Advanced Digital Image Processing

Chapter 3: Image Enhancement in the Spatial Domain

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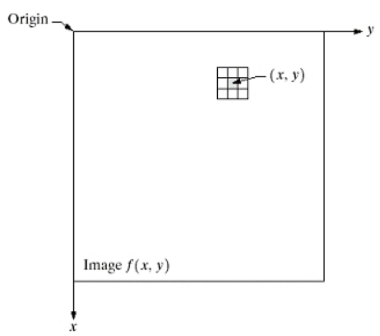
Outline

Spatial Domain Mask processing



Definition

______________________________:

$$g(x,y) = T[f(x,y)]$$

- $f(x,y)$: Input image
- $g(x,y)$: Output image
- T: Operator on f, called ______________________________ (say, 3×3 in figure)

- The mask is moved from pixel to pixel starting at the top left corner.
- The operator T is applied at each location (x,y) to yield the output g at that location, and only utilize pixel in the

Point Processing

Definition

______________ is a special kind of mask procesing with 1×1 mask, i.e., no neighborhood involved.

$$s = T(r)$$

- r: Gray level of input image $f(x,y)$
- s: Gray level of input image $g(x,y)$
- T: ______________________________.

Point Processing
Contrast Enhancement

Example

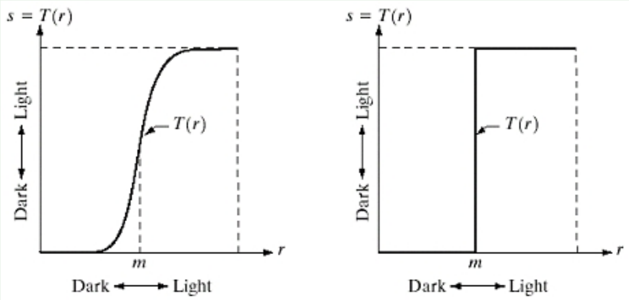
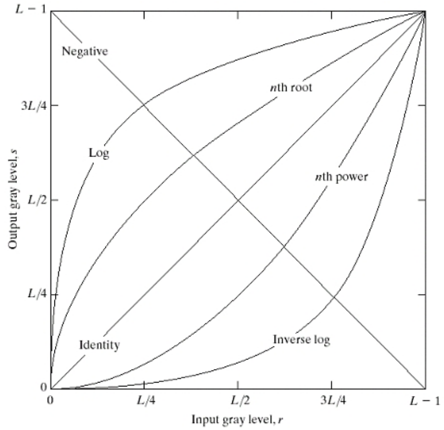


Image Negatives



Definition

__________ with gray levels in the range $[0, L-1]$ has gray-level transformation,

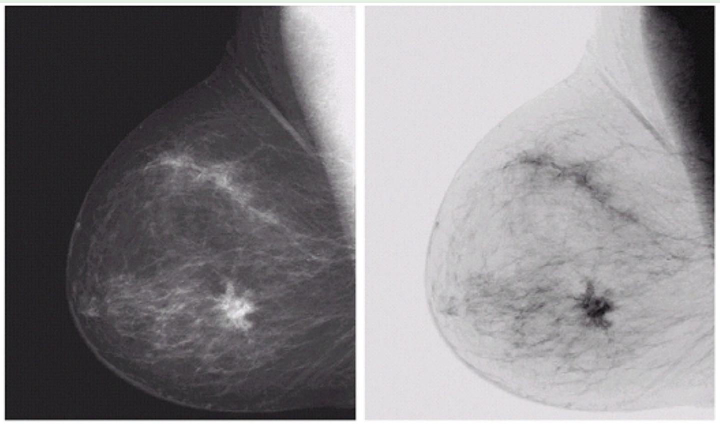
$$s = L - 1 - r$$

- r: Input gray level
- s: Output gray level

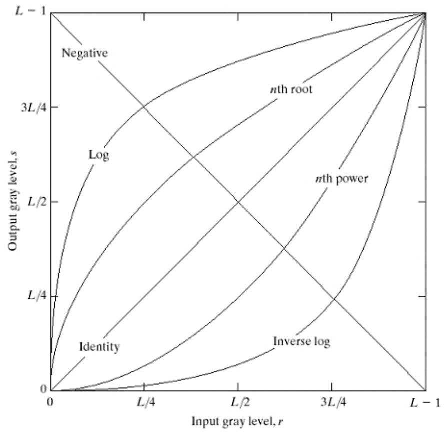
- Equivalent of photographic negative.
- Suited for enhancing white or gray detail embedded in dark regions, where black areas are dominant in size

Image Negatives: Example

Example



Log Transformation



Definition

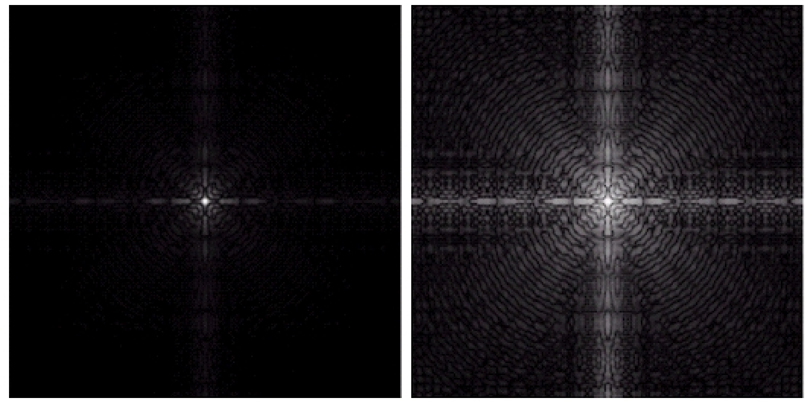
__________ : $s = c \cdot log(1 + r)$

- r: input gray level, $r \geq 0$
- s: output gray level
- c: constant

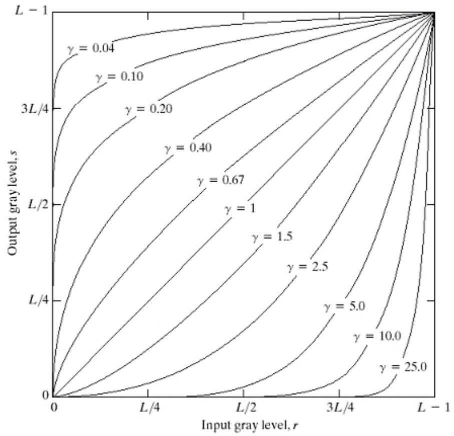
- Exapnd the value of dark pixels while compressing the higher-level value.
- Less versatile in spreading / compressing of gray levels than power-law transformation (2 variables).

Log Transformation: Example

Example



Power-Law Transformation



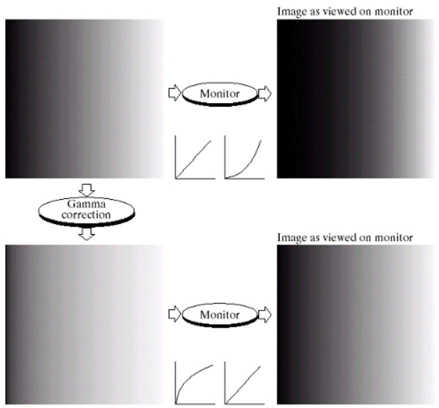
Definition

__________: $s = c \cdot r^{\gamma}$

- r: input gray level, $r \geq 0$
- s: output gray level
- variable 1 c: positive constant
- variable 2 γ: gamma, positive constant.

- $r = 1$: __________
  $r < 1$: Expand dark
  $r > 1$: Expand white.
- Sometimes $s = c \cdot (r + \epsilon)^{\gamma}$, ϵ is offset for display calibration.

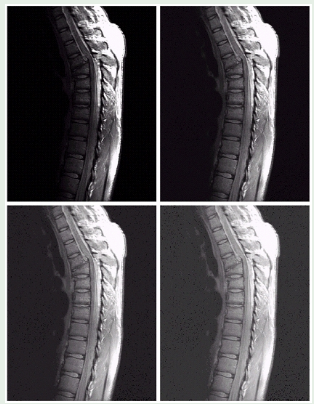
Gamma correction



Gamma correction become increasingly important as use of digital image over the Internet has increased.(e.g. Web images)

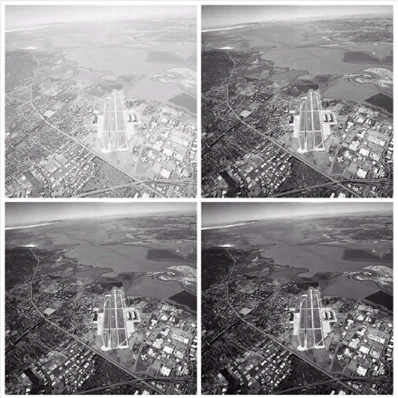
Power Law Transform ($\gamma < 1$)

Example



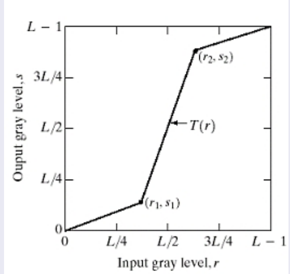
Power Law Transform ($\gamma > 1$)

Example



Piecewise-Linear Transformation

Definition



- Locations (r_1, s_1) and (r_2, s_2) control the shape of the transform.
- $r_1 = s_1$ and $r_2 = s_2$:

  ____
- $r_1 = r_2, s_1 = 0$ and $s_2 = L - 1$:

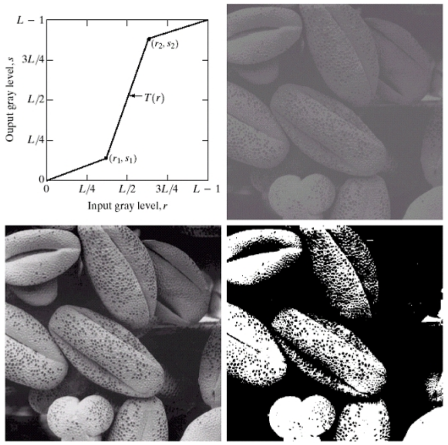
  ____

- Advantage:
  - Can be arbitrarily complex.
  - Practical implementation of some transformations can be formulated only as piecewise functions.
- Disadvantage: Require considerably more user input.

Piecewise-Linear Transformation

Example

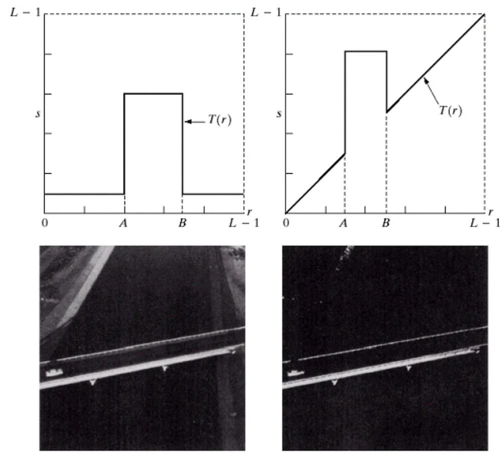
____________________: Increase the dynamic range of the gray levels



Piecewise-Linear Transformation

Gray-level Slicing

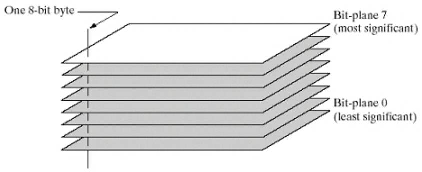
__________________: Highlighting a specific range of gray levels



Piecewise-Linear Transformation
Bit-plane Slicing

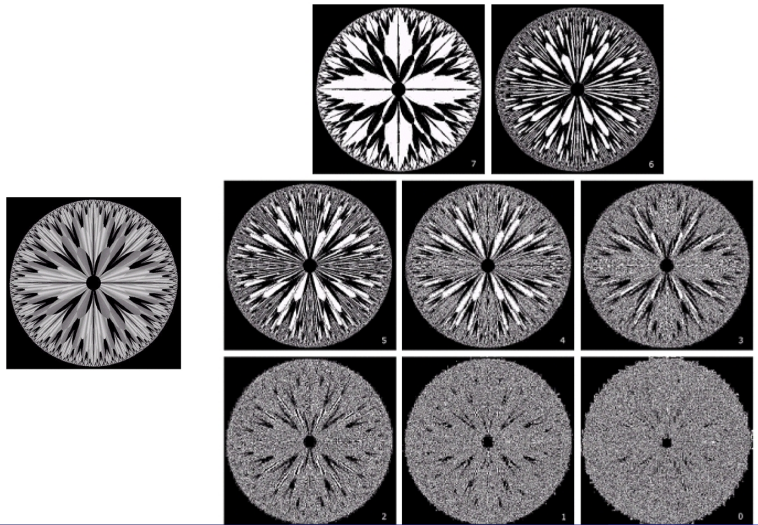
Definition

______: Highlighting the contribution made to total image appearance by specific bits (bit-plane).



- 8-bit image: Bit-plane 7(highest order bit) to bit-plane 0
  - Bit-plane 7: the majority of the visually significant data.
  - Other bit planes: subtle details
- Aids in determineing the number of bits used to quantize each pixel.

Piecewise-Linear Transformation
Bit-plane Slicing



Histogram

Definition

______: $h(r_k) = n_k, k = 0, 1, \ldots, L-1$

where

r_k is the kth gray level

n_k is the number of pixels in the image having gray level r_k.

$\Rightarrow$provide useful real-time image statistics.

$\Rightarrow$processing/compression/segmentaiotn.

Definition

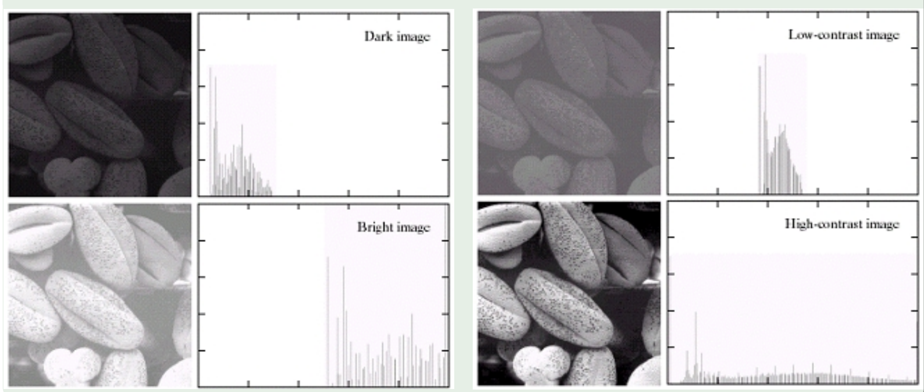
______: $p(r_k) = n_k/n, k = 0, 1, \ldots, L-1$

which is normalized by dividing total number of pixels n in the image.

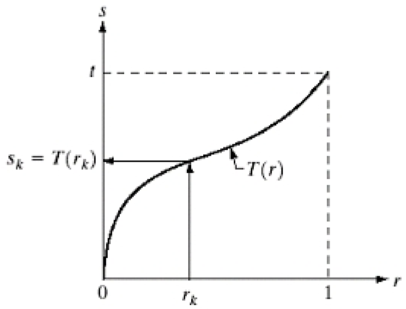
$\Rightarrow$ sum of all components of $p(r_k)$ is equal to 1.

Histograms
Example

Example



Histogram Equalization
Continuous pdf



Assume r is normalzied to $[0, 1]$,
We would like to design gray-level transform $s = T(r)$ with the following properties:

- Input: $0 \leq r \leq 1$
- Output: $0 \leq s = T(r) \leq 1$
- Mapping: $s = T(r)$ is single valued and monotonically increased.

Thus, we will have following property for pdf(or histogram) of s and r,

$$\text{pdf: } \int_0^{s_k} p_s(s)ds = \int_0^{r_k} p_r(r)dr$$
$$p_s(s)ds = p_r(r)dr$$

Histogram Equalization
Continuous pdf

If we design $T(r)$ as the Cumulative Distribution Function (CDF) of random variable r.

$$s = T(r) = \int_0^r p_r(w)dw$$

Then the pdf of output gray level s becomes,

$$\begin{aligned} p_s(s) &= p_r(r)\frac{dr}{ds} \\ &= p_r(r)(\frac{ds}{dr})^{-1} \\ &= p_r(r)\frac{1}{p_r(r)} \\ &= 1, \quad \text{for } 0 \leq s \leq 1 \end{aligned}$$

$\Rightarrow p_s(s)$ (histogram) is uniform distribution [(Continous) Histogram is equalized]

Histogram Equalization
Discrete pdf

Definition

For digital image with discrete histogram (pdf): $p_s(s_k), p_r(r_k)$,
where $k = 0, 1, 2, \ldots, L-1$,
________________ (or ________________) is done by designing $T(r_k)$ as

$$\begin{aligned} s_k &= T(r_k) \\ &= \sum_{j=0}^{k} p_r(r_j) \\ &= \sum_{j=0}^{k} \frac{n_j}{n} \end{aligned}$$

Histogram Equalization
Example: Dark Image

Histogram Equalization
Example: White Image

Histogram Equalization
Example: Low Contrast Image

Histogram Equalization
Example: Good Contrast Image

Histogram Equalization

Conclusion of Examples

The **advantage** of histogram equalization is

- Automatically determines a transformation function
- Produce an ouput image that has a uniform histogram(i.e., covers the entire gray scale)

Histogram Match

Definition

____________________ can be used to generate a processed image that has a specified histogram $p(z)$ (from r to z).

Example

r

Histo Eq: $s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$

$z = G^{-1}(s) = G^{-1}[T(r_k)]$

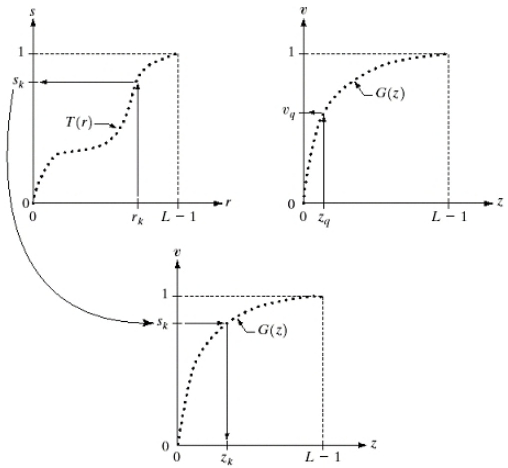
s

Inverse Histo Eq: $z_k = G^{-1}(s_k)$ / Histo Eq: $s_k = \sum_{j=0}^{k} p_z(z_j)$

z

Histogram Match
Non-iterative Scheme

Non-iterative scheme



Histogram Match
Iterative Scheme

________________ can be used to find z from s

Since

- $G(z_k) = s_k \Rightarrow (G(z_k) - s_k) = 0.$
- z_k must be integer,

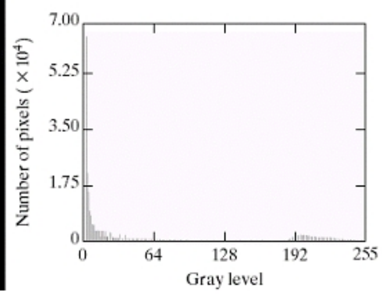
So that we have to find the $\hat{z}$ smallest integer in $[0, L-1]$ such that

$$(G(\hat{z}) - s_k) \geq 0 \qquad k = 0, 1, 2, \ldots, L-1$$

Repeating this proces for all values of k would yield all the requied mapping form s to z.
In practical, we will not start with $\hat{z} = 0$ each time becaue the values of s_k are known to increase monotonically. Thus for $k = k+1$,we would start with $\hat{z} = z_k$ and increment in integer value from there.

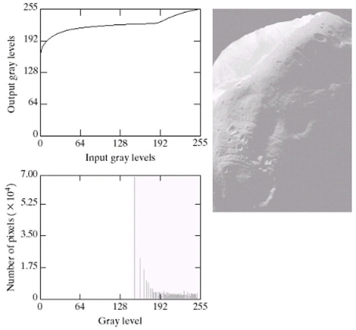
Histogram Match
Example

Original image



Histogram Match
Example

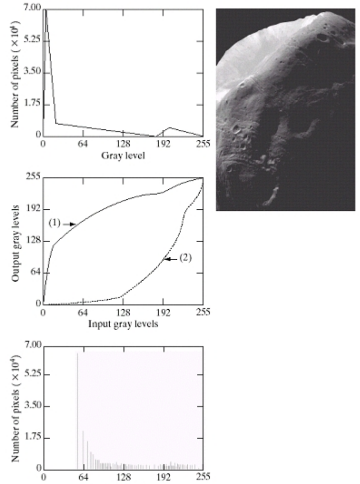
Histogram Equalization



Since numberous pixels in **a very narrow interval of dark levels**, the result is an image with a light, washed-out appearance.

Example

Histogram Match



Local Enhancement

Difference

____________________: Suitable for overall enhancement.
___________________: Enhance details over small areas in an image.

Approach

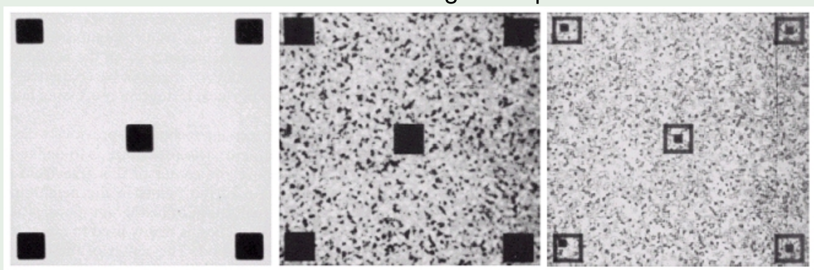
____

1. Define a window of neighborhood and move the center of window from pixel to pixel
2. At each location, the histogram in window is computed to map the gray level of the pixel centered in windows.
3. The center of window is moved to an adjacent pixel repeatly.

Local Enhancement
Example

Example

7×7 window local histogram equalization



Moment

Let $p(r_i)$ denote the **Normalized Histogram**, and thus

Mean value of r: $m = \sum_{i=0}^{L-1} r_i p(r_i)$

Definition

____________ of r about its mean

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

Example

- 0th Moment: $\mu_o = 1$
- 1st Moment: $\mu_1 = 0$ **(not mean)**
- 2nd Moment: $\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \sigma^2(r)$

Moment
Local Moment

Definition

_______________: A measure of average gray level in an image

_____________________: A measure of average contrast in an image.

Definition

Let S_{xy} denote **a neighborhood (subimage)** of specified size

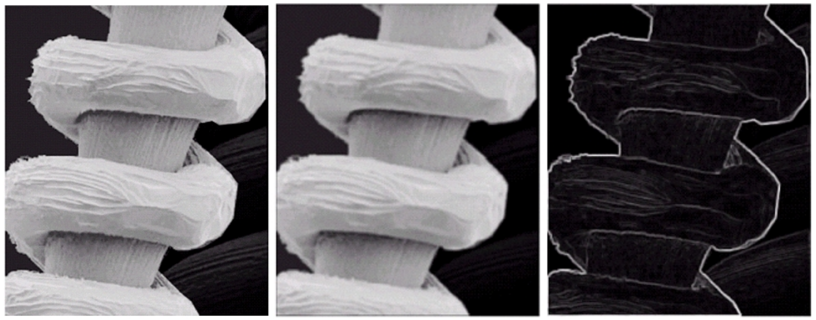
________________: A measure of average gray level **in neighborhood** S_{xy}.

$$m_{S_{xy}} = \sum_{(s,t)\in S_{xy}} r_{s,t} p(r_{s,t})$$

_____________________: a measure of average contrast **in neighborhood** S_{xy}.

$$\sigma^2_{S_{xy}} = \sum_{(s,t)\in S_{xy}} (r_{s,t} - m_{S_{xy}})^2 p(r_{s,t})$$

Moment
Example



Moment
Example

$$g(x,y) = \begin{cases} E \cdot f(x,y) & \text{,if } m_{S_{xy}} \le k_o M_G \text{ (Enhance dark area only)} \\ & \text{and } k_1 D_G \text{ (Limit lowest contrast to enhance)} \le \sigma_{S_{xy}} \le k_2 D_G \text{ (Enhac} \\ f(x,y) & \text{,otherwise} \end{cases}$$

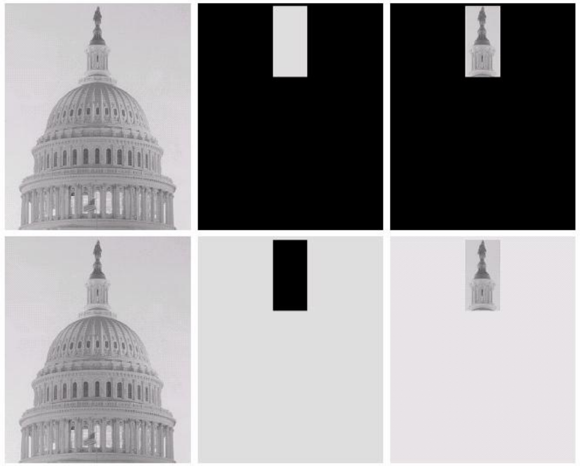
Logic Operations

Definition

Process **one or two images** on an a pixel-by-pixel and bitwise basis with the following operators,

1. ________________ :
   Operates on one image to get the Negative Film
2. ____________________ :
   Operate between two images to get the Masking (or ROI (Region of Interest))

Logic Operations
Example



a b c
d e f

FIGURE 3.27 (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

Arithmetic Operations

Definition

Process **one or two images** on a pixel-by-pixel basis with the following four (two) operations,

1. ________________
2. ____________________

Note

Multiplication operatoin is more general than logical masks.

- Multiplication with a constant
- Multiplication with an image: Mask can be **gray-level**, rather than binary.

Arithmetic Operations
Image Subtraction

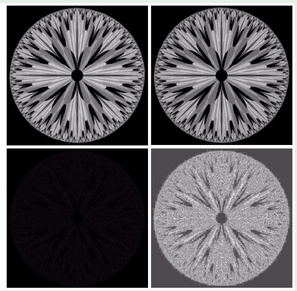
Application

- Enhance the differences between images.
- Image segmentation: ________________ by image substraction.

Example

a b
c d

FIGURE 3.28 (a) Original fractal image. (b) Result of setting the four lower-order bit planes to zero. (c) Difference between (a) and (b). (d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).



Arithmetic Operations
Image Subtraction

Problem

________________: $g(x,y) = f(x,y) - h(x,y)$
will produce output image $g(x,y)$ in range of $[-255, 255]$, but $[0, 255]$ is expected.

Solution

- ________________________________.
  - ☺ Fast and Simple
  - ☹ Full range of display may not be utilized
  - ☹ $/2$ truncation will cause losss in acurracy.
- ________________________________
  - Good results, but more complex and difficult to implement.

Arithmetic Operations
Image Averaging

Application

- Denoising

Consider a noisy image

$$g(x,y) = f(x,y) + \eta(x,y)$$

$\eta(x,y)$ is noise (uncorrelated & zero mean)

The Averaged Image:

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

has the property:

$$\begin{aligned} E\{\bar{g}(x,y)\} &= f(x,y) \\ \sigma^2_{\bar{g}(x,y)} &= \frac{1}{K}\sigma^2_{\eta(x,y)} \end{aligned}$$

Image Averaging
Example

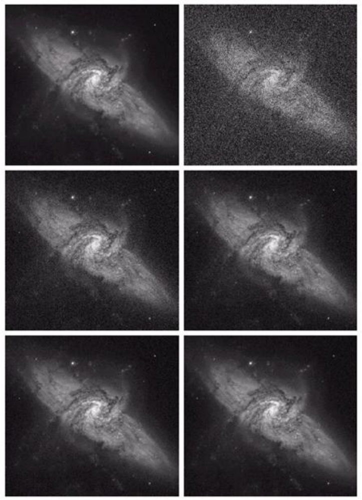
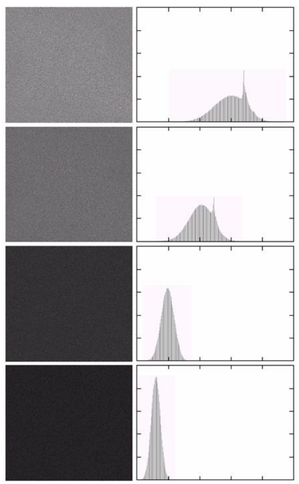


Image Averaging
Example



a b

FIGURE 3.31 (a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.

Image Averaging
Problem

Note
The values in the sum of K, 8-bit images is $[0, 255 \times K]$, and next it is divided by K to scale back to 8-bit image.
$\Rightarrow$ Lost accuracy

Problem
When **noise** is added to an image, the image averaging could be negative.

Solution
- The minimal value is obtained and its negative is added to the image, then multiple by 255/Max

Linear Filtering of An Image

Instead of **point processing**, now we consider bigger window size

Definition
____________:

$$g(x, y) = \sum_{s=-a}^{a} \sum_{s=-b}^{b} w(s, t) f(x + s, y + t)$$

where
- $f(x, y)$: image before filtering (size $M \times N$)
- $w(x, y)$: (size $m(= 2a + 1) \times n(= 2b + 1)$) __________, ______________, ______________

- Linear Filtering is a convolution operation.
- $\Leftrightarrow$ Example: Median Filter

Linear Filtering of An Image

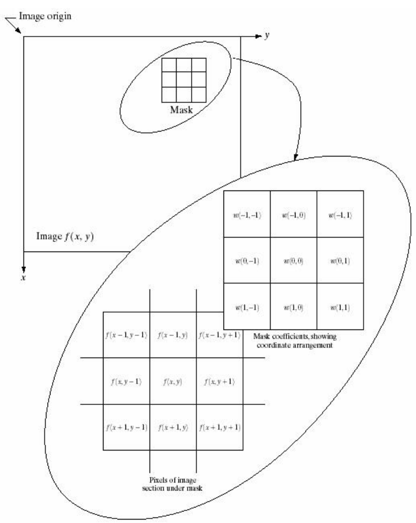


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Linear Filtering of An Image
Border of Image

Problem
When center of mask closes to the border, the mask will locate outside the image.

Solution
1. Limte the excursions of the center of the mask $(n - 1)/2$ from the border. $\Rightarrow$ ☹ The resulting filtered image is smaller than original
2. The pixels near the border is processed with a parital filter mask.
3. Padding the image by adding rows and columns of 0's or constant gray level.
4. Padding the image by replicating rows and columns.
☺ This approach become more prevalent as the size of the mask increases.

Smoothing Linear Filters

Applicatoin

______________________________:

- Noise reduction, but **blurred edge**.
- Smoothing false contour from using an insufficient number of gray levels (skull)
- Blur an image to get a gross representation of objects of interest (____).

Smoothing linear filters
Design

(all coffs are equal)

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

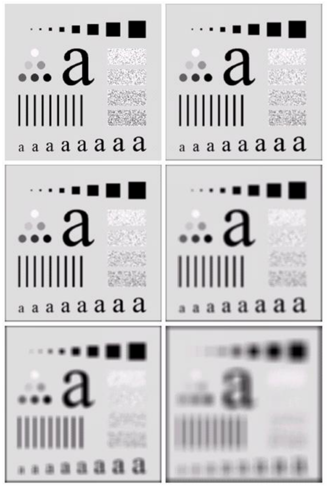
$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

(different coeffs)

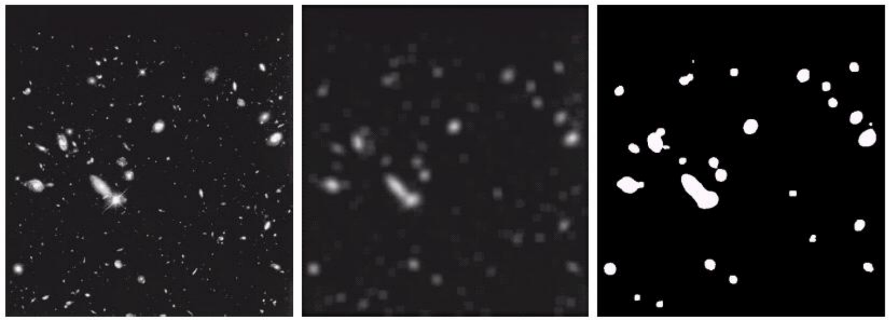
$$\frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

Smoothing linear filters
Example



Smoothing linear filters
Example



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Order-Statistics Filters

Definition

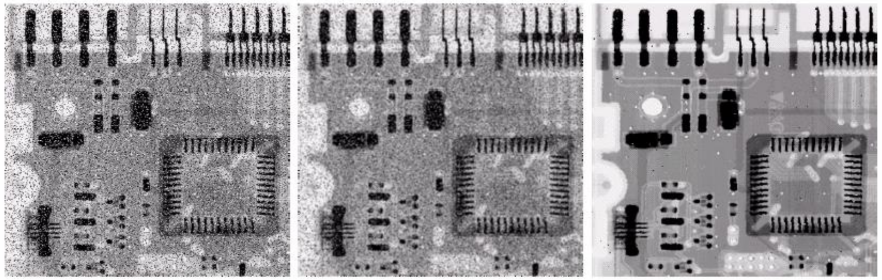
________:

- Nonlinear spatial filter
- Based on ordering (ranking) the pixels in filter image area

Example

- ________
  - Median of 3×3 filter: **5th** largest vlaue.
    $(10, 20, 20, 20, 15, 20, 20, 25, 100)$
    $\Rightarrow$ Sort $(10, 15, 20, 20, 20, 20, 20, 25, 100) \Rightarrow$ Median=20
  - Median of 5×5 filter: **13th** largest vlaue.
  - Remove Impluse noise or Salt-and-pepper noise
- ________

Median Filter
Example



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening spaital filters

Definition

________ is based on the spatial differentiation, and has the following effects,

- Highlight the fine detail
  - Edge and discontinuites will be enhanced.
  - Noise will be also enhanced.
- Enhance details that was blurred, or in error.
  - $\Leftrightarrow$ Smoothing by averaging/integration
- Slowly varying gray-level will be deemphasized.

One-dimensional Spatial Derivative

Definition

____

$$\frac{\partial f}{\partial x} = f(x+1) - f(x).$$

Definition

____

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= (f(x+1) - f(x)) - (f(x) - f(x-1)) \\ &= f(x+1) + f(x-1) - 2f(x)\end{aligned}$$

One-dimensional Spatial Derivative

______:

- 1st order: nonzero in **all** (thick edge)
- 2nd order: nonzero in **onset and end point only** (fine edge)

______________:

- 2nd order is more aggressive than 1st order in **enhancing sharp changes (fine detail or**

One-dimensional Spatial Derivative

Summary

1st order derivatives:

- Edge: Generally produce thicker edges in an image
- Step: Generally have a stronger reponse to a gray-level step.

2nd order derivatives:

- Edge: Generally have a stronger response to fine detail: thin lines, isolated points.
- Step: Generally produce a double reponse at step changes in gray level.

In most applications, the 2nd derivative is better suited than the 1st derivative **for the image enhancement**

$\Longrightarrow$ Because of 2nd order can enhance the fine detail

The Laplacian Enhancement

Definition

___________ is a 2-D 2nd-order derivative filter,

$$\begin{aligned}\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \\ &= [f(x+1,y)+f(x-1,y)-2f(x,y)]+[f(x,y+1)+f(x,y-1)-2f(x,y)] \\ &= [f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]-4f(x,y)\end{aligned}$$

- ______________: Rotation invariant (Rotating image + Filtering = Filtering +Rotating)
- The diagonal direction can be incorporated by adding two more terms into above equation. (isotropic for 45°)

Laplacian Filter Mask

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

Original+Laplacian Filter

Two-pass version

Definition

______________________________________:

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if the center coff of Laplacian mask is negative} \\ f(x,y) + \nabla^2 f(x,y) & \text{if the center coff of Laplacian mask is positive} \end{cases}$$

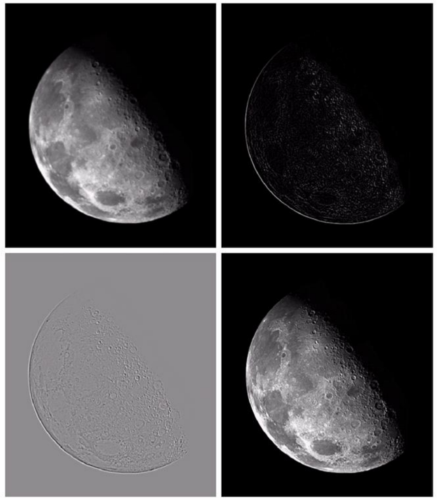
- Functionality: Background features can be “recovered” while still perserving the sharpening effect

Laplacian Filtering

Example

a b
c d

FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)



Original+Laplacian Filter

One-pass Version

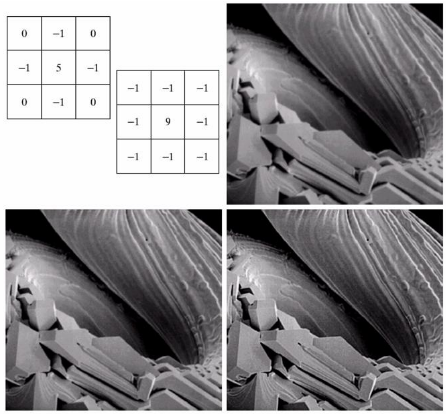
Definition

One pass filtering for Original+Laplacian:

$$\begin{aligned} g(x,y) &= f(x,y) - \nabla^2 f(x,y) \\ &= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y) \\ &= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] \end{aligned}$$

Original+Laplacian Filter

One-pass Version: Example



a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Unsharp Masking and High-boost Filtering

Definition

____________ $f_s(x,y)$ (obtained by ____________) consists of subtracting a blurred version of image from the image itself. (Used in publishing industry)

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

where $\bar{f}(x,y)$ is the **blurred version** of $f(x,y)$;

Definition

____________ $f_{hb}(x,y)$ is a **generalization of unsharp masking**

$$\begin{aligned} f_{hb}(x,y) &= A \cdot f(x,y) - \bar{f}(x,y) \\ &= (A-1) \cdot f(x,y) + f(x,y) - \bar{f}(x,y) \\ &= (A-1) \cdot f(x,y) + f_s(x,y) \end{aligned}$$

High-boost filtering

The above $f_{hb}(x,y)$ is applicable in general and does not state explicitly how the sharp image is obtain.
Thus, we can set $f_s(x,y)$ as Laplacian as below,

$$f_s(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if the center coff of Laplacian mask is negative} \\ f(x,y) + \nabla^2 f(x,y) & \text{if the center coff of Laplacian mask is positive} \end{cases}$$

and plug into $f_{hb}(x,y)$ to obtain the below high-boost filtering

High-boost filtering

Definition

$$f_{hb}(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{if the center coff of Laplacian mask is negative} \\ Af(x,y) + \nabla^2 f(x,y) & \text{if the center coff of Laplacian mask is positive} \end{cases}$$

- If $A = 1$, f_{hb} is Laplacian sharpening
- If A is large enough, f_{hb} = original image multiplied by a constant
- If A increases past 1, the contribution of the sharpening process becomes less and less important.

High-boost filtering
Design

0	−1	0
−1	A + 4	−1
0	−1	0

−1	−1	−1
−1	A + 8	−1
−1	−1	−1

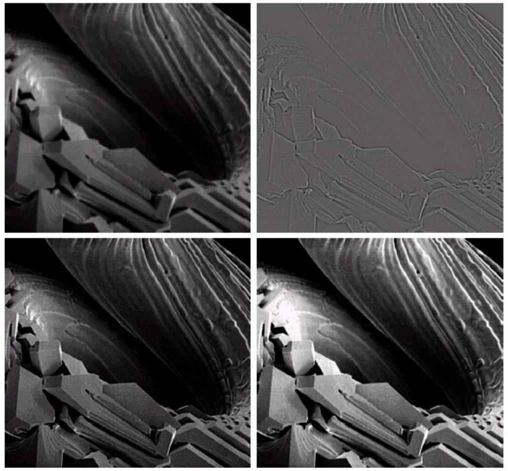
a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

High-boost filtering
Example

a b
c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



Gradient Filtering

Gradient Operation: 2-D 1st-order derivative

Definition

____________: (**Not isotropic but Linear**)

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Definition

_______: Magnitude of Gradient vector (**Isotropic in** $90°$/**Nonlinear**)

$$\begin{aligned} \nabla f &= mag(\nabla \mathbf{f}) \\ &= [G_x^2 + G_y^2] \, (\approx |G_x| + |G_y|) \\ &= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right] \end{aligned}$$

Gradient Filtering
Roberts Cross-gradient Operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Definition

____________________:

$$\begin{aligned} \nabla f &\approx |G_x| + |G_y| \\ &= |z_9 - z_5| + |z_8 - z_6| \end{aligned}$$

- The coffs are summed to 0: give a response of 0 in an area of constant gray level
- Even size is awkward to implement

Gradient Filtering
Sobel Cross-gradient Operators

Definition

____________________:

$$\begin{aligned} \nabla f &\approx |G_x| + |G_y| \\ &= |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \\ &\quad +|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)| \end{aligned}$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

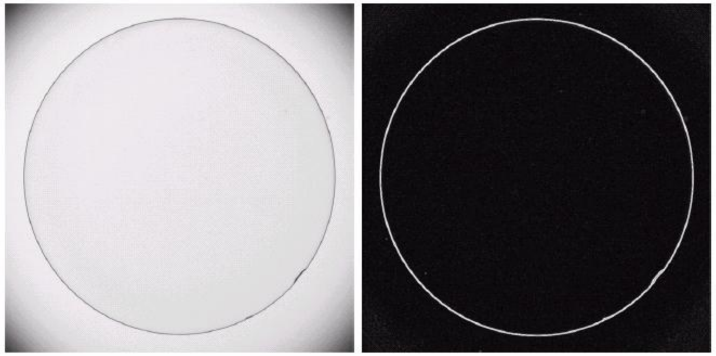
−1	−2	−1
0	0	0
1	2	1

[illegible]	[illegible]	[illegible]
−2	0	2
−1	[illegible]	[illegible]

- The coffs are summed to 0: give a response of 0 in an area of constant gray level
- The weight value of 2 is to achieve some smoothing by **giving more improtantce to the center point.**

Edge Detection Application

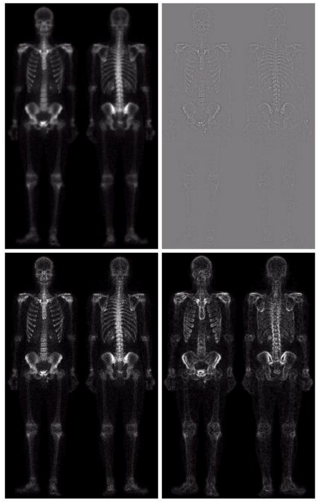
Automated inspection:



a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

Edge Detection Application



Edge Detection Application