

自然语言处理第二次作业

卢雨轩 19071125

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假设 (X, Y) 服从如下联合概率分布：

Y \ X	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

求 $H(X), H(Y), H(X|Y), H(Y|X), H(X, Y)$

Y \ X	1	2	3	4	P(Y)
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{4}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{4}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
4	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$
P(X)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	

P(Y X) \ X	1	2	3	4
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
2	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
3	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
4	$\frac{1}{2}$	0	0	0

P(X Y) \ Y	1	2	3	4
1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$
3	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
4	1	0	0	0

$$\begin{aligned}
 H(X) &= - \sum_{x \in X} p(x) \log_2 p(x) \\
 &= - \left(\frac{1}{2} \times \log_2 \frac{1}{2} + \frac{1}{4} \times \log_2 \frac{1}{4} + 2 \times \frac{1}{8} \times \log_2 \frac{1}{8} \right) \\
 &= \frac{7}{4} \text{ bit}
 \end{aligned}$$

$$\begin{aligned}
H(Y) &= - \sum_{x \in Y} p(x) \log_2 p(x) \\
&= -4 \times \frac{1}{4} \times \log_2 \frac{1}{4} \\
&= 2 \text{ bit}
\end{aligned}$$

$$\begin{aligned}
H(X|Y) &= - \sum_{x \in X} \sum_{y \in Y} p(x, y) \log_2 P(x|y) \\
&= 1.375 \text{ bit}
\end{aligned}$$

$$\begin{aligned}
H(Y|X) &= - \sum_{x \in X} \sum_{y \in Y} p(x, y) \log_2 P(y|x) \\
&= 1.625 \text{ bit}
\end{aligned}$$

$$H(X, Y) = H(X) + H(Y|X) = 3.375 \text{ bit}$$