

Relatório 1

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1 Gauss-Legendre Quadratures

1.1 Desenvolvimento do polinômio de Legendre de grau 4

$$\begin{aligned}P_4(s) &= \frac{1}{2^4 \cdot 4!} \cdot \frac{d^4}{ds^4} (s^2 - 1)^4 \\P_4(s) &= \frac{1}{16 \cdot 24} \cdot \left(\frac{d}{ds} \left(\frac{d}{ds} \left(\frac{d}{ds} \left(\frac{d}{ds} (s^2 - 1)^4 \right) \right) \right) \right) \\P_4(s) &= \frac{8}{384} \cdot \left(\frac{d}{ds} \left(\frac{d}{ds} \left(\frac{d}{ds} s(s^2 - 1)^3 \right) \right) \right) \\P_4(s) &= \frac{1}{48} \cdot \left(\frac{d}{ds} \left(\frac{d}{ds} (s^2 - 1)^2 (7s^2 - 1) \right) \right) \\P_4(s) &= \frac{1}{48} \cdot \left(\frac{d}{ds} 6s(7s^4 - 10s^2 + 3) \right) \\P_4(s) &= \frac{6}{48} \cdot (35s^4 - 30s^2 + 3) \\P_4(s) &= \frac{1}{8} \cdot (35s^4 - 30s^2 + 3)\end{aligned}$$

Desenvolvimento das derivadas que foram utilizadas acima:

$$\begin{aligned}\frac{d}{ds} (s^2 - 1)^4 &= 4(s^2 - 1)^3 \cdot 2s = 8s \cdot (s^2 - 1)^3 \\ \frac{d}{ds} s(s^2 - 1)^3 &= 3s(s^2 - 1)^2 \cdot 2s + (s^2 - 1)^3 = (s^2 - 1)^3 (7s^2 - 1) \\ \frac{d}{ds} (s^2 - 1)^3 (7s^2 - 1) &= (s^2 - 1)^2 \cdot 14s + (7s^2 - 1)(s^2 - 1) \cdot 4s = s(s^2 - 1)(42s^2 - 18) = 6s(7s^4 - 10s^2 + 3) \\ \frac{d}{ds} 6s(7s^4 - 10s^2 + 3) &= 6 \left(\frac{d}{ds} (7s^5 - 10s^3 + 3s) \right) = 6(35s^4 - 30s^2 + 3)\end{aligned}$$

1.2 Cálculo das raízes do polinômio

Raízes do polinômio P_4 de Legendre:

$$35s^4 - 30s^2 + 3 = 0$$

Seja $x = s^2$, temos:

$$35x^2 - 30x + 3 = 0$$

$$\Delta = (-30)^2 - 4 * 3 * 35$$

$$\Delta = 480$$

Assim teremos:

$$x = 30 \pm \sqrt[2]{480}$$

$$x_1 = 0.7415$$

$$x_2 = 0.1156$$

Dessa forma:

$$s = \pm 0.8611$$

$$s = \pm 0.34$$

1.3 Desenvolvimento do polinômio de Lagrange para as raízes do polinômio de Legendre

1.3.1 Ponto 1

$$L_1 = \frac{s - (-0.34)}{-0.8611 - (-0.34)} \cdot \frac{s - 0.34}{-0.8611 - (-0.34)} \cdot \frac{s - 0.8611}{-0.8611 - 0.8611}$$

$$L_1 = \frac{s^3 - 0.8611s^2 - 0.1156s + 0.099}{-1.078}$$

1.3.2 Ponto 2

$$L_2 = \frac{s - (-0.8611)}{-0.34 - (-0.8611)} \cdot \frac{s - 0.34}{-0.34 - (-0.34)} \cdot \frac{s - 0.8611}{-0.34 - 0.8611}$$

$$L_2 = \frac{s^3 - 0.34s^2 - 0.7415s + 0.2521}{0.4256}$$

1.3.3 Ponto 3 e 4

$$L_3 = L_1$$

$$L_4 = L_2$$

1.4 Cálculo dos pesos associados

1.4.1 Ponto 1

$$\int_{-1}^1 L_1 ds = \frac{1}{-1.078} \cdot \left(\int_{-1}^1 s^3 ds - 0.8611 \int_{-1}^1 s^2 ds - 0.1156 \int_{-1}^1 s ds + 0.099 \int_{-1}^1 ds \right)$$
$$\int_{-1}^1 L_1 ds = \frac{1}{-1.078} \cdot \left(-0.8611 \cdot \frac{2}{3} + 0.099 \cdot 2 \right) = 0.3488$$

1.4.2 Ponto 2

$$\int_{-1}^1 L_2 ds = \frac{1}{0.4256} \cdot \left(\int_{-1}^1 s^3 ds - 0.8611 \int_{-1}^1 s^2 ds - 0.1156 \int_{-1}^1 s ds + 0.099 \int_{-1}^1 ds \right)$$
$$\int_{-1}^1 L_2 ds = \frac{1}{0.4256} \cdot (-0.2267 + 0.5042) = 0.6520$$

1.4.3 Resumo

$$L_1 = 0.3488$$

$$L_2 = 0.6520$$

$$L_3 = 0.3488$$

$$L_4 = 0.6520$$