Advanced Digital Image Processing

Chapter 5: : Image Restoration

Tien-Ying Kuo
E-mail: tykuo@ee.ntut.edu.tw

Department of Electrical Engineering
National Taipei University of Technology

Outline

Image Restoration

Definition

__________________ is,

- An objective process
- To reconsruct or recover an image that has been degraded by using a priori knowledge of the degradation phenomenon.

Example

Removal of Image Blur

Image Restoration

Problem Formation

Problem Formation

Given g(x,y) ang some knowledge about $H(x,y), \eta(x,y)$
$\Rightarrow$ The goal is to obtain an estimate $\hat{f}(x,y)$ of original image $f(x,y)$ (the more knowledge, the closer)

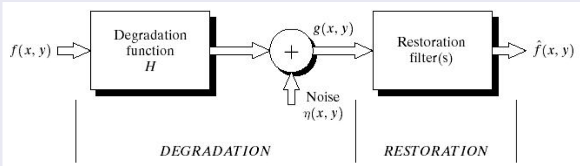


FIGURE 5.1 A model of the image degradation/restoration process.

- ____________: $g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$
  (Other notation: $g(x,y) = H[f(x,y)] + \eta(x,y)$)
- ________________: $G(u,v) = H(u,v)F(u,v) + N(u,v)$
- $h(x,y), H(u,v)$: __________________, set as identify operator if only dealing with noise degradation.

Noise
Cause and Characteristics

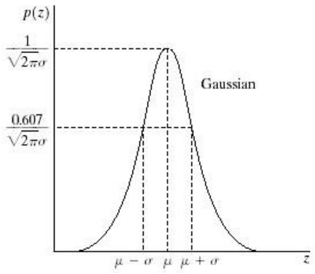
Source of Noise

- ____________________
  - **Quality of the sensor**: ccd vs cmos
  - **Environmental Condition**: Light levels, sensor temperature.
- ____________: Wireless under lightning, atmospheric disturbance.

Characteristics

- ___________: Fourier specturm of noise is constant, i.e., containing nearly all freqs in the visible spectrum in equal proportions.
- _______________: No correlation between pixel values and values of noise components, i.e., noise is independent of spatial coordinates.
- cf. ⇔_____________: Correlated noise

Noise Probability Density Function
Gaussian Noise



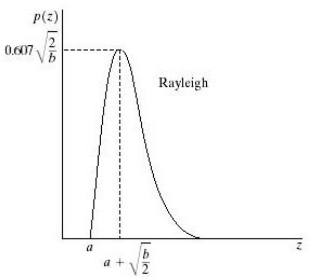
_______________:
(Electronic ckt,Sensor noise under poor light/temp)

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

where
z: Gray level
μ: Mean of z
σ: Standard deviation of z, σ^2: variance of z

- z: 70% in $[(\mu-\sigma, \mu+\sigma)]$
  95% in $[(\mu-2\sigma, \mu+2\sigma)]$
- **Popularly used** due to tractablility (spatial↔freq)

Noise Probability Density Function
Rayleigh noise



_______________: (Range imaging)

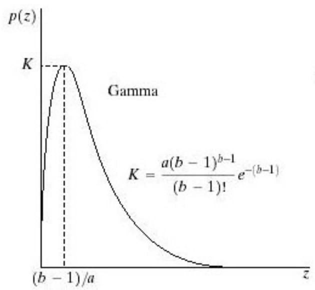
$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

where

$$\begin{cases} \mu = a + \sqrt{\pi b/4} \\ \sigma^2 = \frac{b(4-\pi)}{4} \end{cases}$$

- Basic shape of this density is skewed to the right.
- Useful for approximating skewed histogram.

Noise Probability Density Function
Erlang (Gamma) Noise



_____________________: (Laser imaging)

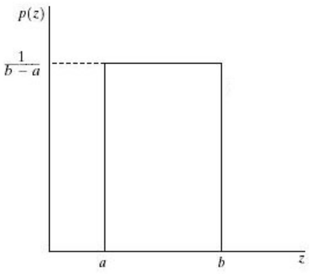
$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

where

$$\begin{cases} \mu = \frac{b}{a} \\ \sigma^2 = \frac{b}{a^2} \end{cases}$$

- $a > 0$, b is a positive integer.
- Gamma noise is namely because the denominaotr is gamma function $\Gamma(b)$

Noise Probability Density Function
Uniform Noise



__________:

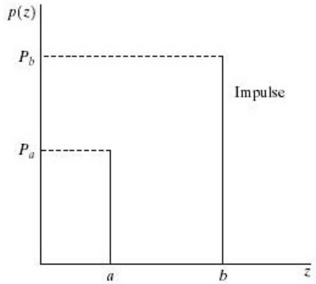
$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

where

$$\begin{cases} \mu = \frac{a+b}{2} \\ \sigma^2 = \frac{(b-a)^2}{12} \end{cases}$$

Noise Probability Density Function
Impulse (Salt-and-pepper) Noise

__________:
(Quick transition/Faulty switching)



$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{Otherwise} \end{cases}$$

where

a, b: Two tones of noise (can be positve/negative before normalized)

- Either P_a or P_b is zero:
  ____
- If $P_a \approx P_b \neq 0$: __________
- Impulse correction usually is larger with the strength of image signal, a, b is saturized, say, $a = 0$ and $b = 255$. (black and white)

Noise example
example

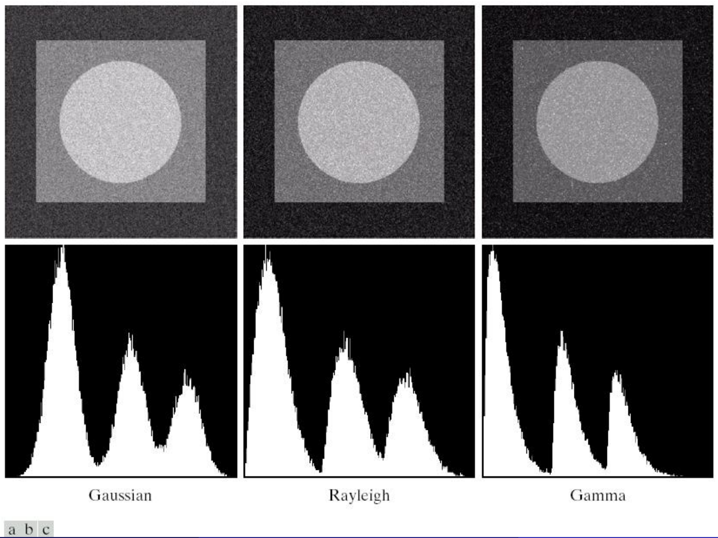


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Noise Probability Density Function
example

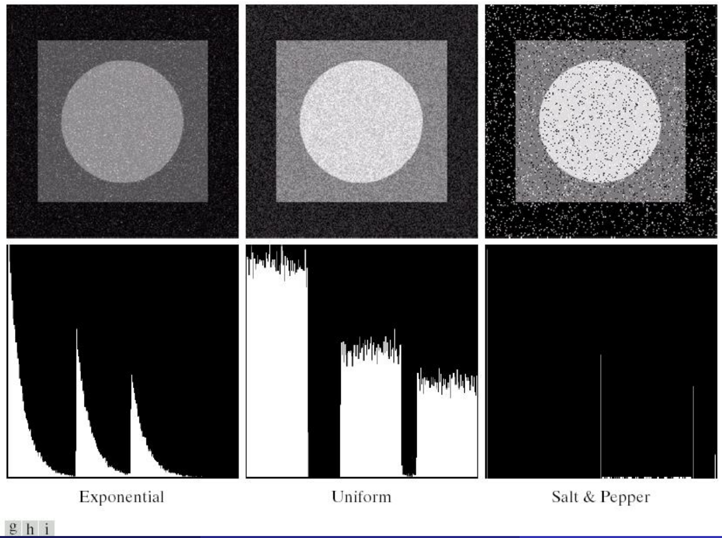
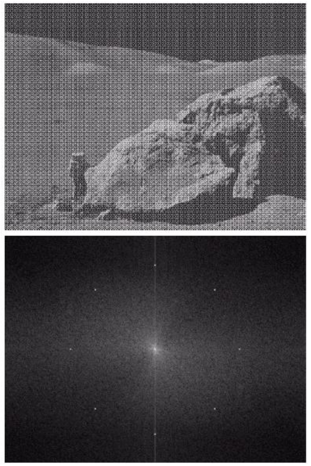


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

Periodic Noise
Example

a
b
FIGURE 5.5
(a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

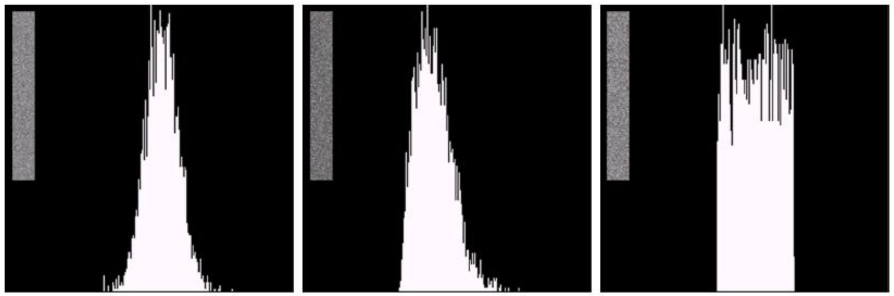


________:
- Spatially dependent noise
- From electrical or electromechanical interference during image acquisition.
- Sinusoids in Spatial $\Rightarrow$ Two peaks in Freq

Estimation of Noise Parameter

- ________________:
  - From FFT peaks: Easier
  - From spatial: Only for simple case
- ________________:
  - From histogram distribution (shape)
  - From histogram mean and variance $\begin{cases} \mu = \sum_{z_i \in S} z_i p(z_i) \\ \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 p(z_i) \end{cases}$
  - ________: Estimate μ and σ^2 only.
  - ______________________: a and b from μ and σ^2
  - _______: P_a and P_b by the height of black and white peaks.

Estimation of Noise Parameter
Example



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Degradation by Noise Only
Model

Problem

Model for considering only ______________,
Time domain:

$$g(x,y) = f(x,y) + \eta(x,y)$$

Frequency domain:

$$G(u,v) = F(u,v) + N(u,v)$$

Distorted image $g(x,y)$ is given, and how to solve oringal image $f(x,y)$?

Solution

- $\eta(x,y)$ **is unknown** so that the substraction to get $f(x,y)$ does not work. $\Rightarrow$ However, for ____________, the $\eta(x,y)$ can be obtained.
- ____________ works well when only additive noise presented.

Mean Filters I

Definition

____________________:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(x,y)$$

- Can be implemented by convolution mask $\frac{1}{mn}$.
- Noise is removed by blurring (Smoothing local variatins)

Definition

____________________:

$$\hat{f}(x,y) = \left(\prod_{(s,t)\in S_{xy}} g(x,y) \right)^{\frac{1}{mn}}$$

- Lose less detail than arithmetic.

Mean Filters II

Definition

____________________:

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(x,y)}}$$

- Work well salt noise, Guassian, etc. But fails for pepper noise.

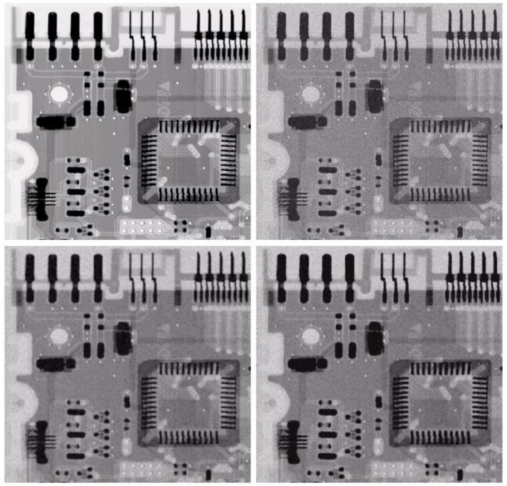
Definition

________________________:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(x,y)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(x,y)^{Q}}$$

- Good for salt-and-pepper noise. Order $Q > 0$: **Pepper nosie**, $Q < 0$: **Salt noise**, but cannot eliminate both simultaneously
- $Q = 0$ Arithmetic, $Q = -1$ Harmonic

Mean Filters
Example



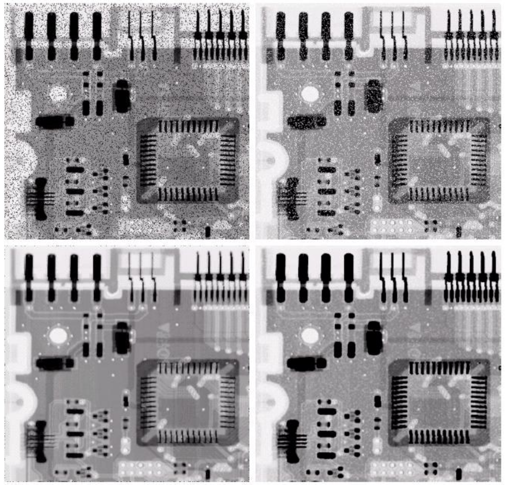
a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

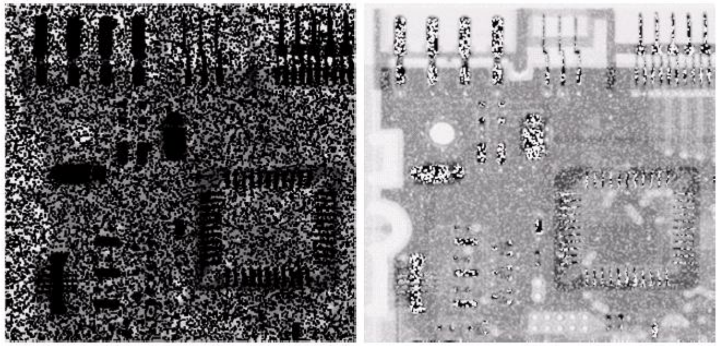
Mean Filters
Example

a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



Mean Filters
Selecting Wrong Sign in Contraharmonic Fitlering



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Mean Filter
Summary

- ____________________________________:
  Well suited for random nosie, such as Gaussian or Unifrom noise
- ____________________:
  Impulse Noise, but it must be known the noise is dark or light for selecting Q.

Order-Statistics Filters I

Definition

__________: $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$

- Most popular: Effective in **both bipolar and unipolar impulse** noise.
- Less blurred than linear smoothing filters of similar size.

Definition

________: $\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$

- Useful for finding **brightest point**: Remove pepper noise

Definition

________: $\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$

- Useful for finding **darkest** point: Remove Salt noise

Order-Statistics Filters II

Definition

______________: $\hat{f}(x,y) = \frac{1}{2}\left(\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right)$

- Combine **order statistics and averaging**
- Best for **randomly distributed noise**(ex: Gaussian or unifrom noise.)

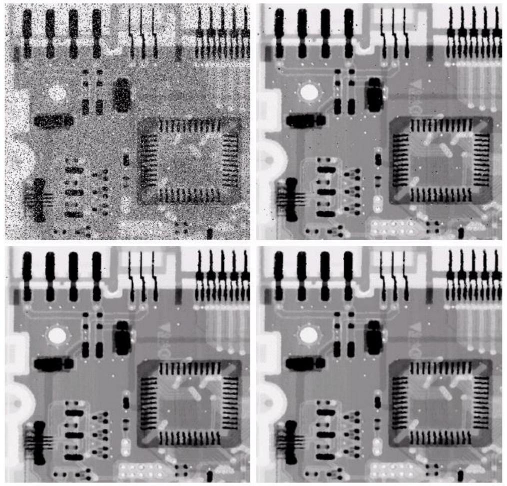
Definition

________________________: $\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$ where

$0 \le d \le m-1$

- $d/2$ lowest and $d/2$ highest gray-level in S_{xy} are deleted.
- $d = 0$: ______________, $d = (mn-1)/2$: _______
- d other value: useful for **mutliptle types of noise** (ex: impluse+Gaussian)

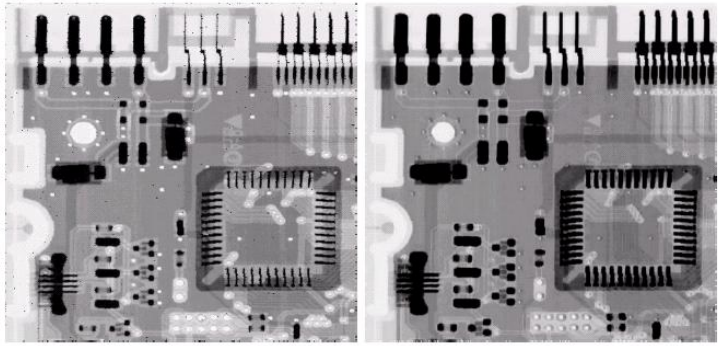
Order-Statistics Filters
Example



a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

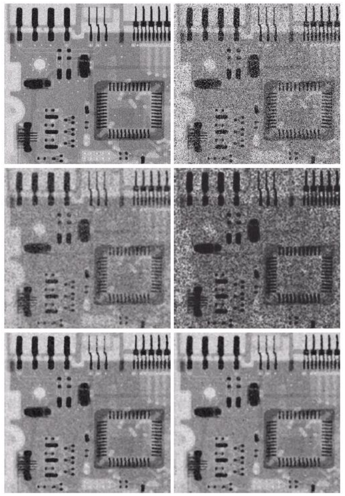
Order-Statistics Filters
Example



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Order-Statistics Filters
Example



Adaptive Filters

Definition

____________ is to design its **behavior to change based on statistical characteristics** of the image inside the filter reigion $m \times n$ window S_{xy}.

- Statistical characteristics: μ, σ^2, z_{max}, z_{min}, z_{mid} $\cdots$ etc.

Example

- ________________________
- ______________

Adaptive Local Noise Reduction Filter

Definition

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}(g(x,y) - m_L)$$

where σ_η^2: Variance of noise
σ_L^2: Local variance of pixels in S_{xy}
m_L^2: Local mean of pixels in S_{xy}

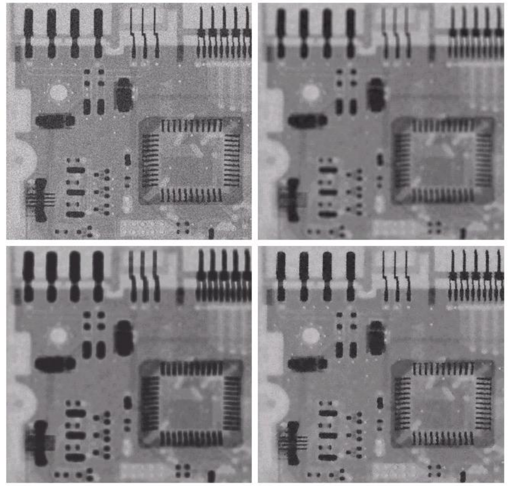
- $\sigma_\eta^2 = 0, f(x,y) = g(x,y)$.
- $\sigma_\eta^2 < \sigma_L^2$: Edge should be preserved.
- $\sigma_\eta^2 = \sigma_L^2$: Local area has same properties as overall image. m_L.
- $\sigma_\eta^2 > \sigma_L^2$: set $\frac{\sigma_\eta^2}{\sigma_L^2} = 1$ to avoid negative pixels/nolinear.
- **Estimation of** σ_η^2 will affect the results.

Adaptive Local Noise Reduction Filter

Example

a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.



Adaptive Mean Filter

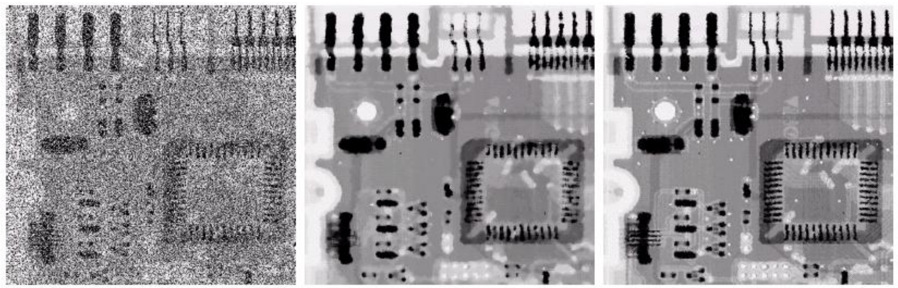
Adaptive Mean Filter:

- Preserve detail while smoothing nonimpulse noise.
- Adaptively changes the size of mask S_{xy}

```
Level A:

   if (Zmin < Zmed < Zmax)
       goto Level B //Zmed nonimpluse
   else
       increase the window size

   if (window size <= Smax)
       repeat Level A
   else
       output Zxy //may or maynot be impulse:
                  //large Sxy or small Pa&Pb->noimpulse
```

```
Level B:

   if ( Zmin < Zxy < Zmax)
       output Zxy //Zxy nonimpulse
   else
       output Zmed
```


Adaptive Mean Filter



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7 × 7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Bandreject Filters

Definition

________: $H(u,v) = \begin{cases} 1 & \text{if } D(u,v) < D_0 - \frac{W}{2} \\ 0 & \text{if } D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & \text{if } D(u,v) > D_0 + \frac{W}{2} \end{cases}$

Definition

________: $H(u,v) = \frac{1}{1+\left(\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right)^{2n}}$

Definition

________: $H(u,v) = 1 - e^{-\frac{1}{2}\left(\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right)^2}$

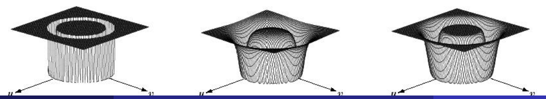
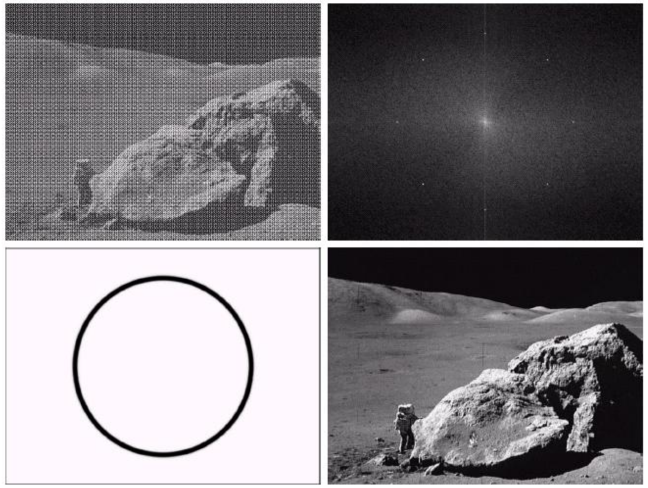


FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Bandreject Filters

Example



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Bandpass Filters

Definition

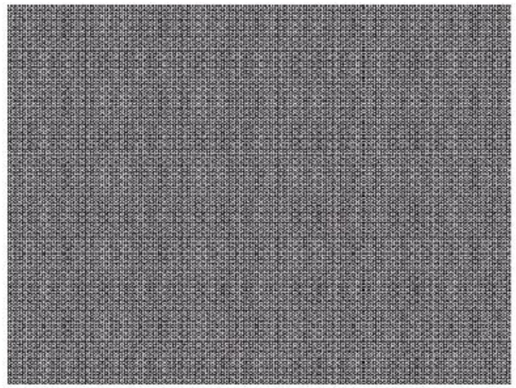
Bandpass filter $H_{bp}(u,v)$:

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

where $H_{br}(u,v)$ is band-reject filter.

Bandpass Filters: Example

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Notch (Reject) Filters

Definition

________________: $H(u,v) = \begin{cases} 0 & \text{if } D_1(u,v) \leq D_0 \text{or } D_2(u,v) \leq D_0 \\ 1 & \text{otherwise} \end{cases}$

Definition

________________: $H(u,v) = \frac{1}{1+[\frac{D_0^2}{D_1(u,v)D_2(u,v)}]^n}$

Definition

________________: $H(u,v) = 1 - e^{\frac{1}{2}[\frac{D_1(u,v)D_2(u,v)}{D_0^2}]}$

where

$$D_1(u,v) = [(u - M/2 - u_0)^2 + (v - M/2 - v_0)^2]$$
$$D_2(u,v) = [(u - M/2 + u_0)^2 + (v - M/2 + v_0)^2]$$

Note: $u_0 = v_0 = 0 \rightarrow$ HPF

Notch Pass Filter

Definition

________________:

$$H_{np}(u,v) = 1 - H_{nr}(u,v)$$

Note: $u_0 = v_0 = 0 \rightarrow$ LPF

Notch Filter

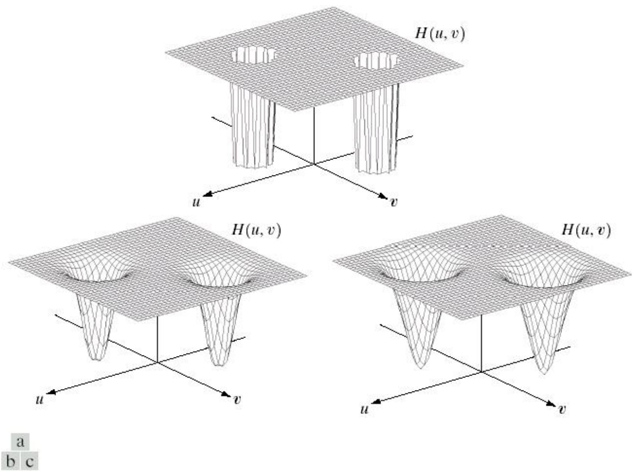
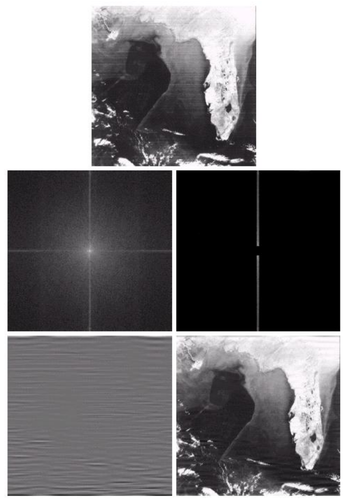


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

Notch Filter
Example



Optimum Notch Filtering
Problem Formulation

Problem

- Several interferences are present: Preceding approach may remove too much image information in the filtering process.
- The interferences are generally are not single frequency bursts, and those broad skirts may not easily to detect.
- Therefore, $\eta(x,y)$ **may not present all inteferences, and the directly substraction may not work well.**

Optimum Notch Filtering
Solution

Solution

Using ______________________:

- First step is to extract the principal freq component of the interference pattern with a notch pass filter at the location of each spike. This result is only the approximation of true pattern of noise.
- The effect not presetnt in the estimate of noise can be minimize by the following. Let $w(x,y)$ be a weighting or modulation funciton.

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$

Optimization: To select $w(x,y)$ so that the local variance of the estimated $\hat{f}(x,y)$ is minimized.

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

Optimum Notch Filtering
Proof

Proof.

Let $\bar{\hat{f}}(x,y)$ be the *average value* of $\hat{f}$ in the neighborhood

$$\bar{\hat{f}}(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{-a}^{a} \sum_{-b}^{b} \hat{f}(x+s,y+t)$$

Then, the *local variance* of $\hat{f}(x,y)$:
(assuming $w(x,y)$ constant in neighborhood)

$$\begin{aligned} \sigma^2(x,y) &= \frac{1}{(2a+1)(2b+1)} \sum_{-a}^{a} \sum_{-b}^{b} [\hat{f}(x+s,y+t) - \bar{\hat{f}}(x,y)]^2 \\ &= \frac{1}{(2a+1)(2b+1)} \sum_{-a}^{a} \sum_{-b}^{b} \{[g(x+s,y+t) \\ &\quad -w(x,y)\eta(x+s,y+t)] - [\bar{g}(x,y) - w(x,y)\bar{\eta}(x,y)]^2 \end{aligned}$$

$\square$

Optimum Notch Filtering
Proof (Cont.)

Proof.

Optimization: To select $w(x,y)$ so that the *local variance* of the estimated $\hat{f}(x,y)$ is minimized.
That is,

$$\frac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0$$

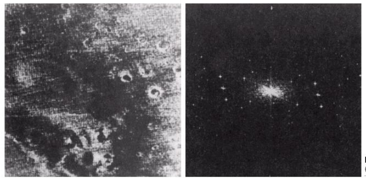
Obtaining

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

$\square$

Note: It is unnecessary to computing $w(x,y)$ for every (x,y) in image. As $w(x,y)$ is assumed to be constant in a neighborhood, it is only need to compute $w(x,y)$ for one point in each nonoverlapping neighorhood (preferably the center point)

Optimum Notch Filtering
Example



a b
FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

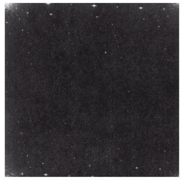
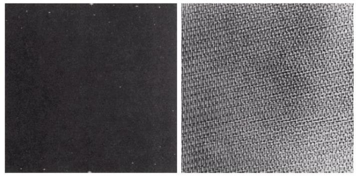


FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)



a b
FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

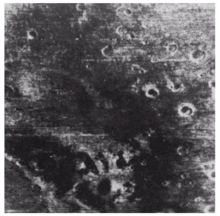


FIGURE 5.23 Processed image. (Courtesy of NASA.)

System Model and Impulse Response
Spatial Domain: General vs LPI

Considering continuous system:

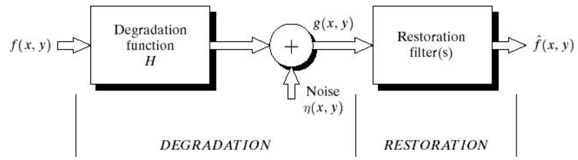


FIGURE 5.1 A model of the image degradation/restoration process.

________________ of degradation function:

$$H[\delta(x-\alpha, y-\beta)] = h(x, \alpha, y, \beta)$$

Note:

- ________________________: in optics, the impluse becomes point of light and __________ is commonly referred to as the PSF.
- If H is ________________: $H[\delta(x-\alpha, y-\beta)] = h(x-\alpha, y-\beta)$

Linear, Position-Invariant Degradations
Spatial Domain: LPI

For **linear** degradation model, H is linear

$$g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta)h(x, \alpha, y, \beta)d\alpha d\beta + \eta(x, y)$$

which is called the

____.

If H is **position invariant**

$$g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta)h(x-\alpha, y-\beta)d\alpha d\beta + \eta(x, y)$$

which is called the __________________

☺ Many types of degradations can be approximated by linear,position-invariant processes.

Linear, Position-Invariant Degradations
Frequency Domain & Summary

Frequency Domain,

$$\begin{aligned} g(x, y) &= h(x, y) * f(x, y) + \eta(x, y) \\ G(u, v) &= H(u, v)F(u, v) + N(u, v) \end{aligned}$$

Summary

- **Degratatoins** ⇔ The result of **Convolution**.
- **Linear image restoration**⇔ **Image De-convolution**
- **Filters used in the resotration process** often are called ____________________.

Estimating the Degradation Function For Image Restoration

Three principle ways

- ____________
- ________________
- _____________

Since the **true** degradation funciton is estimated and seldom known completely, this kind of process of restoring an image is called

____

Estimating the Degradation Function

Esitmation by Image Observation

Assuming H is structurelinear position-invarient system, and say, the image is blurred.

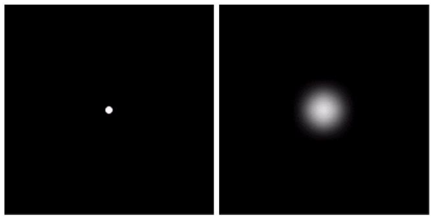
- Look at the small areas of simple structure with strong signal content (noise is negligible).
- Using observed sample subimage $f_s(u, v)$ to contruct an unblurred image $g_s(x, y)$.
- $H_s(u, v) = \dfrac{G_s(u, v)}{\hat{F}_s(u, v)}$

Estimating the Degradation Function

Esitmation by Experimentation

If **equipment** similar to the equipment used to acquire the degraded image is **available**,

- Feed in the impluse with strength A (a bright dot of light) to the equipment.
- Measure the output $G(u, v)$
- The degradation function is $H(u, v) = \dfrac{G(u, v)}{A}$
- The degradation function is possibly an accurate estimate.



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Estimating the Degradation Function

Esitmation by Modeling: Ex.1 Atomspheric Turbulence

Degradaton model for atomspheric turbulence (Hufnagel and Stanley[1964]).

$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$

- Similar to Gaussian LPF (except $5/6$ term): Blurring model.
- k $\uparrow \Rightarrow$ Sever turbulence.

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Estimating the Degradation Function
Estimation by Modeling: Ex.2 Motion Blur

If $f(x,y)$ undergoes planar motion $(x_0(t), y_0(t))$, the blurred output $g(x,y)$ is the totoal exposure when the shutter is open,

$$g(x,y) = \int_0^T f[x - x_0(t), y - y_0(t)]dt$$

Performing Fourier Transform

$$\begin{aligned} G(v,u) &= F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \\ &= F(u,v)H(u,v) \end{aligned}$$

where

Estimating the Degradation Function
Estimation by Modeling: Ex.2 Motion Blur

Example

If the image undergoes uniform linear motion $x_0(t) = at/T$, $y_0(t) = 0$.

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt = \frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$

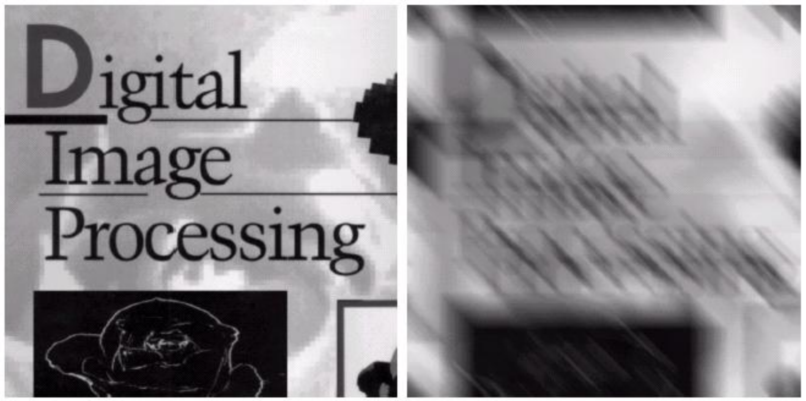
Note: H vanishes at $u = n/a$. (n is integer)

Example

If the image undergoes uniform linear motion $x_0(t) = at/T$, $y_0(t) = bt/T$.

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt = \frac{T}{\pi(ua+vb)}\sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$

Estimating the Degradation Function
Estimation by Modeling: Ex.2 Motion Blur



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Inverse Filtering

Supposed that the degradation function $H(u,v)$ is given or obtained, $F(u,v)$ can be estimated by

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Problem

Howevever, when $H(u,v)$ is small or zero,

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

The $\frac{N(u,v)}{H(u,v)}$ can dominate the output.

Solution

Since $H(0,0)$ DC compoment usually is the highest values, we can limit the analysis to frequencies near the origin to reduce the probability encountering $H(u,v)$ zero value.

Inverse Filtering
Example

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

- Gaussian Shape could produce very small H.
- Inverse filtering will give poor performance in general

MMSE/LSE/Wiener Filtering

Problem

Let image and noise be random processes and uncorrelated each other.
The **objective** is to find an estimate $\hat{f}$ of the uncorrpted image by minimizing error measure (N. Wiener [1942])

$$e^2 = E(f - \hat{f})^2$$

Solution

The solution is (Let $S(u,v)$ denotes power spectrum)

$$\begin{aligned}\hat{F}(u,v) &= \left(\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right)G(u,v)\\ &= \left(\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right)G(u,v)\\ &= \left(\frac{1}{H(u,v)}\cdot\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right)G(u,v)\end{aligned}$$

MMSE/LSE/Wiener Filtering

vs. Inverse Filter

- Better than Inverse Filter (problem in $H = 0$) **unless** both $H(u,v)$ and $S_\eta(u,v)$ are zero for the same value(s) of u and v.
- If noise is zero, i.e., $S_\eta(u,v) = 0$, then Wiener filter reduces to **Inverse Filter**.

$S_\eta(u,v)$

- If white noise ($S_\eta(u,v)$ is constant), equation becomes simple.
- But, usually, $S_\eta(u,v)$ is seldom known.
  The ______________ can be estimated by

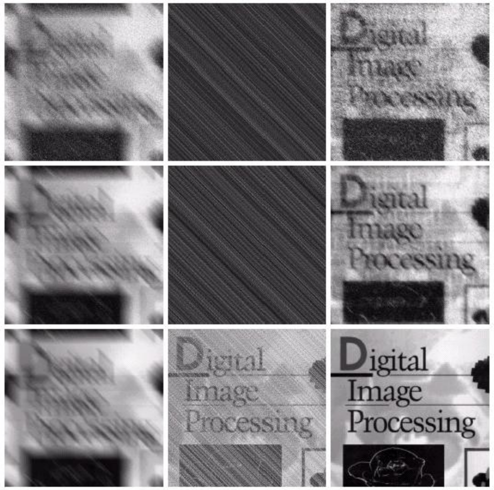
$$F(\hat{u},v) = \left(\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right)G(u,v)$$

MMSE/LSE/Wiener Filtering
Example

a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

MMSE/LSE/Wiener Filtering
Example



a b c
d e f
g h i

sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Why Constrained LSF?

Problem

Disadvantage of Wiener Filter:

- Have to know $S_\eta(x,y) \Rightarrow$ Difficult. Or,
- Assume $\dfrac{S_\eta(x,y)}{S_f(x,y)} = k$ constant $\Rightarrow$ not always true.
- Optimal in *average* sense.

Solution

____

- Only requires knowlege of mean and variance of noise.
- Optimal for *each* image to which it is applied.

Constrained LSF
Matrix Form of Degradation System

Re-write $g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$ (image dimension $M \times N$) into matrix form:

$$\mathbf{g} = \mathbf{Hf} + \eta$$

where

$\mathbf{g}, \mathbf{f}, \eta$ are $M \times N$
$\mathbf{H}$ are $MN \times MN$ (obtained by convol formula)

Note: The matrix form is only for analysis but not trivil for manipulating. It because:

- Say, $M = N = 512$, $\mathbf{H}$ is $262,144 \times 262,144$. $\Rightarrow$ **Huge**
- $\mathbf{H}$ is slighly sensitive to **noise**.

Constrained LSF
Derivation

____: Alleviate the noise sensitivity of $\mathbf{H}$ by basing optimality of restoration on a measure of *smoothness (Laplacian)*.

________: Minimize a criterion function

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\nabla^2 f(x,y)]^2$$

subject to the constraint

$$\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\eta\|^2$$

Constrained LSF
Derivation

Solution

____

$$\hat{F}(u,v) = \left(\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right) G(u,v)$$

where

γ: a parameter adjusted to satisfy constraint $\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\eta\|^2$.

$P(u,v)$: Fourier Transform of *padded Laplacian*

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Constrained LSF
Manually adjust γ

- γ is selected manually to yield best visual results.
- Why better than Wiener: γ is scalar, while K in Wiener is an approximation to the ratio, which seldom is constant.

Example

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

Constrained LSF:
Finding the best γ

Problem

Define a "residual" vector $\mathbf{r}$ as $\mathbf{r} = \mathbf{g} - \mathbf{H}\hat{\mathbf{f}}$

What we want is to adjust γ (i.e., $\hat{f}$) so that

$$\|\mathbf{r}\|^2 = \mathbf{r}^{\mathbf{T}}\mathbf{r} = \|\eta\|^2 \pm a$$

where a is an ______________. (we want $a = 0$)

Solution

This solution requires the quantities $\|\mathbf{r}\|^2$ and $\|\eta\|^2$ (Talk later)

- Specify an initial value of γ.
- Compute $\|\mathbf{r}\|^2$.
- Stop if $\|\mathbf{r}\|^2 = \|\eta\|^2 \pm a$ satisfied. Otherwise, return to Step 2 after increasing γ if $\|\mathbf{r}\|^2 < \|\eta\|^2 - a$ or after increasing γ if $\|\mathbf{r}\|^2 > \|\eta\|^2 + a$
- Newton-Raphson can be used to improve the speed of convergence.

Constrained LSF
Computing $\|\mathbf{r}\|^2$ and $\|\eta\|^2$

The above algorithm requires the quantities $\|\mathbf{r}\|^2$ and $\|\eta\|^2$.

Computing $\|\mathbf{r}\|^2$

$$\|\mathbf{r}\|^2 = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} r^2(x,y)$$

where $r(x,y)$ is from I.F.T. of $R(u,v) = G(u,v) - H(u,v)\hat{F}(u,v)$

Computing $\|\eta\|^2$

$$\|\eta\|^2 = MN[\sigma_\eta^2 + m_\eta^2] \quad \text{where} \quad m_\eta = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\eta(x,y)$$

(Because $\sigma_\eta^2 = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\eta(x,y) - m_\eta]^2$ and $\|\eta\|^2 = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\eta^2(x,y)$)

Constrained LSF
Example

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.

Geometric Mean Filter

Definition

____________________ is with a single equation representing a family of filters.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha}\left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha}$$

where α and β are postive, real constants.

- $\alpha = 1$: ____________
- $\alpha = 0$: ______________________
  - $\beta = 1$: ___________________
- $\alpha = 1/2$: _________________________________
- $\alpha = 1/2$ and $\beta = 1$: __________________________
  - $\beta = 1, \alpha < 1/2$: behave more like **Inverse filter**.
  - $\beta = 1, \alpha > 1/2$: behave more like **Wiener Filter**.

Geometric Transformation

Definition

Geometric Transformation is to **modify the spatial relationships between pixels in an image**, and achieved by **two steps**:

- ______________________: "Rearrangement" of pixels on the image plane.
- ________________________: Assignment of gray levels to pixels in the spaitally transformed domain.

Also called ________________________: viewed as "print" image on a sheet and stretching this sheet according predfined set of rules.

Spatial Transformation

Definition

______________________ is to spatially transform $f(x,y)$ to an image $g(x',y')$ by

$$\begin{cases} x' = r(x,y) \\ y' = s(x,y) \end{cases}$$

Example

$r(x,y) = x/2$ and $s(x,y) = y/2 \Rightarrow$ Shrinking $1/2$

- If spatial transform $r(x,y)$ and $s(x,y)$ were known analytically, recovering $f(x,y)$ from distorted image $g(x',y')$ is possible.
- But r and s are generally not known over the entire image $\Rightarrow$ Tiepoints.

Spatial Transformation
Tiepoint Mapping

Tiepoint Mapping

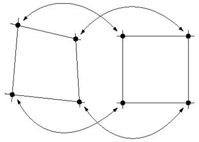


FIGURE 5.32 Corresponding tiepoints in two image segments.

Example

________________ for geometric distorition within the **quadrilateral** region.

$$\begin{cases} x' = r(x,y) = c_1x + c_2y + c_3xy + c_4 \\ y' = s(x,y) = c_5x + c_6y + c_7xy + c_8 \end{cases}$$

- Transform **all** pixels within the quadrilateral
- Need enough tiepoints/coefficients to generate a set of quadrilaterals that cover the entire image.

Gray-Level Interpolation

The Spatial Transformation may yield **noniteger values for** x' **and** y', where the gray levels of $G(x', y')$ are **not defined**.

In this case, we need Gray-level interpolation.

Gray-level Interpolation

- ________________ simplest but producing artifacts. (distoriton of stright eges in high-resol image.)
- ________________: Use **four nearest neighbors**. $v(x', y') = ax' + by' + cx'y' + d$. (4 unknowns from 4 eqs).
- ________________: fits **a surface of the** $\frac{sin(z)}{z}$ **type** through a much larger number of neighbors(say, 16)/**High computational burden** $\Rightarrow$ Use in area requires smooth approximations: **3-D graphics, medical imaging**.

Gray-Level Interpolation

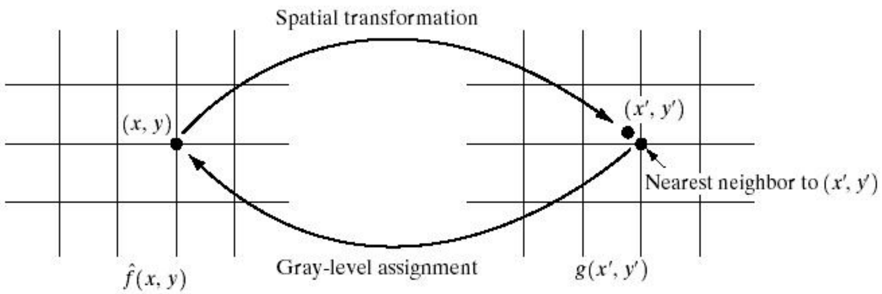
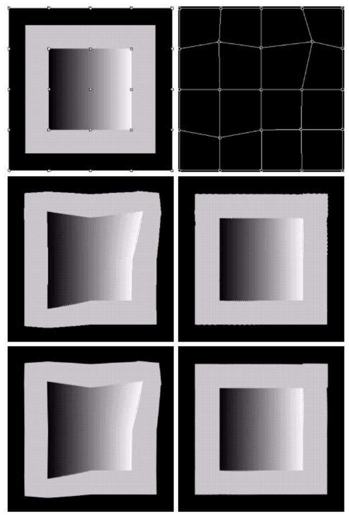


FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

Geometric Transformation
Example 1



Geometric Transformation
Example 2

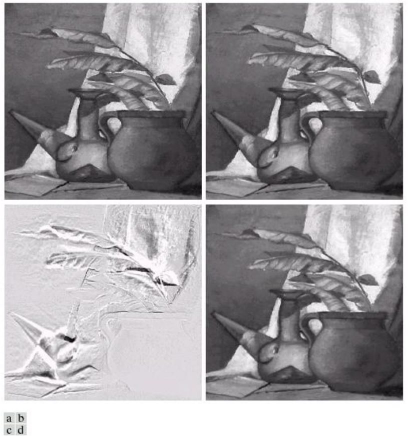


FIGURE 5.35 (a) An image before geometric distortion. (b) Image geometrically distorted using the same parameters as in Fig. 5.34(e). (c) Difference between (a) and (b). (d) Geometrically restored image.