Morphological Image Processing

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Morphology

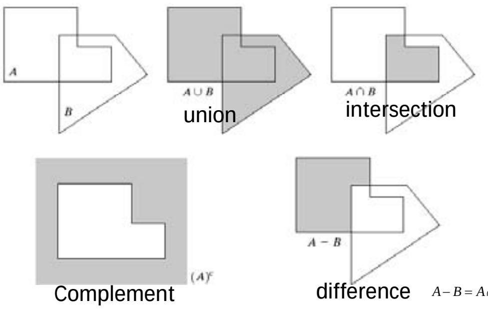
- Morphology in biology: Form and structure of animals and plants.
- Mathematical morphology: a tool for extracting image components
  - The representation and description of region shape
    - Boundaries
    - Skeleton
    - Convex Hull
  - Applications: Extract “meaning” from an image
    - Mophological filtering, thinning and pruning

Preliminaries

Set Theory

- Set Theory: Set = Object in image
- Binary Image:
  - $\{(x_1,y_1),(x_2,y_2),\ldots\}$
  - Z^2 tupple: The set of all black pixels is a complete morphological description of image
- Gray-scale Image:
  - $\{(x_1,y_1,I_1),(x_2,y_2,I_2)\ldots\}$
  - Z^3 tupple
- Higher dimensional tupple:
  - Contain other image attributes:
    - Color, Time varying component

Set Theory

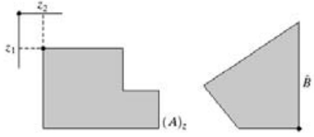


a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

$A - B = A \cap B^c$

Set Theory



a b

FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

$(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$

translation

$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$

reflection

Logic Operations Involving Binary Image

- AND, OR and NOT:
  - funtionally complete: can be combined to form any other logic operation
  - In binary image: AND=intersection,…

TABLE 9.1 The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

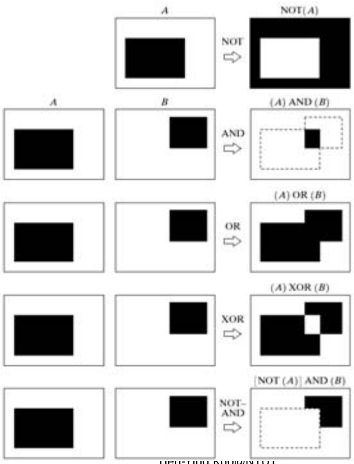
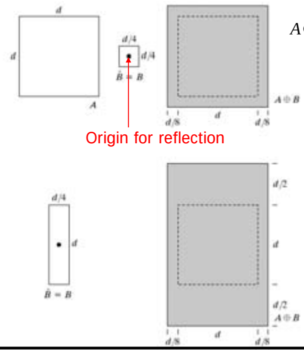


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Dilation and Erosion (binary image)

Dilation

FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.



$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$
$$= \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

- B:structuring Element
- B: viewed as a convolution mask
- Dilation of A by B: The set of all displacements, z, such that $\hat{B}$ and A overlap by at least one element

Dilation

- Application: Bridging Gaps
  - Known: Max break length=2 pixel
    - To Design B
  - Result directly in a binary image
    - Better than LPF: I/P binary => O/P gray-scale

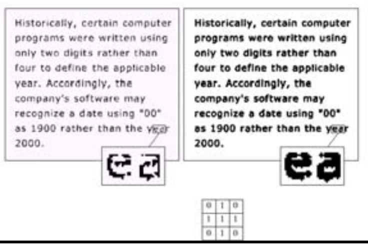
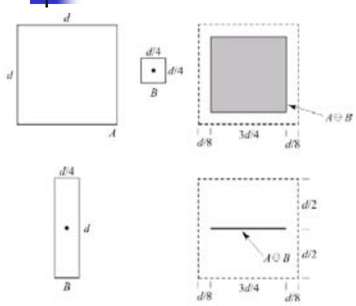


FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Erosion



Eroded to a line

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

- Erosion of A by B
  - More than one def
- Further displacement of the origin of B would cease being completely contained in A
- Dilation and Erosion are dual with respect to complementation and reflection

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A\}^c$$
$$= \{z \mid (B)_z \cap A^c = \varnothing\}^c$$
$$= \{z \mid (B)_z \cap A^c \neq \varnothing\}$$
$$= A^c \oplus \hat{B}$$

Erosion

- Application: Eliminating irrelevant detail (in terms of size)
  - Size of structuring element < Size of objects to keep
  - After erosion:
    - Restore the kept objects by dilation
    - But dilation does not fully restore eroded objects in general (because: erode+dilate=opening)

Erosion

- To keep 15x15, use B: 13x13

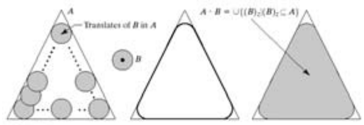
a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening and Closing (binary image)

Opening

- Opening: $A \circ B = (A \ominus B) \oplus B$
- Effects: $= \cup\{(B)_z \mid (B)_z \subseteq A\}$
  - Smooth the contour of an object
  - Break narrow isthmuses
  - Eliminate thin protrusion



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

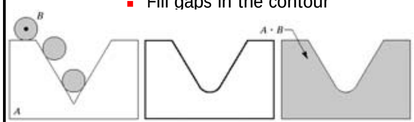
- B: Rolling Ball
- Geometric Fitting Property: The boundary of opening is the points in B that reach the farthest into the boundary of A as B is rolled around the inside of the boundary

Opening

- Property:
  - $A \circ B$ is a subset (subimage) of A
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  - Multiple opening have no effect after the operator has been applied once: $(A \circ B) \circ B = A \circ B$

Closing

- Closing: $A \bullet B = (A \oplus B) \ominus B$
- Effects:
  - Smooth the contour of an object
  - Fuse narrow breaks and long thin gulfs
  - Eliminate small holes
  - Fill gaps in the contour



a b c

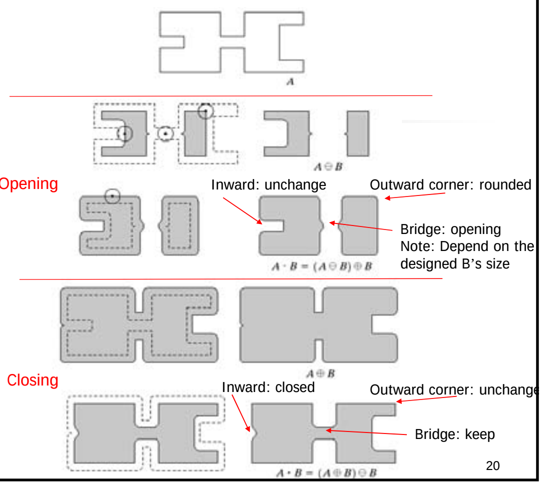
FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

- Dual:
  - $(A \bullet B)^c = (A^c \circ \hat{B})$
  - Rolling B on the outside of the boundary

Closing

- Property:
  - A is a subset (subimage) of $A \bullet B$
  - If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
  - Multiple opening have no effect after the operator has been applied once:
    $(A \bullet B) \bullet B = A \bullet B$

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



Opening and Closing: Eliminate noise

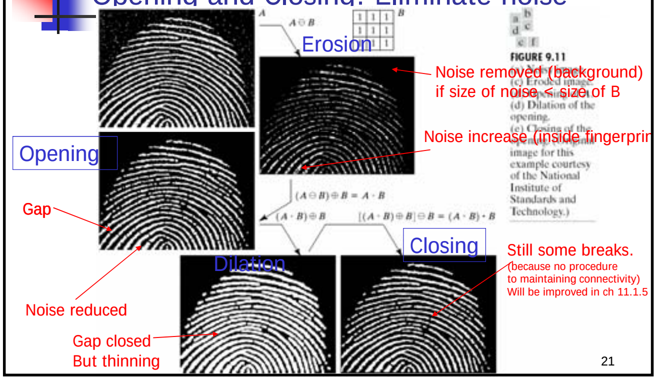


FIGURE 9.11

Hit-or-Miss (binary image)

Hit-or-Miss

- Morphological Hit-or-Miss Transfrom:

$$A * B = (A \ominus X) \cap [A^c \ominus (W - X)]$$
$$= (A \ominus B_1) \cap (A^c \ominus B_2) \text{ where } B_1 = X, B_2 = W - X$$
$$= (A \ominus B_1) \cap (A \oplus \hat{B}_2) \text{ by dual}$$

- $B=(B_1,B_2)$
  - B: The set composed of X and its background
  - B_1: X, The set formed form elements of B associated with an object
  - B_2: W-X, The set of elements B associated with the corresponding background
- Shape Detection

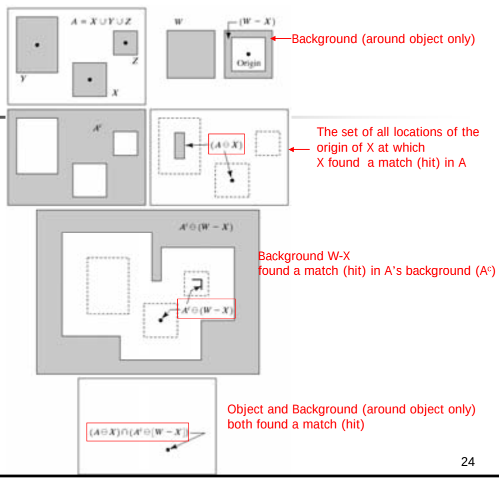


FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of X with respect to W, (W − X). (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by (W − X). (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

Hit-or-Miss

- Assumption:
  - Two or more objects are
    - Distinct
    - Disjoint, i.e., each object has at least a one-pixel-thick background around it.
- In some application, we are only interested in detecting some pattern within a set, then
  - Background matching is not required (second term)
  - Hit-or-miss is reduced to simple erosion (first term)

Some basic morphological Algorithm (binary image)

Some basic morphological Algorithm

- Boundary Extraction
- Region Filling
- Extraction of Connected Components
- Convex Hull
- Thinning
- Thickening
- Skeletons
- Pruning

Boundary Extraction

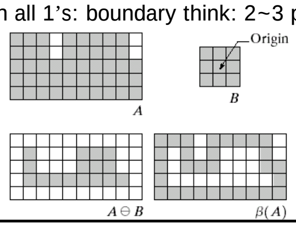
- The boundary of a set A:

  $\beta(A) = A - (A \ominus B)$

  - Structuring Element B: not unique
    - Ex: 5x5 with all 1's: boundary think: 2~3 pixels

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



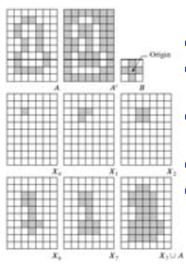
Boundary Extraction

a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

B: 3x3 with all 1's: result in 1 pixel thick boundary

Region Filling



- Region Filling:

  $X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \ldots$

  - Structuring Element B: as shown
  - X_0=p, where p is a point inside boundary
  - Conditional Dilation($\cap A^c$): Limit to inside the ROI
  - Iteration Termination: Until X_k=X_{k-1}
  - More subsets: Just need to give a point inside each boundary

Region Filling

a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

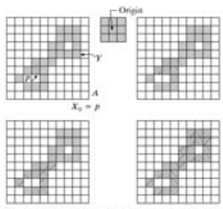
It must be known whether black points are background points or sphere inner points

Extract of Connected Components

- Extract of Connect Components:

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

- Structuring Element B: as shown for 8-connectivity between pixels
- X_0=p, where p is a point of a connected component
- Conditional Dilation($\cap A^c$): Limit to inside the ROI
- Iteration Termination: Until X_k=X_{k-1}
- More subsets: Just need to give a point in each connected component
- Diff from Region Filling: A vs A^c
  - Because elements sought (i.e., connected components) are labled 1



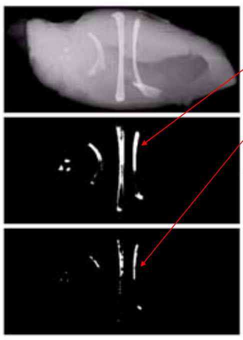
a b c
d e

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Extract of Connected Components

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



- Automated inspection

1. Threshold is used to identify the bone

2. Only objects of "significant" size remain by eroding (5x5 structuring element of 1's) the thresholded image

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

3. To analyze the size of the remained objects

Convex Hull

- S is Convex: The straight line segment joining any two points in S lies entirely within S
- Convex Hull C(A) of A: The smallest convex set containing the set A
- Convex Deficiency of A: Set difference C(A)-A

Convex Hull

- The Convex Hull of A

$$C(A) = \bigcup_{i=1}^{4} D_i = \bigcup_{i=1}^{4} X^i_{converge}$$

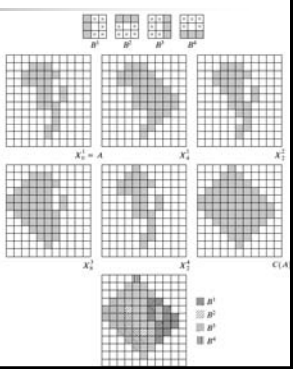
$$X^i_k = (X_{k-1} * B^i) \cup A \quad i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3\ldots$$

$$X^i_0 = A$$

- B^i is a clockwise rotation of B^{i-1} by 90 degree
- x in B^i: don't care
- A pattern match occus when the center of the 3x3 region in A is 0

a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



Convex Hull

- C(A) can grow beyond the minimum dimensions required to guarantee convex
  - Sol 1: To limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points
  - Sol 2: limit in vertical, horizontal and diagonal directions
    - Require more complexity

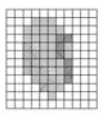


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

Thinning

- Thinning $A \otimes B = A - (A * B)$

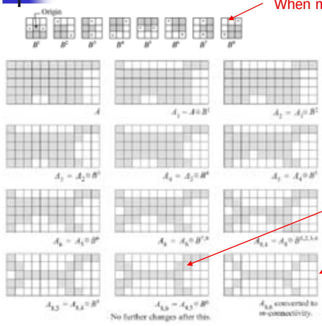
$$= A \cap (A * B)^c$$

  - *: here no background operation is required, because we are interested only in pattern matching with the structuring elements
- Thinning by a sequence of structuring element

$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$

  - B^i is a rotated version of B^{i-1}: To thin symmetrically
  - $\Rightarrow B^1 \Rightarrow B^2 \Rightarrow \ldots \Rightarrow B^n \Rightarrow B^1 \Rightarrow B^2 \Rightarrow \ldots \Rightarrow B^n \Rightarrow$ until no further change

Thinning



Thickening

- Thickening

$$A \odot B = A \cup (A * B)$$

  - *: here no background operation is required, because we are interested only in pattern matching with the structuring elements
- Thickening by a sequence of structuring element

$$A \odot \{B\} = ((...((A \odot B^1) \odot B^2)...) \odot B^n)$$

  - B^i is a rotated version of B^{i-1}: To thicken symmetrically
  - B has the same form as thinning, but with all 1's and 0's exchange (dual)
  - $\Rightarrow B^1 \Rightarrow B^2 \Rightarrow \ldots \Rightarrow B^n \Rightarrow B^1 \Rightarrow B^2 \Rightarrow \ldots \Rightarrow B^n \Rightarrow$ until no further change
- Thickening is the morphological dual of thinning

Thickening

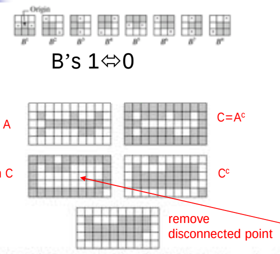


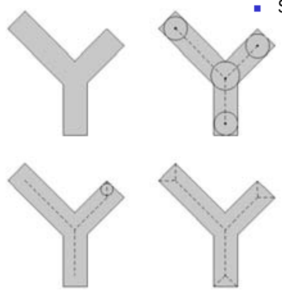
FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

- Another Approach is more popular in practical: Use background thinning to accomplish thickening
- To thicken a set A
  - Form $C=A^c$
  - Thin C
  - Form C^c
  - Post-processing C^c to remove disconnected point

Side-Benefit: Thinned background forms a boundary for the thickening process

Skeleton

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.



- Skeleton S(A):
  - If z is a point of S(A) and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A. The disk $(D)_z$ is called a maximum disk
  - The disk $(D)_z$ touches the boundary of A at two or more different places.

Skeleton

- Skeleton:

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = (...(A \ominus B) \ominus B) \ominus ...) \ominus B$$

$$K = \max\{k \mid (A \ominus kB) \neq \varnothing\}$$

- $S_k(A)$: Skeleton subset
- K: The last iterative step before A erodes to an empty set
- S(A) guarantee connectivity of output image
- If the skeleton must be max thin, connected, and min eroded: See Ch 11.1.5

Skeleton ← → Reconstruction

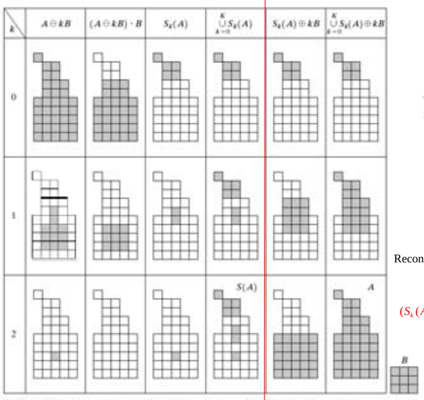


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Skeleton:

$$S(A)=\bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A)=(A\ominus kB)-(A\ominus kB)\circ B$$

Reconstruction:

$$A=\bigcup_{k=0}^{K}(S_k(A)\oplus kB)$$

$$(S_k(A)\oplus kB)=((\ldots(S_k(A)\oplus B)\oplus B)\oplus\ldots)\oplus B$$

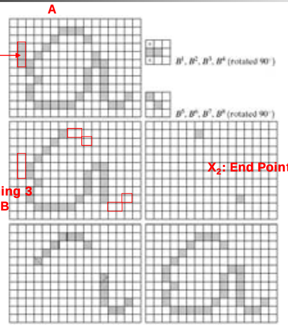
Pruning

- Why Pruning:
  - Thinning & Skeletoning tend to leave parasitic components (spurs) that need to be "cleaned up" by postprocessing
  - Automated recognition of hand-printed character requires to analyze the skeleton without spurs
- Pruning:
  - Need to know the length of spur
  - Suppress a spur by successively eliminating its end point

Pruning

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Spur to remove



X_1: Thinning 3 times by B

X_2: End Point

X_3: Dilation of End Point X_2 3 times

Pruned Image: $X_4=X_1\cup X_2$

- Assume: spur lenght=3
  - So run 3 times to get X_1 and X_3
- X_1: Thinning 3 times:

$$X_1=A\otimes\{B\}$$

- X_2: To get End Point

$$X_2=\bigcup_{k=1}^{8}(X_1 * B^k)$$

- X_3: Dilate 3 times (grow back non-spur portion)

$$X_3=(X_2\oplus H)\cap A$$

H is 3x3 of 1's

- X_4: Pruned image

$$X_4=X_1\cup X_3$$

Summary

- Five Basic types of structuring elements

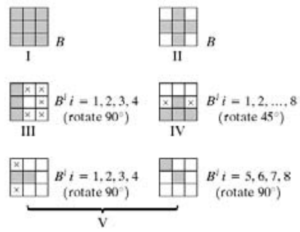


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

Extensions of Gray-Scale Images

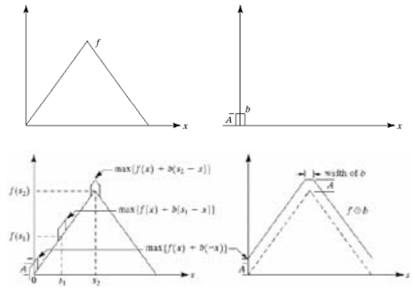
Extensions to Gray-Scale Images

- Binary Image:
  - $\{(x_1,y_1),(x_2,y_2),\ldots\}$
  - Z^2 tupple: The set of all black pixels is a complete morphological description of image
- Gray-scale Image:
  - $\{(x_1,y_1,I_1),(x_2,y_2,I_2)\ldots\}$
  - Z^3 tupple

Dilation

- Gray Scale Dilation of f by b
  $(f \oplus b)(s,t)$
  $= \max\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$
- f(x,y): input image
- b(x,y): structuring element
- D_f: Domain of f
- D_b: Domain of b
- Two sets have to overlap at least one element
- Cf: 2-D convolution:
  - max ⇔Σ
  - + ⇔ *
- f shifts here (Cf: b shfits in binary image) to get a simpler math form than b shift version

Dilation

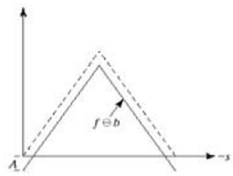


Erosion

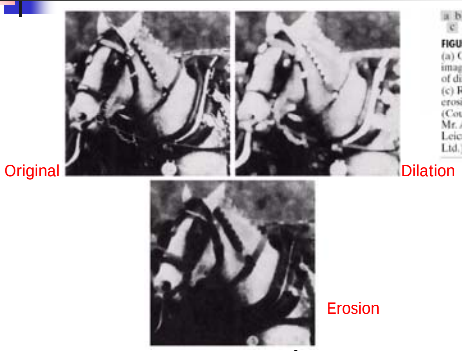
- Gray Scale Erosion of f by b
  $(f \ominus b)(s,t)$
  $= \min\{f(s+x,t+y)-b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$
- f(x,y): input image
- b(x,y): structuring element
- D_f: Domain of f
- D_b: Domain of b
- Two sets have to overlap at least one element
- Cf: 2-D convolution:
  - min ⇔Σ
  - - ⇔ *
- f shifts here (Cf: b shfits in binary image) to get a simpler math form than b shift version

Erosion

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



Dilation and Erosion



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Dilation and Erosion

- Dilation effects:
  - If all values of b > 0, the output image tends to be brighter than the input
  - Dark details either are reduced or eliminated depends on the values and shapes of f related to b
- Erosion effects:
  - If all values of b > 0, the output image tends to be darker than the input
  - Brighter details in f are smaller in area than b is reduced, with the degree of reduction being determined by the gray-level values surrounding the bright detail and by the shape and amplitude values of b
- Dual: with respect to function complementation and reflection

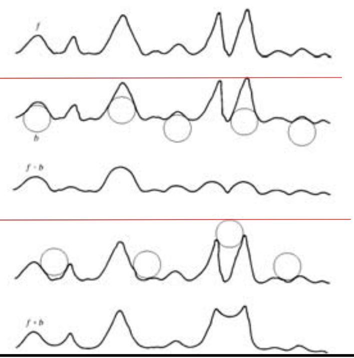
$$(f \ominus b)^c(s,t) = (f^c \oplus \hat{b})(s,t)$$

Opening and Closing

- Opening: $f \circ b = (f \ominus b) \oplus b$
- Closing: $f \bullet b = (f \oplus b) \ominus b$
- Dual: $(f \bullet b)^c = (f^c \circ \hat{b})$
- Because $f^c = -f(x, y)$

  $-(f \bullet b) = (-f \circ \hat{b})$
- Rolling ball operation

Opening and Closing



a b c d e

FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

Opening and Closing

- Opening:
  - Remove small (with respect to b's size) light detail, while leaving the overall gray levels and larger bright features relatively unchanged
- Closing:
  - Remove dark details from an image, while leaving bright features relatively unchanged

Opening and Closing

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Opening:
Remove small light details

Closing:
Remove small dark details

Opening and Closing

- Properties of Opening:

  (i) $(f \circ b) \dashv f$

  (ii) If $f_1 \dashv f_2$, then $(f_1 \circ b) \dashv (f_2 \circ b)$

  (iii) $(f \circ b) \circ b = f \circ b$
- Properties of Closing:

  (i) $f \dashv (f \bullet b)$

  (ii) If $f_1 \dashv f_2$, then $(f_1 \bullet b) \dashv (f_2 \bullet b)$

  (iii) $(f \bullet b) \bullet b = f \bullet b$
- The notation $e \dashv r$ is used to indicate that the domain of e is a subset of the domain of r, and also that $e(x,y) \leq r(x,y)$ for any (x,y) in the domain of e

Some Applications of Gray-Scale Morphology

- Morphological Smoothing
- Morphological Gradient
- Top-Hat Transform
- Textural Segmentation
- Granulometry

Morphological Smoothing

- Opening then Closing
- Remove or attenuate both bright and dark artifacts or noise

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Morphological Gradient

- $g=(f\oplus b)-(f\ominus b)$
- Highlight sharp gray-level transition
- Cf. Sobel or Robert Gradient:
  - Using symmetrical structuring elements tend to depend less on edge directionality

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Top-Hat Transform

- $h=f-(f\circ b)$
- Enhance detail in the presence of shading
- Top-Hat: Use a cylindrical or parallelepiped structuring element with a flat top

Enhance detail in the presence of shading

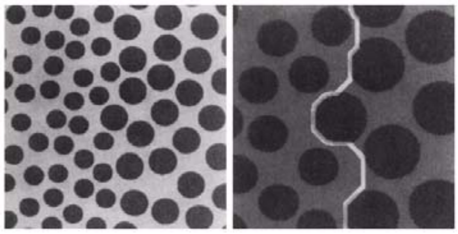
FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Textural Segmentation

- Objective: Find the boundary between the two regions based on their texture content

a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)



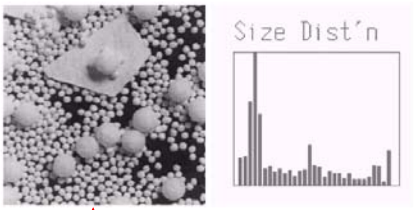
Textural Segmentation

- Produce a light region on the left:
  - Close the input image by using successively larger structuring elements b. (Closing operation is to remove dark details, or blobs here)
  - When the size of b corresponds to that of small blobs, the blobs is removed, and leaving
    - Left: light background
    - Right: larger blobs & light background between the large blobs
- Produce a dark region on the right:
  - A single opening is performed with b that is large in relation to the separation between the large blobs.
  - Then the light background between larger blobs is removed as the dark region
- Threshold yields the boundary

Granulometry

- Determine the size distribution of particles in an image
- Basic idea:
  - Opening operations of a particular size have the most effect on regions of the input image that contain particles of similar size
  - Thus, a measure of the relative number of such particles is obtained by computing the difference between input and output images
- Steps:
  - Opening
  - Difference(orginal, opening) is computed after each pass when a different structuring element is completed.
  - Normalize the difference to construct a histogram of particle-size distribution

Granulometry



a b
FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

3 Objects: Overlapping & too clustered