - Information Theory
- Gustav Herdan (1954, 1956, 1960, 1962, 1964 1966, 1969), Rajmund.G. Piotrowski (1959, 1968, 1979) and Walter Meyer-Eppler (1959)
- Gabriel Altmann, Reinhard Köhler, Paul Menzerath, Juhan Tuldava, Peter Grzybek, Wilhelm Fucks, ...
- today, Quantitative Linguistics is a well-developed scientific discipline with a broad applicational impact

Laws in Quantitative Linguistics

- Zipf-Mandelbrot - frequency and rank are inversely related
- Menzerath - the size of linguistic constituents decreases as the size of the corresponding construct increases
- Heaps (Herdan) - relation between lexical size and text size
- Piotrowski - development of new units or forms over time

Laws in Quantitative Linguistics (Universität Trier) http://lql.uni-trier.de
Glottopedia http://www.glottopedia.org/

Structuralism

Structural linguistics

- Ferdinand de Saussure - *Course in General Linguistics* in 1916.
- theoretical paradigm: elements must be understood in terms of their relationship to a larger, overarching system or structure
- linguistic signs were composed of two parts: *signifier* and *signified*
- linguistic levels: phonemes, morphemes, lexical categories, noun phrases, verb phrases, and sentence types

"En matière de langue on s'est toujours contenté d'opérer sur des unités mal définies"[1] (de Saussure, 1916).

[1] In language's matter it has always been sufficient to operate on ill-defined units.

Language Structure - Categorization

The categorization process in each language is different, although some common aspects are observed in all/most of them → different sounds and rules.

Vowels systems.

English	o ɔ w ə i ɪ e ɛ æ a u ʊ ʌ ɑ
Swedish	ɔ iː yː ɪː ʏ eː øː ɛ ɛː œ a uː ʊ oː ɑː ɵ ʉ̟
Japanese	ɔ w i ɛ a ɯ
Portuguese	o ɔ i e ɛ a u

There is considerable distinction between vowels that receive the same label (Disner, 1983).

Vowel system: Yoruba x Italian

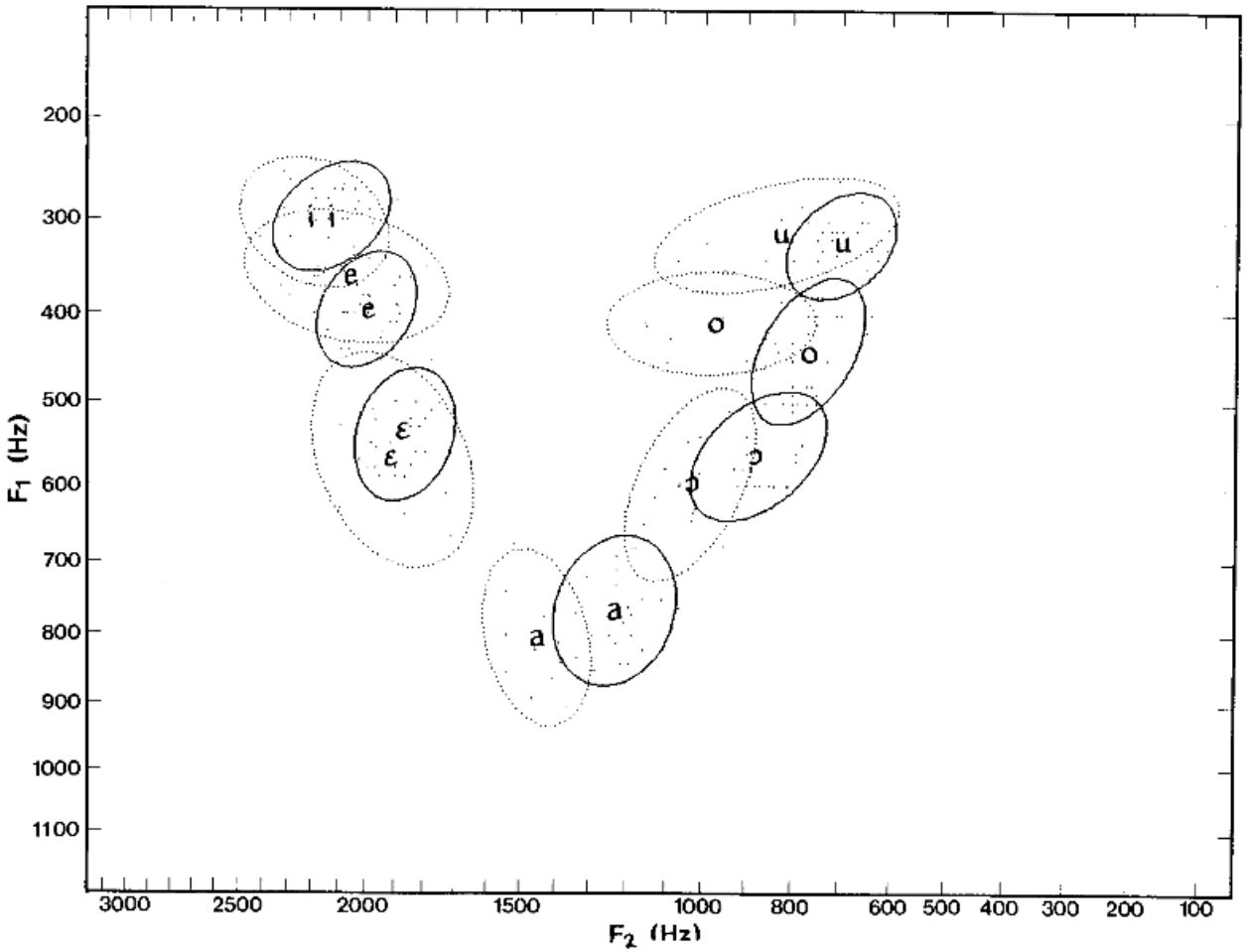


Figure: Shared vowels of Yoruba (dotted) and Italian (solid) (Disner, 1983).

Vowel system: German x Dutch

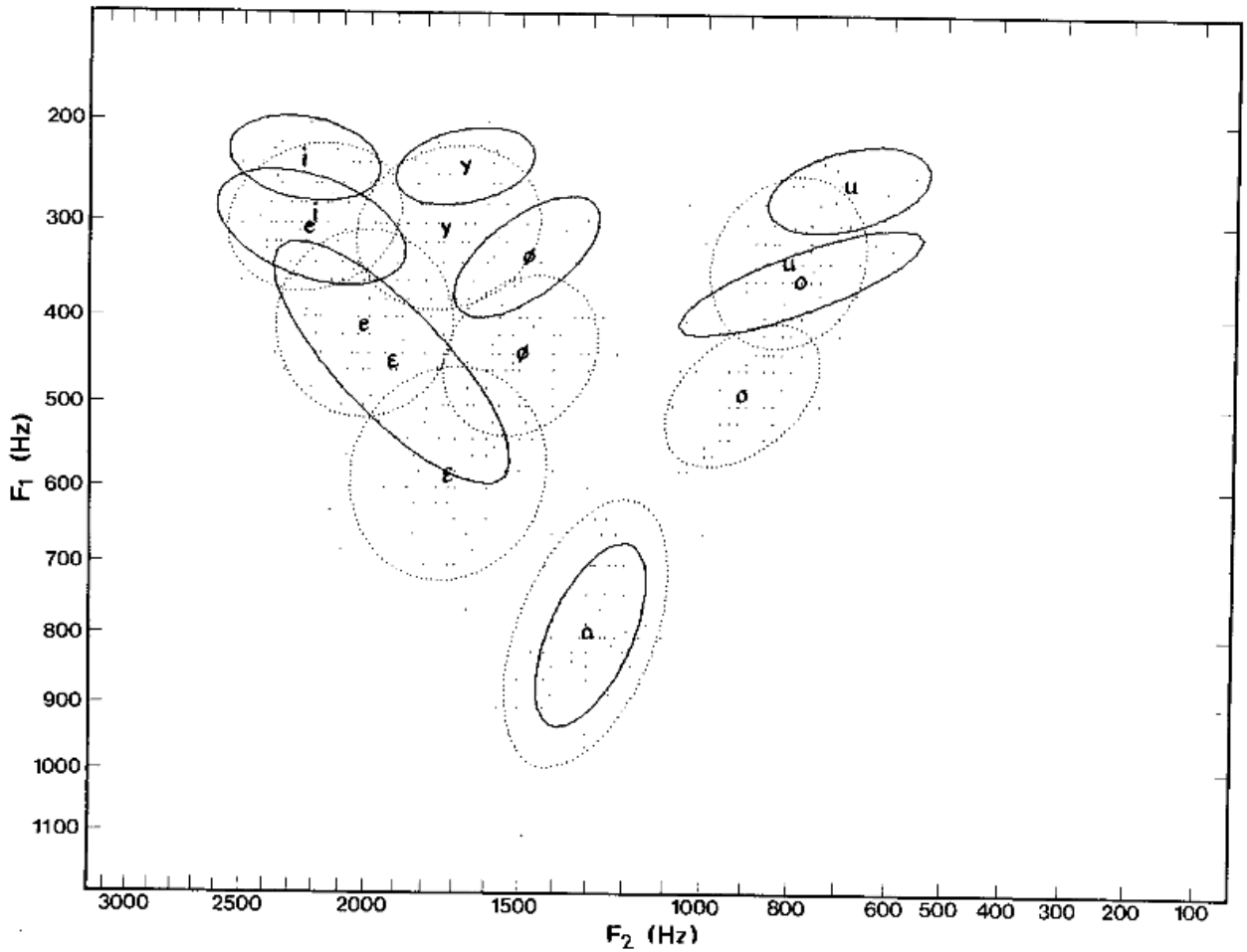


Figure: Shared vowels of German (solid) and Dutch (dotted) (Disner, 1983).

Language Structure - Combinations

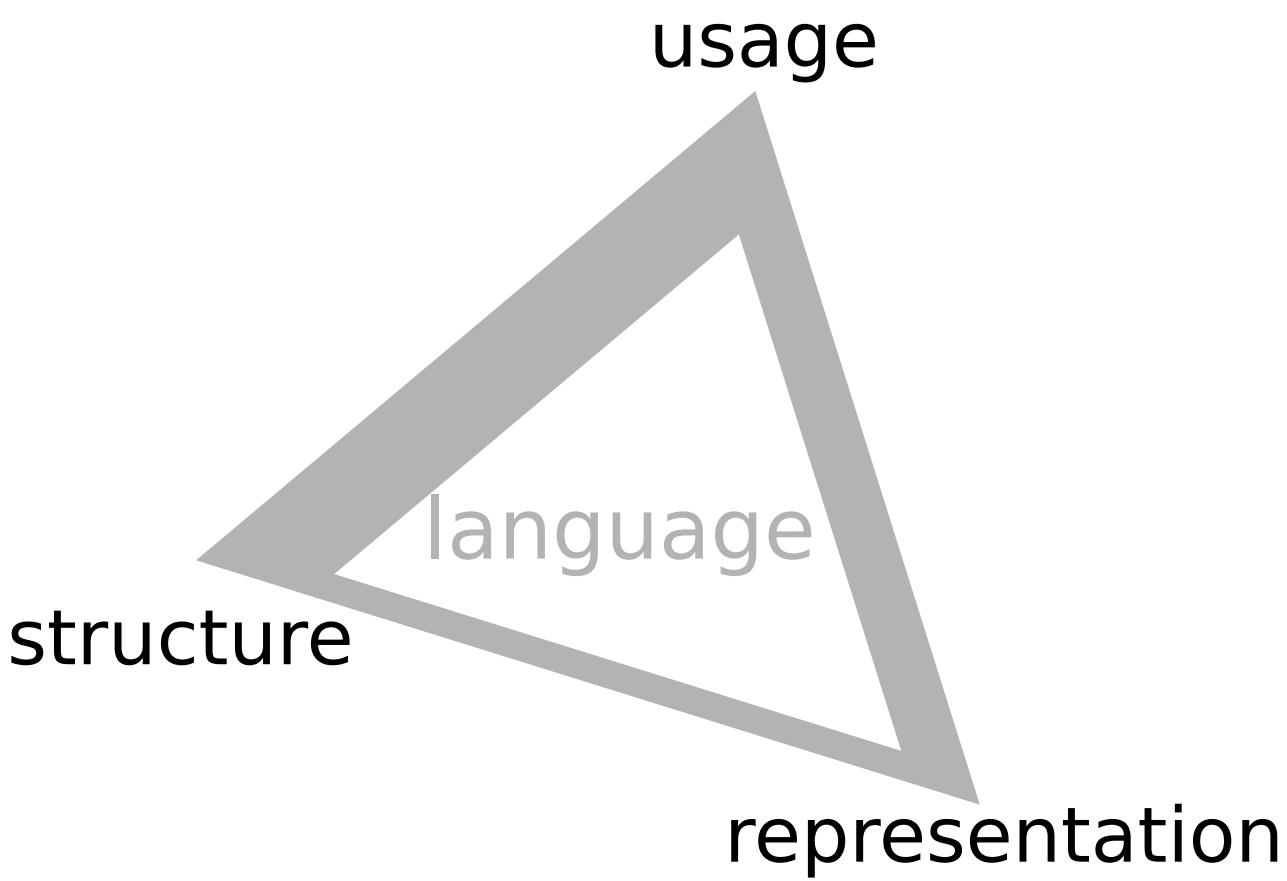
Phonemic Rules

The consonantal cluster [ts] in

- German is allowed.
  examples: 'Konferenz' ([kɔnfe'rɛnts]), 'Zeit' ([tsaɪt]), 'umziehen' (['ʔʊmtsiːən])
- English is allowed only in word final.
  examples: 'cats' ([kæts]), 'splits' ([splits])

Classical Arabic no multiconsonant onsets are allowed at all.

Language Structure, Usage and Representation



Frequency and Complexity

Frequency of occurrence is inversely proportional to complexity (Zipf, 1949).

- words
- phones, clusters
- within a language or among various languages of the world

Complexity and occurrence are intrinsically related to the way languages change.

Outline

UCLA Phonological Segment Inventory Database

The UCLA Phonological Segment Inventory Database (or UPSID) is a statistical survey of the phoneme inventories in 451 of the world's languages. The database was created by American phonetician Ian Maddieson for the University of California, Los Angeles (UCLA) in 1984 and has been updated several times.

`http://www.linguistics.ucla.edu/faciliti/sales/software.htm`

Book: Patterns of sounds (Maddieson, 1984).

UPSID - Number of Segments (919)

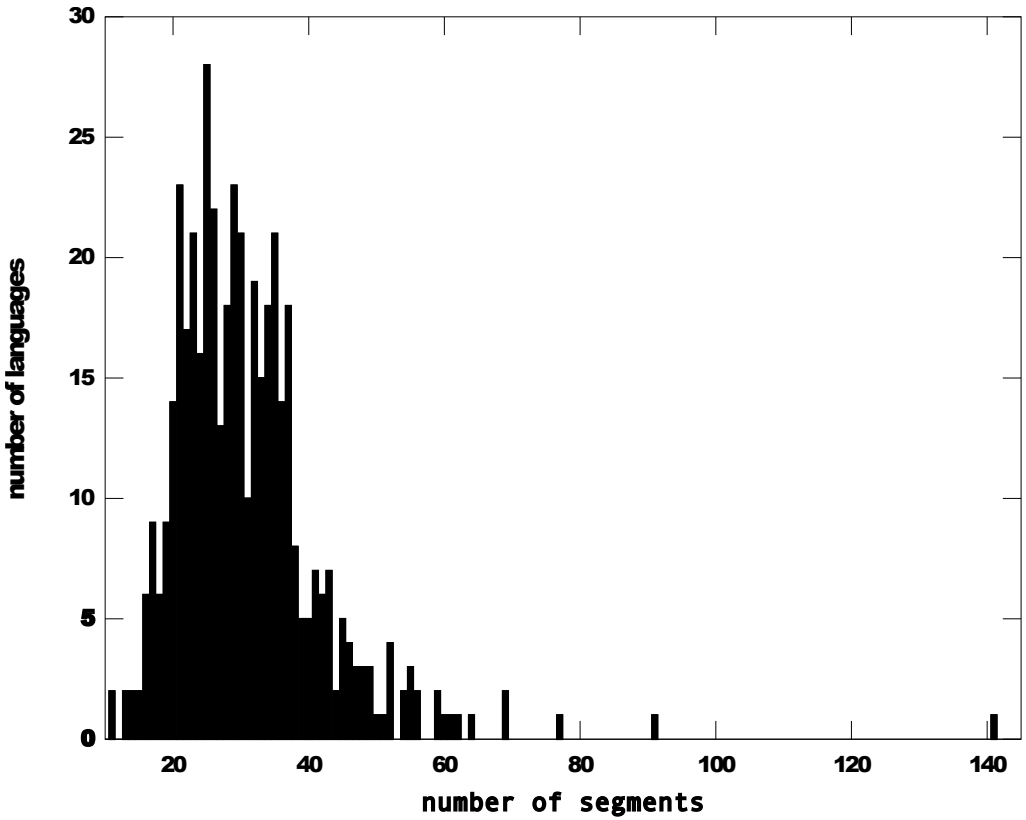


Figure: Number of phones in various languages of the world.

UPSID - Segments in Languages (451)

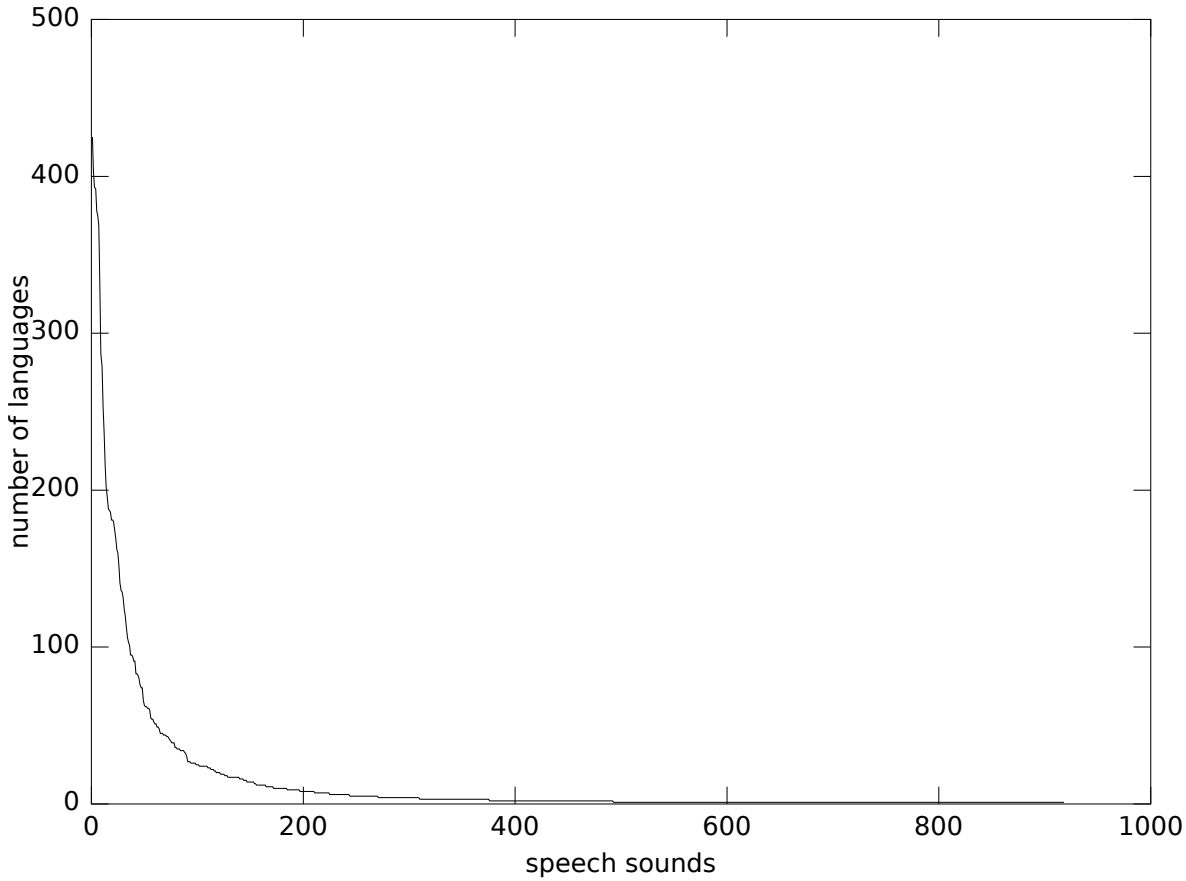


Figure: Number of languages where a give speech sounds occurs.

UPSID - Segments in Languages

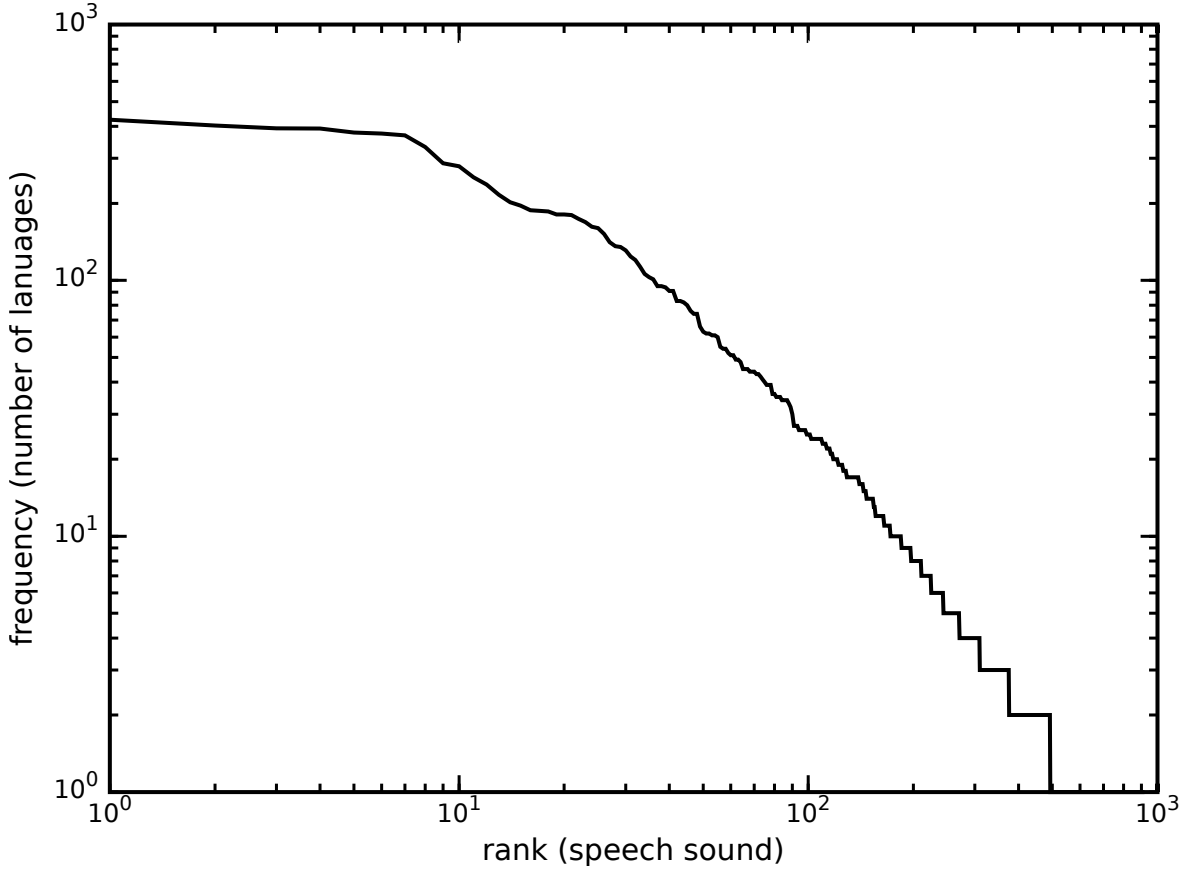


Figure: Number of languages for a given speech sound.

UPSID - Number of Vowels (269)

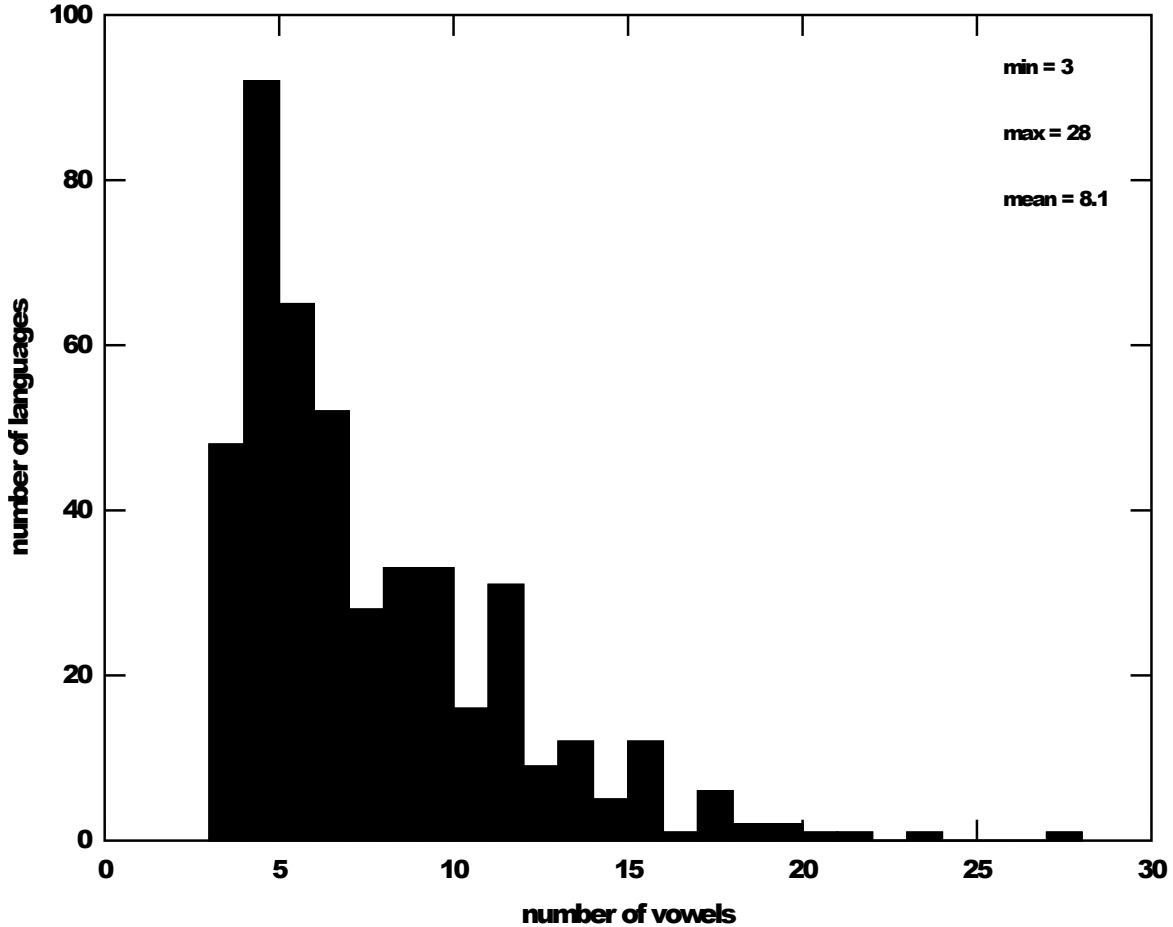


Figure: Number of vowels in various languages of the world.

UPSID - Number of Consonants (652)

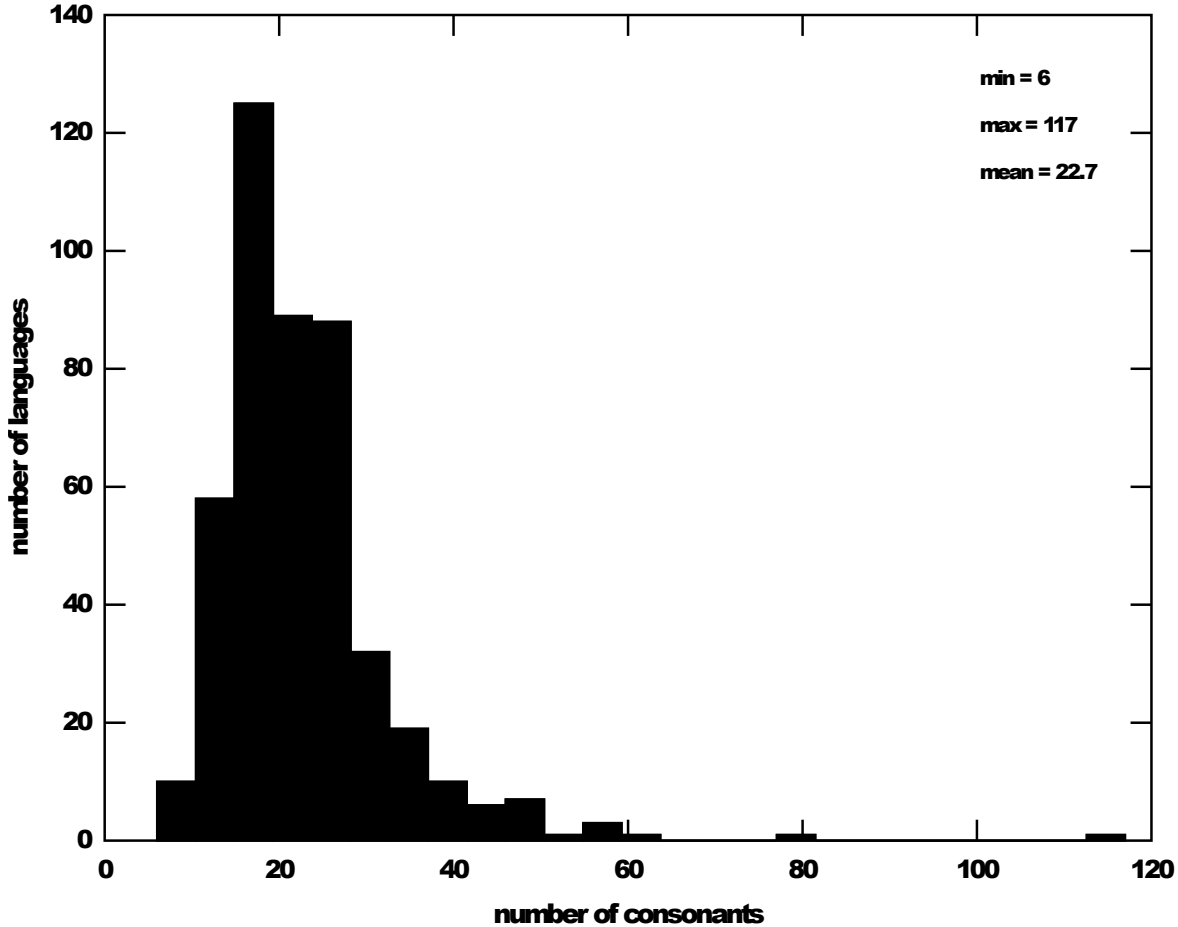


Figure: Number of consonants in various languages of the world.

UPSID - Consonant-Vowels Ratio

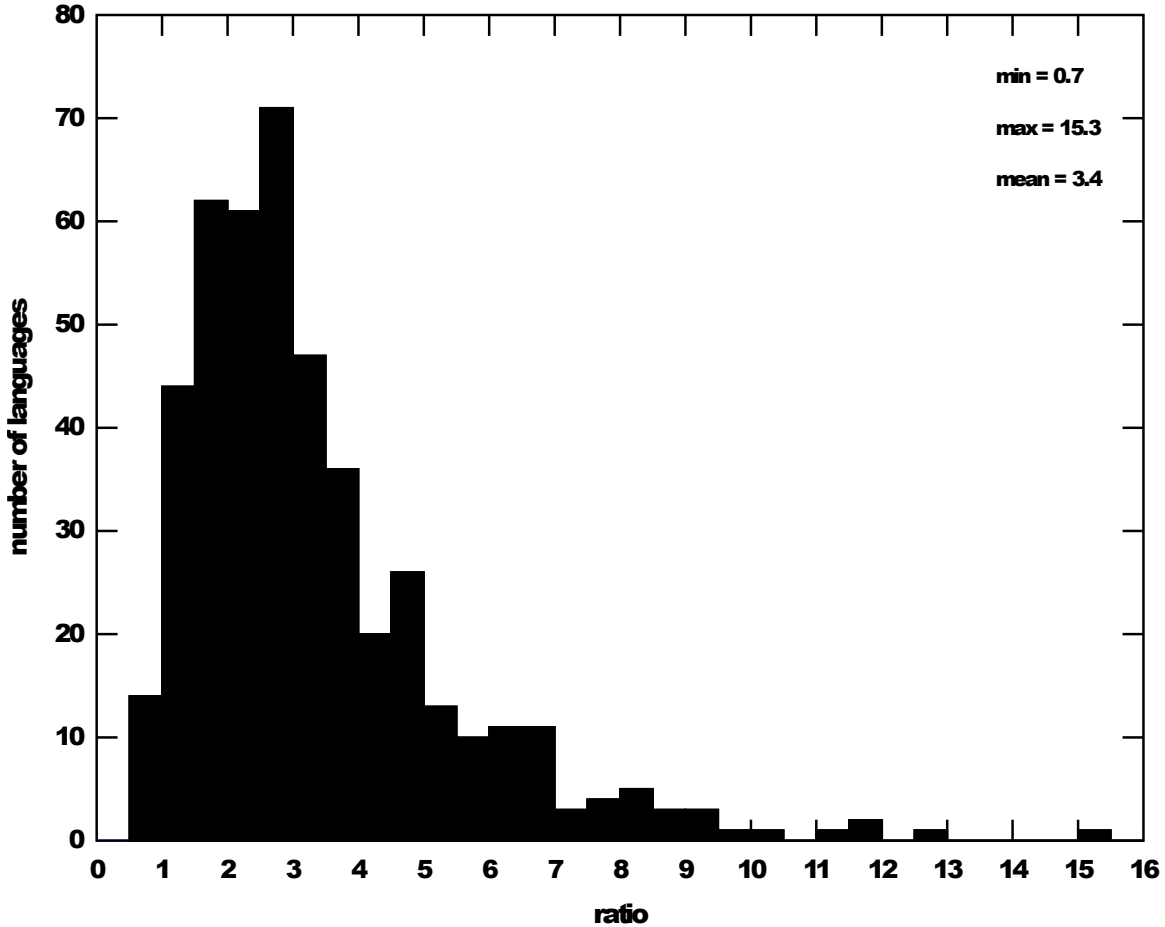


Figure: Consonants to Vowel ratio in various languages of the world.

UPSID - Consonant-Vowels Ratio CDF

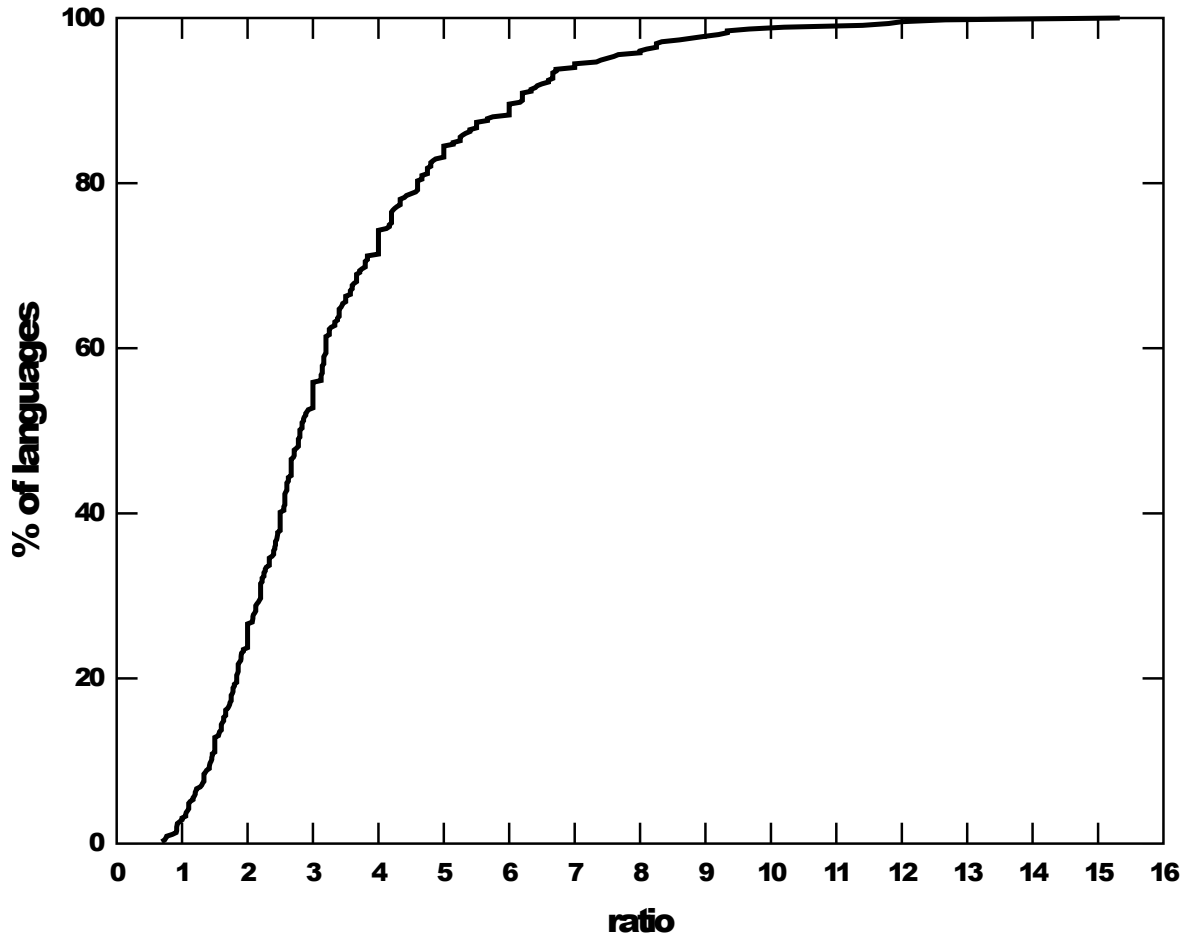


Figure: Cumulative distribution of the Consonant-Vowel ratio.

UPSID - Consonants

Table: List of the 20 most frequent consonants in UPSID.

consonant	m	k	j	p	w	b	h
n. of languages	425	403	378	375	332	287	279
frequency	94.2	89.4	83.8	83.2	73.6	63.6	61.9
consonant	g	ŋ	ʔ	n	s	tʃ	ʃ
n. of languages	253	237	216	202	196	188	187
frequency	56.1	52.6	47.9	44.8	43.5	41.7	41.5
consonant	t	f	l	n̪	t̪	ɲ	
n. of languages	181	180	174	160	152	141	
frequency	40.1	39.9	38.6	35.5	33.7	31.3	

UPSID - Vowels

Table: List of the 10 most frequent vowels in UPSID.

vowel	i	a	u	ɛ	o/ɔ
n. of languages	393	392	369	186	181
frequency	87.1	86.9	81.8	41.2	40.1
vowel	e/ɛ	ɔ	o	e	a
n. of languages	169	162	131	124	83
frequency	37.5	35.9	29.0	27.5	18.4

UPSID - number of phones vs. frequency index

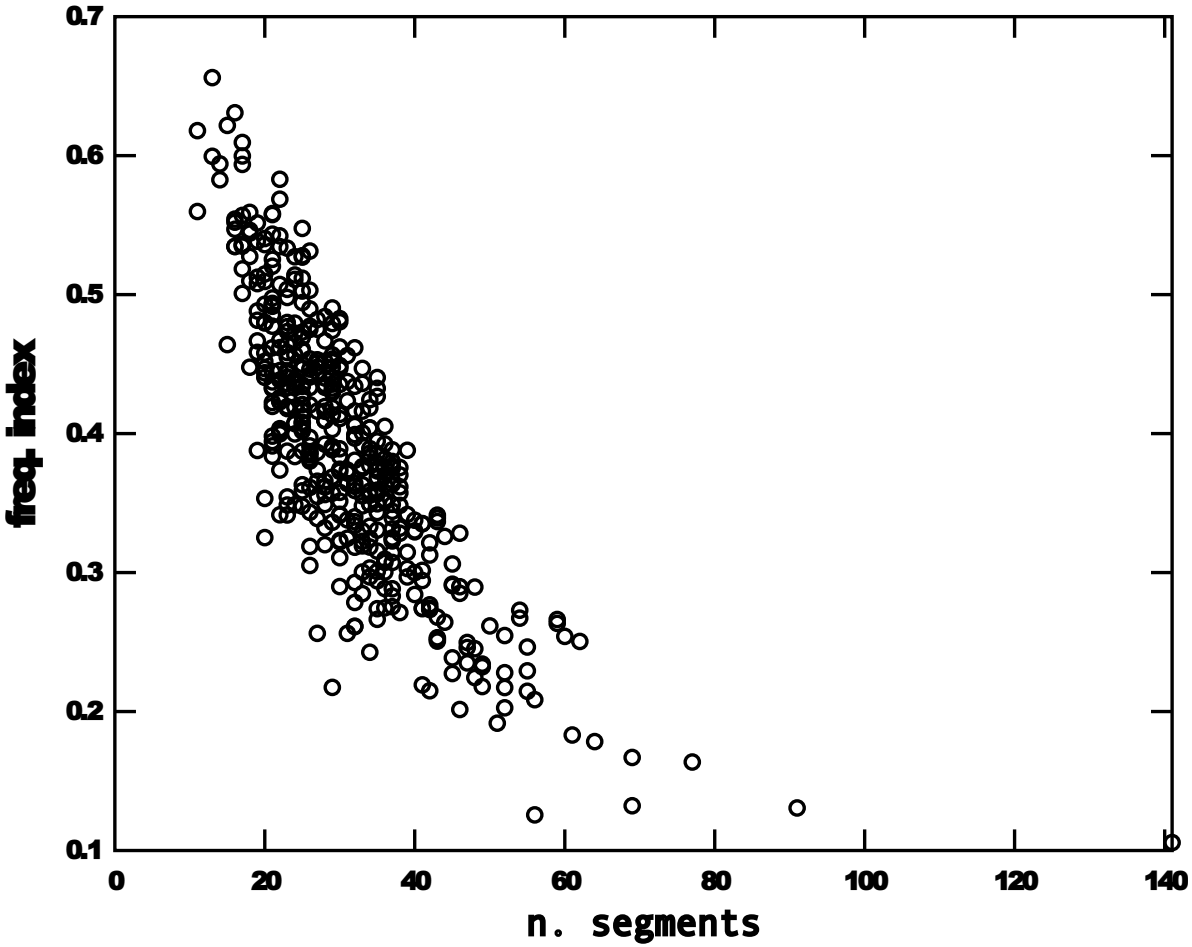
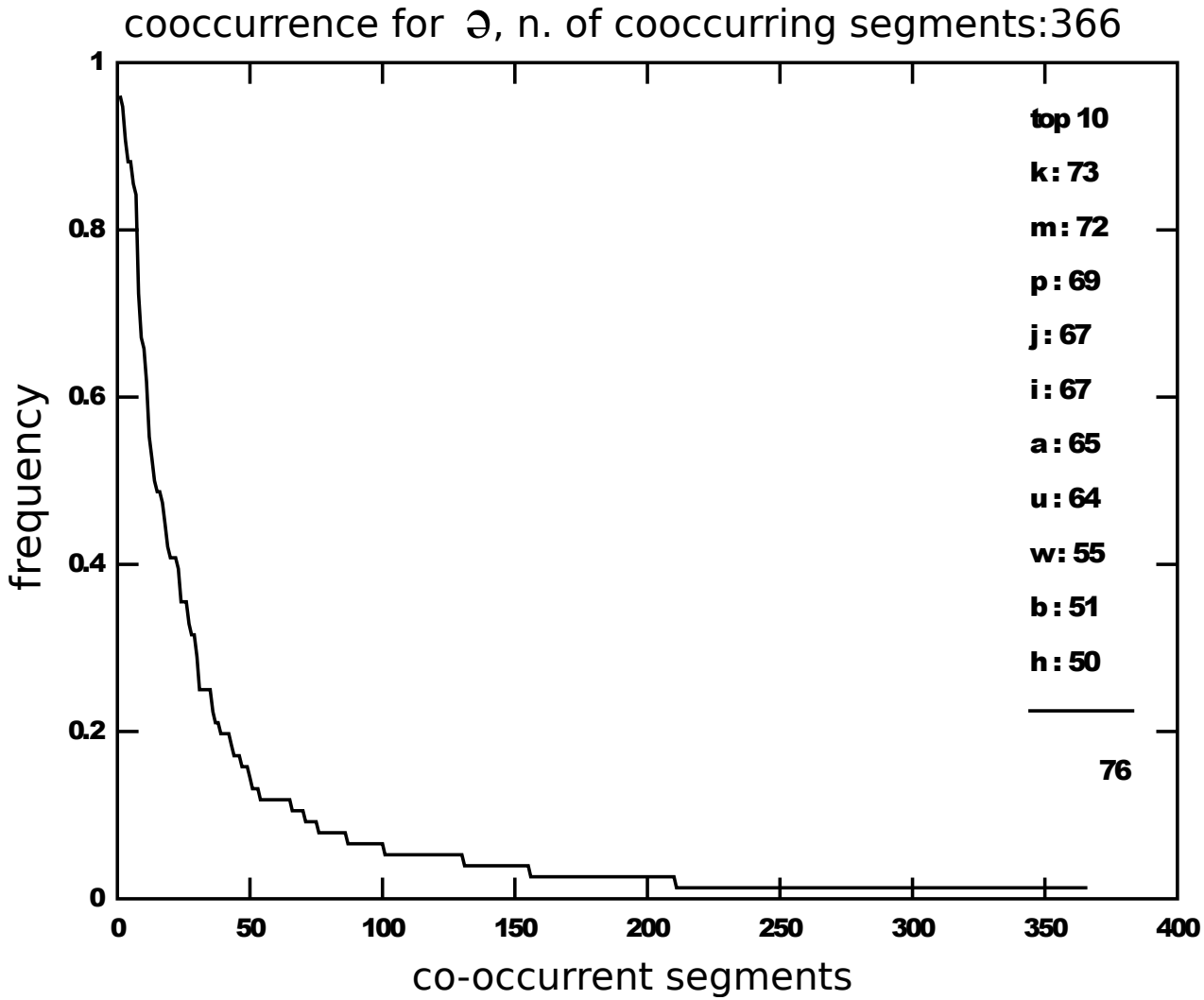


Figure: Relation between the frequency index and the number of phones in a language. (Data from UPSID)

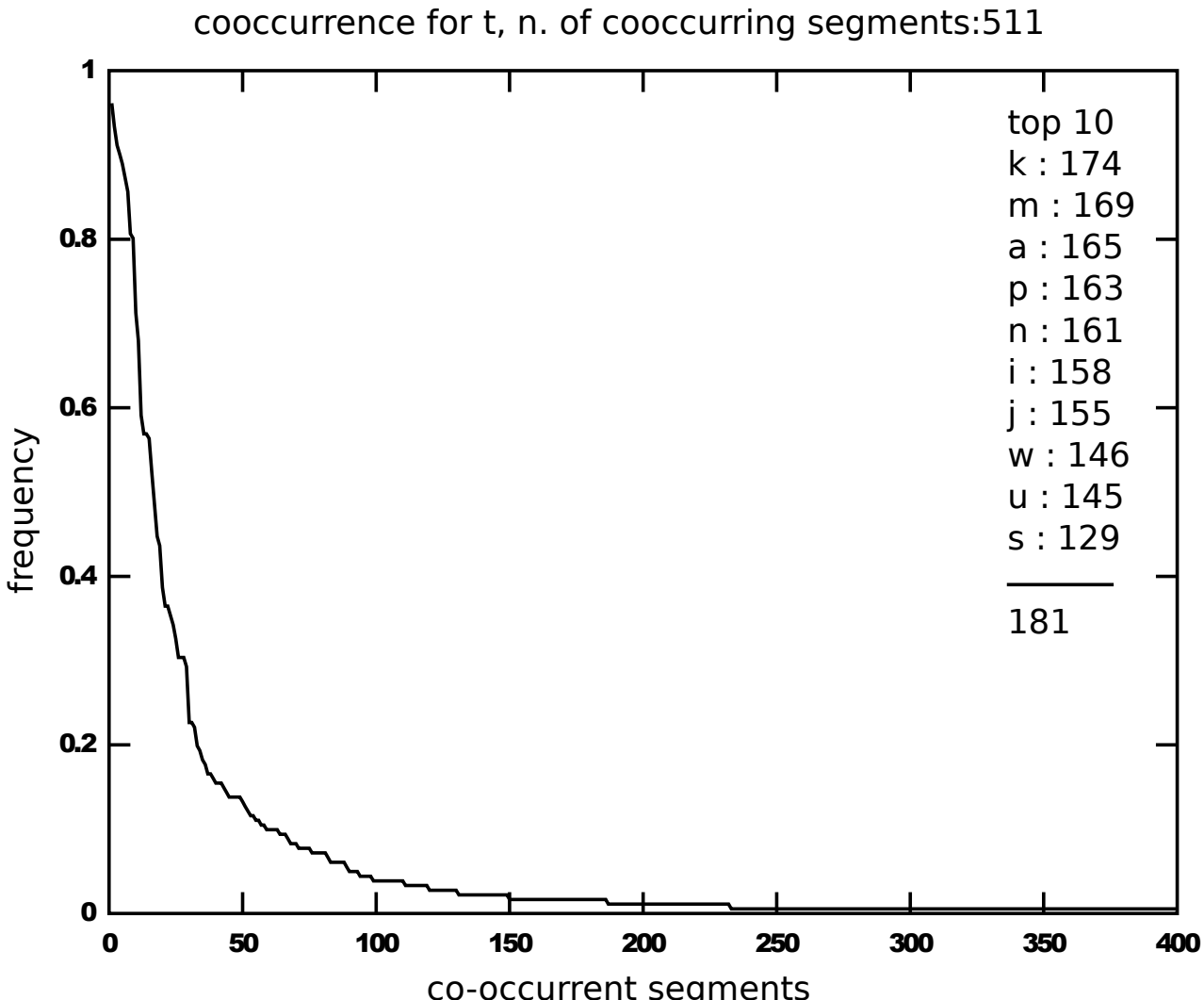
UPSID - cooccurrence of phones in a language

Figure:
Co-occurrence frequency for phones in relation to [ə]. [ə] occurs in 76 languages (16.8%) and has 366 cooccurring phones (39.8%). Cooccurring phones:
[k] (96.0%),
[m] (94.7%),
[p] (90.7%), etc.



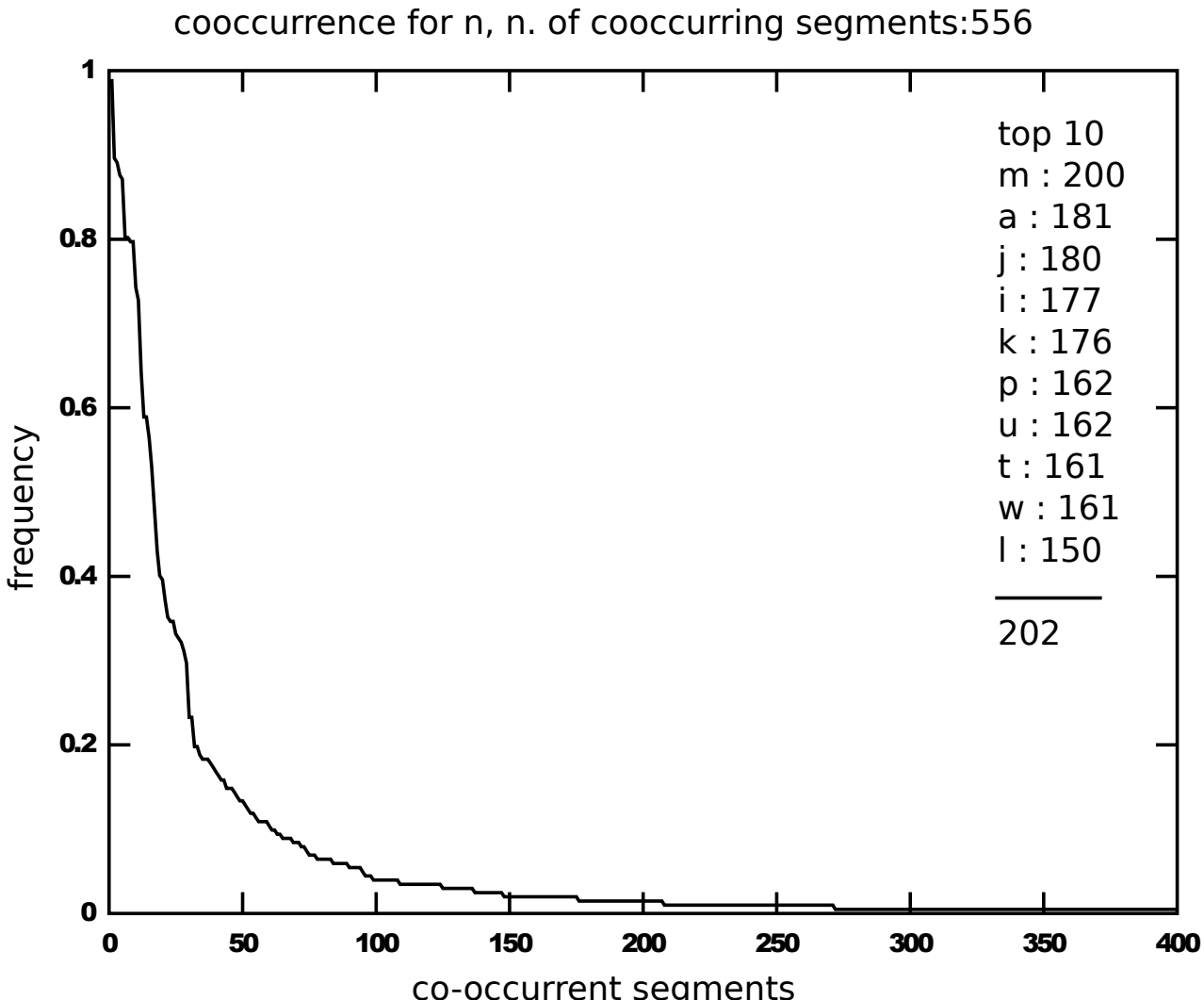
UPSID - cooccurrence of phones in a language

Figure:
Co-occurrence frequency for phones in relation to [t]. [t] occurs in 181 languages (40.1%) and has 511 cooccurring phones (55.6%).
Cooccurring phones:
[k] (96.1%),
[m] (93.3%),
[a] (91.2%), etc.



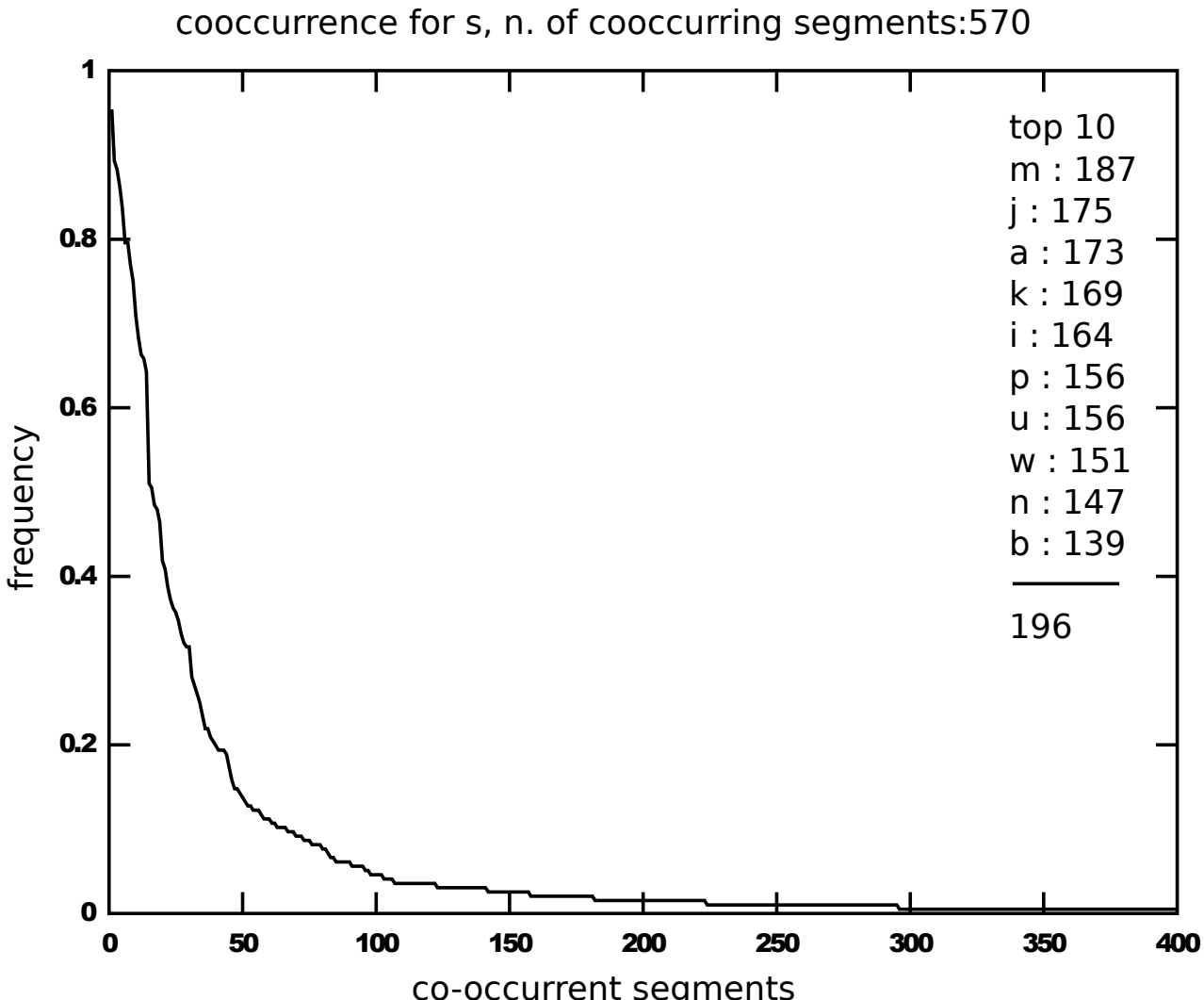
UPSID - cooccurrence of phones in a language

Figure:
Co-occurrence frequency for phones in relation to [n]. [n] occurs in 202 languages (44.7%) and has 556 cooccurring phones (60.5%).
Cooccurring phones:
[m] (99.0%),
[a] (89.6%),
[j] (89.1%), etc.



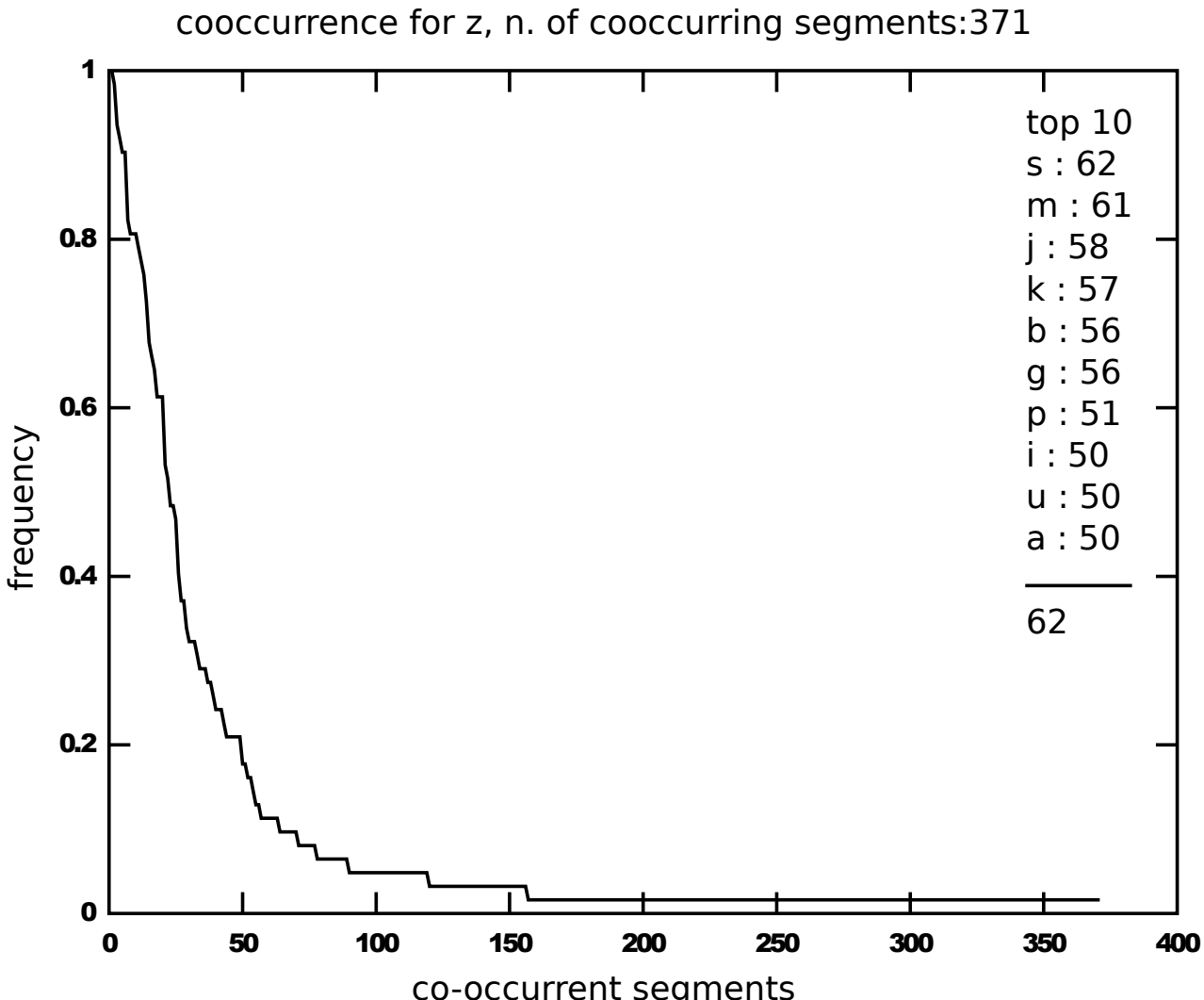
UPSID - cooccurrence of phones in a language

Figure:
Co-occurrence frequency for phones in relation to [s]. [s] occurs in 196 languages (43.5%) and has 570 cooccurring phones (62.0%).
Cooccurring phones:
[m] (95.4%),
[j] (89.3%),
[a] (88.3%), etc.



UPSID - cooccurrence of phones in a language

Figure:
Co-occurrence frequency for phones in relation to [z]. [z] occurs in 62 languages (13.7%) and has 371 cooccurring phones (40.4%).
Cooccurring phones:
[s] (100%),
[m] (98.4%),
[j] (93.5%), etc.



UPSID - cooccurrence of phones in a language

Table: List of phones and their top 8 co-occurring pairs with their relative frequency of occurrence (data from UPSID).

phone	co-occurring phone with their respective relative frequency (%)							
ə	k : 96.1	m : 94.7	p : 90.8	j : 88.2	i : 88.2	a : 85.5	u : 84.2	w : 72.4
t	k : 96.1	m : 93.4	a : 91.2	p : 90.1	n : 89.0	i : 87.3	j : 85.6	w : 80.7
n	m : 99.0	a : 89.6	j : 89.1	i : 87.6	k : 87.1	p : 80.2	u : 80.2	t : 79.7
ɪ	m : 97.3	j : 91.9	k : 86.5	a : 83.8	p : 83.8	ʊ : 74.3	w : 71.6	b : 66.2
s	m : 95.4	j : 89.3	a : 88.3	k : 86.2	i : 83.7	p : 79.6	u : 79.6	w : 77.0
z	s : 100.0	m : 98.4	j : 93.5	k : 91.9	b : 90.3	g : 90.3	p : 82.3	i : 80.6
d	b : 96.7	m : 94.2	i : 91.7	a : 90.8	j : 90.0	n : 89.2	g : 87.5	u : 86.7
l	m : 98.9	j : 89.1	k : 86.8	n : 86.2	a : 86.2	i : 85.6	p : 81.0	w : 79.9
i	m : 93.9	u : 91.6	k : 89.6	a : 89.1	p : 82.7	j : 82.7	w : 73.8	b : 65.1
d̪	b : 97.5	m : 96.2	g : 93.8	j : 88.8	k : 85.0	t̪ : 83.8	i : 80.0	p : 76.2
m	k : 89.2	i : 86.8	a : 86.6	j : 85.2	p : 82.6	u : 81.6	w : 74.1	b : 64.2
n	m : 99.0	a : 89.6	j : 89.1	i : 87.6	k : 87.1	p : 80.2	u : 80.2	t : 79.7
k	m : 94.0	p : 91.3	i : 87.3	a : 86.6	j : 83.4	u : 82.1	w : 73.2	b : 62.8
g	b : 96.4	m : 95.3	i : 89.3	j : 87.4	k : 86.2	u : 84.2	a : 83.0	p : 76.7
p	k : 98.1	m : 93.6	a : 87.2	i : 86.7	j : 82.9	u : 81.3	w : 71.7	b : 60.5
b	m : 95.1	i : 89.2	k : 88.2	j : 86.8	g : 85.0	u : 84.3	a : 84.3	p : 79.1
ʃ	m : 94.7	j : 91.4	k : 88.8	i : 84.0	a : 82.9	p : 80.2	u : 78.1	w : 74.3
ʒ	ʃ : 95.1	m : 95.1	j : 90.2	k : 90.2	b : 82.0	g : 80.3	i : 78.7	p : 78.7

UPSID - cooccurrence of phones in a language

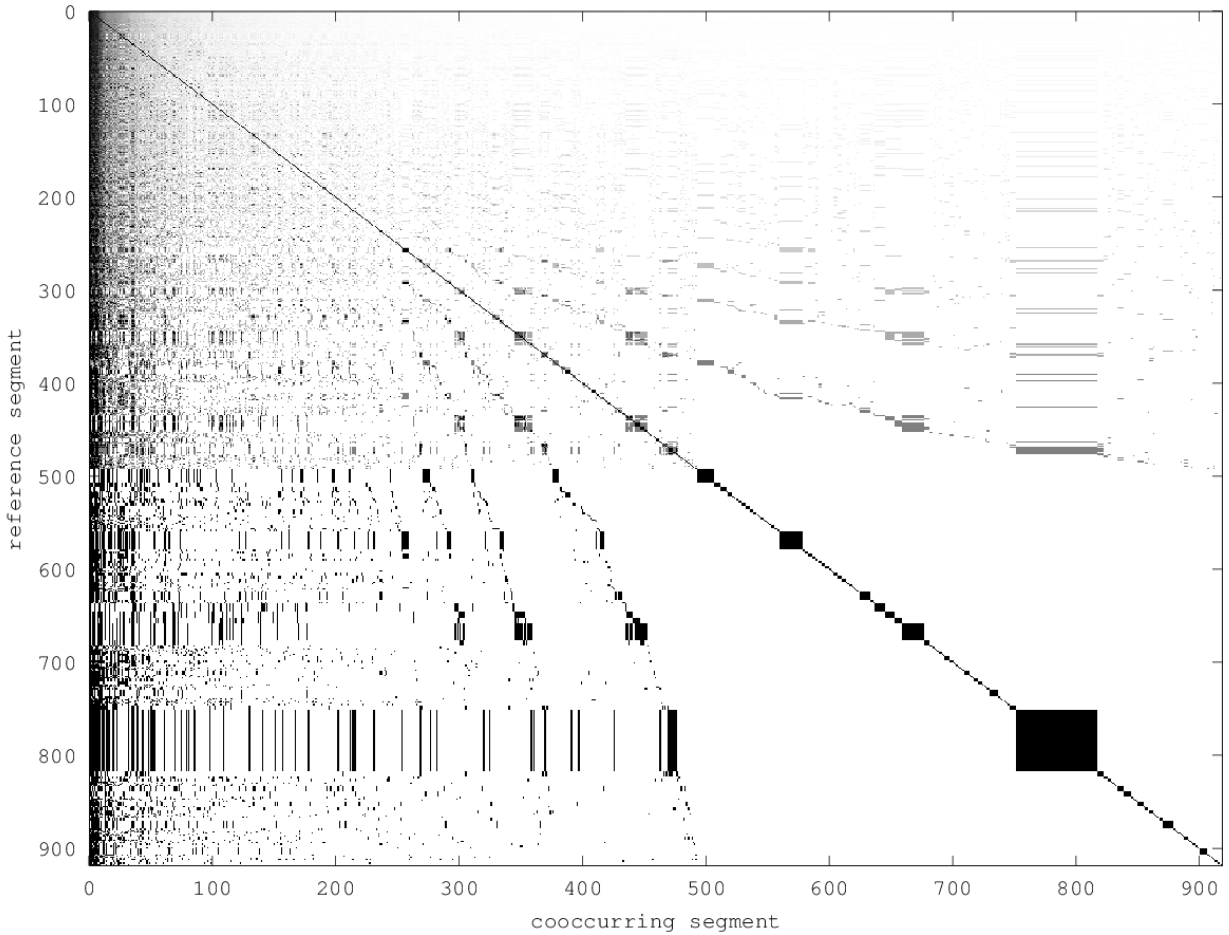
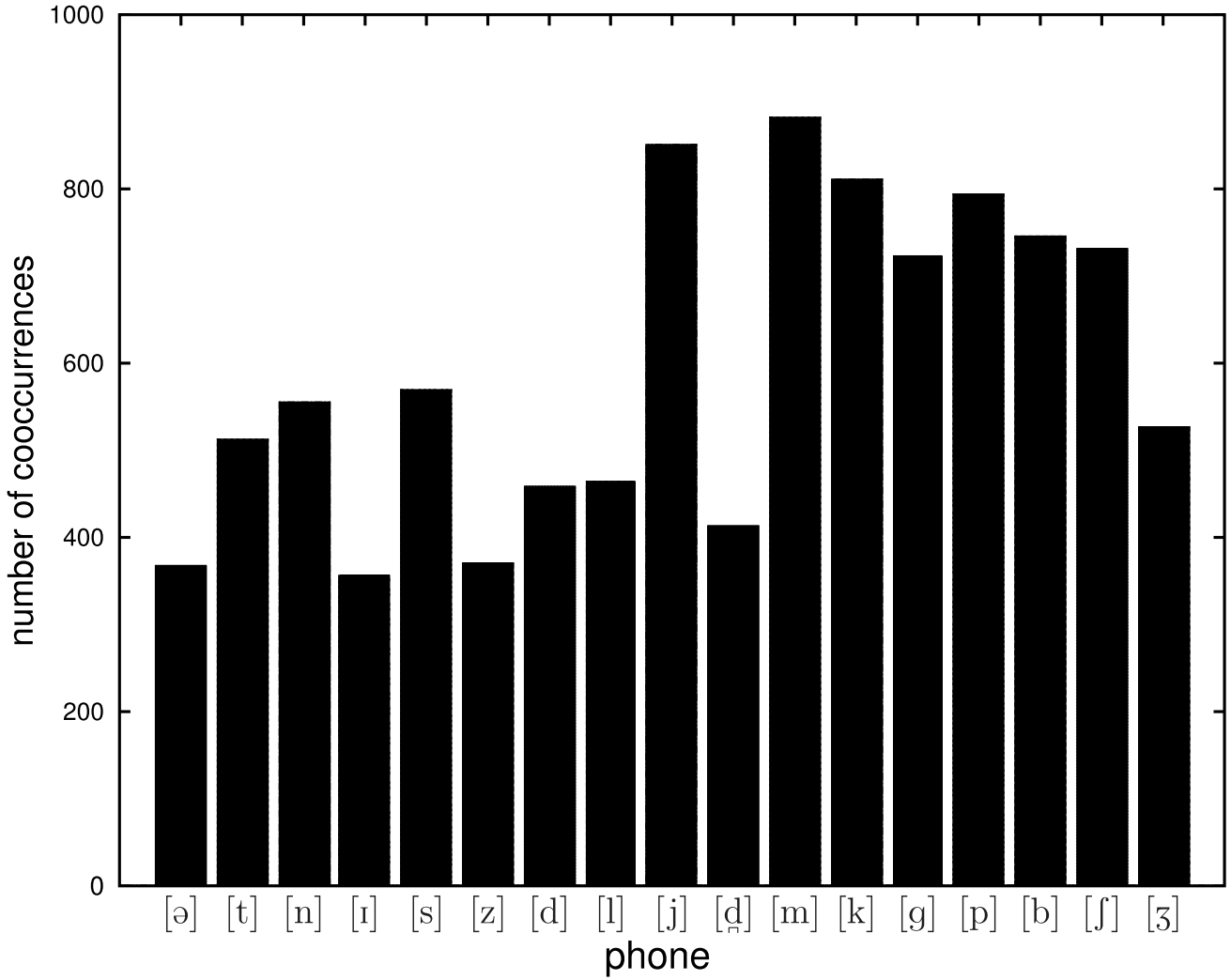
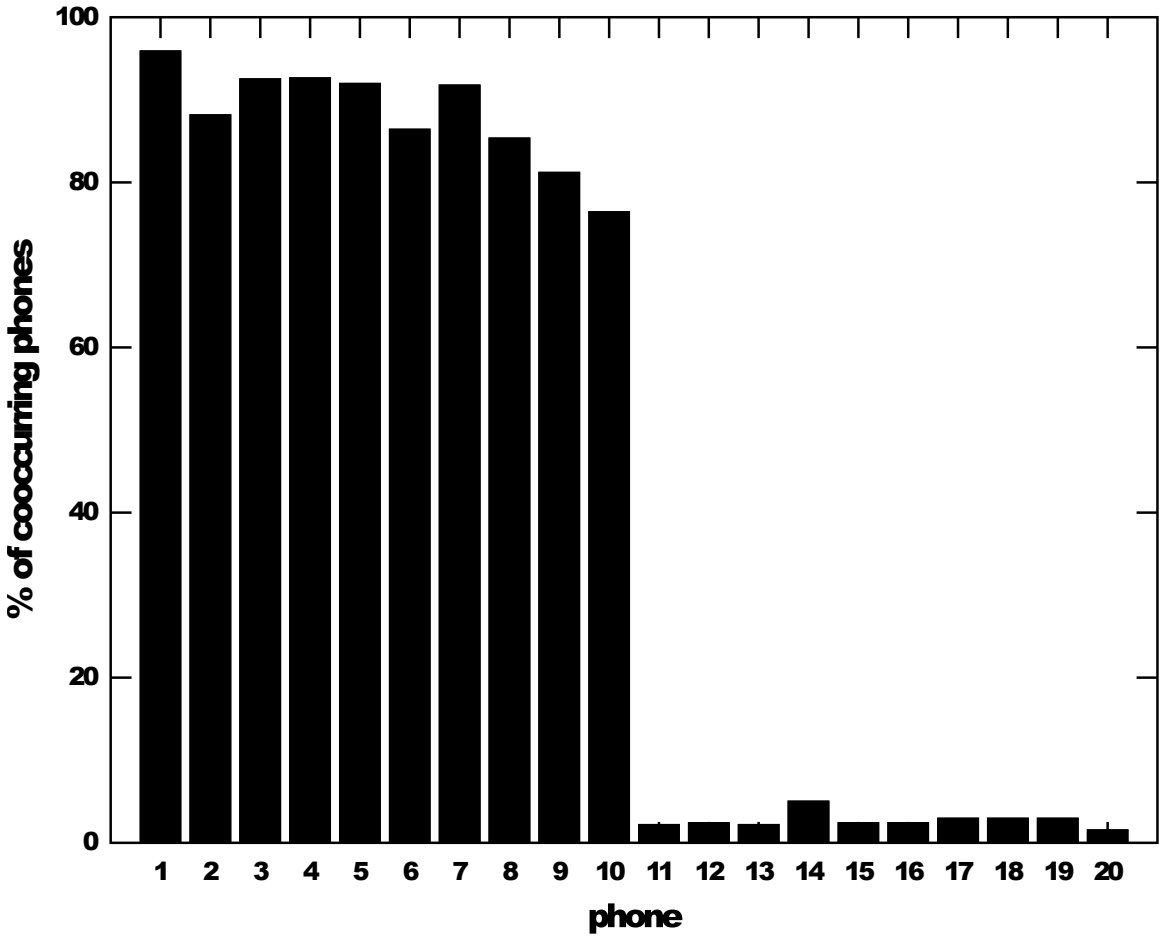


Figure: Number of cooccurring phones for each phone in UPSID. The ordered by frequency of occurrence in languages.

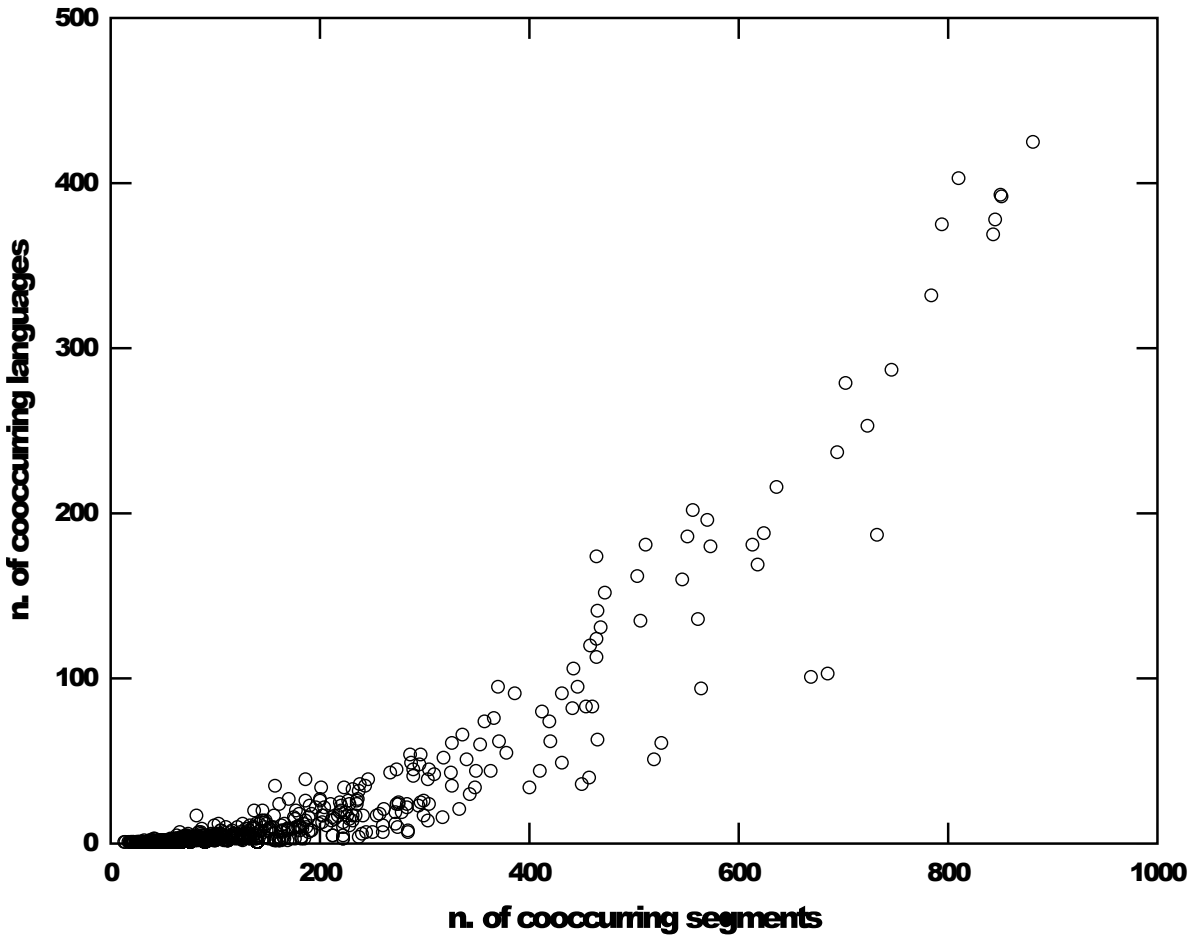
Cooccurrence for a given phone



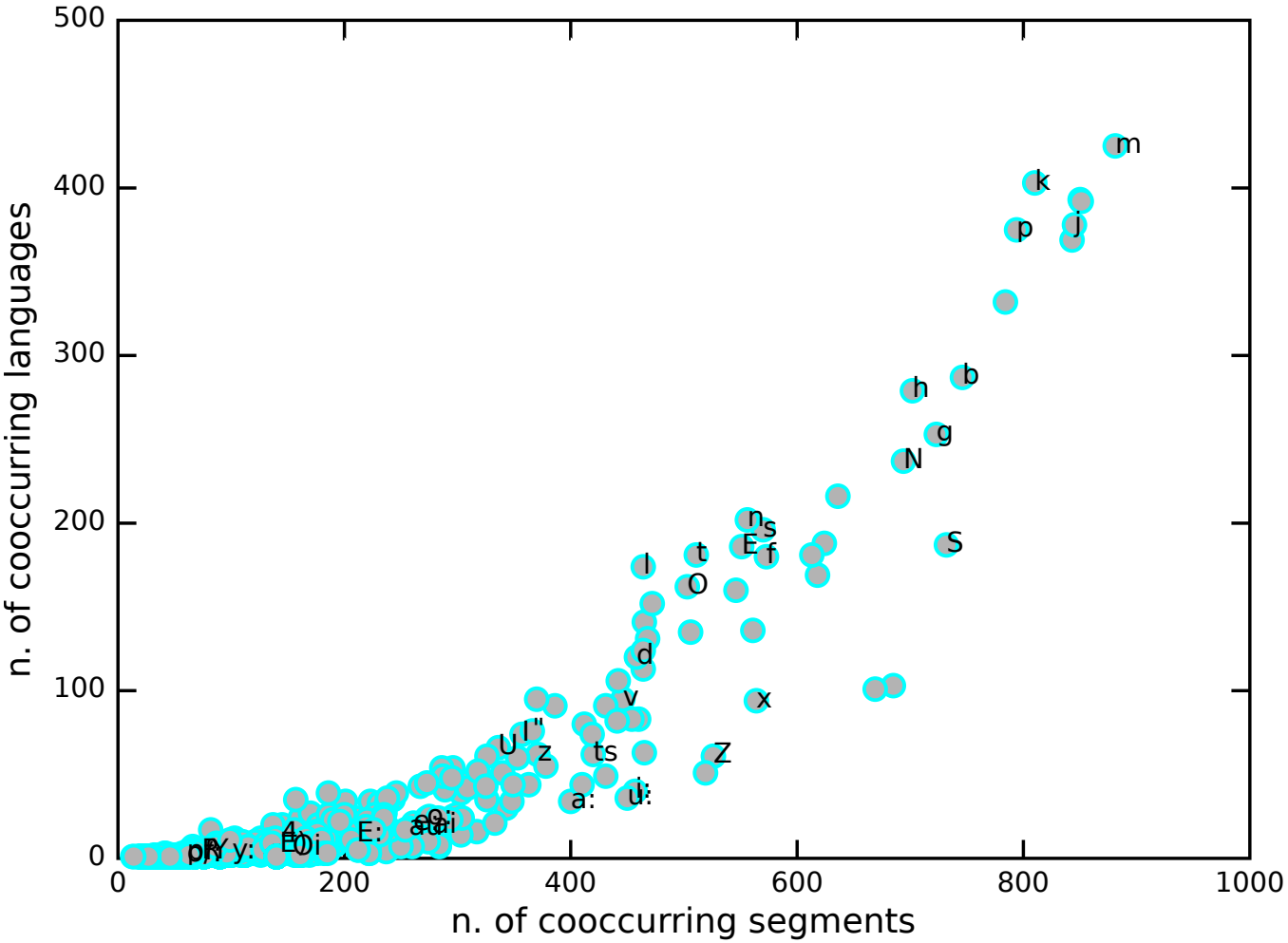
Cooccurrence: most frequent vs least frequent



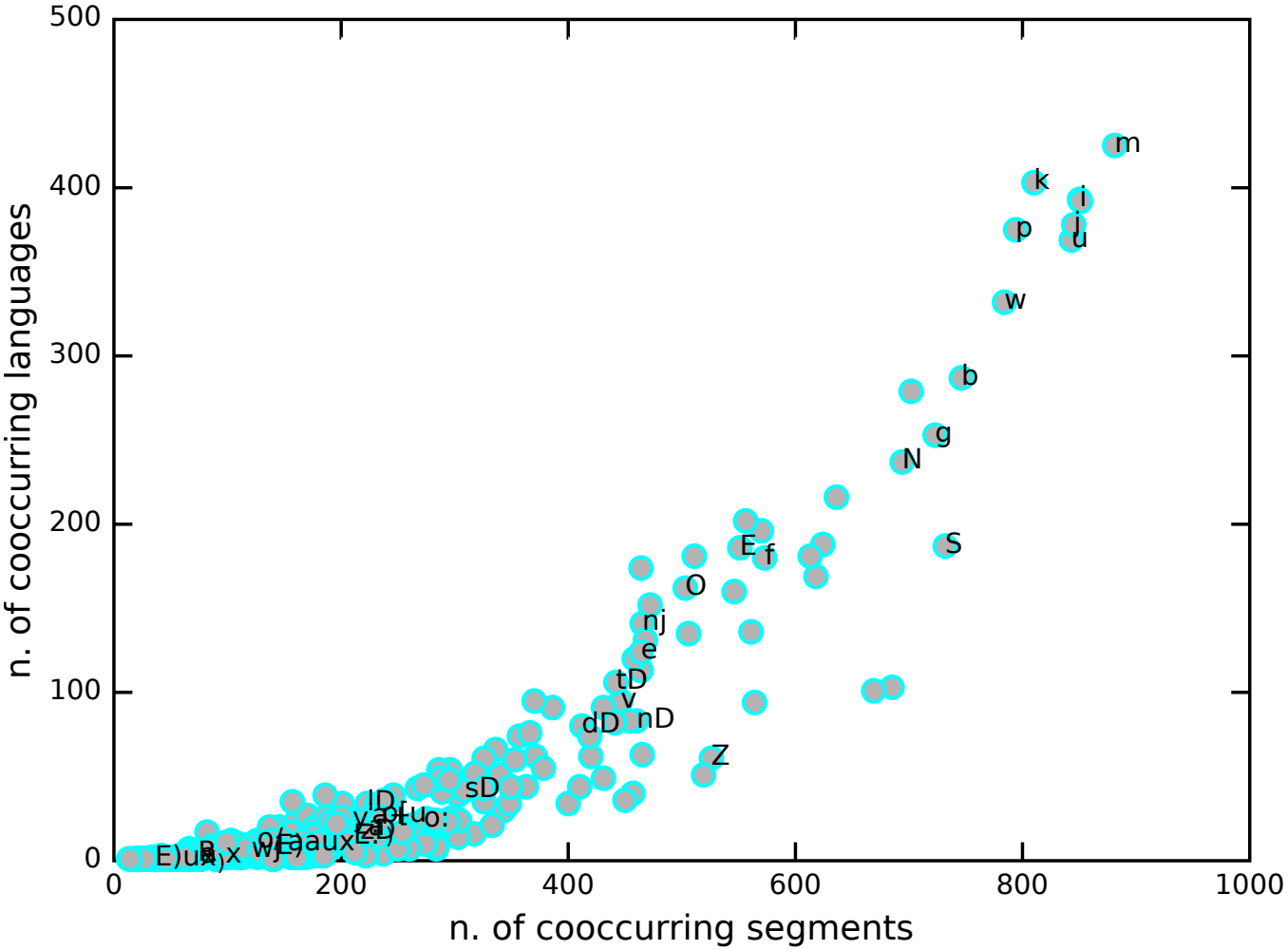
Cooccurrence : languages and segments



Cooccurrence : languages and segments - German



Cooccurrence : languages and segments - French



Outline

Approach

- synchrony
- written corpus
- pronouncing dictionary
- statistical analysis

Text Database

Project Gutenberg is a volunteer effort to provide free dgital access to cultural works. It was founded by Michael S. Hart in 1971 and is the oldest digital library. Most of its items are public domain books. Project Gutenberg claimed over 42,000 items in its collection (March 2013) (Project Gutenberg, 2013).

Google Ngram is a large database of n-grams (words combinations) based originally on 5.2 million books, published between 1500 and 2008, containing 500 billion words in American English, British English, French, German, Spanish, Russian, or Chinese (Michel et al., 2011b).

Pronouncing Dictionary

Dictionary : Carnegie Mellon University Pronouncing Dictionary - is a machine-readable pronunciation dictionary for North American English that contains over 125,000 words and their transcriptions (Weide, 2008).

Phoneme set : 39 phonemes.

Symbols : ARPAbet.

Phoneme	IPA	Example	Translation	IPA Translation
AE	[æ]	at	AE T	[æt]
B	[b]	be	B IY	[bi]
S	[s]	sea	S IY	[si]

Words Frequency

(1)	the : 775911	(16)	for : 107245	(31)	by : 63944	(46)	if : 38421
(2)	and : 471916	(17)	as : 102009	(32)	which : 63051	(47)	there : 38209
(3)	of : 414499	(18)	not : 96636	(33)	she : 57839	(48)	we : 37944
(4)	to : 350613	(19)	be : 86896	(34)	they : 57770	(49)	when : 37385
(5)	a : 277321	(20)	but : 81643	(35)	from : 56128	(50)	their : 36721
(6)	in : 226505	(21)	had : 80327	(36)	or : 52089	(51)	who : 36109
(7)	i : 200689	(22)	at : 76688	(37)	so : 51617	(52)	an : 35485
(8)	that : 173083	(23)	her : 75761	(38)	said : 50040	(53)	your : 33401
(9)	he : 162183	(24)	on : 75493	(39)	no : 48930	(54)	would : 32582
(10)	it : 145364	(25)	my : 73879	(40)	are : 45831	(55)	do : 31225
(11)	was : 130804	(26)	him : 72258	(41)	one : 43822	(56)	out : 30165
(12)	his : 129300	(27)	have : 68463	(42)	what : 41575	(57)	then : 29682
(13)	you : 118473	(28)	this : 67572	(43)	them : 41320	(58)	been : 29502
(14)	with : 114122	(29)	all : 65960	(44)	were : 40475	(59)	up : 28860
(15)	is : 112640	(30)	me : 64560	(45)	will : 39733		. . .

Words Frequency and Zipf Law

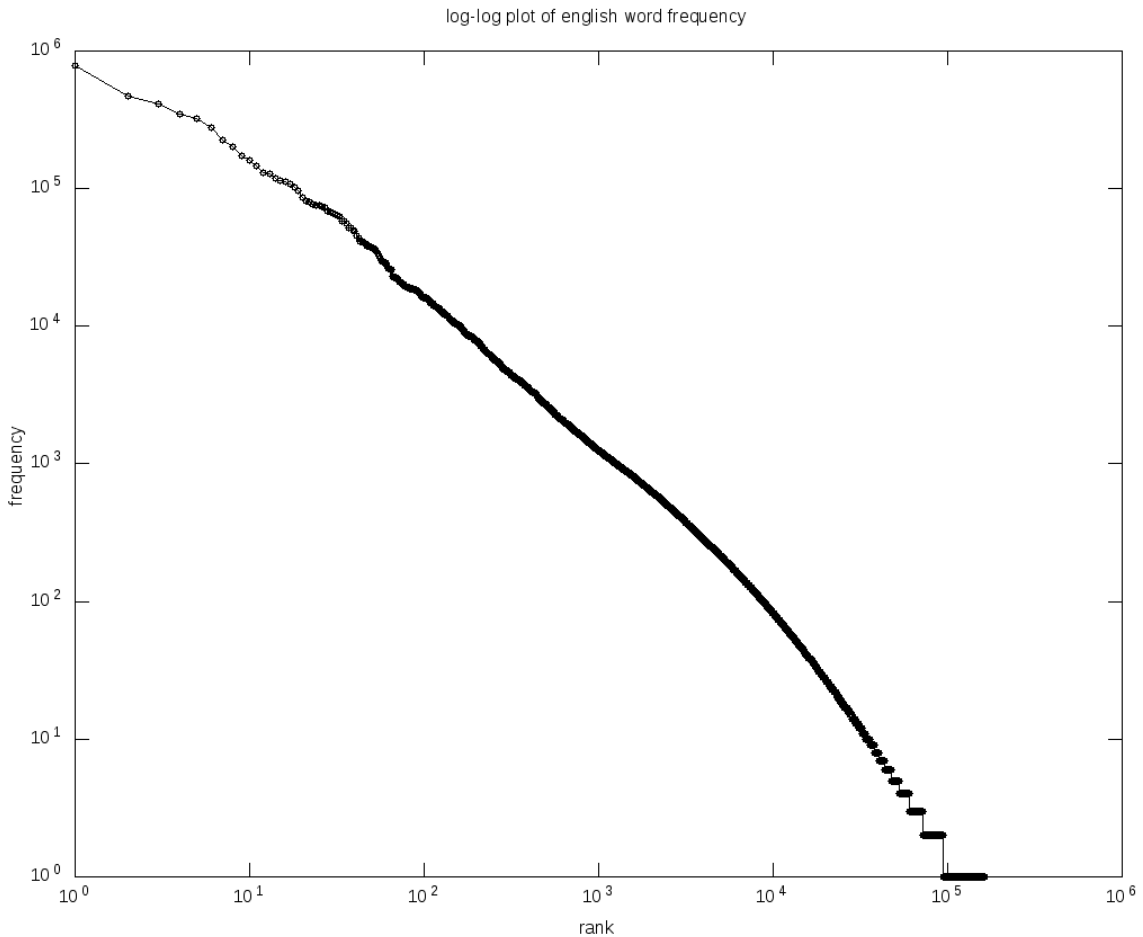


Figure: Log-log plot of words rank versus frequency of occurrence.

Analysis of Smaller Units

Word is considered the smallest free form that can be uttered or written and carries a meaning (Bloomfield, 1926).

Basic units of speech perception:

- phones (Pisoni, 1982)
- diphones (Klatt, 1979)
- triphones (speech synthesis)
- syllables (Studdert-Kennedy, 1976)
- demisyllables (Fujimura and Lovins, 1978)

Letters Frequency

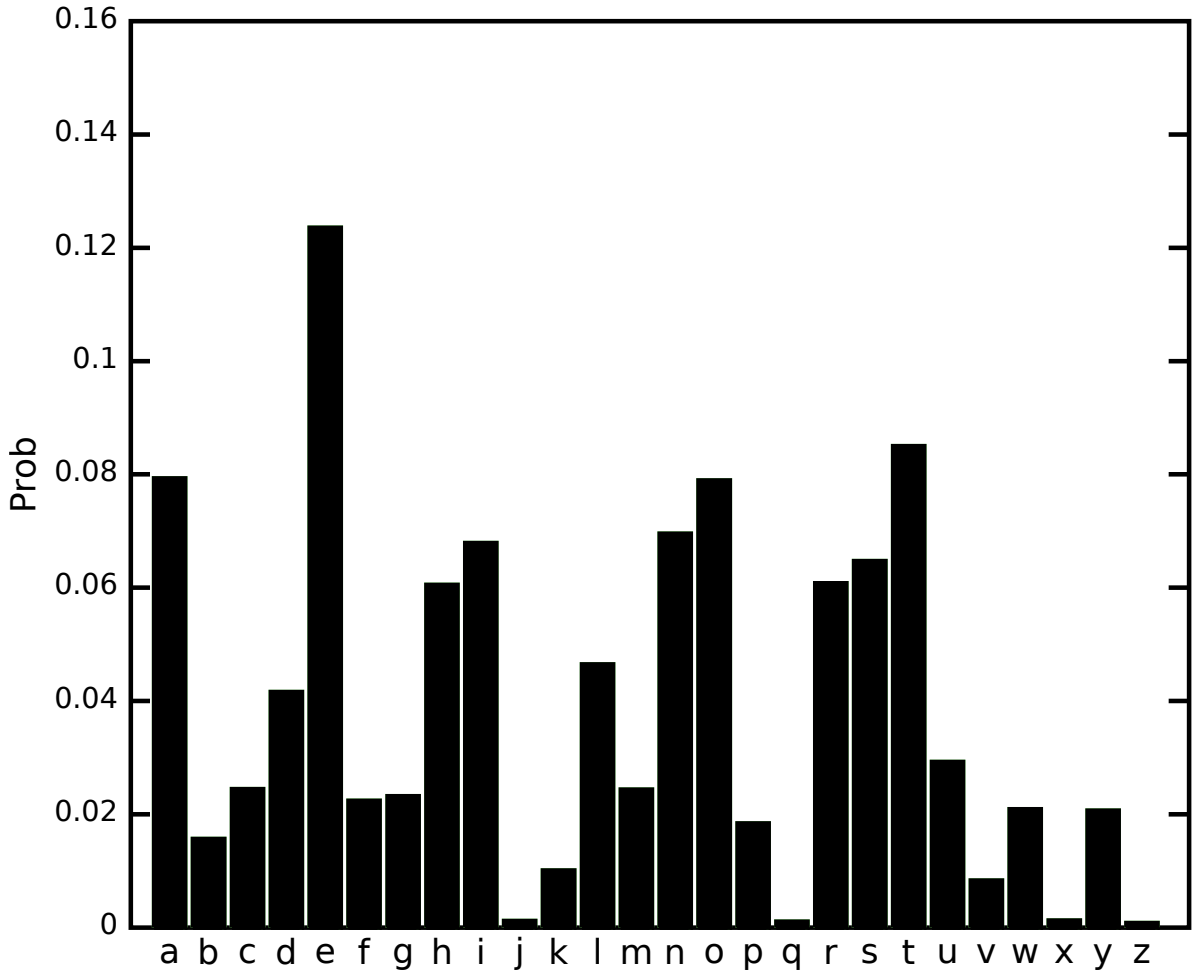


Figure: Relative frequency of letters in text.

Letters Frequency

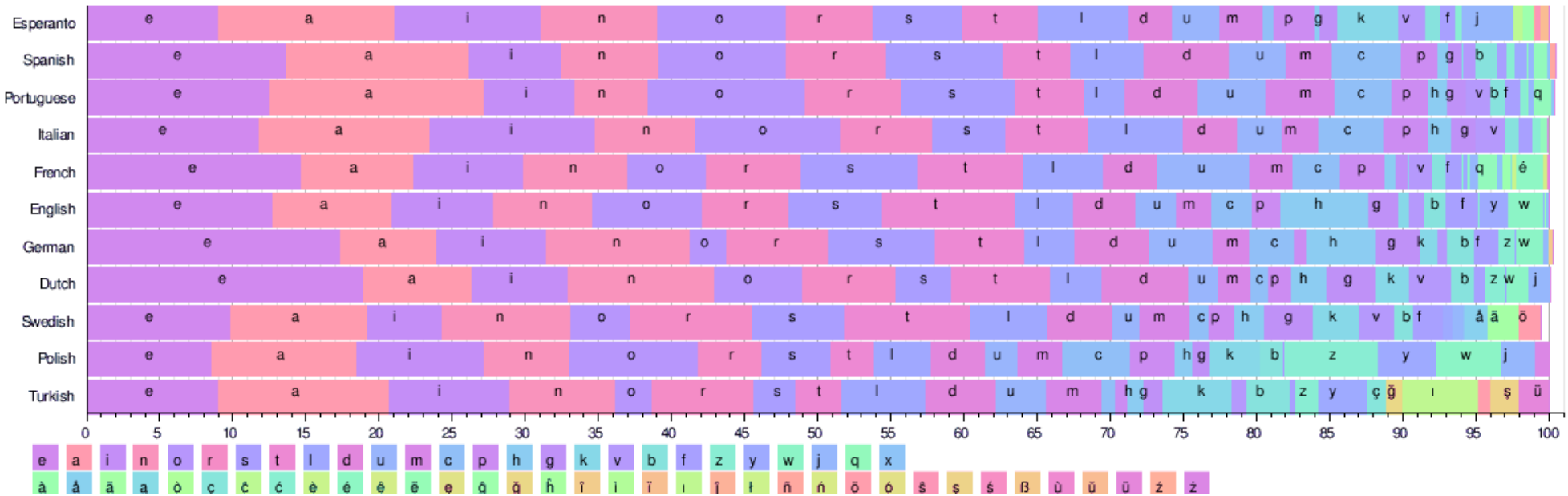


Figure: Frequency distributions of the 26 most common Latin letters across some languages (source: Wikipedia).

Words Frequency - Transcribed

(1) DH AH : 775911
(2) AH N D : 471916
(3) AH V : 414499
(4) T UW : 350613
(5) AH : 277321
(6) IH N : 226505
(7) AY : 200689
(8) DH AE T : 173083
(9) HH IY : 162183
(10) IH T : 145364
(11) W AA Z : 130804
(12) HH IH Z : 129300
(13) Y UW : 118473
(14) W IH DH : 114122
(15) IH Z : 112640
(16) F AO R : 107245
(17) AE Z : 102009
(18) N AA T : 96636
(19) B IY : 86896
(20) B AH T : 81643
(21) HH AE D : 80327
(22) AE T : 76688
(23) HH ER : 75761
(24) AA N : 75493
(25) M AY : 73879
(26) HH IH M : 72258
(27) HH AE V : 68463
(28) DH IH S : 67572
(29) AO L : 65960
(30) M IY : 64560
(31) B AY : 63944
(32) W IH CH : 63051
(33) SH IY : 57839
(34) DH EY : 57770
(35) F R AH M : 56128
(36) AO R : 52089
(37) S OW : 51617
(38) S EH D : 50040
(39) N OW : 48930
(40) AA R : 45831
(41) W AH N : 43822
(42) W AH T : 41575
(43) DH EH M : 41320
(44) W ER : 40475
(45) W IH L : 39733
(46) IH F : 38421
(47) DH EH R : 38209
(48) W IY : 37944
(49) W EH N : 37385
(50) DH EH R : 36721
(51) HH UW : 36109
(52) AE N : 35485
(53) Y AO R : 33401
(54) W UH D : 32582
(55) D UW : 31225
(56) AO L AW T : 30165
(57) DH EH N : 29682
(58) B IH N : 29502
(59) AH P : 28860
. . .

Phones Frequency

(1) ə : 44539
(2) t : 33131
(3) n : 31928
(4) ɪ : 28845
(5) s : 21928
(6) d : 20032
(7) r : 18563
(8) i : 16482
(9) l : 15816
(10) ɛ : 13896
(11) m : 13072
(12) ɝ : 12640
(13) k : 12308
(14) w : 11107
(15) z : 10744
(16) ð : 10720
(17) v : 10407
(18) h : 10009
(19) f : 9391
(20) ɑ : 8744
(21) æ : 8635
(22) b : 8390
(23) u : 7972
(24) p : 7501
(25) ɔ : 7429
(26) eɪ : 6196
(27) aɪ : 6148
(28) oɯ : 5283
(29) ʃ : 4915
(30) ŋ : 4861
(31) g : 3351
(32) tʃ : 2501
(33) j : 2462
(34) θ : 2309
(35) ʊ : 2276
(36) aʊ : 2242
(37) dʒ : 2100
(38) ɔɪ : 326
(39) ʒ : 314

Phones Frequency - Log-log plot

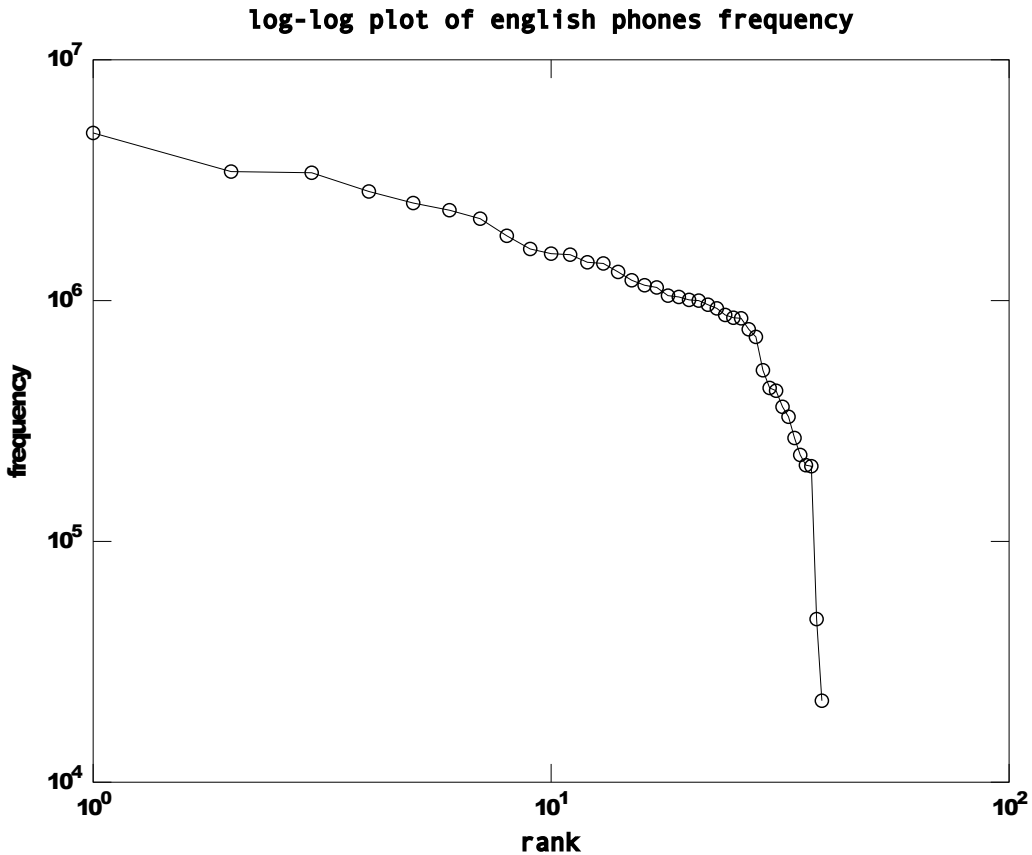


Figure: Log-log plot of phones rank versus frequency of occurrence.

Probability of occurrence of [ə] across words rank

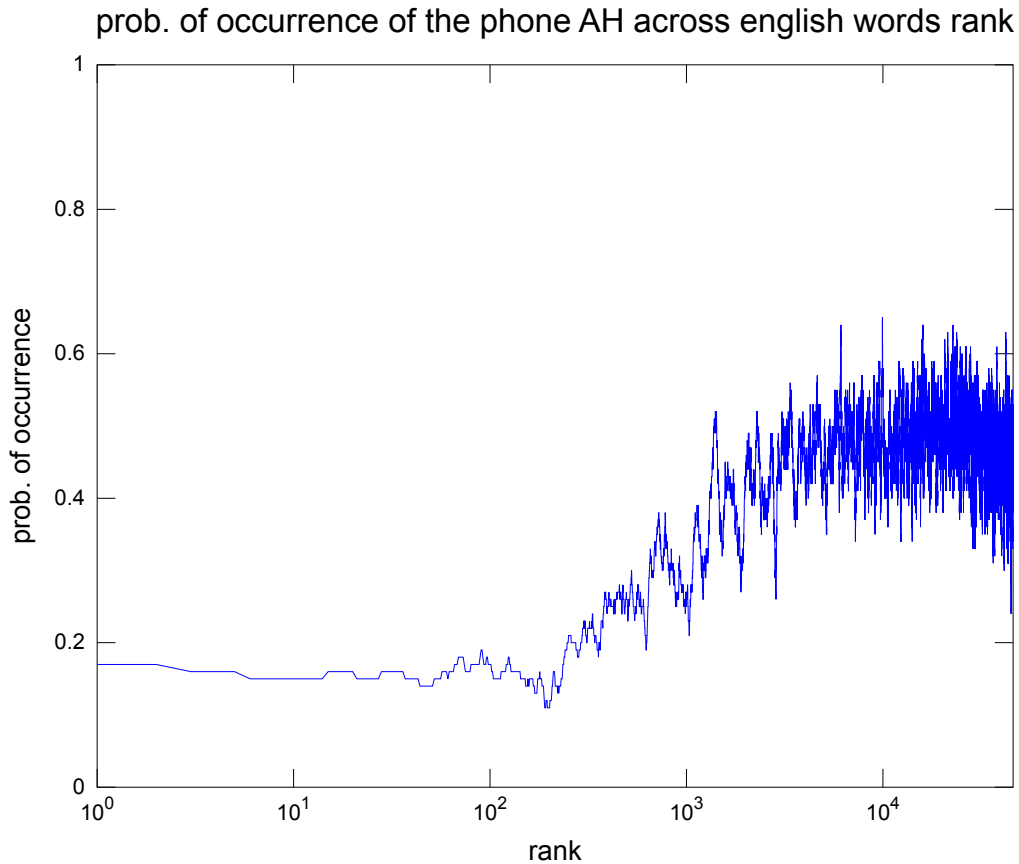


Figure: Probability of occurrence of [ə] in words versus words rank.

Probability of occurrence of [t] across words rank

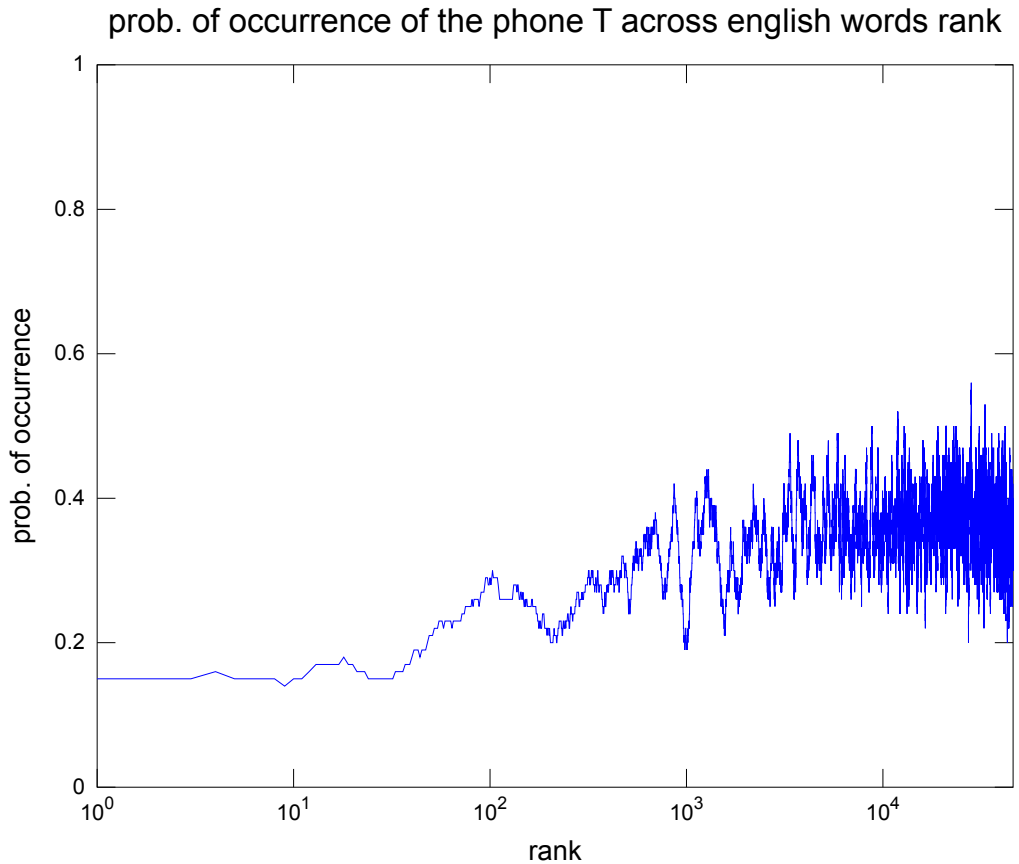


Figure: Probability of occurrence of [t] in words versus words rank.

Probability of occurrence of [n] across words rank

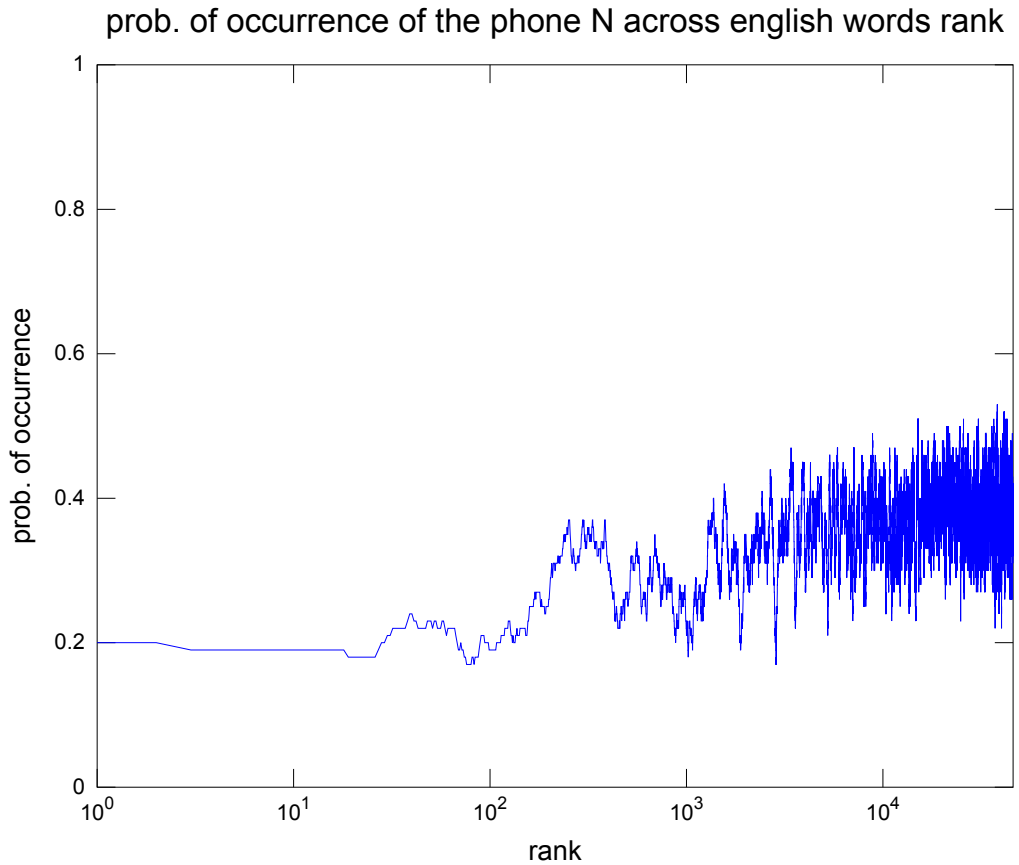


Figure: Probability of occurrence of [n] in words versus words rank.

Probability of occurrence of [ʊ] across words rank

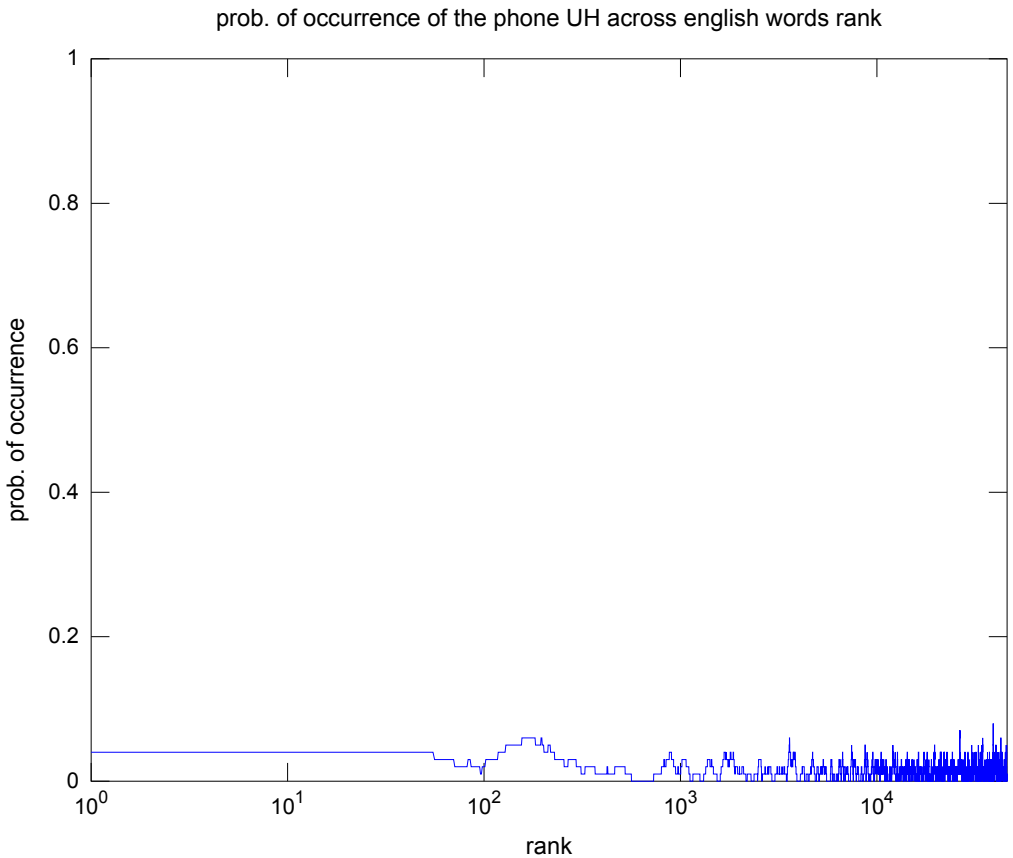


Figure: Probability of occurrence of [ʊ] in words versus words rank.

Probability of occurrence of [ɔɪ] across words rank

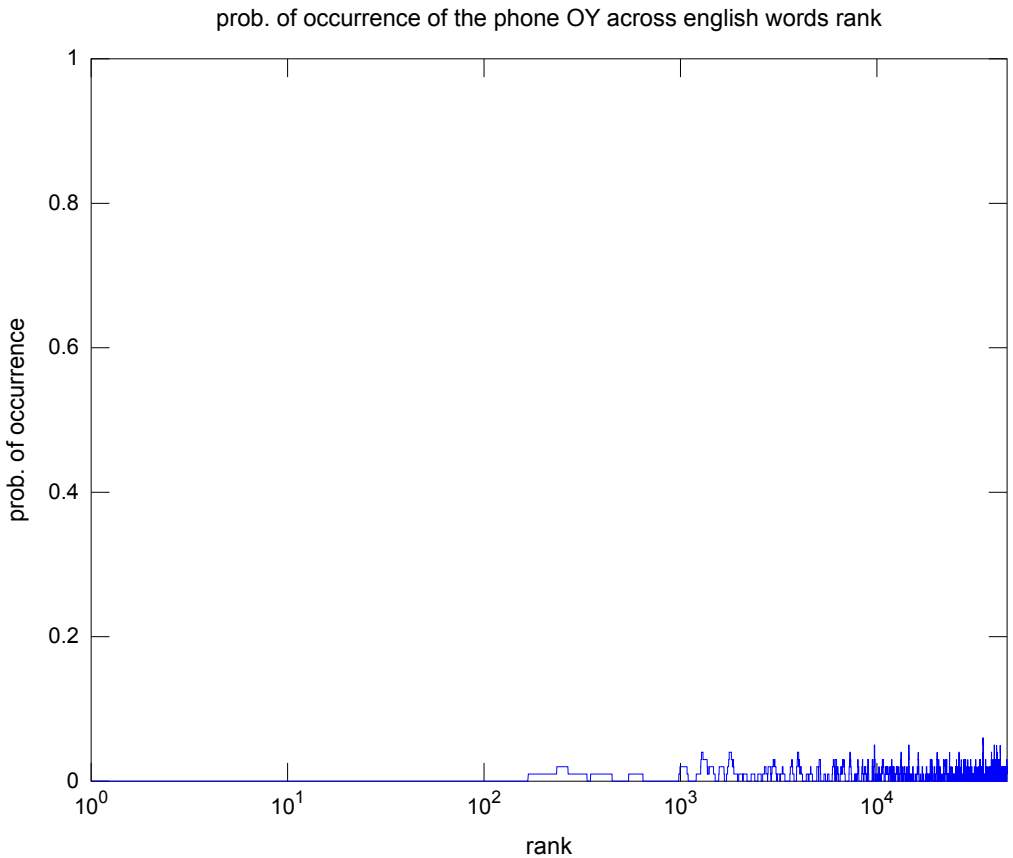


Figure: Probability of occurrence of [ɔɪ] in words versus words rank.

Probability of occurrence of [ʒ] across words rank

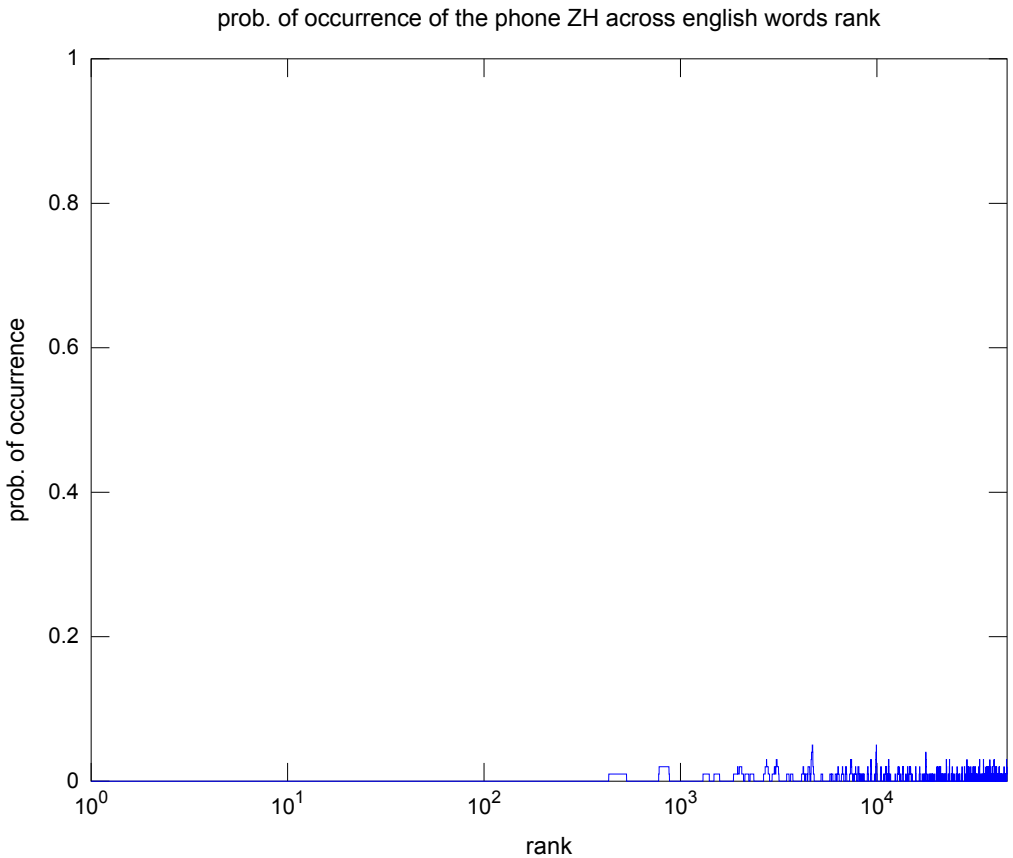


Figure: Probability of occurrence of [ʒ] in words versus words rank.

Diphones : Frequency of occurrence

(1) ən : 1296408
(2) ðə : 785354
(3) nd : 784028
(4) st : 651129
(5) əv : 489267
(6) ɔr : 472069
(7) ɪn : 470069
(8) tu : 425544
(9) tə : 420825
(10) ɪŋ : 387096
(11) ɪz : 380357
(12) əl : 372847
(13) ɛn : 349905
(14) nt : 337193
(15) æt : 284460
(16) wɪ : 282526
(17) ət : 265396
(18) ɛr : 264293
(19) rə : 261447
(20) ɪt : 260544
(21) əs : 246996
(22) ju : 245715
(23) hæ : 237724
(24) hɪ : 237490
(25) əm : 233793
(26) ri : 219997
(27) li : 219652
(28) ɑn : 213755
(29) æn : 213169
(30) sə : 210385
(31) ɪs : 208619
(32) ðæ : 194602
(33) əf : 193570
(34) ɑr : 193525

⋮

(1120) uaɪ : 1
(1121) ɔɪɑ : 1
(1122) bʃ : 1
(1123) pv : 1
(1124) iʊ : 1
(1125) ɛoʊ : 1

Diphones Frequency - Log-log plot

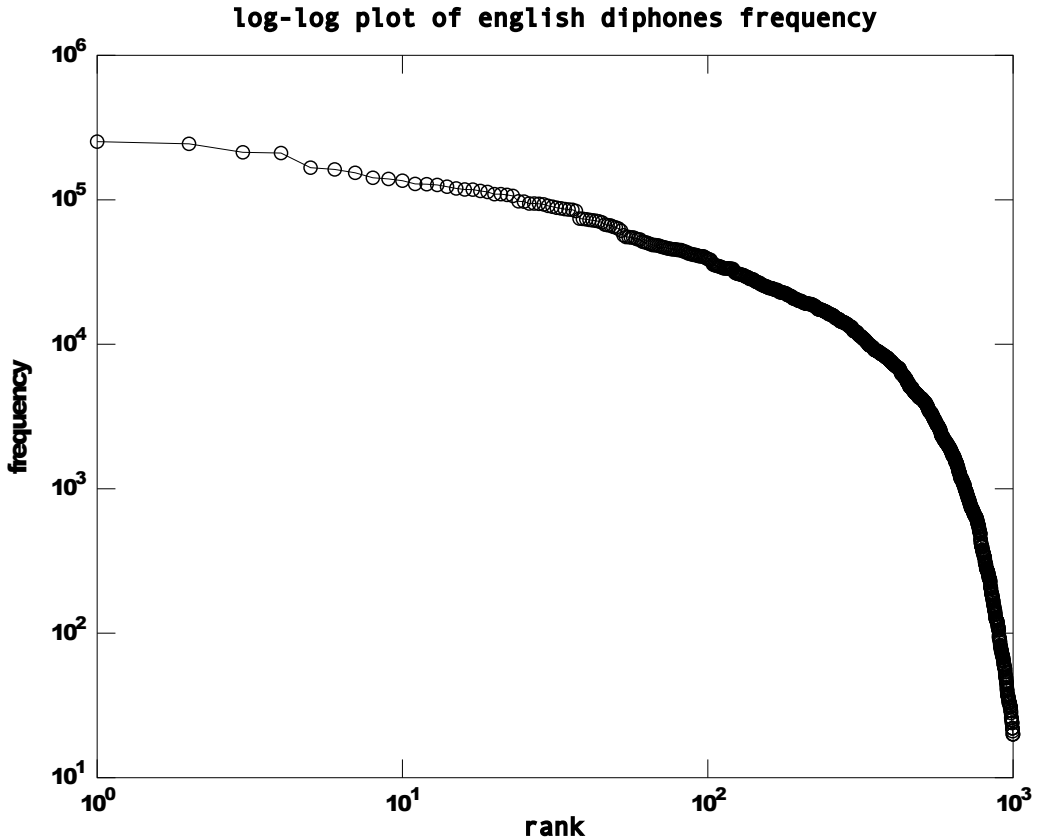


Figure: Log-log plot of the diphones frequency of occurrence versus their rank.

Diphones : phone-normalized frequency of occurrence

(1)	ʒə	(5)	dʒə	(9)	ju	(13)	ətʃ	(17)	əp	(1122)	aıh				
(2)	ðə	(6)	bə	(10)	əm	(14)	nd		⋮	(1123)	ɛoʊ				
(3)	əv	(7)	əl	(11)	ɔr	(15)	kə	(1120)	zs	(1124)	ddʒ				
(4)	ʃə	(8)	əf	(12)	əb	(16)	əg	(1121)	uaı	(1125)	tv				

Diphones Normalized Frequency - Log-log plot

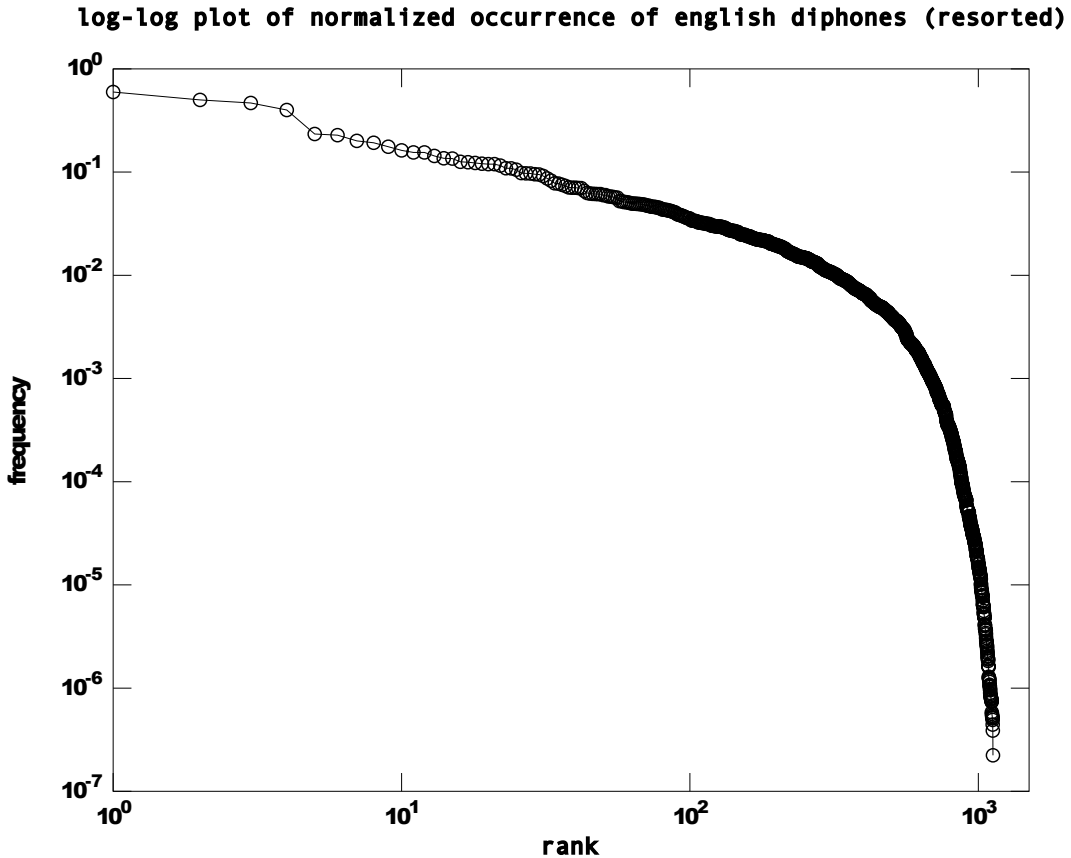


Figure: Log-log plot of the diphones normalized frequency of occurrence versus their rank. The normalization is made using the frequency of occurrence of each phone in the pair.

Diphones Conditional Probability

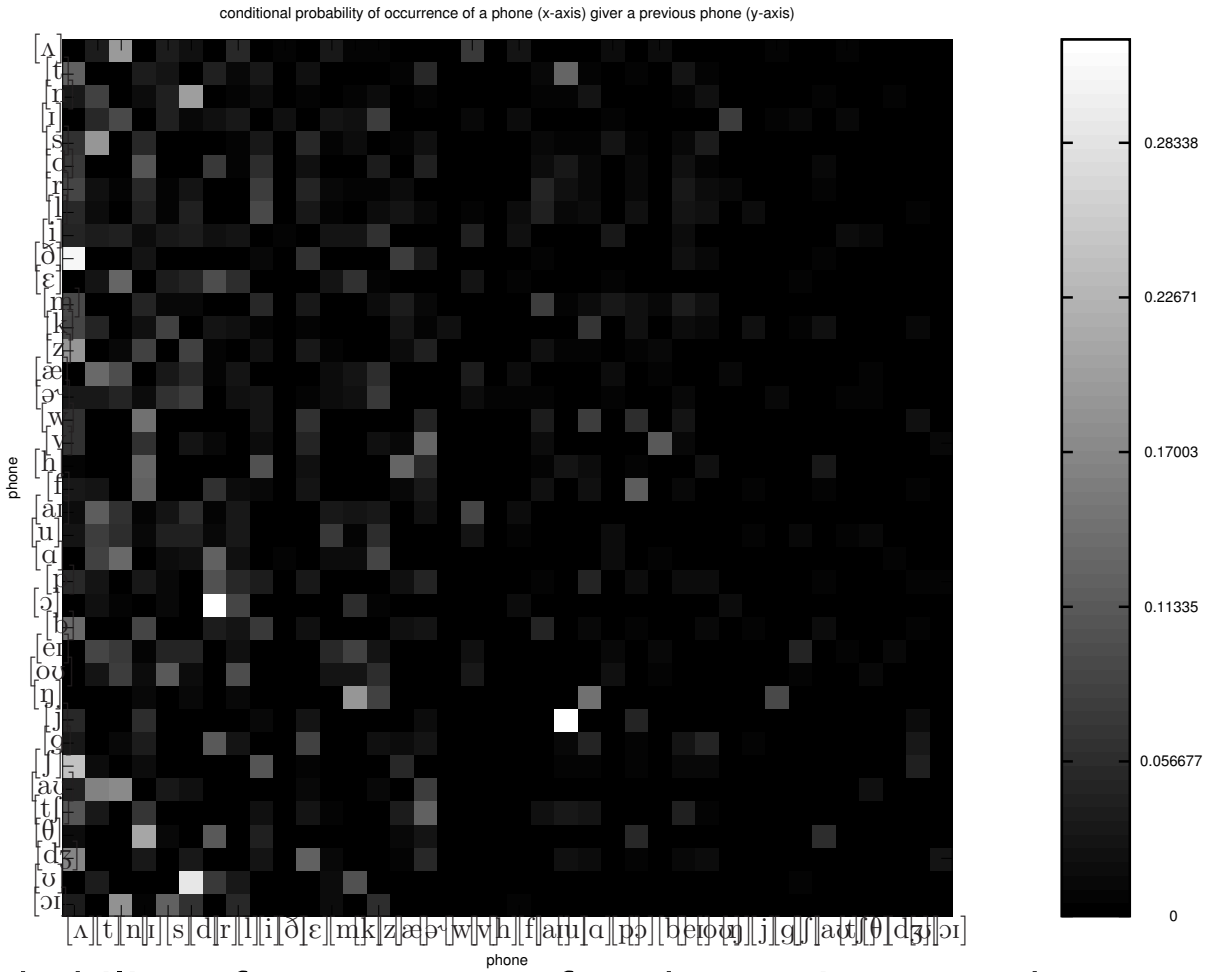


Figure: Probability of occurrence of a phone given another previous phone.

Triphone Probabilities

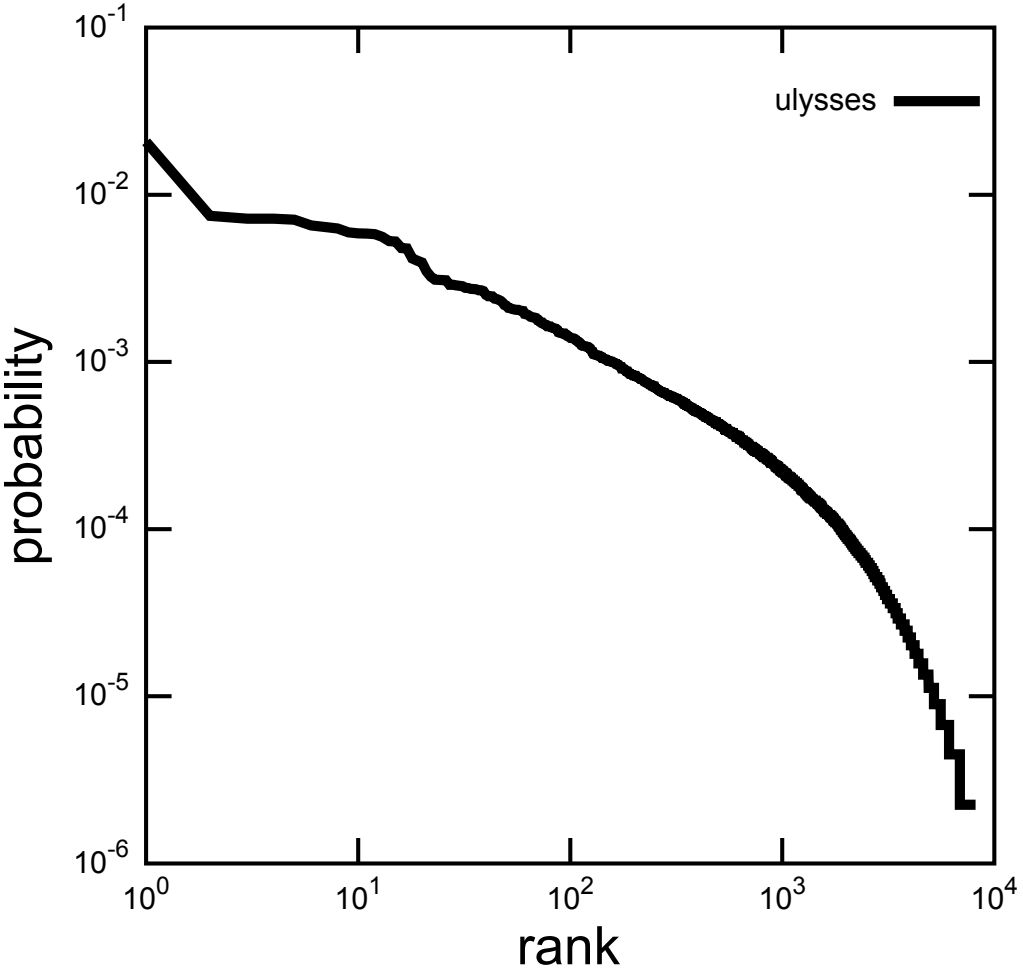


Figure: Triphone Probability (from *Ulysses*).

Syllable Probabilities

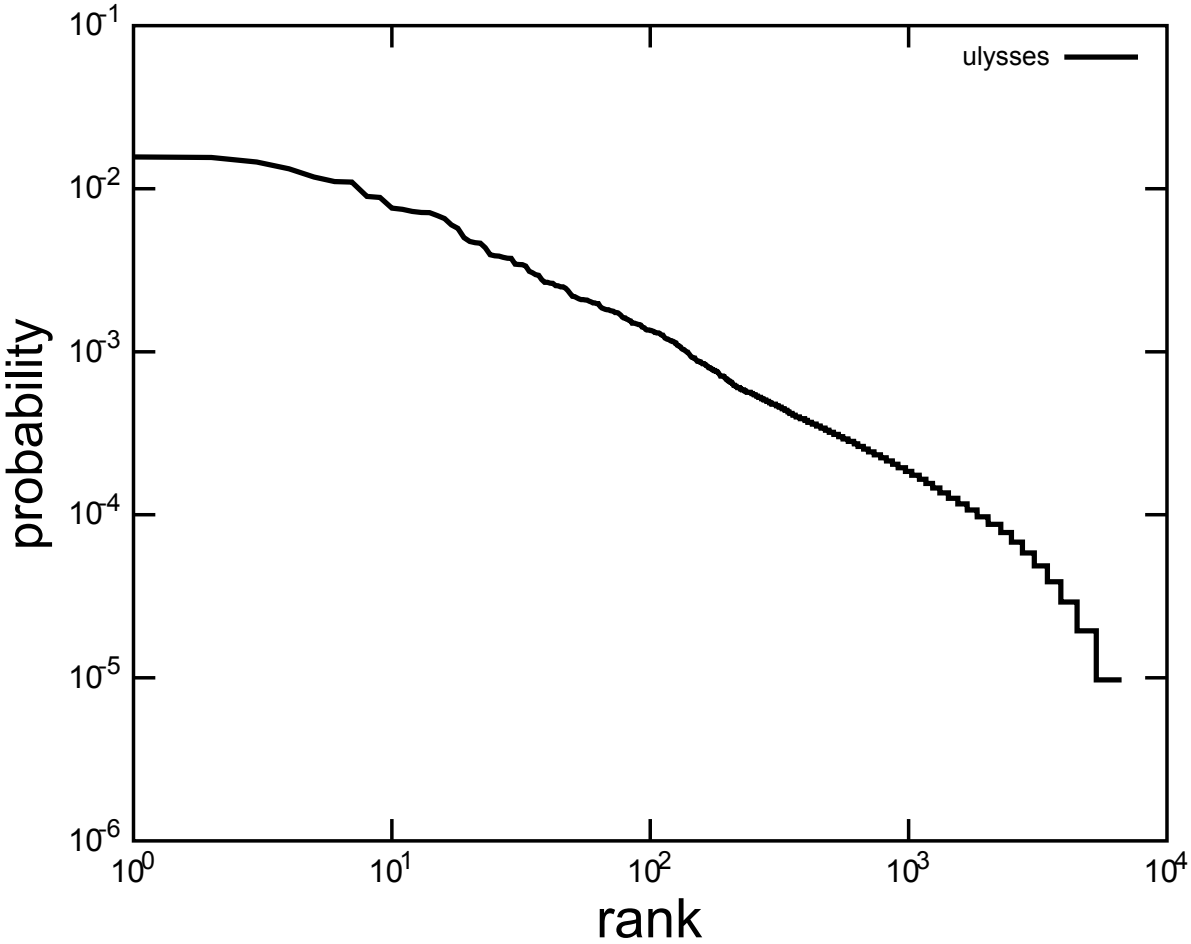


Figure: Syllable Probability (from *Ulysses*).

Words Length

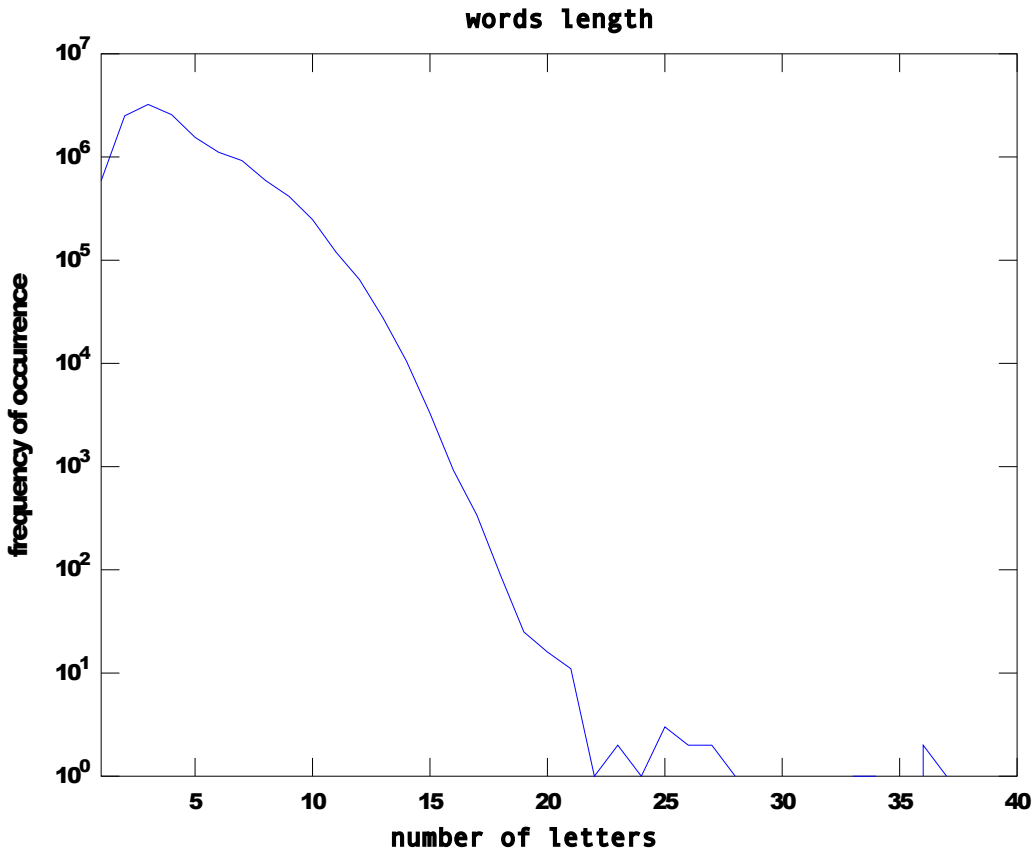


Figure: Frequency of occurrence of words of a given length (letters).

Words Phones Length

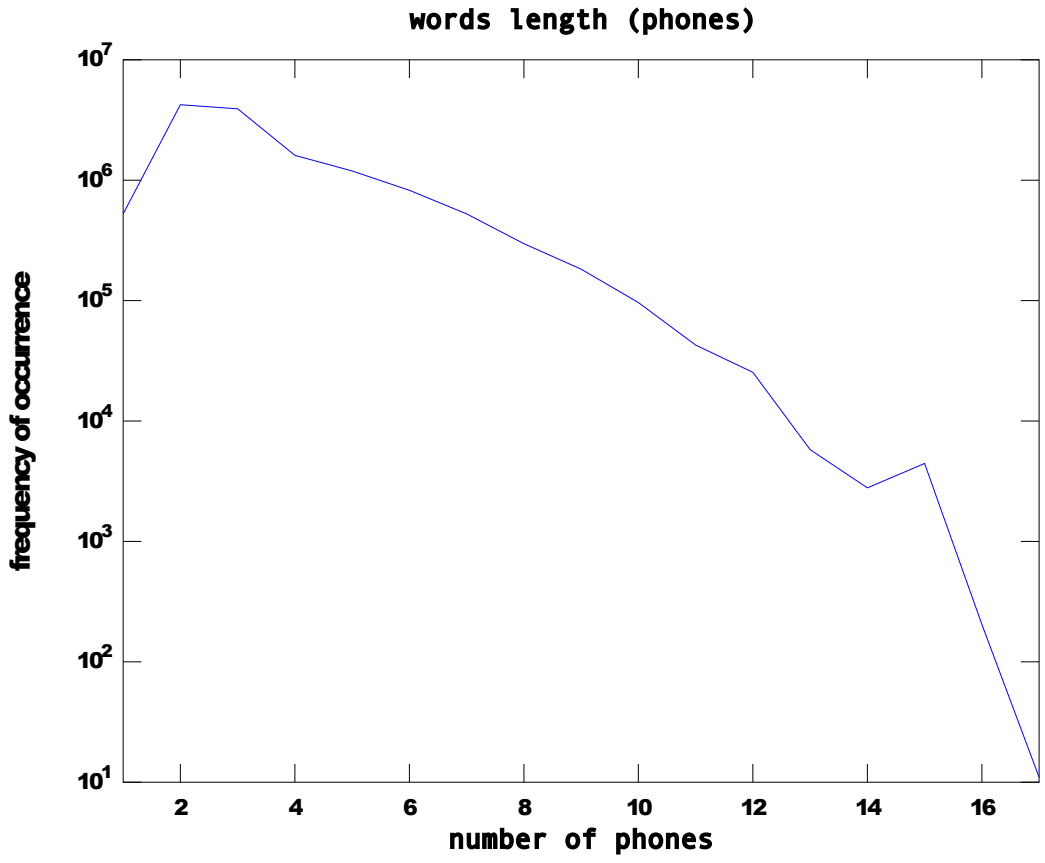


Figure: Frequency of occurrence of words of a given length (phones).

Average word length (letters)

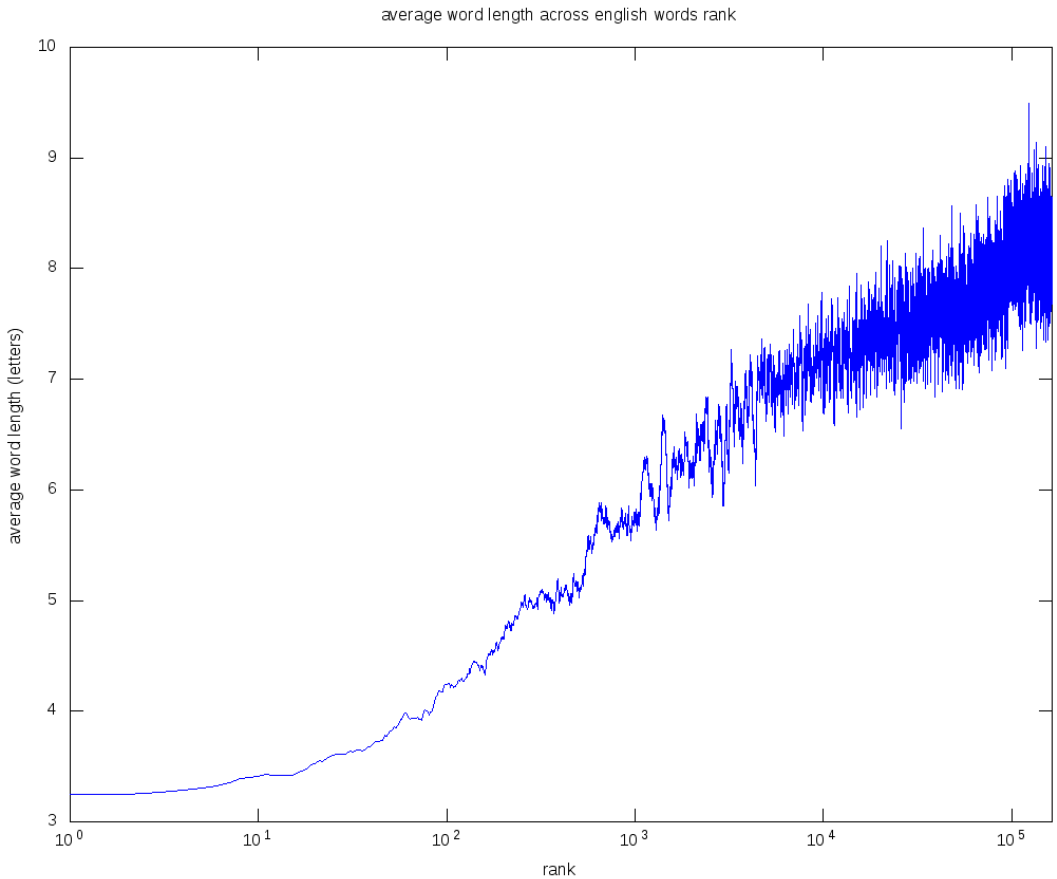


Figure: Average word length (letters) across word rank.

Average word length (phones)

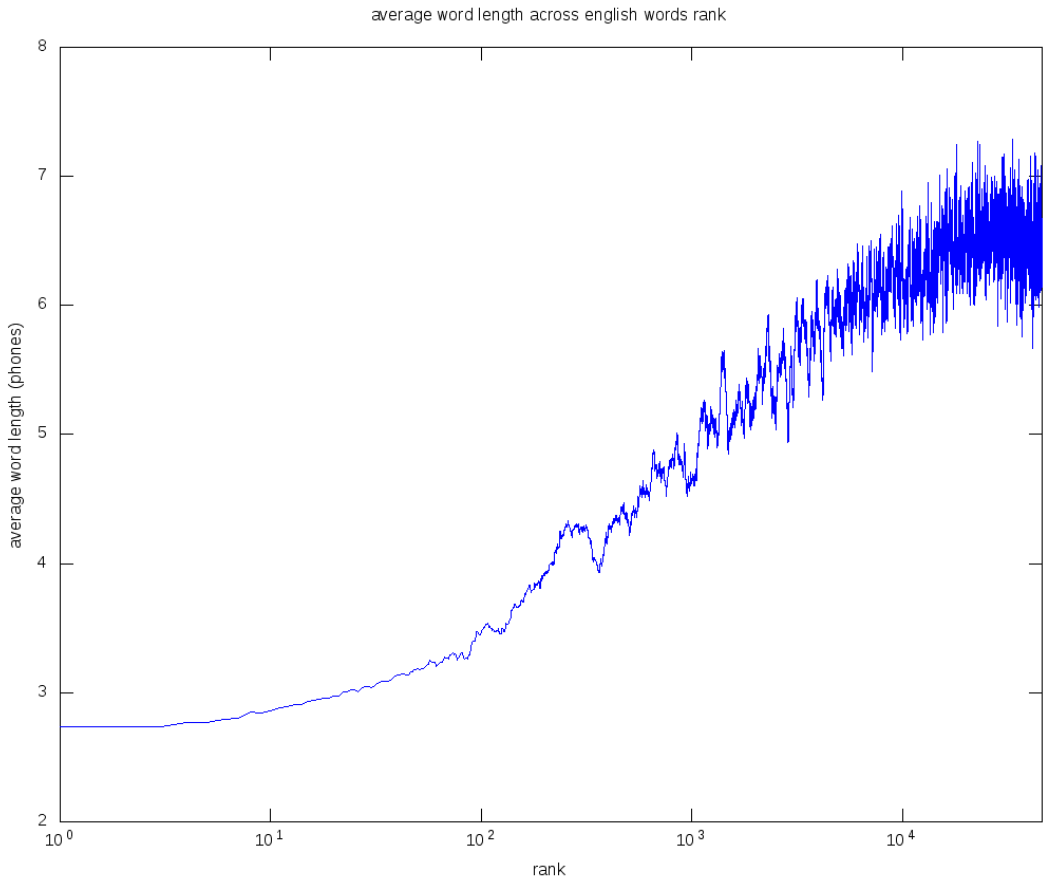


Figure: Average word length (phones) across word rank.

Frequency of occurrence and frequency index

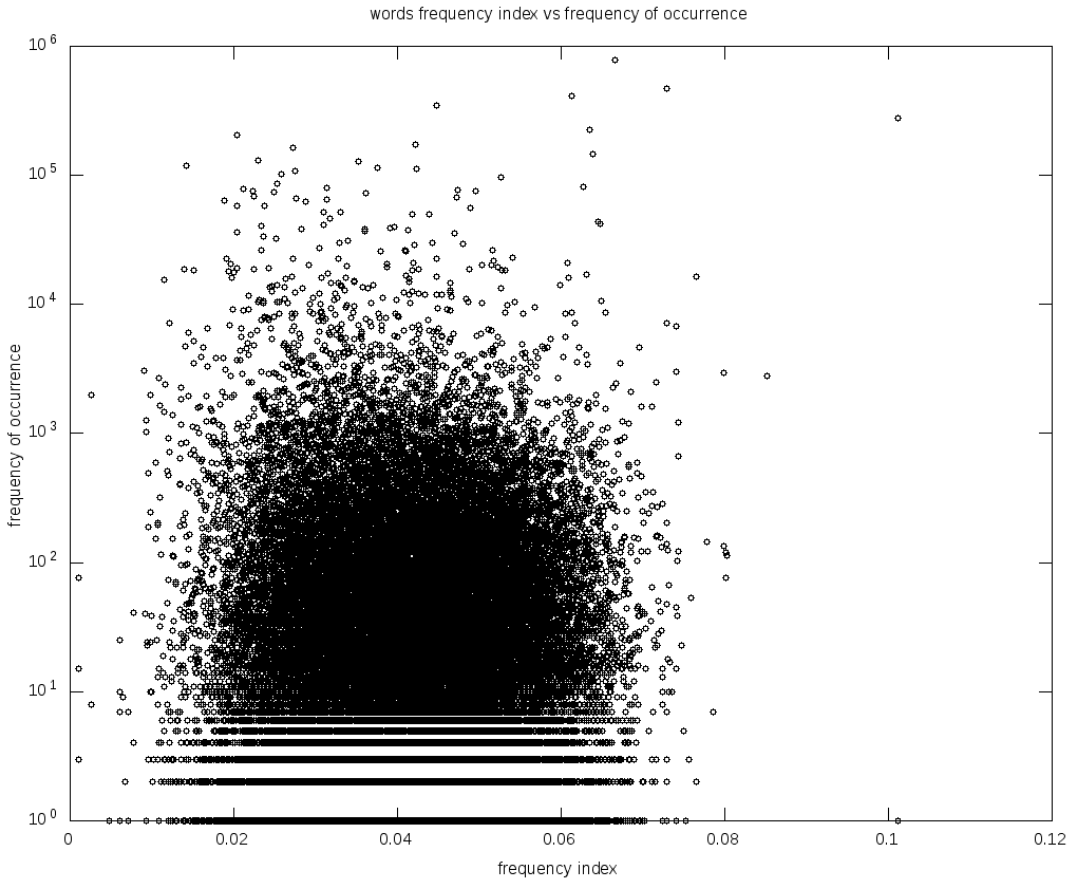


Figure: Each spot represents a word with a given frequency of occurrence and a frequency index. For a better visualization the frequency of occurrence is displayed in a logarithm scale.

Frequency of occurrence vs frequency index (density plot)

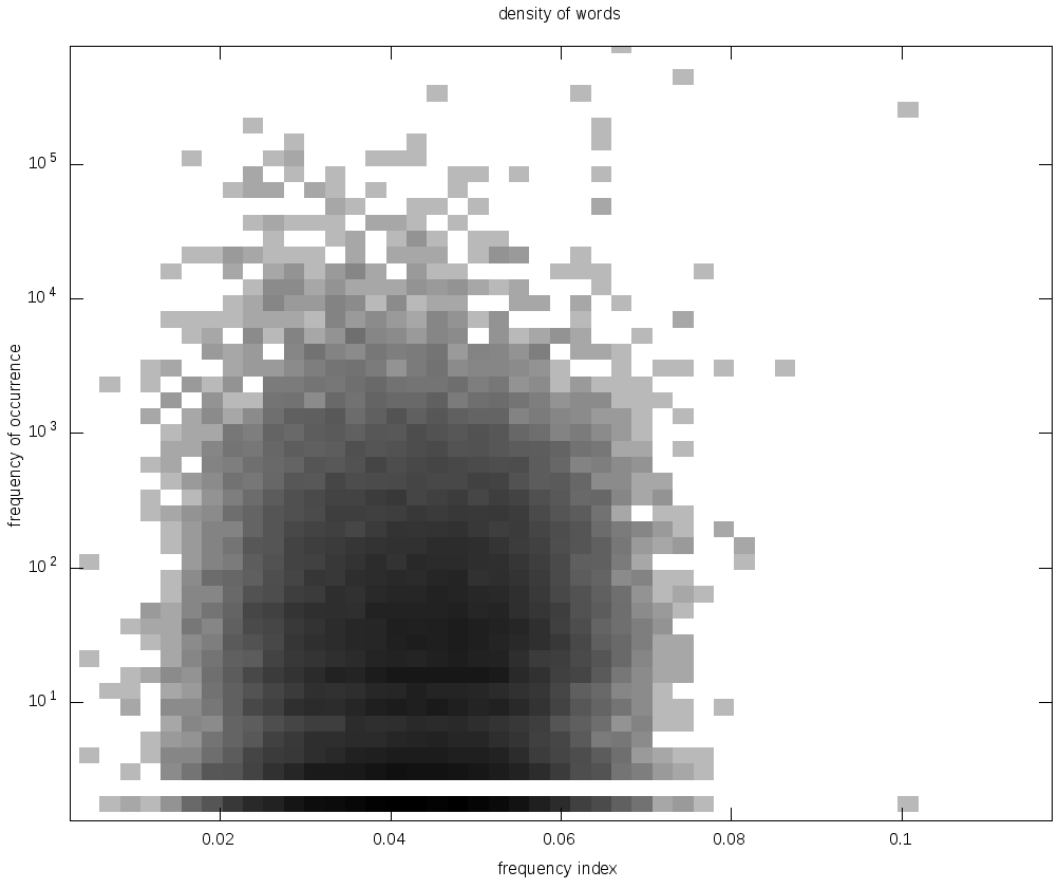


Figure: This picture is the density of words in each partition on the frequency of occurrence vs. frequency index space. The largest number is displayed in black and it refers to 578 words. White represents no word found in a spot.

Distinctive Features

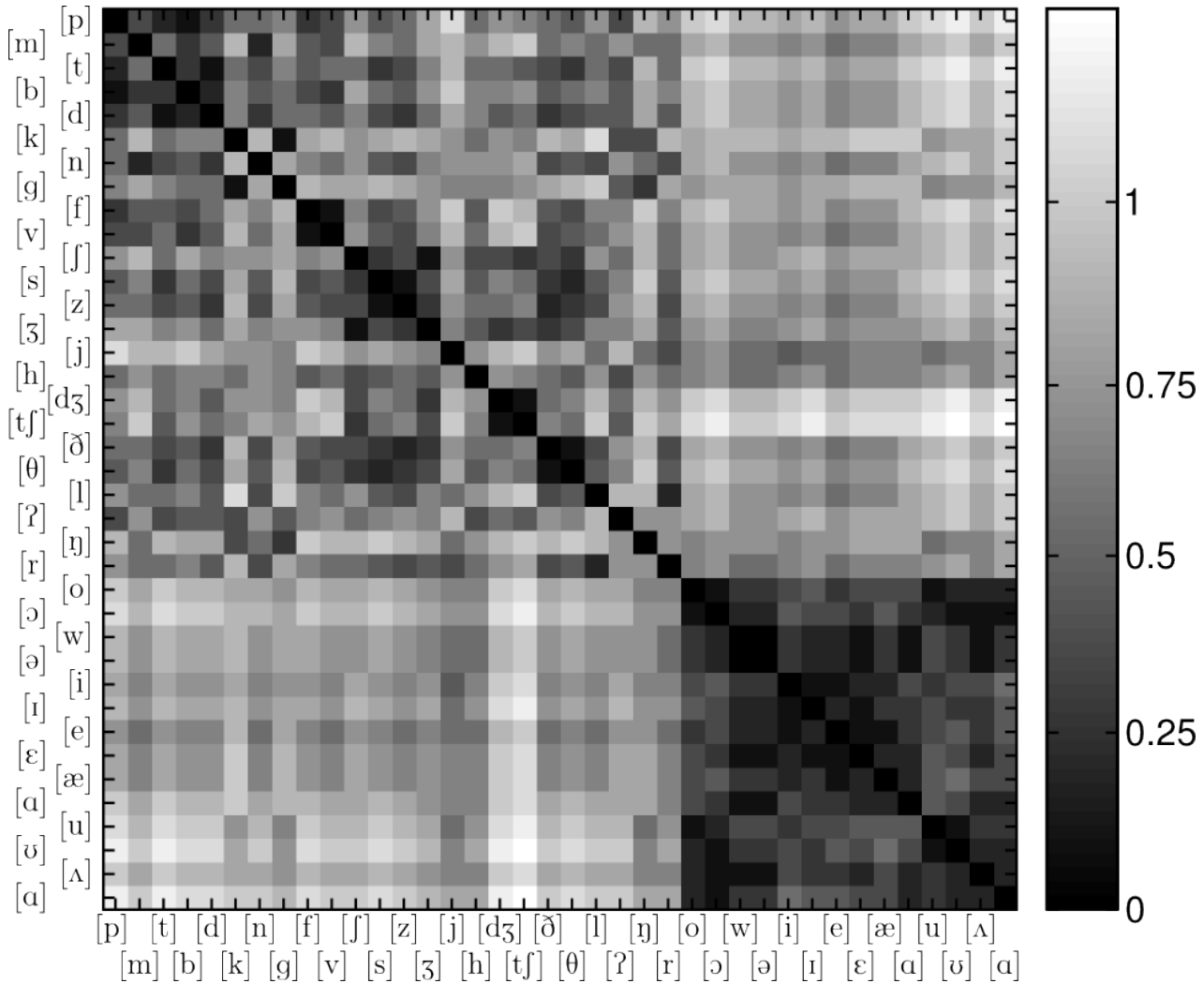
Figure: English Distinctive Features Table.

Distinctive Feature Distance

definition: distance measure between two segments

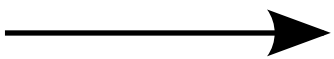
we define a distance measure of two segments as the number of features not shared between them (that dissimilarity definition resembles the natural class definition of Flemming (2005))

Dissimilarity Matrix



Multidimensional Scaling (MDS)

	cph	aar	ode	aal
cph	0	93	82	133
aar	93	0	52	60
ode	82	52	0	111
aal	133	60	111	0



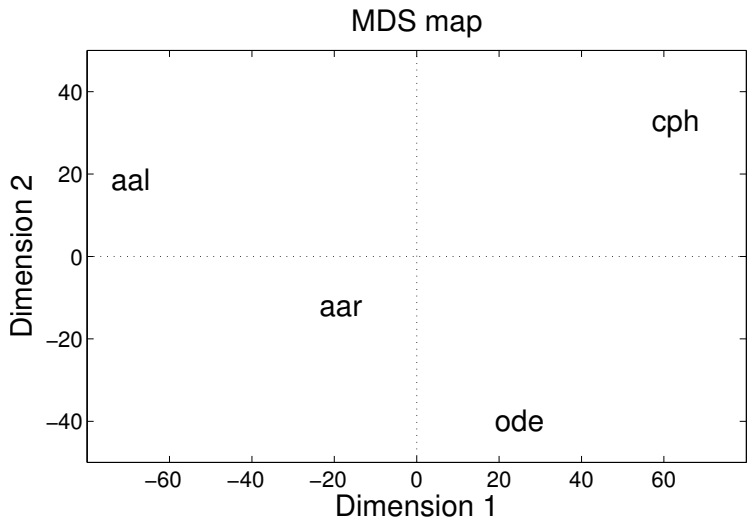


Figure: Simple Multidimensional Scaling Example.

MDS - Vowels

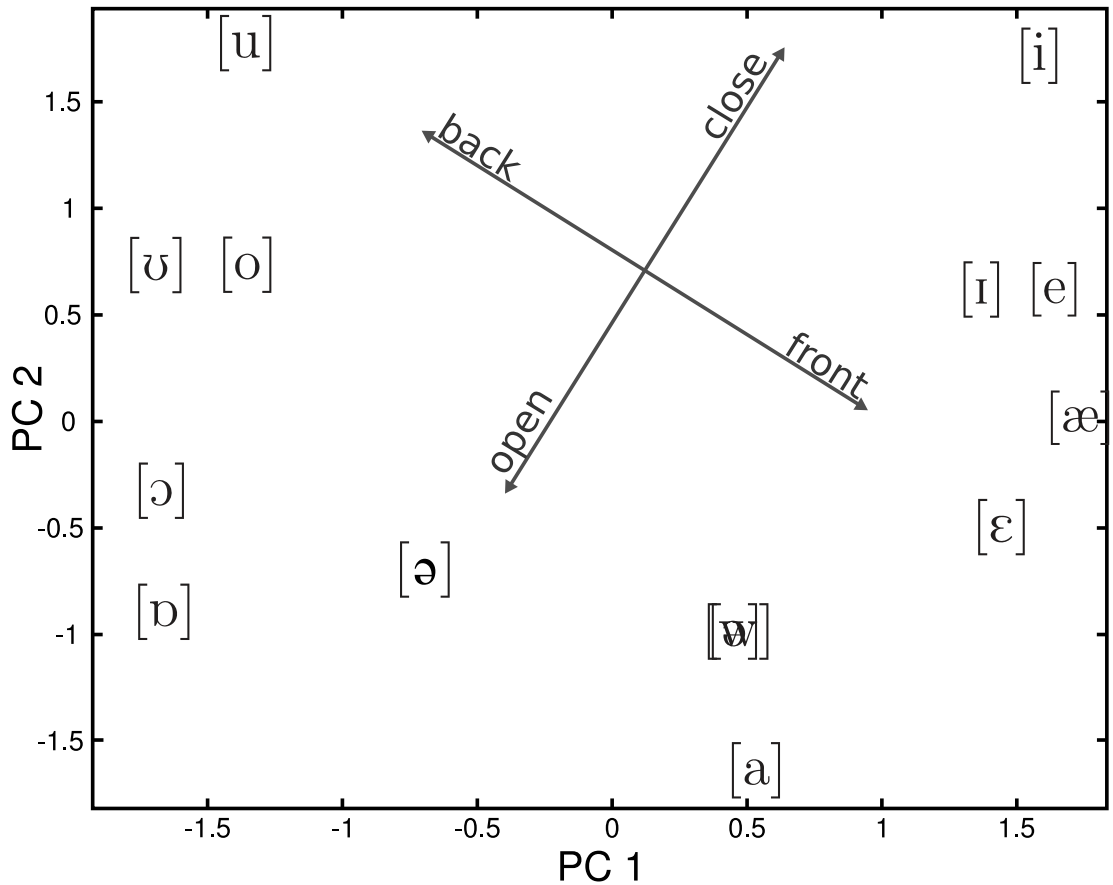


Figure: MDS for English vowels. The first two PCs account for 63% of variance.

MDS - Consonants

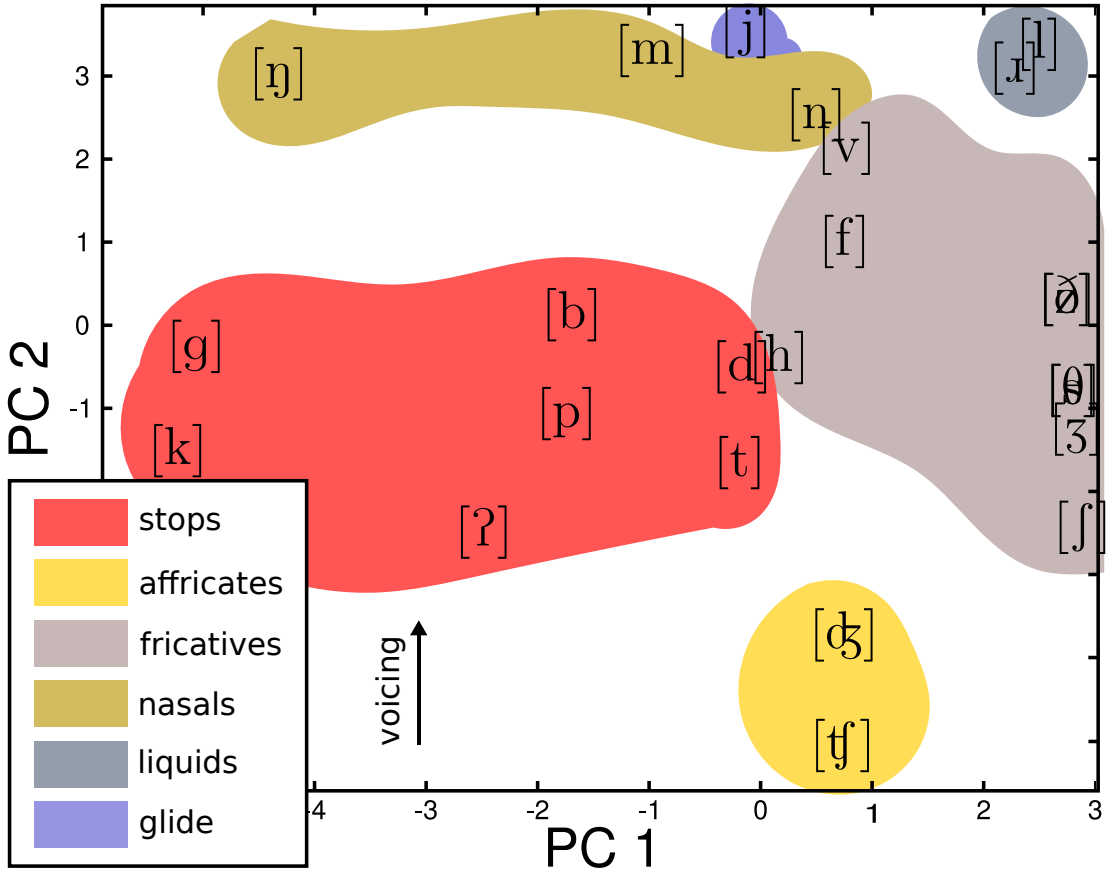


Figure: MDS for English consonants. The first two PCs account for 46% of variance.

Intradistances - triphones

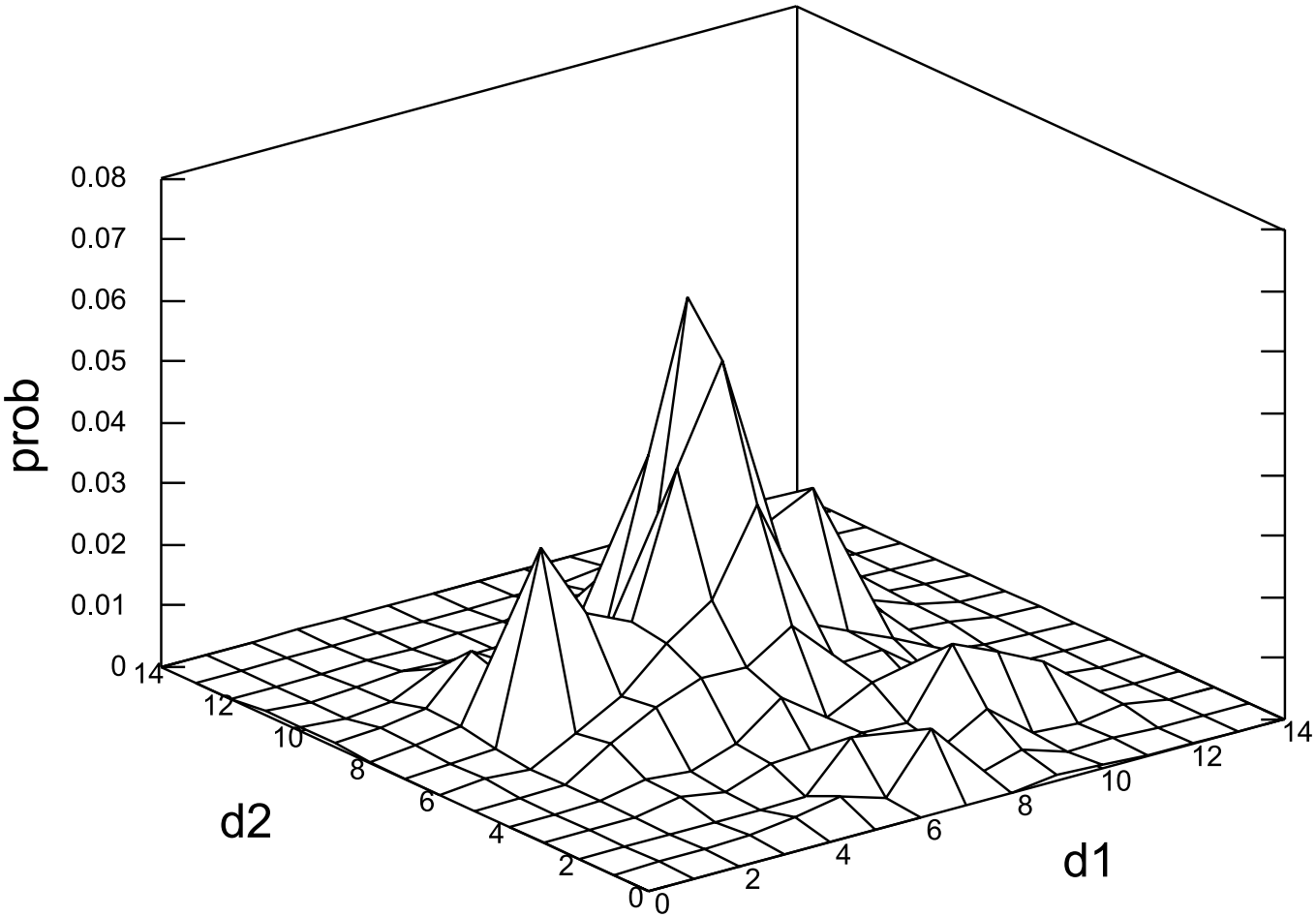


Figure: Probability of triphones given its intradistances.

Frequency of occurrence of words in logarithmic scale

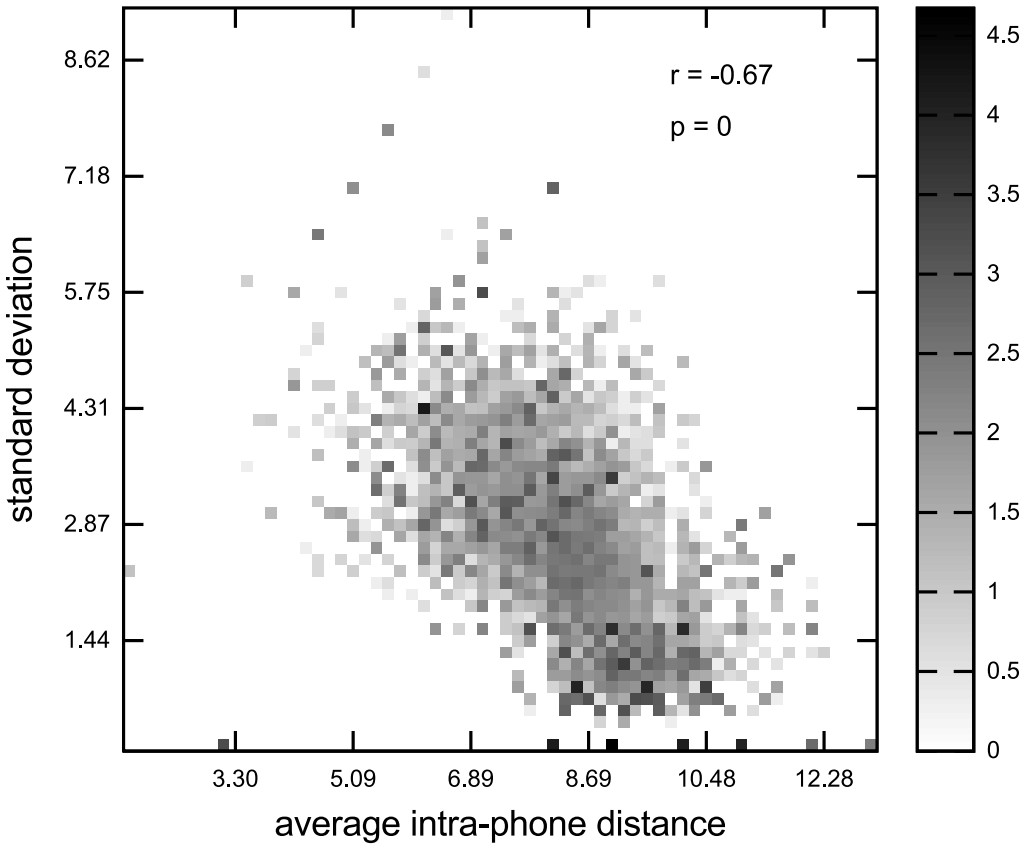


Figure: Relation in words between frequency of occurrence, average intra-phone distances and standard deviation of these distances.

Words Frequency and Zipf Law

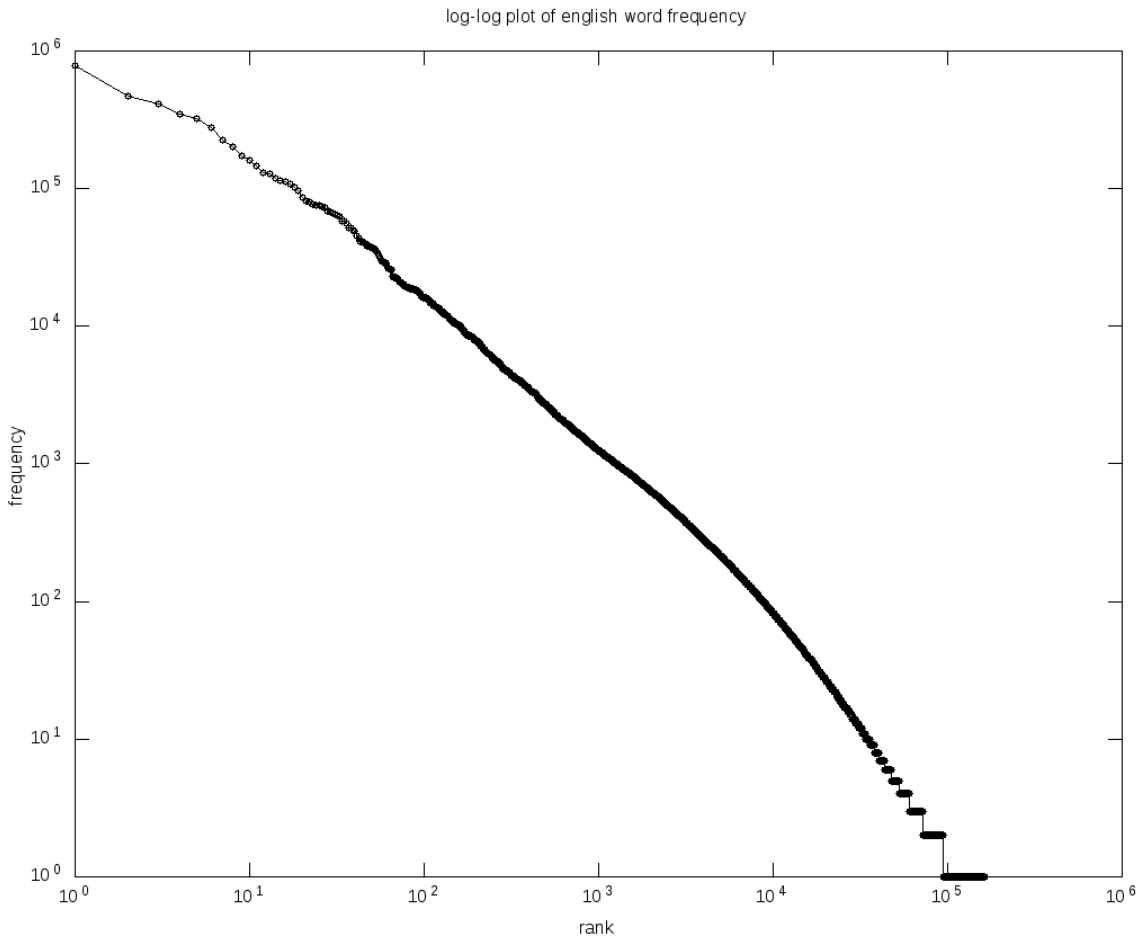


Figure: Log-log plot of words rank versus frequency of occurrence.

Zipf's Law

Zipf law states that there is a relationship between the word's frequency of appearance in texts and its rank, the product of them is roughly a constant.
Power law relation:

$$f(k; s, N) = Ck^{-s} = \frac{k^{-s}}{\sum_{n=1}^{N} n^{-s}} \tag{1}$$

- f frequency
- k rank
- N number of elements in the set
- s characterizing exponent

Zipf's Law

The normalizing constant C in Zipf's law might also be written as

$$C = \frac{1}{H_{N,s}} \tag{2}$$

where $H_{N,s}$ is known as the generalized harmonic number

$$H_{N,s} = \sum_{n=1}^{N} \frac{1}{n^s} \ . \tag{3}$$

(as $N \to \infty$, converges for $s > 1$)

Zipf's exponent

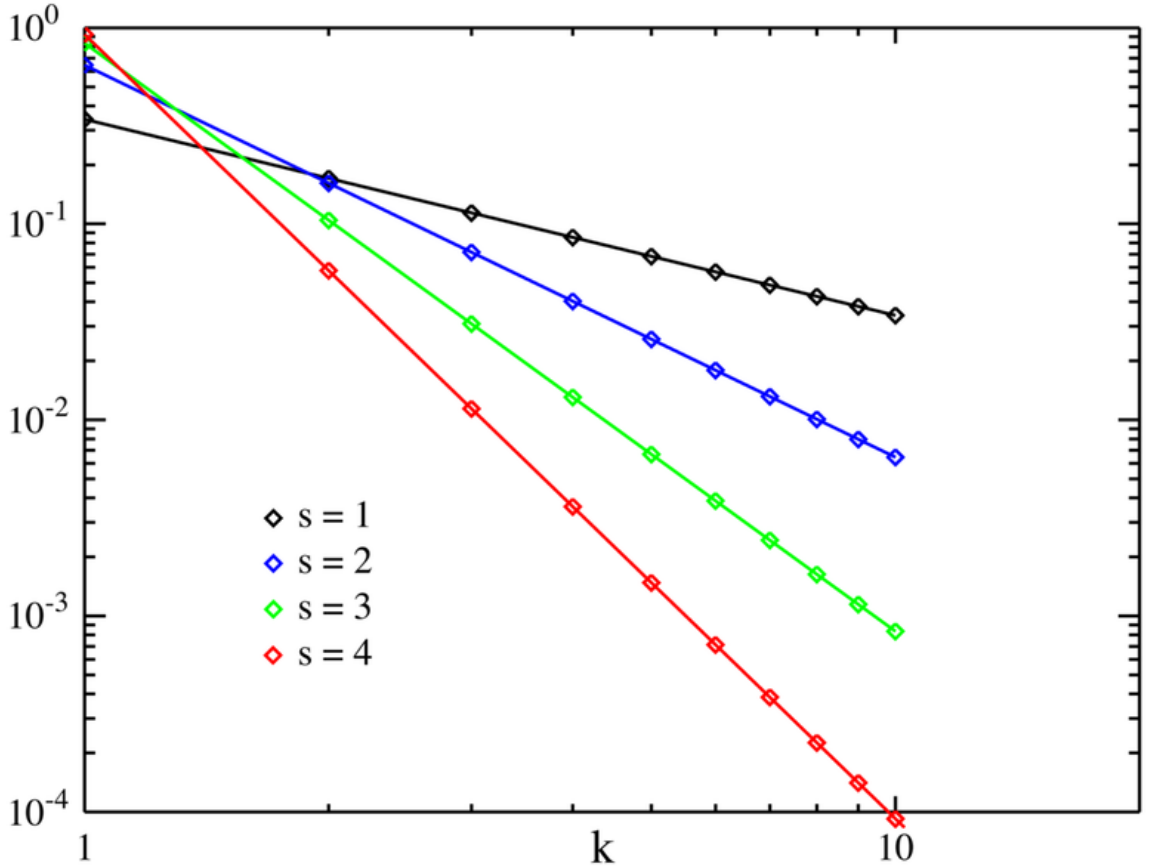


Figure: Probability mass function with $N = 10$. (Wikipedia)

The ubiquity of Zipf's law

This seemingly universal law is present in various phenomena:

(1) earthquakes
(2) avalanches
(3) populations
(4) firms
(5) stocks
(6) game of life
(7) lifespan of genera
(8) etc

Gutenberg-Richter law for earthquakes

Relation between magnitude m of a earthquake and the number N of earthquakes with magnitude greater or equal to m:

$$\log(N) = -bm + a \ . \quad (4)$$

$$m = \log_{10}\left(\frac{A}{A_0(\delta)}\right) \quad (5)$$

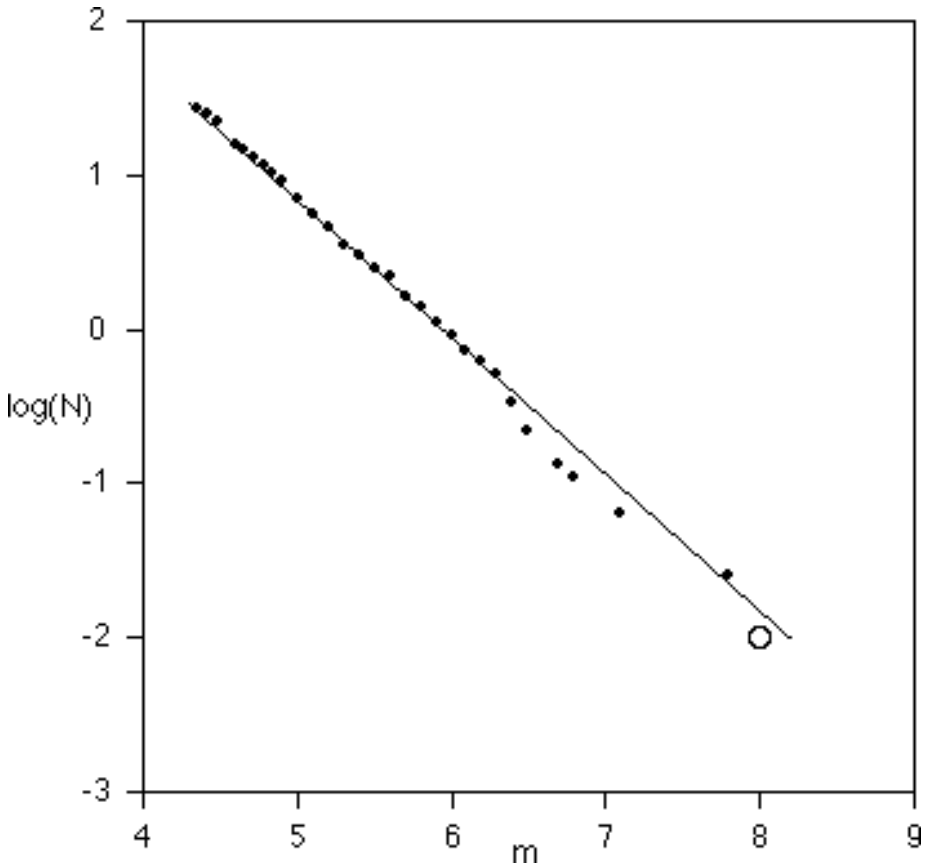


Figure: Data from the South of California from 1932 to 1972 (Turcotte, 1989).

Sandpile

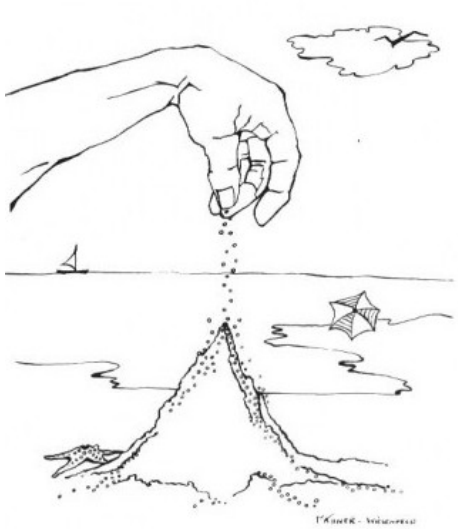


Figure: Sandpile avalanches (Bak, 1999).

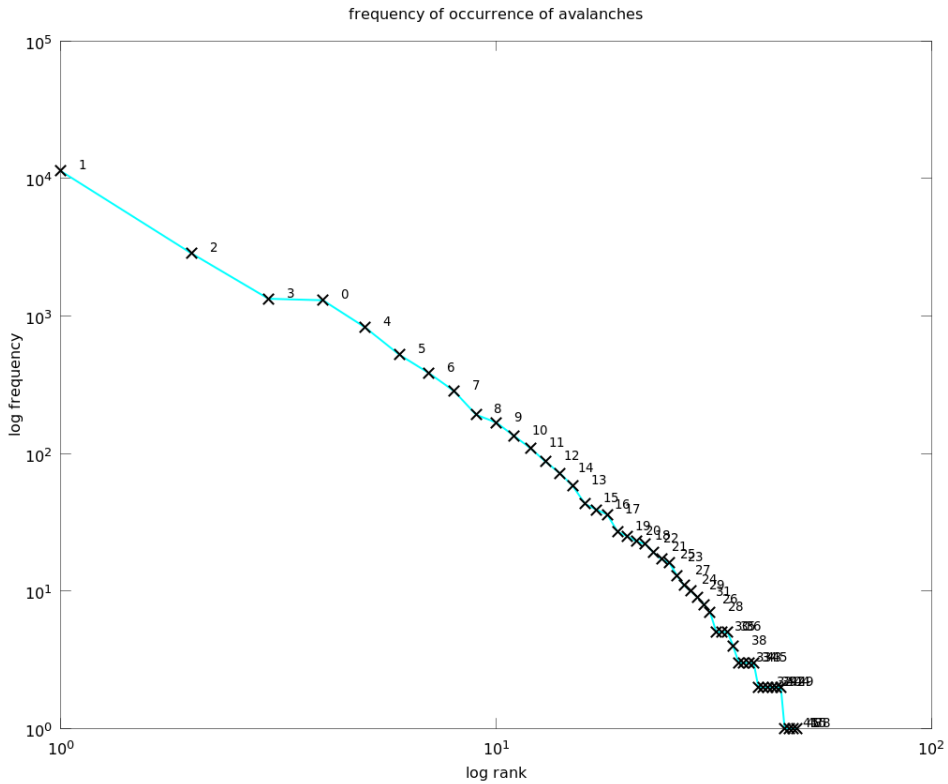
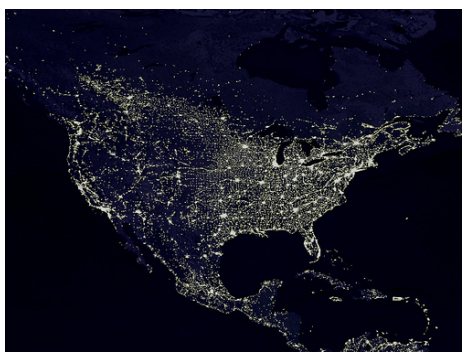


Figure: Frequency of avalanches.

Population



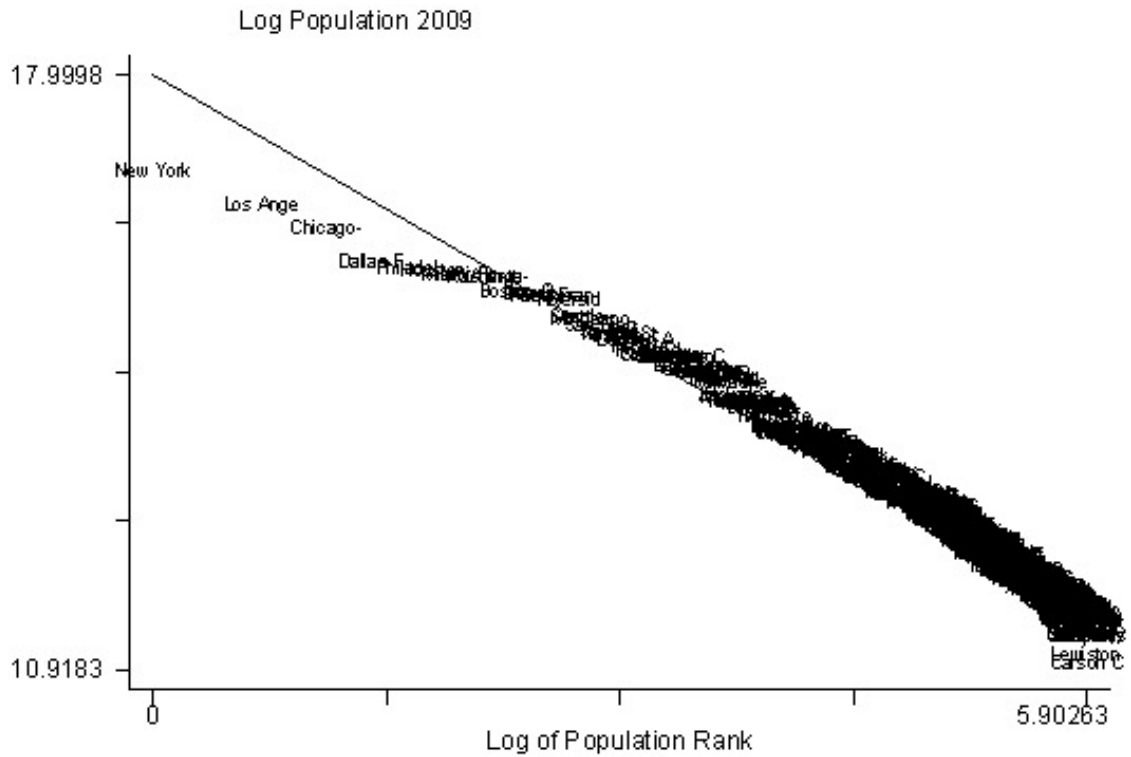
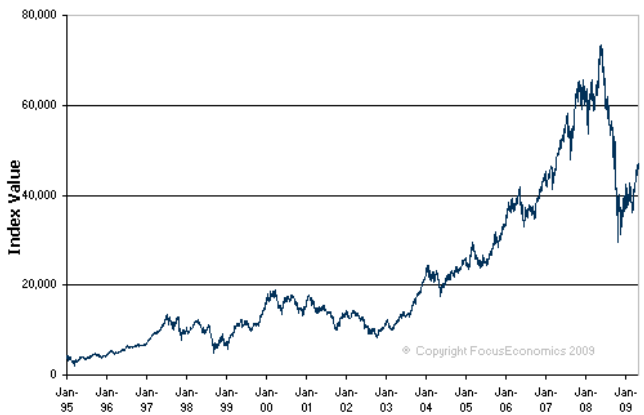


Figure: Population of American metropolitan areas (Edward L. Glaeser).

Stocks



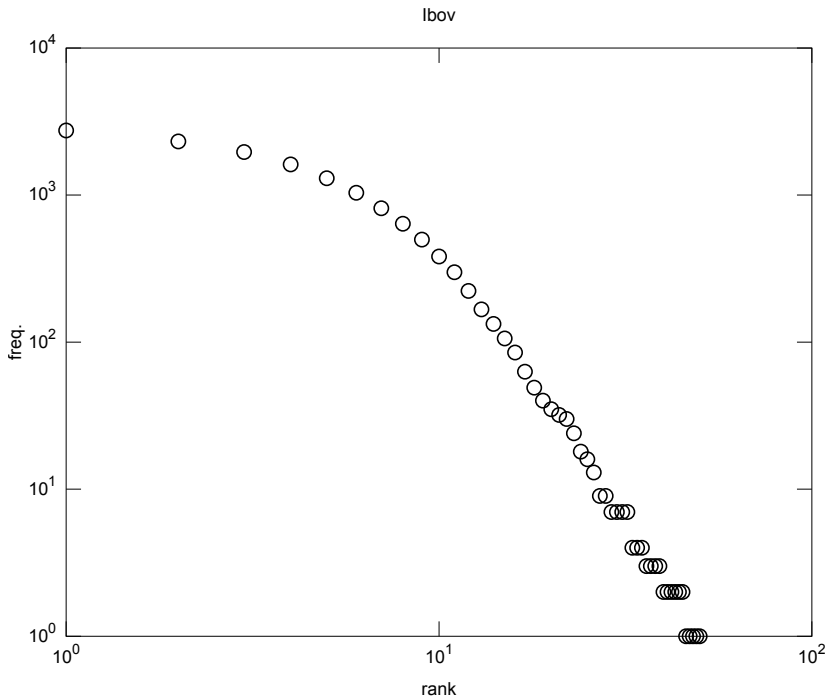
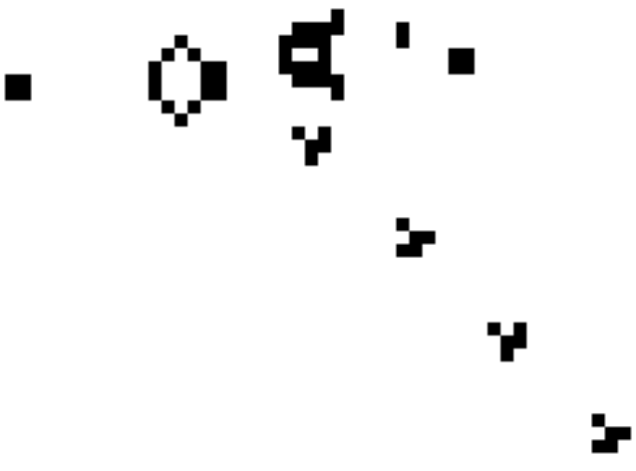


Figure: Variations on the Ibov index, using the idea presented by Mandelbrot (1963).

Game of Life

- cellular automata (Stanisław Ulam, John von Neumann, 1940)



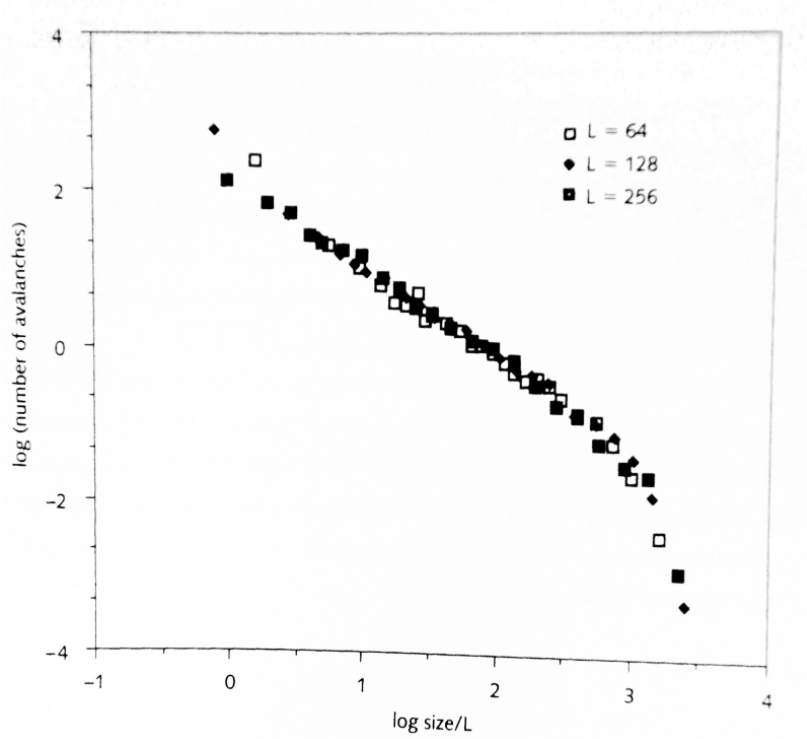


Figure: Avalanches (number of births and deaths until a static configuration is reached) in the game of life (Maslov et al., 1994).

Lifespans of Genera

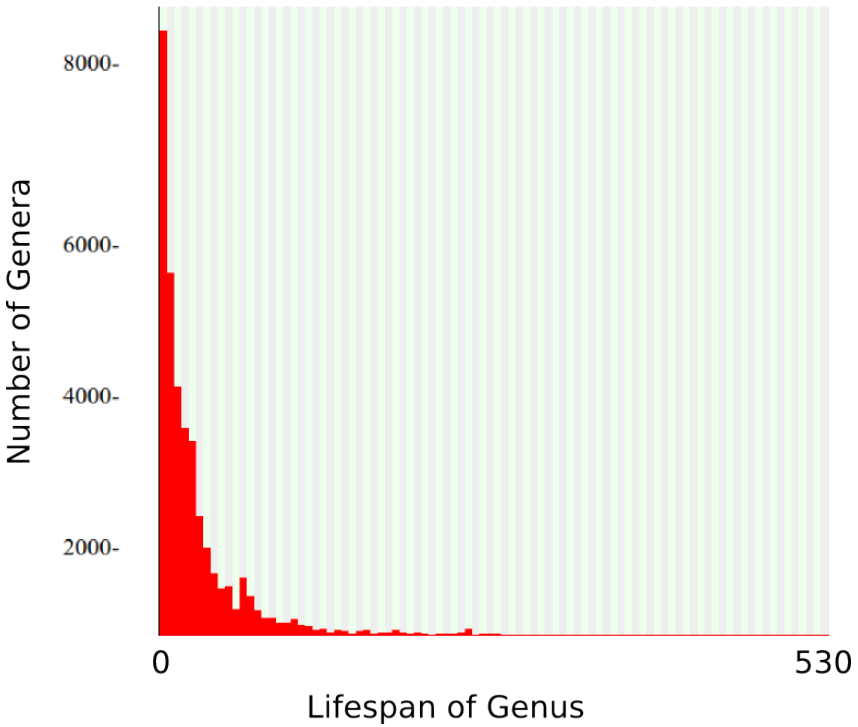


Figure: Distribution of length of lifespans of genera (in millions of years).

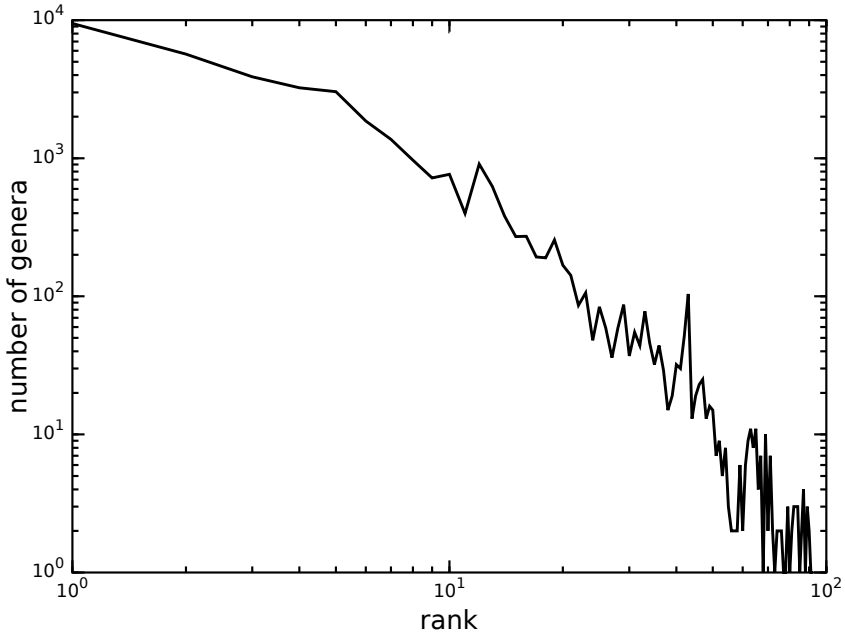


Figure: Loglog plot of the lifespans (data from Sepkoski Fossil Marine Genera Data Warehouse).

Random Texts

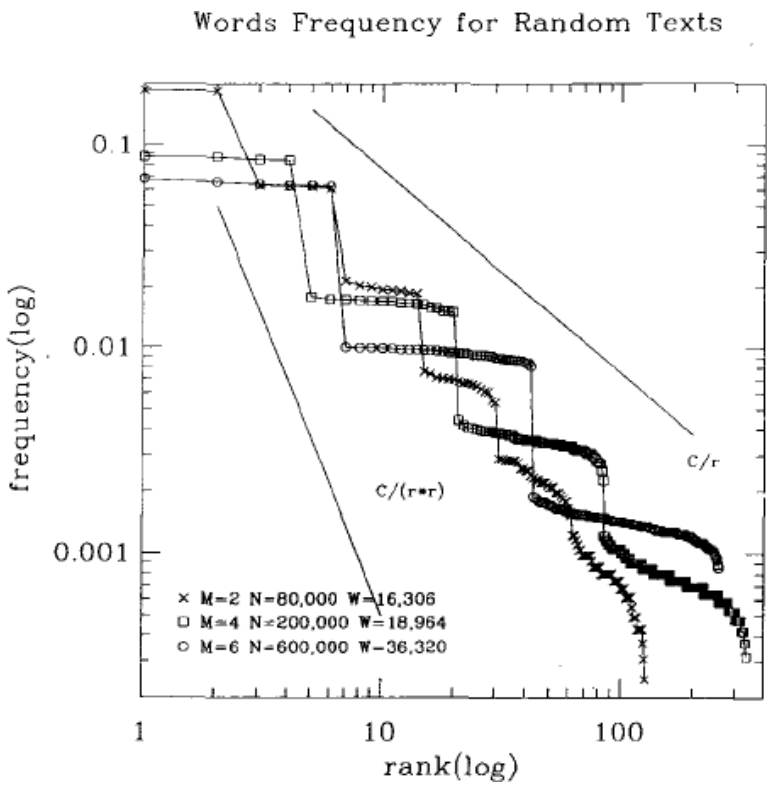


Figure: Word frequency vs *rank* for random generated symbols equally distributed. Symbol set size: $M = 2$, 4, and 6. Reference Zipf's law for $s = 1$ and 2 are displayed. (Li, 1992; Miller, 1957)

Random Text vs Ulysses

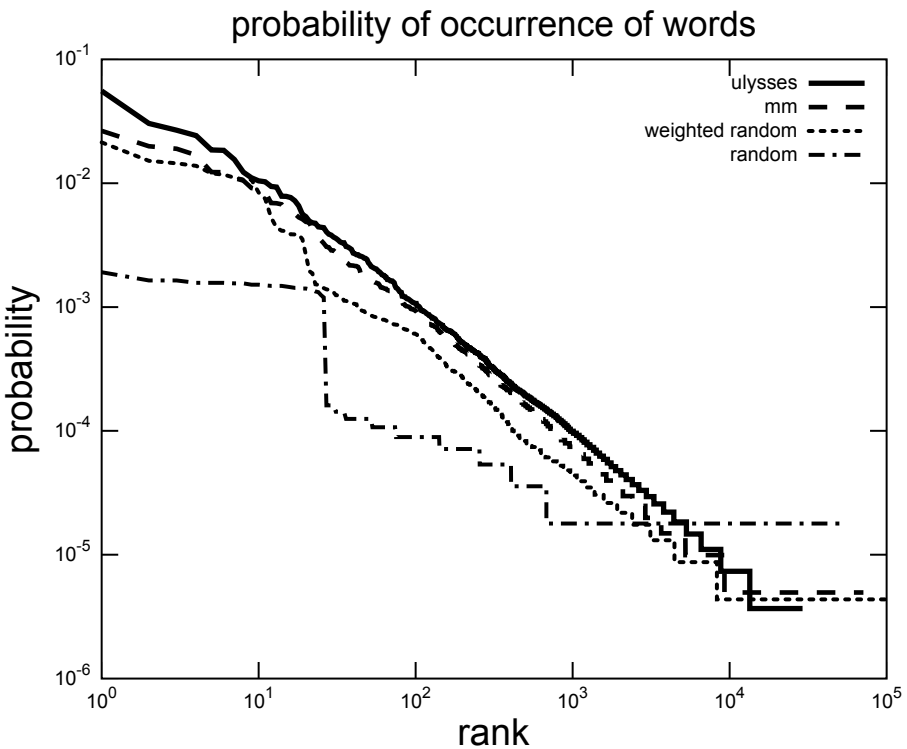


Figure: Random text and *Ulysses* compared.

Random Texts vs Ulysses: word length

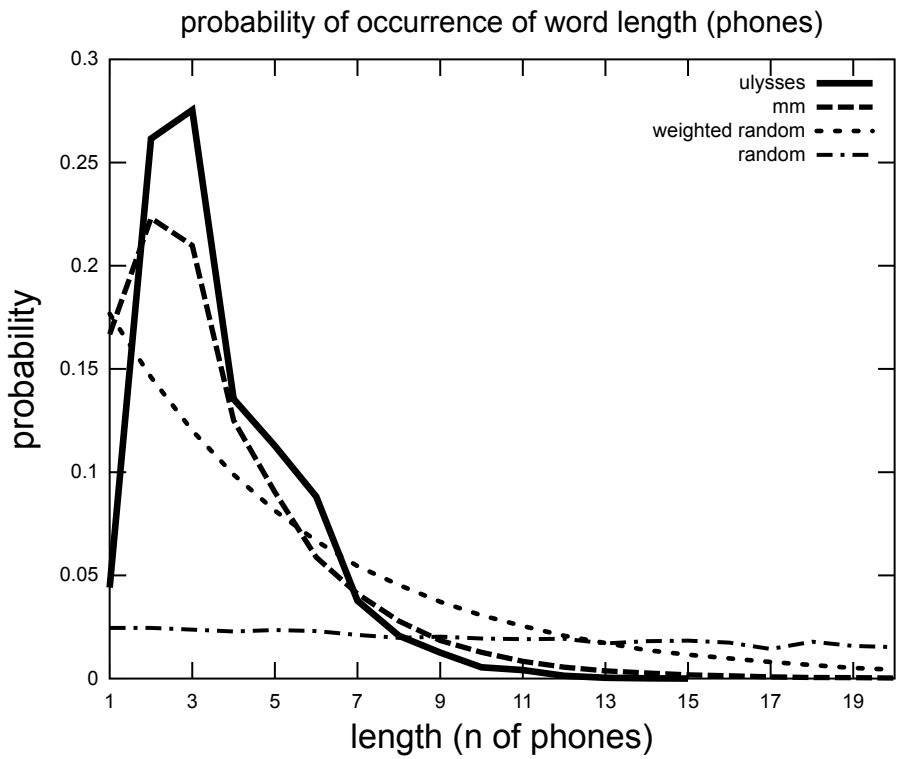


Figure: Word probability for a given length.

Markov Processes and Zipf's law

Kanter and Kessler (1995) shows that a 2-parameter random Markov process constructed with N states and biased random transitions gives rise to a stationary distribution where the probabilities of occurrence of the states exhibit a rank-ordered frequencies of occurrence of words given by Zipf's law.

Random Text Model - Biemann

Biemann (2007) proposes a model of Random Text generation that takes the properties of neighboring co-occurrence into account and introduces the notion of sentences in random text.

It is basically composed of: a word generator that produces random words composed of letters and a sentence generator that composes random sentences of words.

Random Text Model - Biemann

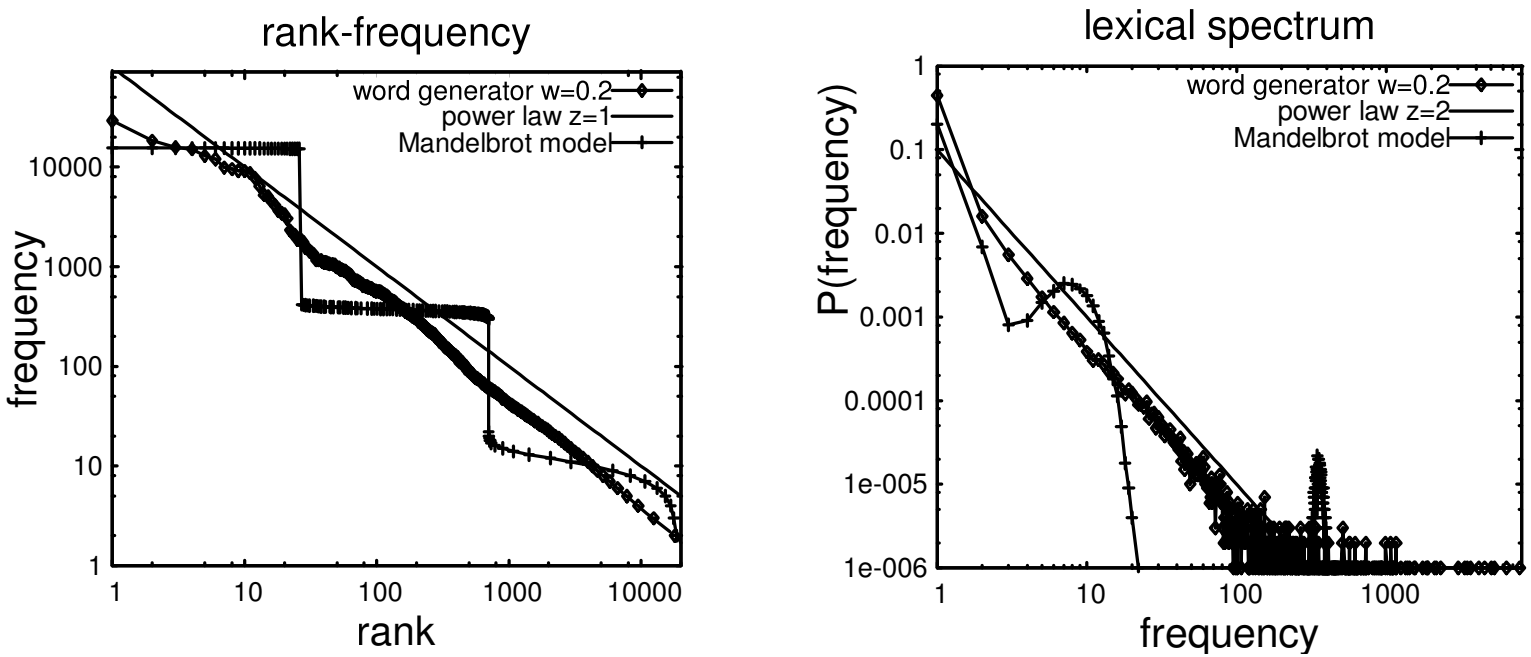


Figure: Rank-frequency distribution and lexical spectrum for the word generator in comparison to the Mandelbrot model (Biemann, 2007).

Zipf Exponent

The Zipf's exponent characterizes the source's distribution.

- natural phenomena $1 \leq s \leq 2$ (Baek et al., 2011)
- natural languages $s \approx 1$ (Piotrovskii et al., 1994)
- children's speech $s \approx 1.66$ (Piotrovskii et al., 1994)
- military combat text $s \approx 1.42$ (Kolguškin, 1960)
- adult and infant dolphins $s \approx 1.1$ and $s \approx 0.87$, respectively (McCowan et al., 1999)
- authorship attribution (Havlin, 1995)
- noncoding DNA sequence $s \approx 0.36$ and coding DNA sequence $s \approx 0.20$ (Mantegna et al., 1994)

Zipf Fit

Probability density function for a Zipf distribution:

$$f(k|s,N) = \frac{1/k^s}{\sum_{n=1}^{N} n^{-s}} = \frac{1/k^s}{H_{N,s}}, \tag{6}$$

Maximum likelihood estimation (MLE).

$$\begin{aligned} L(s|k_1,\cdots,k_M,N) &= f(k=(k_1,k_2,\cdots,k_M)|s,N) \\ &= \prod_{m=1}^{M} f(k_m|s,N) \\ &= \left(\frac{1}{H_{N,s}}\right)^M \prod_{m=1}^{M} \frac{1}{k_m^s}. \end{aligned} \tag{7}$$

Zipf Fit

The logarithm of the likelihood function:

$$\ln L(s|k_1, \cdots, k_M, N) = -M \ln H_{N,s} - s \sum_{m=1}^{M} \ln k_m. \qquad (8)$$

Solution: the parameter s such that

$$\frac{\partial \ln L(s|k, N)}{\partial s} = 0 \ , \qquad (9)$$

if

$$\frac{\partial^2 \ln L(s|k, N)}{\partial s^2} < 0 \ . \qquad (10)$$

Zipf Fit

We need to solve

$$\frac{\partial \ln L(s|k_1, \cdots, k_M, N)}{\partial s} = -M\frac{G_{N,s}}{H_{N,s}} - \sum_{m=1}^{M} \ln k_m = 0 \ , \qquad (11)$$

given that

$$\frac{\partial^2}{\partial s^2} \ln L(s|k_1, \cdots, k_M, N) = M\frac{G_{N,s}^2 - I_{N,s}H_{N,s}}{H_{N,s}^2} < 0 \ . \qquad (12)$$

where

$$G_{N,s} = -\sum_{n=1}^{N} n^{-s} \ln n \ , \qquad (13)$$

$$I_{N,s} = \sum_{n=1}^{N} n^{-s} \ln^2 n \ . \qquad (14)$$

Zipf Fit

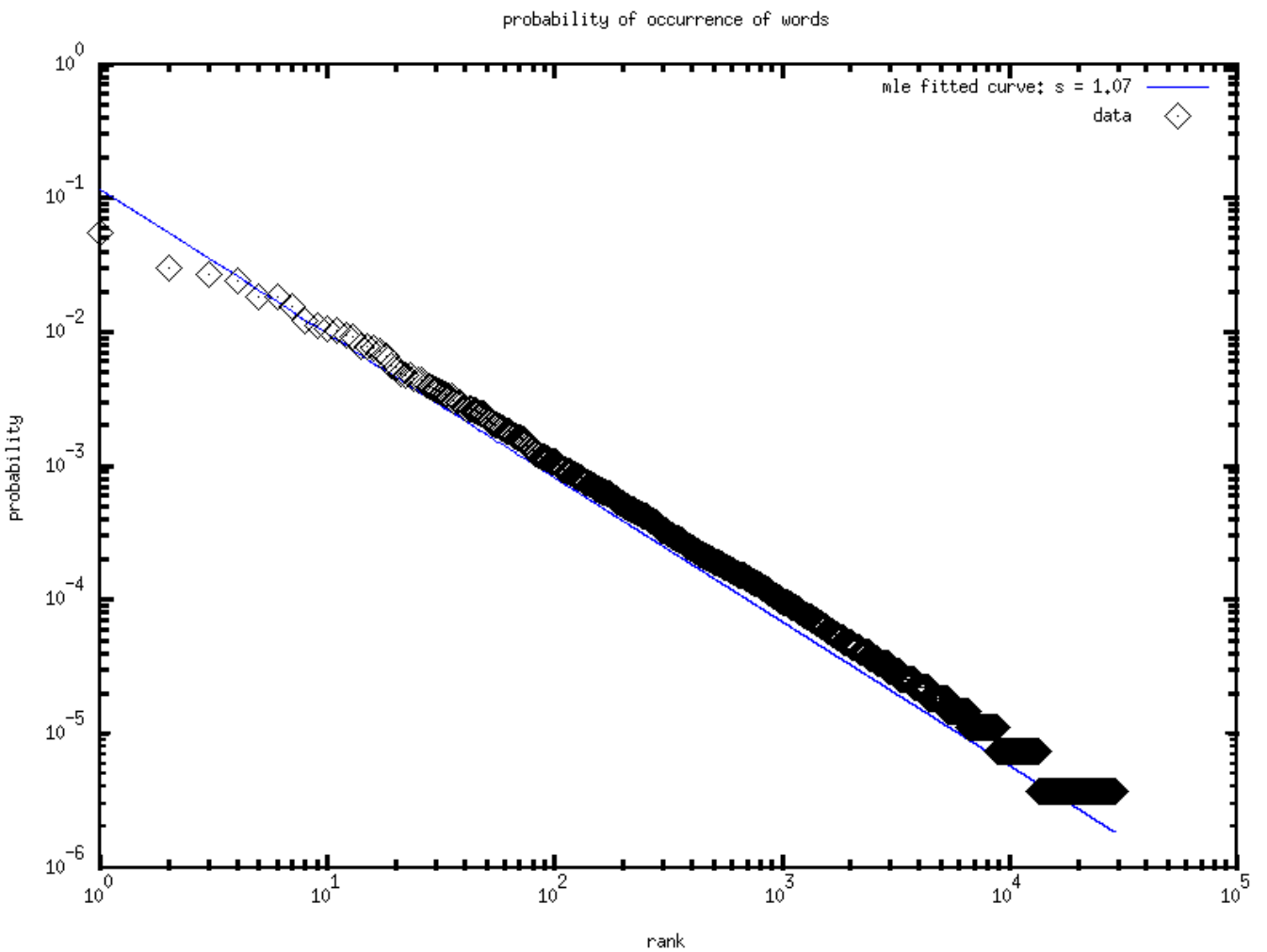


Figure: Zipf fit to *Ulysses* words ($s_{mle} = 1.0738$).

Zipf-Mandelbrot Fit

The Zipf-Mandelbrot distribution is given by

$$f(k|s,q,N) = \frac{1/(k+q)^s}{\sum_{n=1}^{N}(n+q)^{-s}} = \frac{1/(k+q)^s}{H_{N,s,q}} \ . \qquad (15)$$

The logarithm of the likelihood is

$$\ln L(s|k_1,\cdots,k_M,N) = -M \ln H_{N,s,q} - s \sum_{m=1}^{M} \ln(k_m+q) \ . \qquad (16)$$

Zipf-Mandelbrot Fit

Now we need to solve:

$$\frac{\partial \ln L(s|k_1,\cdots,k_M,N)}{\partial s} = -M\frac{G_{N,s,q}}{H_{N,s,q}} - \sum_{m=1}^{M} \ln(k_m+q) = 0 \ , \quad (17)$$

$$\frac{\partial^2 \ln L(s|k_1,\cdots,k_M,N)}{\partial s^2} = M\frac{G^2_{N,s,q} - I_{N,s,q}H_{N,s,q}}{H^2_{N,s,q}} < 0 \ . \quad (18)$$

$$\frac{\partial \ln L(s|k_1,\cdots,k_M,N)}{\partial q} = sM\frac{H_{N,s+1,q}}{H_{N,s,q}} - sH_{N,1,q} = 0 \ , \quad (19)$$

$$\frac{\partial^2 \ln L(s|k_1,\cdots,k_M,N)}{\partial q^2} = s^2 M\frac{H^2_{N,s+1,q}}{H^2_{N,s,q}} - s(s+1)M\frac{H_{N,s+2,q}}{H_{N,s,q}} + sH_{N,2,q} < 0 \ . \quad (20)$$

Zipf-Mandelbrot Fit

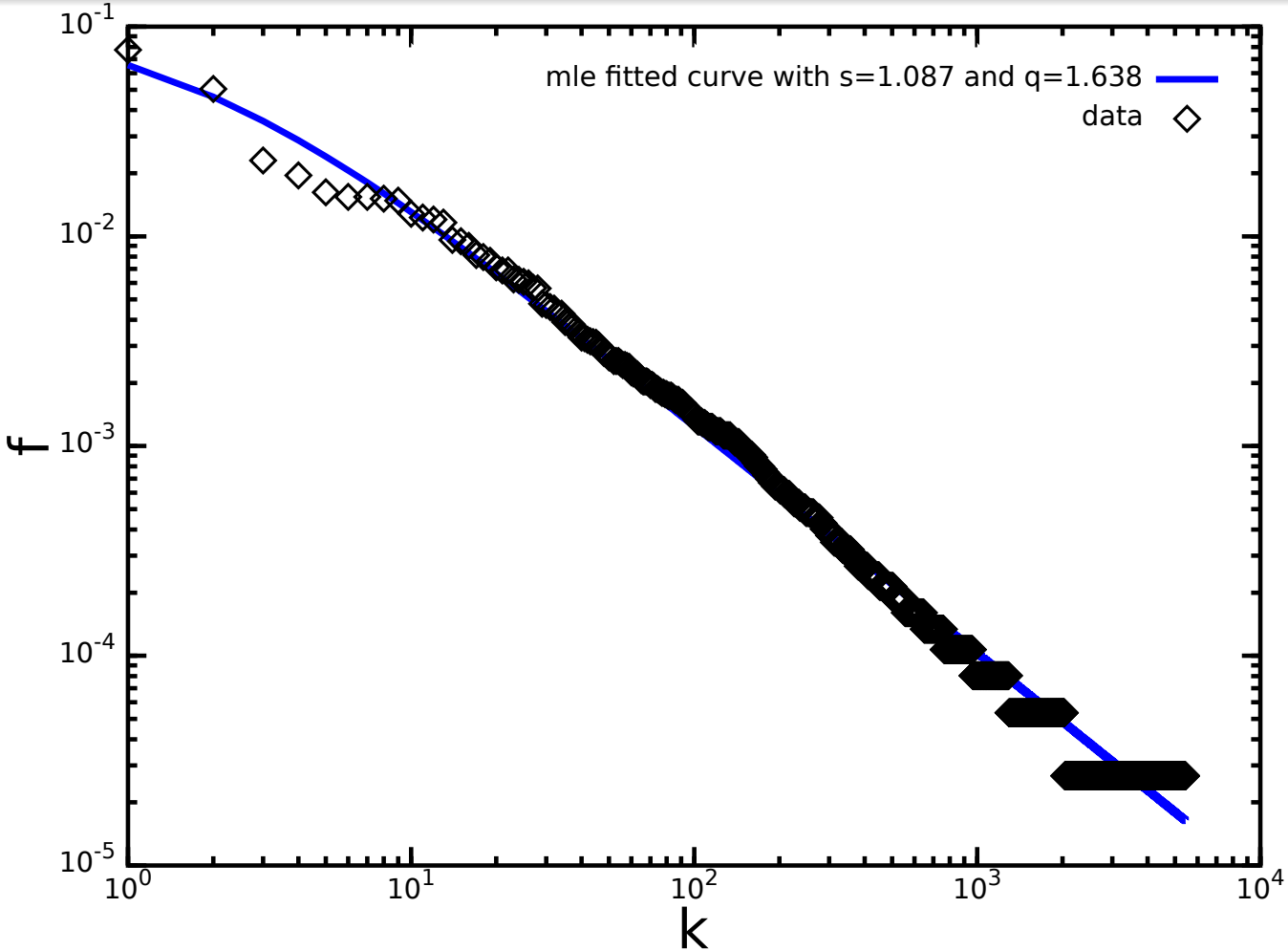


Figure: Zipf-Mandelbrot fit to *Hamlet* words ($s_{mle} = 1.087$ $q_{mle} = 1.638$).

Lexical Spectrum or Inverse Zipf

"Understanding the origins and evolution of language requires an appropriate identification of its universal features. One of the most obvious is the statistical distribution of word abundances. The second form is called the lexical spectrum or the inverse Zipf's distribution" (Ferrer-i-Cancho and Solé, 2002).

Inverse Zipf

The number of words for a given frequency of occurrence

$$N(f) = af^{-\beta} \tag{21}$$

where

$N(f)$ number of words with a given frequency f

f frequency of occurrence

a and β parameters

The exponents are related by

$$\beta = \frac{1}{s} + 1 \ . \tag{22}$$

Inverse Zipf

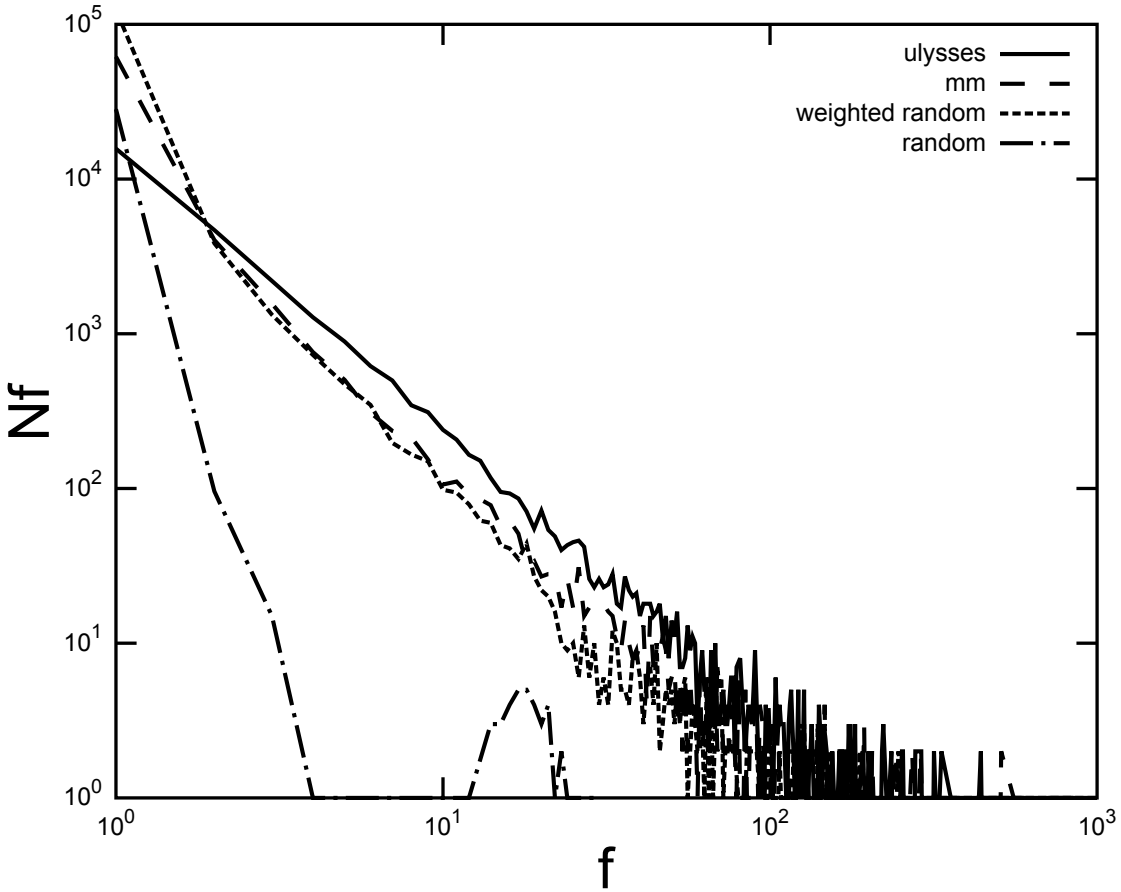


Figure: Inverse Zipf. Natural and Artificial Texts compared.

Smoothing

The maximum likelihood estimator predicts that the probability of a word not seen in the corpus is zero.

Smoothing is used to overcome this problem.

(1) *Add-one* estimator (Laplace, 1902)

(2) Good-Turing estimator (Good, 1953)

(3) Simple Good-Turing estimator (Gale and Sampson, 1995)

Good-Turing

The Good-Turing method states that $N_0 = N_1$ (the total probability of all unseen events is equal to the sum of probabilities of all events that occur only once) and the number of observations are adjusted by

$$f^* = (f + 1)\frac{E[N_{f+1}]}{E[N_f]} \tag{23}$$

$E[\cdot]$ expectation of a random variable

f^* adjusted number of observations

f observed frequency of occurrence

N_f number of types observed f times

Simple Good-Turing

Gale and Sampson (1995) proposes a simple way to do a Good-Turing estimation by choosing $E[\cdot]$ so that

$$E[N_{f+1}] = E[N_f]\left(\frac{f}{f+1}\right)\left(1 - \frac{E[N_1]}{N}\right) \tag{24}$$

leading to

$$p_f^* = p_f\left(1 - \frac{E[N_1]}{N}\right) \ . \tag{25}$$

Simple Good-Turing

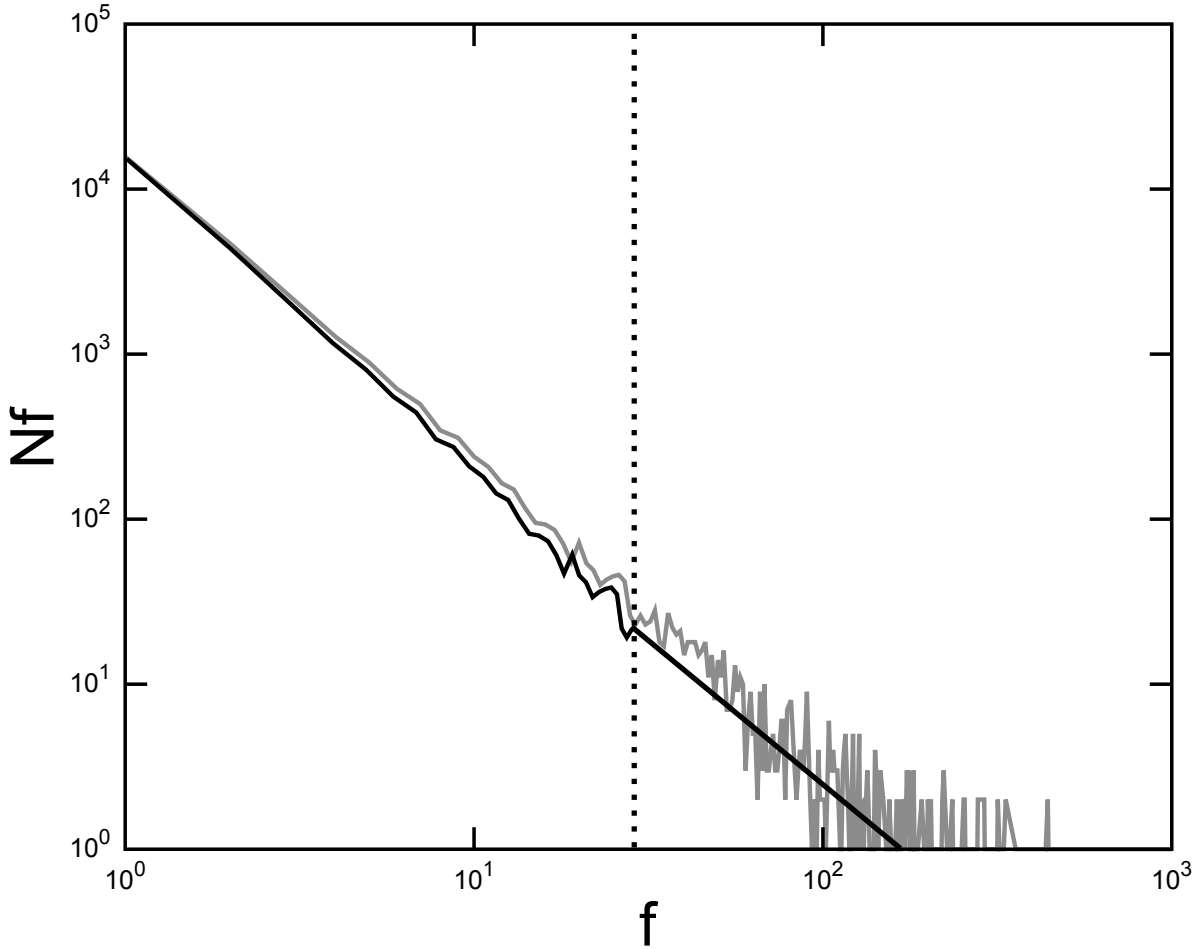


Figure: Simple Good-Turing.

How Zipf's law emerges

According to Ferrer-i-Cancho and Solé (2003); Ferrer-i-Cancho (2005a), Zipf's law is a manifestation of a complex system operating between order and disorder.

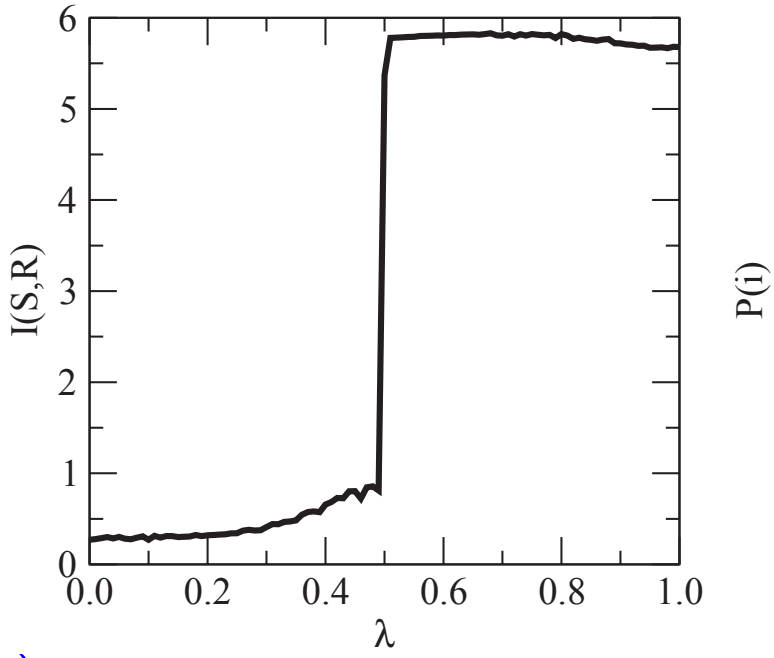
A communication system should minimize Ω

$$\Omega(\lambda) = -\lambda I(S,R) + (1-\lambda)H(S) \ , \tag{26}$$

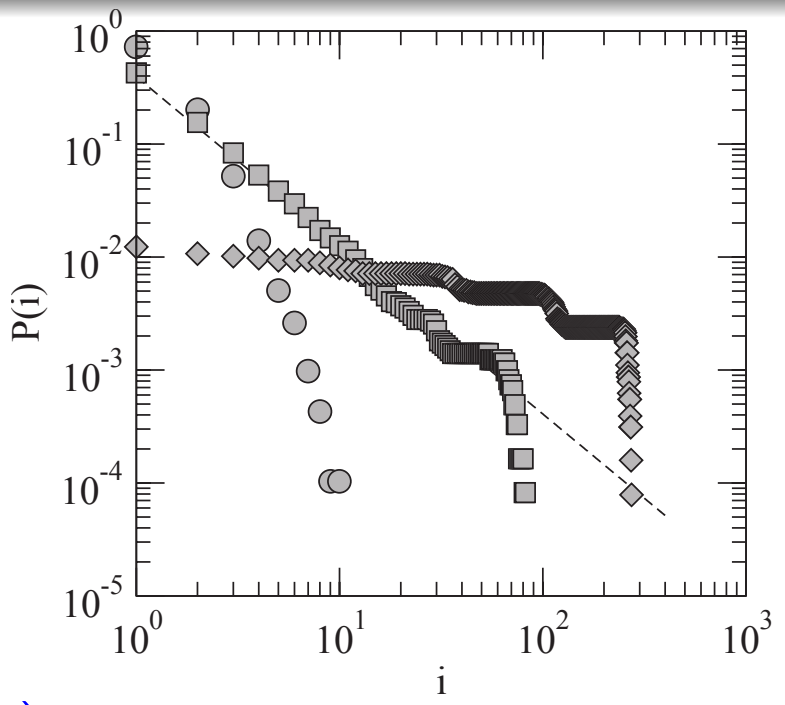
where

- S set of signs (words)
- R set of estimula
- I(S,R) transmitted information
- H(S) cost (entropy of words)

How Zipf's law emerges



(a) $I(S,R)$, the information transfer between words and meanings, versus λ, the parameter regulating the balance between maximizing $I(S,R)$ and minimizing the entropy of words.



(b) $P(i)$, the probability of the i-th most likely word in the system for $\lambda = 0.49$ (circles), $\lambda = 0.498$ (squares) and $\lambda = 0.5$ (diamonds). The dashed line contains the theoretical curve for $\lambda = 0.498$.

Figure: Some computational results on the model where meaning probabilities are governed by the internal structure of the communication system. The size of the system is $n = m = 400$ (i.e. 400 words and meanings). Figure reproduced from Ferrer-i-Cancho (2005b).

Entropy of a Zipfian Source

Entropy of a Zipfian distributed source:

$$\begin{aligned} \overline{H} &= -\frac{1}{\ln 2}\sum_{k=1}^{N} Ck^{-s}\ln(Ck^{-s}) \\ &= \frac{sC}{\ln 2}\sum_{k=1}^{N}\frac{\ln k}{k^{s}} - \frac{\ln C}{\ln 2}\ . \end{aligned} \tag{27}$$

We propose lower and upper bounds to estimate the entropy

$$B_l \leq \sum_{k=1}^{N} k^{-s}\ln k \leq B_u. \tag{28}$$

Entropy of a Zipfian Source

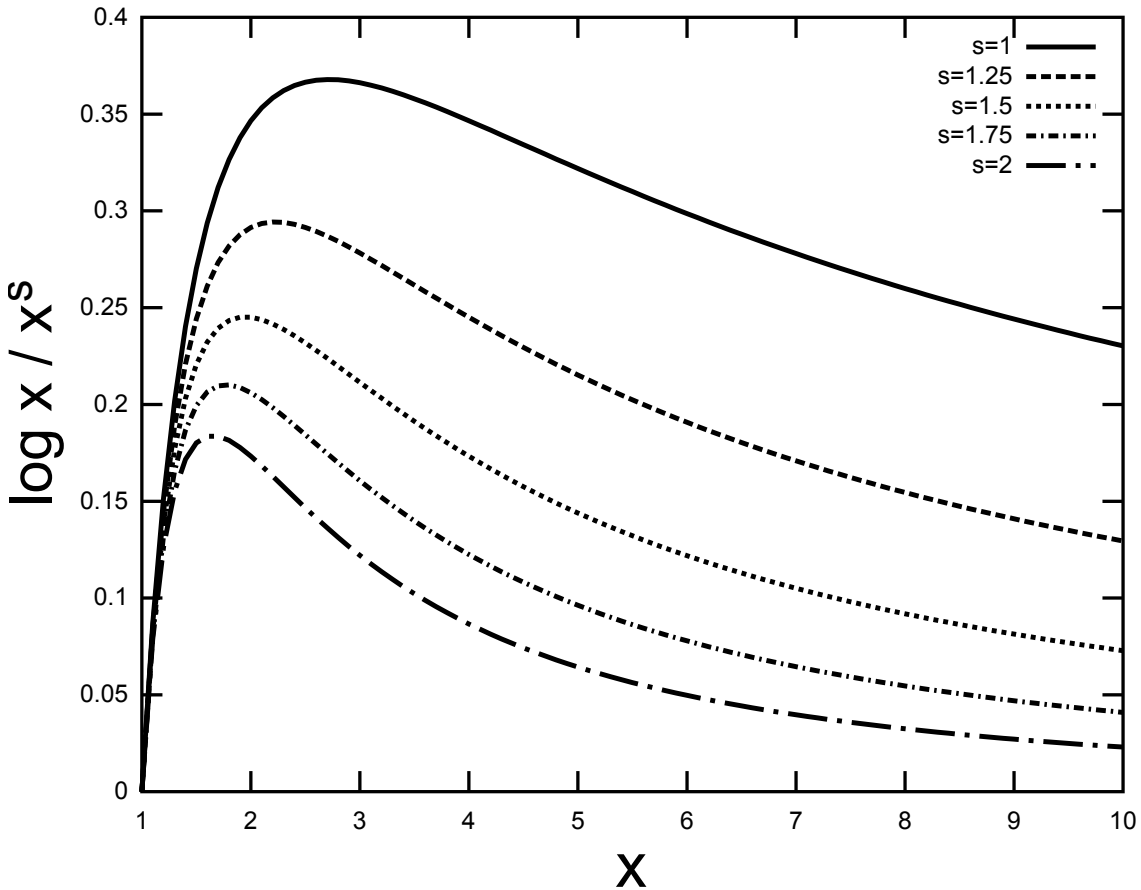


Figure: Function $f(x) = \ln x / x^s$ for different values of s.

Entropy of a Zipfian Source

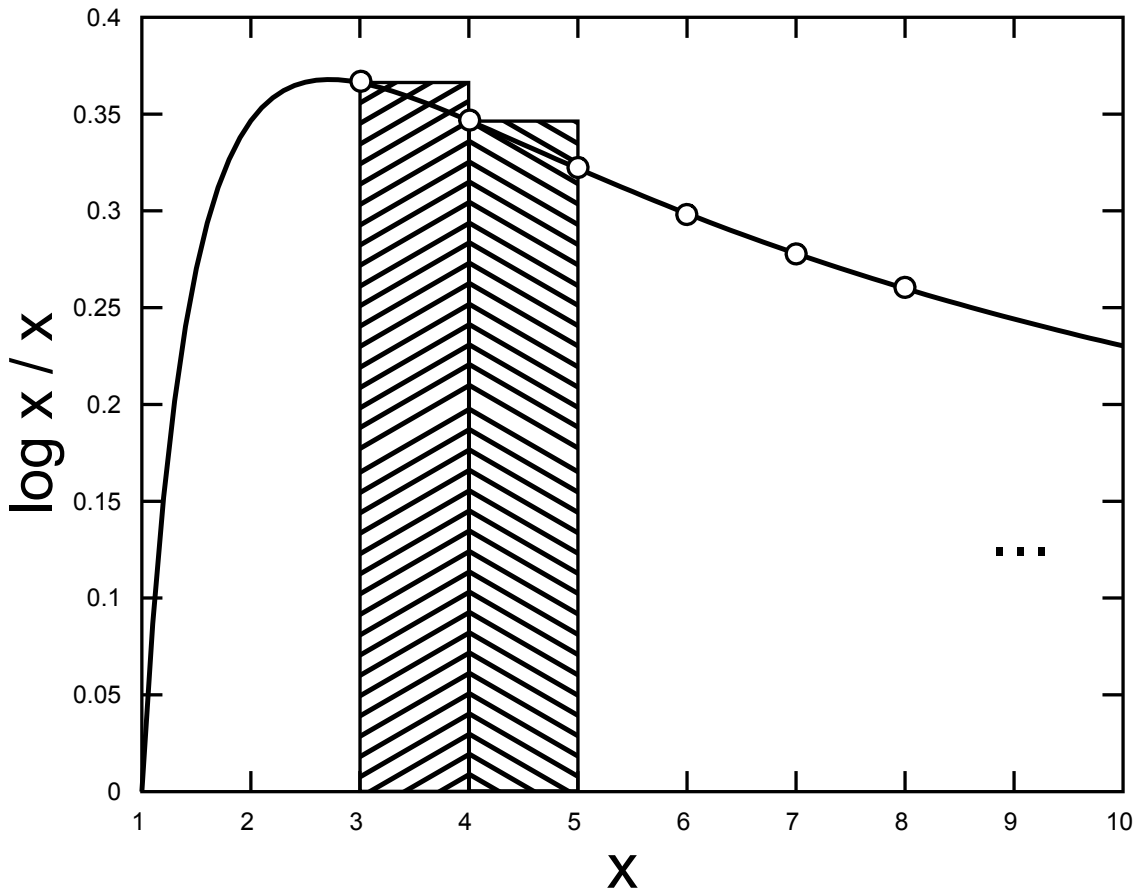


Figure: Left Riemann sum approximation of the integral.

Entropy of a Zipfian Source

The bounds are given by

$$\begin{aligned} B_l &= \int_3^N \frac{\ln x}{x^s} dx + \frac{\ln 2}{2^s} + \frac{\ln N}{N^s} \\ &\leq \sum_{n=1}^{N} \frac{\ln n}{n^s} \\ &\leq \int_3^{N-1} \frac{\ln x}{x^s} dx + \frac{\ln 3}{3^s} + \frac{\ln 2}{2^s} + \frac{\ln N}{N^s} = B_u \ , \end{aligned} \tag{29}$$

and the estimated Entropy is bounded by

$$\frac{sC}{\ln 2} B_l - \frac{\ln C}{\ln 2} = H_l \leq \overline{H} \leq H_u = \frac{sC}{\ln 2} B_u - \frac{\ln C}{\ln 2} \ . \tag{30}$$

Entropy of a Zipfian Source

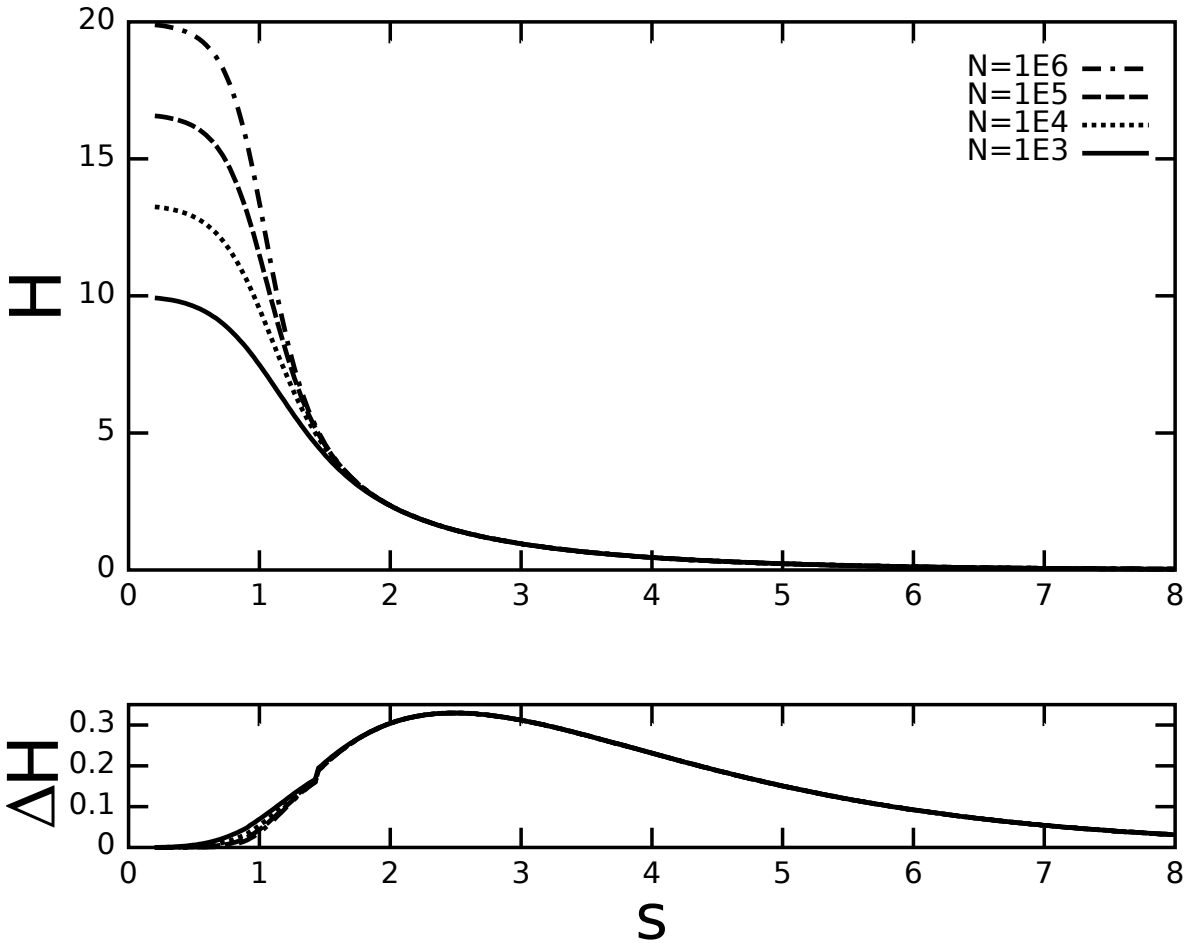


Figure: Entropy H (in bits) as a function of the Zipf exponent s and the number of types N. The upper plot presents the average Entropy estimated and the lower plot presents the difference between the upper and lower bounds of the entropy estimated.

Entropy of a Zipfian Source

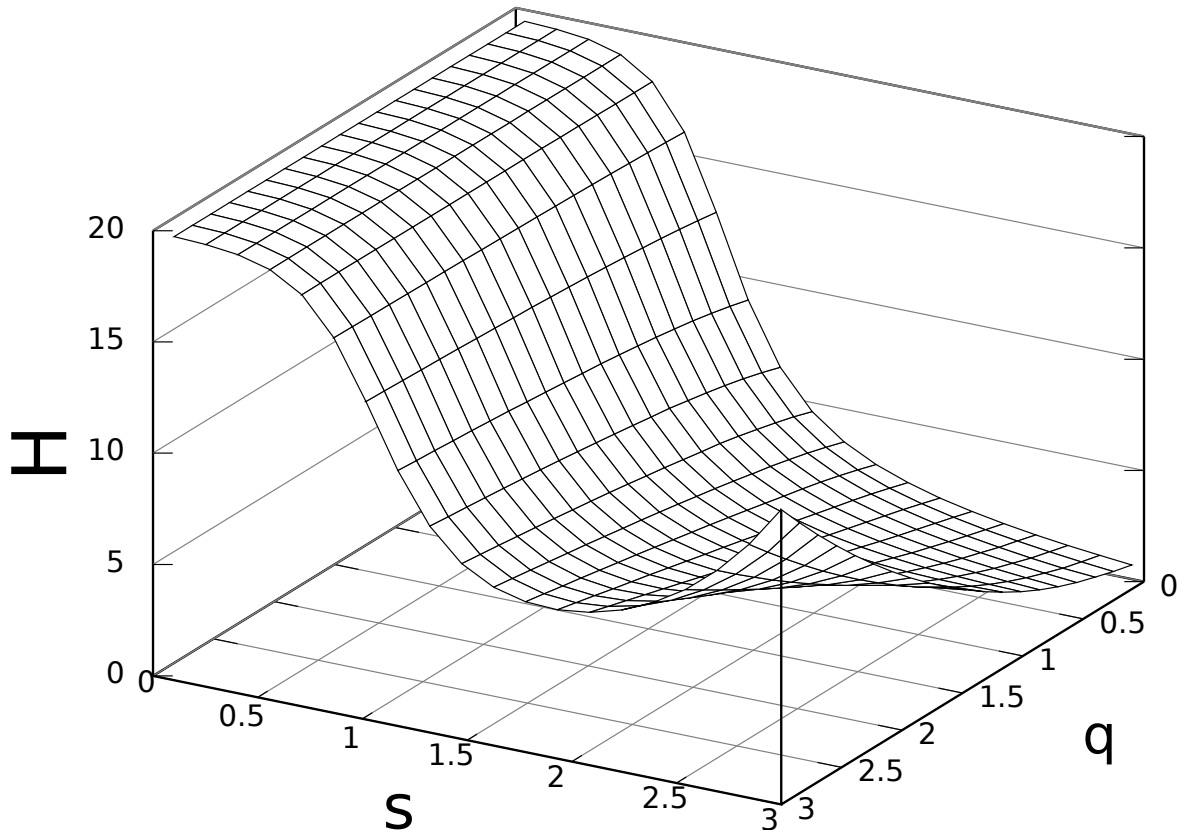


Figure: Effect of the parameters s and q on the entropy H (in bits) for a Zipf-Mandelbrot distribution.

Entropy of a Zipfian Source

Table: Entropy of real texts (bits), with and without SGT smoothing, compared with the estimated entropy (bits) using the parameter N (number of types) found in the text, parameter s (Zipf exponent) found by a Maximum Likelihood Estimation (MLE) and the flattening parameter q, also found by MLE.

source	N	estimated parameters			entropy		estimated entropy	
		Zipf	Zipf-Mandelbrot		normal	sgt	Zipf	Zipf-Mandelbrot
		s	s	q				
Alice	3016	0.992	1.172	3.27	8.49	8.79	8.55	8.73
Hamlet	5447	0.991	1.087	1.64	9.04	9.08	9.09	9.13
Macbeth	4017	0.969	1.009	0.56	9.00	9.00	9.02	9.04
Shakespeare	29847	1.060	1.172	2.33	9.52	9.57	9.60	9.69
Ulysses	34391	1.025	1.085	1.18	10.19	10.25	10.22	10.25

Heaps' law (Herdan's law)

There is a relation between

T text size

V_T vocabulary length

given by

$$V_T \propto T^{\alpha} \ , \quad \alpha < 1 \ . \tag{31}$$

Leijenhorst et al. (2005) shows that a Zipfian distributed source presents this relation between text size and vocabulary length.

Heaps' law (Herdan's law)

The expected number of types is monotonically increasing with the sample size. It converges to the length of the underlying lexicon.

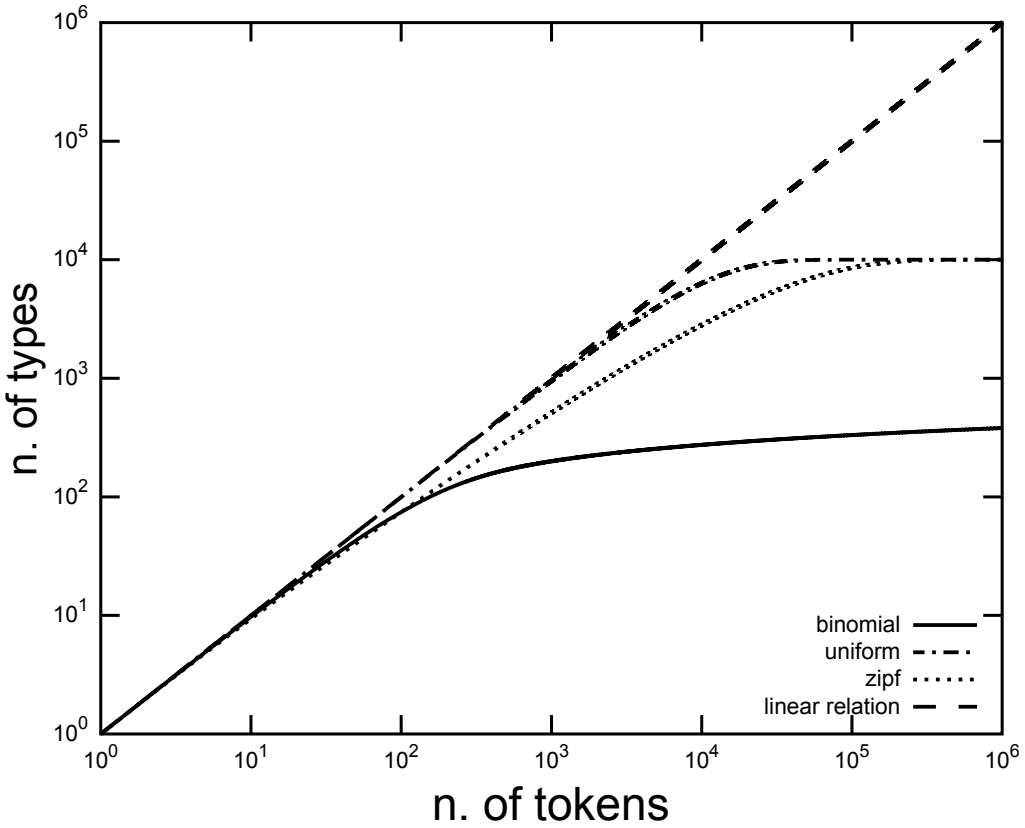


Figure: The recurrence equation is used to estimate the expected number of types for a sample with a certain number of tokens.

Heaps' law (Herdan's law)

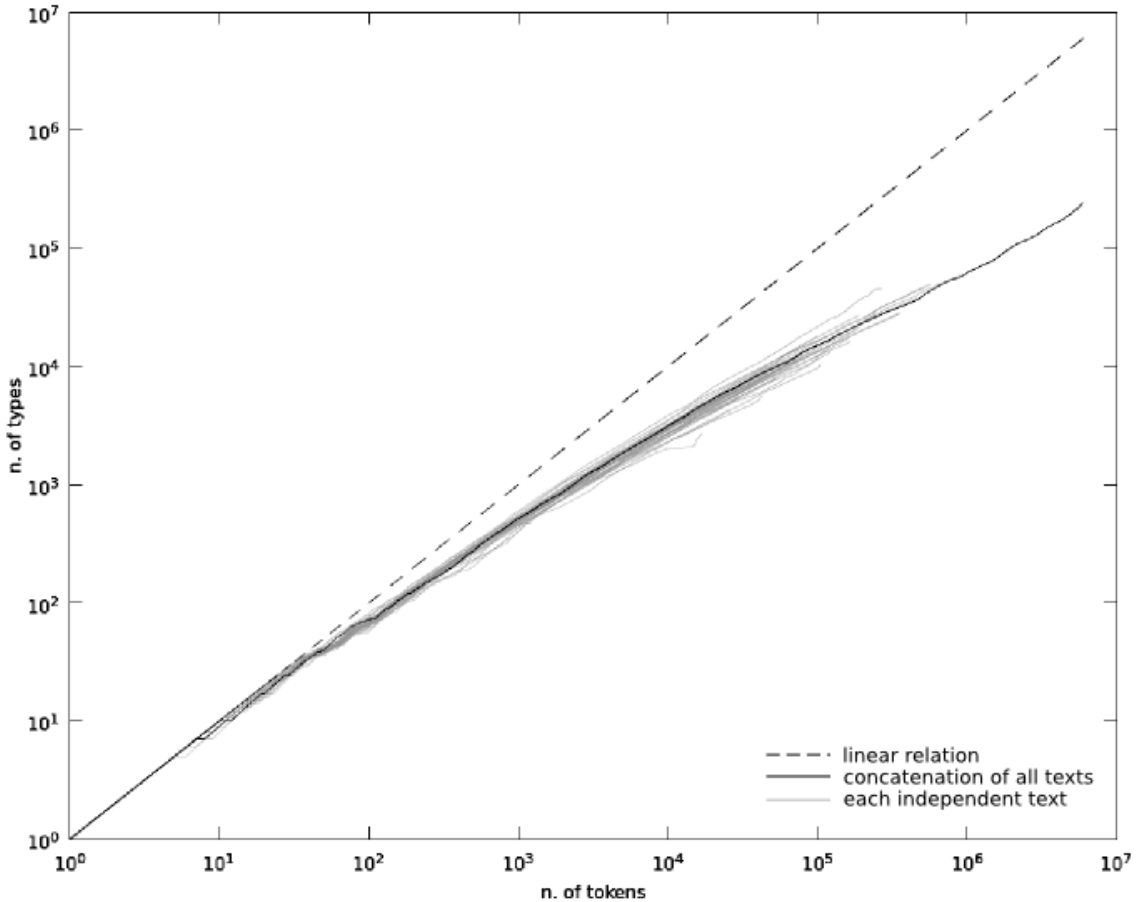


Figure: The relation between the number of tokens and types in 35 books from Gutenberg Database is presented in gray.

Menzerath's law

The longer the whole, the smaller the parts.
Menzerath (1928) observed the decreasing relation between syllable's length and length of a word (number of syllables).

The change relative rate on the length of the constituent is inversely proportional to the length of the construct.

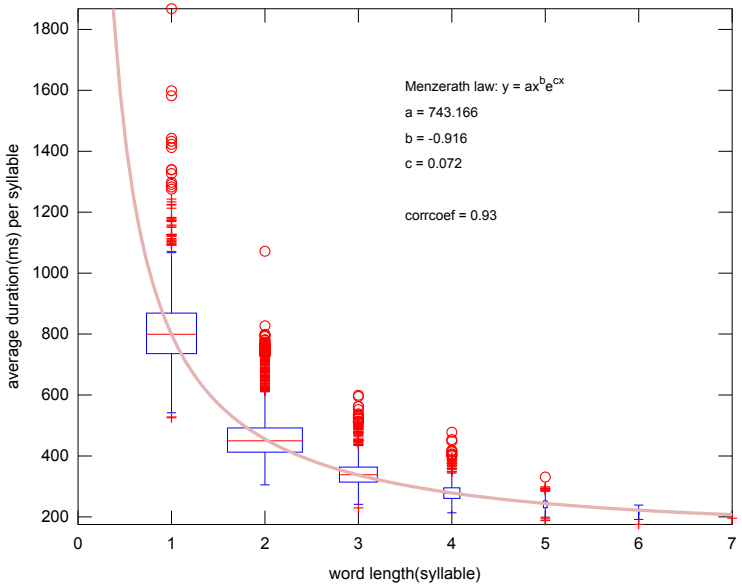
$$\frac{y'}{y} = \frac{b}{x}. \tag{32}$$

y constituent length

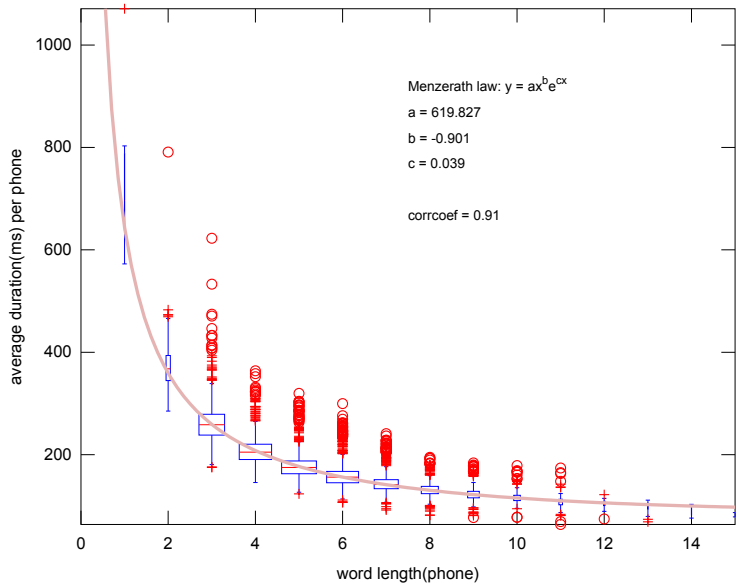
x construct length

Solution: $y = ax^b$.

Menzerath's law



(a) Average syllable duration as words get longer (number of syllables).



(b) Average phone duration as words get longer (number of phones).

Menzerath's law

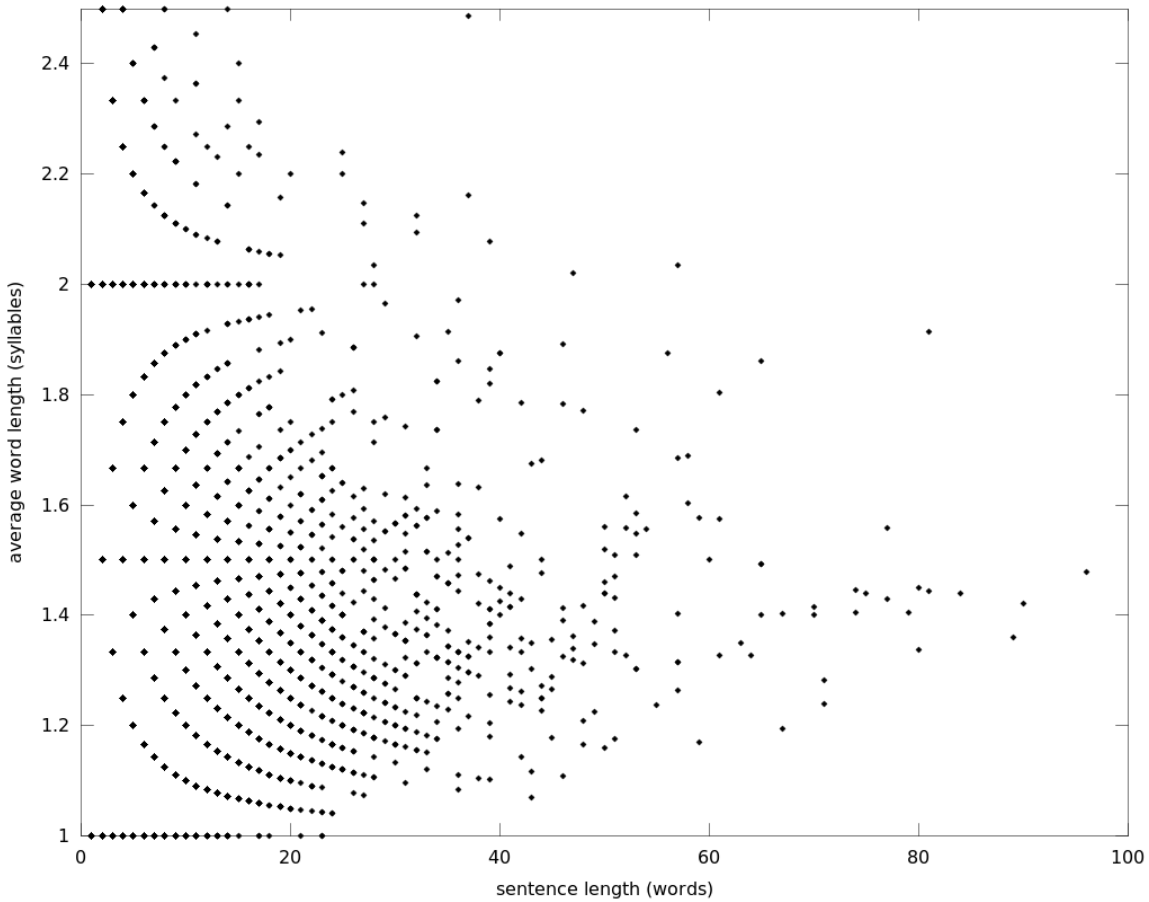


Figure: Relation between sentence length (number of words) and average words length (number of syllables) in a sentence.

Conclusions I

- usage-based driven language (Bybee, 2001, 2003, 2006; Ellis, 2002)
- humans track co-occurrence patterns and statistical regularities (Saffran et al., 1996a, 1999; Saffran and Wilson, 2003)
- regularities among several languages
  - some speech segments are very common (example: [m] appear in 94.2% of the languages)
  - 427 out of 919 speech sounds appear just once among all 425 languages
  - 90% of the languages use at most 6 times more consonants than vowels

Conclusions II

  - a few phones have many co-occurring segments in their languages and are also found in many different languages (wildcards)
- ubiquity of scaling laws in nature
- a compared study of 'intermittent silence', Markov model and natural text
- compare different sources using the Zipf-Maldelbrot model
- mle method to fit the best model
- entropy estimation estimation of a Zipf-Maldelbrot source
- Menzerath's law and Heap's law
- distance measure between phones using Feature Theory

Outline

Abe, S. and Suzuki, N. (2005). Scale-free statistics of time interval between successive earthquakes. *Physica A: Statistical Mechanics and its Applications*, 350(2-4):588–596.

Adamic, L. A. and Huberman, B. A. (2002). Zipf's law and the internet. *Glottometrics*, 3:143–150.

Aldrich, J. (1997). R. A. Fisher and the Making of Maximum Likelihood 1912-1922. *Statistical Science*, 12(3):162–176.

Alexander, L., Johnson, R., and Weiss, J. (1998). Exploring zipf's law. *Teaching Mathematics Applications*, 17(4):155–158.

Altmann, G. (1980). Prolegomena to menzerath's law. *Glottometrika*, 2:1–10.

Altmann, G. and Arens, H. (1983). *Allgemeine Sprachwissenschaft, Sprachtypologie und Textlinguistik. Festschrift für Peter Hartmann*, chapter ,,Verborgene Ordnung" und das Menzerathsche Gesetz, pages 31–39. Narr, Tübingen.

Altmann, G., Schwibbe, M., and Kaumanns, W. (1989). *Das Menzerathsche Gesetz in informationsverarbeitenden Systemen*. Olms.

Andrews, S. (1989). *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 15(5):802–814.

Aoyama, K., Flege, J. E., Guion, S. G., Akahane-Yamada, R., and Yamada, T. (2004). Perceived phonetic dissimilarity and l2 speech learning: the case of japanese /r/ and english /l/ and /r/. *Journal of Phonetics*, 32:233–250.

Apostel, L., Mandelbrot, B., and Morf, A. (1957). *Logique, Langage et Théorie de L'Information*. Presses Universitaires de France, Paris.

Baayen, R. H. (2001). *Word Frequency Distributions*. Kluwer, Dordrecht.

Baayen, R. H. (2003). *Probabilistic Linguistics*, chapter Probabilistic Approaches to Morphology. MIT Press.

Baddeley, A. D., Thomson, N., and Buchanan, M. (1975). Word length and the structure of short-term memory. *Journal of Verbal Learning and Verbal Behavior*, 14(6):575–589.

Baek, S. K., Bernhardsson, S., and Minnhagen, P. (2011). Zipf's law unzipped. *New Journal of Physics*, 13(4).

Bak, P. (1999). *How Nature Works: The Science of Self-organized Criticality*. Copernicus Series. Springer.

Balasubrahmanyana, V. and Naranan, S. (1996). Quantitative linguistics and complex system studies. *Journal of Quantitative Linguistics*, 3(3):177–228.

Biemann, C. (2007). A random text model for the generation of statistical language invariants. In *Proceedings of HLT-NAACL-07*, Rochester, NY, USA.

Bisani, M. and Ney, H. (2003). Multigram-based grapheme-to-phoneme conversion for lvcsr. In *In Proc. Eurospeech*, pages 933–936.

Bloomfield, L. (1926). A set of postulates for the science of language. *Language 2*.

Boas, F. (1911). *Handbook of American Indian languages*. Government PrintingOffice.

Bod, R., Hay, J., and Jannedy, S. (2003). *Probabilistic Linguistics*. MIT Press.

Bond, Z. S. and Garnes, S. (1980). *Perception and Production of Fluent Speech*, chapter Misperception of fluent speech. Erlbaum, N.J.

Bornstein, M. H. and Korda, N. O. (1984). Discrimination and matching within and between hues measured by reaction times: some implications for categorical perception and levels of information processing. *Psychological research*, 46(3):207–222.

Brent, M. R. and Cartwright, T. A. (1996). Distributional regularity and phonotactic constraints are useful for segmentation. *Cognition*, 61:93–125.

Brown, G. D. A. and Neath, I. (2007). A temporal ratio model of memory. *Psychological Review*, 114(3):539–576.

Buk, S. and Rovenchak, A. A. (2007). Menzerath-altmann law for syntactic structures in ukrainian. *CoRR*.

Bybee, J. L. (2001). *Phonology and Language Use*. Cambridge University Press.

Bybee, J. L. (2003). *The evolution of language out of pre-language*, chapter Sequentiality as the basis of constituent structure, pages 109–132. John Benjamins.

Bybee, J. L. (2006). *Frequency of Use And the Organization of Language*. Oxford University Press.

Capek, M. J. (1983). Phoneme theory and umlaut: A note on the creation of knowledge. *Monatshefte*, 75(2):126–130.

Charoenpornsawat, P. and Schultz, T. (2006). Example-based grapheme-to-phoneme conversion for thai. In *INTERSPEECH 2006*.

Chen, S. F. and Goodman, J. (1998). An empirical study of smoothing techniques for language modeling. Technical report, Computer Science Group, Harvard University.

Chomsky, N. (1957). *Syntactic Structures*. Mouton.

Chomsky, N. (1968). *Language and Mind*. Harcourt, Brace and World, New York.

Chomsky, N. (1969). *Aspects of the Theory of Syntax*. The MIT Press.

Chomsky, N. and Halle, M. (1968). *The Sound Pattern of English*. Harper & Row.

Church, K. W. and Gale, W. A. (1991). A comparison of the enhanced good-turing and deleted estimation methods for estimating probabilities of english bigrams. *Computer Speech and Language*, 5:19–54.

Cole, R. A. and Jakimik, J. (1980). *Perception and Production of Fluent Speech*, chapter A model of speech perception, pages 133–163. Lawrence Erlbaum Associates, Hillsdale, NJ.

Coleman, J. (2002). *Phonetics, phonology and cognition*, chapter Phonetic representations in the mental lexicon, pages 96–130. Oxford University Press.

Copelli, M. and Campos, P. R. A. (2007). Excitable scale free networks. *The European Physical Journal B: Condensed Matter and Complex Systems*, 56:273–278.

Coulmas, F. (2003). *Writing Systems: An introduction to their linguistic analysis*. Cambridge University Press.

Cover, T. M. and Thomas, J. A. (1991). *Elements of Information Theory*. Wiley-Interscience, New York, USA.

Cristelli, M., Batty, M., and Pietronero, L. (2012). There is more than a power law in Zipf. *Scientific Reports*, 2.

Cristófaro-Silva, T., de Almeida, L. S., and Fraga, T. (2005). Aspa: a formulacao de um banco de dados de referência da estrutura sonora do português contemporâneo. In *XXV Congresso da Sociedade Brasileira de Computação*, pages 2268–2277.

Davidson, D. (2005). *Truth, Language, and History*. Oxford University Press.

de Boer, B. (2000). Self-organization in vowel systems. *Journal of Phonetics*, 28(4):441–465.

de Saussure, F. (1916). *Cours de linguistique générale*. Payot.

Deacon, T. (1997). *The Symbolic Species: The Co-Evolution of Language and the Brain*. Science: Linguistics. W.W. Norton.

Disner, S. F. (1983). *Vowel Quality: The relation between Universal and Language-specific Factors*. PhD thesis, UCLA: Department of Linguistics.

Dresher, B. E. (2011). *The Blackwell Companion to Phonology*, volume 1, chapter The Phoneme, pages 241–266. John Wiley & Sons.

Düring, B., Matthes, D., and Toscani, G. (2008). A boltzmann-type approach to the formation of wealth distribution curves. Technical Report 08-05, Center of Finance and Econometrics, University of Konstanz.

Ellis, N. C. (2002). Frequency effects in language processing: A review with implications for theories of implicit and explicit language acquisition. *Studies in Second Language Acquisition*, 24(2):143–188.

Etcoff, N. L. and Magee, J. J. (1992). Categorical perception of facial expressions. *Cognition*, 44:227–240.

Everett, D. (2005). Cultural constraints on grammar and cognition in pirahã: Another look at the design features of human language. *Current Anthropology*, 46:621–646.

Ferrer-i-Cancho, R. (2005a). The variation of zipf's law in human language. *European Physical Journal B*, 44(2):249–257.

Ferrer-i-Cancho, R. (2005b). The variation of zipf's law in human language. *European Physical Journal B*, 44(2):249–257.

Ferrer-i-Cancho, R. (2006). *Exact methods in the study of language and text. In honor of Gabriel Altmann*, chapter On the universality of Zipf's law for word frequencies, pages 131–140. Gruyter.

Ferrer-i-Cancho, R. and Elvevåg, B. (2010). Random texts do not exhibit the real zipf's law-like rank distribution. *PLoS ONE*, 5(3):e9411+.

Ferrer-i-Cancho, R. and Gavaldà, R. (2009). The frequency spectrum of finite samples from the intermittent silence process. *Journal of the American Society for Information Science and Technology*, 60(4):837–843.

Ferrer-i-Cancho, R. and Solé, R. V. (2002). Zipf's law and random texts. *Advances in Complex Systems*, 5(1):1–6.

Ferrer-i-Cancho, R. and Solé, R. V. (2003). Least effort and the origins of scaling in human language. *PNAS*, 100(3):788–791.

Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philosophical Transactions of the Royal Society of London*, 222:309–368.

Flemming, E. (2005). Deriving natural classes in phonology. *Lingua*, 115(3):287–309.

Frank, M. C., Everett, D. L., Fedorenko, E., and Gibson, E. (2008). Number as a cognitive technology: Evidence from pirahã language and cognition. *Cognition*.

Freedman, D. A. and Wang, W. (1996). Language polygenesis: A probabilistic model. *Anthropological Science*, 104(2):131–137.

Frisch, S. (1996). *Similarity and frequency in phonology*. PhD thesis, Northwestern University.

Fujimura, O. and Lovins, J. (1978). *Syllables and segments*, chapter Syllables as concatenative phonetic elements, pages 107–120. North Holland, Amsterdam.

Fujiwara, Y. (2004). Zipf law in firms bankruptcy. *Physica A: Statistical and Theoretical Physics*, 337(1-2):219–230.

Fukada, T. and Sagisaka, Y. (1997). Automatic generation of a pronunciation dictionary based on a pronunciation network. In *European Conference on Speech Communication and Technology (EuroSpeech'97*, pages 2471–2474.

Gabaix, X. (1999). Zipf's law for cities: An explanation. *Quarterly Journal of Economics*, 114(3):739–67.

Gale, W. (1994). Good-turing smoothing without tears. *Journal of Quantitative Linguistics*, 2.

Gale, W. A. and Church, K. W. (1994). What's wrong with adding one. In *Corpus-Based Research into Language. Rodolpi*.

Gale, W. A. and Sampson, G. (1995). Good-turing frequency estimation without tears. *Journal of Quantitative Linguistics*, 2:217–237.

Gernsbacher, M. A. (1994). *Handbook of Psycholinguistics*. Academic Press.

Givón, T. (1979). *On understanding grammar*. Academic Press.

Good, I. J. (1953). The population frequencies of species and the estimation of population parameters. *Biometrika*, 40(3-4):237–264.

Good, J. (2008). *Linguistic Universals and Language Change*. Oxford University Press.

Grainger, J. (1990). Word Frequency and Neighborhood Frequency Effects in Lexical Decision and Naming. *Journal of Memory and Language*, 29:228–244.

Greenberg, J. H., Osgood, C. E., and Jenkins, J. J. (1966). *Universals of Language*, chapter Memorandum concerning language universals. M.I.T. Press.

Gregory, M. L., Raymond, W. D., Bell, A., Fosler-lussier, E., and Jurafsky, D. (1999). The effects of collocational strength and contextual predictability in lexical production.

Grignetti, M. (1964). A note on the entropy of words in printed english. *Information and Control*, 7:304–306.

Grotjan, R. and Altmann, G. (2007). *Contributions to Quantitative Linguistics*, chapter Modelling the Distribution of Word Length: Some Methodological Problems, pages 141–153. Springer Publishing Company, Incorporated.

Grzybek, P., Stadlober, E., and Kelih, E. (2006). The relationship of word length and sentence length: The inter-textual perspective. pages 611–618.

Hagège, C. (1986). *La Structure des langues*. Presses universitaires de France.

Halle, M. (1985). Speculations about the representation of words in memory. pages 101–114, Orlando, Florida. Academic Press.

Hancock, J. T., Curry, L. E., Goorha, S., and Woodworth, M. T. (2004). Lies in conversation: An examination of deception using automated linguistic analysis. In *Proceedings of the 26th Annual Conference of the Cognitive Science Society*, volume 26, pages 534–539, Mahwah, NJ: LEA.

Harris, C. M. and Wolpert, D. M. (1988). Signal-dependent noise determines motor planning. *Nature*, 394:780–784.

Hartley, R. V. L. (1928). Transmission of information. *Bell System Technical Journal*.

Havlin, S. (1995). The distance between zipf plots. *Physica A: Statistical Mechanics and its Applications*, 216(1):148–150.

Hockett, C. F. (1960). *A course in modern linguistics*. Macmillan.

Hopper, P. J. and Thompson, S. A. (1980). Transitivity in grammar and discourse. *Language*, 56(2):281–299.

Hopper, P. J. and Thompson, S. A. (1984). The discourse basis for lexical categories in universal grammar. *Language*, 60(4):703–752.

Hualde, J. I. (2004). Quasi-phonemic contrasts in spanish. In Chand, V., Kelleher, A., Rodríguez, A. J., and Schmeiser, B., editors, *WCCFL 23: Proceedings of the 23rd West Coast Conference on Formal Linguistics*, pages 374–398, Somerville, MA. Cascadilla Press.

Hurford, J. (1989). Biological evolution of the saussurean sign as a component of the language acquisition device. *Lingua*, 77(2):187–222.

Iverson, P. and Kuhl, P. K. (2000). Perceptual magnet and phoneme boundary effects in speech perception: Do they arise from a common mechanism? *Perception & Psychophysics*, 62(4):874–886.

Jakobson, R. (1942). *On Language*, chapter The Concept of Phoneme, pages 218–241. Harvard University Press. reprinted.

Jakobson, R. and Waugh, L. R. (2002). *The Sound Shape of Language*. Mouton de Gruyter.

Jeffreys, H. (1939). *Theory of probability*. International series of monographs on physics. The Clarendon press.

Johnson, W. (1932). Probability: deductive and inductive problems. *Mind*, 41:421–423.

Jurafsky, D. (1996). A probabilistic model of lexical and syntactic access and disambiguation. *Cognitive Science*, 20:137–194.

Jurafsky, D. (2003). *Probabilistic Linguistics*, chapter Probabilistic Modeling in Psycholinguistics: Linguistic Comprehension and Production. MIT Press.

Jurafsky, D., Bell, A., Gregory, M., and Raymond, W. D. (2001). *In Frequency and the emergence of linguistic structure*, chapter Probabilistic relations between words: Evidence from reduction in lexical production, pages 229–254. John Benjamins.

Jurafsky, D. and Martin, J. H. (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. Prentice Hall Series in Artificial Intelligence. Pearson Prentice Hall.

Kanter, I. and Kessler, D. A. (1995). Markov processes: Linguistics and zipf's law. *Phys. Rev. Lett.*, 74(22):4559–4562.

Kay, P. and Regier, T. (2003). Resolving the question of color naming universals. *Proceedings of the National Academy of Sciences*, 100:9085–9089.

Kello, C. T., Brown, G. D., Ferrer-i-Cancho, R., Holden, J. G., Linkenkaer-Hansen, K., Rhodes, T., and Orden, G. C. V. (2010). Scaling laws in cognitive sciences. *Trends in Cognitive Sciences*, 14(5):223–232.

Khmaladze, E. V. (1987). The statistical analysis of large number of rare events. Technical report, Dept. Math. Statist. CWI, Amsterdam.

Klatt, D. (1979). Speech perception: A model of acoustic-phonetic analysis and lexical access. *Journal of Phonetics*, 7:279–312.

Klatt, D. H. (1977). Review of the arpa speech understanding project. *The Journal of the Acoustical Society of America*, 62(6):1345–1366.

Köhler, R. (1986). *Zur Linguistischen Synergetik: Struktur und Dynamik der Lexik*. Brockmeyer.

Köhler, R., Altmann, G., and Piotrovskiĭ, R. (2005). *Quantitative Linguistik /Quantitative Linguistics: Ein Internationales Handbuch /An International Handbook*. De Gruyter.

Kolguškin, A. N. (1960). *Linguistic and Engineering Studies in Automatic Language Translation of Scientific Russian Into English: Technical Report Phase II*. University of Washington Press.

Kruskal, J. B. and Wish, M. (1978). *Multidimensional Scaling*. Sage.

Laboissiere, R. (2010). preprint.

Labov, W., Ash, S., and Boberg, C. (2006). *The Atlas of North American English: Phonetics, Phonology, and Sound Change : a Multimedia Reference Tool*. Number v.1 in Topics in English Linguistic Series. Mouton De Gruyter.

Lacquaniti, F., Terzuolo, C., and Viviani, P. (1983). The law relating the kinematic and figural aspects of drawing movements. *Acta Psychologica*, 54:115–130.

Ladd, D. R. (2002). Distinctive phones in surface representation. In *8th Conference on Laboratory Phonology*, New Haven, Connecticut.

Ladefoged, P. and Maddieson, I. (1996). *The Sounds of the World's Languages*. Wiley-Blackwell.

Lamel, L. and Adda, G. (1996). On designing pronunciation lexicons for large vocabulary continuous speech recognition. In *ICSLP 96. Proceedings*, volume 1, pages 6–9.

Laplace, P. S. (1902). *A philosophical essay on probabilities*. John Wiley & Sons.

Leijenhorst, D. C. v., Weide, T. P. v. d., and Grootjen, F. (2005). A formal derivation of heaps' law. *Information Sciences*, 170(2-4):263–272.

Li, W. (1992). Random texts exhibit zipf's-law-like word frequency distribution. *IEEE Transactions on Information Theory*, 38(6):1842–1845.

Liberman, A. M. (1970). The grammars of speech and language. *Cognitive Psychology*, 1(4):301–323.

Liberman, A. M., Cooper, F. S., Shankweiler, D. P., and Studdert-Kennedy, M. (1967). Perception of the speech code. *Psychological Review*, 74:431–461.

Liberman, A. M., Cooper, F. S., Shankweiler, D. P., and Studdert-Kennedy, M. (1972). *Human Communications: A Unified View*, chapter Perception of the Speech Code. McGraw-Hill.

Liberman, A. M., Harris, K., Hoffman, H., and Griffith, B. C. (1957). The discrimination of speech sounds within and across phoneme boundaries. *Journal of Experimental Psychology*, 54:358–368.

Lidstone, G. J. (1920). Note on the general case of the bayes-laplace formula for inductive or a posteriori probabilities. *Transactions of the Faculty of Actuaries*, 8:182–192.

Lü, L., Zhang, Z.-K., and Zhou, T. (2010). Zipf's law leads to heaps' law: Analyzing their relation in finite-size systems. *PLoS ONE*, 5(12):e14139.

MacDonald, M. C. (1993). The interaction of lexical and syntactic ambiguity. *Journal of Memory and Language*, 32:692–715.

Maddieson, I. (1984). *Patterns of Sounds*. Cambridge University Press.

Mandelbrot, B. (1954). Structure formelle des textes et communications. *Word*, 10:1–27.

Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4):394–419.

Mandelbrot, B. B. (1953). An informational theory of the statistical structure of languages. In Jackson, B. W., editor, *Communication Theory*, pages 486–502.

Mandelbrot, B. B. (1965). Information theory and psycholinguistics. In Wolman, B. B. and Nagel, E. N., editors, *Scientific Psychology: Principles and Approaches*, pages 550–562. Basic Books Publishing.

Mandelbrot, B. B. (1982). *The Fractal geometry of Nature*. Freeman, New York.

Mandelbrot, B. B. (1999). *Multifractals and 1/f noise: wild self-affinity in physics (1963-1976)*. Selected works of Benoit B. Mandelbrot. Springer.

Manin, D. Y. (2008). Zipf's law and avoidance of excessive synonymy. *Cognitive Science: A Multidisciplinary Journal*, 32(7):1075–1098.

Manin, D. Y. (2009). Mandelbrot's model for zipf's law: Can mandelbrot's model explain zipf's law for language? *Journal of Quantitative Linguistics*, 16(3):274–285.

Manning, C. D. and Schütze, H. (1999). *Foundations of Statistical Natural Language Processing*. Mit Press.

Mantegna, R. N., Buldyrev, S., Goldberger, A., Havlin, S., Peng, C., Simons, M., and Stanley, H. (1994). Linguistic features of noncoding dna sequences. *Physical review letters*, 73(23):3169–3172.

Martínez-Celdrán, E. (2004). Problems in the classification of approximants. *Journal of the International Phonetic Association*, 34:201–210.

Maslov, S., Paczuski, M., and Bak, P. (1994). Avalanches and 1/f noise in evolution and growth models. *Physical Review Letters*, 73:2162–2165.

McCowan, B., Hanser, S. F., and Doyle, L. R. (1999). Quantitative tools for comparing animal communication systems: information theory applied to bottlenose dolphin whistle repertoires. *Animal Behaviour*, 57:409–419.

Mendenhall, T. C. (1887). The characteristic curves of composition. *Science*, 9(214):237–249.

Mendenhall, T. C. (1901). A mechanical solution of a literary problem. *Popular Science*, 9:97–105.

Menzerath, P. (1928). über einige phonetische probleme. In *Actes du premier Congres International de Linguistes*, pages 104–105, Sijthoff: Leiden.

Menzerath, P. (1954). *Die Architektonik des deutschen Wortschatzes*. F. Dümmler.

Michel, J.-B., Shen, Y. K., Aiden, A. P., Veres, A., Gray, M. K., Team, T. G. B., Pickett, J. P., Hoiberg, D., Clancy, D., Norvig, P., Orwant, J., Pinker, S., Nowak, M., and Lieberman-Aiden, E. (2011a). Quantitative analysis of culture using millions of digitized books. *Science*, 331:176–182.

Michel, J.-B., Shen, Y. K., Aiden, A. P., Veres, A., Gray, M. K., Team, T. G. B., Pickett, J. P., Hoiberg, D., Clancy, D., Norvig, P., Orwant, J., Pinker, S., Nowak, M. A., and Aiden, E. L. (2011b). Quantitative analysis of culture using millions of digitized books. *Science*, 331(6014):176–182.

Mielke, J. (2005). Modeling distinctive feature emergence. In *West Coast Conference on Formal Linguistics XXIV*.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, 63(2):81–97.

Miller, G. A. (1957). Some effects of intermittent silence. *The American Journal of Psychology*, 70(2):311–314.

Miller, G. A. (1965). *The Psycho-biology of language: an introduction to dynamic philology*, chapter Introduction. The MIT Press.

Miller, G. A. and Nicely, P. E. (1955). An analysis of perceptual confusions among some english consonants. *The Journal of the Acoustical Society of America*, 27(2):338–352.

Miller, G. A. and Taylor, W. G. (1948). The perception of repeated bursts of noise. *The Journal of the Acoustical Society of America*.

Montemurro, M. A. and Zanette, D. H. (2011). Universal entropy of word ordering across linguistic families. *PLoS ONE*, 6(5):e19875.

Myung, I. J. (2003). Tutorial on maximum likelihood estimation. *Journal of Mathematical Psychology*, 47(1):90–100.

Nádas, A. (1985). On turing's formula for word probabilities. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 33(6):1414–1416.

Nairne, J. S. (1988). A framework for interpreting recency effects in immediate serial recall. *Memory & Cognition*, 16:343–352.

Nairne, J. S. (1990). A feature model of immediate memory. *Memory & Cognition*, 18:251–269.

Neath, I. and Nairne, J. S. (1995). Word-length effects in immediate memory: Overwriting trace decay theory. *Psychonomic Bulletin & Review*, 2(4):429–441.

Nowak, M. A., Komarova, N. L., and Niyogi, P. (2002). Computational and evolutionary aspects of language. *Nature*, 417:611–617.

Nowak, M. A., Plotkin, J. B., and Jansen, V. A. A. (2000). The evolution of syntactic communication. *Nature*, 404:495–498.

Odden, D. (2005). *Introducing Phonology*. Cambridge University Press.

Olson, D. (1994). *The World on Paper*. Cambridge University Press, Cambridge.

Peterson, G. E. and Barney, H. L. (1952). Control methods used in a study of the vowels. *The Journal of the Acoustical Society of America*, 24(2):175–184.

Pierpont, W. G. (2002). *The Art and Skill of Radio-Telegraphy*. NOHFF.

Pierrehumbert, J. (1994). *Papers in laboratory phonology, vol. 3: Phonological structure and phonetic*, chapter Syllable structure and word structure: A study of triconsonantal clusters in English, pages 168–190. Cambridge University Press, Cambridge.

Pierrehumbert, J. (2001). *Frequency and the emergence of linguistic structure*, chapter Exemplar dynamics: Word frequency, lenition, and contrast, pages 137–157. John Benjamins, Amsterdam.

Pierrehumbert, J. B. (2003). *Probabilistic Linguistics*, chapter Probabilistic Phonology: Discrimination and Robustness. MIT Press.

Pinker, S. (2003). *The Language Instinct: The New Science of Language and Mind*. Penguin Books Limited.

Piotrovskii, R. G., Pashkovskii, V. E., and Piotrovskii, V. R. (1994). Psychiatric linguistics and automatic text processing. *Nauchno-Tekhnicheskaya Informatsiya, Seriya 2*, 28(11):21–25.

Pisoni, D. B. (1982). *Project SCAMP 1981: Acoustic Phonetics and Speech Modeling*, chapter In defense of segmental representations in speech processing. Institute for Defense Analyses, Communications Research Division, Princeton, NJ.

Pollack, I. (1952). The informationof elementaryauditorydisplays. *The Journal of the Acoustical Society of America*, 24(6):745–749.

Pombo, E. L. (2002). Visual construction of writing in the medieval book. *Diogenes*, 49(196):31–40.

Port, R. (2006). *Second Language Speech Learning: The Role of Language Experience in Speech Perception and Production*, chapter The graphical basis of phones and phonemes. John Benjamins, Amsterdam.

Port, R. (2007). How are words stored in memory? beyond phones and phonemes. *New Ideas in Psychology*, 25:143–170.

Port, R. and Leary, A. (2005). Against formal phonology. *Language*.

Project Gutenberg (2013). Project Gutenberg. http://www.gutenberg.org/.

Pulvermüller, F. (2003). *The Neuroscience of Language: On Brain Circuits of Words and Serial Order*. Cambridge University Press.

Reetz, H. (2010). Simple upsid interface. [online] `http://web.phonetik.uni-frankfurt.de/upsid.html`.

Saenger, P. (1991). *Literacy and Orality*, chapter The separation of words and the physiology of reading. Cambridge University Press.

Saenger, P. H. (1997). *Space between words: the origins of silent reading*. Stanford University Press.

Saffran, J. R., Aslin, R. N., and Newport, E. L. (1996a). Statistical learning by 8-month-old infants. *Science*, 274:1926–1928.

Saffran, J. R., Johnson, E. K., Aslin, R. N., and Newport, E. L. (1999). Statistical learning of tone sequences by human infants and adults. *Cognition*, 70:27–52.

Saffran, J. R., Newport, E. L., and Aslin, R. N. (1996b). Word segmentation: The role of distributional cues. *Journal of memory and language*, 35:606–621.

Saffran, J. R. and Wilson, D. P. (2003). From syllables to syntax: Multi-level statistical learning by 12-month-old infants. *Infancy*, 4(2):273–284.

Samuelsson, C. (1996). Relating turing's formula and zipf's law. *CoRR*, cmp-lg/9606013.

Sapir, E. (1929). The status of linguistics as a science. *Language*, 5(4):207–214.

Schneier, B. (1996). *Applied cryptography: protocols, algorithms, and source code in C*. Wiley.

Sedelow, S. Y. and Sedelow, W. A. (1966). *The computer and literary style*, chapter A preface to computational stylistics, pages 1–13. Kent State University Press.

SendImeier, W. F. (1995). Featere, phoneme, syllable or word: How is speech mentally represented. *Phonetica*, 52:131–143.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal*.

Shannon, C. E. (1951). Prediction and entropy of printed english. Technical Report 30, The Bell System Technical Journal.

Shoup, J. E. (1980). Phonological aspects of speech recognition. In Lea, W. A., editor, *Trends in Speech Recognition*, pages 125–138. Prentice Hall, Englewood Cliffs.

Silva, D. C., de Lima, A. A., Maia, R., Braga, D., ao F. de Moraes, J., ao A. de Moraes, J., and Jr., F. G. V. R. (2006). A rule-based grapheme-phone converter and stress determination for brazilian portuguese natural language processing. In *International Telecommunications Symposium*, pages 550–554.

Siravenha, A. C., Neto, N., Macedo, V., and Klautau, A. (2008). Uso de regras fonológicas com determinação de vogal tônica para conversão grafema-fone em português brasileiro. In *7th International Information and Telecommunication Technologies Symposium*.

Steels, L. (1997). Synthesising the origins of language and meaning using co-evolution, self-organisation and level formation.

Stevens, S. S. (1957). On the psychophysical law. *Psychological Review*, 64:153–181.

Studdert-Kennedy, M. (1976). *Contemporary issues in experimental phonetics*, chapter Speech perception, pages 243–293. Academic Press, New York.

Trubetzkoy, N. S. (1939). *Grundzüge der Phonologie*. Vandenhoeck und Ruprecht.

Tuldava, J. (1996). The frequency spectrum of text and vocabulary. *Journal of Quantitative Linguistics*, pages 38–50.

Tuldava, J. (2004). The development of statistical stylistics (a survey). *Journal of Quantitative Linguistics*, 11(1–2):141–151.

Turcotte, D. (1989). Fractals in geology and geophysics. In Scholz, C. and Mandelbrot, B., editors, *Fractals in Geophysics*, pages 171–196. Birkhauser Verlag, Basel.

von Humboldt, W. (1836). *The Heterogeneity of Language and its Influence on the Intellectual Development of Mankind (Über die Verschiedenheit des menschlichen Sprachbaus und ihren Einfluss auf die geistige Entwicklung des Menschengeschlechts)*. Cambridge University Press.

Vygotsky, L. S. (1934). *Thought and Language*. MIT Press, Cambridge, MA.

Wang, W. S.-Y. (1991). *Explorations in Language*. Pyramid Press.

Wang, W. S.-Y., Ke, J., and Minett, J. W. (2004). Computational studies of language evolution. In Huang, C. and Lenders, W., editors, *Computational linguistics and Beyond*. Academia Sinica: Institute of Linguistics.

Weide, R. L. (2008). Carnegie mellon pronouncing dictionary, release 0.7a. `http://www.speech.cs.cmu.edu/`.

Whaley, C. P. (1978). Word-nonword classification time. *Journal of Verbal Learning and Verbal Behavior*, 17:143–154.

Whorf, B. (1940/1956). *Language, Thought, and Reality: Selected Writings of Benjamin Lee Whorf*, chapter Science and linguistics, pages 207–219. MIT Press, Cambridge, MA.

Wickelgren, W. A. (1969). *Information Processing in the Nervous System*, chapter Context-sensitive coding in speech recognition, articulation, and development, pages 85–95. Springer-Verlag, New York.

Wilden, A. (2001). *System and Structure: Essays in Communication and Exchange*. International Behavioural and Social Sciences Classics from the Tavistock Press, 96. Routledge.

Yoshida, K., Watanabe, T., and Koga, S. (1989). Large vocabulary word recognition based on demi-syllable hidden markov model using small amount of training data. In *International Conference on Acoustics, Speech, and Signal Processing*, volume 1, pages 1–4.

Young, G. and Householder, A. (1938). Discussion of a set of points in terms of their mutual distances. *Psychometrika*, 3:19–22.

Zipf, G. K. (1935). *The Psycho-biology of language: an introduction to dynamic philology*. The MIT Press.

Zipf, G. K. (1942). Children's speech. *Science*, 96:344–345.

Zipf, G. K. (1949). *Human Behaviour and the Principle of Least Effort: An Introduction to Human Ecology*. Hafner Pub. Co.

Ziv, J. and Lempel, A. (1978). Compression of individual sequences via variable-rate coding. *IEEE Transactions on Information Theory*, 24(5):530–536.